Precision tests of unitarity in leptonic mixing

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Overview

- Status of the standard model
- Sterile neutrinos
- Precision observables
- Precision test of the standard model
- Precision test of standard model plus sterile neutrinos
- Summary and conclusion

Importance of the LHC results

- The **standard model Higgs** boson has been discovered.
- No new physics, carrying standard model charges at the weak scale, appears to be present.
- Therefore only limited extensions of the standard model are possible.

Theory predictions:

- Precision predictions are sensitive to radiative corrections dependent on m_H.
- Higgs mass before the LHC: 110 GeV $\leq m_H \leq$ 160 GeV.
- The knowledge of m_H fixes the radiative corrections.
- The quantitative comparison of precision data with predictions is now possible at a much higher level than ever before !

Sterile neutrinos

The model:

- *n* neutral (sterile) fermions (Dirac or Majorana)
- Mixing with left-handed neutrinos of the standard model
- *PMNS* matrix is a part of the general mixing matrix (Pontecorvo-Maki-Nakagawa-Sakata)

Motivation:

- Provide dark matter candidates
- Baryogenesis via leptogenesis
- Essentially invisible at the LHC
- Right-handed neutrinos and *PMNS* matrix exist

The *PMNS* matrix

- Mass eigenstates and flavour basis ($\alpha = e, \mu, \tau$): $\{\nu_i = \nu_{L_\alpha},\ N_n\}$ expressed via a unitary $(3+n) \times (3+n)$ matrix:

$$\begin{pmatrix} \nu_1 \\ \vdots \\ \nu_{3+n} \end{pmatrix} = \begin{pmatrix} PMNS & \mathcal{W} \\ \mathcal{W}^\dagger & \mathcal{V} \end{pmatrix} \begin{pmatrix} \nu_{L_e} \\ \vdots \\ N_n \end{pmatrix} .$$

- Unitarity of *PMNS* as submatrix not generally true
- Definition of the ϵ parameters:

$$\epsilon_\alpha = \sum_{i>3} |\mathcal{U}_{\alpha i}|^2 = 1 - \sum_{\beta} |\mathcal{U}_{\alpha\beta}|^2 .$$

Low energy parameters

The theory prediction for meson decays is dependent on the ratio:

$$\frac{g_\alpha}{g_\beta} = 1 - \frac{\epsilon_\alpha - \epsilon_\beta}{2}. \tag{1}$$

The epsilon parameters modify the Fermi constant via the following relation:

$$G_\mu^2 = G_F^2(1 - \epsilon_e)(1 - \epsilon_\mu)\,, \tag{2}$$

with G_μ the Fermi constant measured in muon decay, and G_F the theoretical Fermi parameter.
They also affect the unitarity of the Cabibbo-Kobayashi-Maskawa matrix:

$$CKM = 1 + \epsilon_\mu\,, \tag{3}$$

High energy parameters

$$\frac{M_W}{[M_W]_{SM}} = 1 + 0.11\,(\epsilon_e + \epsilon_\mu) + 0.0056\,T \quad (4)$$

$$\frac{\Gamma_{inv}/\Gamma_{lept}}{\left[\Gamma_{inv}/\Gamma_{lept}\right]_{SM}} = 1 - 0.76\,(\epsilon_e + \epsilon_\mu) - 0.67\,\epsilon_\tau - 0.0015\,T \quad (5)$$

$$\frac{\Gamma_{lept}}{\left[\Gamma_{lept}\right]_{SM}} = 1 + 0.60\,(\epsilon_e + \epsilon_\mu) + 0.0093\,T \quad (6)$$

$$\frac{\sin^2\theta_{eff}^{lept}}{\left[\sin^2\theta_{eff}^{lept}\right]_{SM}} = 1 - 0.72\,(\epsilon_e + \epsilon_\mu) - 0.011\,T. \quad (7)$$

Observable	Experiment	standard model
$(g_\mu/g_e)_\tau$	1.0020(16)	1.0
$(g_\tau/g_e)_\tau$	1.0029(21)	1.0
$(g_\mu/g_e)_\pi$	1.0021(16)	1.0
$(g_\tau/g_\mu)_\pi$	0.9965(33)	1.0
CKM	0.9999(6)	1.0
M_W (GeV)	80.385(15)	80.359(11)
$\Gamma_{\rm inv}/\Gamma_{\rm lept}$	5.942(16)	5.9721(2)
$\Gamma_{\rm lept}$ (MeV)	83.984(86)	84.005(15)
$s^{2,\rm lept}_{\rm eff}$	0.23113(21)	0.23150(1)
$s^{2,\rm hadr}_{\rm eff}$	0.23222(27)	0.23150(1)

Table: Experimental results and standard model prediction for lepton universality and electroweak observables.

Observable	χ^2_{SM}	χ^2_T	χ^2_ϵ	$\chi^2_{\epsilon+T}$
$(g_\mu/g_e)_\tau$	19.8	18.8	17.5	17.4
$(g_\tau/g_e)_\tau$	20.3	19.3	14.0	13.5
$(g_\mu/g_e)_\pi$	19.7	18.6	17.4	17.2
$(g_\tau/g_\mu)_\pi$	20.0	19.0	17.3	17.3
CKM	21.3	20.3	15.9	15.2
M_W (GeV)	19.4	19.4	16.9	11.6
$\Gamma_{\mathrm{inv}}/\Gamma_{\mathrm{lept}}$	17.8	16.9	15.8	15.4
Γ_{lept} (MeV)	21.4	20.2	17.6	17.5
$s^{2,\mathrm{lept}}_{\mathrm{eff}}$	18.2	18.1	16.2	16.0
$s^{2,\mathrm{hadr}}_{\mathrm{eff}}$	14.2	10.5	5.3	5.3
Total χ^2	21.3	20.3	18.0	18.0

Table: The χ^2 for the standard model (χ^2_{SM}), the minimum with unitarity violation (χ^2_ϵ), with unitarity violation and the T parameter ($\chi^2_{\epsilon+T}$), and the T parameter only, are evaluated *excluding* the entry on each line. The total χ^2 (considering all entries) is given for reference.

Hypothesis testing: Standard Model

Selected set of precision data

High energy observables: $\left\{ M_W, \frac{\Gamma_{inv}}{\Gamma_{lept}}, \Gamma_{lept}, s^{2,\text{lept}}_{\text{eff}}, s^{2,\text{hadr}}_{\text{eff}} \right\}$

Low energy observables: $\left\{ \left(\frac{g_\mu}{g_e}\right)_\tau, \left(\frac{g_\tau}{g_e}\right)_\tau, \left(\frac{g_\mu}{g_e}\right)_\pi, \left(\frac{g_\tau}{g_\mu}\right)_\pi, \text{CKM} \right\}$

Statistical χ^2 analysis:

- Fit result: $\chi^2/\text{dof} = 21.3/10$
- Likelihood of data described by prediction: less than 2%

Alternative (controversial) fit:

- Removing one data point in turn
- Best fit for $\{s^{2,\text{hadr}}_{\text{eff}}\}$ removed: $\chi^2/\text{dof} = 14.2/9$
- Likelihood of data minus $\{s^{2,\text{hadr}}_{\text{eff}}\}$ described by prediction: 13%.

A word on statistics

Data and uncertainty:

- Weighting of data via uncertainty^{-2}
- Large uncertainty means tiny contribution to the χ^2
- NuTeV, W and kaonic decays have large errors

$\Rightarrow$ Inclusion of **all** data leads to a **dilution** of the $\chi^2/d.o.f$

Removing a data point:

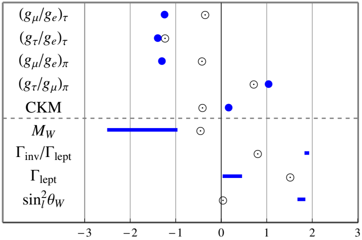
- $s_{\mathrm{eff}}^{2,\mathrm{hadr}}$ inferred from hadronic measurements
- A posteriori justification: Considerable change of χ^2
- Hint: underlying systematics (e.g. underestimated uncertainty)

1. **Standard model disfavoured by precision data**
2. **however good overall consistency with data**
3. **Sterile neutrinos can remedy 1. without spoiling 2.**

Hypothesis testing: Non-unitary lepton mixing

Analysis with ϵ parameters:

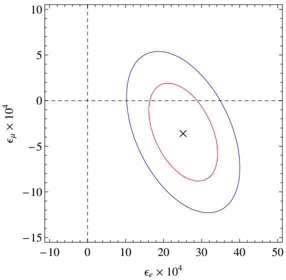
- Total fit: $\chi^2/\mathrm{dof} = 18.0/7$
- Corresponding likelihood: 1.5%
- Best fit for $\{s^{2,\mathrm{hadr}}_{\mathrm{eff}}\}$ removed
- $\chi^2/\mathrm{dof} = 5.3/5$

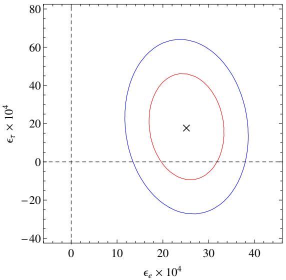


$\Rightarrow$ Likelihood that data without $\{s^{2,\mathrm{hadr}}_{\mathrm{eff}}\}$ is described by the Standard Model plus non-unitary lepton mixing is 50%.

$\Rightarrow$ Inclusion of oblique parameters barely improves the fit.

The unitarity violation parameters



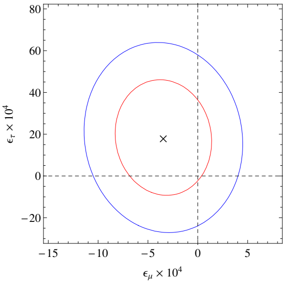


Quantify mixing and universality:

ϵ_e non zero at $\sim 3\sigma$

ϵ_μ small, compatible with zero

ϵ_τ not well constrained



Experimental constraints on sterile neutrinos

Probing the model further:

- $\epsilon_e + \epsilon_\mu + \epsilon_\tau \neq \left(UU^\dagger\right)_{e\mu} + \left(UU^\dagger\right)_{e\tau} + \left(UU^\dagger\right)_{\mu\tau}$
- Rare decays like $\mu \to e\,\gamma$ cannot be assessed
- New constraint on models with lepton unitarity violation
- Direct neutrino mixing experiments still too imprecise

Neutrinoless double beta decay:

- No constraints if Dirac fermions
- Masses $\mathcal{O}(100\ \text{TeV})$ and/or *PMNS* cancellations if Majorana

See-saw models:

- Mixing $\sim \epsilon_e$ *too large* for type-I see-saw
- Strong cancellations in the *PMNS* matrix required

Summary and Conclusions

- Measurement of the Higgs boson mass makes precision tests meaningful.
- Standard Model cannot explain discrepancies in precision data.
- Removing $s_{eff}^{2,hadr}$ improves consistency between data and theory.
- 3.0σ evidence for lepton unitarity violation of $\mathcal{O}(10^{-3})$.
- Indication for mixing of left-handed neutrinos with sterile neutrinos.
- Additional oblique corrections are unnecessary.

Outlook

- Clarification of the discrepancy between $s_{eff}^{2,hadr}$ and $s_{eff}^{2,lept}$, Mainz, JLab;
- Tau-factories: improved precision of τ–decays, Peking;
- LHC: improved measurement of M_W;
- Higher order theoretical calculations;
- new beamdump experiment at CERN, snoopy.

$\Rightarrow$ More than 5σ for ϵ_e possible.

$\Rightarrow$ Sterile neutrino model becomes *predictive*.

Precision = Discovery !!