# Models for proton-proton scattering at LHC: asymptotic limits, black-disk limit and geometrical scaling

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# Outline

- Definitions
- State of the art : before and after LHC and AUGER
- Moliere,Heisenberg, Froissart : what did they predict?
- QCD : which QCD?
- Minijets: are mini-jets QCD?
- Losing faith: an empirical approach (10’)
- More Empiricism: Geometrical scaling (5’)
- Prospects?

# Definitions and general theorems

- Total cross-section : Optical theorem from conservation of probability, unitarity

$$\sigma_{tot} = 4\pi \Im m \mathcal{A}(s, t=0)$$

- asymptotic behaviour: Froissart bound

$$\sigma_{tot} \lesssim \frac{\pi}{m_\pi^2} (\ln s/s_0)^2$$

# State-of-the-art of total cross-section : before LHC and AUGER

1973
Barger

VOLUME 33, NUMBER 17 PHYSICAL REV

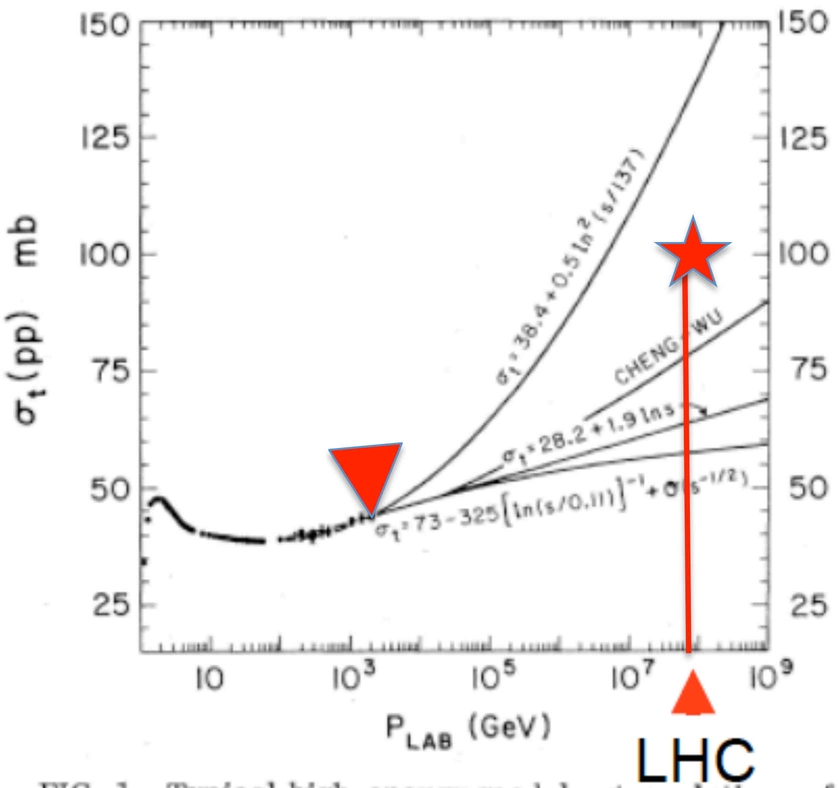
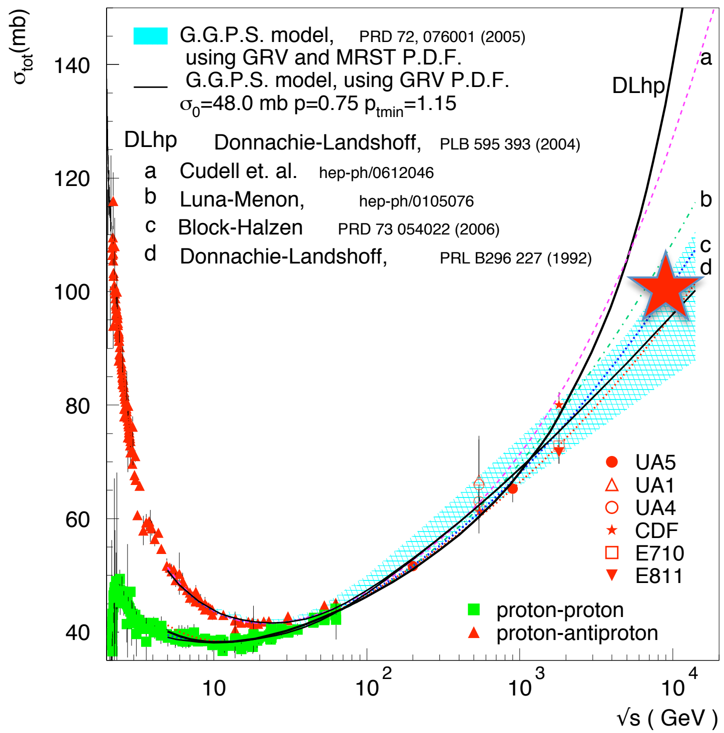



FIG. 1. Typical high-energy model extrapolations of the proton-proton total cross section to the energy range accessible to extensive-air-shower experiments.

2008, PLB, GP etc.



# The total pp cross-section now:

$$\sqrt{s} \sim (0.002 - 57)\ TeV$$

5 decades in energy!
~ 50 years of measurements

# The total cross-section after LHC and AUGER

- Past ISR energies

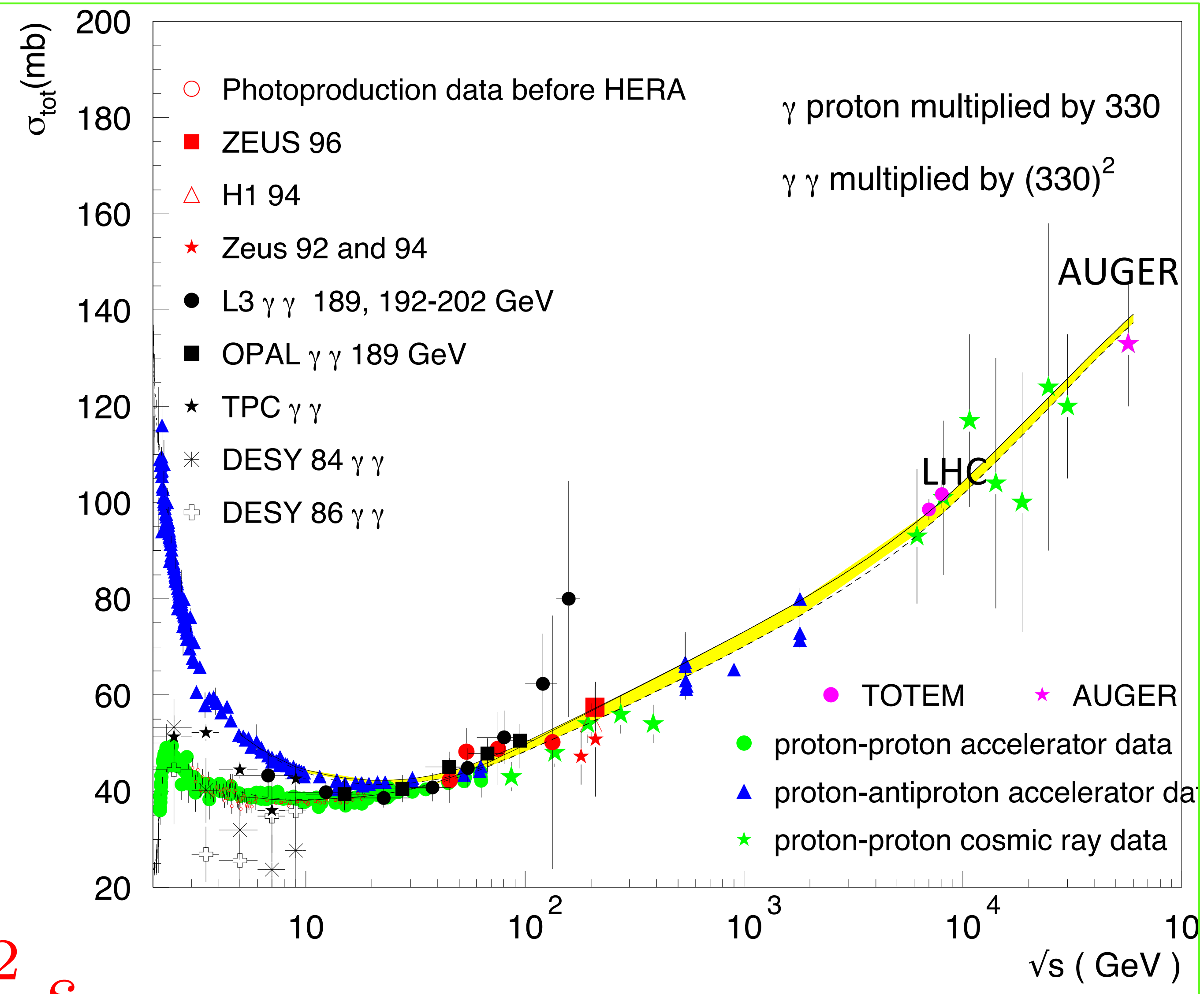
all total cross-sections rise

- The rise is fast at

the beginning

- General theorems

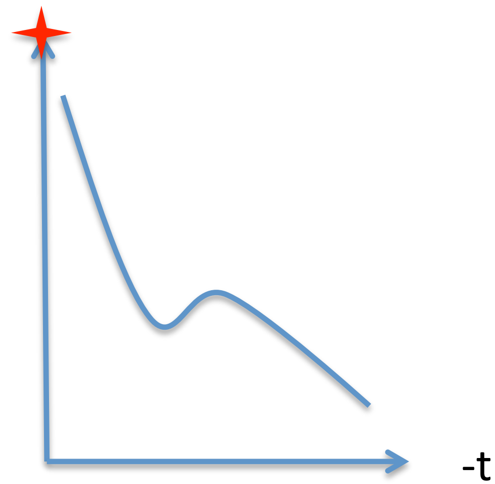
restrict the rate at which the cross-section rises




$$\sigma_{total} \lesssim \log^2 s$$

# The measurements

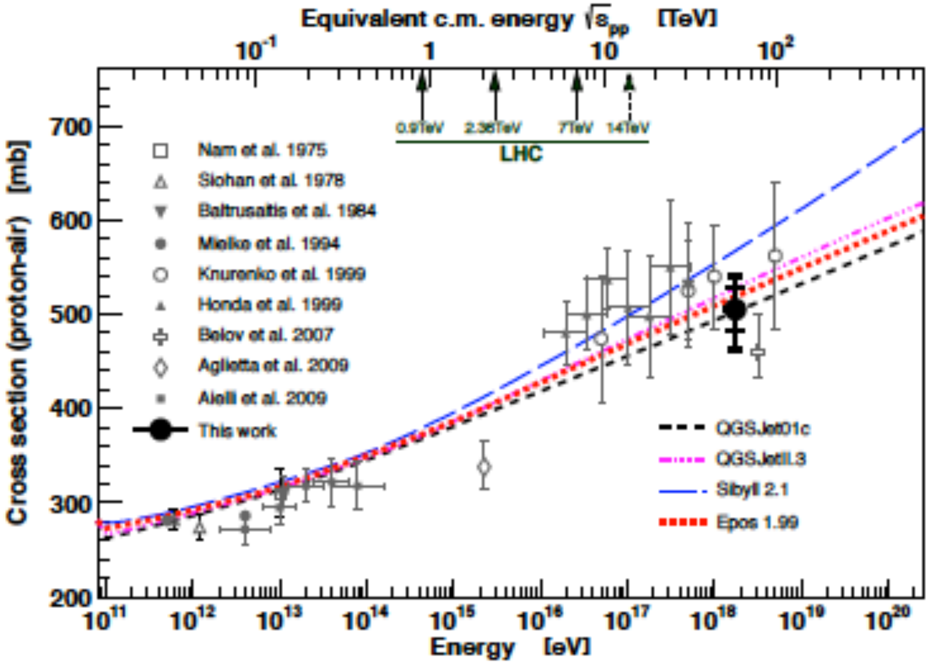
- LHC (and after ISR) : optical point

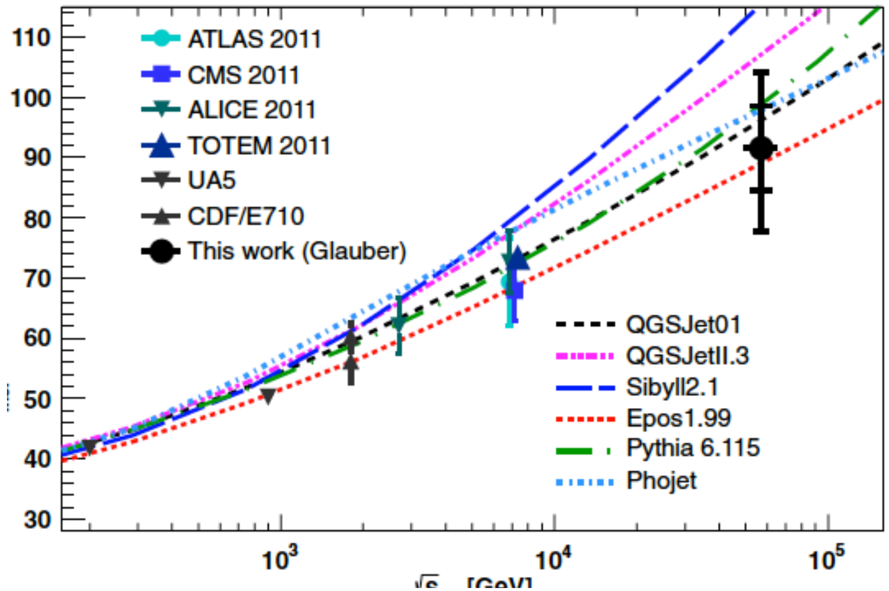



- Can we trust the cosmic ray measurements, i.e. how to you get it from cosmic ray experiments?
  - Only with a rather large error

# $\sigma_{tot}^{pp}$ From cosmic ray experiments

$$\Lambda_{air-showers} \rightarrow \lambda_{p-air} \rightarrow \sigma_{inel/prod}^{p-air} \rightarrow \sigma_{tot/inel}^{pp}$$




$$\sigma_{p-air} = (505 \pm 22^{+38}_{-36})mb$$

$$\sigma_{tot}^{pp} = [133 \pm 13(stat)^{+17}_{-20}(syst) \pm 16(Glauber)]\ mb$$

# State of the art : theoretically , after LHC and AUGER

- Do we understand the total cross-section? i.e.:can we calculate normalization and energy dependence?
  - we know it is related to large distances, i.e. to confinement, so the answer is

Not really because

$$\alpha_{strong}(Q^2) \to ??? \quad Q^2 \to 0$$

# Moliere, Heisenberg,Froissart : what did they say about total x-section?

- Moliere 1948 multiple scattering theory

$$f(\theta, t) = \int_0^\infty \eta d\eta J_0(\eta\theta) e^{\Omega(\eta)-\Omega_0}$$

- Heisenberg 1952 :cut off in space determined by pion cloud

$$\sigma_{total} \simeq \frac{\pi}{m_\pi^2} (\ln \frac{\sqrt{s}}{< E_0 >})^2$$

- Froissart 1961 : based on cut-off in angular momentum,

->i.e impact parameter, with pion mass as scale

$$l_{max} = kb_{max}$$

1. cut off in distance (confinement),
2. analyticity in Lehman ellipse
   and thus finite lowest mass

# Models since ~1950

- Heisenberg 1952 $constant \simeq \sigma_{total} \lesssim \log^2 s$
- Froissart limit $cut-off\ in\ b-space$
- Regge theory + optical theorem
- Eikonal models a' la Glauber
- Regge+eikonal
- Pomeron 1+2+3 … BFKL Pomeron
- QCD Minijets (w/o soft gluon resummation)

# Basic tension between Regge vs. eikonal: total x-section

- Regge + optical theorem:

t-space

$$\mathcal{A}(s,t) \simeq i\beta(t)s^{\alpha(t)-1}$$

$$\sigma_{tot} = 4\pi \Im m \mathcal{A}(s,t=0)$$

$$\simeq s^{\alpha(0)-1}$$

- Rise ~ $\alpha(0) = 1 + \epsilon > 1$

Donnachie Landshoff

$$\sigma_{total} = X s^{-\eta} + Y s^{\epsilon}$$

- NO Froissart bound

- Eikonal models:b-space

$$\mathcal{A}(s,t) = \int d^2\mathbf{b} e^{i\mathbf{q}\cdot\mathbf{b}}[1 - e^{i\chi(b,s)}]$$

Simplest : Black Disk Limit

$$i\chi(b,s) = -\theta(R(s) - b)$$

$$\sigma_{total} = \pi R^2(s)$$

Expanding radius ~ log s
Froissart bound OK because of cut-off in b-space

# QCD : which QCD?

- Quarks and gluon interact, gluon reggeizes, and the result is called a Pomeron exchange and here the Russian School provided the BFKL equation for the Pomeron trajectory
- Then you can iterate this exchange, using eikonal formalism
- But because of $\alpha_{strong}(Q^2)$ nobody really knows how to go on

# Regge +eikonal

$$\mathcal{A}(s,0) = \int d^2\mathbf{b}[1 - e^{-A_{pomeron}(b,s)}]$$

$$A_{pomeron}(b,s) = \mathcal{F}[s^{\alpha_P(t)-1}] \sim s^{\epsilon} f(b)$$

+ various models and
parametrizations for
f(b)

Soft Pomeron $\alpha_P(t) - 1 \sim 0.1$

Hard Pomeron $\alpha_P(t) - 1 \sim 0.3 - 0.4$

# Our QCD model for the total cross-section

R. Godbole, A. Grau, GP, YN Srivastava

$$\sigma_{total} \simeq 2 \int d^2\vec{b}[1 - e^{-\chi_I(b,s)}]$$

$$2\chi_I(b,s) = \sigma_{soft} + A(b,s)\sigma_{jet}$$

- Minijets to drive the rise
- Soft kt-resummation to tame the rise and introduce the cut-off needed to satisfy theFroissart bound

- Phenomenological singular but integrable soft gluon coupling to relate confinement with the rise
- Interpolation between soft and asymptotic freedom region

PLB1996-PRD2005

# Minijets: are mini-jets QCD?

$$\sigma_{jet}(s, p_{tmin}; PDF) \sim s^{\epsilon}$$

From DGLAP evolution

Hard Pomeron

$$s^{\epsilon P}$$

YES : minijets are a phenomenological realization of the BFKL Pomeron

# Impact parameter dependence?

- Our expression is obtained from resummation except that

# Our proposal for running $\alpha_s(k_t)$ in the infrared region

$$V_{one\ gluon\ exchange} \sim r^{2p-1}$$

$$\propto k_t^{-2p} \qquad k_t << \Lambda$$

To reconcile with asymptotic Freedom

$$\propto \frac{1}{\log k_t^2/\Lambda^2} \qquad k_t >> \Lambda$$

A phenomenological interpolation

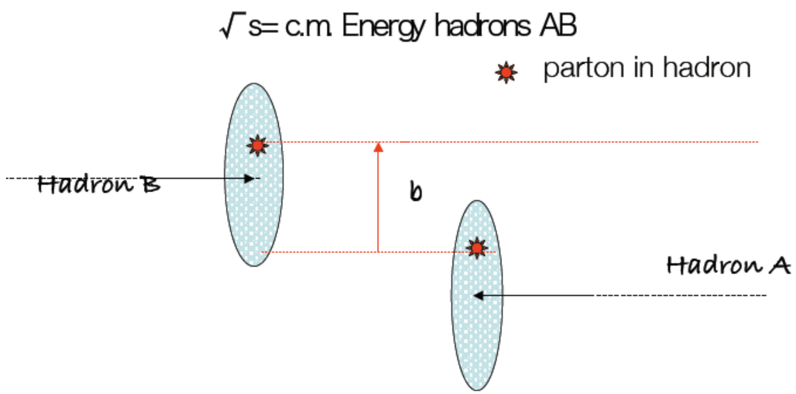
$$\alpha_{eff}(k_t) = \frac{12\pi}{11N_c - 2N_f} \frac{p}{\log[1 + p(k_t/\Lambda_{QCD})^{2p}]}$$

We model the impact parameter distribution as the Fourier-transform of ISR soft $k_t$ distribution and thus obtain a cut-off at large distances : Froissart bound?

$$A_{BN}(b,s) = N \int d^2\mathbf{K}_\perp \; e^{-i\mathbf{K}_\perp\cdot\mathbf{b}} \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}\; e^{-h(b,q_{max})}}$$

$$h(b,E) = \frac{16}{3\pi}\int_0^{q_{max}} \frac{dk_t}{k_t}\alpha_{eff}(k_t)\ln\left(\frac{2q_{max}}{k_t}\right)[1-J_0(bk_t)]$$

$$\alpha_{eff}(k_t\to 0) \sim k_t^{-2p}$$




$$A_{BN}(b,s) \sim e^{-(b\bar{\Lambda})^{2p}}$$

$q_{tmax}$ ? Fixed by single gluon emission kinematics

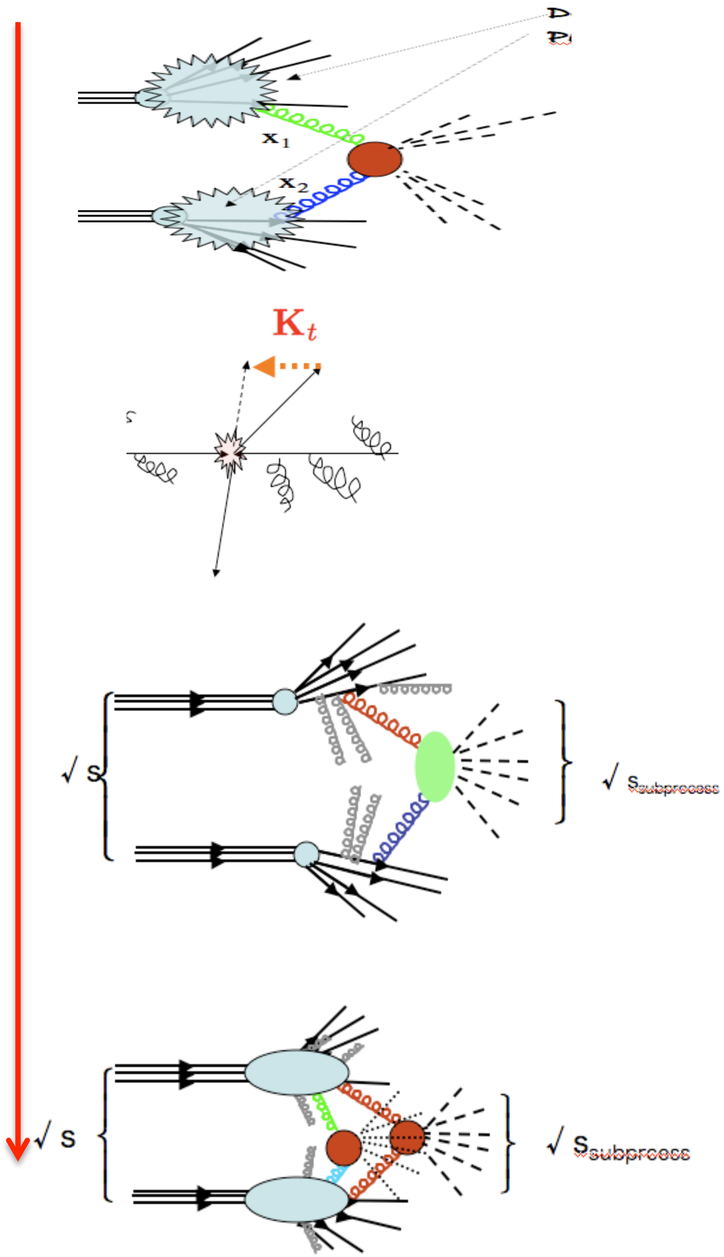
In our model, the emission of singular infrared gluons tames low-x gluon-gluon scattering ( mini-jets) and restores the Froissart bound

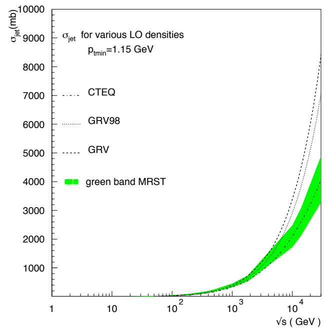
$s^{\varepsilon}$

$$\sigma_{tot}(s) \approx 2\pi \int_0^\infty db^2 [1 - e^{-C(s) e^{-(b\bar{\Lambda})^{2p}}}]$$

$$\sigma_{tot}(s) \to [\varepsilon \ln(s)]^{(1/p)} \qquad \frac{1}{2} < p < 1$$

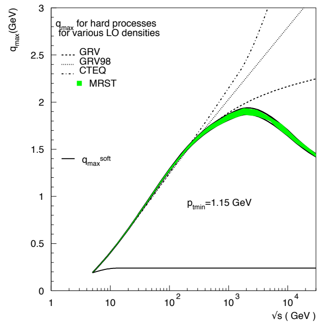
Grau, Godbole,GP,Srivastava,PLB682 2009



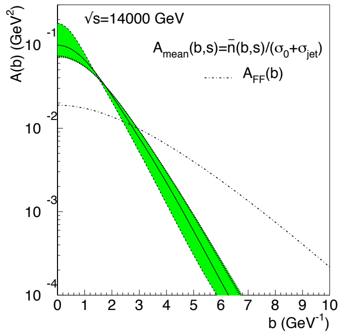

1. Calculate mini-jet cross-section Choosing densities and ptmin

$$\sigma_{mini-jet} \simeq s^{\epsilon}$$
$$\epsilon \simeq 0.3 - 0.4$$



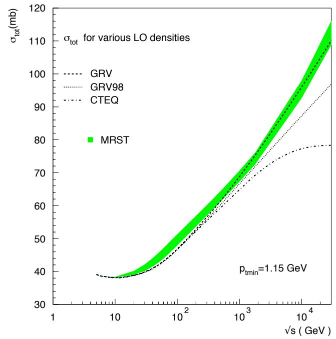

2. Calculate qmax: single soft gluon upper scale, for given PDF, ptmin

$$q_{max} \simeq p_{tmin}$$
$$\lesssim 2 - 3\ GeV$$




3. Calculate impact parameter distribution for given qmax and given infrared parameter p

$$\chi(b,s) = \chi_{low\ energy} + $$
$$+ A(b, qmax)\sigma_{jet}$$




4. Eikonalize

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\chi(b,s)}]$$

# The inelastic total cross-section

# The inelastic total cross-section in eikonal models

$$\sigma_{total} = \sigma_{elastic} + \sigma_{inelastic}$$

$$-t \geq 0$$

$$t = 0$$

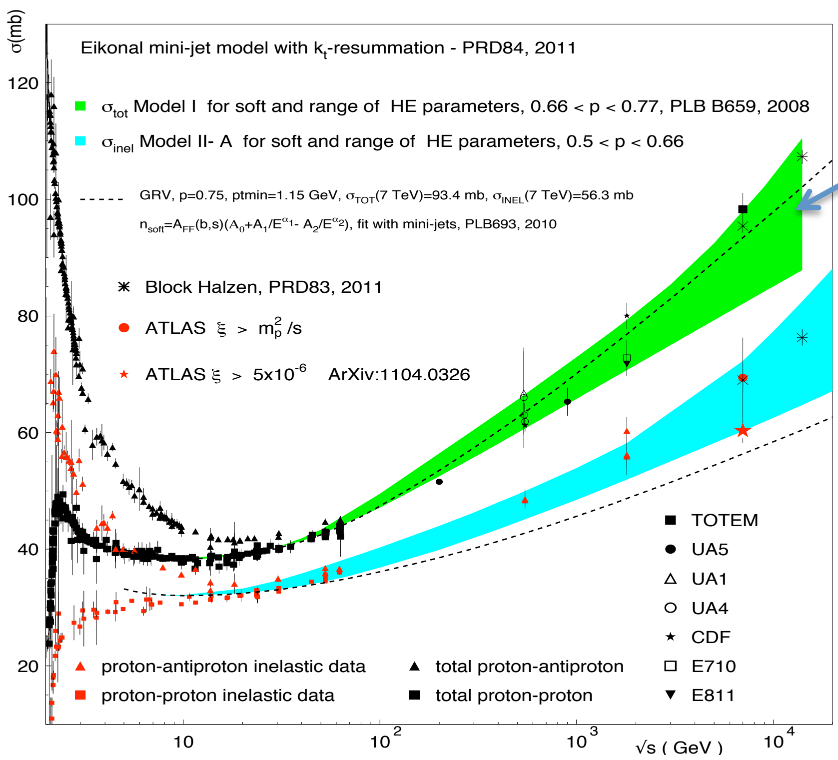
$$\mathcal{A}(s,t)$$

$$\sigma_{inelastic} = \int d^2\mathbf{b}[1 - e^{-2\Im m\chi(b,s)}]$$

Only Poisson – independent-collisions-nodiffraction

# The inelastic cross-section

PLB 2008
Band
for the total

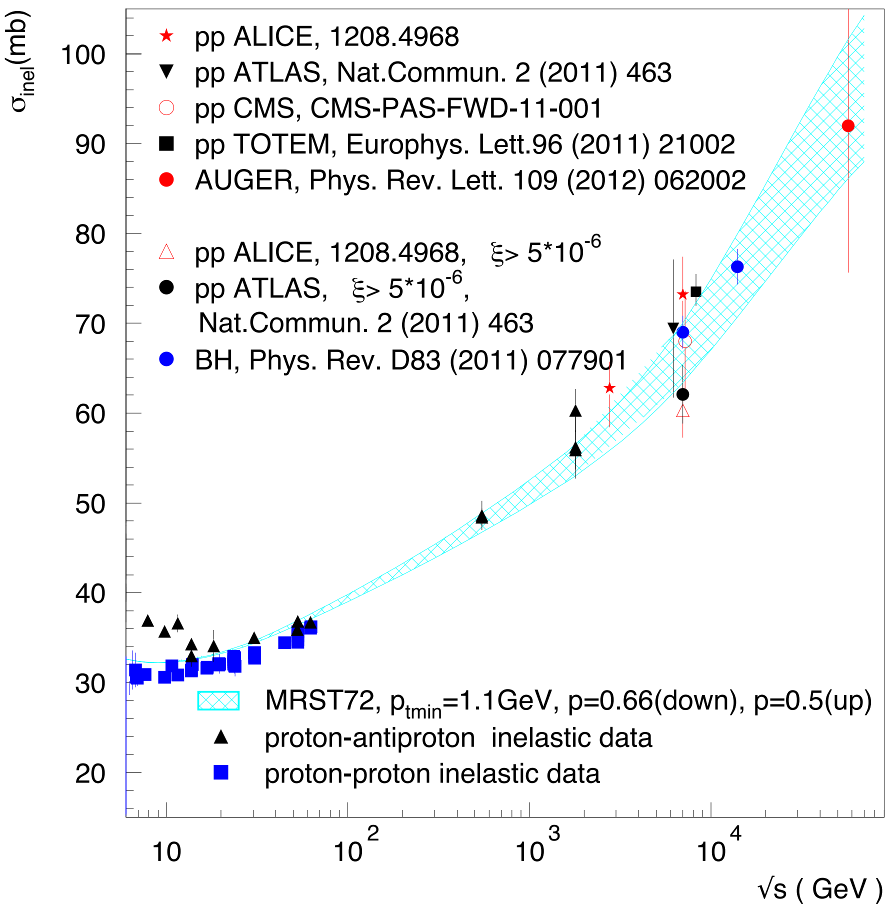


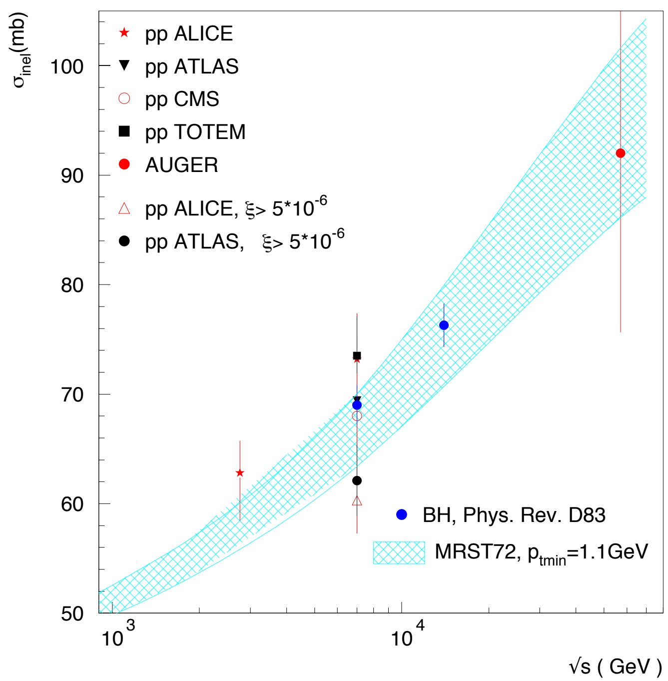

Inelastic cross-section PRD2012

It is not so clear experimentally nor theoretically: need cuts and models or parametrizations for diffraction

# Update of PRD2012 analysis

With Olga Shekhovtsova




Why the uncertainty in the inelastic? Models for diffraction

$$\sigma_{tot} = \sigma_{elastic} + \sigma_{inelastic}$$

- Elastic cross-section: *pp* amplitude $-t \neq 0$

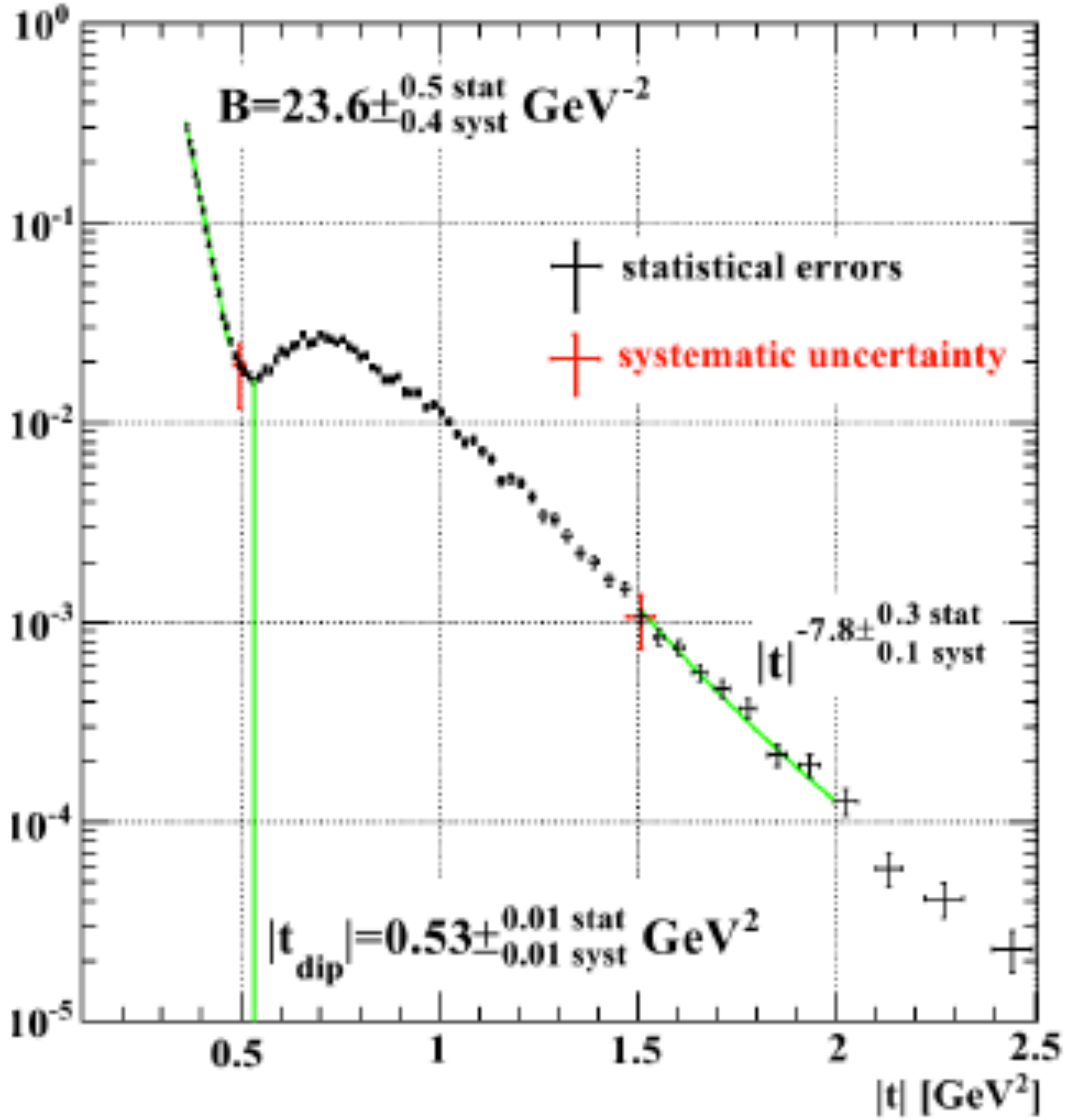
  well defined both theoretically and experimentally

$$\int_0^\infty dt\{[\Im m\mathcal{A}(s,t)]^2 + [\Re e\mathcal{A}(s,t)]^2\}$$

- Inelastic : what is not elastic!!! Yes, but not so simple, diffractive, central, large mass,small mass

# The elastic cross-section

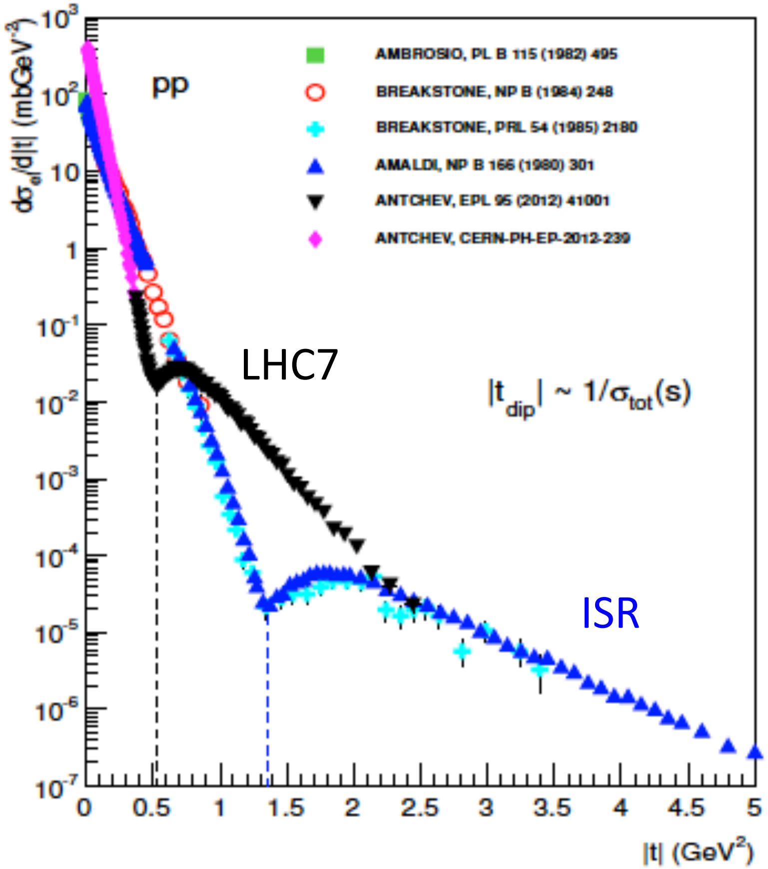
# TOTEM data 2011: the elastic differential cross-section



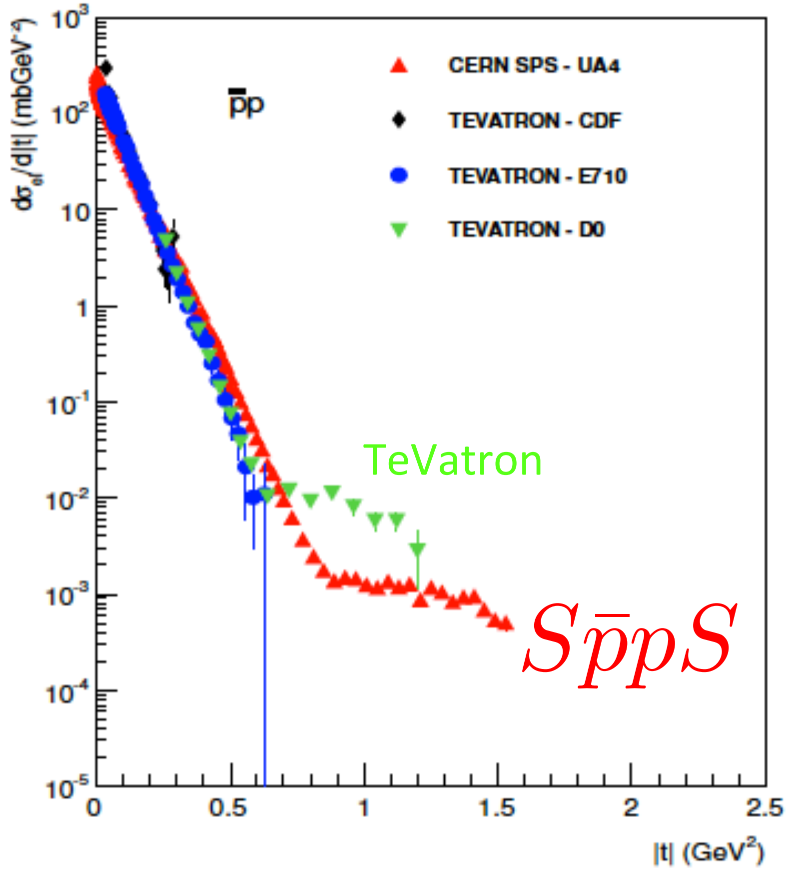

The return of the **dip!** It had not been seen since ISR

[because it not really present in  pbarp]

# Elastic ISR,LHC $pp$     S $p\bar{p}$ S,Tevatron $p\bar{p}$



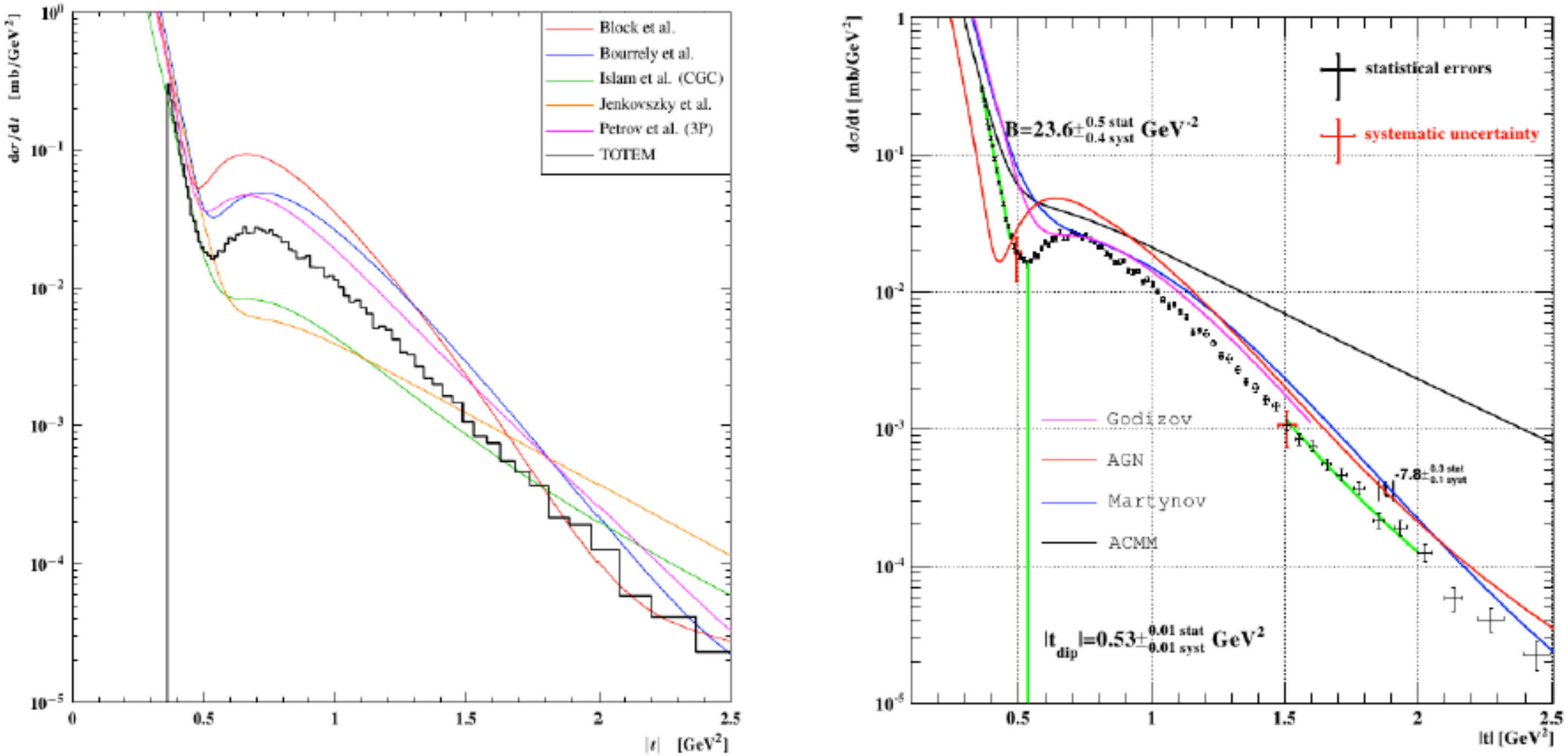

dip




Faint dip

From D. Fagundes, DIS 2013 Marseille

## LHC run at $\sqrt{s} = 7$ TeV from TOTEM[2]




none of the representative models reproduce the data

[2] *Left*: G. Antchev *et al.*, Europhys.Lett. 95 (2011) 41001. *Right*: A.A. Godizov, PoS (IHEP-LHC-2011) 005.

# The 4 components of the elastic scattering amplitude :

- The optical point

$$\frac{d\sigma}{dt}|_{t=0} \propto \sigma_{tot}^2$$

- The forward precipitous descent

$$\frac{d\sigma}{dt}|_{t\sim 0} \propto e^{-Bt}$$

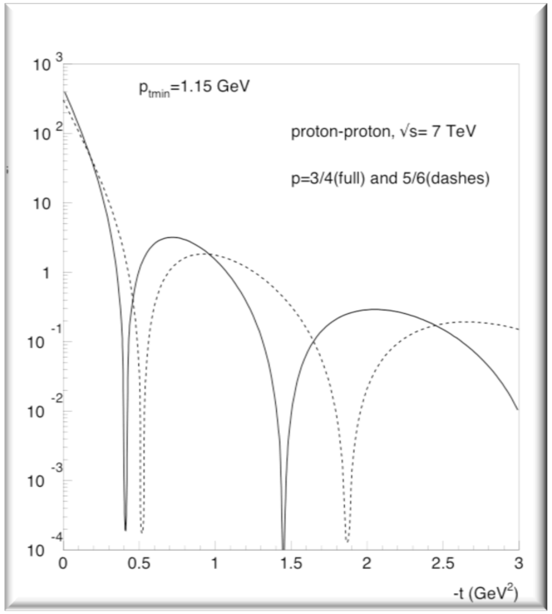
- The dip in pp (and not in pbarp)

$$a\ phase\ ?$$

- The tail

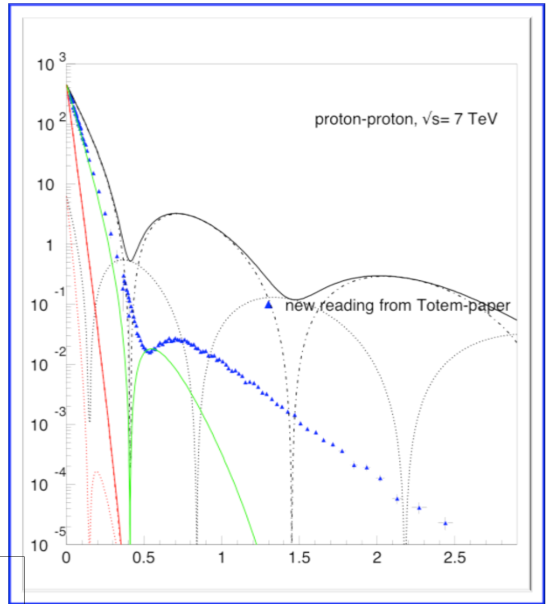
$$\frac{d\sigma}{dt} \sim t^{-(7\div 8)}$$

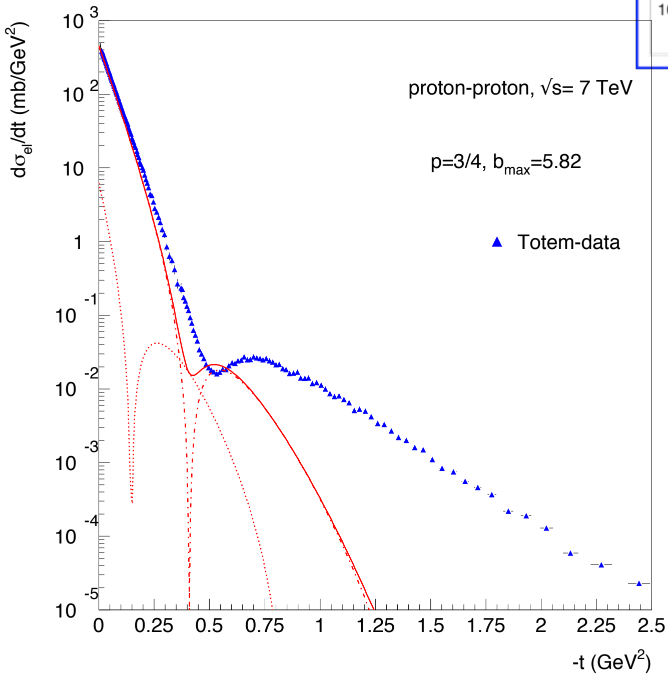
# Our QCD one-channel eikonal model with mini-jets and resummation




Purely imaginary eikonal

With real part a' la Martin

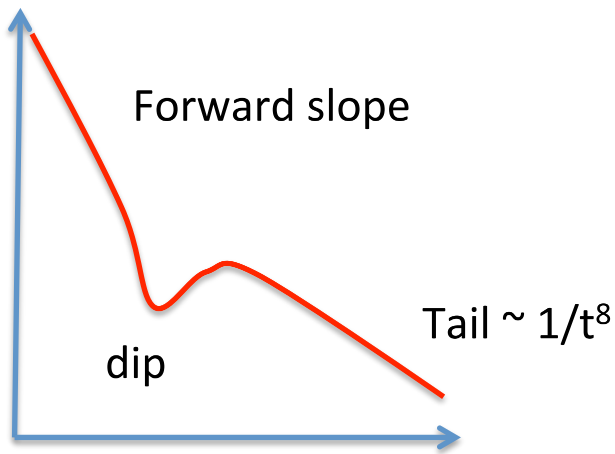



Complex eikonal
With a gaussian cutoff
In b-space: but the dip and tail are still wrong
Parameters could be added
But choose to change strategy

# The one eikonal does not work

- Optical point : total cross-section
- Forward slope? Regge?
- The dip? ??
- The tail? 3 gluons perhaps




Resort to an EMPIRICAL MODEL to try to understand the building blocks

# Losing patience :

# an empirical approach

# Empirical model for pp scattering from ISR to LHC, from the optical point to past the dip

$$\mathcal{A}(s,t) = i[G(s,t)\sqrt{A(s)}e^{B(s)t/2} + e^{i\phi(s)}\sqrt{C(s)}e^{D(s)t/2}].$$

$$G(s,t) \equiv 1$$

Barger-Phillis 1973
ISR data

Grau, GP,Pacetti,Srivastava 2012
ISR & LHC7

This work, 2013, with D. Fagundes

$$G(s,0) = 1$$

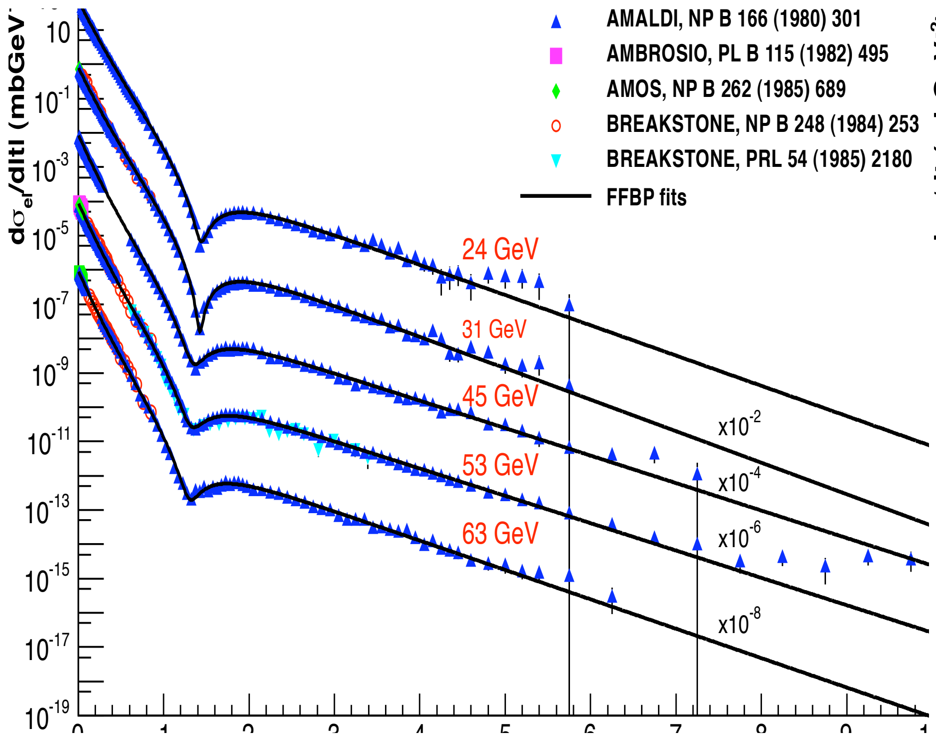
Pion-loop singularity
Anselm&gribov, KMR,
Jenkovszki

$$G(s,t) = [\frac{1}{(1-t/t_0)^2}]^2$$

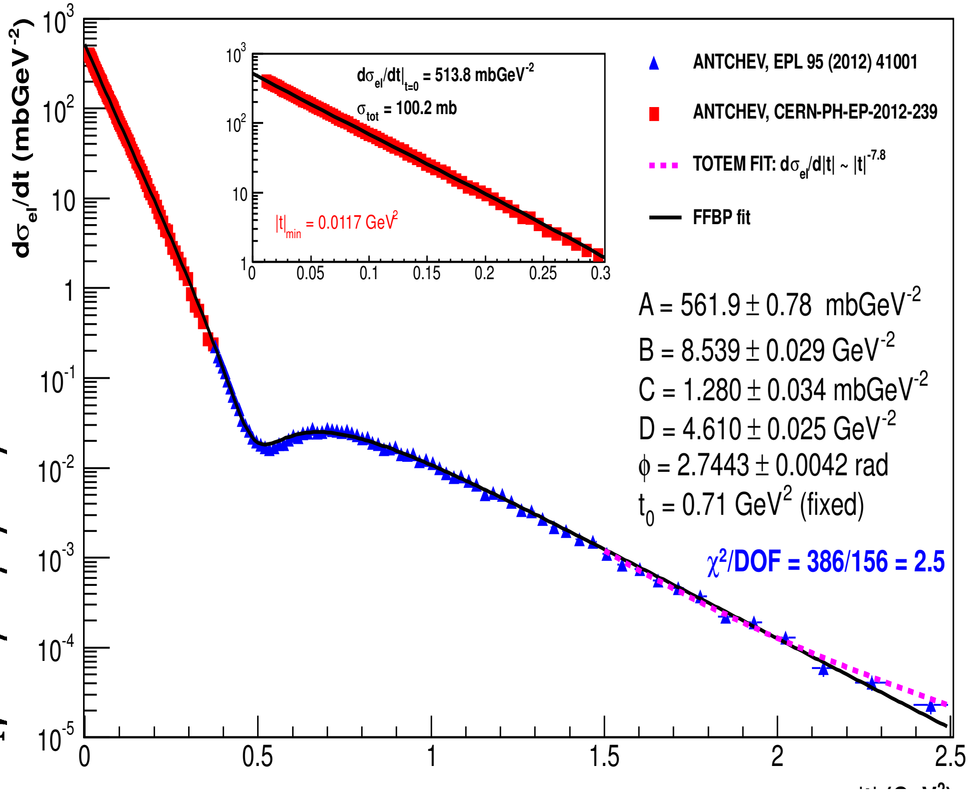
Proton form factor

# BP model with Proton Form Factor

**ISR for pp**




**TOTEM LHC7 for pp**

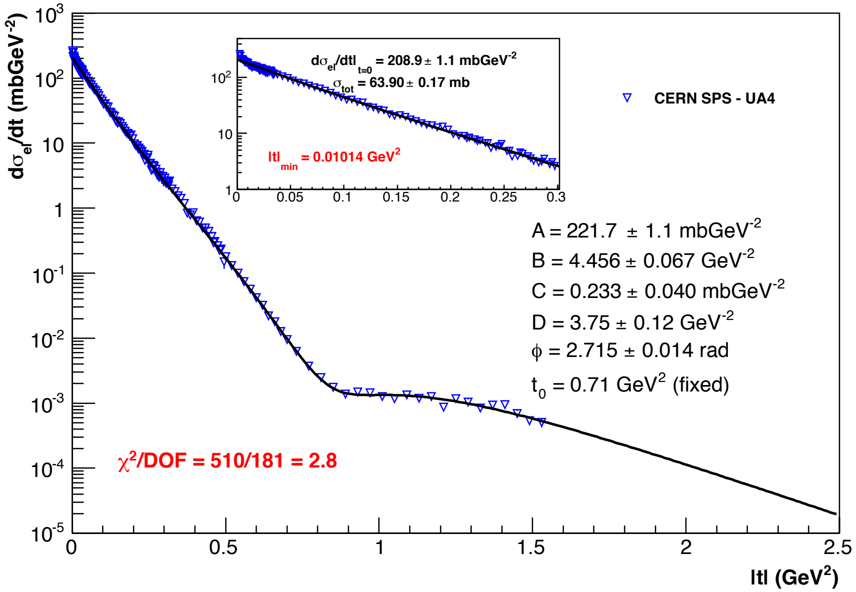




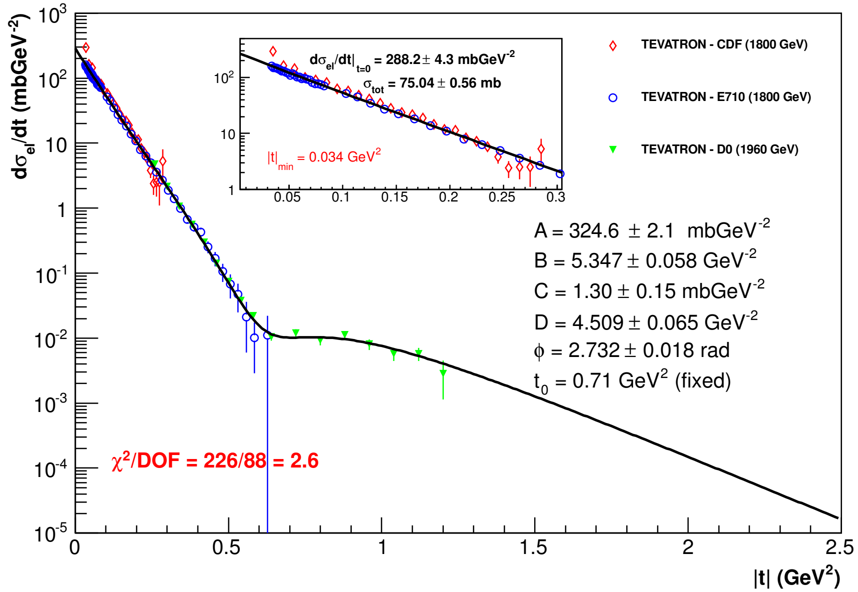

Fagundes, Grau, GP, Srivastava PRD 2013

# Empirical model applied to pbarp

## UA4 data




## CD-E710-D0 data



# Can one make predictions?

## An asymptotic model of maximal saturation

Fagundes, Grau, Pacetti, GP, Srivastava arXiv:1306.0452 **(to be published)**

- Froissart-Martin bound

- Khuri-Kinoshita

- Total absorption at b=0

# How about physical meaning and predictions for higher energies?

Leading term

Non leading at t=0

$$\mathcal{A}(s,t) = i[G(s,t)\sqrt{A(s)}e^{B(s)t/2} + e^{i\phi(s)}\sqrt{C(s)}e^{D(s)t/2}].$$

Form factor

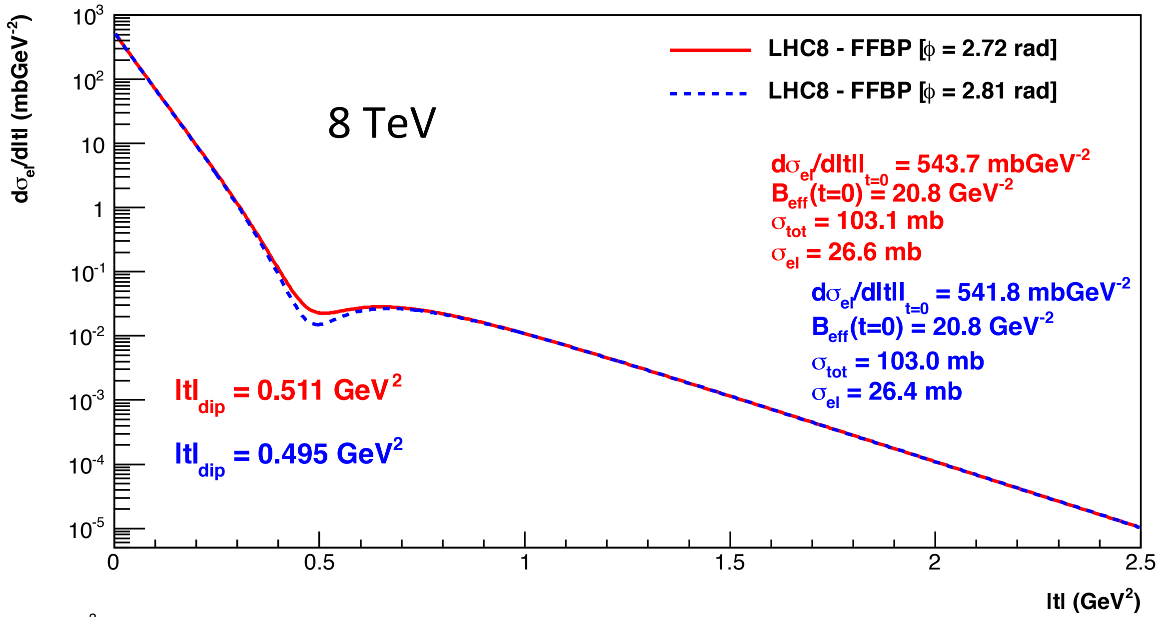
Total x-section

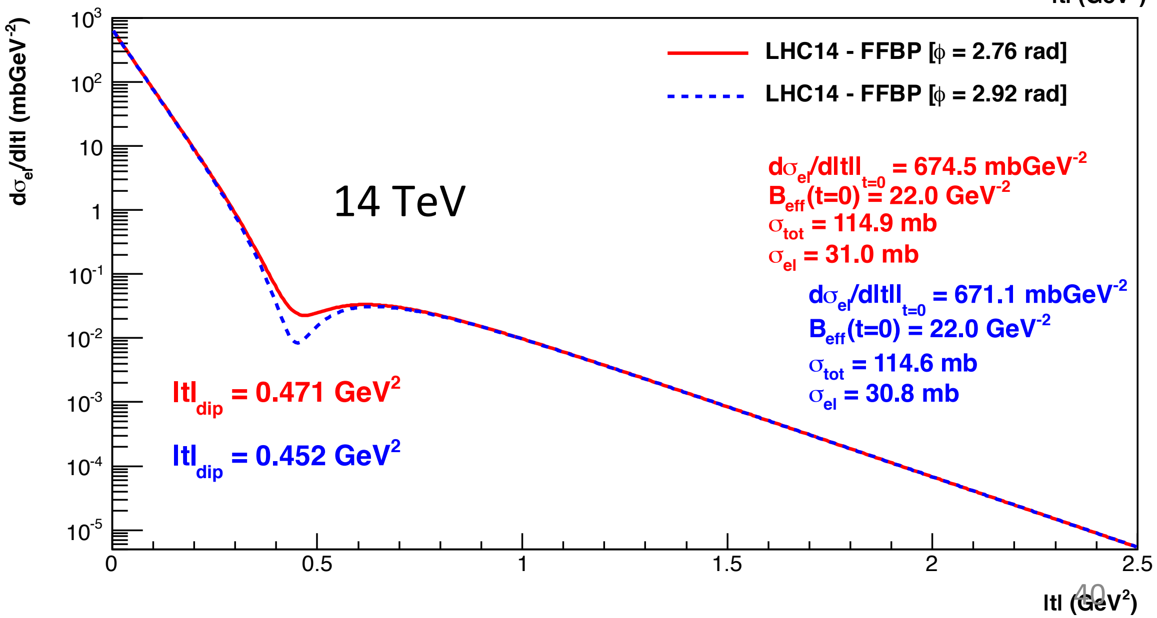
Mixture of
C=+1 and C=-1

Forward slope

# Predictions from asymptotic model

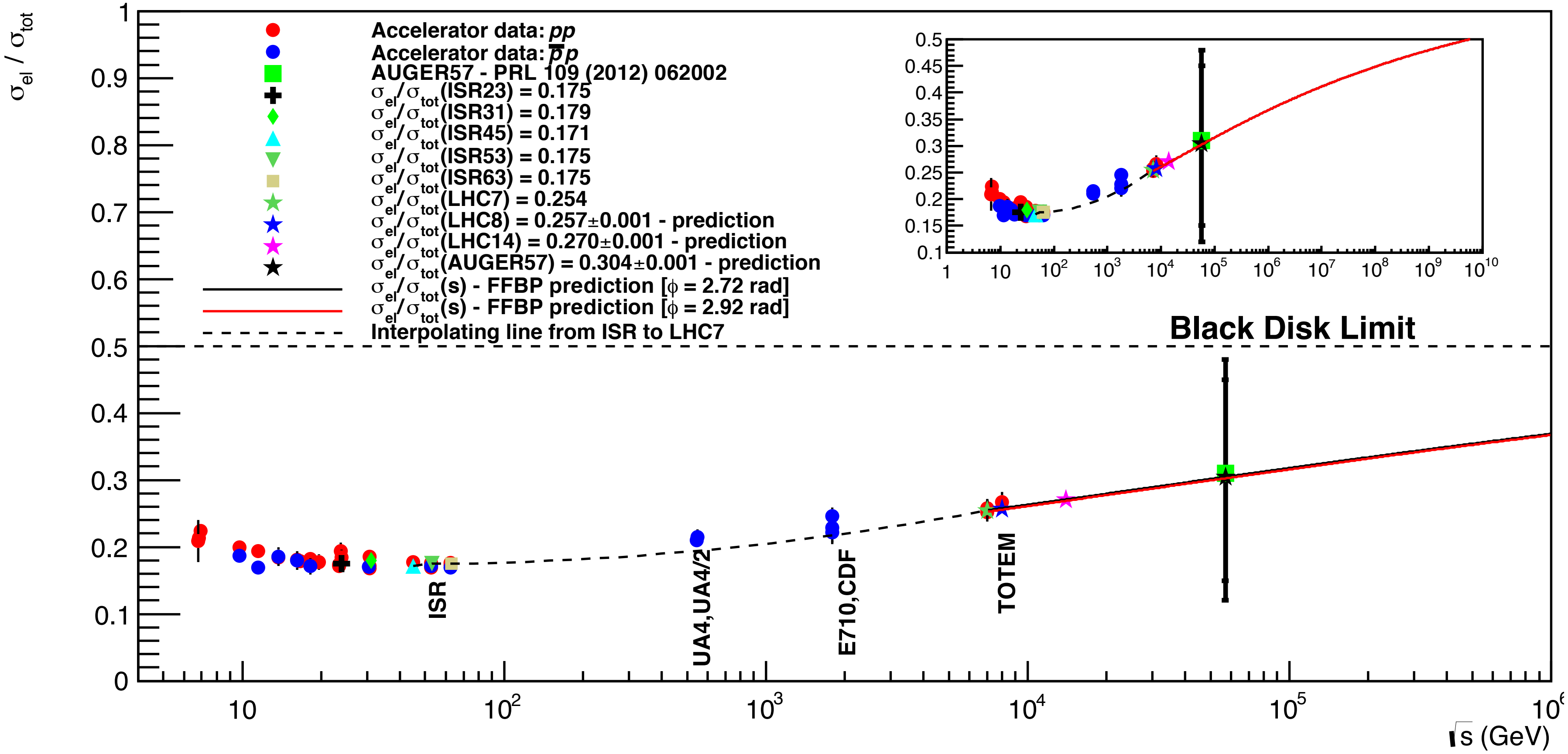
At any given energy
the difference
Is in the phase,
Which is so far
unconstrained



## The black disk limit in the asymptotic model

$$R_{el} = \frac{\sigma_{elastic}}{\sigma_{total}}$$




The black disk limit in this asymptotic extrapolation is not reached until

$$\sqrt{s} \sim 10^5\ TeV$$

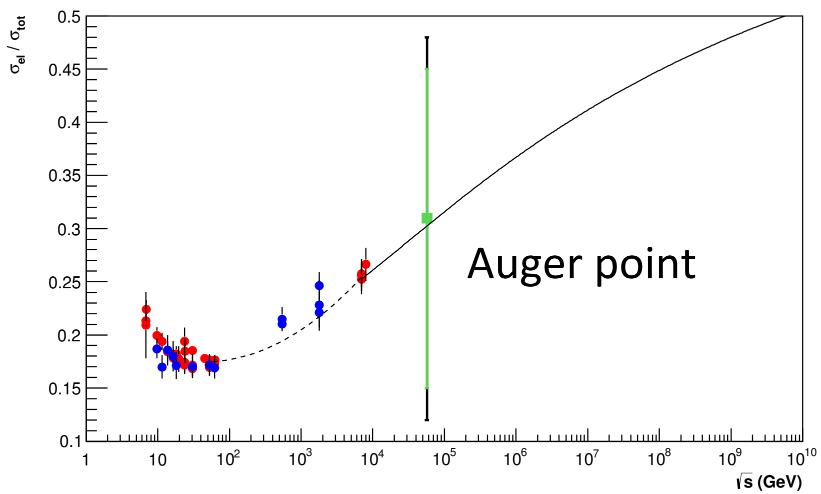
# A lesson from the empirical model

- The black disk limit is very far away

$$\sigma_{total}^{blackdisk} = \pi R^2(s)$$

$$\sigma_{elastic}^{blackdisk} = 2\pi R^2(s)$$

$$\mathcal{R}(s) = \frac{\sigma_{elastic}}{\sigma_{total}} \neq \frac{1}{2}$$



# More Empiricism: a lesson for Geometrical Scaling (GS)

- When $\mathcal{R}_{elastic} = 1/2$ only one energy scale is relevant and thus

- Geometrical scaling : a scaling variable at asymptotic energies for the elastic differential cross-section

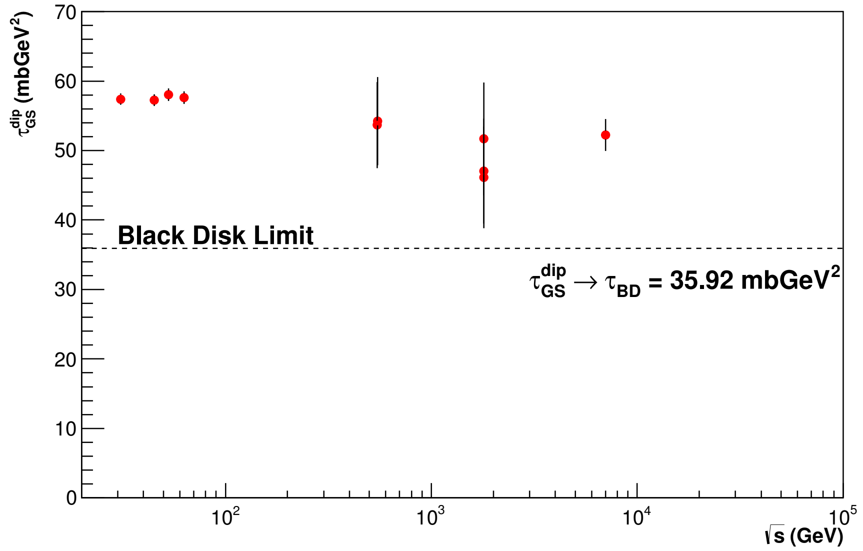
$$\tau_{GS}(-t, s) = -t\sigma_{total}$$

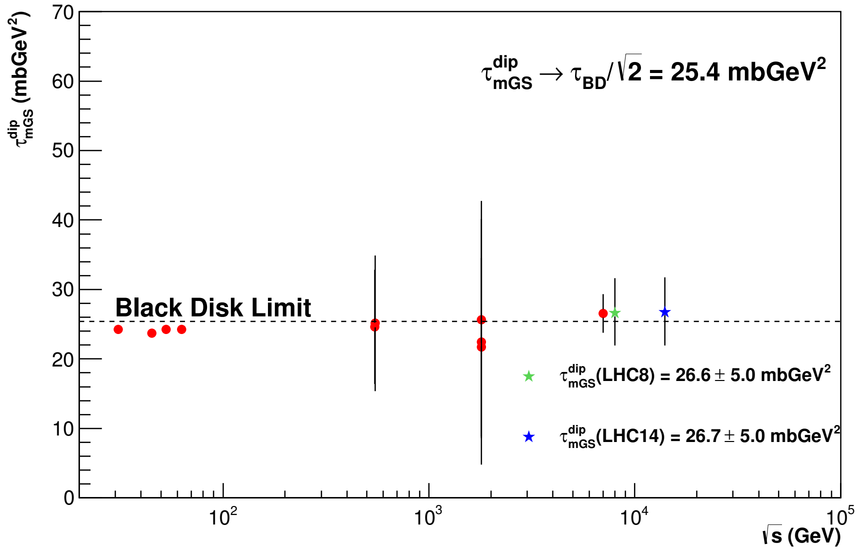
But at present, there are still 2 scales,

$$\sigma_{elastic} \quad \sigma_{total}$$

NO GS

# Prospects? The position of the dip can be predicted by a new scaling law




$$\tau_{GS}(-t,s) = -t\sigma_{total}$$

$$\tau_{GS}^{mean}(-t,s) = -t\sqrt{\sigma_{el}\sigma_{total}}$$

# Abstract

- Models for proton-proton scattering at LHC: asymptotic limits, black-disk limit and geometrical scaling *45'*
  We discuss recent LHC measurements for the total, elastic and inelastic cross-sections, by TOTEM, CMS, ATLAS and ALICE collaborations in the light of present phenomenological predictions. We present result of various models, including an empirical model for elastic pp scattering at LHC, which indicates that the asymptotic black-disk limit R=(elastic cross section)/(total cross section) =1/2 is far from being reached, and discuss the implications on classical geometrical scaling behaviour. We propose a geometrical scaling law for the position of the dip in the differential elastic cross-section. The new scaling law allows to make predictions valid both for intermediate and asymptotic energies.