How well can we describe statistical decay with GEMINI

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COURTESY: JOE ANGELES/WUSTL PHOTO

Statistical Model

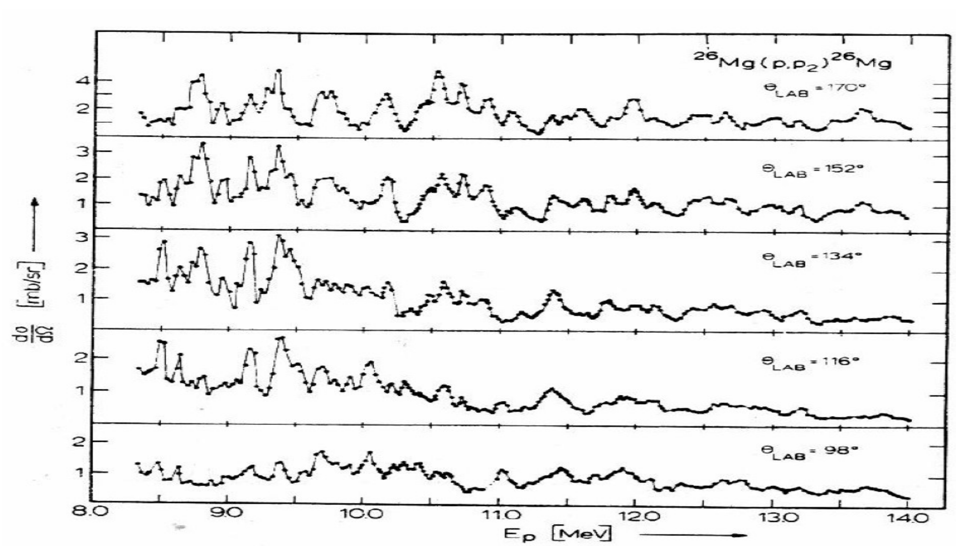
a) Compound-nucleus hypothesis – decay is independent of formation.

b) Applies to region of strongly overlapping levels, i.e., average level spacing is much less that average level width ($1/\rho << \Gamma$)

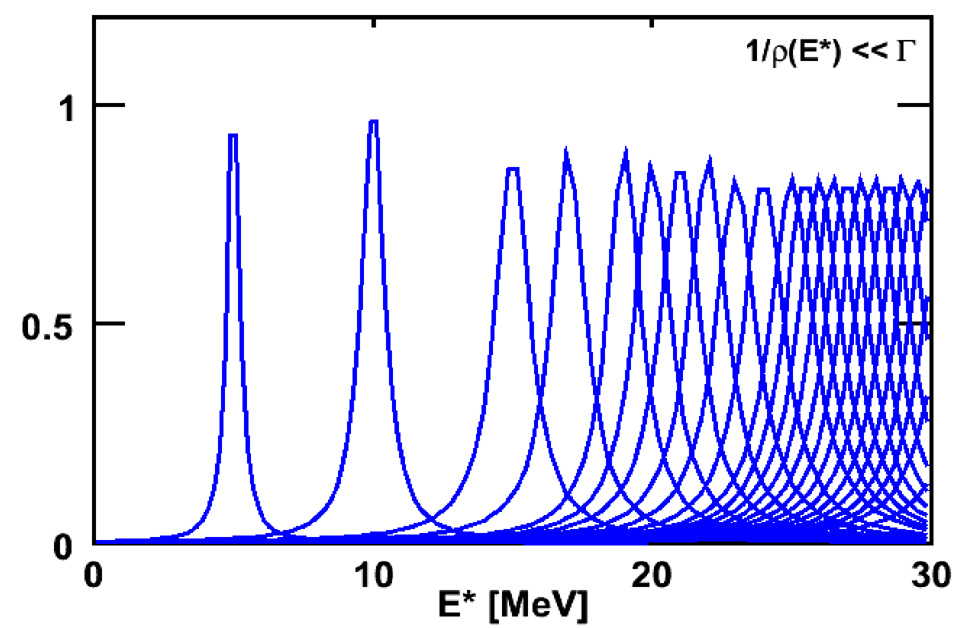
c) Statistical model predicts average cross sections over a small energy interval.

d) Fluctuations around this average “Ericson Fluctuations” can be seen at moderate excitation energies with excellent energy resolution.

e) All decay channels specified by a set of quantum numbers and a particular phase-space cell are equally likely if there is no barrier, otherwise weighted by barrier penetration probably. We just need to count all the ways a compound nucleus can decay.



O. Häusser, A. Richter, W. von Witsch, W. J. Thompson, Nucl. Phys. A 109 (1968) 329



A Brief History

1936 Bohr's independence hypothesis for the formation and decay of a compound nucleus

1937-1940 Weisskopf and Ewing - evaporation
1939 Bohr and Wheeler – statistical fission

1952 Hauser and Feshbach – full treatment of angular momentum

1975 Morreto complex-fragment emission

1986 GEMINI

2010 GEMINI++
Large number of statistical-model code available

PACE, JULIAN, EDCATH, MB2, Alice, EVAP, CASCADE, ALERT, LILITA, ABLA

GEMINI – Handles high angular momentum.
Includes complex-fragments emission.
Easy to add as an “afterburner” to a dynamical code

Evaporation of light particles

Weisskopf-Ewing Formalism S_0~0 (faster)

$$\Gamma_i = \frac{(2S_1+1)}{2\pi\rho_{CN}(E^*)} \int_0^\infty d\varepsilon \;\; \varepsilon\,\sigma_{inv.}(\varepsilon)\,\rho(E^* - B_i - \varepsilon)$$

$$\varepsilon\,\sigma_{inv}(\varepsilon) \propto \sum_{l=0}^{\infty} (2l+1)\,T_l(\varepsilon) \qquad \sigma_{inv}: \quad \text{inverse cross section}$$

Hauser-Feshbach Formalism (full angular-momentum coupling, slow)

$$\Gamma_i = \frac{1}{2\pi\rho_{CN}(E^*, S_0)} \int_0^\infty d\varepsilon \sum_{s_2=0}^{\infty} \sum_{J=|S_0-S_2|}^{S_0+S_2} \sum_{l=|J-S_1|}^{J+S_1} T_l(\varepsilon)\,\rho(E^* - B_i - \varepsilon, S_2)$$

$\rho(E^*, J)$: spin-dependent level density of daughter

$\rho_{CN}(E^*, J)$: level density of parent

$T_l(\varepsilon)$: transmission coefficients

ε, l, S_1: energy, orbital AM, and spin of evaporated particle

B_i: separation energy of particle

S_0, S_2: spins of compound nucleus and daughter

$$\vec{S}_0 = \vec{S}_2 + \vec{S}_1 + \vec{l}\,, \; \vec{J} = \vec{S}_1 + \vec{l}$$

Need level densities, transmission coefficients, and separation energies.

Symmetric fission

Bohr-Wheeler Formalism (Transition-State Formalism)
borrowed from chemical reaction-rate theory

$$\Gamma_{BW} = \frac{1}{2\pi\rho(E^*, S_0)} \int_0^\infty d\varepsilon \ \rho_{saddle}[E^* - \varepsilon, S_0]$$

ε: energy in fission degree of freedom

ρ_{saddle}: level density at saddle point

Ingredients: level densities at saddle-point (transition state)

Intermediate mass fragment emission

Morreto Formalism - extension of the Bohr-Wheeler transition-state Formalism

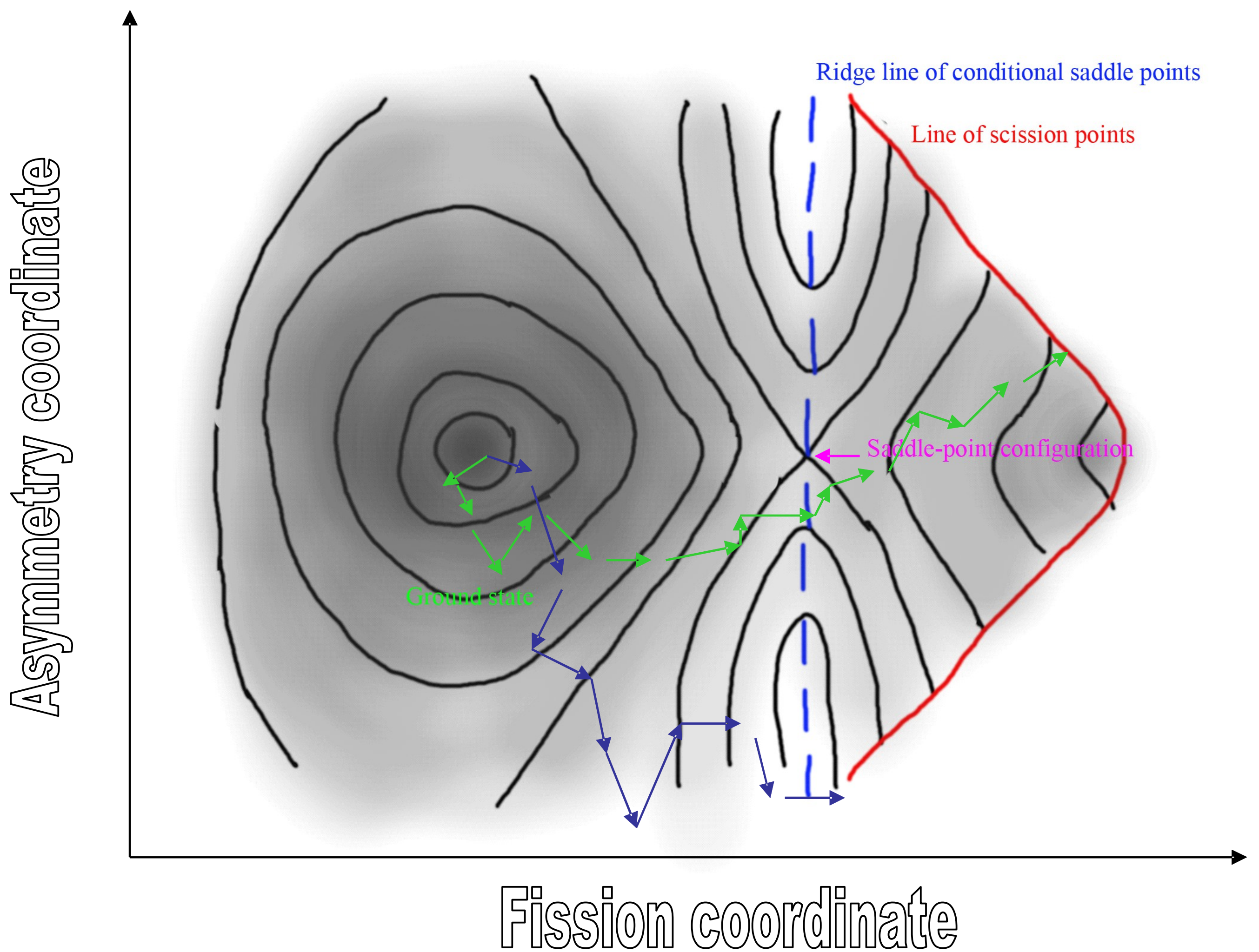
$$\Gamma_Z = \frac{1}{2\pi\rho(E^*, S_0)} \int_0^{E^* - E_Z(S_0)} d\varepsilon \ \rho_Z[E^* - \varepsilon, S_0]$$

ε: energy in binary-decay degree of freedom

ρ_Z: level density at the conditional saddle point for a particular charge split

$E_Z(S_0)$: conditional saddle point energy

Potential energy surface



For large saddle-to-scission distance asymmetry at saddle may not be preserved at scission

GEMINI/GEMINI++ ingredients

a) evaporation - Hauser-Feshbach or Weisskopf-Ewing
n, *p*, *d*, *t*, ^{3}He, α, ^{6,7,8,9}Li*, ^{7,8,9,10,11}Be* - excited states included for Li and Be

b) complex fragment emission – Morreto

c) fission - Bohr-Wheeler

d) gamma-ray E1 and E2 only

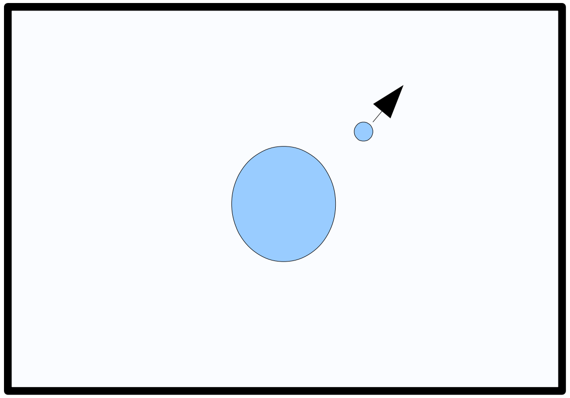
AIMS

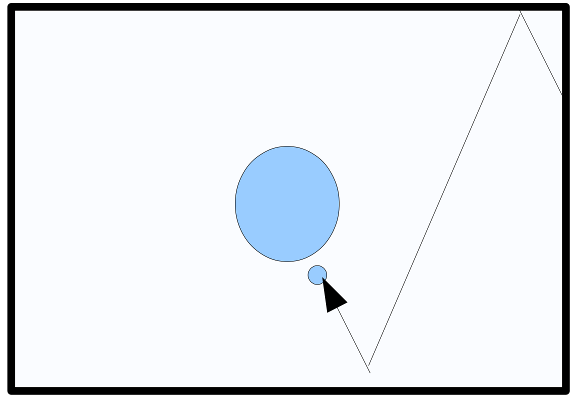
1) With the Statistical model we can learn about the properties of warm and hot nuclei, for example via extracting the penetration factors and level densities and their evolution with excitation energy

2) fit lots of data to constrain the statistical-model parameters and thus provide a model with predictive power. Can be used in fusion reactions and as an “afterburner” in more dynamical reaction models.

e.g, couple GEMINI++ to BUU, AMD, QMD, INCL, Isabel, ….

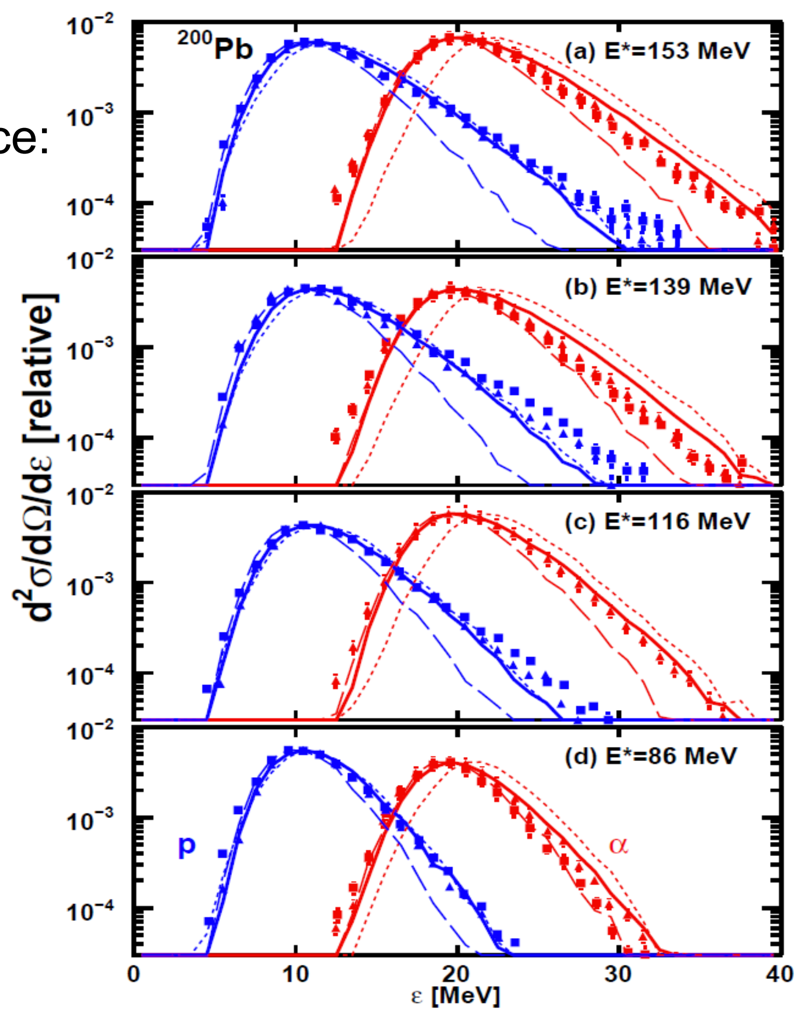
Barrier penetration factors





Weisskopf formalism is easy derived from detailed balance:
rate of evaporation = rate of fusion

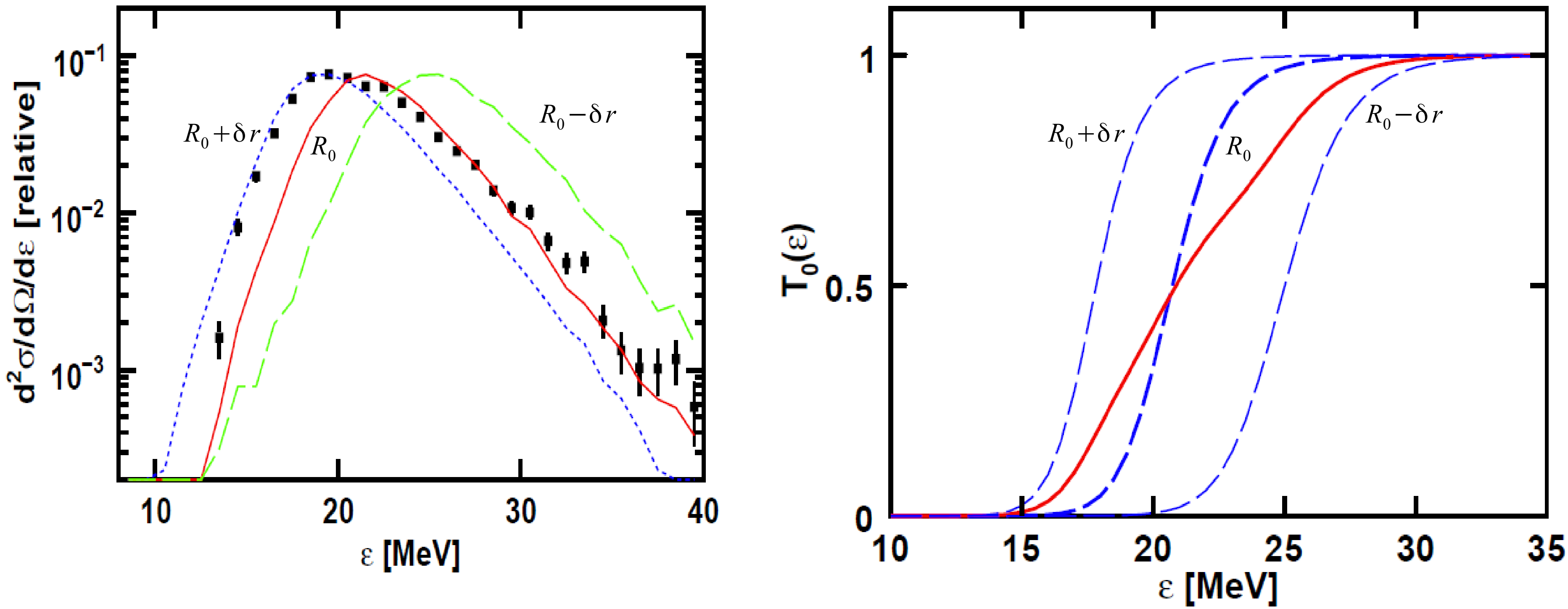
hence σ_{inv} and $T_l(\varepsilon)$ are for the inverse *fusion* process comomonly obtained from global optical-model fits to experimental data of the inverse scattering process.



For the emission of alpha and other heavier particles the Coulomb barrier appears lower in energy than the OM predictions.

The true inverse process is scattering from an excited nucleus, not a ground state nuclei. So we can deduce how the Coulomb barrier changes with excitation energy

CN = ^{193}Tl, alpha emission



A simple shifting of the barrier does not work, we need more diffuse transmission coeff.
a) narrower barrier ?
b) distribution of barriers – deformation (static or fluctuating), fluctuating surface diffusenesses

In GEMINI a simple barrier distribution is used

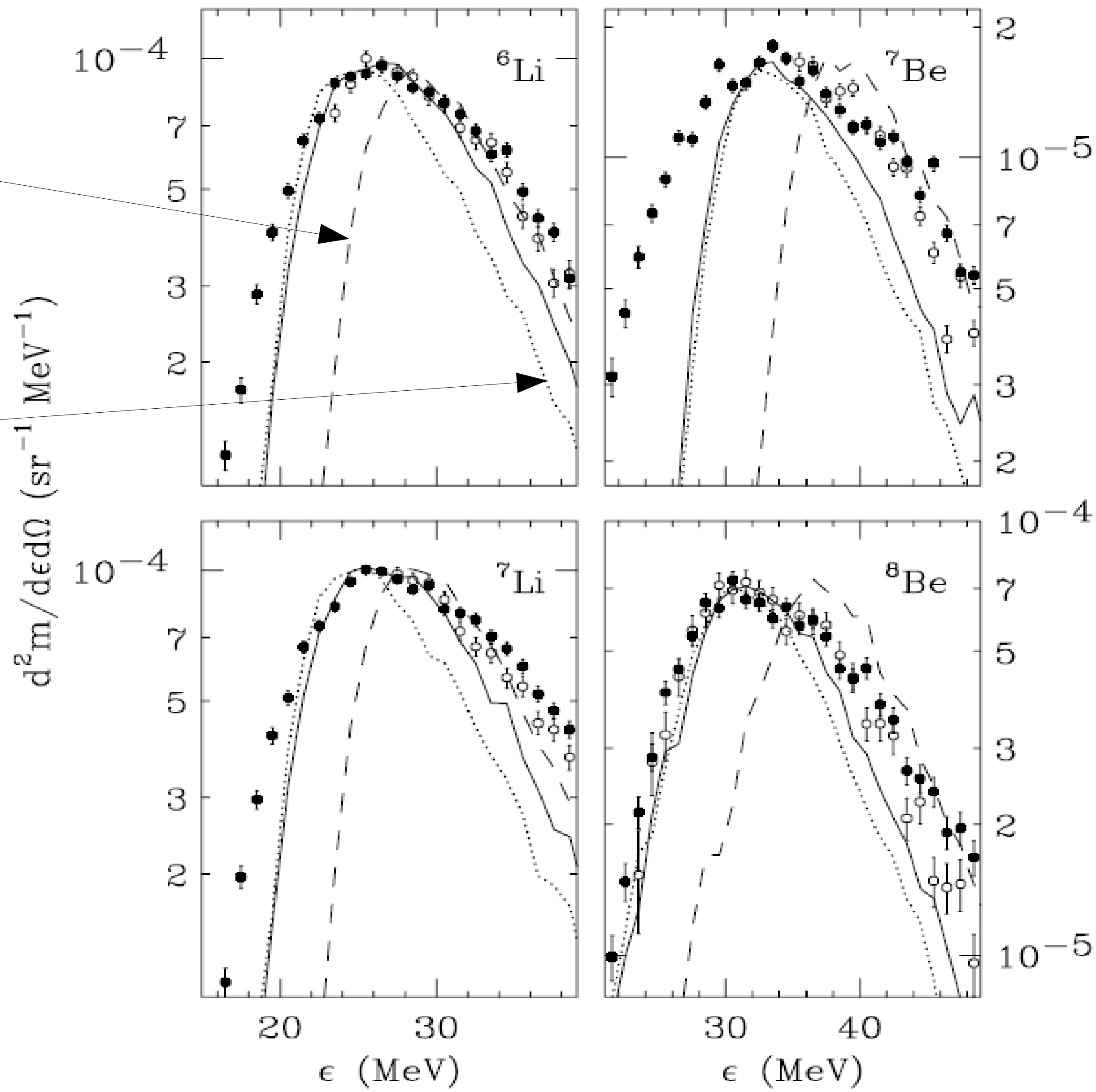
$$T_l(\varepsilon) = \frac{T_l^{R_0-\delta r}(\varepsilon) + T_l^{R_0}(\varepsilon) + T_l^{R_0+\delta r}(\varepsilon)}{3}$$

$$\delta r = w\sqrt{T}, w = 1\ fm$$

Nuclear structure changes significantly between the ground state and E*/A~0.6 to 1.5 MeV
Would take substantial deformations or changes to surface diffuseness to produce
the required transmission coefficients.

Transmission coef's from optical model fits to inverse reaction.

Simple barrier shift

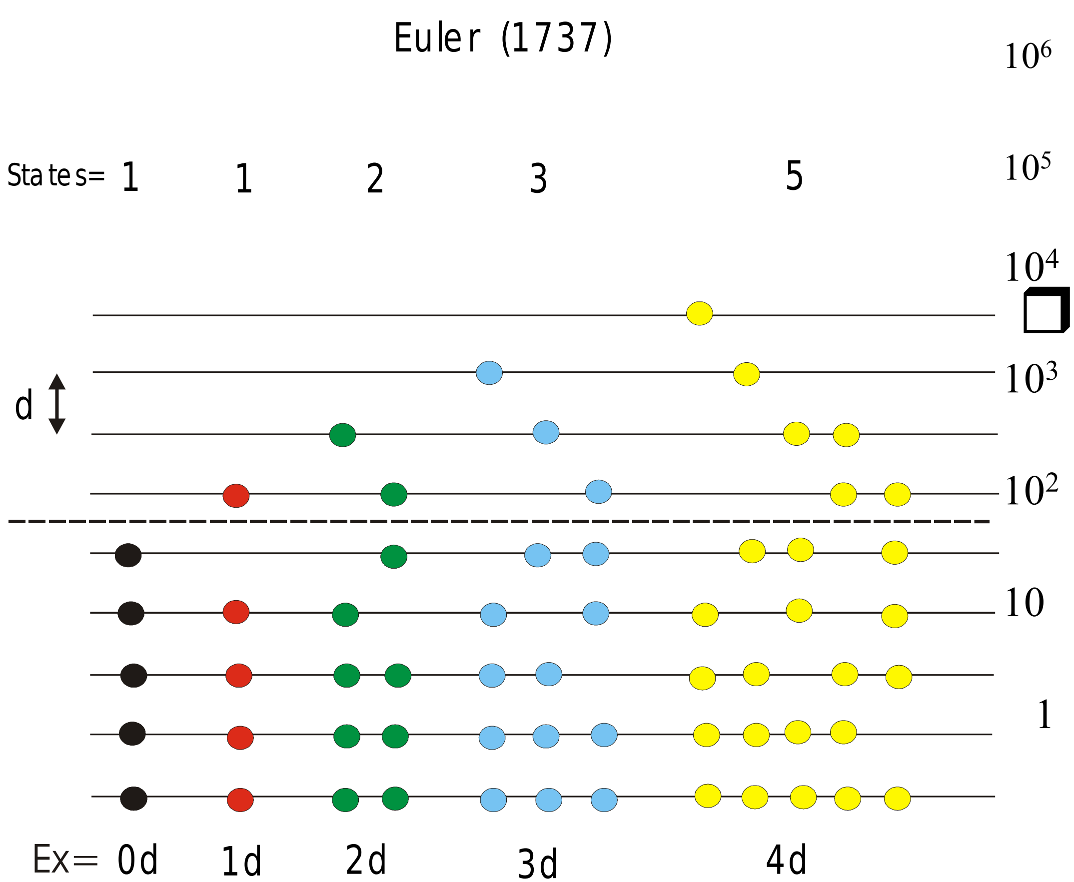


Evaporation spectra from E/A=11 MeV ^{60}Ni+^{100}Mo fusion reactions

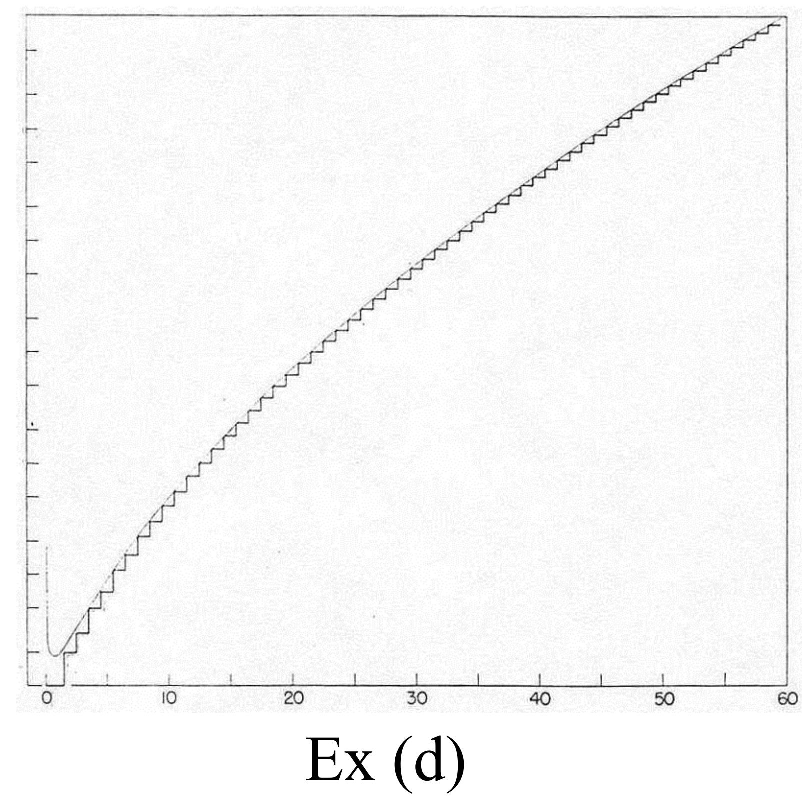
PRC 63 (2001) 024611

Level Densities

Considered the simple combinatorial problem with equally-spaced levels and one particle type - equivalent to problem considered by Euler



Number of ways or level density



Only one particle type and equally spaced levels

Level Densities from single-particle excitations

a) Fermi-gas Level Density from constant single-particle level density)

$$\rho(E^*) = \frac{\sqrt{\pi}}{12}\frac{\exp(2\sqrt{a E^*})}{a^{1/4} E^{*5/4}}, \text{ where } a = \frac{\pi^2}{6} g(\varepsilon_F)$$

$g(\varepsilon_F)$ is single-particle level density at Fermi energy

b) Backshifted Fermi-gas Level Density

Constant single-particle level density + pairing interactions (BCS)

At excitation energy U_{crit}, the gap Δ vanishes and level density becomes

$$\rho(E^*) \propto \exp(2\sqrt{a(E^* + \delta P)}), \text{ for } E^* > U_{crit}$$

where δP is the pairing correction to the liquid-drop-mass formula

c) Using shell-model single-particle levels

$g(\varepsilon)$ has oscillations due to shell gaps. As we average over a larger range of ε either side of the Fermi energy with increasing excitation energy

$a_{shell} \rightarrow \tilde{a}$ (semi classical)

$$a(E^*) = \tilde{a}\left[1 - \frac{\delta W}{E^*} k(E^*)\right]$$ Ignatyuk et al. empirical formula

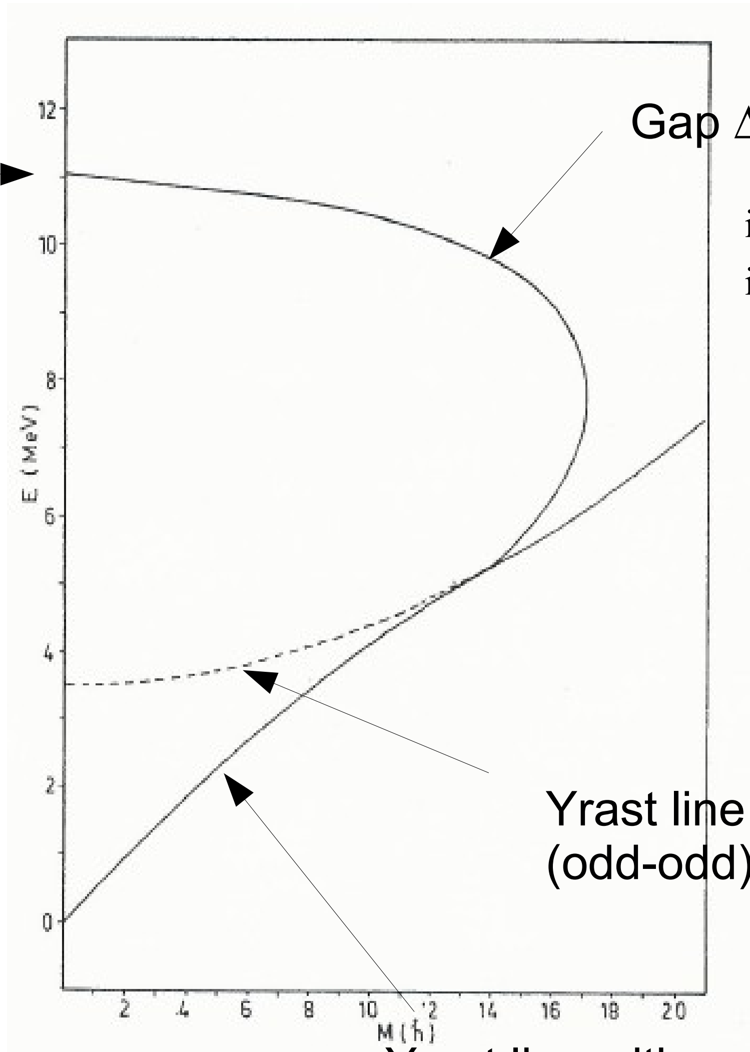
fade out factor $k(0) = 1$ and $k(\mathrm{E}) \rightarrow \infty$ as $E^* \rightarrow \infty$ and δW shell correction to liquid-drop mass

$$\exp(2\sqrt{a E^*}) = \exp\left(2\sqrt{\tilde{a}\left[1 + k(E^*)\frac{\delta W}{E^*}\right] E^*}\right) = \exp(2\sqrt{\tilde{a}\ (E^* + k(E^*)\delta W)})$$

equivalent to an excitation-energy dependent shift

BCS model
Morreto NP A203 578 (1973)

U^0_{crit}



Gap $\Delta = 0$

if $U^0_{crit} < B_i$ then pairing only comes in to play for last-chance particle emission in GEMINI++ at the moment $U^0_{crit}=9$ and $J_{crit}=14$, but not a lot of thought has been given to these values

Yrast line without pairing (odd-odd)

Yrast line with paring even-even or odd even

$$U = E^* - E_{rot}^{macro}(J) + \delta P(J)$$

$$\delta P(J) = \begin{cases} \delta P_{g.s.}\left[1-\left(\dfrac{U}{U_{crit}(J)}\right)^2\right], U_{crit}=U^0_{crit}\left[1-\left(\dfrac{J}{J_{crit}}\right)^2\right] & \textit{for } J<J_{crit} \text{ and } U<U_{crit} \\ \delta P_{g.s.} & \textit{otherwise} \end{cases}$$

From Finite range model of Moeller and Nix Atomic Data Nucl. Data tables 59 (1995) 185

$$\delta P_{g.s.}^* = \begin{cases} \dfrac{4.8}{N^{1/3}}+\dfrac{4.8}{Z^{1/3}}-\dfrac{6.6}{A^{2/3}}+\dfrac{30}{A} & \textit{for } N=Z=odd \\ \dfrac{4.8}{N^{1/3}}+\dfrac{4.8}{Z^{1/3}}-\dfrac{6.6}{A^{2/3}} & odd-odd \\ \dfrac{4.8}{Z^{1/3}} & odd\ Z-even\ N \\ \dfrac{4.8}{N^{1/3}} & even\ Z-odd\ N \\ 0 & even-even \end{cases}$$

$$\delta P_{g.s.} = \delta P_{g.s}^* - \delta P_{g.s.}^*(odd-odd)$$

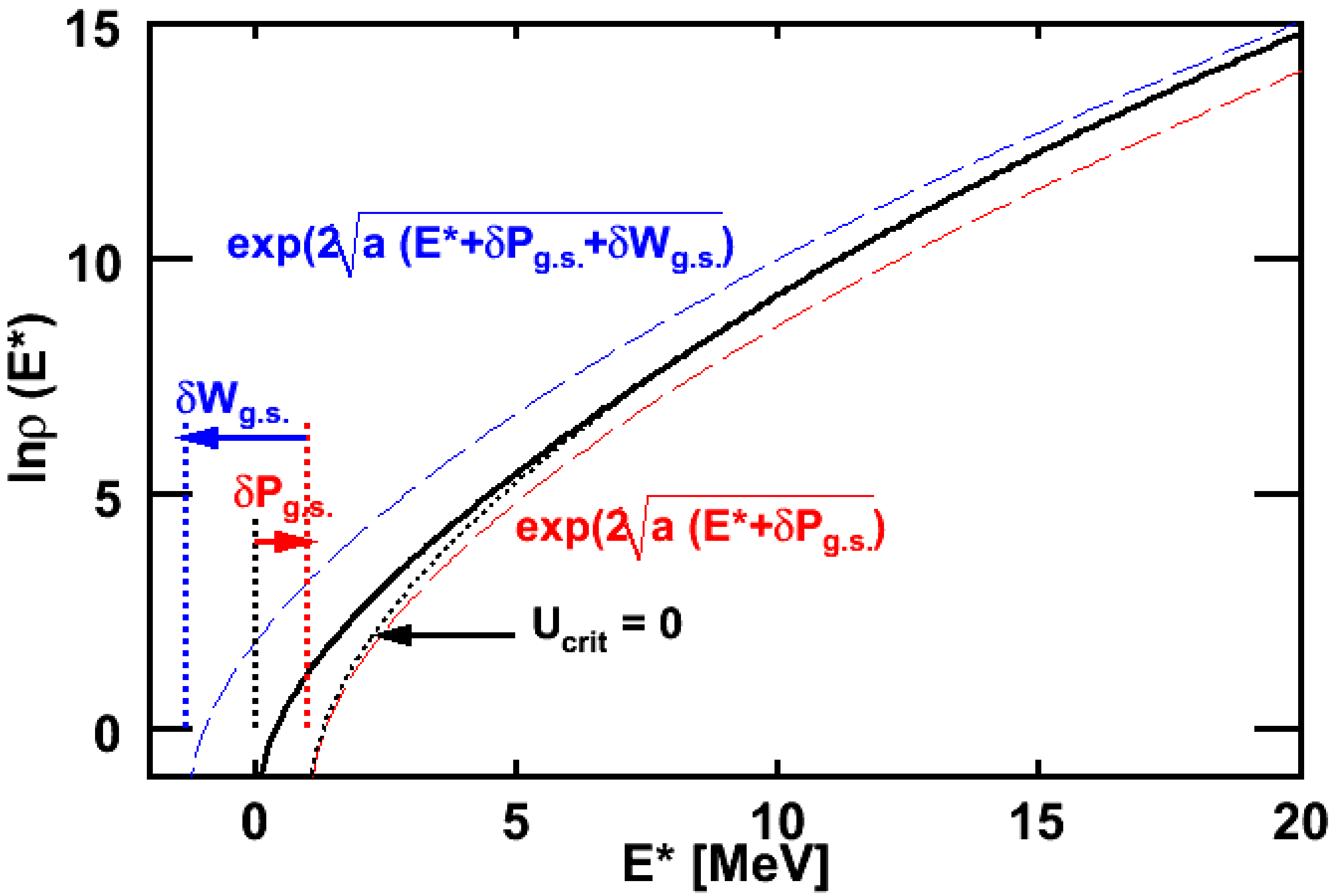
note $\delta P \leqslant 0$

$$\rho(E^*,J) \propto (2J+1)\exp\left[2\sqrt{\tilde{a}\;\left(U+\delta P_{g.s.}\,h(U)+\delta W_{g.s.}\,k(u)\right)}\right]$$

$$U = E^* - E_{rot}^{macro}(J)$$

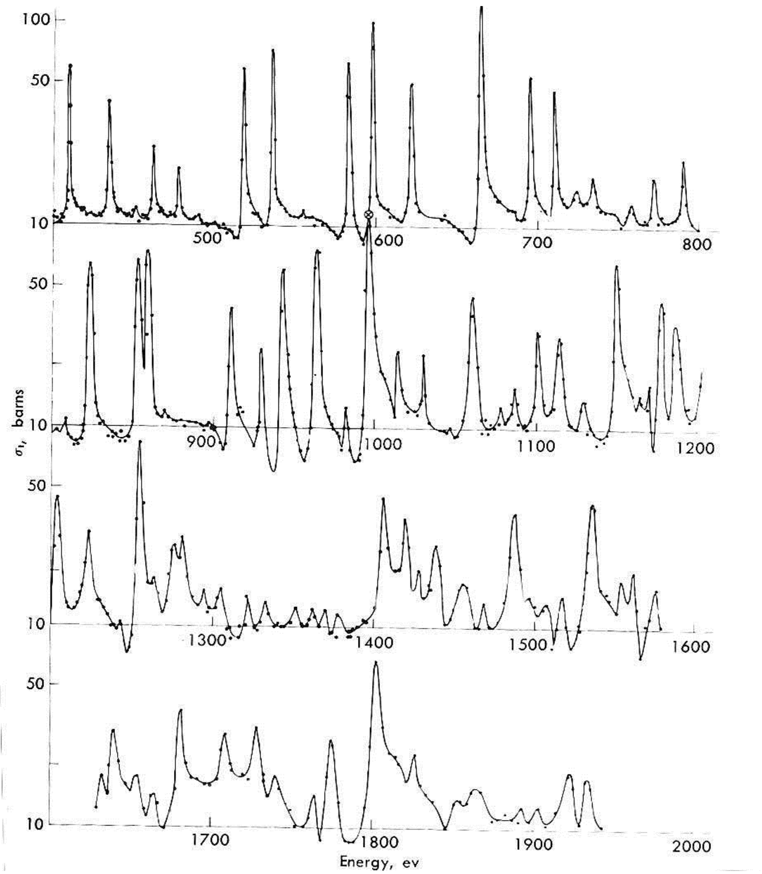
$$h(U) = 1-\left(1-\frac{U}{U_{crit}(J)}\right)^2 \text{ fadeout factor for pairing } U_{crit}(J) = U_{crit}^0\left[1-\left(\frac{J}{J_{crit}}\right)^2\right]$$

$$k(U) = 1+\tanh\left(\frac{U}{\eta}+\frac{J}{\chi}\right) \text{ fadeout factor for shell effects}$$

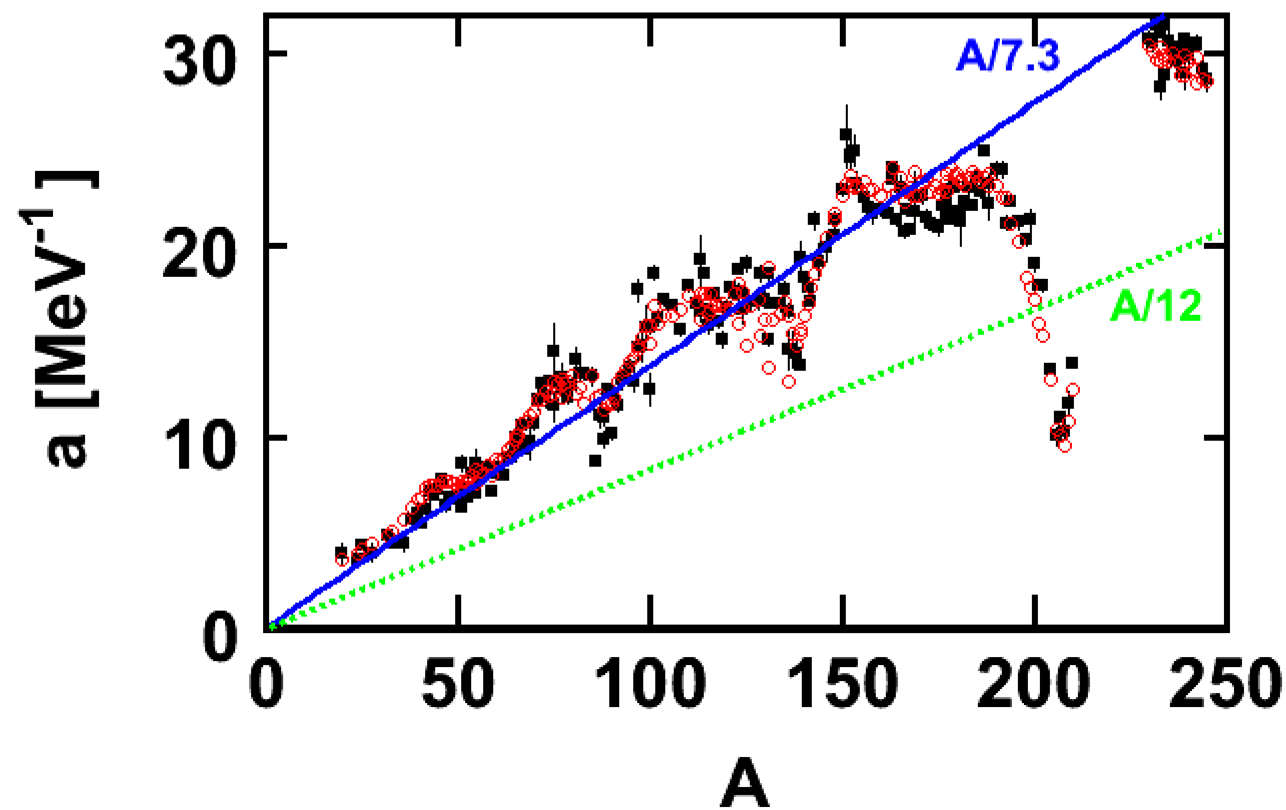


Low-energy behavior of level-density parameter
can be determined from neutron-resonance counting

This gives us absolute level densities at the neutron separation energy B_n for S=0, i.e. $\rho(B_n,0)$. Pairing is fully washed out at this excitation energy, but shells effects are still significant



n+^{208}Pb



This is the only information we have on absolute level densities.
From fit we use $\tilde{a} = A/7.3\ MeV^{-1}$ for lower excitation energies.

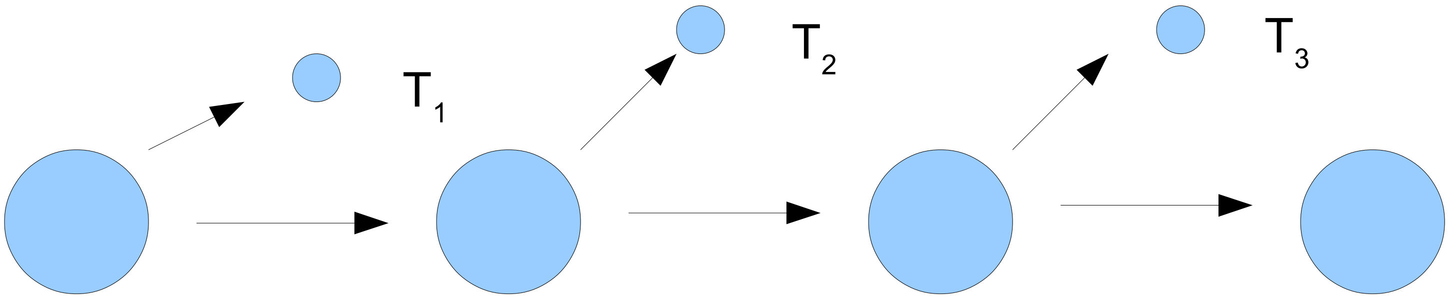
Level-density parameter at high excitation energies

$$\Gamma(\varepsilon) \propto \varepsilon\, \sigma_{inv}(\varepsilon)\, \rho(E^* - B_i - \varepsilon)$$
$$\propto \varepsilon\, \sigma_{inv}(\varepsilon) \exp 2\sqrt{\tilde{a}(E^* - B_i - \varepsilon)}$$
$$\propto \varepsilon\, \sigma_{inv}(\varepsilon) \exp 2\sqrt{\tilde{a}(E^* - B_i)} \exp\frac{-\varepsilon}{T}, \quad \frac{1}{T} = \frac{d \log\rho}{dE}$$
$$\text{as } \varepsilon \to \infty, \sigma_{inv} \to \text{constant, and hence } \Gamma(\varepsilon) \propto \varepsilon \exp\frac{-\varepsilon}{T}$$

So from the exponential tails of kinetic-energy spectra of evaporated particles we can obtain information on the derivative of the level density.

More complicated in reality: multi-chance emission and angular-momentum effects
Compare spectra to statistical-model predictions

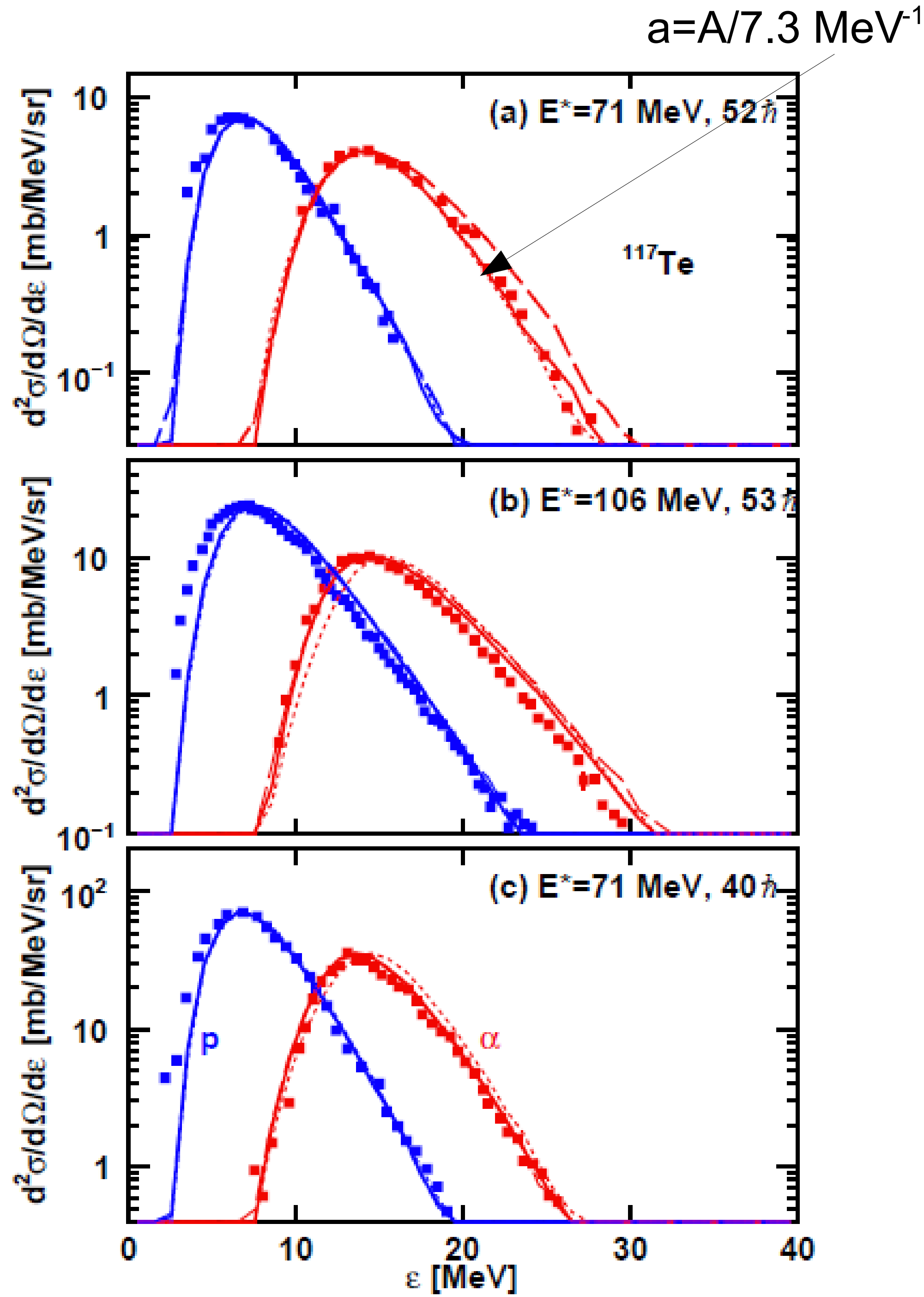
$$\tilde{a} = \frac{A}{k_0 + \kappa \frac{U}{A}}, \quad k_0 = 7.3\,\text{MeV from neutron-resonance counting}$$



For A_{CN} < ~120

a = A/7.3 MeV^{-1} no dependence on excitation energy needed.

Consistent with level counting



$^{14}N+^{103}Rh$, $^{40}Ar+^{77}Se$ fusion reactions
Compound nucleus ^{117}Te

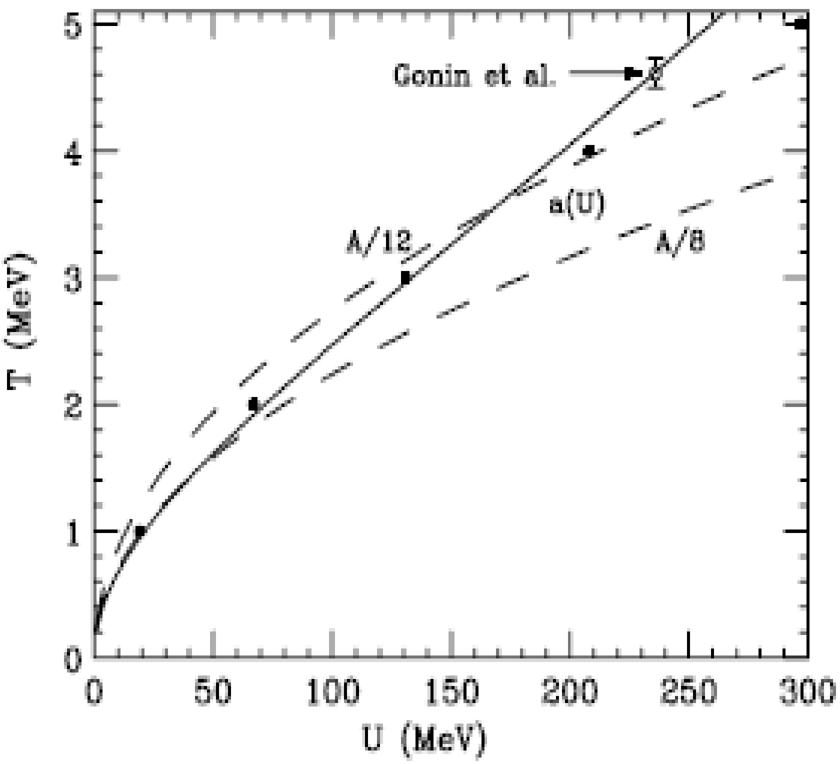
Data = Galin et al PRC 9 (1974) 1113
PRC 9 (174) 1113

^{60}Ni + ^{100,92}Mo fusion reactions

$$\tilde{a} = \frac{A}{k_\infty - (k_\infty - k_0)\exp\frac{-\kappa}{k_\infty - k_0}\frac{U}{A}}$$

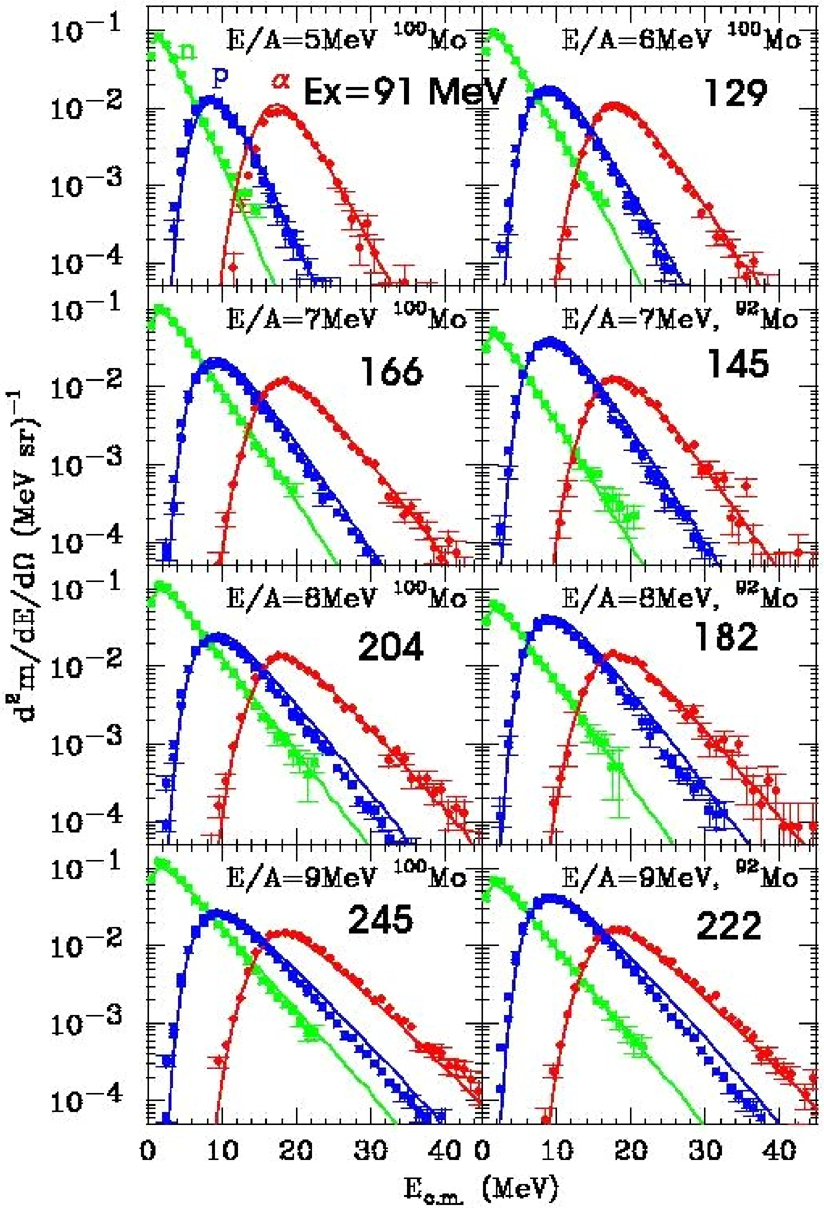
$$= \frac{A}{k_0 + \kappa\frac{U}{A}}, \text{ for } E^* \text{ of interest}$$

$$k_0 = 7.3\,MeV,\ k_\infty = 12\,MeV,\ \kappa = 1.3$$



Is this due to changes in the effective mass.

Statistical-model fits to evaporation spectra measured in coincidence with evaporation residues



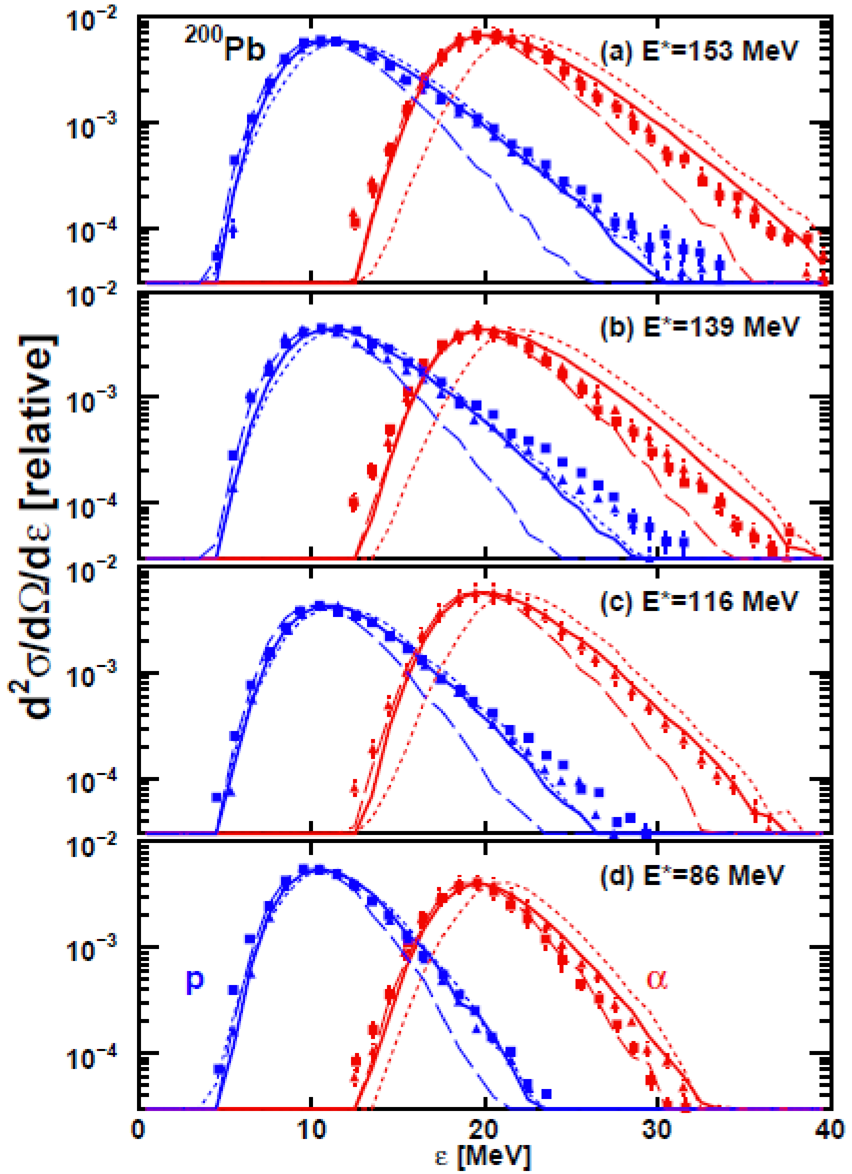
^{60}Ni+^{100,92}Mo fusion reactions

A~200

Can be fit with either a constant a=A/12 or a=A/(8 + κ U/A). κ=2-3

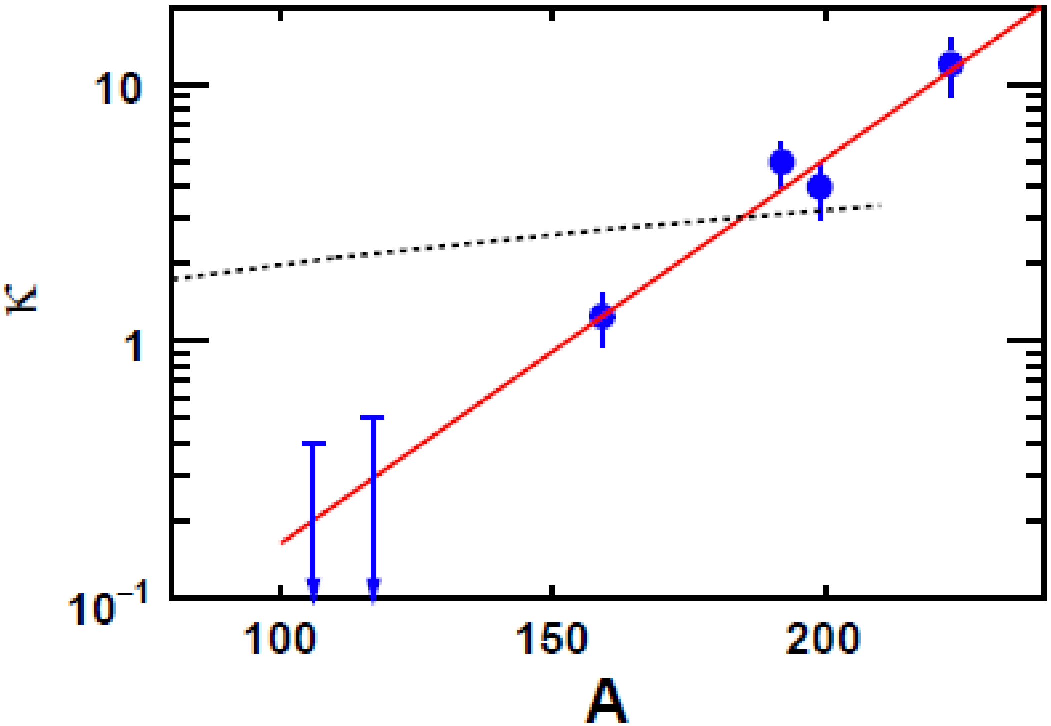
Similar conclusion obtained for the ^{193}Tl Compound system,
Fineman et al PRC 50(1994) 1991

Smaller range of excitation energy probed compared to Yb system.

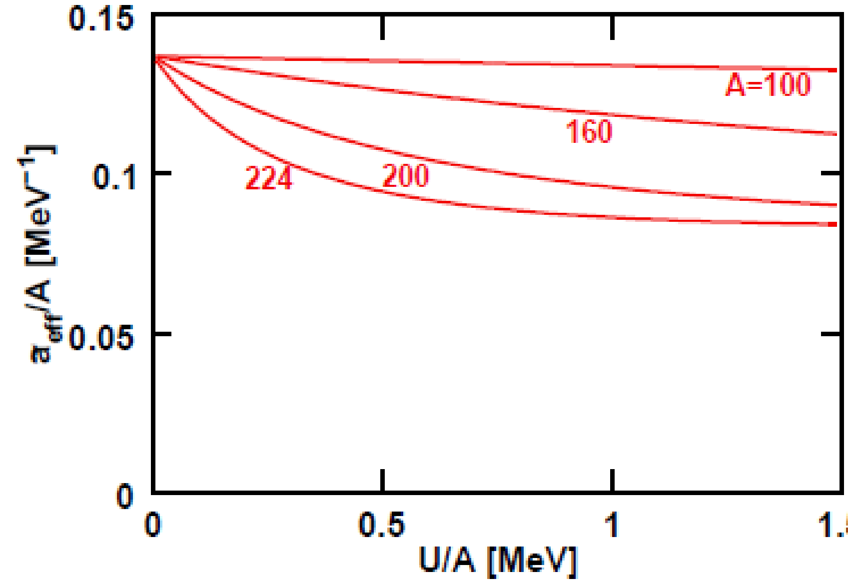


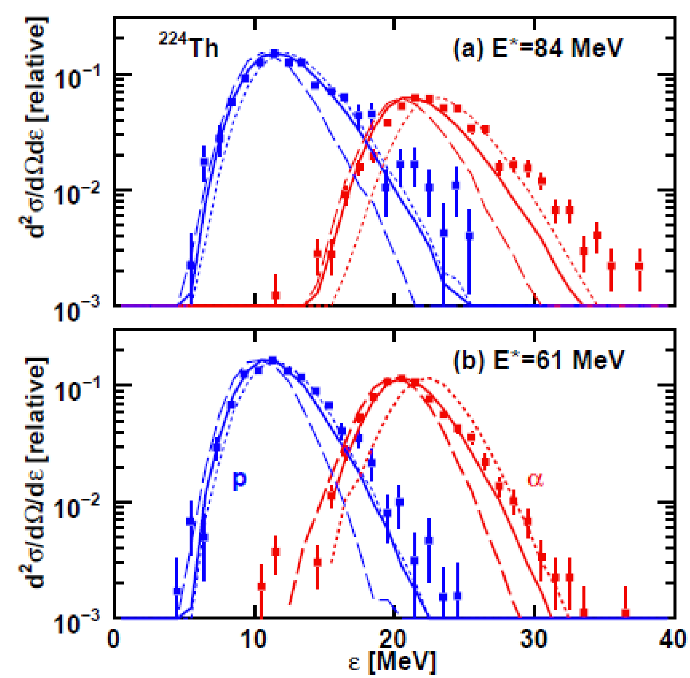
Data Caraley et al
^{19}F+^{181}Ta → ^{200}Pb fusion data
PRC 62 (2000) 054612

$a = A/(7.3 + \kappa\, U/A)$



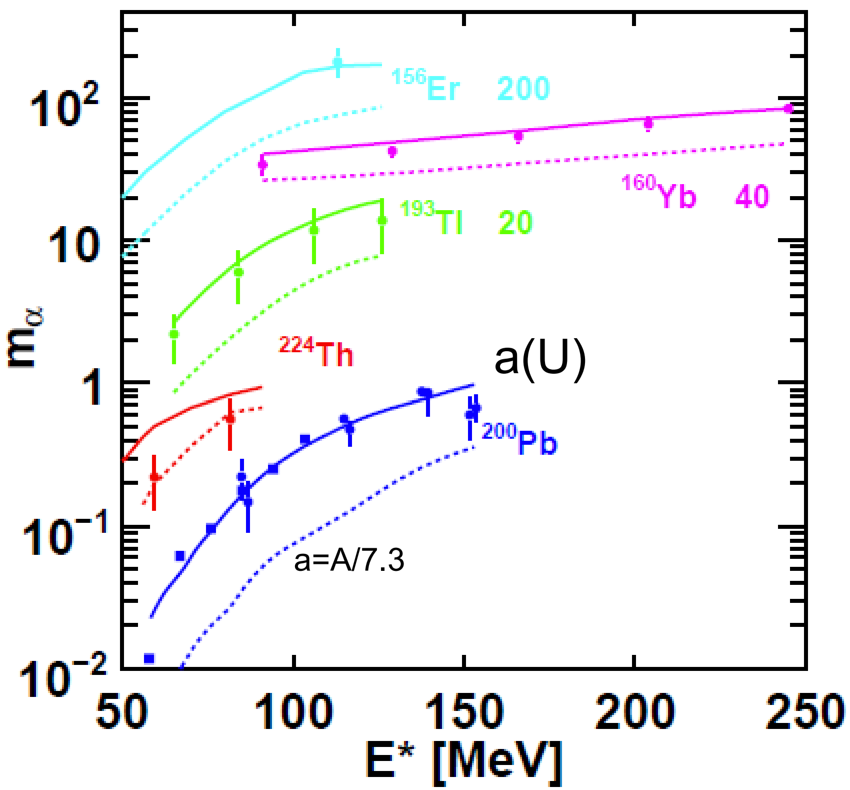
A dependence of κ from fusion reactions.





1) exponential increase of κ with A – what is the physics behind this?
2) κ is effectively zero for A<130, $a = A/7.3\ \mathrm{MeV}^{-1}$
3) superheavies: κ will be very big, what are the consequences?
4) κ most uncertain for the highest A . better measurements needed.

Consistency with alpha multiplicities and evaporation-residue excitation functions

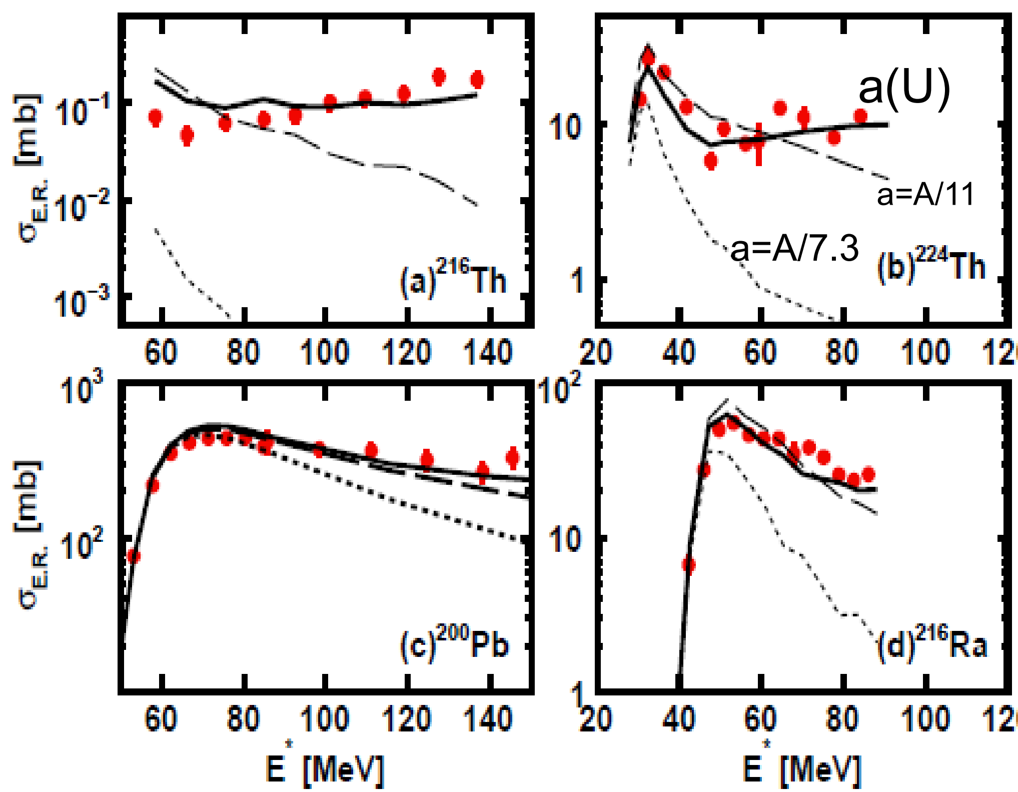


$^{19}F+^{181}Ta \rightarrow ^{200}Pb$

$^{19}F+^{197}Au \rightarrow ^{216}Ra$

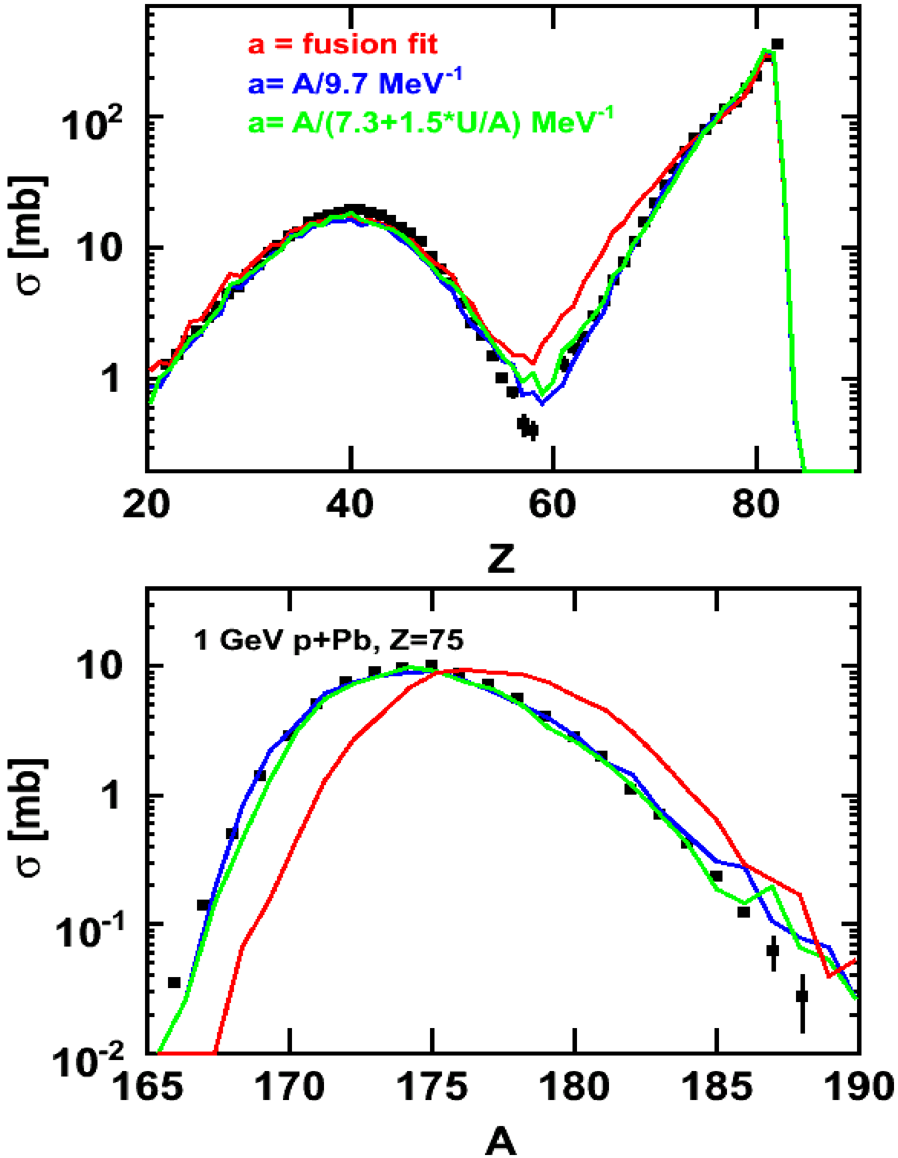
$^{32}S+^{184}W \rightarrow ^{216}Th$

$^{16}O+^{208}Pb \rightarrow ^{224}Th$

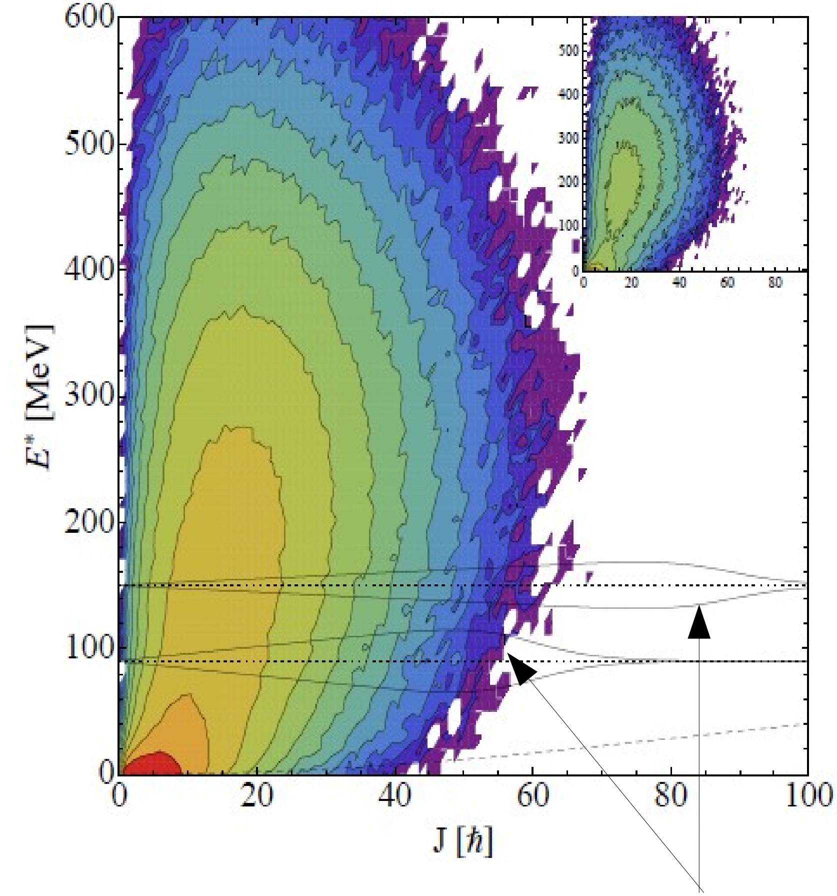


Inconsistency on level-density parameter between fusion and spallation data?

INCL + GEMINI++



E/A=1 GeV ^{208}Pb+p GSI data
Enqvist et al NPA 686 (2001)481



^{19}F+^{181}Ta fusion

Get similar results with
Isabel + GEMINI++

Calculations by Mancusi, Charity, Cugnon

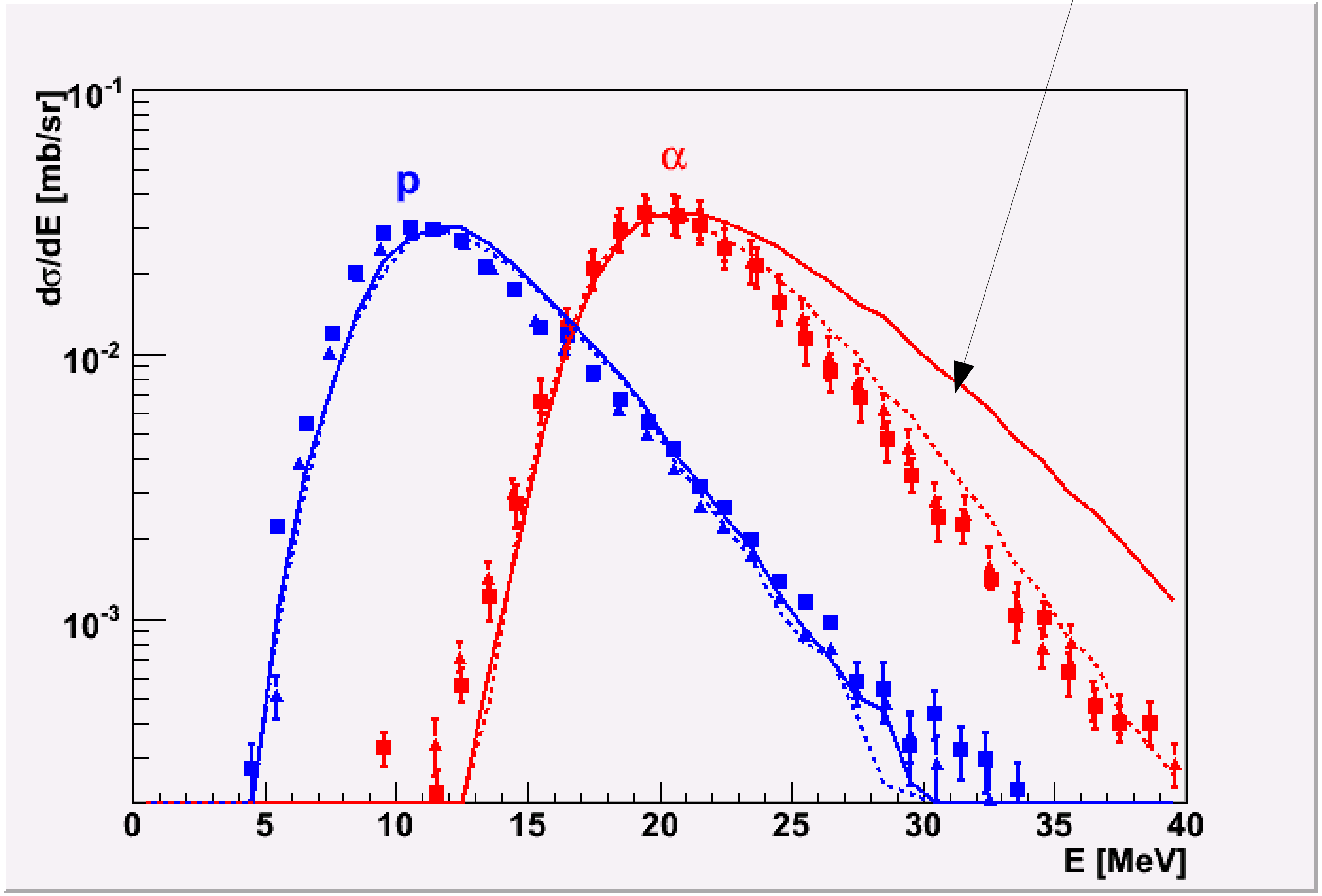
p+^{208}Pb spallation (low spin) κ = 1.5
^{19}F+^{181}Ta fusion (high spin) κ = 4.5

Is κ spin dependent, try κ = 1.5 + 0.114 J

No alpha spectrum very sensitive to the spin dependence of level density.

κ=1.5+0.114J

E_{lab} = 195 MeV ^{19}F+^{181}Ta



Level Densities Conclusions

a) there clearly is an evolution of the level-density parameter
With mass beyond the a=A/k linear dependence.

b) Only for the Yb compound nucleus was an excitation-energy dependence
definitely needed.

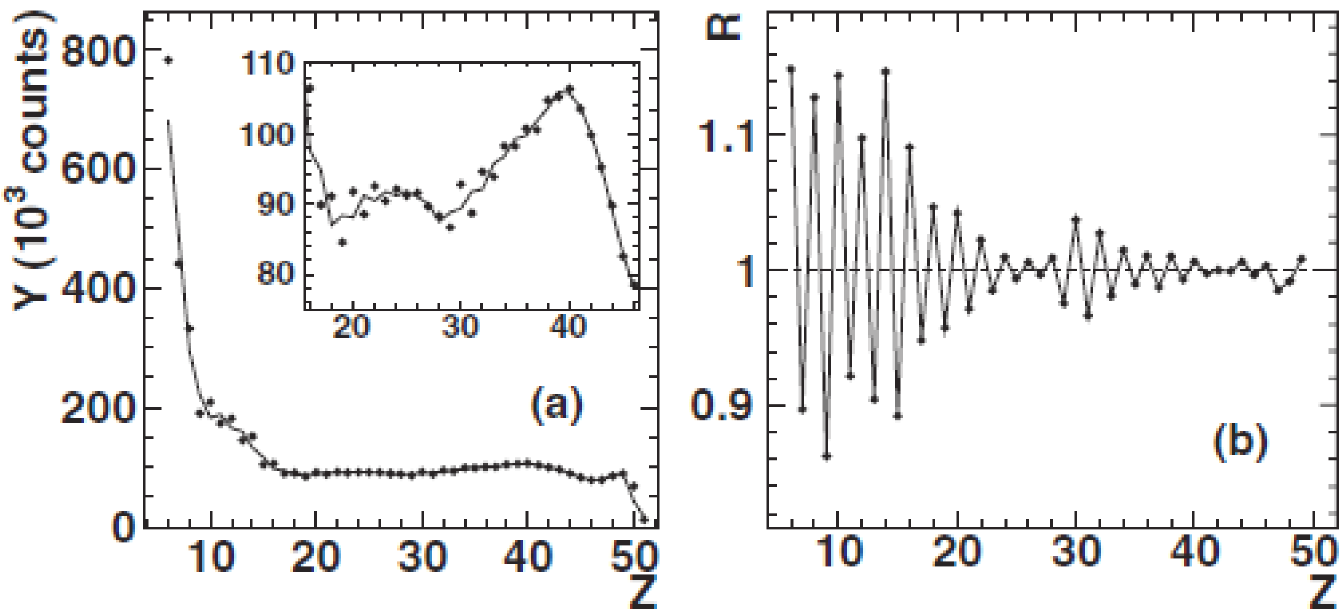
c) the prescription obtained from fitting fusion reaction, did not work
for the higher energy spallation data. Possibly the excitation-energy
dependence is not as steep as used in the prescription.

d) This is all data driven, would help to have some theory to guide us.

c) More data would help map out the mass and excitation energy
dependencies

e) what about (N-Z)/A and spin dependencies

Odd-even effects in Z and N distributions



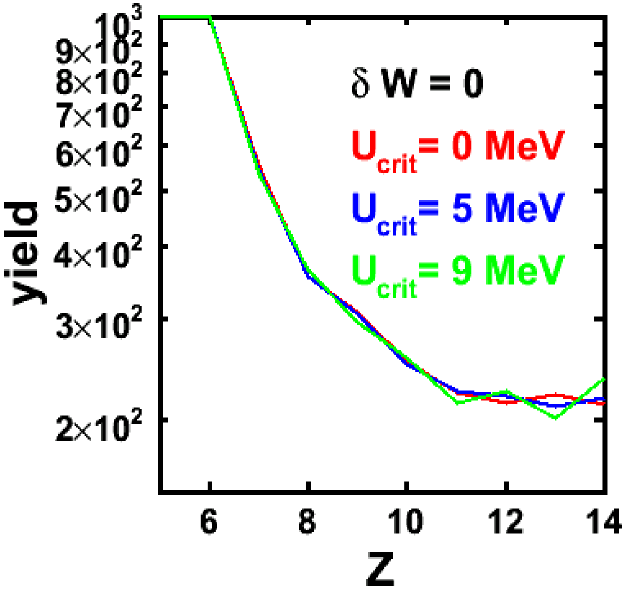
E/A=35 MeV $^{112}Sn+^{58}Ni$

Casini et al
PRC 86 (2012) 011602

Possible causes of odd- even oscillations in IMF yields

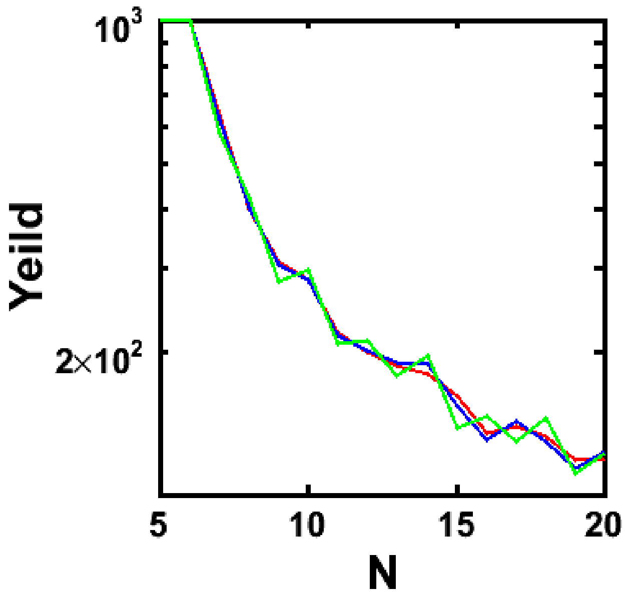
a) δP_{saddle} (to get measurable IMF yields, need significant excitation energy at saddle, so pairing should wash out?)

b) sequential evaporation from the primary fragments (last-chance emission)



GEMINI++ calculation with
no shell corrections in the masses.
Just the smooth macroscopic masses
plus pairing correction.

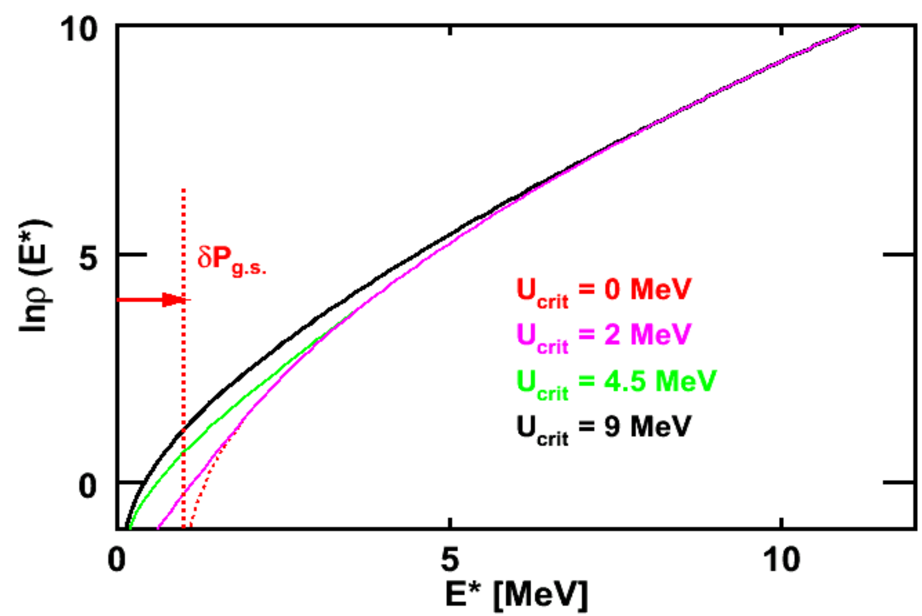
$^{9}Be+^{93}Nb \rightarrow ^{102}Rh$ fusion reaction
at E/A= 17.7 MeV
Looking at IMF emission

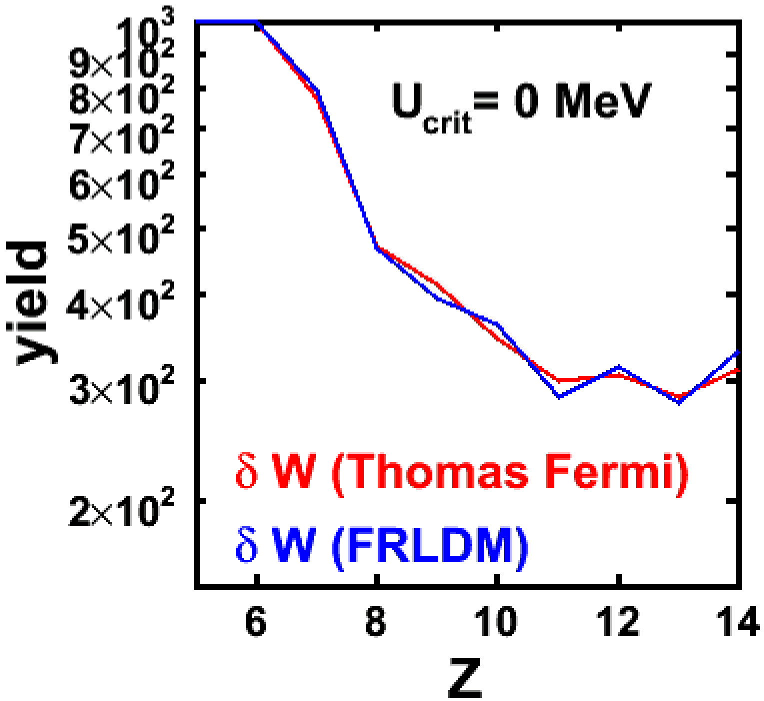


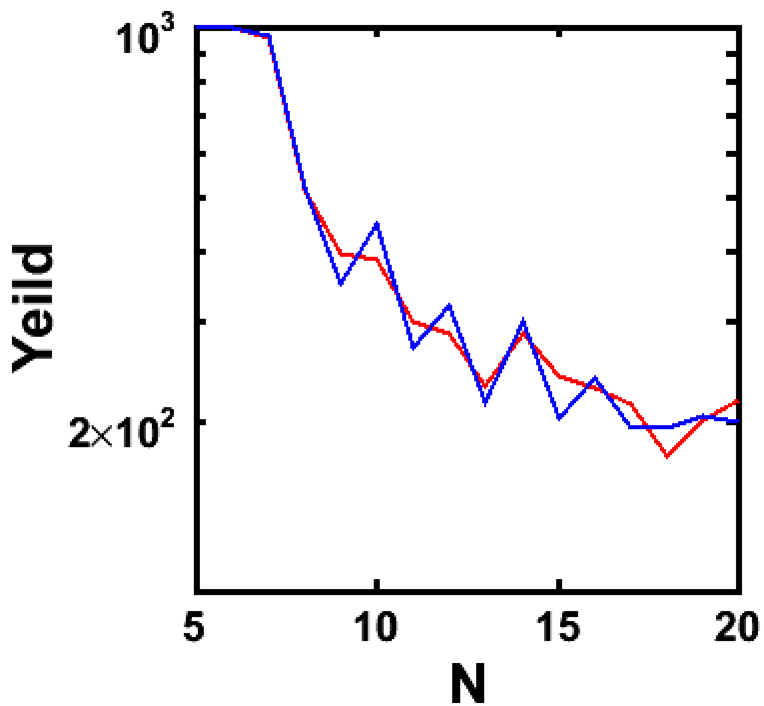
Odd-even oscillation from last-chance
n or *p* emission from excited primary IMF'S

Oscillations are small.

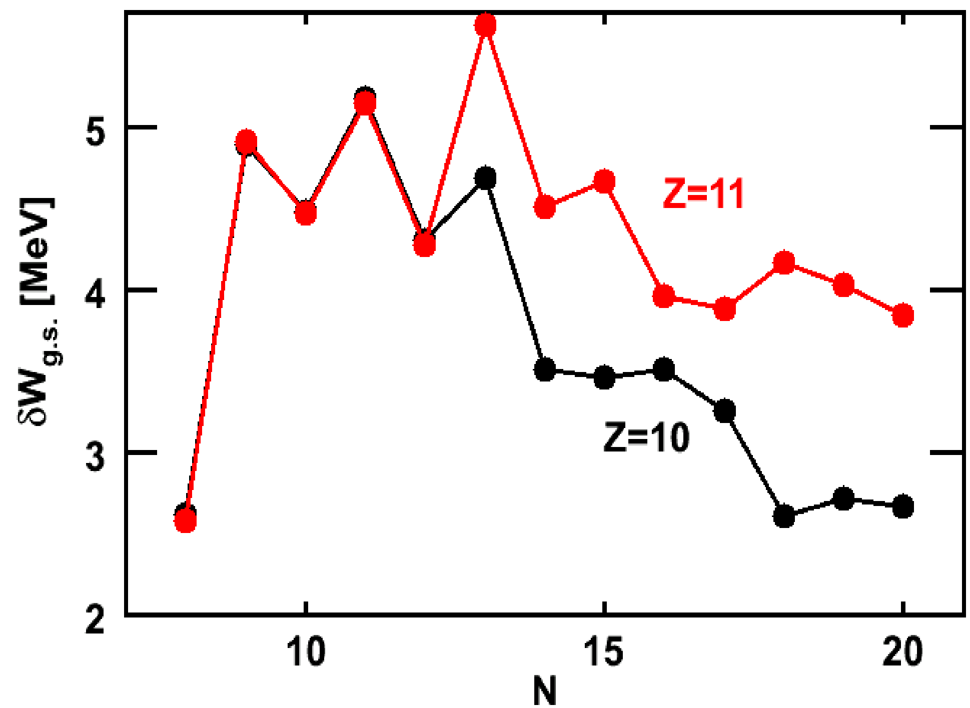
Expected behavior







Shell corrections from FRLDM



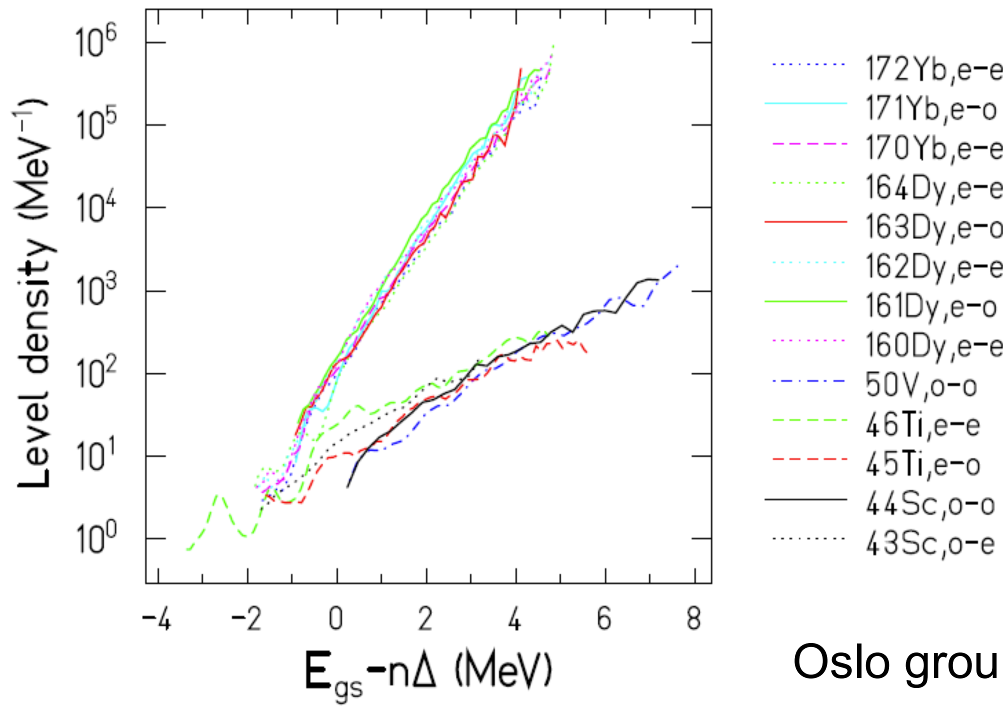
In present version of GEMINI++, the most of Odd-even oscillations come from the odd-even variations of the shell corrections in the FRLDM of Moeller et al?

The Thomas-Fermi mas model of Myers and Swiatecki gives much smaller effect.

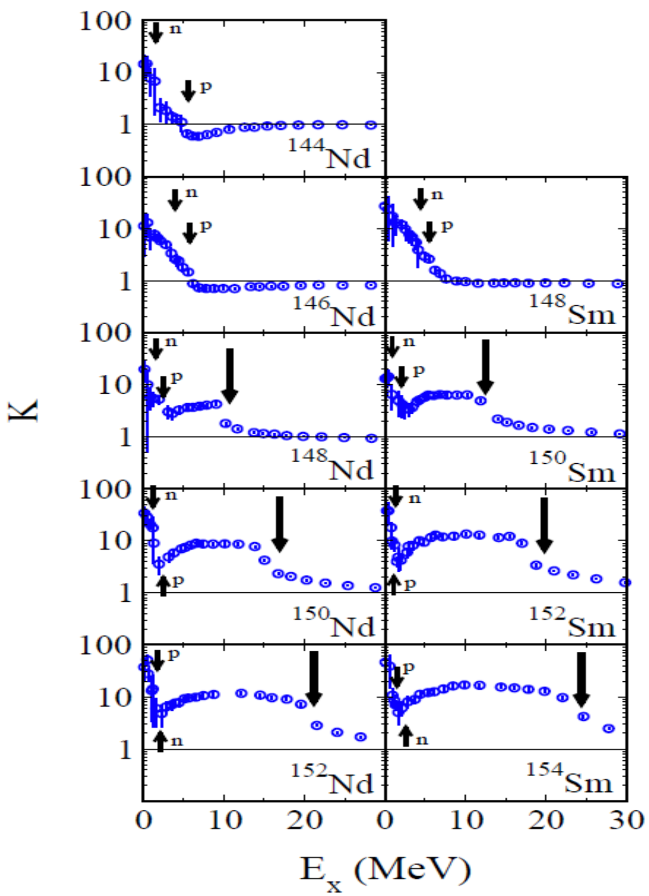
Fermi-gas model fails at low Excitation energy.

Experimentally we find "constant temperature" Level densities

$$\rho = \frac{1}{T}\exp\left(\frac{E^* + \delta P}{T}\right)$$



Oslo group



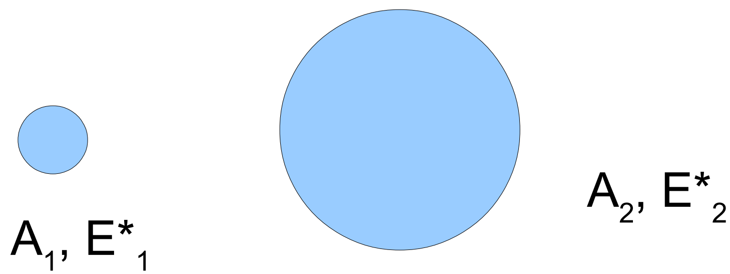
Fermi-Gas model only accounts for single-particle excitations. There should also be collective enhancement factors which wash out with excitation energy which may contribute to this

$$\rho(E^*) = \rho_{FG}(E^*) K_{vib}(E^*) K_{rot}(E^*)$$

Alhassid et al (Shell-Model Monte Carlo Method)

ArXiv::1305.5605

Excitation energy division in IMF emission



$E^*_{tot} = E^*_1 + E^*_2$

$$prob(E^*_1) = \rho_1(E^*_1)\rho_2(E^*_{tot} - E^*_1)$$

if $\rho_1(E) \propto \exp(2\sqrt{a_1 E^*})$ and $\rho_2 \propto \exp(2\sqrt{a_2 E^*})$ then

$prob$ is a maximum at $\dfrac{E^*_1}{E^*_2} = \dfrac{a_1}{a_2}$ (equal temperatures)

if $a_1 = A_1/k$, $a_2 = A_2/k$, then $\dfrac{E^*_1}{E^*_2} = \dfrac{A_1}{A_2}$

if $\rho_1(E^*) = \dfrac{1}{T_1}\exp(E^*/T_1)$, $\rho_2(E^*) = \dfrac{1}{T_2}\exp(E^*/T_2)$, $T_1 > T_2$ then equal temperatures is not possible.

$prob$ is a maximum at $E^*_1 = 0, E^*_2 = E^*_{tot}$ (energy sorting Schmidt and Jurado PRL 104 (2010) 212501)

In energy sorting the product $\rho_1(E^*_1)\rho_2(E^*_{tot}-E^*_1)$
Is greatest if any unpaired nucleons are put into the heavy fragment, as the light fragment is always in the ground state

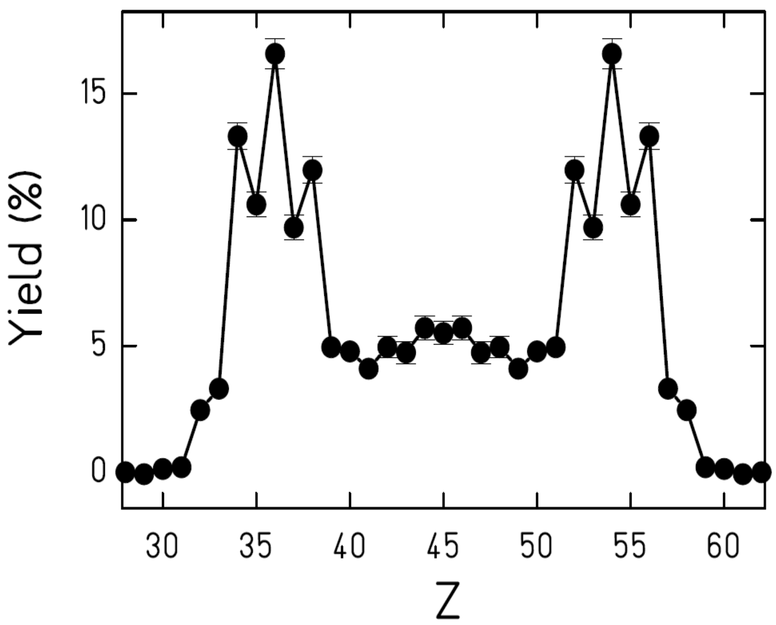
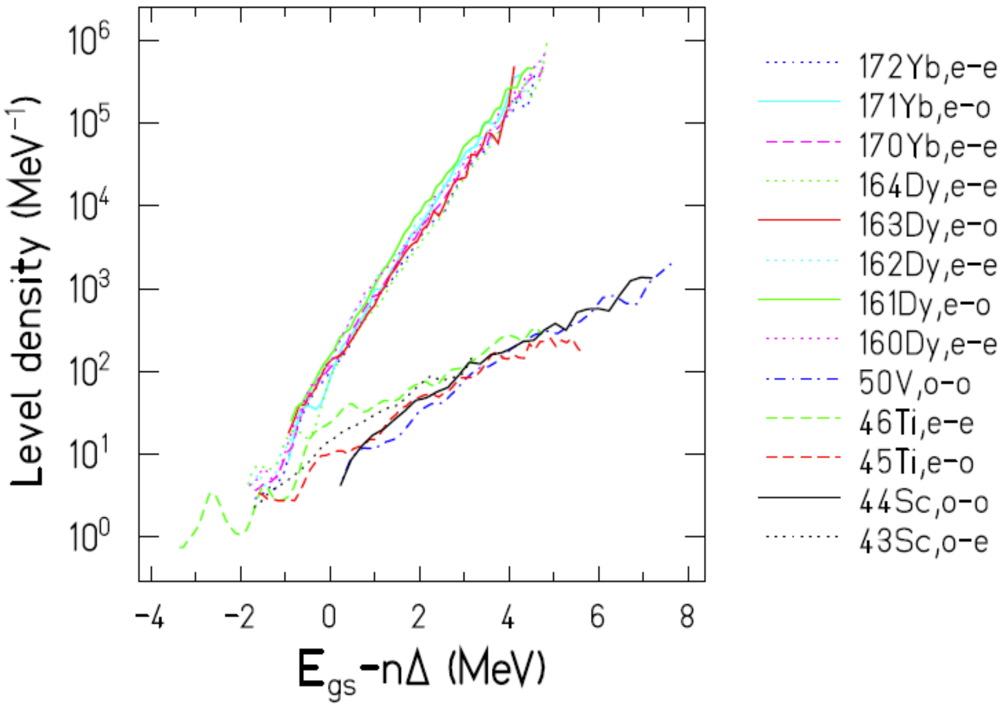


Figure 1. Element distribution observed in the electromagnetic-induced fission of ^{229}Th [6]



Cannot get equal temperatures

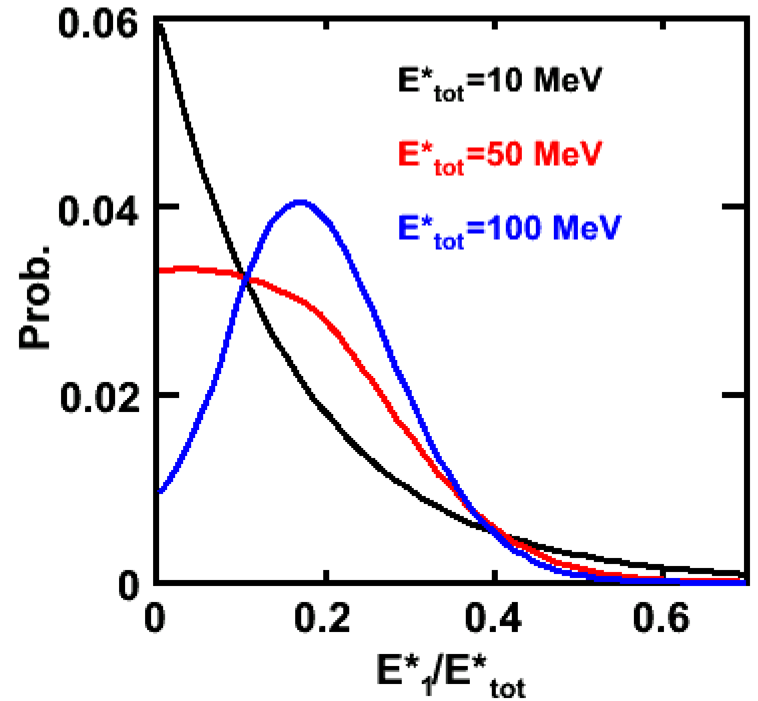
Transition from “energy sorting” to equal temperatures.

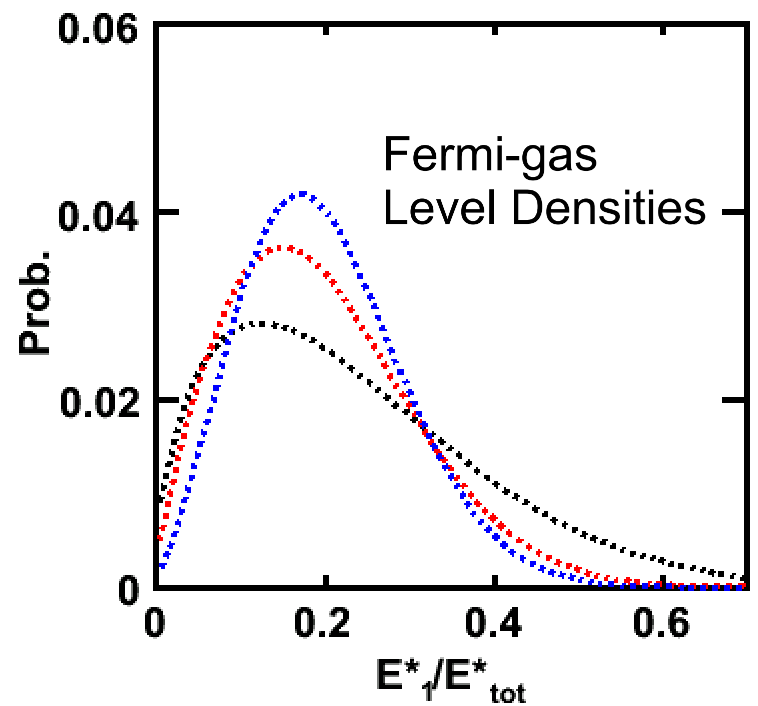
For E*<10 MeV “const T”
E* > 10 MeV Fermi Gas matched
Like Gilbert and Cameron LD's

$A_1 = 20$, $A_2 = 80$

What is the correct change-over energy?
Voinov *et al* see “const T” behavior up to E*=20 MeV in A=60
PRC 79 (2009) 031301

In order for GEMINI++ to better predict odd-even oscillations need better level densities prescription at low excitation energies.
How do these effects depend on spin





Morreto Formalism - extension of the Bohr-Wheeler transition state Formalism

$$\Gamma_Z = \frac{1}{2\pi\rho(E^*, S_0)} \int_0^{E^* - E_Z(S_0)} d\varepsilon \;\; \rho_Z\left[E^* - \varepsilon, S_0\right]$$

ε: energy in binary-decay degree of freedom

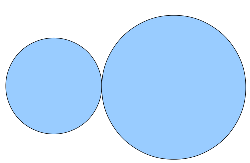
ρ_Z: level density at the conditional saddle point for a particular charge split

$E_Z(S_0)$: conditional saddle point energy

in GEMINI++

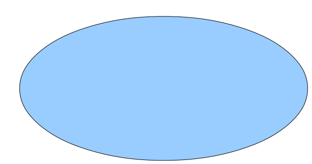
$$\Gamma_{Z,A} = \frac{1}{2\pi\rho(E^*, S_0)} \int_0^{E^* - E_{Z,A}(S_0)} d\varepsilon \;\; \rho_{Z,A}(E^* - \varepsilon, S_0)$$

$$\Gamma_Z = \sum_A \Gamma_{Z,A}$$



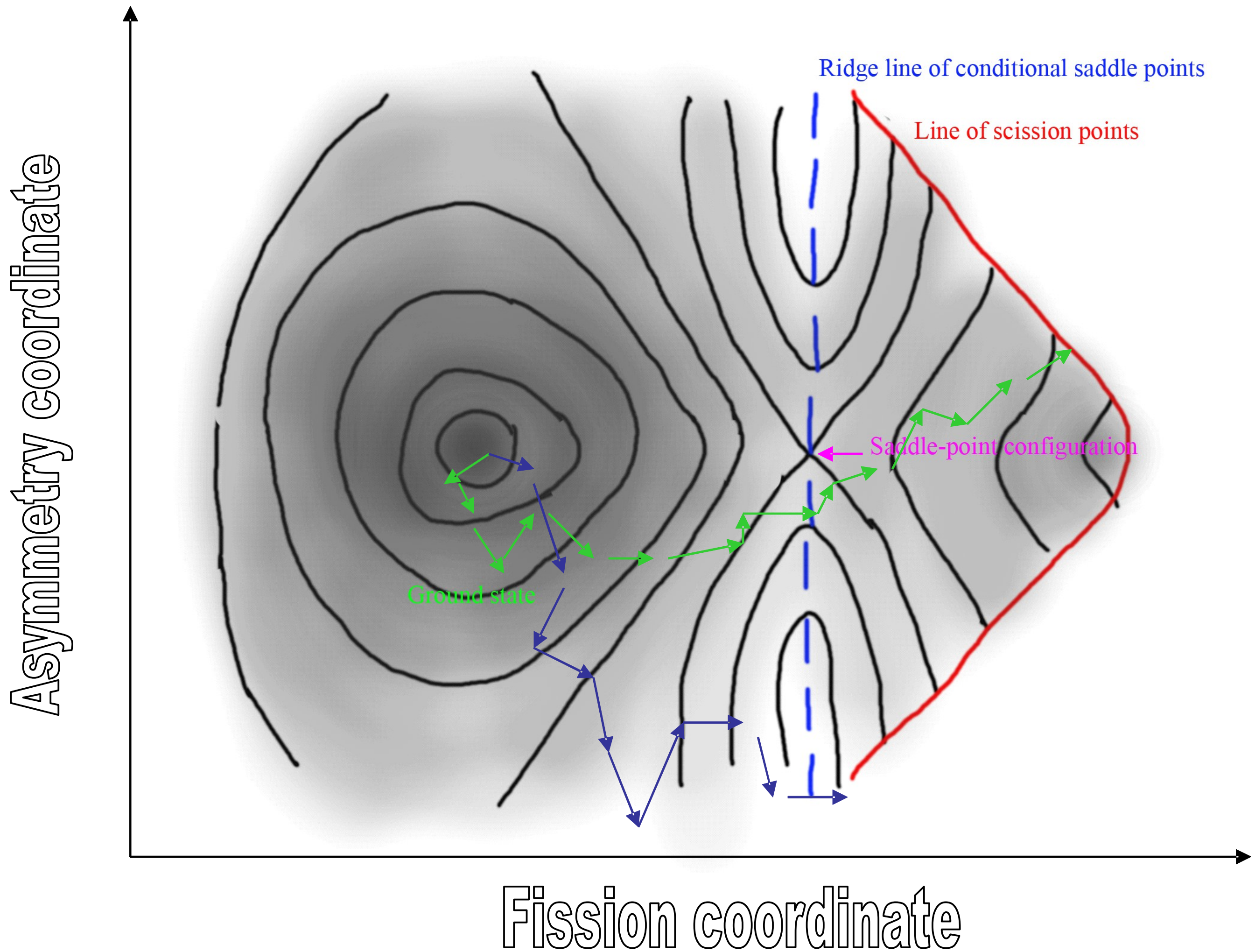
Z,A $\quad$ $Z_{CN}-Z, A_{CN}-A$

Conditional saddle point

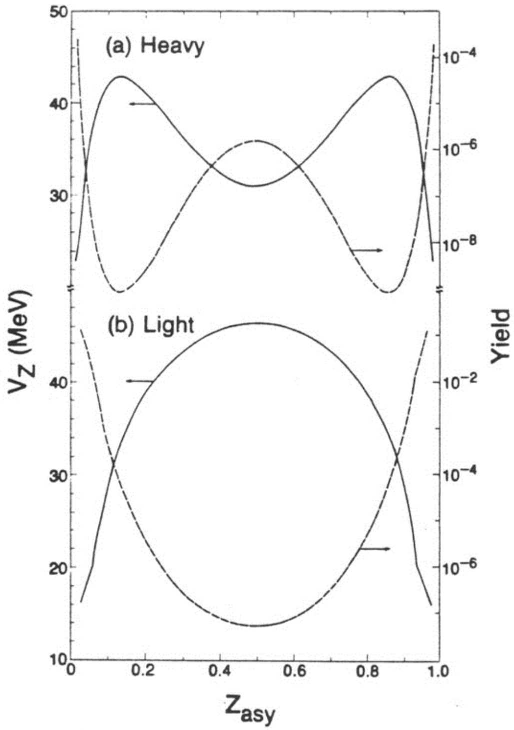


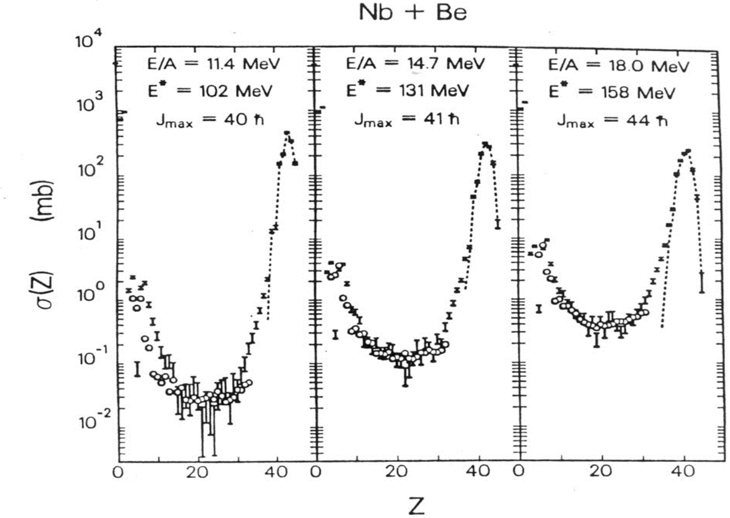
charge and mass asymmetry
are only defined if saddle-point
has significant neck.
True for asymmetric divisions,
Not always true for symmetric divisions

Potential energy surface

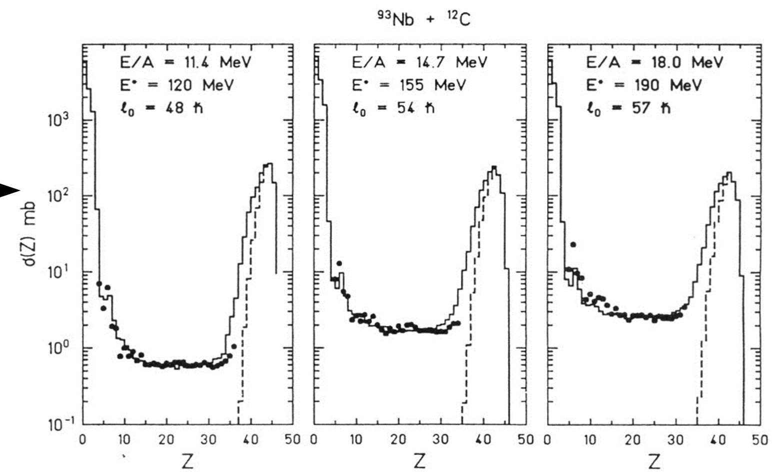


For large saddle-to-scission distance asymmetry at saddle may not be preserved at scission





Businaro-Gallone point



Increasing angular momentum has the same effect as increasing the mass

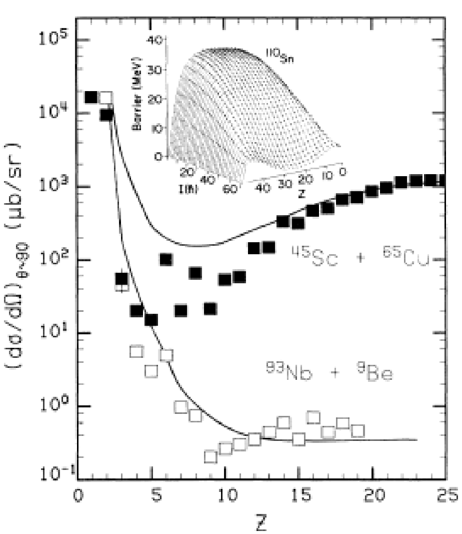
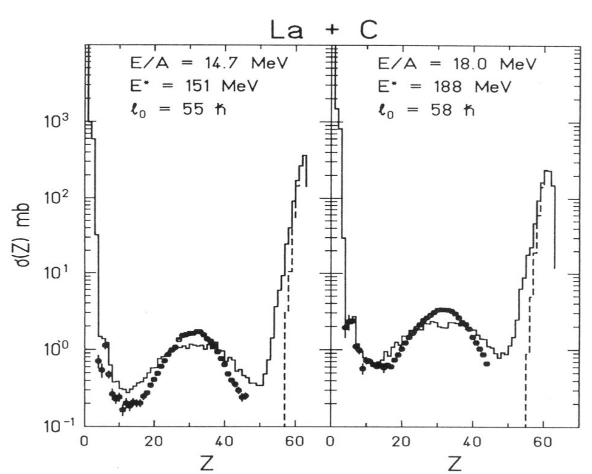
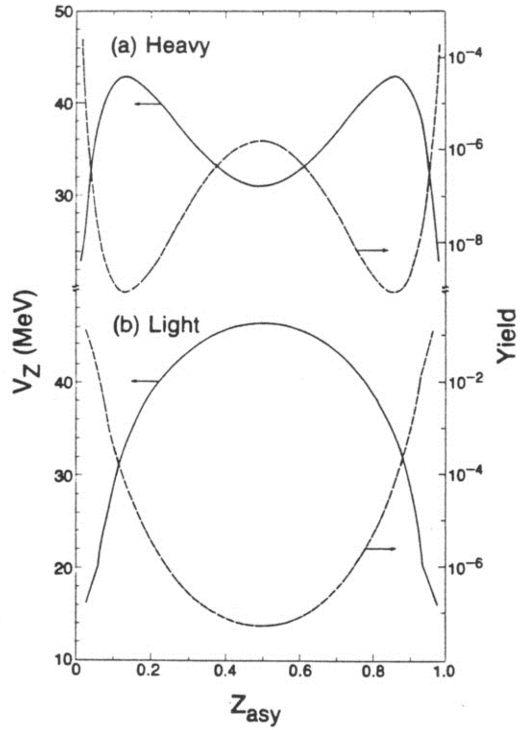


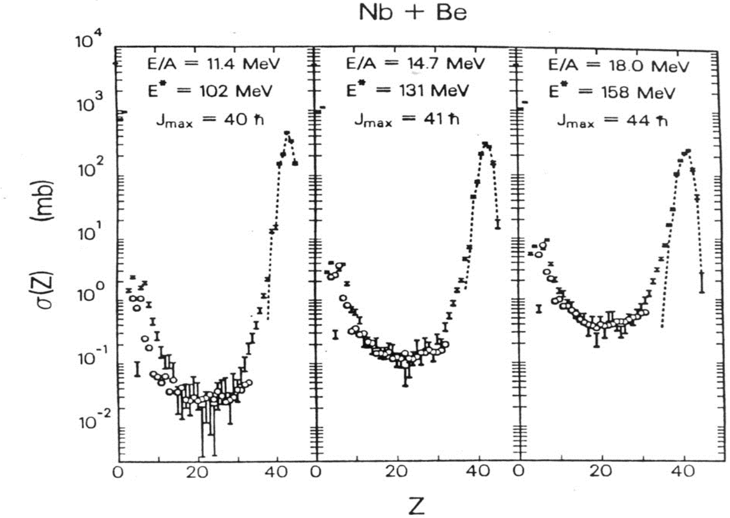
TABLE I. Quantities characterizing the reactions of interest.

System	E_{lab} (MeV)	CN	E^* (MeV)	l_{crit}[a] ($\hbar$)	l_m[b]	x[c]	y[d]
$^{93}Nb+^{9}Be$	782	^{102}Rh	78	34	43	0.40	0.05
$^{45}Sc+^{65}Cu$	200	^{110}Sn	94	70	80	0.45	0.17

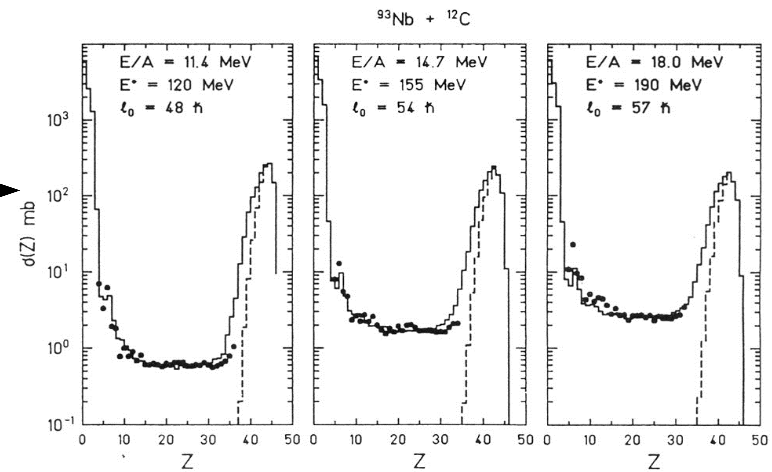
Sobotka et al PRC **36**, 2713 (1987)



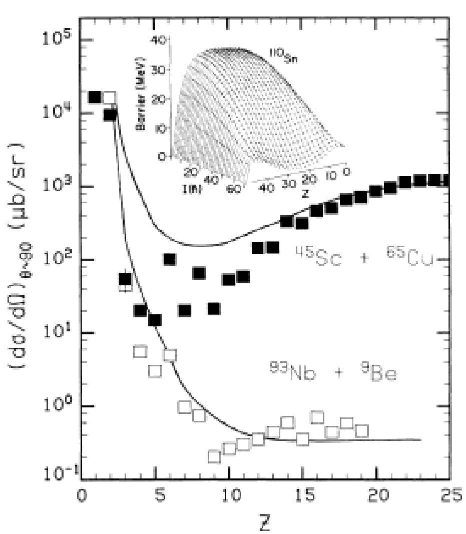




Businaro-Gallone point



Increasing angular momentum has the same effect as increasing the mass



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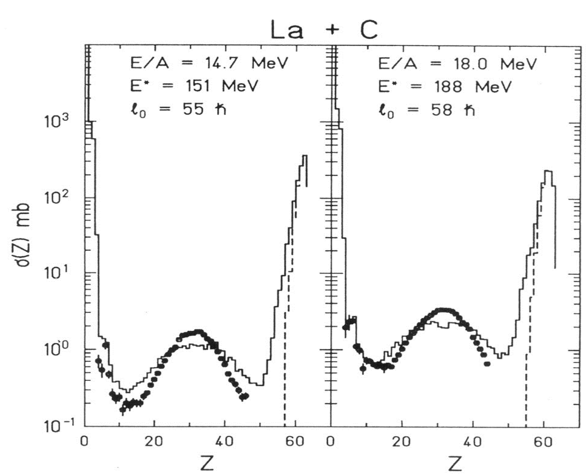
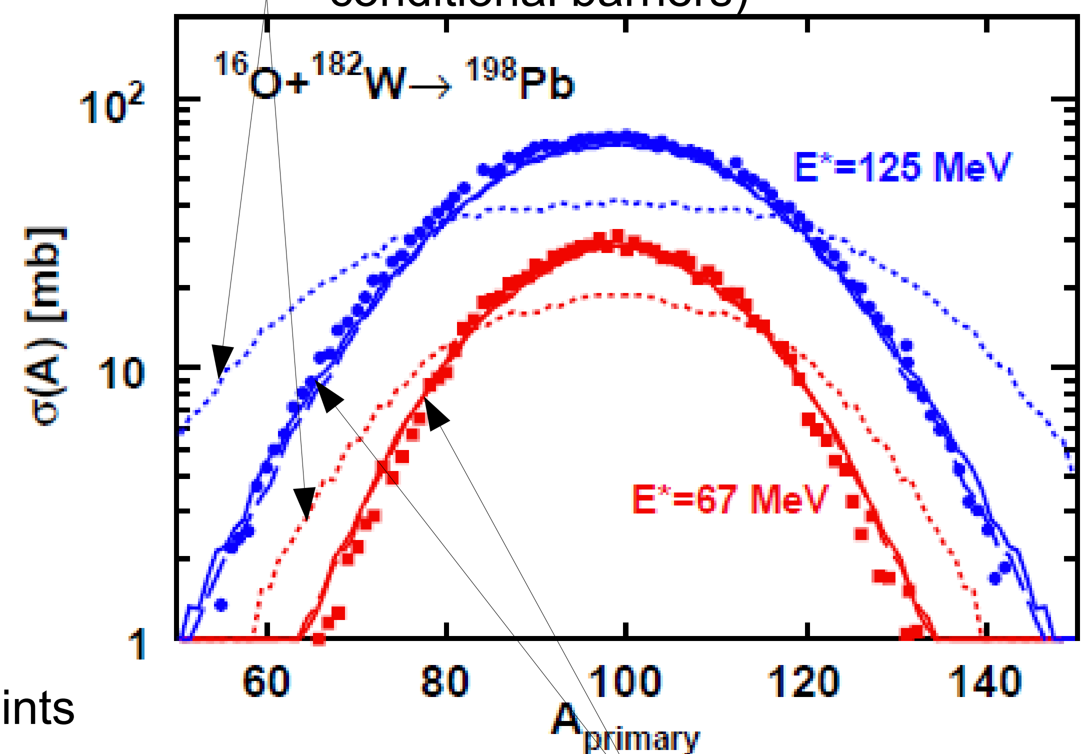


Why use the Bohr-Wheeler Formalism at all, why not just use the Morreto formalism for all non-evaporative binary decays

Morreto (using Sierk's finite-range conditional barriers)

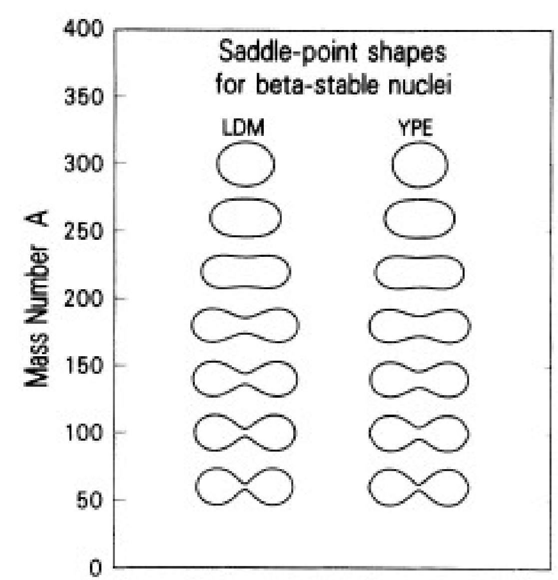


Rusanov systematics

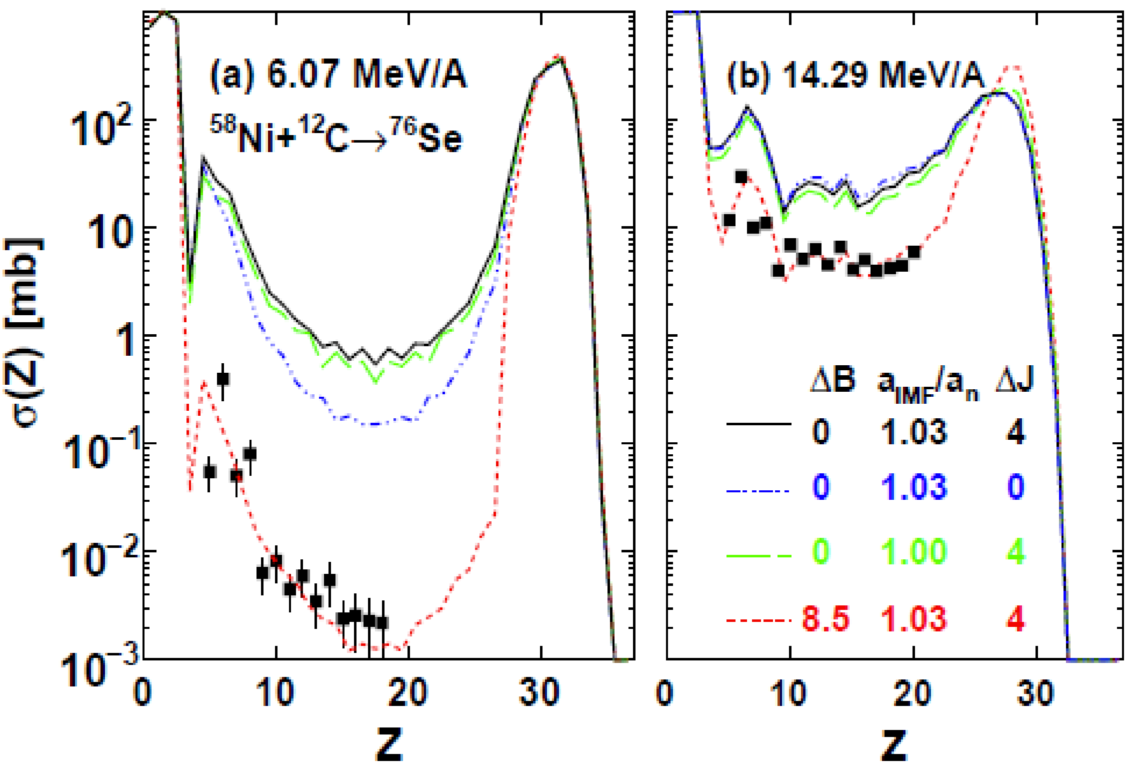
a) predicts too-wide fission mass distributions above the Businaro-Gallone point(using Sierk's Finite-range conditional barriers)

b) asymmetry not defined for very heavy system

Morreto assumes saddle and scission points are degenerate. Almost true below the Businaro-Gallone point, However there can be modifications to charge asymmetry in decent from saddle to scission for heavier systems.

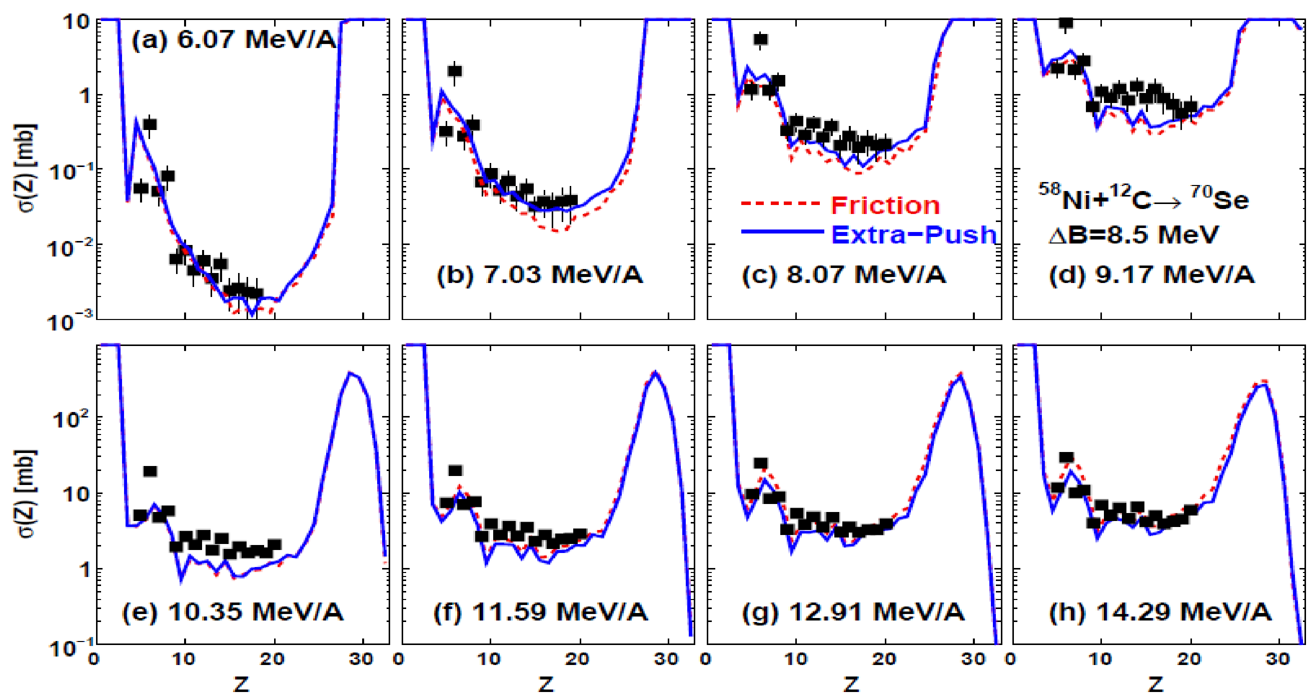


So in GEMINI++, we use the Morreto Formalism for all binary decays below the Businaro-Gallone point. Above the Businaro-Gallone point, the symmetric fission is calculated from the Bohr-Wheeler Formalism, with mass and charge distributions from the Rusanov systematics



Cannot fit IMF yields with Sierks, Finite-range conditional barriers.

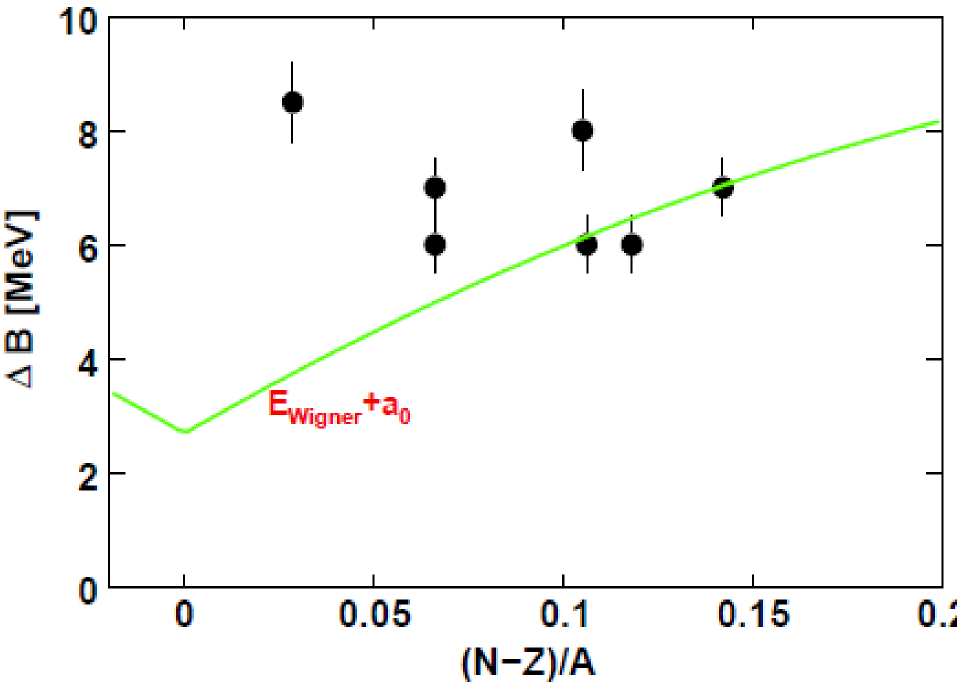
$$\rho_Z(E^*, S_0) \propto \exp\left(2\sqrt{a_{IMF}(E^* - E_Z(S_0))}\right)$$
$$E_Z(s_0) = E_Z^{Sierk}(S_0) + \Delta B$$

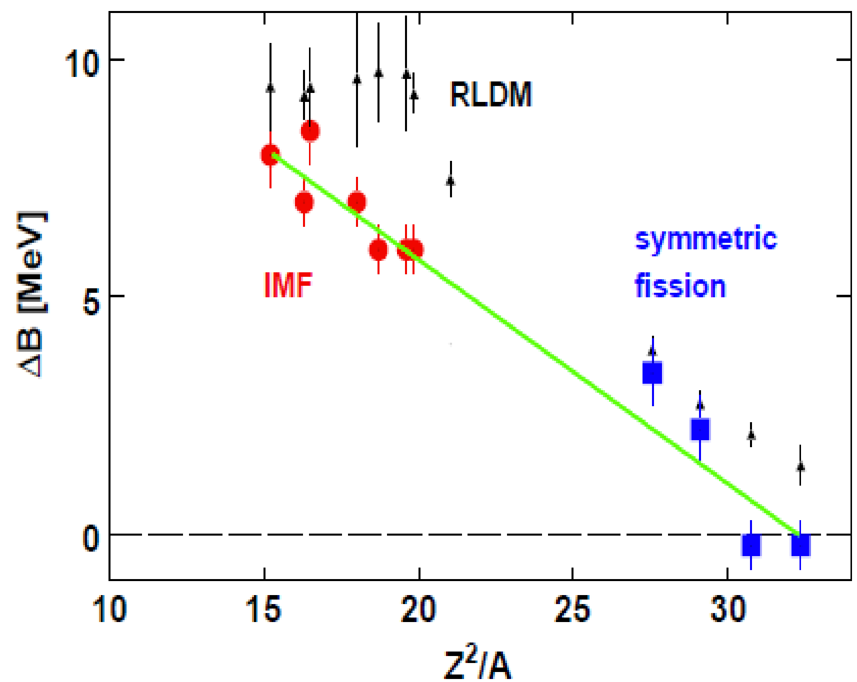


Data :Fan et al, NPA 679 (2000) 121
IMF emission in fusion reactions

Systematics of ΔB

$$E_Z = E_Z^{Sierk} + \Delta B$$





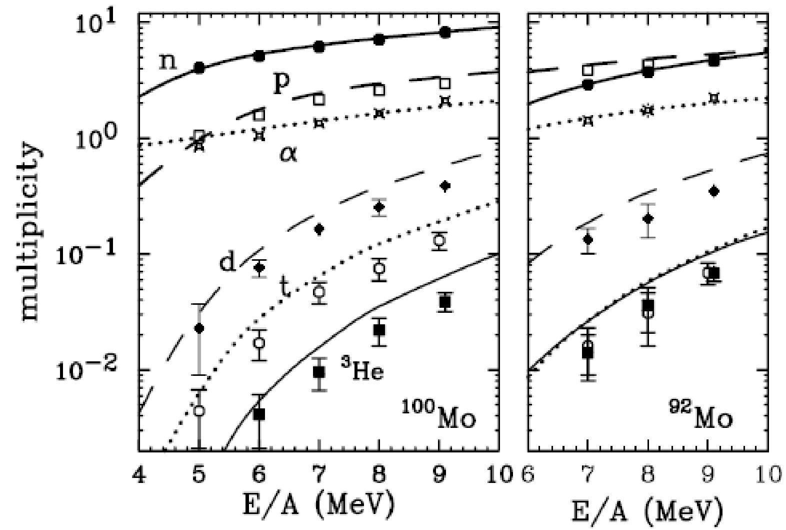
$^{58}Ni + ^{12}C \rightarrow ^{70}Se$
$^{64}Ni + ^{12}C \rightarrow ^{76}Se$
$^{63}Cu + ^{12}C \rightarrow ^{75}Br$
$^{78}Kr + ^{12}C \rightarrow ^{90}Mo$
$^{82}Kr + ^{12}C \rightarrow ^{94}Mo$
$^{86}Kr + ^{12}C \rightarrow ^{98}Mo$
$^{93}Nb + ^{9}Be \rightarrow ^{102}Rh$

Analysis of data from Morreto's Group

At the moment in GEMINI++, ΔB=7 MeV

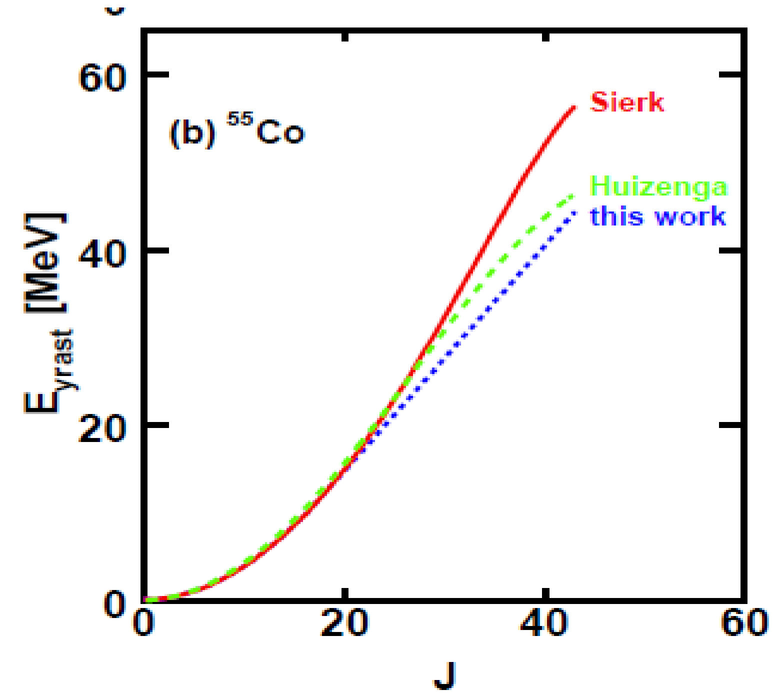
Other points

a) *d*, *t*, ^{3}He evaporation.
Very few experimental studies, but yields are
~ 0.5 of statistical model predictions for A~160
More investigations are needed?
GEMINI++ uses a 0.5 “preformation factor” for *d*,*t*, ^{3}He



Ni + Mo fusion reactions

b) Finite-range yrast line is too steep for light nuclei.
GEMINI++ contains a modification to the FR yrast line determined from data.



c) fission transients?
The effect of a fission delay on the fission probability is similar to that of a_f/a_n, ie. decreasing the level-density parameter at the saddle point.
Not Included in GEMINI++ at the moment.

d) Should the Wigner-energy fade out?
The Wigner has been explained due to neutron-proton pairing

$$W_{Wigner} = 30\left|\frac{N-Z}{A}\right| + \begin{cases} 1/A & \text{if } Z \text{ and } Z \text{ odd and even} \\ 0 & \text{otherwise} \end{cases}$$

Conclusions

Still lots of uncertainty is obtaining a global set of statistical-model parameters.

a) What is the mass and excitation energy dependence of the level-density parameter.
Need more proton, alpha and neutron kinetic energy spectra for heavy nuclei.

b) Can we obtain a better global prescription of the level density at very low excitation energies. Also need the J dependence.

c) what is the mass-dependence of the barrier shift for IMF emission in light nuclei.

A lot of the variation in the statistical-model parameters discussed are data driven.
We lack theoretical models to help understand these changes and allow us to extrapolate.
All the data I showed are below the multi-fragmentation regime where large expansion effects maybe important. We may need to understand these lower-energy phenomena In order to fully understand multi-fragmentation.

Level densities (back-shifted Fermi-gas level densities)

$$\rho(E^*, J) \propto (2J+1) \exp 2\sqrt{aU}$$
a : level density parameter
U : thermal excitation energy

for standard nuclei, include fadeout of shell and pairing effects

$$U = E^* - E_{rot}^{macro}(J) + \delta P(J)$$

$$\delta P(J) = \begin{cases} \delta P_{g.s.}\left[1 - \left(\frac{U}{U_{crit}(J)}\right)^2\right], U_{crit} = U_{crit}^0\left[1 - \left(\frac{J}{J_{crit}}\right)^2\right] & \textit{for } J < J_{crit} \text{ and } U < U_{crit} \\ \delta P_{g.s.} & \textit{otherwise} \end{cases}$$

$$a = \tilde{a}\left[1 + \tanh(U/\eta + J/\chi)\frac{\delta W_{g.s.}}{U}\right]$$

$E_{rot}^{macro}(J)$: Macroscopic rotational energy from finite-range model of Sierk

where $\delta P_{g.s.}$ and $\delta W_{g.s.}$ are ground-state shell and paring corrections

$$\eta = 18.52\,\text{MeV}, \chi = 50$$

for large E^*: $\quad aU \to \tilde{a}(E^* - E_{rot}^{macro}(J) + \delta P_{g.s.} + \delta W_{g.s.})$

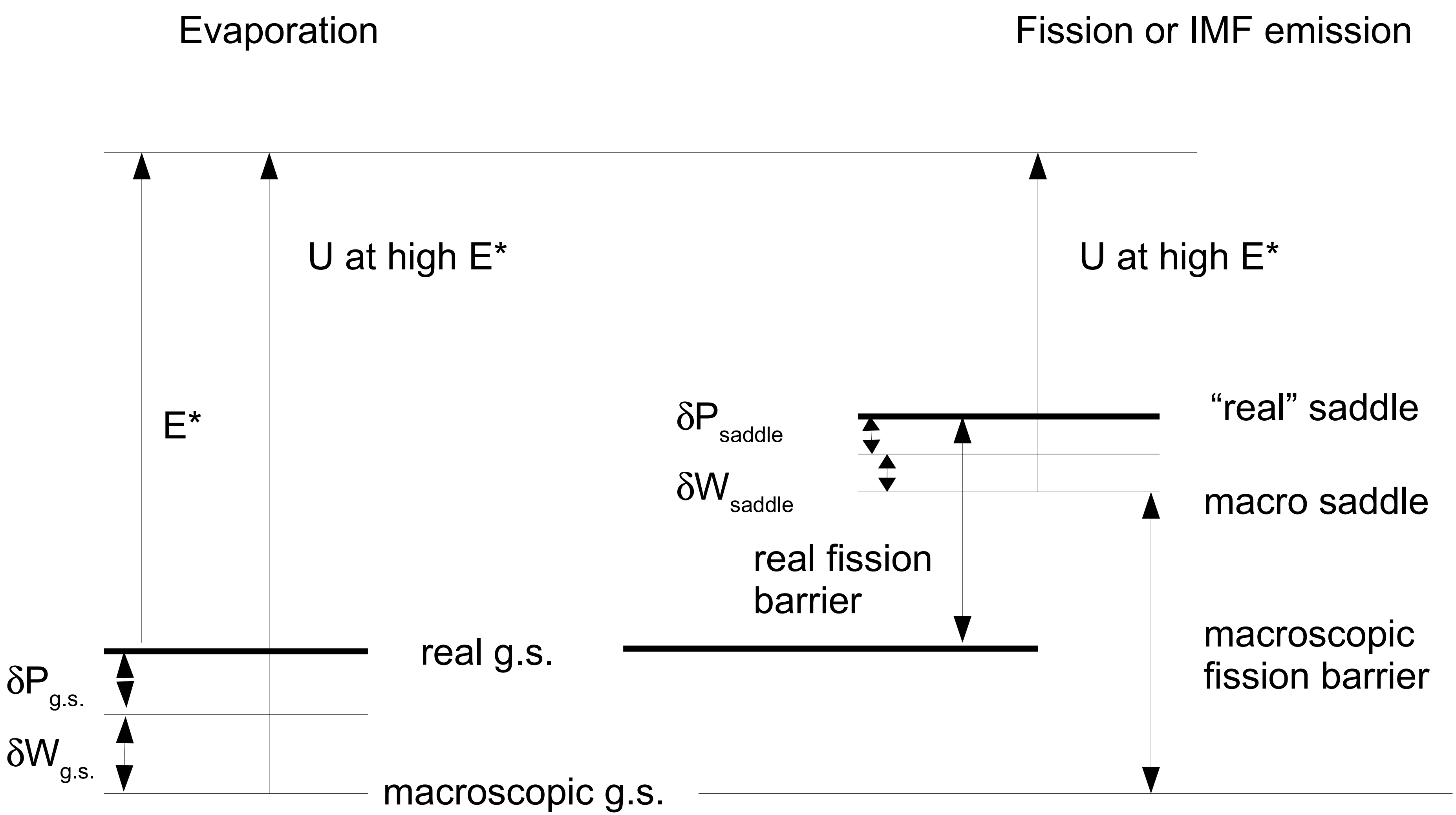
for small E^*: $\quad aU \to \tilde{a}(E^* - E_{rot}^{macro}(J))$

Saddle-point configurations

$$U = E^* - E_{saddle}^{macro}(J) + \delta P_{g.s.} + \delta W_{g.s.}$$

$E_{saddle}^{jacro}(J)$: macroscopic saddle-point energy from finite-range model of Sierk

Shell and pairing corrections at the saddle-point not considered, i.e. $\delta P_{saddle} = \delta W_{saddle} = 0$



Macroscopic-microscopic model $M = M_{macro} + \delta P_{g.s.} + \delta W_{g.s.}$

$\delta P_{g.s.}$ And $\delta W_{g.s.}$ from finite-range model (P. Moeller et al), except
$\delta P_{g.s.}$ shifted such that $\delta P_{g.s.}=0$ for odd-odd (BCS theory)