Longitudinal gradient super-bends and anti-bends for compact low emittance light source lattices

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Recall: paths to low emittance
Recall: the TME cell
The LGAB cell
Longitudinal gradient bends
Anti-bends
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References

AS & Albin Wrulich, *Compact low emittance light sources based on longitudinal gradient bending magnets*, submitted to NIM A

Recall: paths to low emittance

\[ \varepsilon_{xo} [\text{m} \cdot \text{rad}] = \bar{C}_q \gamma^2 \frac{I_5}{I_2 - I_4} \]

\[ \sigma_\delta^2 = \bar{C}_q \gamma^2 \frac{I_3}{2I_2 + I_4} \]

\[ \Delta E [\text{keV}] = \bar{C}_\gamma \gamma^4 I_2 \]

Equilibrium beam parameters of a flat lattice

- \( \varepsilon_{xo} \): natural horizontal emittance
- \( \sigma_\delta \): rms relative momentum spread, \( \delta = \Delta p/p \)
- \( \Delta E \): energy loss per turn

\( I_2, I_3, I_4, I_5 \): synchrotron radiation integrals

\( \bar{C}_q = 3.83 \cdot 10^{-13} \text{ m} \quad \bar{C}_\gamma = 9.60 \cdot 10^{-13} \text{ keV} \)

\( 3.18 \)
\( \varepsilon_{xo}[m \cdot \text{rad}] = \tilde{C}_q \gamma^2 \frac{I_5}{I_2 - I_4} \quad \sigma_\delta^2 = \tilde{C}_q \gamma^2 \frac{I_3}{2I_2 + I_4} \quad \Delta E[\text{keV}] = \tilde{C}_\gamma \gamma^4 I_2 \)

**Optics**

\[ I_5 = \int |b|^3 \mathcal{H} \, ds \rightarrow \text{min} \]

- horizontal focus in each dipole
- many small dipoles of angle \( \Phi \ll 1 \)

*multibend achromat (MBA) lattice*

**Power**

\[ I_3 = \int |b|^3 \, ds \]

\[ I_2 = \int b^2 \, ds \rightarrow \text{max...} \]

- *damping wigglers (DW)*

**Damping**

\[ I_4 = \int b \eta (b^2 + 2k) \, ds \rightarrow -I_2 \]

\[ J_x = 1 - \frac{I_4}{I_2} \rightarrow 2 \]

- *gradient bends* for vertical focusing \( (bk < 0) \)

orbit curvature \( b = 1/\rho = B/(p/e) \)
dispersion’s betatron amplitude \( \mathcal{H} = [\eta^2 + (\alpha \eta + \beta \eta')^2] / \beta \)
dispersion \( \eta \), derivative \( \eta' \)
hor. betafunction \( \beta \), \( \alpha = -\beta' / 2 \)
transverse gradient \( k \) \( (k > 0 \) hor. foc.)

**MBA lattice without wigglers**

\[ \varepsilon_{xo}[m \cdot \text{rad}] = \frac{\tilde{C}_q \gamma^2}{12\sqrt{15}} \frac{\Phi^3}{J_x} \cdot F \]

\( F = 1 \Leftrightarrow \text{TME} \)

(theoretical minimum emittance)

**MBA & DW**

need space!
Recall: the TME cell

- Lowest emittance of a conventional lattice cell
  - homogenous (constant $b$), short ($\Phi = bL \ll 1$) bending magnet
  - set $\alpha_0 = \eta'_o = 0$ at bend center (symmetry); find minimum $H(\beta_o, \eta_o)$:
    \[ \Rightarrow \text{theoretical minimum emittance (TME) for} \]
    \[ \beta_o^{\text{TME}} = \frac{L}{2\sqrt{15}} \quad \eta_o^{\text{TME}} = \frac{\Phi L}{24} \rightarrow F = 1 \rightarrow \varepsilon_{xo}^{\text{TME}} [\text{m} \cdot \text{rad}] = \frac{\tilde{C}_q}{12\sqrt{15}} \gamma^2 \]

- Periodic symmetric cell:
  $\alpha = \eta' = 0$ at ends
  \[ \Rightarrow \text{matching problem} \]
  \[ \mu^{\text{TME}} = 284.5^\circ \]

- 2nd focus, useless
- long cell
- overstrained optics
- Deviations from TME conditions

\[ F = \frac{\varepsilon_{xo}}{\varepsilon_{xo}^{\text{TME}}} \quad r = \frac{\beta_o}{\beta_o^{\text{TME}}} \quad d = \frac{\eta_o}{\eta_o^{\text{TME}}} \]

- Ellipse equations for emittance

\[ \frac{5}{4} (d - 1)^2 + (r - F)^2 = F^2 - 1 \]

- Cell phase advance

\[ \tan \frac{\mu}{2} = \frac{6}{\sqrt{15}} \frac{r}{(d - 3)} \]

- Real cells: \( \mu < 180^\circ \Rightarrow F \sim 3.6 \)

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How to get \( F < 1 \) and \( \mu < 180^\circ \)?

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The LGAB cell

- Detuned TME cell vs. longitudinal-gradient/anti-bend cell
  - both: angle 6.7°, $E = 2.4$ GeV, $L = 2.36$ m, $\Delta \mu_x = 160°$, $\Delta \mu_y = 90°$, $J_x \approx 1$

**TME**: $F = 3.4$, $\varepsilon = 990$ pm

**LGAB**: $F = 0.69$, $\varepsilon = 200$ pm

- $\beta_x$, $\beta_y$, $\eta$
- dipole field, quad field, total field
- at $R = 13$ mm

\[\text{longitudinal gradient bend} \quad \text{anti-bend}\]
Longitudinal gradient bends

\[ I_5 = \int_L \left| b(s) \right|^3 \mathcal{H}(s) \, ds \quad b(s) = B(s)/(p/e) \]

\[ \mathcal{H} = \eta^2 + (\alpha \eta + \beta \eta')^2 / \beta \]

- Longitudinal field variation \( b(s) \) to compensate \( \mathcal{H}(s) \) variation

- Beam dynamics in bending magnet
  - Curvature is source of dispersion: \( \eta''(s) = b(s) \rightarrow \eta'(s) \rightarrow \eta(s) \)
  - Horizontal optics \( \sim \) like drift space: \( \beta(s) = \beta_0 - 2\alpha_0 s + \frac{1+\alpha_0^2}{\beta_0} s^2 \)
  - Assumptions: no transverse gradient \((k = 0)\); rectangular geometry

- Variational problem: find extremal of \( \eta(s) \) for
  \[ I_5 = \int_L f(s, \eta, \eta', \eta'') \, ds \rightarrow \min \text{ with functional } f = \mathcal{H}(s, \eta, \eta', \eta'') \mid \eta''' \mid^3 \]
  - too complicated to solve
    - mixed products up to \( \eta^{(4)} \) in Euler-Poisson equation...

\rightarrow use special function \( b(s) = f(s, \{ a_k \}) \) with parameters \( \{ a_k \} \):
  - variational problem \( \rightarrow \) minimization problem for \( \{ a_k \} \)

\rightarrow numerical optimization: find extremal; suggest functions \( f \)
**Half** bend in $N$ slices: curvature $b_i$, length $\Delta s_i$

Knobs for minimizer: 
$\{b_i\}, \beta_0, \eta_0$

Objective: $I_5$ ( or $\varepsilon \sim I_5 / I_2$ )

Constraints:
- length: $\Sigma \Delta s_i = L / 2$
- angle: $\Sigma b_i \Delta s_i = \Phi / 2$
- [ field: $b_i < b_{\text{max}}$ ]
- [ optics: $\beta_0, \eta_0$ ]

Results:
- hyperbolic field variation
  (for symmetric field, dispersion suppressor bend is different)
- $I_5 / I_5^{\text{hom}} = 0.34, \ I_2 / I_2^{\text{hom}} = 2.5 \rightarrow \varepsilon / \varepsilon^{\text{hom}} = 0.13$
- Trend: $b_0 \rightarrow \infty, \ \beta_0 \rightarrow 0, \ \eta_0 \rightarrow 0$
Analytical optimization

- Given function \( b(s) = f(s, \{a_k\}) \), parameters \( \{a_k\} \)
  - \( \partial I_5/\partial \beta_0, \partial I_5/\partial \eta_0 = 0 \) → emittance & matching
  - \( \{\partial I_5/\partial a_k\} = 0 \) → optimum parameters

- Useful simple functions for field profiles:
  - high field magnets: \( \textbf{hyperbola} \) \( b(s) = \frac{b_0}{(1 + hs)^p} \)
    (parameters \( h, p \))
    → superbends
    → hard X-ray photons from field peak!
  - low field magnets: \( \textbf{step function} \)
    \( b(s) = \begin{cases} b_0 & \text{for } 0 < s < m \\ b_1 & \text{for } m < s < L/2 \end{cases} \)
    (parameter \( m \))
    → most simple design

numerical optimization results
Numerical optimization of field profile for fixed $\beta_0$, $\eta_0$

- Emittance ($F$) vs. $\beta_0$, $\eta_0$ normalized to data for TME of hom. bend

Small (~0) dispersion at centre required, but tolerant to large beta function
Anti-bends

- General problem of dispersion matching:
  - dispersion production in dipoles → “defocusing”: $\eta'' > 0$
- Quadrupoles in conventional cell:
  - dispersion is horizontal trajectory: quads treat $\eta$ and $\beta_x$ in same way.
  - over-focusing of horizontal beta function $\beta_x$
  - insufficient focusing of dispersion $\eta$
  - striking example: the TME cell
  → disentangle $\eta$ and $\beta_x$!
- use negative dipole: anti-bend
  - kick $\Delta\eta' = \psi$, angle $\psi < 0$
  - out of phase with main dipole
  - negligible effect on $\beta_x$, $\beta_y$
- Side effects on emittance:
  - main dipole angle increase by $2|\psi|$
  - anti-bend located at large $H$
  → in total, still lower emittance

relaxed TME cell, 5°, 2.4 GeV, $J_x \approx 2$
Emittance: 500 pm / 200 pm
Half quad anti-bend

- Recall: emittance reduction via $I_4$
  → get $\approx$ half emittance

  $$I_4 = \int b \eta (b^2 + 2k) \, ds \rightarrow -I_2 \Rightarrow J_x = 1 - \frac{I_4}{I_2} \rightarrow 2$$

- $2k \gg b^2, \quad \eta > 0$
  → $b > 0, \quad k < 0$
  defocusing gradient bend

  → $b < 0, \quad k > 0$
  focusing gradient anti-bend

- need horizontal focusing at anti-bend location anyway
  (out of phase with main bend).

- convenient magnet design:
  anti-bend = half quadrupole
Plans for an upgrade of the Swiss Light Source (SLS)
- SLS emittance now: **5500 pm**

M. Ehrlichman, *First studies on a possible SLS upgrade*, Wednesday 10:10

SLS constraints:
288 m, 12 straights, 2.4 GeV → rather **compact** lattice!

**→ LGAB-HMBA lattice**
*hybrid multibend achromat incorporating longitudinal gradient bends and anti-bends*

- **100 – 200 pm** emittance: factor **50 – 25** improvement.
- hard X-rays (100 keV) from LG-superbend field peak.
a) most aggressive design

- ultra-low emittance: $\varepsilon = 73$ pm! (\(\approx 18\) m / 30° arc at 2.4 GeV)
- \(\approx\) feasible magnets, \(\approx\) sufficient dynamic aperture
- quasi isochronous (MCF $\alpha = -5 \cdot 10^{-5}$) and nonlinear
- too short bunches, insufficient energy acceptance
- large normalized chromaticities $-\xi/Q = 3.9 / 4.3$
b) compromise design

- Acceptable emittance: \( \varepsilon = 183 \text{ pm} \)
- \( \approx \) feasible magnets, \( \approx \) sufficient dynamic aperture
- Large MCF (\( \alpha = +1.3 \cdot 10^{-4} \) ) \( \Rightarrow \) bunch length & E-acceptance
- Large normalized chromaticities \( -\xi/Q = 4.1 / 6.5 \)
- Only partial exploitation of LGAB scheme
c) negative alpha design

- Acceptable emittance: $\varepsilon = 162$ pm
- Large negative MCF ($\alpha = -1.0 \cdot 10^{-4}$)
- Low normalized chromaticities $-\xi/Q = 2.0 / 2.9$
- Full exploitation of LGAB scheme: relaxed focusing.

→ work in progress...
Conclusions

- **Longitudinal gradient bends** ...
  - ... provide lower emittance than the TME for homogenous bends.
  - ... offer the double use to provide low emittance and hard X-rays.
  - ... can be described well by hyperbolae (high field) or step functions (low field).
  - ... require very small dispersion at focus,
  - ... but tolerate large values of horizontal beta function at focus.

- **Anti-bends** ...
  - ... disentangle dispersion and horizontal beta function,
  - ... are thus well suited to provide the matching for LG bends.
  - ... introduce negative momentum compaction.

- **The LGAB cell** ...
  - ... combines longitudinal gradient bends and anti-bends.
  - ... offers a lattice solution for compact low emittance rings.