Analytical considerations for reducing the emittance with longitudinally variable bends

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Stefania Papadopoulou[*+], Yannis Papaphilippou[*]
[*]CERN, [+] University of Crete

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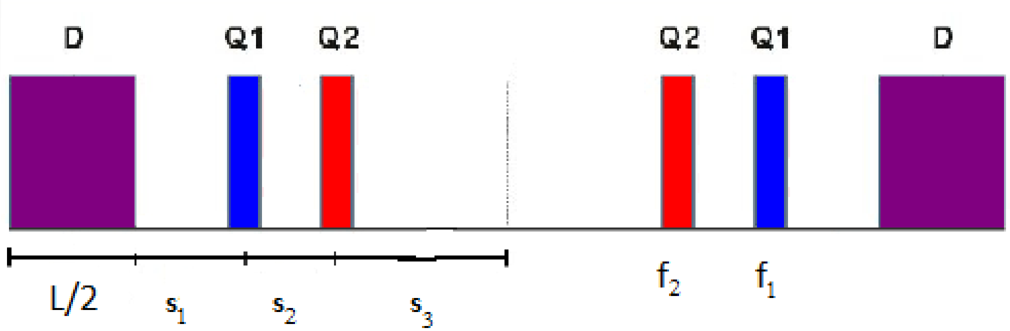
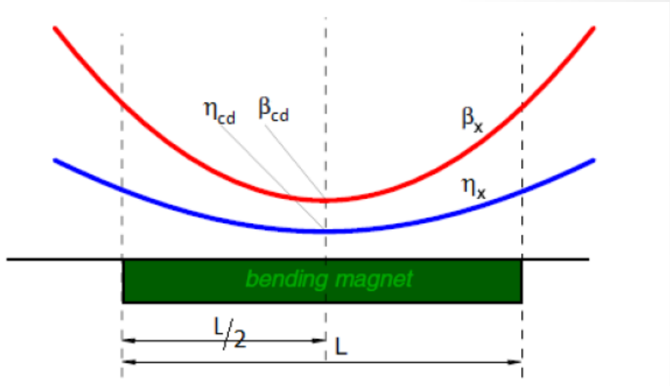
The TME cell



$$L_{cell} = 2(s_1 + s_2 + s_3 + 2l_q) + L$$



The balance between radiation damping and quantum excitation results in the equilibrium betatron emittance. Using a theoretical minimum emittance, TME cell, low emittance values can be achieved. The horizontal emittance of the beam can be generally expressed as:

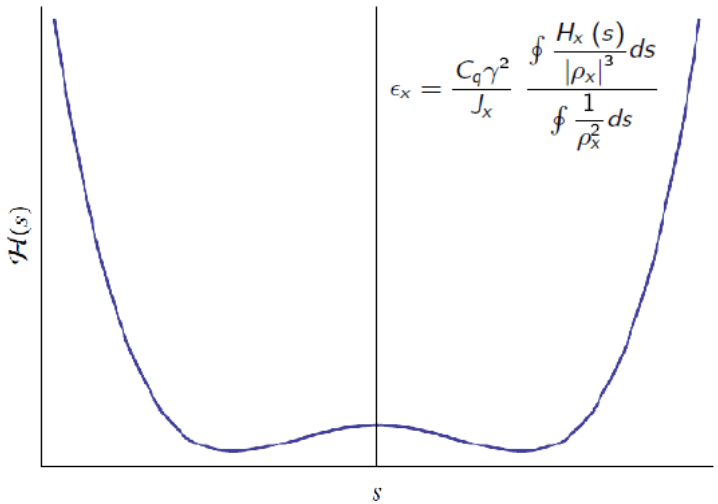
$$\epsilon_x = \frac{C_q \gamma^2}{J_x} \frac{\langle \frac{H_x}{|\rho_x|^3} \rangle}{\langle \frac{1}{\rho_x^2} \rangle} = \frac{C_q \gamma^2}{J_x} \frac{\frac{1}{C}\int_0^C \frac{\mathcal{H}_x}{|\rho_x|^3} ds}{\frac{1}{C}\int_0^C \frac{1}{\rho_x^2} ds}$$

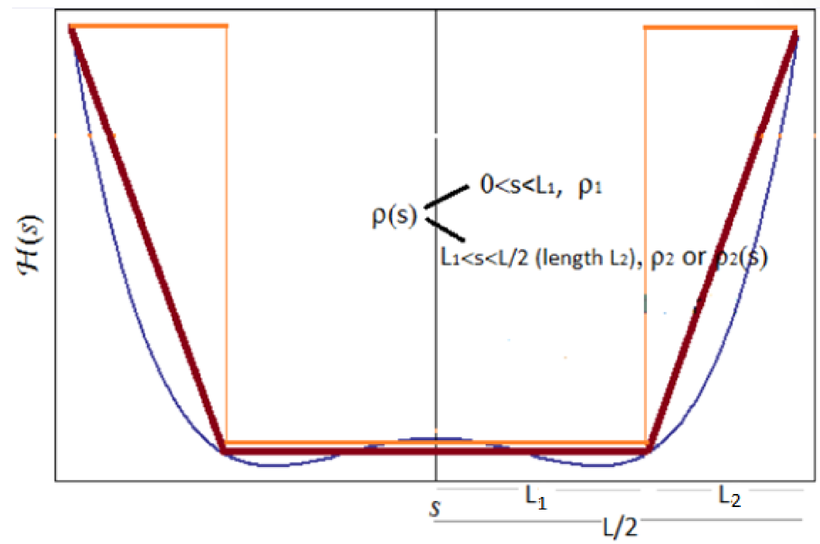
$$\beta(s) = \beta_{cd} - 2\alpha_{cd}s + \gamma_{cd}s^2,\ \alpha(s) = \alpha_{cd} - \gamma_{cd}s,\ \gamma(s) = \gamma_{cd},\ \eta(s) = \eta_{cd} + \eta'_{cd}s + \tilde{\theta}(s),\ \eta'(s) = \eta'_{cd} + \theta(s)$$

$$\mathcal{H}(s) = \gamma(s)\eta(s)^2 + 2\alpha(s)\eta(s)\eta(s)' + \beta(s)\eta(s)'^2$$

Longitudinally variable bends

Approaching the evolution of the uniform dipole's dispersion invariant means approaching its emittance behaviour in order to reduce it. The evolution of the dispersion invariant along the dipole guides the dipole profile choice for the emittance reduction.





Considering only the half dipole for simplicity (from 0 till L/2) as the other is symmetric and then dividing the dipole into two parts of different bending radii can be expressed as:

$$\text{length } L_1 \text{ with bending radius } \rho_1(s),\ 0 < s < L_1$$

$$\text{length } L_2 \text{ with bending radius } \rho_2(s),\ L_1 < s < L_1 + L_2 = L/2$$

Bending angle of half dipole: $\theta = \int_0^{L_1} \frac{1}{\rho_1(s)}ds + \int_{L_1}^{L_1+L_2} \frac{1}{\rho_2(s)}ds$

References: J. Guo and T. Raubenheimer, (EPAC02), Y.Papaphilippou, P. Elleaume, PAC'05, R. Nagaoka, A.F. Wrulich, (NIM A575, 2007), C.-x Wang (PRST-AB, 2009)

Dipole profiles

Bending radii ratio $\rho = \frac{\rho_1}{\rho_2}$ Lengths ratio $\lambda = \frac{L_1}{L_2}$ $(\rho < 1 \; as \; \rho_2 > \rho_1 \; and \; \lambda > 0 \; as \; L_1, L_2 > 0)$

Emittance reduction factor

$$F_{TME} = \frac{\epsilon_{TME_{uni}}}{\epsilon_{TME_{var}}}$$

$$F_{TME} > 1$$

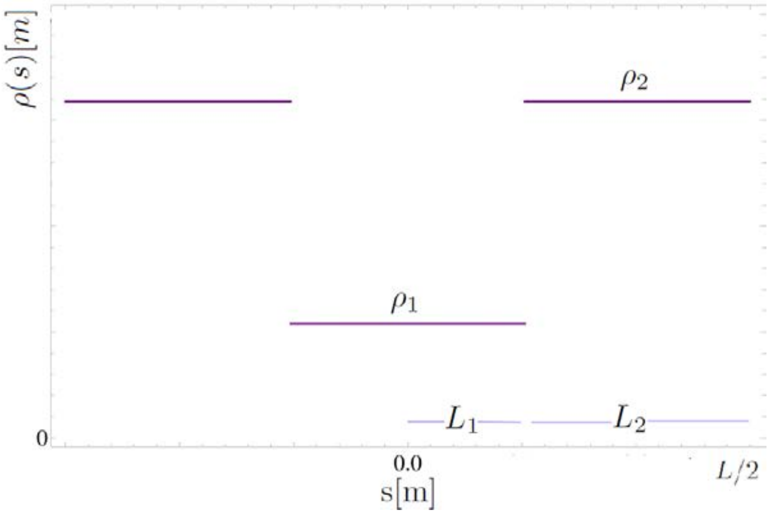
F_{TME} depends only on ρ and λ as the bending radii of the uniform and of the chosen profile are the same (as well as their length) and thus are simplified .

If the dipole's characteristics are not fixed F_{TME} is a function of ρ and λ. Contour-plots that give this dependence will be shown for each dipole profile studied.

Fixing the dipole's characteristics in accordance to the **CLIC DR constraints** leads to the dependence of F_{TME} either on ρ or λ . In this way the highest F_{TME} value for a specific design can be found.

Bending angle of the dipole θ	**Dipole's length L**	**Minimum bending radius ρ_1 (maximum magnetic field 1.8T)**
2π/100	0.6 m	5.4 m

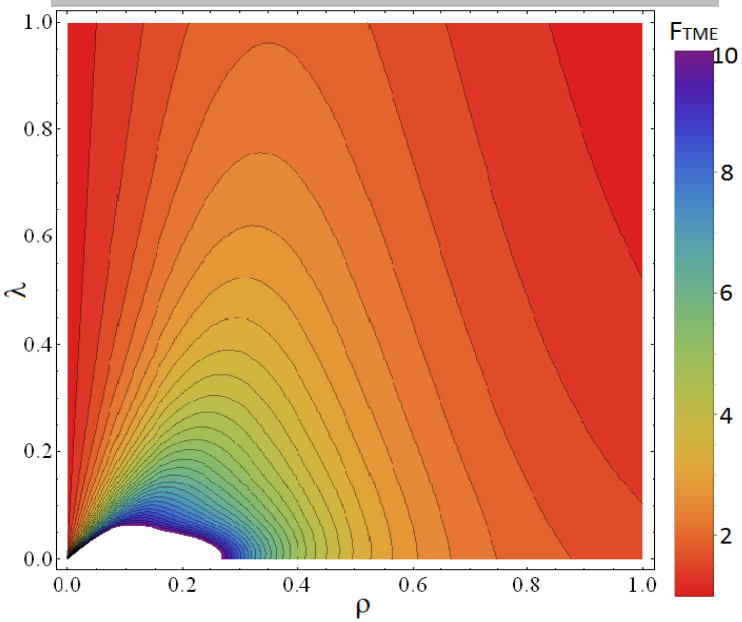
Step profile



$$\rho_2(s) = \rho_2, \quad L_1 < s < L_1 + L_2$$

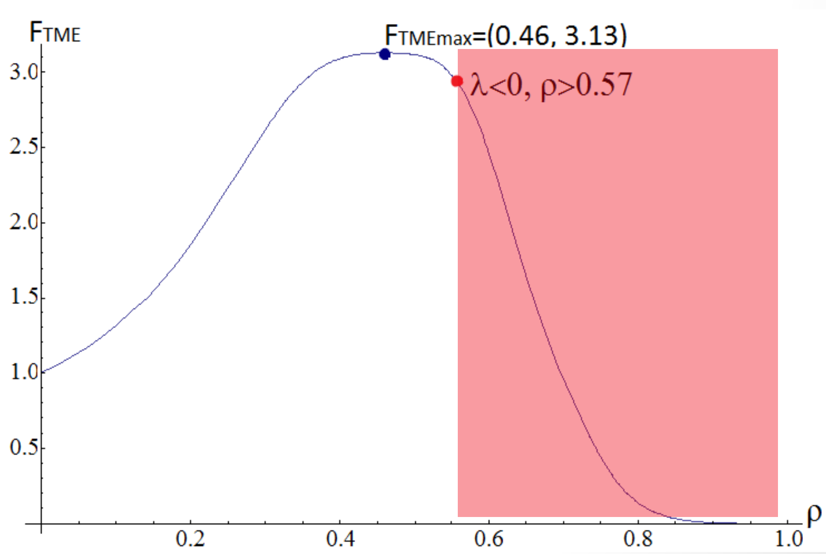
$$\rho_1(s) = \rho_1, \quad 0 < s < L_1$$

General parameterization



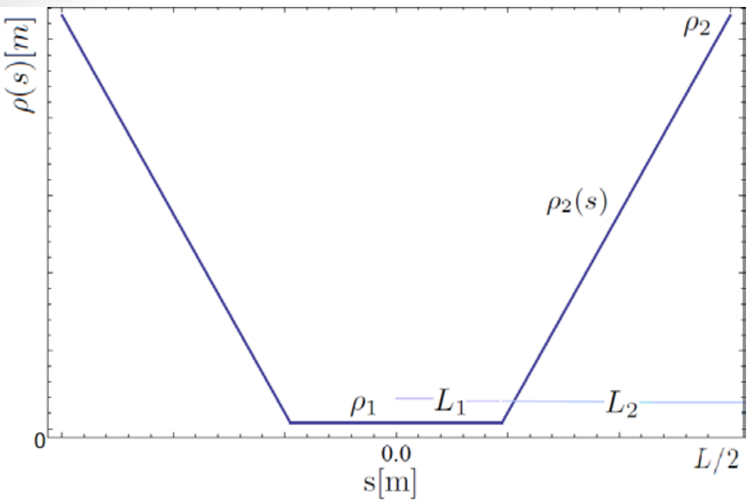
The parameterization of the reduction factor FTME with the bending radii ratio ρ and the lengths ratio λ.

CLIC design constraints



The reduction factor FTME as a function of ρ, when fixing λ according to CLIC design parameters

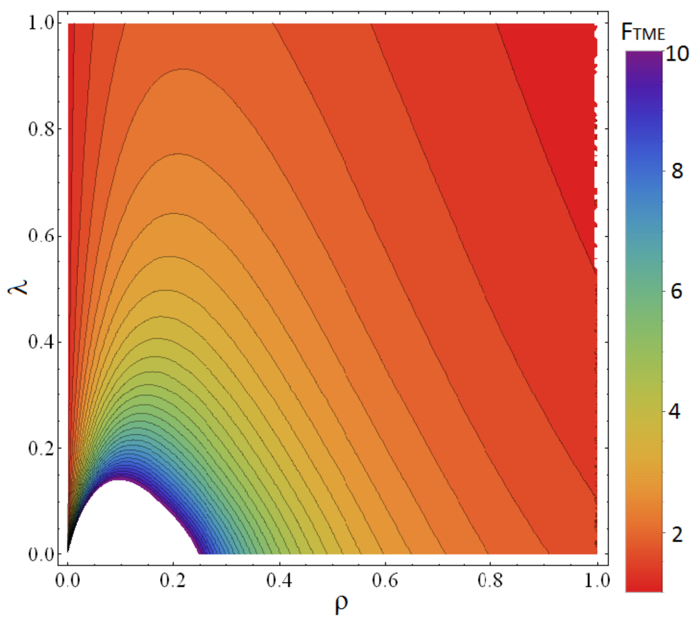
Trapezium profile



$$\rho_2(s) = \rho_1 + (L_1 - s)(\rho_1 - \rho_2)/L_2 \ , \quad L_1 < s < L_1 + L_2$$

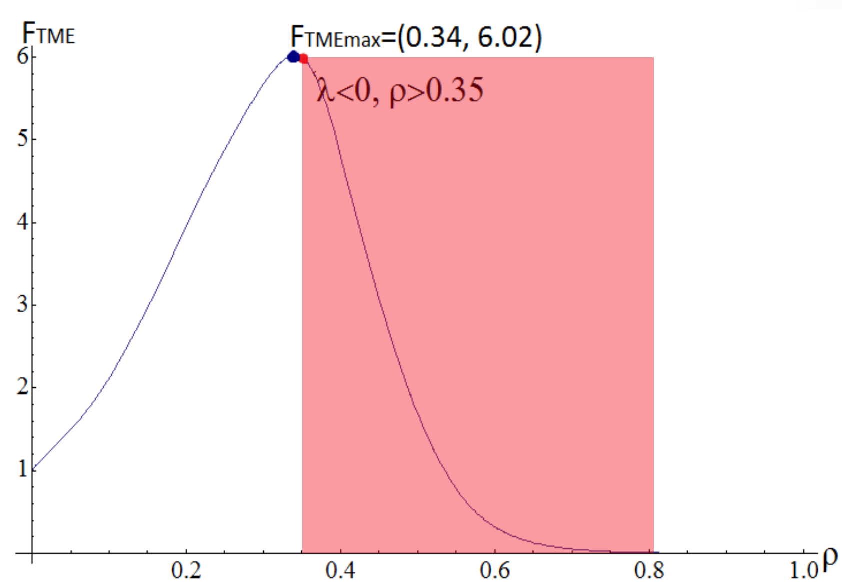
$$\rho_1(s) = \rho_1 \ , \quad 0 < s < L_1$$

General parameterization



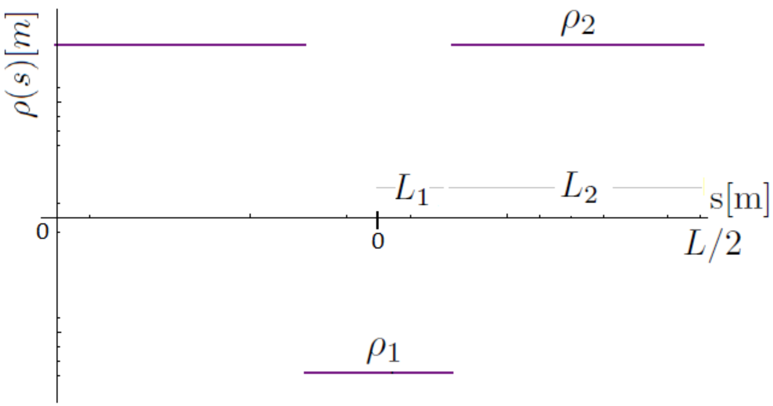
The parameterization of the reduction factor FTME with the bending radii ratio ρ and the lengths ratio λ.

CLIC design constraints



The reduction factor FTME as a function of ρ, when fixing λ according to CLIC design parameters

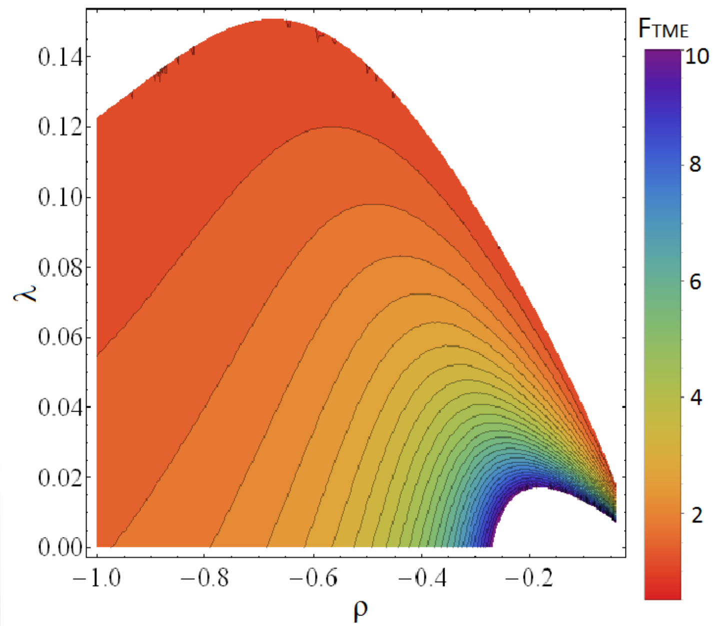
Step (negative bend) profile



$$\rho_2(s) = \rho_2, \quad L_1 < s < L_1 + L_2$$

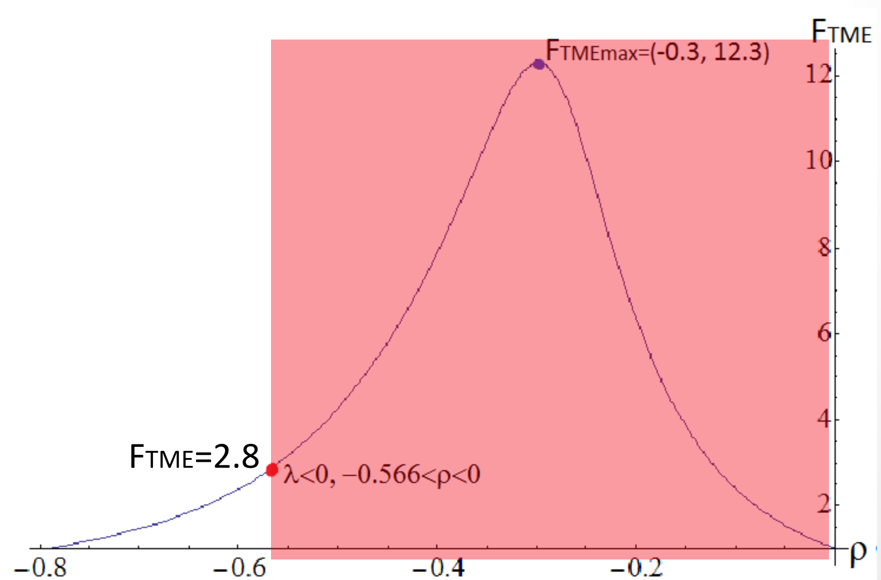
$$\rho_1(s) = -\rho_1, \quad 0 < s < L_1$$

General parameterization



The parameterization of the reduction factor FTME with the bending radii ratio ρ and the lengths ratio λ.

CLIC design constraints

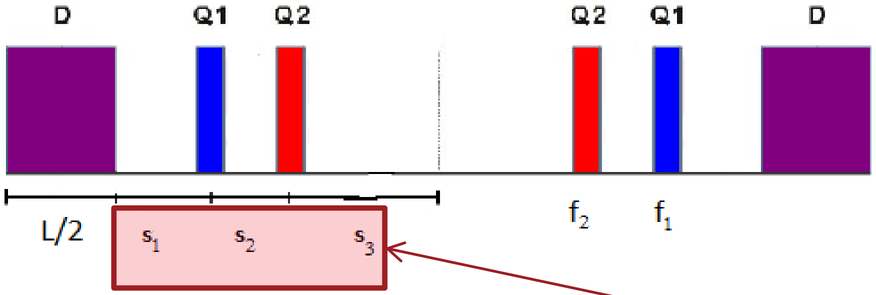


The reduction factor FTME as a function of ρ, when fixing λ according to CLIC design parameters

Analytical parameterization of a variable bend TME cell

Knowing the dipole's characteristics it is important to fix some more parameters in order to produce the numerical results for the CLIC DR lattice design:

- The quadrupoles' length is set to l_q = 0.2m.
- The maximum dipole field is set to 1.8T (minimum bending radius = 5.4m)
- The maximum pole tip field of the quadrupoles and the sextupoles is Bmaxq = 1.1T and Bmaxs = 0.8T respectively.
- The required output normalized emittance for N_d = 100 dipoles is 500nm and the operational energy of the CLIC Damping Rings complex of 2.86 GeV.



Fixing those parameters the free parameters left are the drift space lengths s_1, s_2, s_3 and the emittance. The stability criterion is governing every result and is included in the feasibility constraints:

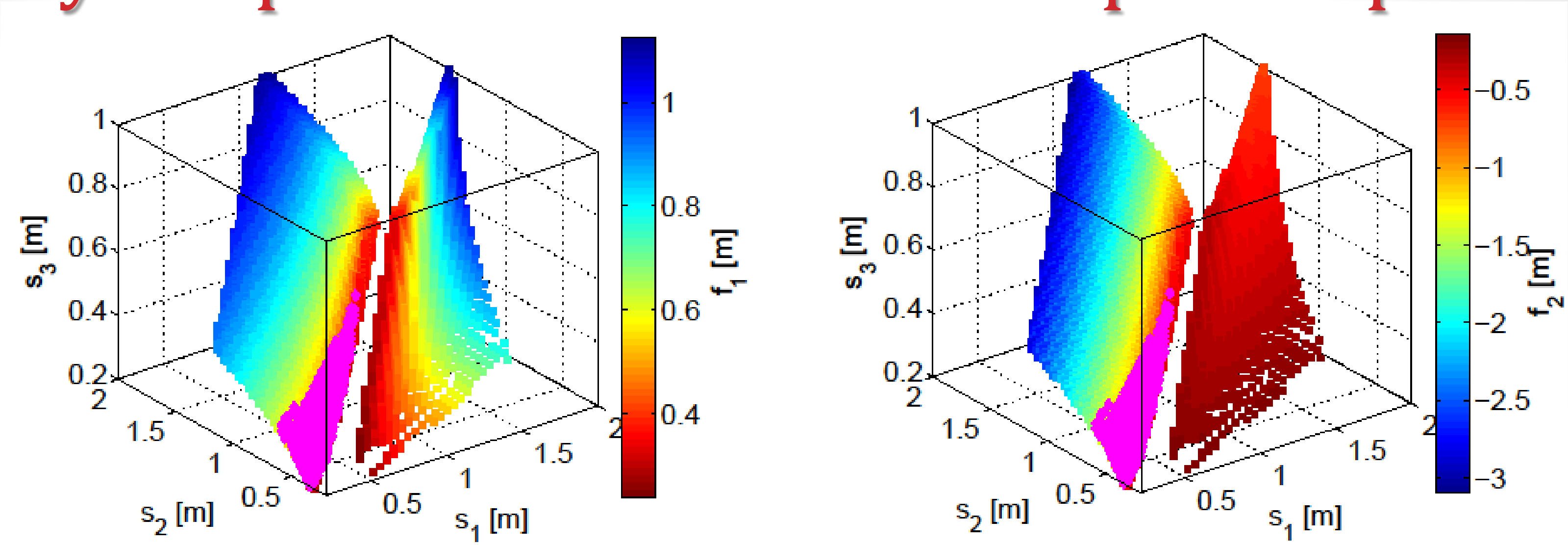
$$|cos\varphi_{x,y}| < 1$$

$$k = \frac{1}{fl_q} =\leq \frac{1}{(B\rho_x)}\frac{B_q^{\max}}{R_{\min}}$$

$$S \leq \frac{2B_s^{\max}}{R_{\min}^2}\frac{1}{(B\rho_x)}$$

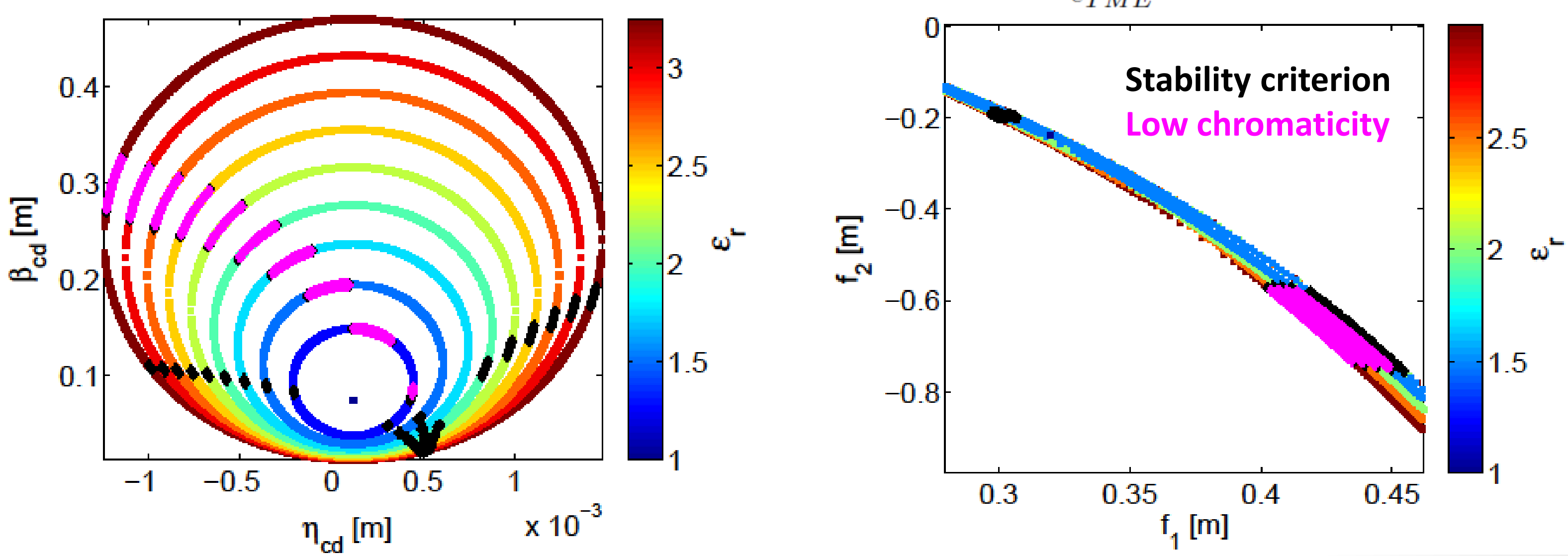
Reference: F. Antoniou and Y. Papaphilippou, *PRSTAB*, **17**, 064002 , 23 June 2014

Analytical parameterization : the trapezium profile



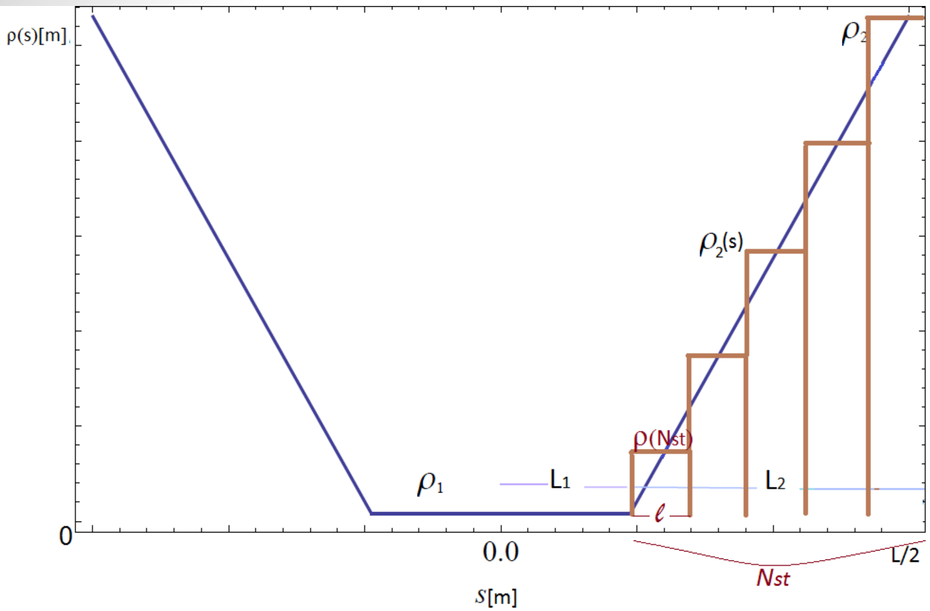
The focal lengths f_1 (Q_1 quadrupole) and f_2 (Q_2 quadrupole) are parameterized with the drift lengths s_1, s_2, s_3

Emittance detuning factor: Emittance deviation from the absolute TME $\epsilon_r = \frac{\epsilon_x}{\epsilon_{TME}}$



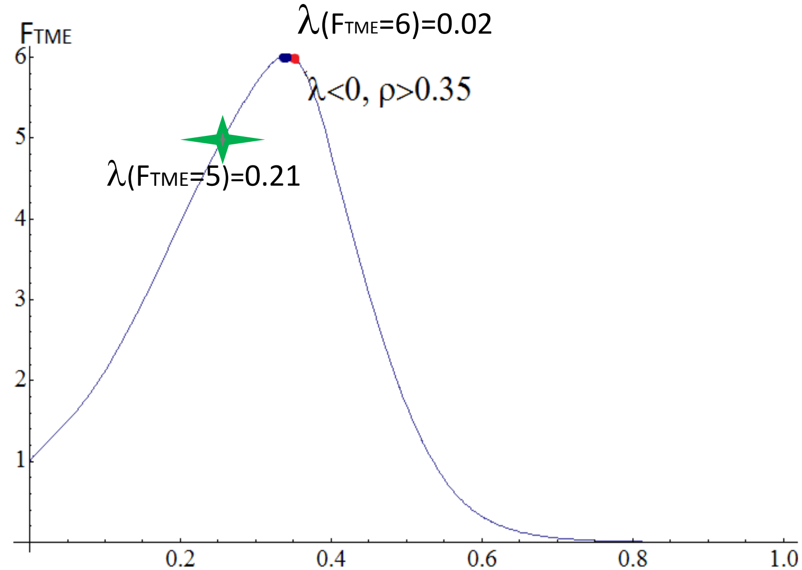
Parameterization of the beta and dispersion functions at the dipole center β_{cd}, η_{cd} and of the focal lengths f_1,f_2 with the detuning factor

Comparison with MADX code



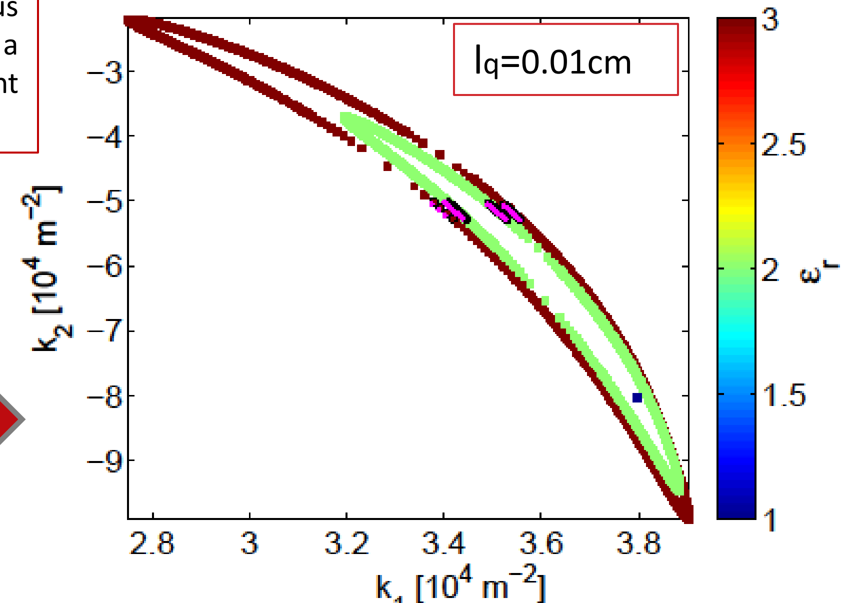
The dipole's bending radius evolution is being approached by a sequence of consecutive dipoles with the same length ℓ (their total number is Nst).

The number of steps used here to approach the bending radius evolution is Nst=10, increasing this number may seems to be a better approximation but actually the improvement is insignificant even when having Nst=100.



In order to get a good agreement with MADX, instead of keeping the maximum reduction (F_{TMEmax}=6.02) a reduction F_{TME}=5 providing λ=0.21 (10 times larger the one for the F_{TMEmax}) is preferred.

The parameterization of the quadrupole strengths k_1, k_2 with the detuning factor is shown for quadrupoles length lq=0.01cm. The black colored solutions assure motion stability, the magenta colored areas give the obtainable MADX stable solutions for detuning factors ε_r=2 and ε_r=3.



Conclusions and next steps

Dipole profiles	Total cell's length Lcell [m]	FTME (CLIC design)	λ (CLIC design)
Step	3.4	3.1	0.25
Trapezium	3.6	5.0	0.21
Step (negative bend)	3.4	2.8	0.003

- All the numerical results have been produced in accordance to the CLIC design parameters, thus the length of the dipole and the bending angle are exactly the same with the design's ones. Also the length of the cell is kept within this limits.
- The highest emittance reduction is given by the trapezium profile, concurrently it provides feasible-low chromaticity solutions for low detuning factors .
- The agreement with the simulation code MADX validates the analytical solutions for the step and the trapezium profile, specially for the thin lens approximation.
- A further improvement of the final emittance values can be achieved when taking into consideration the collective effects, such as the Intrabeam scattering IBS that in the regime of ultralow emittances with high bunch charge has a significant impact on the emittance limits.

Paper in preparation: "Emittance reduction with variable bending magnet strengths: Analytical optics considerations"

Thank you!

Special thanks to F. Antoniou for her valuable help.

Dispersion invariant (1,2 for the individual dipole parts)

$$\mathcal{H}_{1,2}(s) = \gamma_{1,2}{\eta_{1,2}}^2 + 2\alpha_{1,2}\eta_{1,2}\eta'_{1,2} + \beta_{1,2}{\eta'_{1,2}}^2$$

$$(\alpha_{cd} = 0, \eta'_{cd} = 0)$$

Horizontal Emittance

$$\epsilon_x = G\left(\frac{1}{L_1}\int_0^{L_1}\frac{\mathcal{H}_1}{|\rho_1|^3}ds + \frac{1}{L_2}\int_{L_1}^{L_1+L_2}\frac{\mathcal{H}_2}{|\rho_2|^3}ds\right),\ where\ G = \frac{C_q\gamma^2}{J_x}\left(\frac{1}{L_1}\int_0^{L_1}\frac{1}{\rho_1^2}ds + \frac{1}{L_2}\int_{L_1}^{L_1+L_2}\frac{1}{\rho_2^2}ds\right)^{-1}$$

$$\epsilon_x = G\frac{(I_7 + I_8\lambda + (I_1 + I_2\lambda)\beta_{cd}^2 + \eta_{cd}(I_5 + I_6\lambda + (I_3 + I_4\lambda)\eta_{cd}))}{L_1\beta_{cd}}$$

$$I_1 = \int_0^{L_1}\frac{\theta_1^2}{|\rho_1|^3}ds\ 1,\ \ I_2 = \int_{L_1}^{L_1+L_2}\frac{(\theta_2 + \theta_{L_1})^2}{|\rho_2|^3}ds\ ,\ \ I_3 = \int_0^{L_1}\frac{1}{|\rho_1|^3}ds\ ,\ \ I_4 = \int_{L_1}^{L_1+L_2}\frac{1}{|\rho_2|^3}ds\ ,\ \ I_5 = \int_0^{L_1}2\frac{-s\theta_1 + \tilde{\theta}_1}{|\rho_1|^3}ds$$

$$I_6 = \int_{L_1}^{L_1+L_2}2\frac{-s\theta_2 + \tilde{\theta}_2 - L_1\theta_{L_1} + \tilde{\theta}_{L_1}}{|\rho_2|^3}ds\ ,\ \ I_7 = \int_0^{L_1}\frac{(-s\theta_1 + \tilde{\theta}_1)^2}{|\rho_1|^3}ds\ ,\ \ I_8 = \int_{L_1}^{L_1+L_2}\frac{(-s\theta_2 + \tilde{\theta}_2 - L_1\theta_{L_1} + \tilde{\theta}_{L_1})^2}{|\rho_2|^3}ds$$

Beta and dispersion functions for the TME

$$\beta_{TME} = \frac{\sqrt{-I_5^2 - 2I_5I_6\lambda + 4I_3(I_7 + I_8\lambda) + \lambda(-I_6^2\lambda + 4I_4(I_7 + I_8\lambda))}}{2\sqrt{(I_1 + I_2\lambda)(I_3 + I_4\lambda)}}\ \ and\ \ \eta_{TME} = -\frac{I_5 + I_6\lambda}{2(I_3 + I_4\lambda)}$$

$$\epsilon_{TME} = G\frac{(I_1 + I_2\lambda)\sqrt{-I_5^2 - 2I_5I_6\lambda + 4I_3(I_7 + I_8\lambda) + \lambda(-I_6^2\lambda + 4I_4(I_7 + I_8\lambda))}}{L_1\sqrt{(I_1 + I_2\lambda)(I_3 + I_4\lambda)}}$$