Longitudinal gradient super-bends and anti-bends for compact low emittance light source lattices

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References

- AS & Albin Wrulich, *Compact low emittance light sources based on longitudinal gradient bending magnets*, submitted to NIM A
- AS, *The anti-bend cell for ultralow emittance storage rings*, NIM A 737 (2014) 148-154

Recall: paths to low emittance

$$\varepsilon_{xo}[\mathrm{m}\cdot\mathrm{rad}] = \tilde{C}_q \gamma^2 \frac{I_5}{I_2 - I_4}$$

$$\sigma_\delta^2 = \tilde{C}_q \gamma^2 \frac{I_3}{2 I_2 + I_4}$$

$$\Delta E\,[\mathrm{keV}] = \tilde{C}_\gamma \gamma^4 I_2$$

Equilibrium beam parameters of a flat lattice

ε_{xo} natural horizontal emittance

σ_δ rms relative momentum spread, $\delta = \Delta p/p$

ΔE energy loss per turn

I_2 I_3 I_4 I_5 synchrotron radiation integrals

$\tilde{C}_q = 3.83\cdot 10^{-13}\,\mathrm{m}$ $\tilde{C}_\gamma = 9.60\cdot 10^{-13}\,\mathrm{keV}$ constants

$$\varepsilon_{xo}[\mathrm{m\cdot rad}] = \tilde{C}_q\,\gamma^2 \frac{I_5}{I_2 - I_4} \qquad \sigma_\delta^2 = \tilde{C}_q\,\gamma^2 \frac{I_3}{2I_2 + I_4} \qquad \Delta E\,[\mathrm{keV}] = \tilde{C}_\gamma\,\gamma^4 I_2$$

Optics

$$I_5 = \int |b|^3\,\mathcal{H}\,ds \rightarrow \min$$

➔ horizontal focus in each dipole

➔ many small dipoles of angle $\Phi << 1$
multibend achromat (MBA) lattice

orbit curvature $b = 1/\rho = B/(p/e)$
dispersion's betatron amplitude
$\mathcal{H} = [\eta^2 + (\alpha\eta + \beta\eta')^2]/\beta$
dispersion η, derivative η'
hor. betafunction β, $\alpha = -\beta'/2$
transverse gradient k ($k > 0$ hor. foc.)

MBA lattice without wigglers

$$\varepsilon_{xo}[\mathrm{m\cdot rad}] = \frac{\tilde{C}_q\,\gamma^2}{12\sqrt{15}} \frac{\Phi^3}{J_x} \cdot F$$

$F = 1 \Leftrightarrow$ TME
(theoretical minimum emittance)

Power

$$I_2 = \int b^2\,ds \rightarrow \max\ldots \qquad I_3 = \int |b|^3\,ds$$

➔ ***damping wigglers (DW)***

Damping

$$I_4 = \int b\eta(b^2 + 2k)\,ds \rightarrow -I_2 \Rightarrow J_x = 1 - \frac{I_4}{I_2} \rightarrow 2$$

➔ ***gradient bends*** for vertical focusing ($bk < 0$)

MBA & DW need space !

Recall: the TME cell

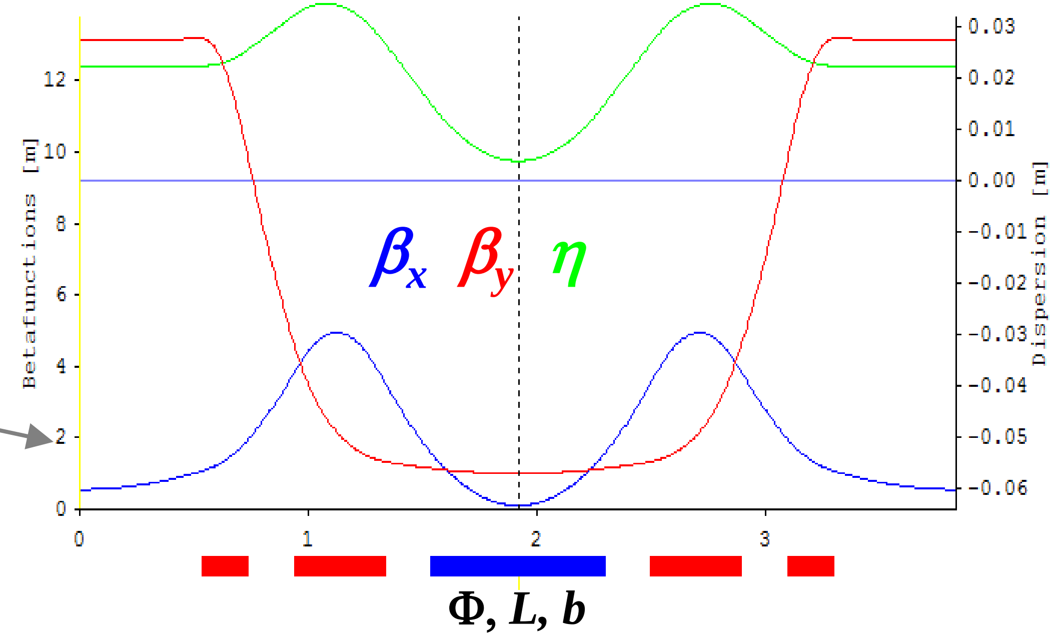
- Lowest emittance of a conventional lattice cell
  - homogenous (constant b), short ($\Phi = bL \ll 1$) bending magnet
  - set $\alpha_o = \eta_o' = 0$ at bend center (symmetry); find minimum $\mathcal{H}(\beta_o, \eta_o)$:

⇨ ***theoretical minimum emittance*** (TME) for

$$\beta_o^{\mathrm{TME}} = \frac{L}{2\sqrt{15}} \qquad \eta_o^{\mathrm{TME}} = \frac{\Phi L}{24} \rightarrow F = 1 \rightarrow \varepsilon_{xo}^{\mathrm{TME}}[\mathrm{m \cdot rad}] = \frac{\tilde{C}_q}{12\sqrt{15}} \gamma^2 \frac{\Phi^3}{J_x}$$

- periodic symmetric cell:
  $\alpha = \eta' = 0$ at ends
  matching problem

⇨ TME phase advance
$\mu^{\mathrm{TME}} = 284.5^\circ$
  - 2nd focus, useless
  - long cell
  - overstrained optics



- Deviations from TME conditions

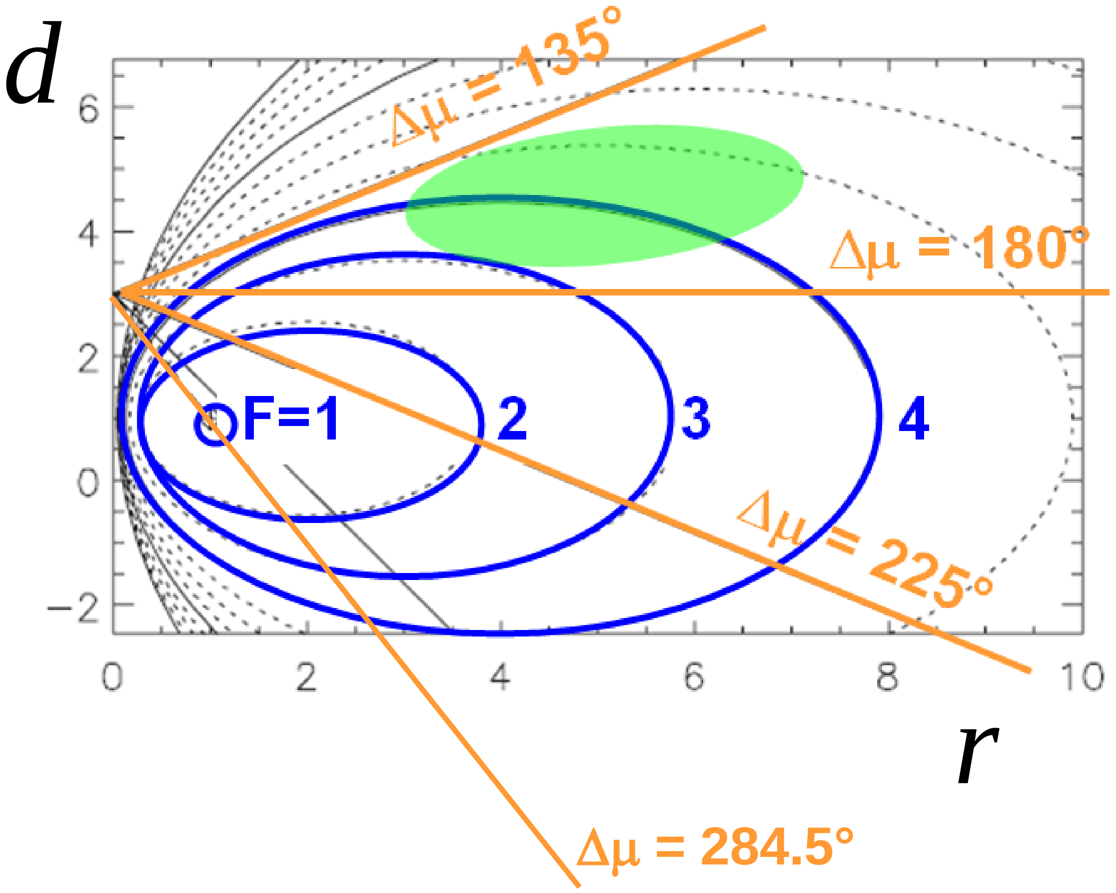
$$F = \frac{\varepsilon_{xo}}{\varepsilon_{xo}^{\mathrm{TME}}} \quad r = \frac{\beta_o}{\beta_o^{\mathrm{TME}}} \quad d = \frac{\eta_o}{\eta_o^{\mathrm{TME}}}$$

- Ellipse equations for emittance

$$\tfrac{5}{4}(d-1)^2 + (r-F)^2 = F^2 - 1$$

- Cell phase advance

$$\tan\frac{\mu}{2} = \frac{6}{\sqrt{15}}\,\frac{r}{(d-3)}$$



- real cells: $\mu < 180°$ ⇨ $F \sim 3..6$

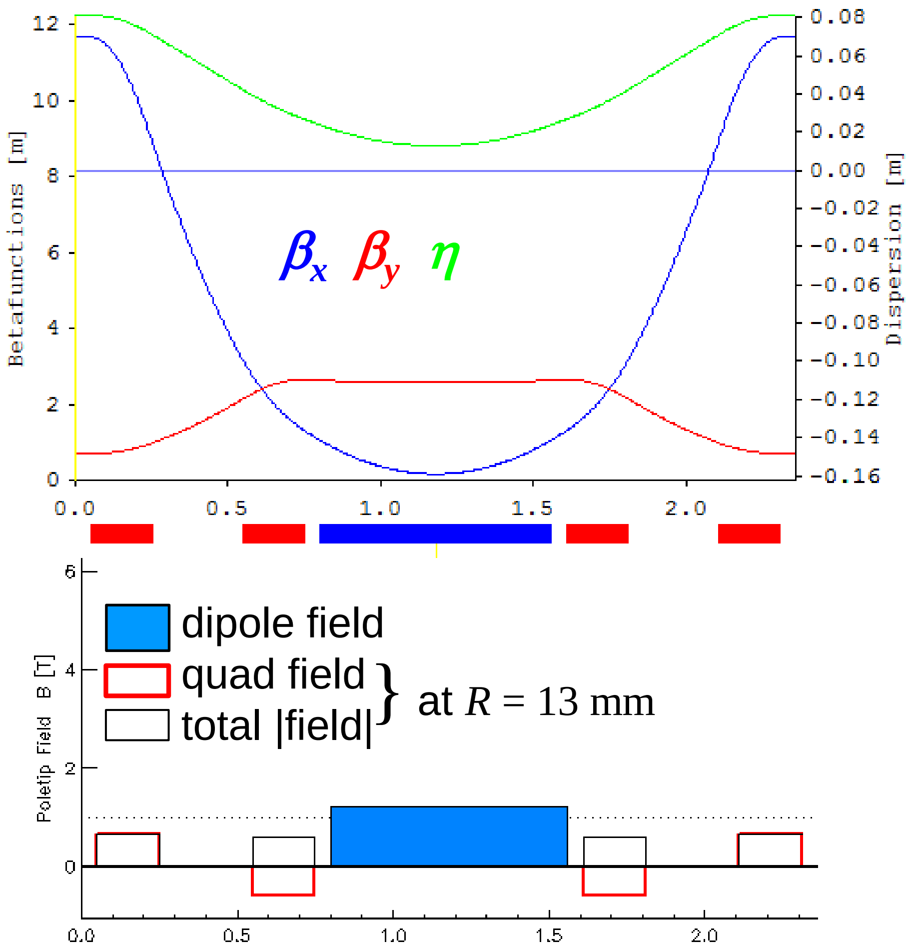
How to get $F < 1$ ***and*** $\mu < 180°$?

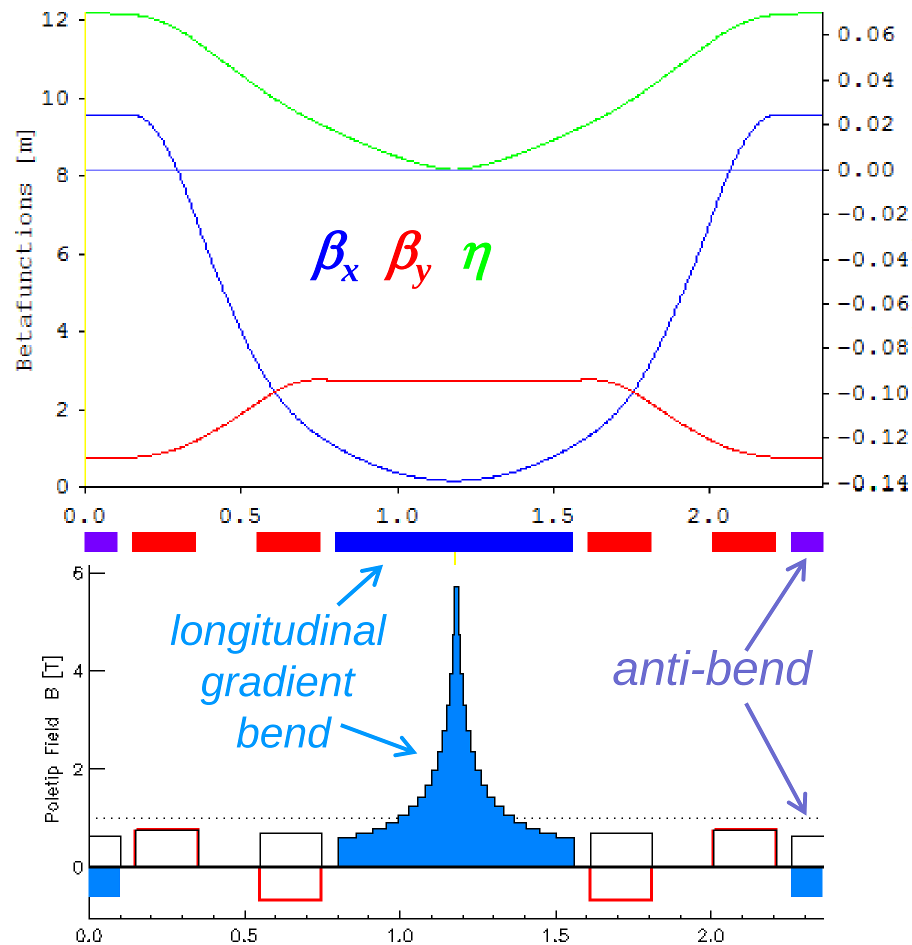
S. Leemann & AS, *Perspectives for future light source lattices incorporating yet uncommon magnets,* PR ST AB, 14, 030701, 844 (2011).

The LGAB cell

- Detuned TME cell vs. longitudinal-gradient/anti-bend cell
  - both: angle 6.7°, E = 2.4 GeV, L = 2.36 m, $\Delta\mu_x = 160°$, $\Delta\mu_y = 90°$, $J_x \approx 1$

TME: F = 3.4, ε = 990 pm **LGAB: F = 0.69, ε = 200 pm**





Longitudinal gradient bends

$$I_5 = \int_L |b(s)|^3 \mathcal{H}(s)\, ds \qquad b(s) = B(s)/(p/e) \qquad \mathcal{H} = \frac{\eta^2 + (\alpha\eta + \beta\eta')^2}{\beta}$$

- Longitudinal field variation $b(s)$ to compensate $\mathcal{H}(s)$ variation
- Beam dynamics in bending magnet
  - Curvature is source of dispersion: $\eta''(s) = b(s) \to \eta'(s) \to \eta(s)$
  - Horizontal optics ~ like drift space: $\beta(s) = \beta_0 - 2\alpha_0 s + \frac{1+\alpha_0^2}{\beta_0} s^2$
  - Assumptions: no transverse gradient ($k = 0$); rectangular geometry
- Variational problem: find extremal of $\eta(s)$ for

  $I_5 = \int_L f(s,\eta,\eta',\eta'')\, ds \to \min$ with functional $f = \mathcal{H}(s,\eta,\eta',\eta'')\,|\eta''|^3$
  - too complicated to solve
    - mixed products up to $\eta^{(4)}$ in Euler-Poisson equation...

→ use special function $b(s) = f(s,\{a_k\})$ with parameters $\{a_k\}$:

variational problem → minimization problem for $\{a_k\}$

→ numerical optimization: find extremal; suggest functions f

Numerical optimization

- *Half* bend in N slices: curvature b_i, length Δs_i
- Knobs for minimizer: $\{b_i\}$, β_0, η_0
- Objective: I_5 (or $\varepsilon \sim I_5 / I_2$)
- Constraints:
  - length: $\Sigma \Delta s_i = L/2$
  - angle: $\Sigma b_i \Delta s_i = \Phi/2$
  - [field: $b_i < b_{max}$]
  - [optics: β_0 , η_0]
- Results:
  - hyperbolic field variation (for symmetric bend, dispersion suppressor bend is different)
  - $I_5 / I_5^{hom} = 0.34, \quad I_2 / I_2^{hom} = 2.5 \ \rightarrow \ \varepsilon / \varepsilon^{hom} = 0.13$
  - Trend: $b_0 \rightarrow \infty$, $\beta_0 \rightarrow 0$, $\eta_0 \rightarrow 0$

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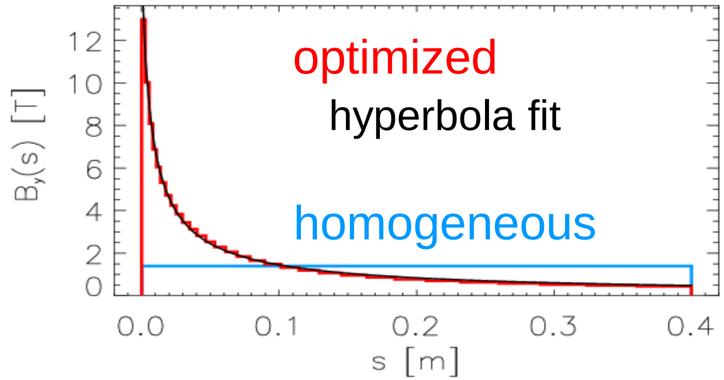
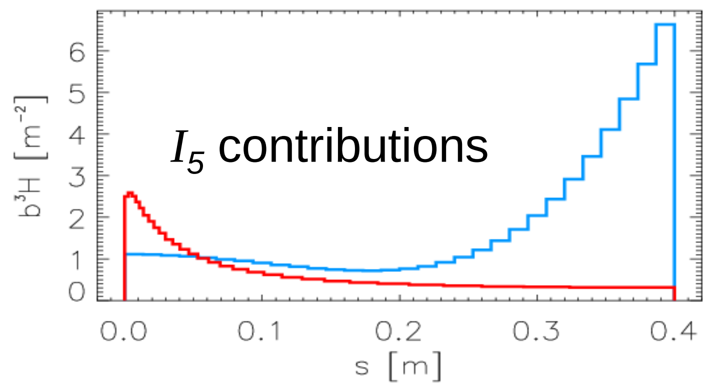
Results for *half* symmetric bend

($L = 0.8$ m, $\Phi = 8°$, 2.4 GeV)



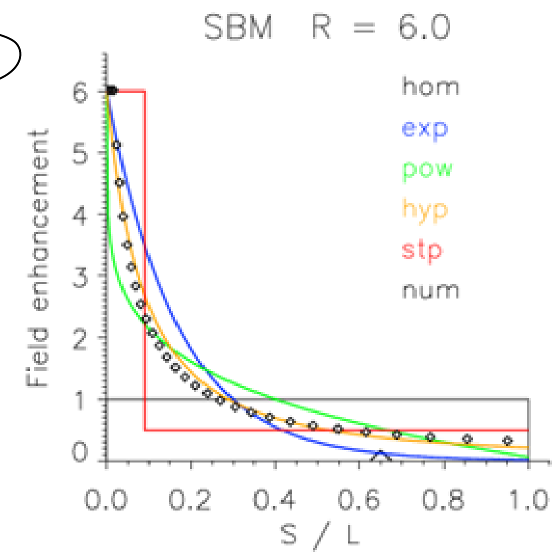
SBM

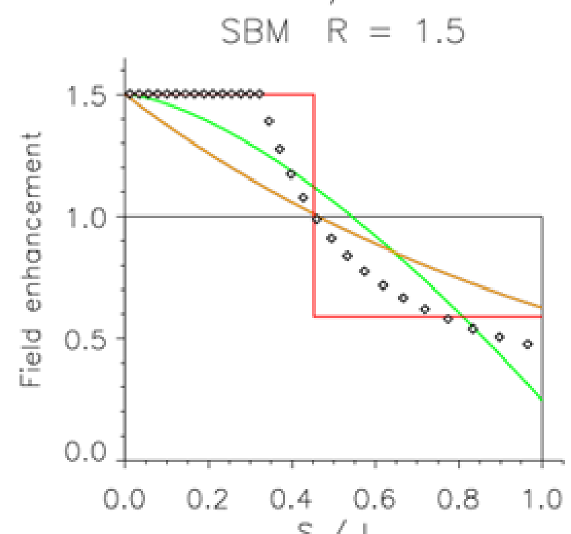


Analytical optimization

- Given function $b(s) = f(s,\{a_k\})$, parameters $\{a_k\}$
  - $\partial I_5/\partial\beta_0,\ \partial I_5/\partial\eta_0 = 0 \ \rightarrow$ emittance & matching
  - $\{\partial I_5/\partial a_k\} = 0 \ \rightarrow$ optimum parameters
- Useful simple functions for field profiles:
  - high field magnets:
    hyperbola
    (parameters h, p)
    $$b(s) = \frac{b_0}{(1+hs)^p}$$
    $\rightarrow$ superbends
    $\rightarrow$ hard X-ray photons from field peak!
  - low field magnets:
    step function
    (parameter m)
    $$b(s) = \begin{cases} b_0 & \text{for } 0 < s < m \\ b_1 & \text{for } m < s < L/2 \end{cases}$$
    $\rightarrow$ most simple design

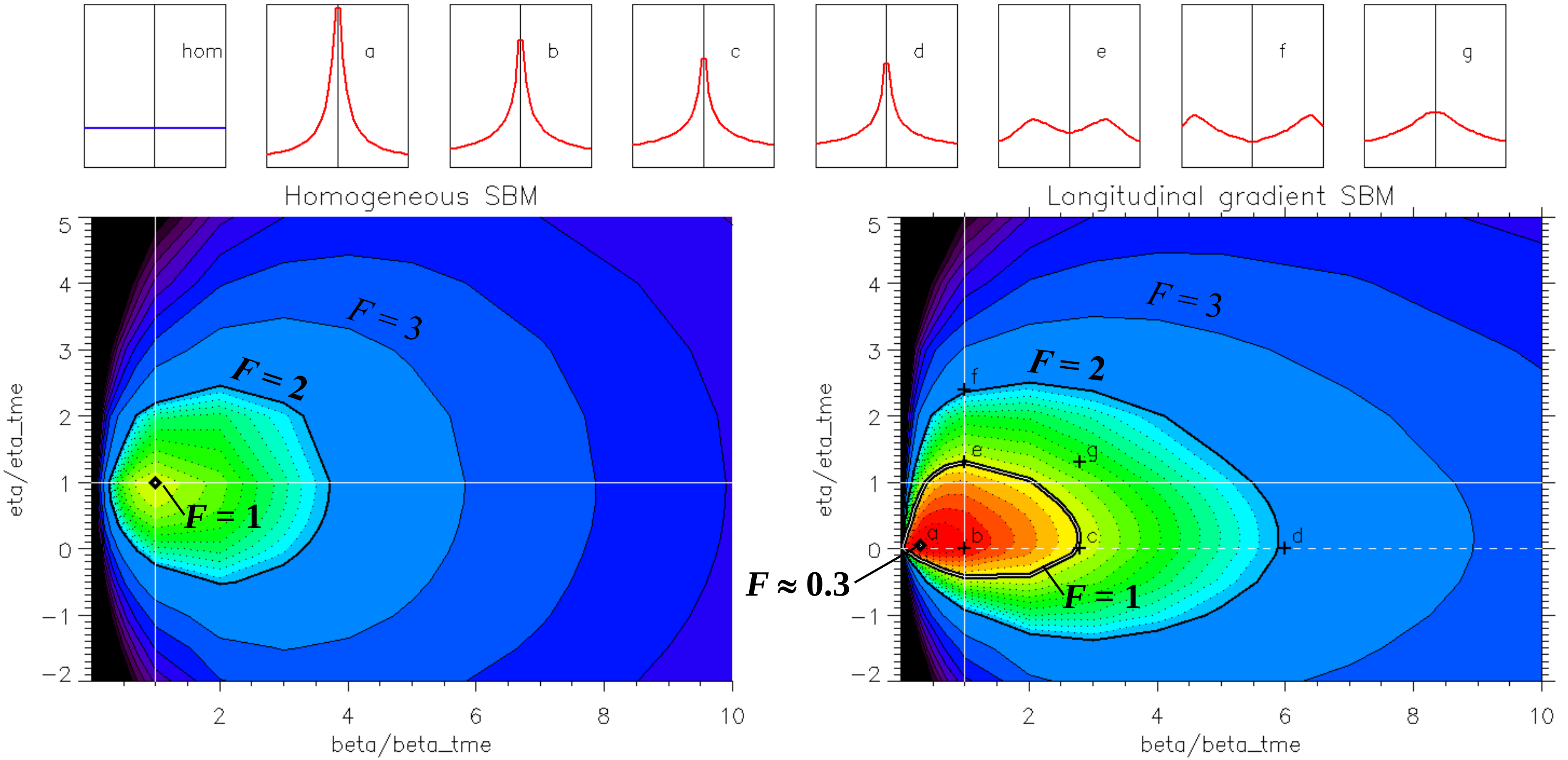
$\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond$ numerical optimization results





Deviations from optimum matching

- Numerical optimization of field profile for fixed β_0, η_0
  - Emittance (F) vs. β_0, η_0 normalized to data for TME of hom. bend



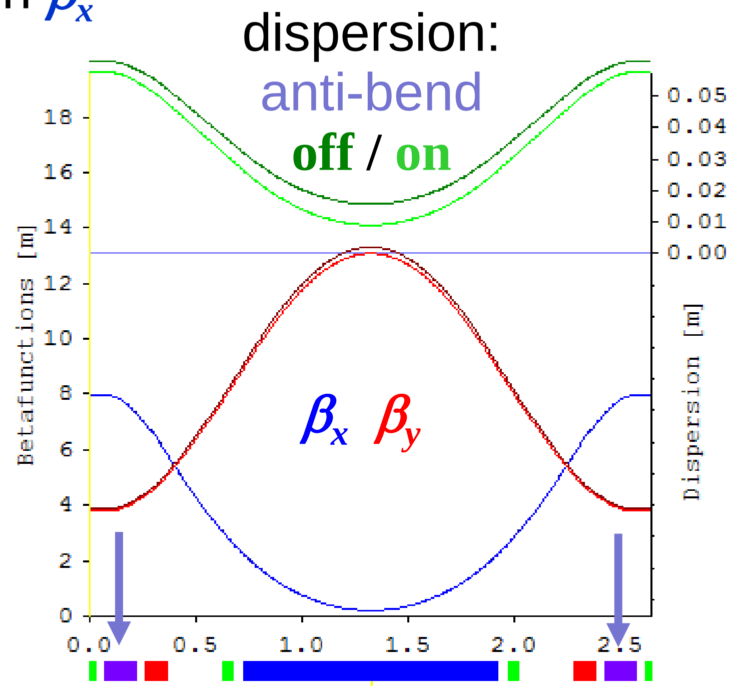
small (~0) dispersion at centre required, but tolerant to large beta function

Anti-bends

- General problem of dispersion matching:
  - dispersion production in dipoles → "defocusing": $\eta'' > 0$
- Quadrupoles in conventional cell:
  - dispersion is horizontal trajectory: quads treat η and β_x in same way.
  - → over-focusing of horizontal beta function β_x
  - → insufficient focusing of dispersion η
  - striking example: the TME cell

→ **disentangle η and β_x !**

- use negative dipole: ***anti-bend***
  - kick $\Delta\eta' = \psi$, angle $\psi < 0$
  - out of phase with main dipole
  - negligible effect on β_x , β_y
- Side effects on emittance:
  - main dipole angle increase by $2|\psi|$
  - anti-bend located at large $\mathcal{H}$
  - → in total, still lower emittance



relaxed TME cell, 5°, 2.4 GeV, $J_x \approx 2$
Emittance: **500 pm** / **200 pm**

Half quad anti-bend

- Recall: emittance reduction via I_4
  → get ≈ half emittance

$$\varepsilon_{xo}[\mathrm{m \cdot rad}] = \tilde{C}_q \gamma^2 \frac{I_5}{I_2 - I_4}$$

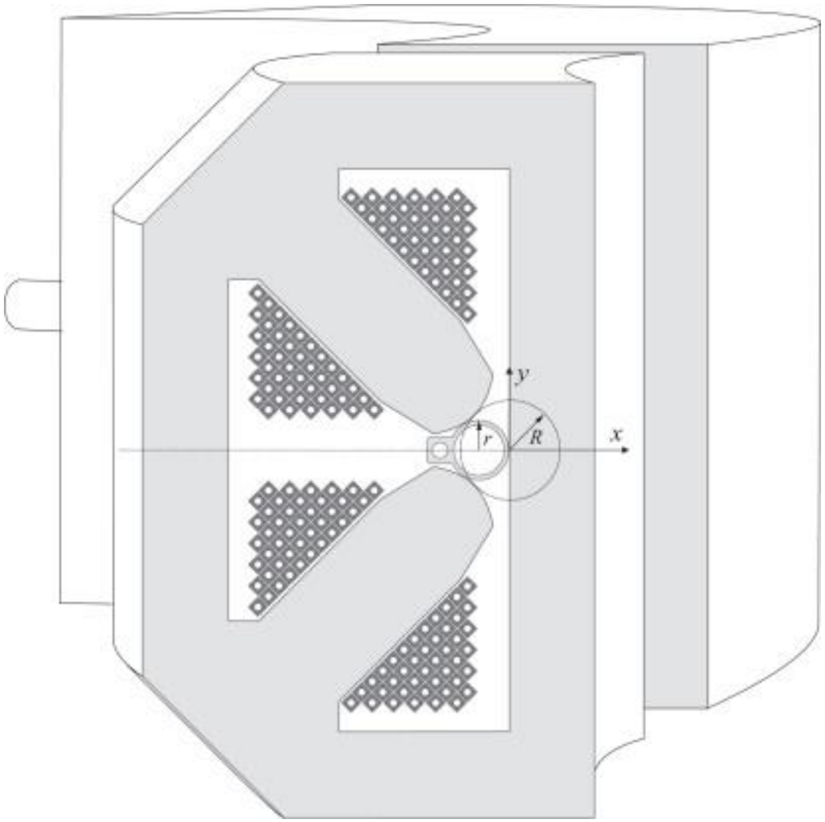
$$I_4 = \int b\eta(b^2 + 2k)\,ds \rightarrow -I_2 \Rightarrow J_x = 1 - \frac{I_4}{I_2} \rightarrow 2$$

- $2k >> b^2, \quad \eta > 0$

  → $b > 0, \; k < 0$
  defocusing gradient bend

  → $b < 0, \; k > 0$
  focusing gradient anti-bend

- need horizontal focusing
  at anti-bend location anyway
  (out of phase with main bend).
- convenient magnet design:
  anti-bend = half quadrupole



Application to SLS upgrade

- Plans for an upgrade of the Swiss Light Source (SLS)
  - SLS emittance now: **5500 pm**

M. Ehrlichman, *First studies on a possible SLS upgrade,* Wednesday 10:10

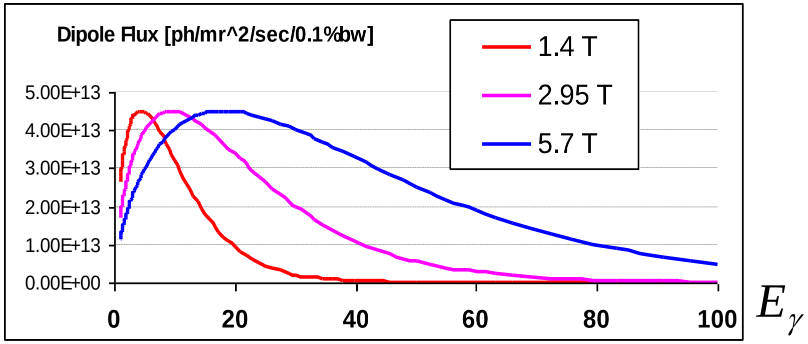
- SLS constraints:
  288 m, 12 straights, 2.4 GeV → rather ***compact*** lattice !

→ LGAB-HMBA lattice

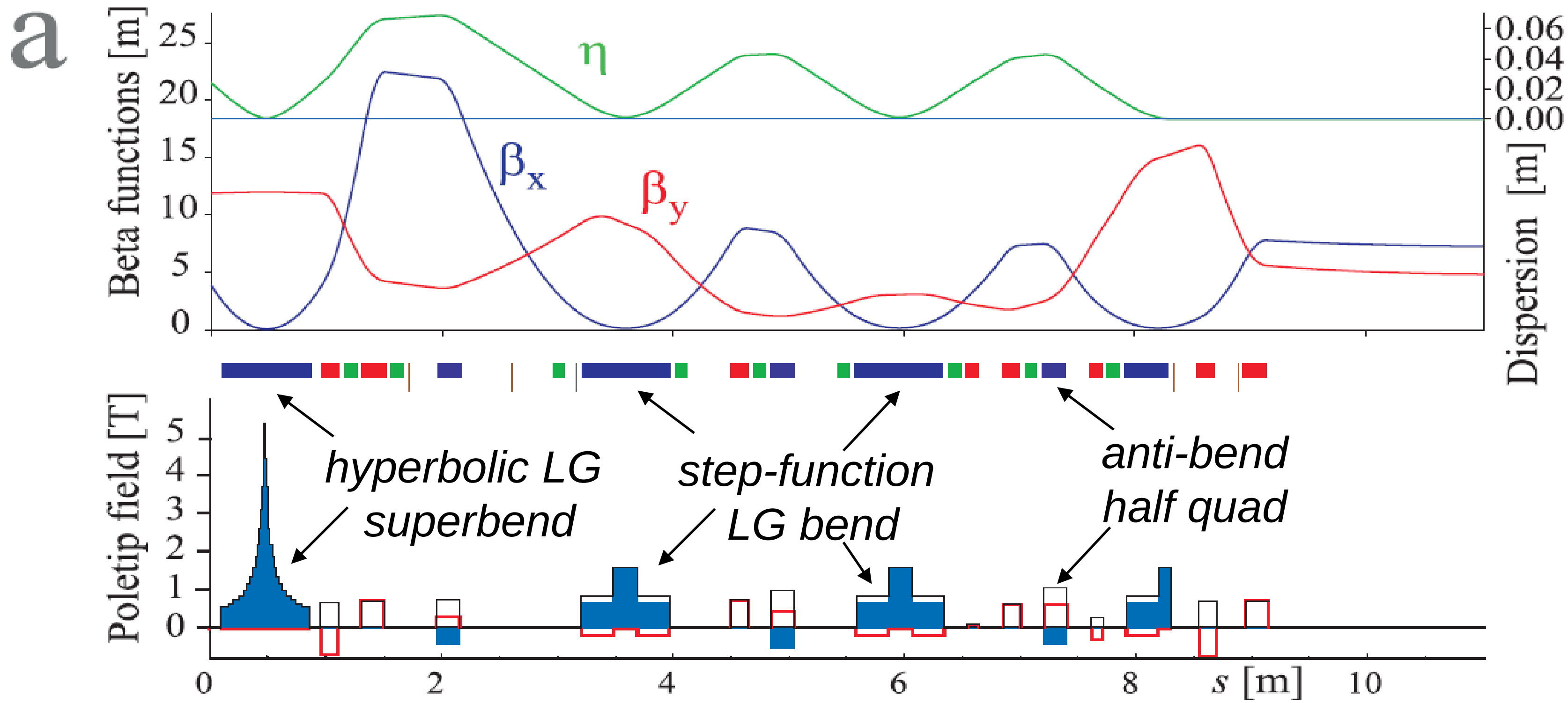
hybrid multibend achromat incorporating longitudinal gradient bends and anti-bends

- **100 – 200 pm** emittance: factor **50 – 25** improvement.
- hard X-rays (100 keV) from LG-superbend field peak.

SLS-1 normal bend
SLS-1 superbend
SLS-2 LG superbend

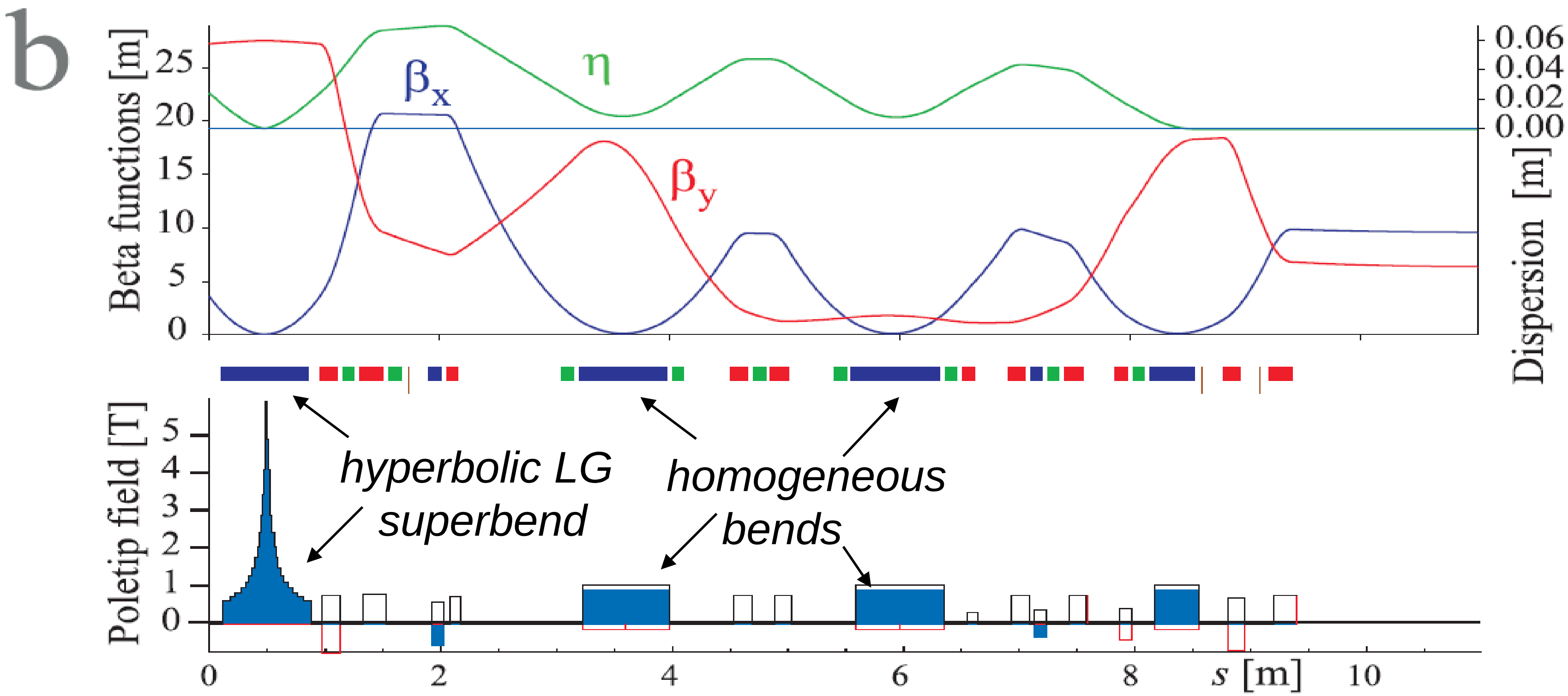


a) most aggressive design



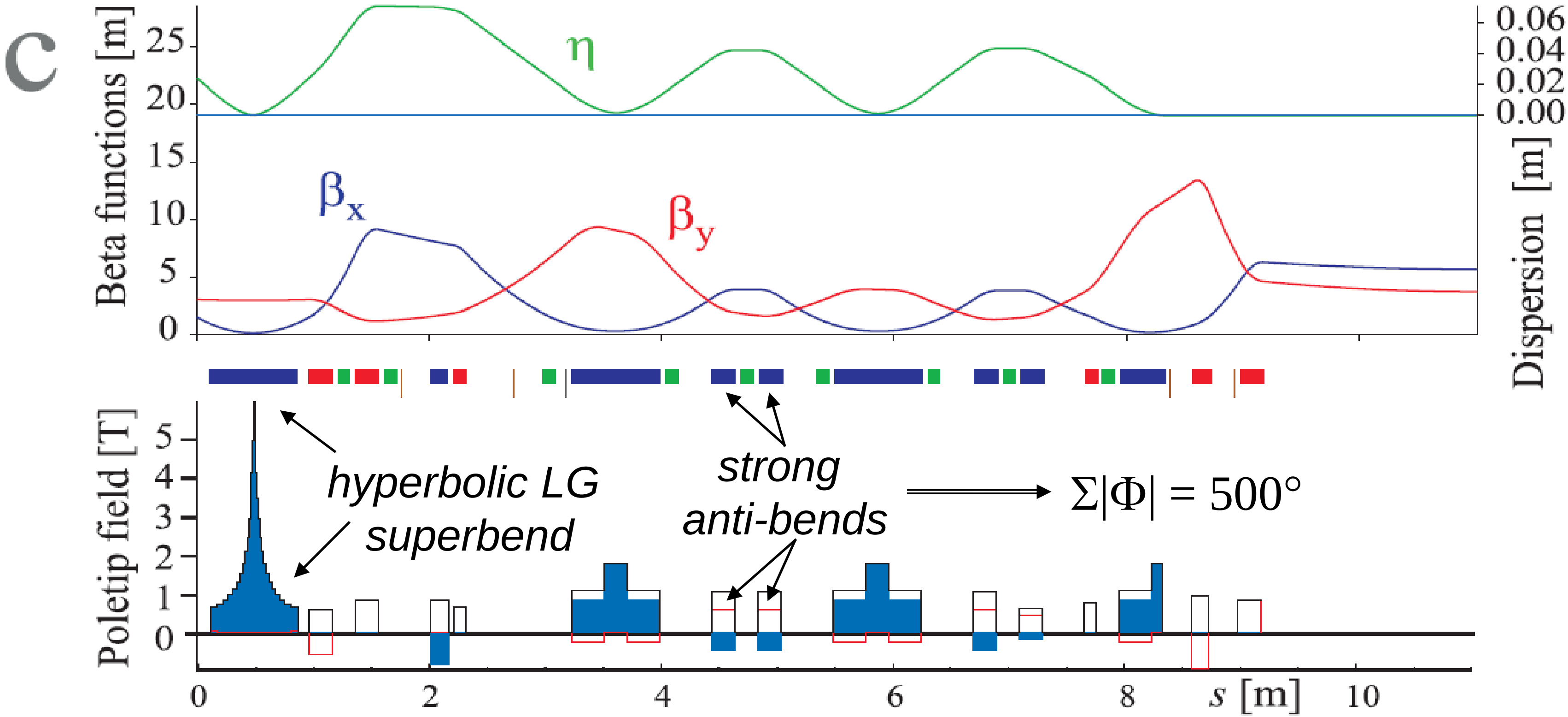
- ☑ ultra-low emittance: ε = **73** pm ! (≈18 m / 30° arc at 2.4 GeV)
- ☑ ≈ feasible magnets, ≈ sufficient dynamic aperture
- ☒ quasi isochronous (MCF $\alpha = -5 \cdot 10^{-5}$) and nonlinear
- ➔ too short bunches, insufficient energy acceptance
- ☒ large normalized chromaticities $-\xi/Q = 3.9$ / 4.3

b) compromise design



- ☑ acceptable emittance: ε = 183 pm
- ☑ ≈ feasible magnets, ≈ sufficient dynamic aperture
- ☑ large MCF ($\alpha = +1.3 \cdot 10^{-4}$) → bunch length & E-acceptance ✓
- ☒ large normalized chromaticities $-\xi/Q$ = 4.1 / 6.5
- ☒ only partial exploitation of LGAB scheme

c) negative alpha design



- ☑ acceptable emittance: ε = 162 pm
- ☑ large *negative* MCF ($\alpha = -1.0 \cdot 10^{-4}$)
- ☑ low normalized chromaticities $-\xi/Q = 2.0 / 2.9$
- ☑ full exploitation of LGAB scheme: relaxed focusing.
- → *work in progress...*

Conclusions

- **Longitudinal gradient bends ...**

  ... provide lower emittance than the TME for homogenous bends.

  ... offer the double use to provide low emittance and hard X-rays.

  ... can be described well by hyperbolae (high field)
  or step functions (low field).

  ... require very small dispersion at focus,

  ... but tolerate large values of horizontal beta function at focus.

- **Anti-bends ...**

  ... disentangle dispersion and horizontal beta function,

  ... are thus well suited to provide the matching for LG bends.

  ... introduce negative momentum compaction.

- **The LGAB cell ...**

  ... combines longitudinal gradient bends and anti-bends.

  ... offers a lattice solution for compact low emittance rings.