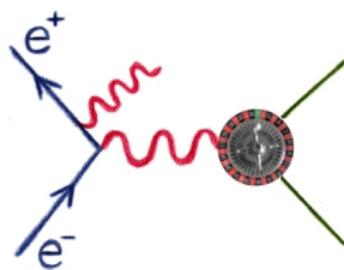


Update on the combined estimate of KLOE ISR measurements

S. E. Müller

Institute of Radiation Physics, Helmholtz-Zentrum Dresden-Rossendorf



*Radio MonteCarLOW Meeting
Institut für Kernphysik Mainz, 11 April 2014*

The KLOE data sets

- **KLOE05:** 60 points between 0.35 and 0.95 GeV^2 ,
based on 141.4 pb^{-1} of data taken in 2001^a
- **KLOE08:** 60 points between 0.35 and 0.95 GeV^2 ,
based on 240.0 pb^{-1} data taken in 2002^b
- **KLOE10:** 75 points between 0.1 and 0.85 GeV^2 ,
based on 232.6 pb^{-1} data taken in 2006^c with $\sqrt{s} = 1.00 \text{ GeV}$
- **KLOE12:** 60 points between 0.35 and 0.95 GeV^2 ,
based on 240.0 pb^{-1} data taken in 2002^d, normalized to muons

^aPhys. Lett. **B606** (2005) 12

^bPhys. Lett. **B670** (2009) 285

^cPhys. Lett. **B700** (2011) 102

^dPhys. Lett. **B720** (2013) 336

The KLOE data sets

- **KLOE05:** 60 points between 0.35 and 0.95 GeV²,
based on 141.4 pb⁻¹ data taken in 2001^a
Superseded by KLOE08!
- **KLOE08:** 60 points between 0.35 and 0.95 GeV²,
based on 240.0 pb⁻¹ data taken in 2002^b
- **KLOE10:** 75 points between 0.1 and 0.85 GeV²,
based on 232.6 pb⁻¹ data taken in 2006^c with $\sqrt{s} = 1.00$ GeV
- **KLOE12:** 60 points between 0.35 and 0.95 GeV²,
based on 240.0 pb⁻¹ data taken in 2002^d, normalized to muons

^aPhys. Lett. **B606** (2005) 12

^bPhys. Lett. **B670** (2009) 285

^cPhys. Lett. **B700** (2011) 102

^dPhys. Lett. **B720** (2013) 336

Recap from last meeting:

- A combined estimate for the data sets of the KLOE08, KLOE10 and KLOE12 analyses was presented
- $195 y_i = \{y_1, \dots, y_{195}\}$ measurements for the 85 observables with true value $X_\alpha = \{X_1, \dots, X_{85}\}$ (the 85 bins between 0.1 and 0.95 GeV 2)
- Combined estimates \hat{x}_α were evaluated using the BLUE (Best Linear Unbiased Estimate) method¹
- Need to construct two matrices $\mathcal{U}_{i\alpha}$ and \mathcal{M}_{ij} to determine the \hat{x}_α

¹A. Valassi, NIM A500 (2003) 391; L. Lyons and D. Gibaut, NIM A270 (1988) 110

Recap from last meeting:

- A combined estimate for the data sets of the KLOE08, KLOE10 and KLOE12 analyses was presented
- 195 $y_i = \{y_1, \dots, y_{195}\}$ measurements for the 85 observables with true value $X_\alpha = \{X_1, \dots, X_{85}\}$ (the 85 bins between 0.1 and 0.95 GeV²)
- Combined estimates \hat{x}_α were evaluated using the BLUE (Best Linear Unbiased Estimate) method¹
- Need to construct two matrices $\mathcal{U}_{i\alpha}$ and \mathcal{M}_{ij} to determine the \hat{x}_α

¹A. Valassi, NIM A500 (2003) 391; L. Lyons and D. Gibaut, NIM A270 (1988) 110

Recap from last meeting:

- A combined estimate for the data sets of the KLOE08, KLOE10 and KLOE12 analyses was presented
- $195 \quad y_i = \{y_1, \dots, y_{195}\}$ measurements for the 85 observables with true value $X_\alpha = \{X_1, \dots, X_{85}\}$ (the 85 bins between 0.1 and 0.95 GeV 2)
- Combined estimates \hat{x}_α were evaluated using the BLUE (Best Linear Unbiased Estimate) method¹
- Need to construct two matrices $\mathcal{U}_{i\alpha}$ and \mathcal{M}_{ij} to determine the \hat{x}_α

¹A. Valassi, NIM A500 (2003) 391; L. Lyons and D. Gibaut, NIM A270 (1988) 110 ↩ ↪ ↫

Recap from last meeting:

- A combined estimate for the data sets of the KLOE08, KLOE10 and KLOE12 analyses was presented
- 195 $y_i = \{y_1, \dots, y_{195}\}$ measurements for the 85 observables with true value $X_\alpha = \{X_1, \dots, X_{85}\}$ (the 85 bins between 0.1 and 0.95 GeV²)
- Combined estimates \hat{x}_α were evaluated using the BLUE (Best Linear Unbiased Estimate) method¹
- Need to construct two matrices $\mathcal{U}_{i\alpha}$ and \mathcal{M}_{ij} to determine the \hat{x}_α

¹A. Valassi, NIM A500 (2003) 391; L. Lyons and D. Gibaut, NIM A270 (1988) 110 ↩ ↪ ↤

The $\mathcal{U}_{i\alpha}$ matrix

is a $(\underbrace{195}_{\text{rows}} \times \underbrace{85}_{\text{cols}})$ matrix linking the measurements y_i to the observables

X_α

$$\mathcal{U}_{i\alpha} = \begin{cases} 1 & \text{if } y_i \text{ is a measurement of } X_\alpha, \\ 0 & \text{if } y_i \text{ is not a measurement of } X_\alpha, \end{cases}$$

The $\mathcal{U}_{i\alpha}$ matrix

$$\mathcal{U} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ \\ 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \mathcal{U} = & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

→ bins in $M_{\pi\pi}^2$ (0.1 - 0.95 GeV 2)

The $\mathcal{U}_{i\alpha}$ matrix

$\mathcal{U} =$

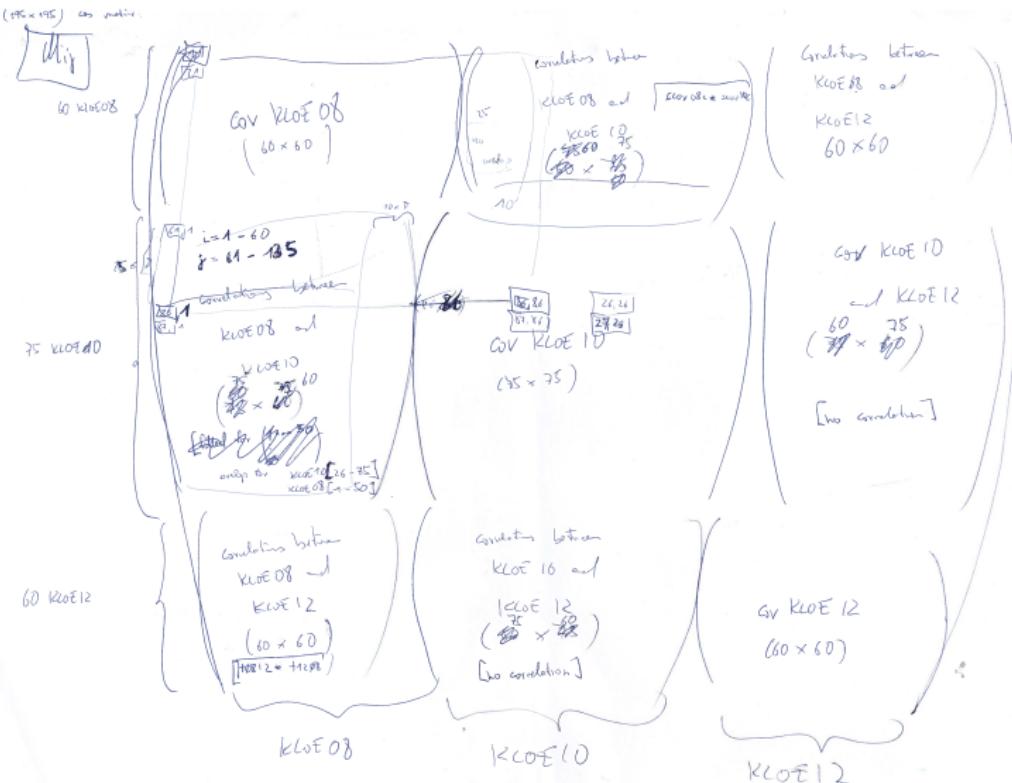
$$\left(\begin{array}{ccccccccc|ccccccccc|cc} 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ \\ 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{array} \right)$$

KLOE08 data KLOE10 data KLOE12 data

→ bins in $M_{\pi\pi}^2$ (0.1 - 0.95 GeV 2)

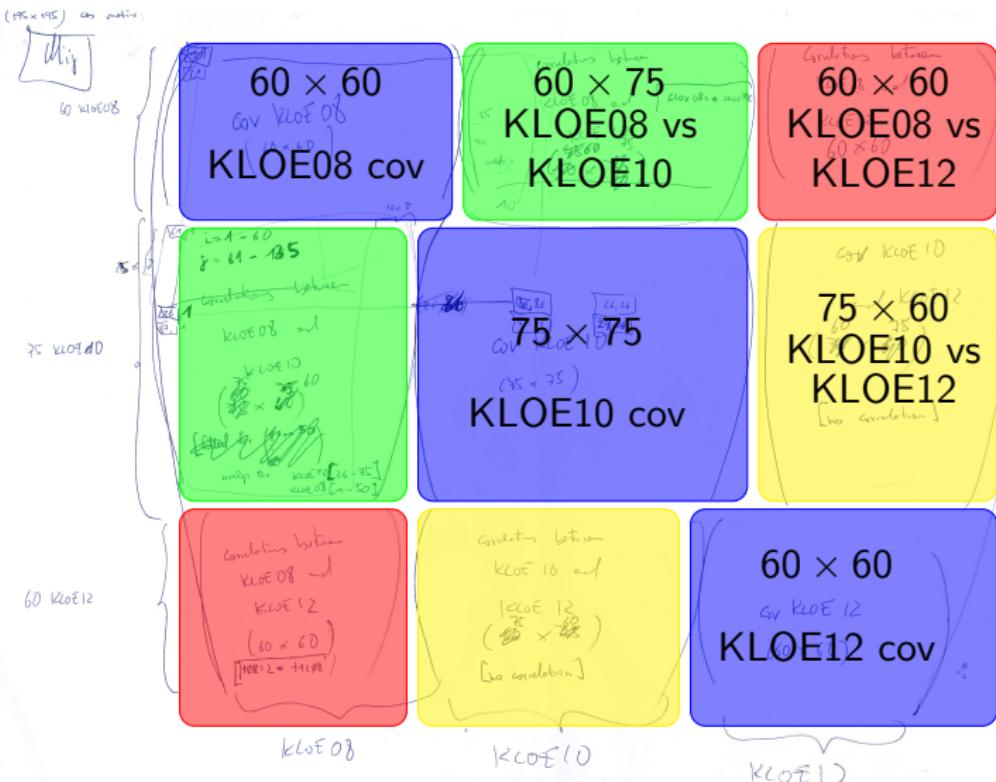
The \mathcal{M}_{ij} matrix

is the covariance matrix for the $60 + 75 + 60 = 195$ data points.



The M_{ij} matrix

is the covariance matrix for the $60 + 75 + 60 = 195$ data points.



A caveat: normalization errors

G. D'Agostini (NIM A346 (1994) 306):

Normalization errors (e.g. errors on scale factors) can create a bias when fitting correlated data

The problem of finding the linear unbiased estimates of minimum variance for the 85 observables X_α is equivalent to the problem of finding the estimates \hat{x}_α minimizing the quantity

$$S = \sum_{i=1}^{195} \sum_{j=1}^{195} [y_i - (\mathcal{U}\hat{x})_i] \mathcal{M}_{ij}^{-1} [y_j - (\mathcal{U}\hat{x})_j] \quad (1)$$

However, only the free parameters \hat{x}_α are varied (within the errors) to find the minimum of S . But in the case of a normalization error, also the elements of \mathcal{M}_{ij} should be scaled accordingly when varying the \hat{x}_α . Therefore, normalization errors lead to a bias in this method.

A caveat: normalization errors

G. D'Agostini (NIM A346 (1994) 306):

Normalization errors (e.g. errors on scale factors) can create a bias when fitting correlated data

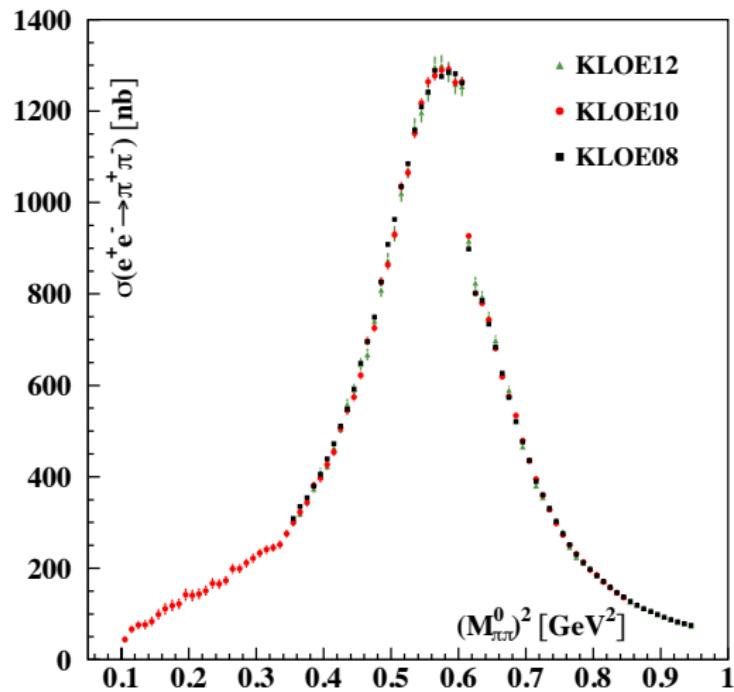
Way out: Two separate covariance matrices:

- $\mathcal{M}_{ij}^{\text{stat}}$ which contains the statistical uncertainties and is used to find the \hat{x}_α
- $\mathcal{M}_{ij}^{\text{syst}}$ which contains all the normalization errors, gives

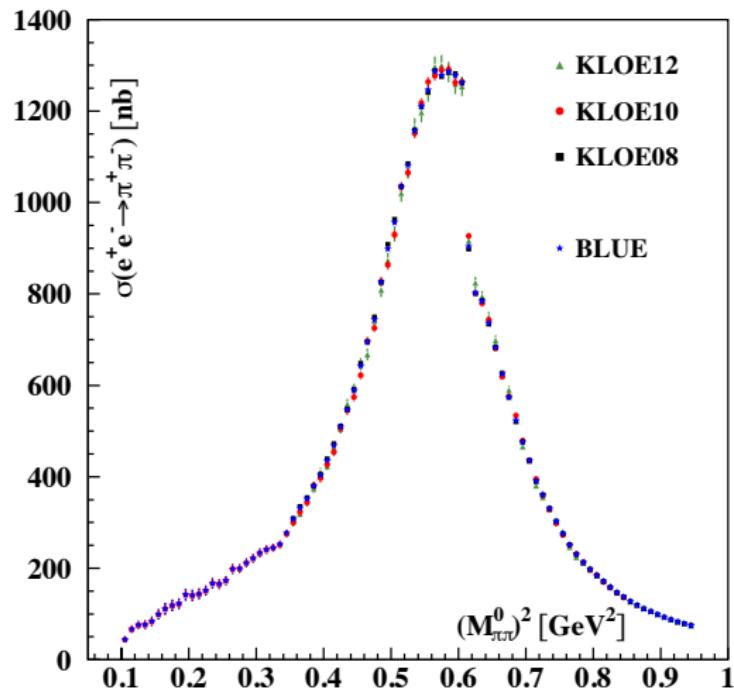
$$\text{cov}^{\text{syst}}(\hat{x}_\alpha, \hat{x}_\beta) = \sum_{i=1}^{195} \sum_{j=1}^{195} \lambda_{\alpha i} \mathcal{M}_{ij}^{\text{syst}} \lambda_{\beta j}, \text{ which can then be added to}$$
$$\text{cov}^{\text{stat}}(\hat{x}_\alpha, \hat{x}_\beta):$$

$$\text{cov}^{\text{tot}}(\hat{x}_\alpha, \hat{x}_\beta) = \text{cov}^{\text{stat}}(\hat{x}_\alpha, \hat{x}_\beta) + \text{cov}^{\text{syst}}(\hat{x}_\alpha, \hat{x}_\beta) \quad (1)$$

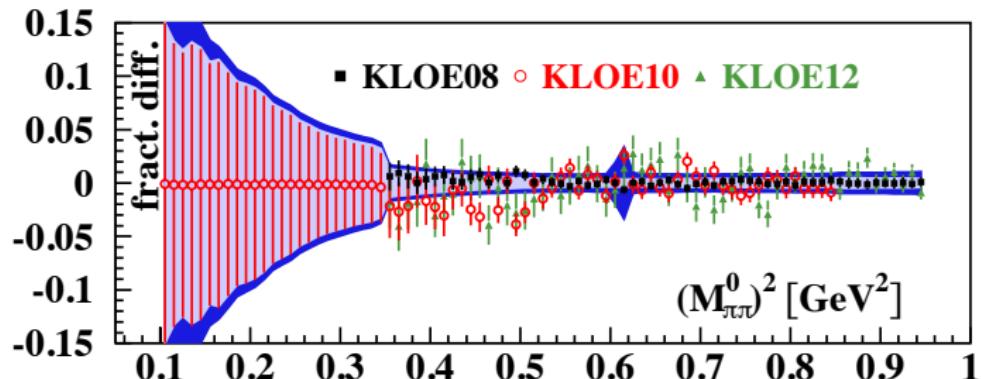
The result: Cross section



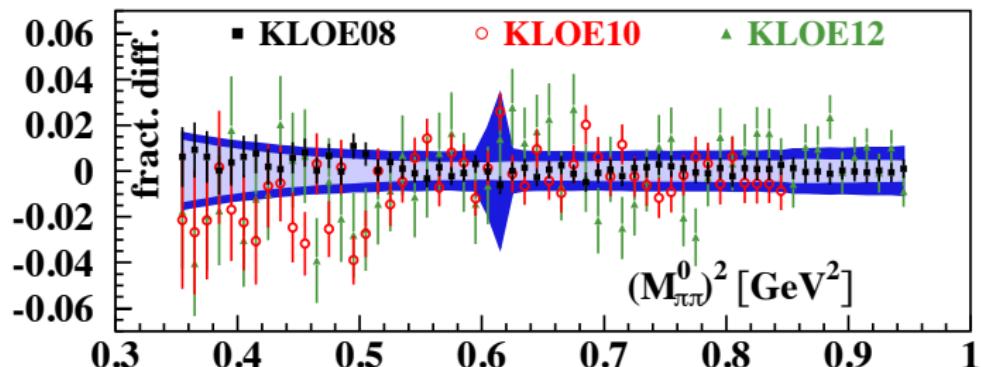
The result: Cross section



The result: Fractional differences

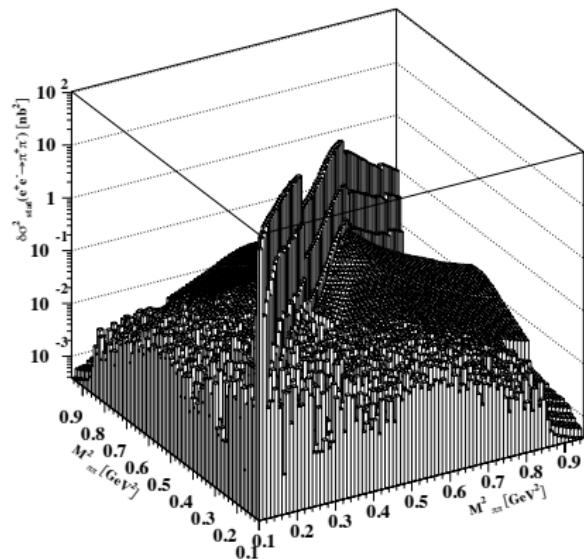


The result: Fractional differences

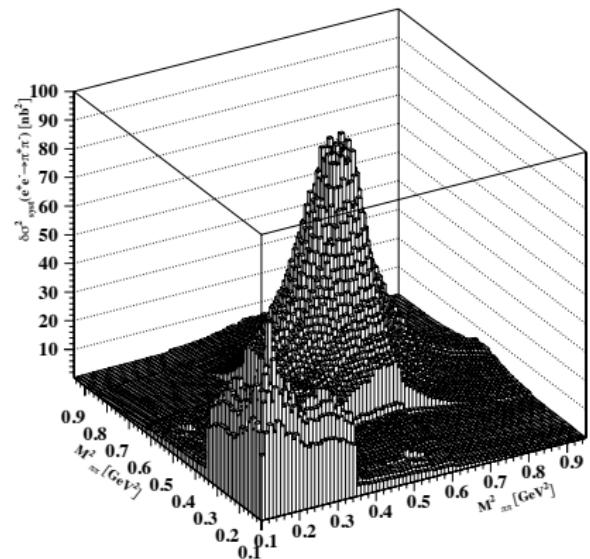


The covariance matrices for the BLUE values

stat. covariance matrix:



syst. covariance matrix:



To obtain the total covariance matrix, simply add the two.

Results on $a_\mu^{\pi\pi}$:

Using a combination of KLOE08 & KLOE10 (KLOE Note 225):

$$\Delta a_\mu^{\pi\pi}[0.10 - 0.95 \text{GeV}^2] = (488.6 \pm 6.0) \times 10^{-10} \quad (2)$$

$$\Delta a_\mu^{\pi\pi}[0.35 - 0.85 \text{GeV}^2] = (379.1 \pm 2.9) \times 10^{-10} \quad (3)$$

Using the BLUE for KLOE08, KLOE10 & KLOE12:

$$\Delta a_\mu^{\pi\pi}[0.10 - 0.95 \text{GeV}^2] = (488.6 \pm 5.7) \times 10^{-10} \quad (4)$$

$$\Delta a_\mu^{\pi\pi}[0.35 - 0.85 \text{GeV}^2] = (378.9 \pm 2.8) \times 10^{-10} \quad (5)$$

Results on $a_\mu^{\pi\pi}$:

Using a combination of KLOE08 & KLOE10 (KLOE Note 225):

$$\Delta a_\mu^{\pi\pi}[0.10 - 0.95 \text{GeV}^2] = (488.6 \pm 6.0) \times 10^{-10} \quad (2)$$

$$\Delta a_\mu^{\pi\pi}[0.35 - 0.85 \text{GeV}^2] = (379.1 \pm 2.9) \times 10^{-10} \quad (3)$$

Using the BLUE for KLOE08, KLOE10 & KLOE12:

$$\Delta a_\mu^{\pi\pi}[0.10 - 0.95 \text{GeV}^2] = (488.6 \pm 5.7) \times 10^{-10} \quad (4)$$

$$\Delta a_\mu^{\pi\pi}[0.35 - 0.85 \text{GeV}^2] = (378.9 \pm 2.8) \times 10^{-10} \quad (5)$$

Using the BLUE for KLOE08, KLOE10 & KLOE12, but correcting KLOE08 and KLOE10 for Fred Jegerlehner's 2012 corrections for vacuum polarization:

$$\Delta a_\mu^{\pi\pi}[0.10 - 0.95 \text{GeV}^2] = (487.8 \pm 5.7) \times 10^{-10} \quad (6)$$

$$\Delta a_\mu^{\pi\pi}[0.35 - 0.85 \text{GeV}^2] = (378.1 \pm 2.8) \times 10^{-10} \quad (7)$$

χ^2 determination

The BLUE method is equivalent to the problem of finding the estimates \hat{x}_α minimizing the sum²:

$$S = -2 \log \mathcal{L} = \sum_{i=1}^n \sum_{j=1}^n [y_i - (\mathcal{U}\hat{x})_i] \mathcal{M}_{ij}^{-1} [y_j - (\mathcal{U}\hat{x})_j] \quad (8)$$

$$= \sum_{\alpha=1}^N \sum_{\beta=1}^N \sum_{i=1}^n \sum_{j=1}^n [\mathcal{U}_{i\alpha}(y_i - \hat{x}_\alpha)] \mathcal{M}_{ij}^{-1} [\mathcal{U}_{j\beta}(y_j - \hat{x}_\beta)] \quad (9)$$

Assuming that errors are Gaussian, the minimum of S should be distributed as a χ^2 with $(n - N) = 195 - 85 = 110$ degrees of freedom.

²A. Valassi, NIM A500 (2003) 391

χ^2 determination

The BLUE method is equivalent to the problem of finding the estimates \hat{x}_α minimizing the sum²:

$$S = -2 \log \mathcal{L} = \sum_{i=1}^n \sum_{j=1}^n [y_i - (\mathcal{U}\hat{x})_i] \mathcal{M}_{ij}^{-1} [y_j - (\mathcal{U}\hat{x})_j] \quad (8)$$

$$= \sum_{\alpha=1}^N \sum_{\beta=1}^N \sum_{i=1}^n \sum_{j=1}^n [\mathcal{U}_{i\alpha}(y_i - \hat{x}_\alpha)] \mathcal{M}_{ij}^{-1} [\mathcal{U}_{j\beta}(y_j - \hat{x}_\beta)] \quad (9)$$

Assuming that errors are Gaussian, the minimum of S should be distributed as a χ^2 with $(n - N) = 195 - 85 = 110$ degrees of freedom.

I find $\chi^2_{\text{tot}}/\text{ndf} = 183/110$ with a χ^2 -probability of $\text{Prob} \simeq 1.5 \cdot 10^{-5}$

²A. Valassi, NIM A500 (2003) 391

χ^2 contributions

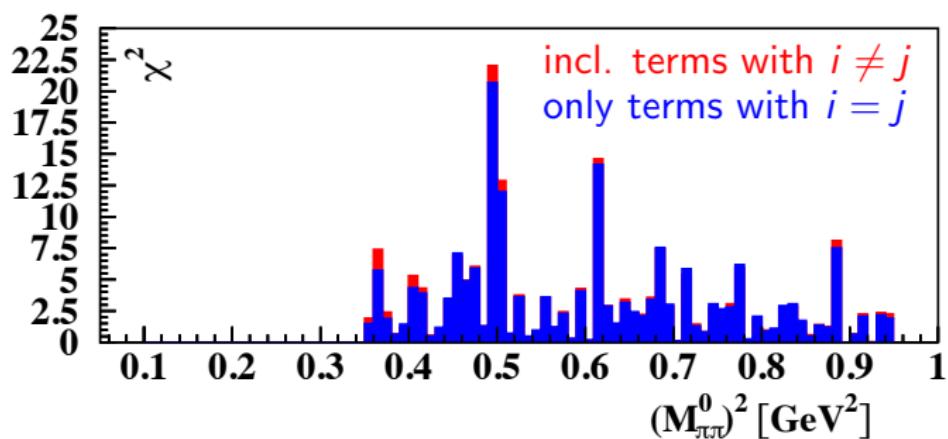
Keeping only the terms with $\alpha = \beta$ in Eq. 9, we can obtain the individual contributions S_α in each bin of $M_{\pi\pi}^2$:

$$S_\alpha = \sum_{i=1}^n \sum_{j=1}^n [\mathcal{U}_{i\alpha}(y_i - \hat{x}_\alpha)] \mathcal{M}_{ij}^{-1} [\mathcal{U}_{j\alpha}(y_j - \hat{x}_\alpha)] \quad (10)$$

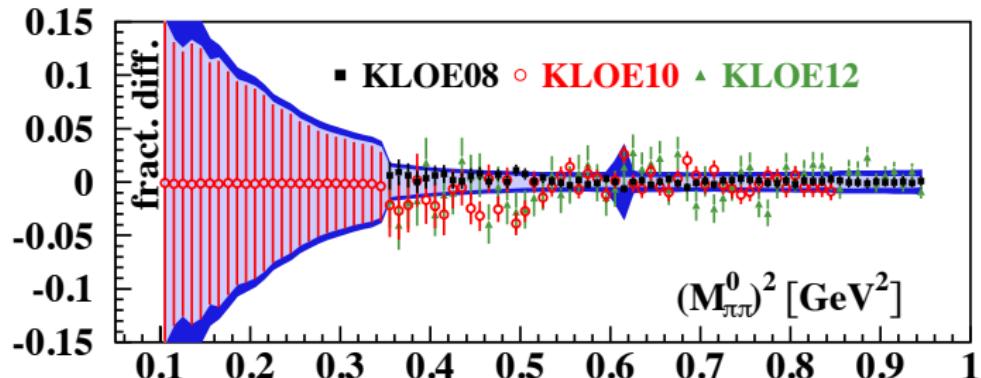
χ^2 contributions

Keeping only the terms with $\alpha = \beta$ in Eq. 9, we can obtain the individual contributions S_α in each bin of $M_{\pi\pi}^2$:

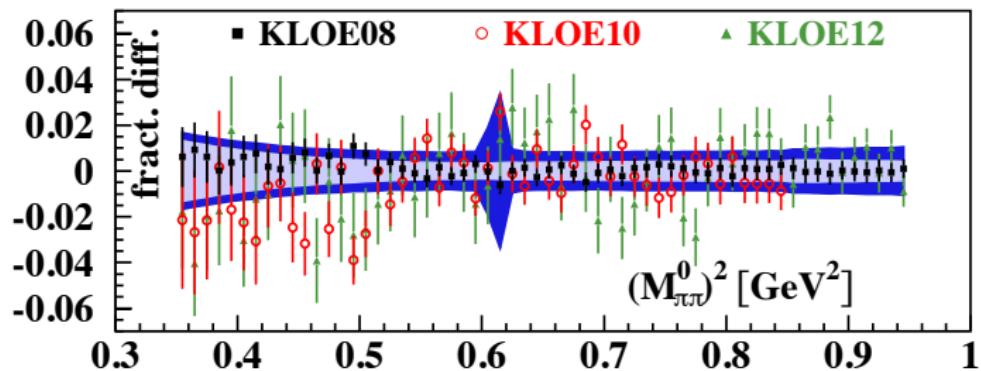
$$S_\alpha = \sum_{i=1}^n \sum_{j=1}^n [\mathcal{U}_{i\alpha}(y_i - \hat{x}_\alpha)] \mathcal{M}_{ij}^{-1} [\mathcal{U}_{j\alpha}(y_j - \hat{x}_\alpha)] \quad (10)$$



The result: Fractional differences (again!)

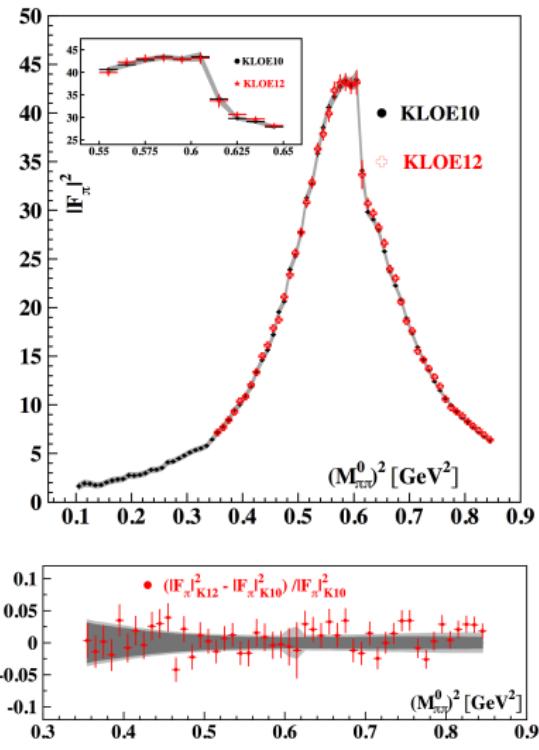
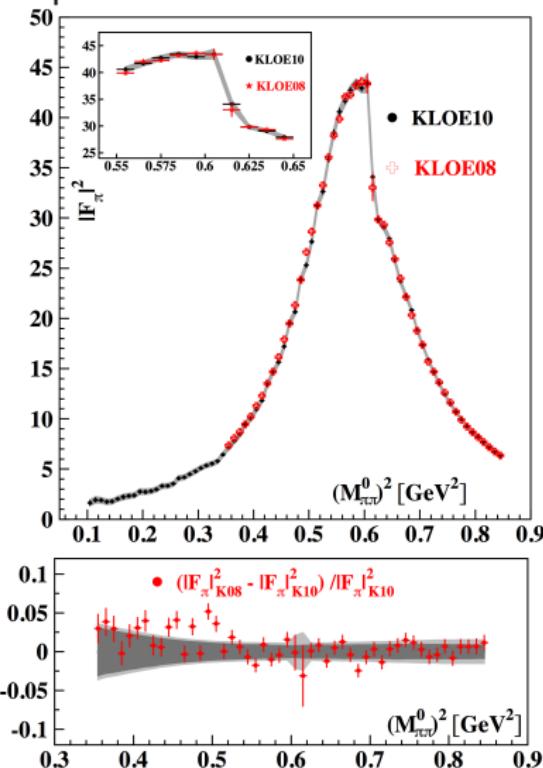


The result: Fractional differences (again!)



KLOE results:

Comparison KLOE10 with KLOE08 and KLOE12:



Correlations between $\pi\pi\gamma$ and $\mu\mu\gamma$ systematic errors in KLOE12 analysis

Given two measured quantities X_1 and X_2 with a common normalization factor f with uncertainty σ_f . Then in the ratio $R = X_1/X_2$, the uncertainty σ_f vanishes.

Cf., if the $\pi\pi\gamma$ and $\mu\mu\gamma$ analyses have common normalization factors, the uncertainty induced by these factors would vanish in the ratio $\pi\pi\gamma/\mu\mu\gamma$

- Uncertainties for KLOE08 $\pi\pi\gamma$ analysis and KLOE12 $\pi\pi\gamma/\mu\mu\gamma$ analysis would be uncorrelated

Correlations between $\pi\pi\gamma$ and $\mu\mu\gamma$ systematic errors in KLOE12 analysis

Given two measured quantities X_1 and X_2 with a common normalization factor f with uncertainty σ_f . Then in the ratio $R = X_1/X_2$, the uncertainty σ_f vanishes.

Cf., if the $\pi\pi\gamma$ and $\mu\mu\gamma$ analyses have common normalization factors, the uncertainty induced by these factors would vanish in the ratio $\pi\pi\gamma/\mu\mu\gamma$

- Uncertainties for KLOE08 $\pi\pi\gamma$ analysis and KLOE12 $\pi\pi\gamma/\mu\mu\gamma$ analysis would be uncorrelated

If $\pi\pi\gamma$ and $\mu\mu\gamma$ analyses have no correlation due to common normalization factors, then

- Uncertainties for KLOE08 $\pi\pi\gamma$ analysis and KLOE12 $\pi\pi\gamma/\mu\mu\gamma$ analysis are fully correlated

Correlations between $\pi\pi\gamma$ and $\mu\mu\gamma$ systematic errors in KLOE12 analysis

Currently, http://www.lng.infn.it/kloe/ppg/ppg_2012.html gives only information on systematic uncertainties on $\sigma_{\pi\pi}$ obtained from the ratio $\pi\pi\gamma/\mu\mu\gamma$ (with “some” correlation between the $\pi\pi\gamma$ and $\mu\mu\gamma$ analyses already taken into account?).

Not clear to me how to include these uncertainties properly in the systematic covariance matrix of the **BLUE** values (at the moment, I take them as fully correlated between **KLOE08** and **KLOE12** - which in principle they are...)

- I need more information on how the systematic errors of the **KLOE12** analysis were obtained (and maybe would be good to have the $\mu\mu\gamma$ uncertainties, too)

Summary and conclusion

- BLUE of KLOE08, KLOE10 and KLOE12 data have been evaluated
- $\chi^2_{\text{tot}}/\text{ndf} = 183/110$ with a χ^2 -probability of $\text{Prob} \simeq 1.5 \cdot 10^{-5}$ was found for the BLUE evaluation
- More information needed to understand treatment of systematic uncertainties in KLOE12 analysis
- Still some work needed, then result will be put on the KLOE ppg-webpage:
<http://www.lng.infn.it/kloe/ppg/>
- Fit of the pion form factor (?)