(Meta) stability of the electroweak vacuum

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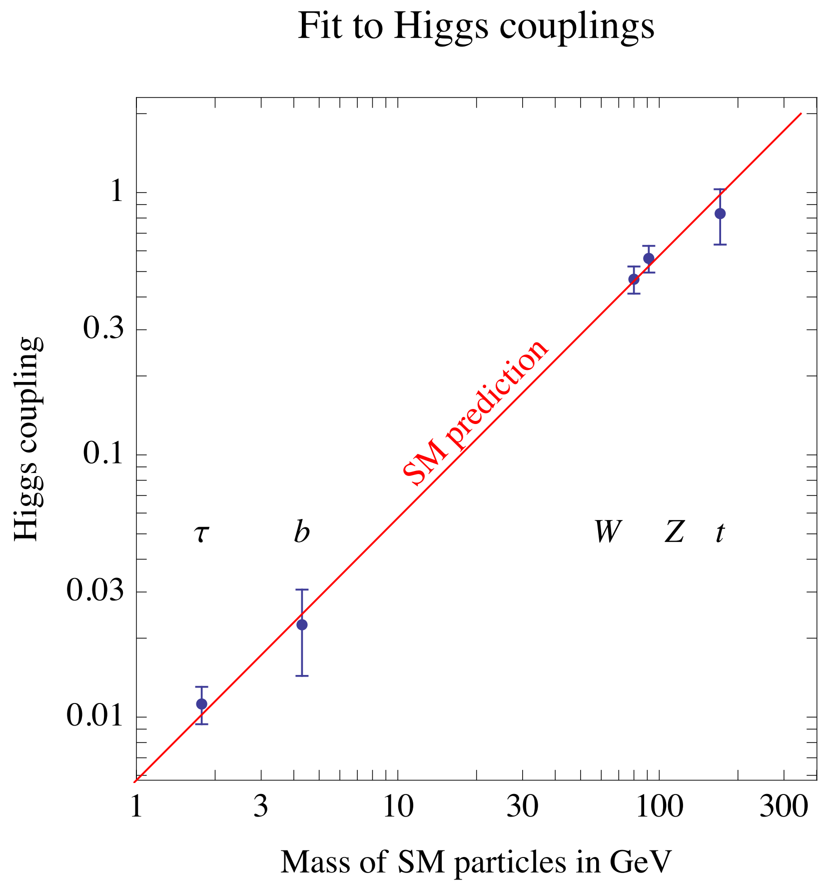
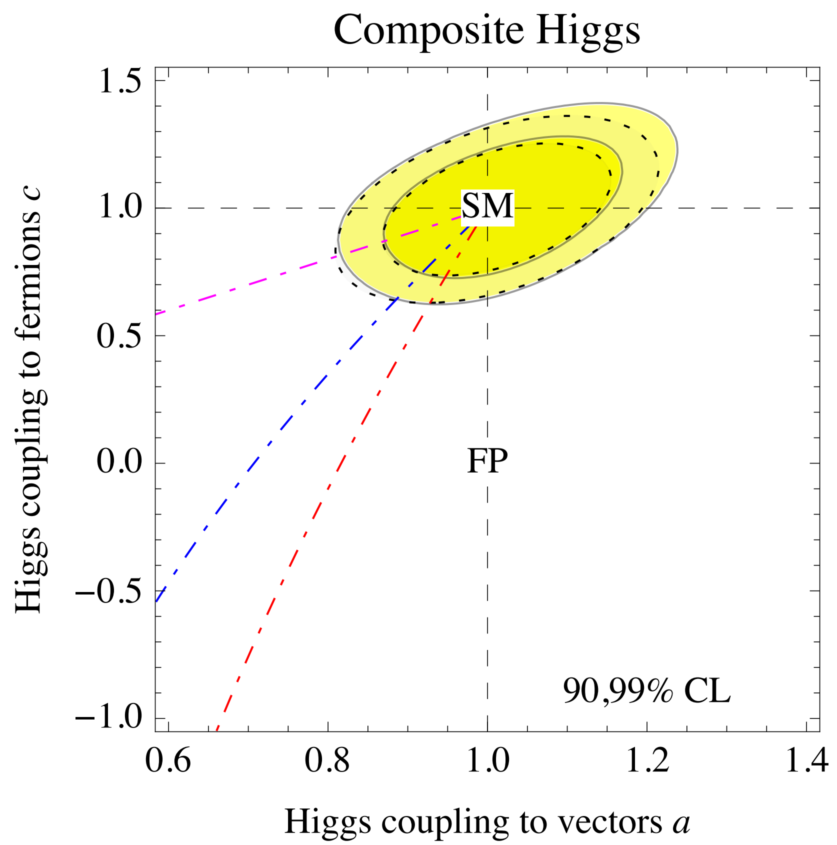
Mainly based on arXiv:1307.3536, D. Buttazzo, G. Degrassi, P.P.G., G. Giudice, F. Sala, A. Salvio, A. Strumia

You already know that:

- The particle found at LHC is a Higgs boson.

- Its characteristics are compatible with the SM Higgs.





- Although we can not exclude the existence of NP at the TeV scale,

- We do not have any evidence of it.

P.P.G., Kannike, Masina, Raidal, Strumia 2013

Assumptions

- Let me assume that there is no NP at the TeV scale,
- and that the SM is the only theory for energies below the Planck scale.
- Deviations from the SM (neutrino masses, dark matter, baryon asymmetry), can be explained without significantly modify the shape of the Higgs potential.

Higgs Potential

$$V(H) \approx \underbrace{\frac{-m^2(\mu)}{2}|H|^2}_{\text{negligible at } \mu \gg v} + \lambda(\mu)|H|^4$$

We are interested in study the shape of the Higgs potential.

The classical shape is modified by quantum corrections. And for high field values (H>>v) it is reduced to

$$V(H) \approx \lambda(\mu \approx H)|H|^4$$

In this way the absolute stability condition simply becomes

$$\lambda(\Lambda) \geq 0$$

For any scale Λ smaller than Planck

λ runs

$$(4\pi)^2 \frac{d\lambda}{d\ln\mu} = \beta_\lambda$$

$$\beta_\lambda = 24\lambda^2 + \lambda(-9g_2^2 - 3g_1^2 + 12y_t^2) + \frac{3}{4}g2^4 + \frac{3}{8}(g1^2 + g2^2)^2 - 6y_t^4$$

+ higher loops

If the Higgs is too heavy λ becomes not perturbative at high energies

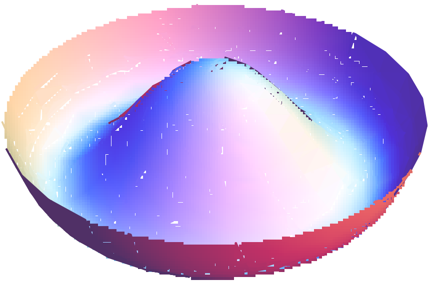
If the Top is too heavy the λ becomes negative at high energies

λ runs

$$(4\pi)^2 \frac{d\lambda}{d\ln\mu} = \beta_\lambda$$

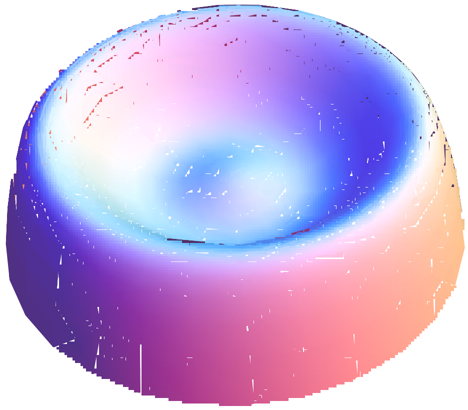
$$\beta_\lambda = 24\lambda^2 + \lambda(-9g_2^2 - 3g_1^2 + 12y_t^2) + \frac{3}{4}g2^4 + \frac{3}{8}(g1^2 + g2^2)^2 - 6y_t^4$$

+ higher loops



your mexican hat

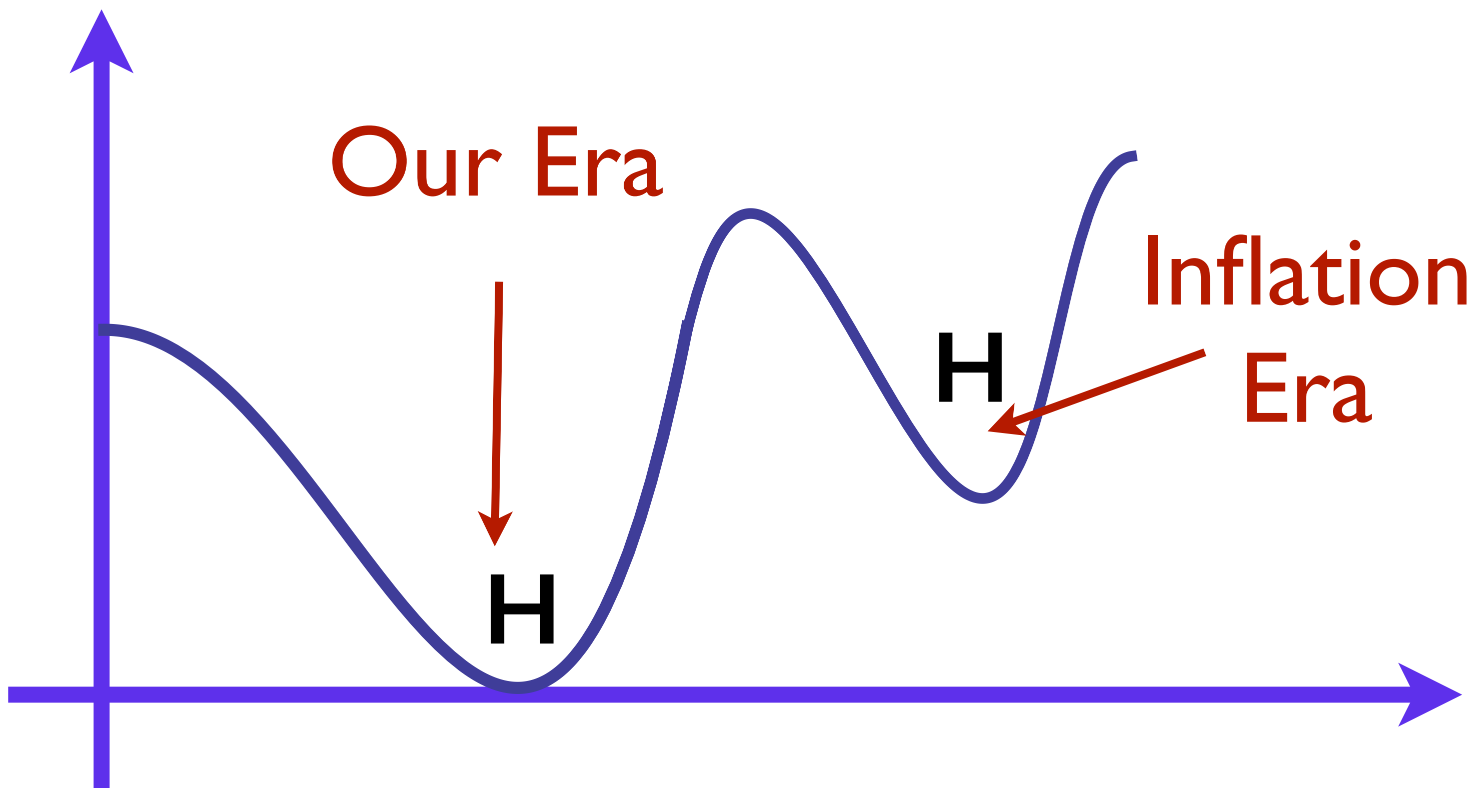
becomes



a dog bowl

Stability

If the top is not too heavy the potential does not become negative and the SM vacuum is stable.

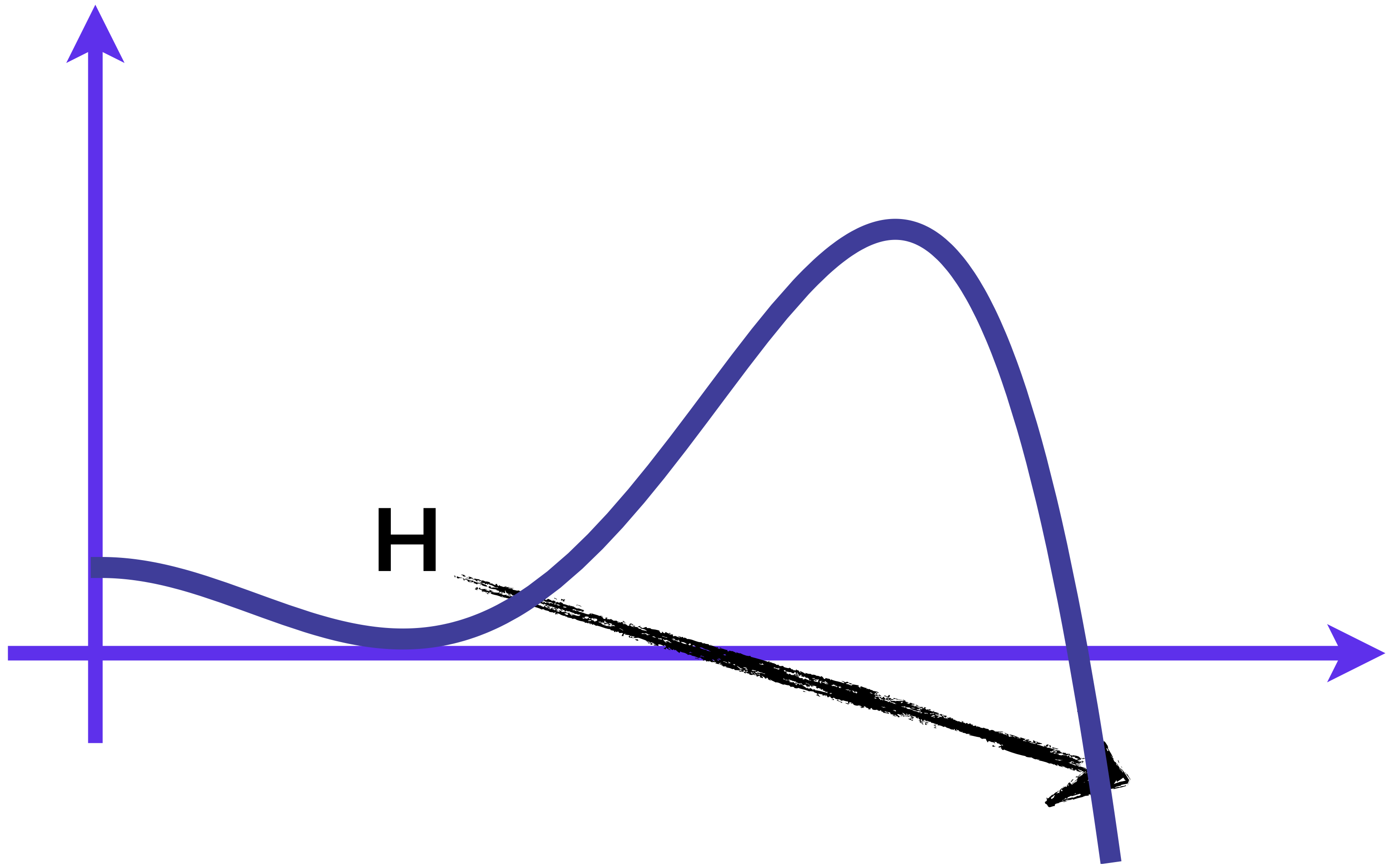


If there is a second minimum we can have inflation. This usually require some other physics at high scale, but it is not necessary.

See, for example: I. Masina and A. Notari, Phys. Rev. D 85 (2012) 123506 [arXiv:1112.2659 [hep-ph]]
I. Masina and A. Notari, Phys. Rev. Lett. 108 (2012) 191302 [arXiv:1112.5430 [hep-ph]]
F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B 659 (2008) 703 [arXiv:0710.3755 [hep- th]]
F. Bezrukov and M. Shaposhnikov, JHEP 0907 (2009) 089 [arXiv:0904.1537 [hep-ph]]

Instability

If the top is too heavy, the beta is negative and stay negative till the Planck scale. The SM vacuum is unstable.

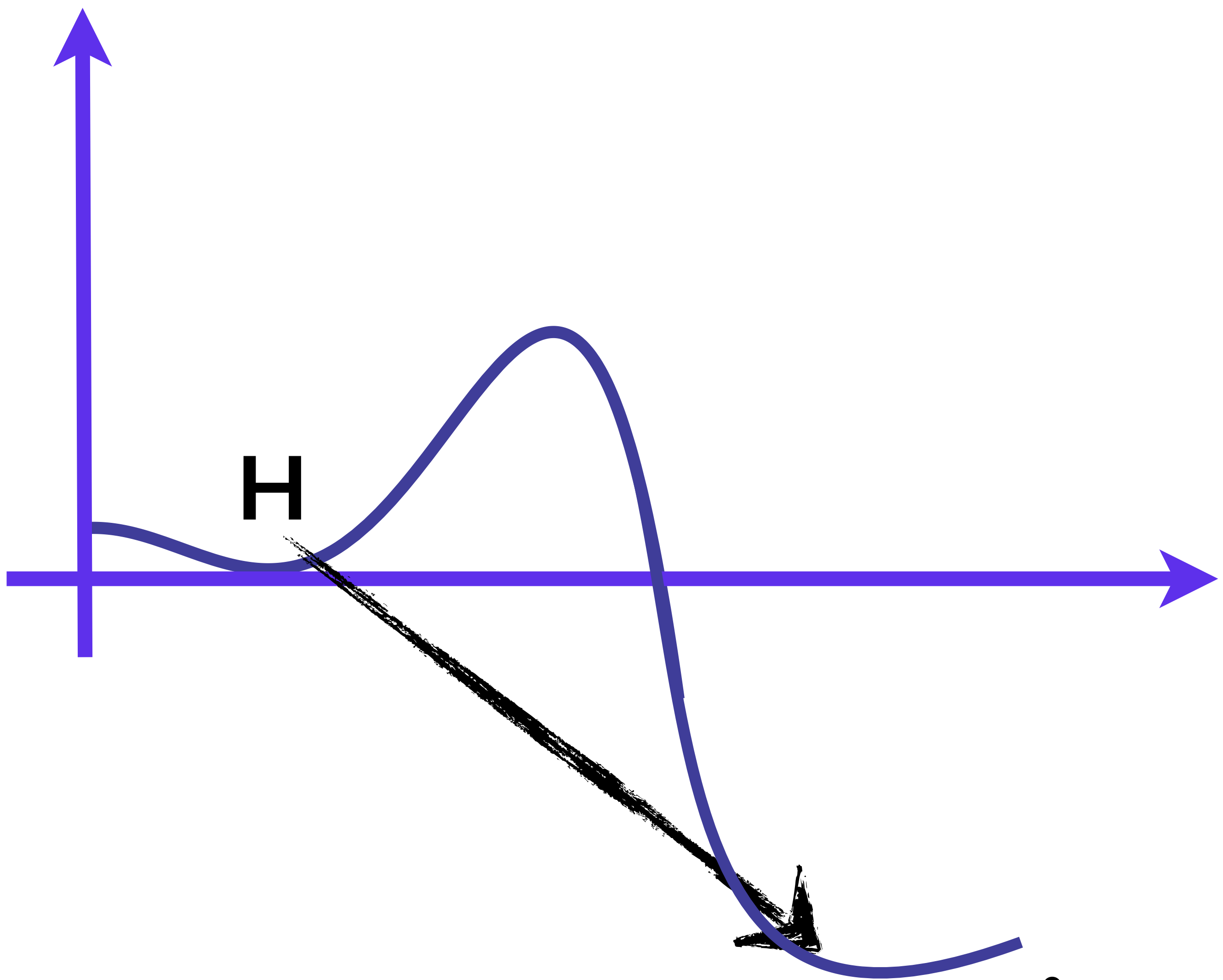


We need N.P. in order to save our Universe…

Tunneling with a $\tau < \tau_{Universe}$

Metastability

If the beta function is negative, but it turns positive at a high scale below the Planck scale, the potential develops a new, deeper minimum and the EW vacuum is unstable, but

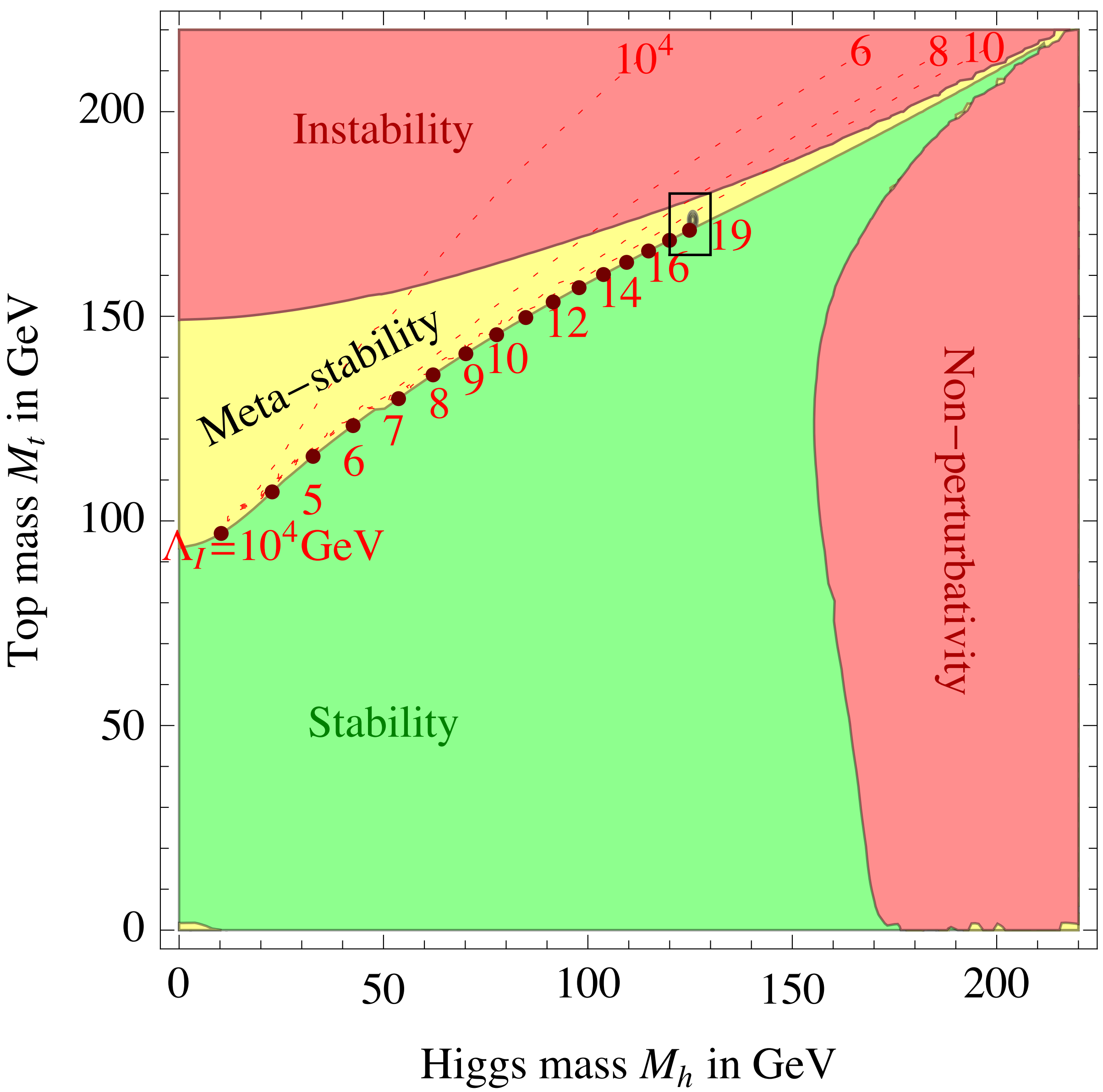


the probability of the decay is very small and we have a meta-stable EW vacuum.

Tunneling with a $\tau > \tau_{Universe}$

Meta stability

The current values of the Higgs and Top masses justify a more precise calculation of the corrections to the Higgs quartic coupling



Renormalisation Group Equations

	LO 1 loop	NLO 2 loop	NNLO 3 loop	NNNLO 4 loop
g_3	full	full	full	$\mathcal{O}(\alpha_3^4)$
$g_{1,2}$	full	full	full	—
y_t	full	full	full	—
λ, m^2	full	full	full	—

Threshold corrections at the weak scale

	LO 0 loop	NLO 1 loop	NNLO 2 loop	NNNLO 3 loop
g_2	$2M_W/V$	full	full	—
g_Y	$2\sqrt{M_Z^2 - M_W^2}/V$	full	full	—
y_t	$\sqrt{2}M_t/V$	full	full	$\mathcal{O}(\alpha_3^3)$
λ	$M_h^2/2V^2$	full	full	—
m^2	M_h^2	full	full	—

λ

The corrections to λ are

Sirlin, Zucchini, 1986

$$\lambda(\mu) = \frac{G_\mu}{\sqrt{2}} M_h^2 - \delta\lambda^{(1)} - \delta\lambda^{(2)}$$

μ decay (analytical)

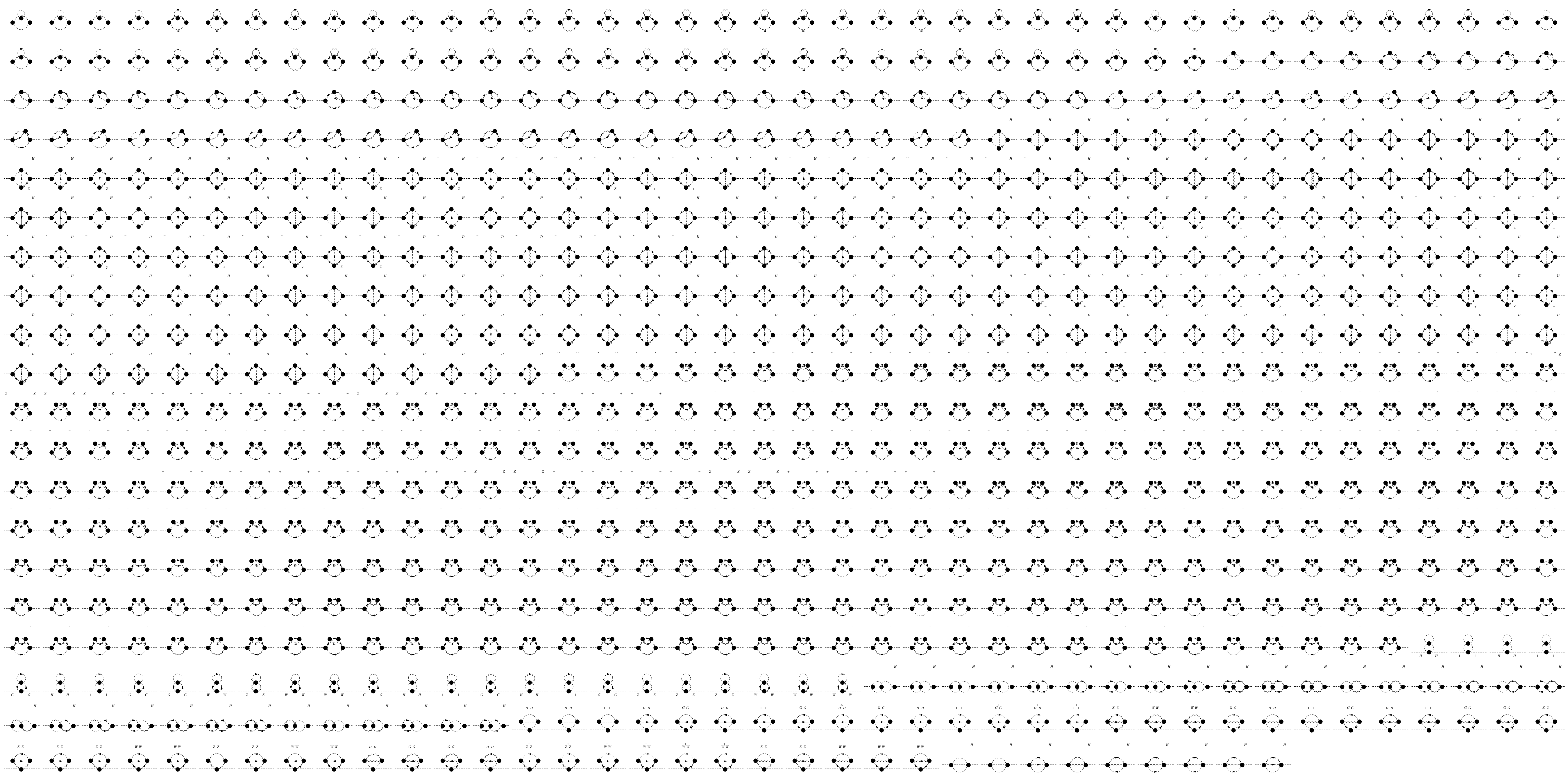
Tadpole (analytical)

Higgs propagator (numerical)

$$\delta\lambda^{(2)} = \frac{G_\mu}{\sqrt{2}} M_h^2 \left\{ \Delta r_0^{(2)} + \frac{1}{M_h^2}\left[\frac{T^{(2)}}{v_{\rm OS}} + {\rm Re}\,\Pi_{hh}^{(2)}(M_h^2)\right] - \frac{\Delta r_0^{(1)}}{M_h^2}\left[M_h^2 \Delta r_0^{(1)} + \frac{3\,T^{(1)}}{2\,v_{\rm OS}} + {\rm Re}\,\Pi_{hh}^{(1)}(M_h^2)\right]\right\}$$

$$\lambda(\mu = M_t) = 0.12711 + 0.00206\left(\frac{M_h}{\rm GeV} - 125.66\right) - 0.00004\left(\frac{M_t}{\rm GeV} - 173.10\right) \pm 0.00030_{\rm th}.$$

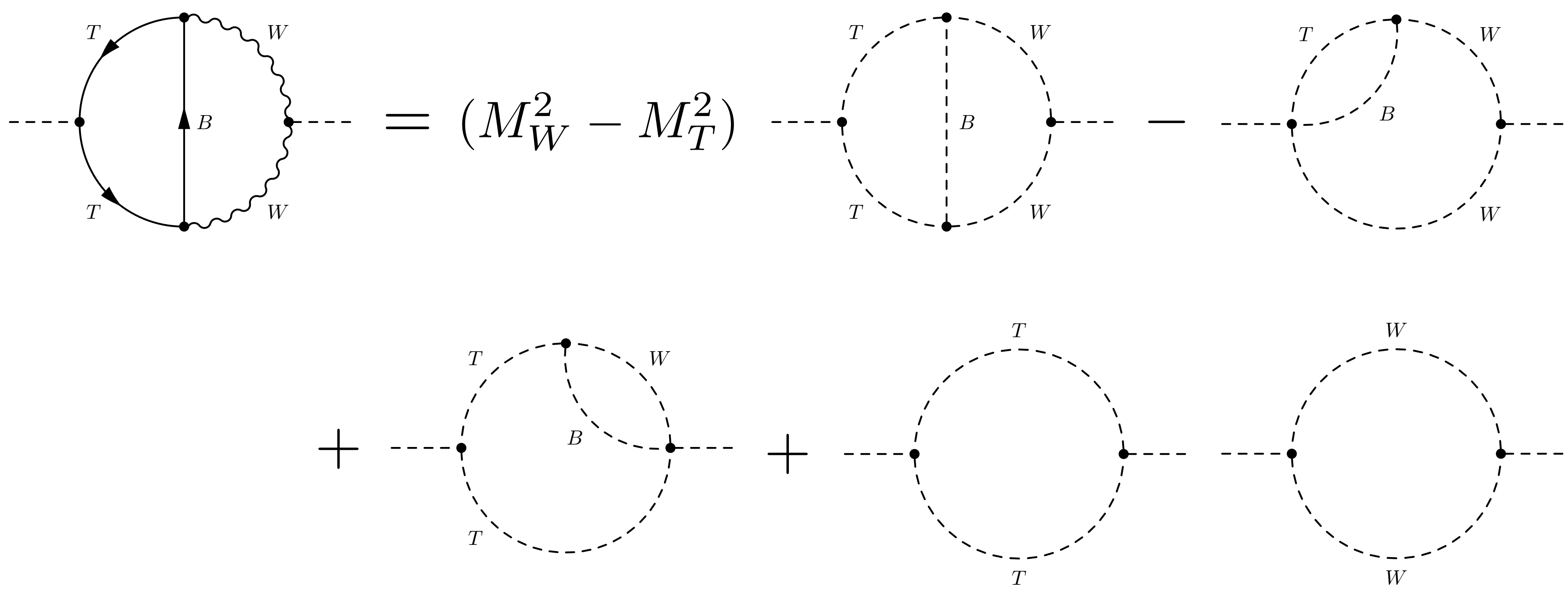
Diagrams



Calculation of two-loops integrals

A generic two loops integral is reduced in terms of Master Integrals using recursion relations.

Tarasov, 1997



Some of these integrals are known analytically.
The others are computed numerically.

Martin, 2003

Numbers...

Our input values of the SM parameters are:

$$
\begin{array}{rcll}
M_W & = & 80.384 \pm 0.014 \text{ GeV} & \text{Pole mass of the } W \text{ boson} \\
M_Z & = & 91.1876 \pm 0.0021 \text{ GeV} & \text{Pole mass of the } Z \text{ boson} \\
M_h & = & 125.66 \pm 0.34 \text{ GeV} & \text{Pole mass of the higgs} \\
M_t & = & 173.10 \pm 0.59 \pm 0.3 \text{ GeV} & \text{Pole mass of the top quark} \\
V \equiv (\sqrt{2} G_\mu)^{-1/2} & = & 246.21971 \pm 0.00006 \text{ GeV} & \text{Fermi constant for } \mu \text{ decay} \\
\alpha_3(M_Z) & = & 0.1184 \pm 0.0007 & \overline{\text{MS}} \text{ gauge SU}(3)_c \text{ coupling (5 flavours)}
\end{array}
$$

The Higgs mass is fundamental,

but the real problem, now, is the mass of the Top.

Uncertainty on Mt

- The Top mass is reconstructed using Monte Carlo methods from its decay products.
- Modeling of the event that contain jets, missing energy and initial state radiation is required.
- The Monte Carlo mass in interpreted as a Top mass with an intrinsic ambiguity of order Λ_{QCD}~250-500 MeV.
- As alternative, we can use the MS mass that can be extracted from production cross sections such as

$$pp \rightarrow t\bar{t} + X$$

S. Alekhin, A. Djouadi, S. Moch, 2013

Uncertainty on Mt

- But, Fermion masses are parameters of the QCD Lagrangian, not of the EW one.

- A MS mass in the EW theory has not a unique definition, and it depends on the definition of the vacuum:

- if the vacuum is the minimum of the tree-level potential, the MS mass is gauge invariant but the EW corrections are large.

Jegerlehner, Kalmykov, Kniehl, 2012

- if the vacuum is the minimum of the radiatively corrected potential, the EW corrections are small, but the MS mass is not gauge invariant.

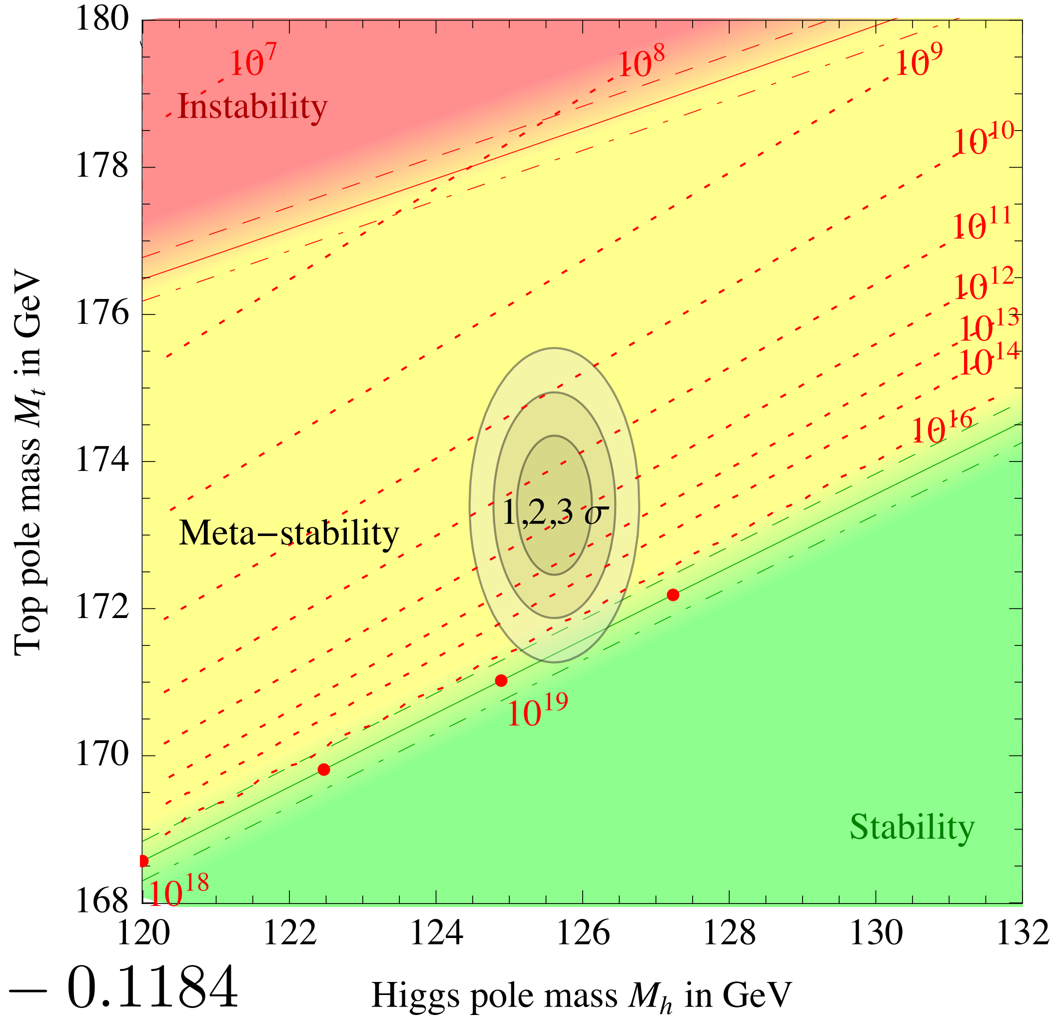
Where do we live?

The stability condition in terms of the top mass is

$$M_t < (171.53 \pm 0.15 \pm 0.23_{\alpha_3} \pm 0.15_{M_h})\,\text{GeV} = (171.53 \pm 0.42)\,\text{GeV}$$

Stability is disfavored at more than 2 σs.

The instability scale (where λ crosses 0) is

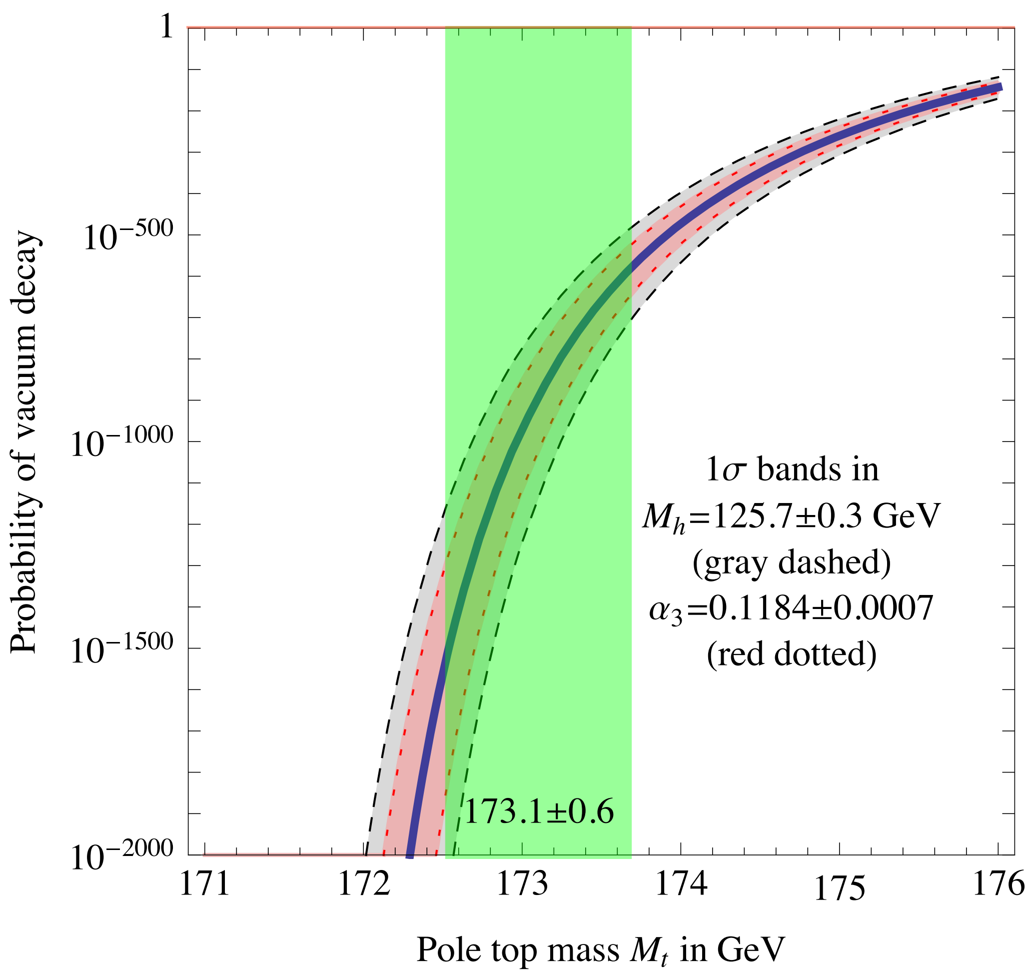


$$\log_{10}\frac{\Lambda_I}{\text{GeV}} = 11.3 + 1.0\left(\frac{M_h}{\text{GeV}} - 125.66\right) - 1.2\left(\frac{M_t}{\text{GeV}} - 173.10\right) + 0.4\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007}.$$

Vacuum decay

If our vacuum is only a local minimum of the potential, quantum tunneling towards the true minimum can happen.

Bubbles of true vacuum can form, and expand throughout the universe converting the false vacuum to true.

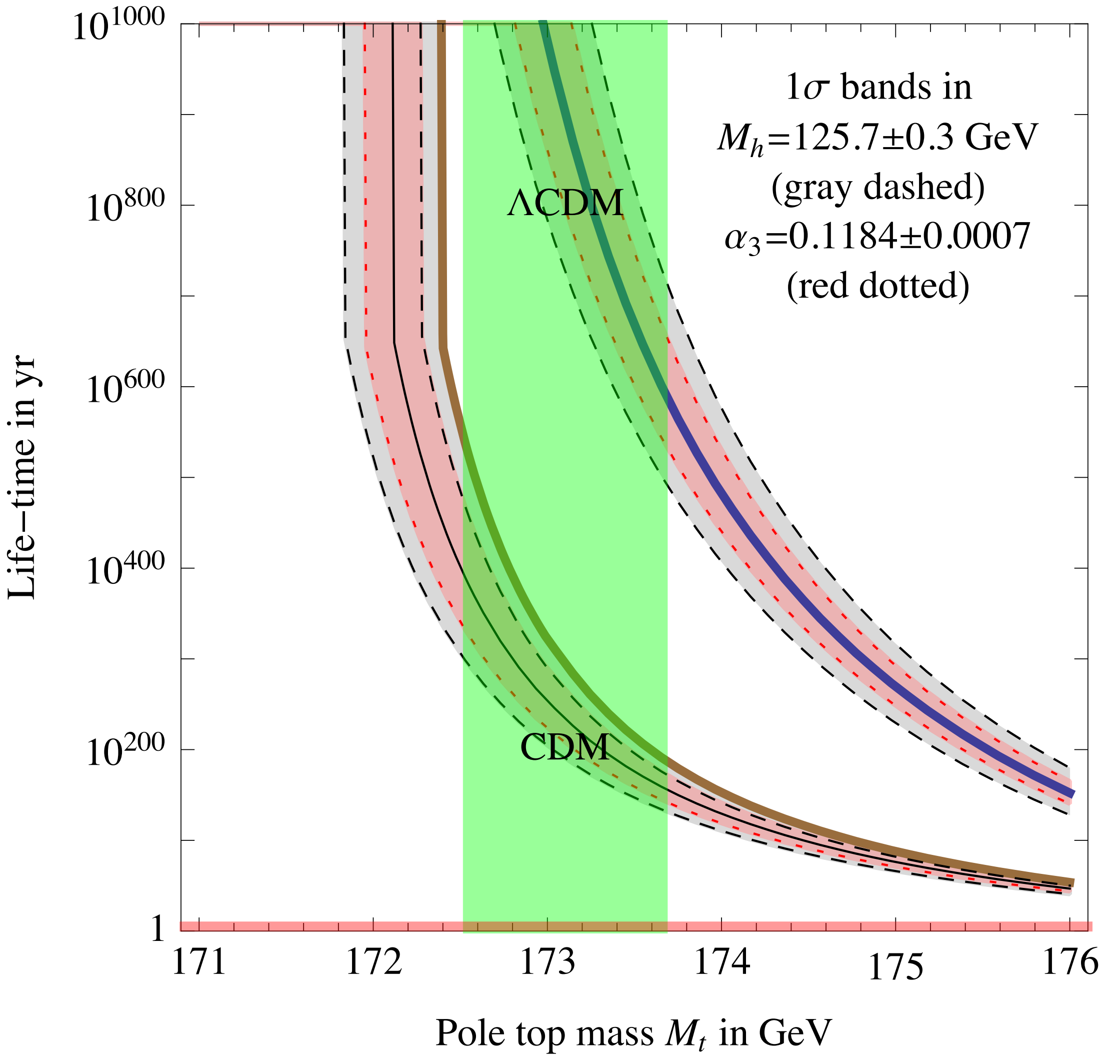


The transition probability is

$$dP = R^{-4}\, e^{-S(R)}\, dt\, dV$$

where R is the radius of the bubble and S is the action of the classical field configuration that interpolates the vacua.

Vacuum decay



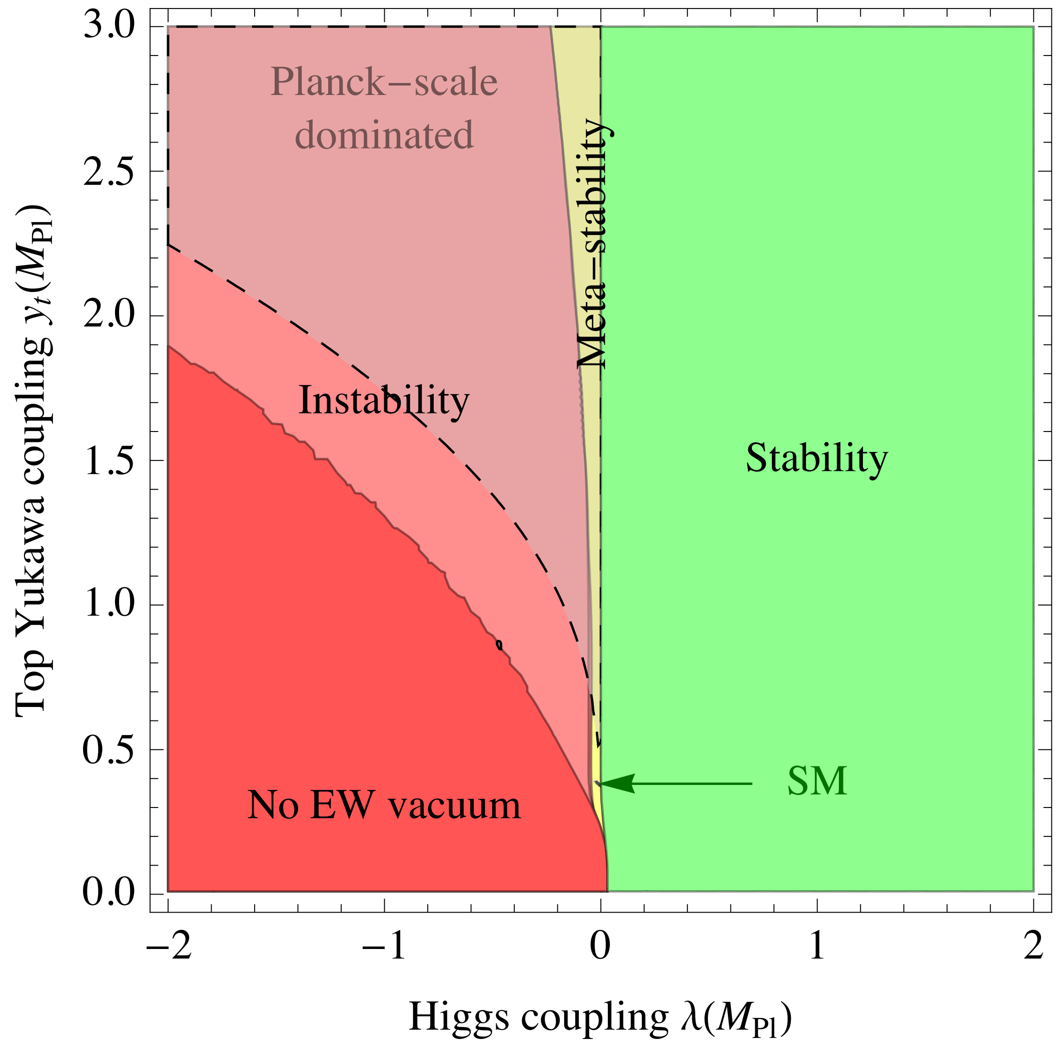
The life-time of the Universe depends on the particular cosmological model, but, in any case the SM vacuum is likely to survive for times that are enormously longer than any significant astrophysical age.

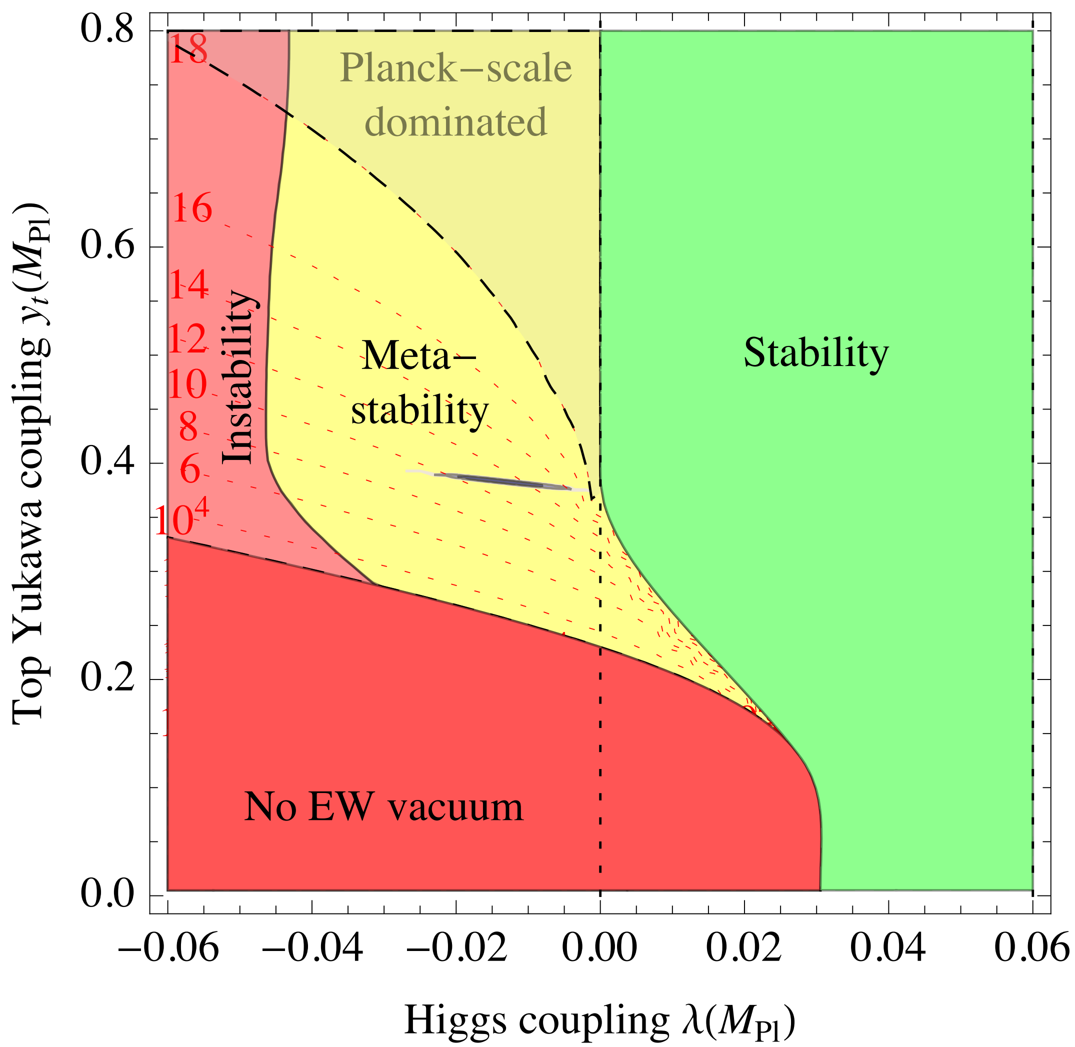
Caveat: unknown Planckian dynamics could affect the tunneling rate.

Branchina, Messina, 2013

Planck scale

It seems that we live in a near-critical condition: right at the border between stability and metastability.



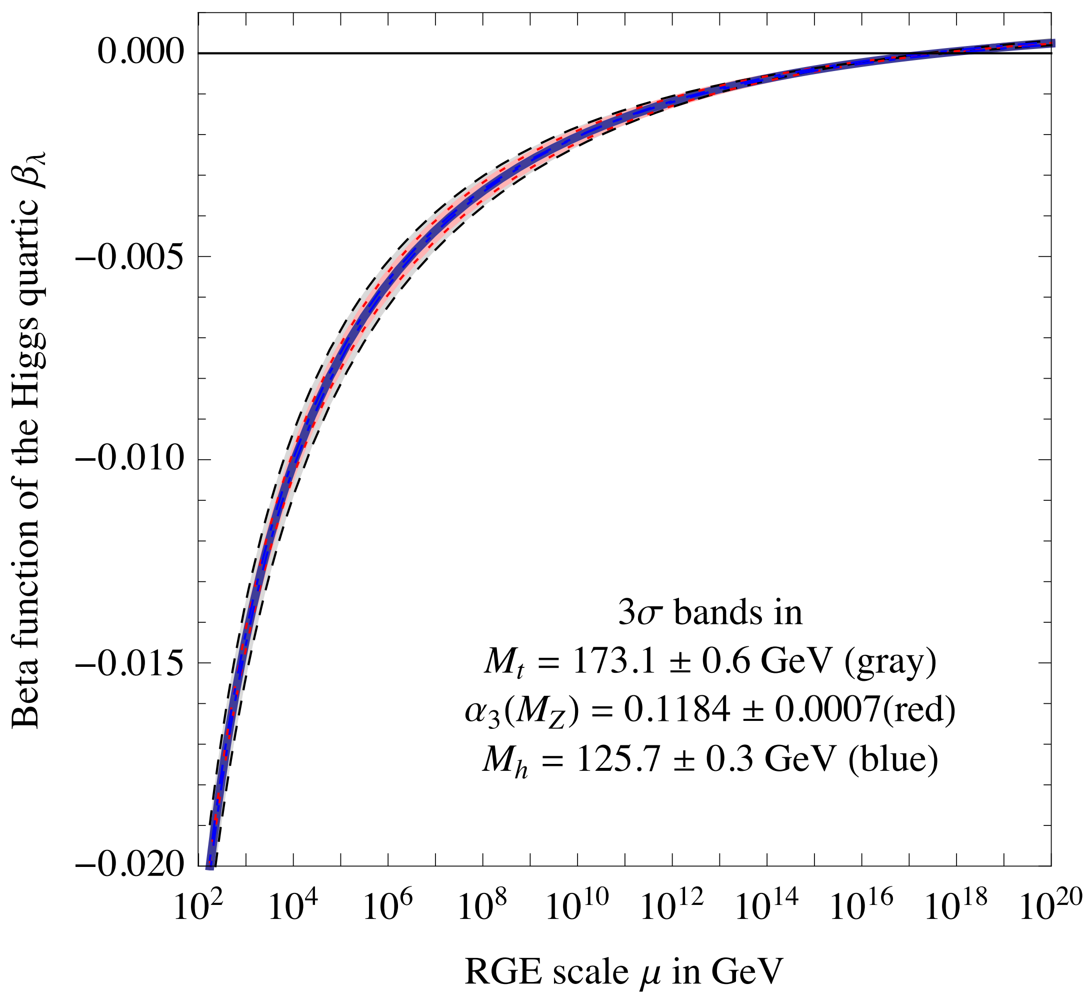


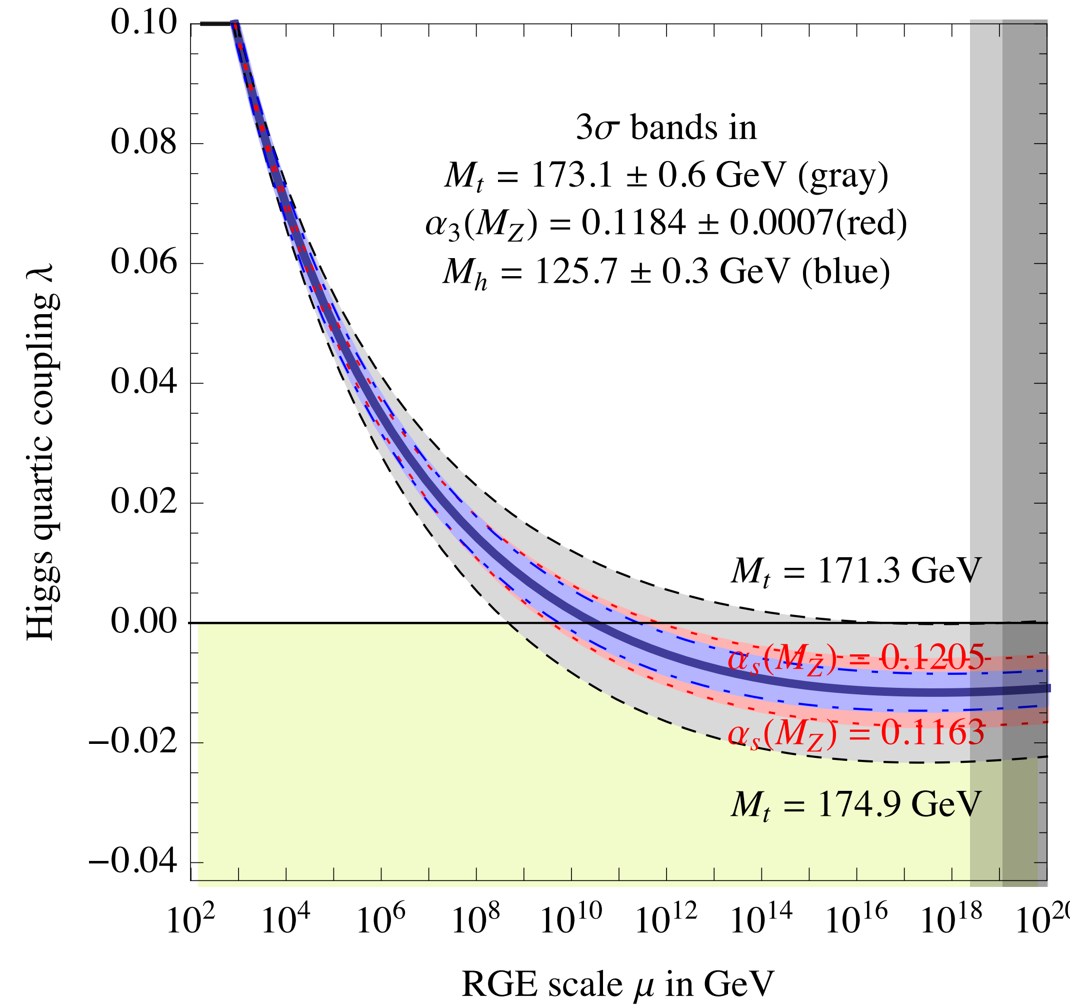
If we recast the plot at the Planck scale, it seems that we live a the end of a funnel

λ andβ

$$\lambda(M_{\rm Pl}) = -0.0113 - 0.0065\left(\frac{M_t}{\rm GeV} - 173.10\right) + 0.0018\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} + 0.0029\left(\frac{M_h}{\rm GeV} - 125.66\right)$$

Also, λ never becomes to negative around Planck. β is also very small.

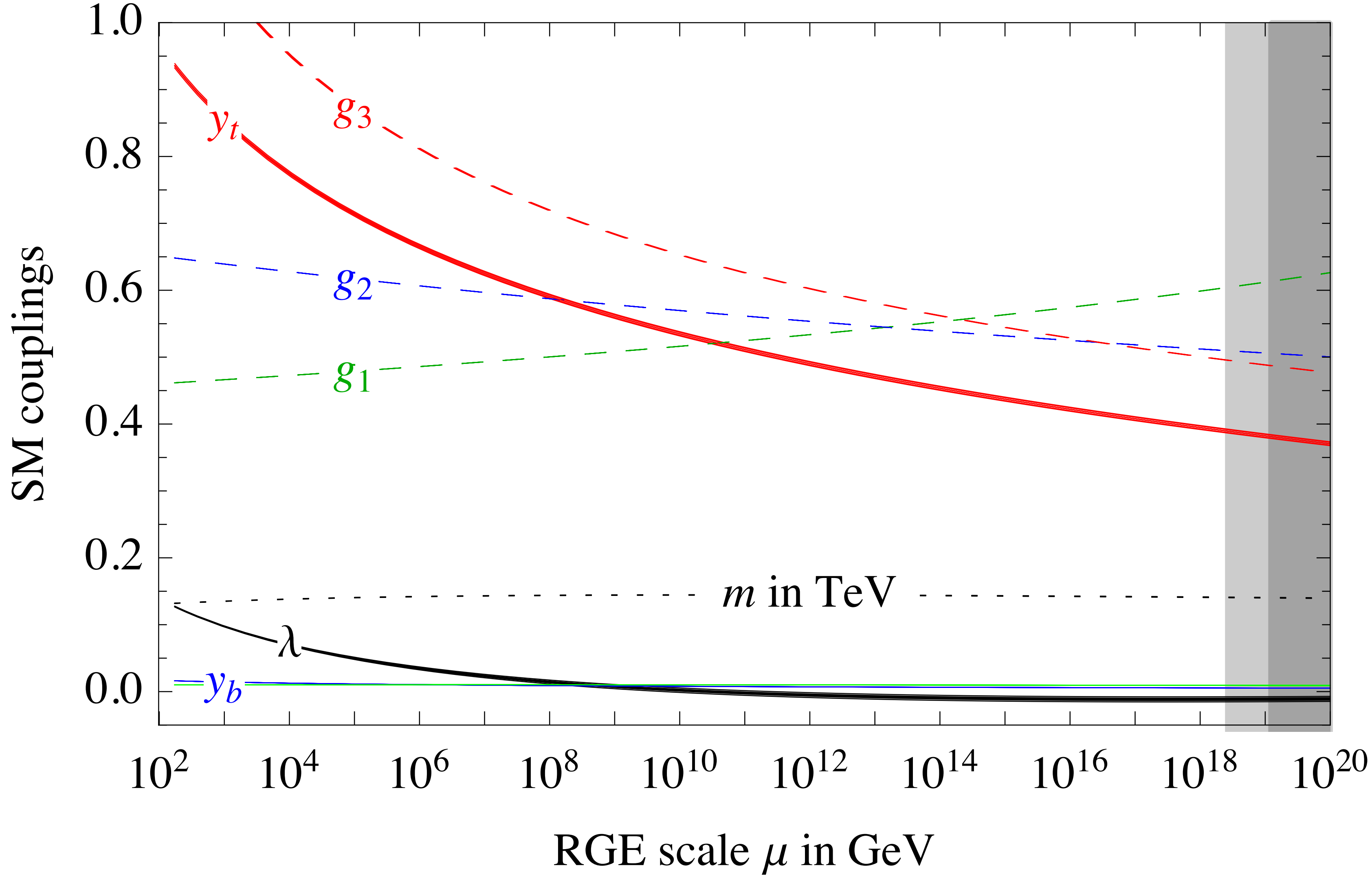




The situation seems rather special

λ andβ

$$\beta_\lambda = 24\lambda^2 + \lambda(-9g_2^2 - 3g_1^2 + 12y_t^2) + \frac{3}{4}g2^4 + \frac{3}{8}(g1^2 + g2^2)^2 - 6y_t^4$$



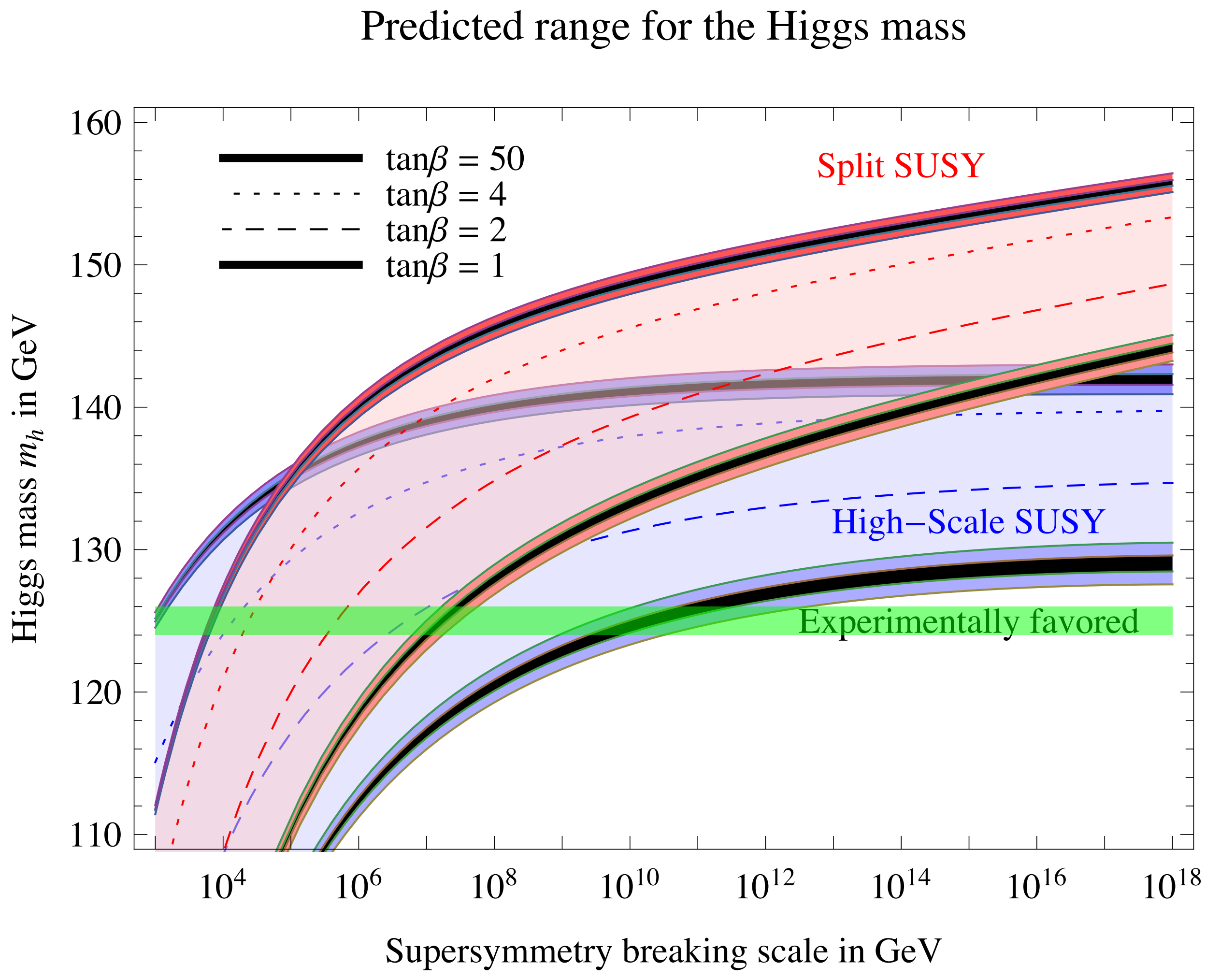
β~0 is due to the cancellation between large contributions.

Matching?

- These results could be only a coincidence, or they could have a deeper meaning.
- Maybe there is some kind of new dynamics that occurs at a very high energy scale.
- For example the value of λ could be due to a matching condition with some high energy new physics.

SUSY

We can add a scalar in order to stabilize the EW potential



We can also consider simple variants of SUSY.
In particular Split SUSY and High Energy supersymmetry, can provide a matching for λ:

$$\lambda(\tilde{m}) = \frac{1}{8}[g^2(\tilde{m}) + g'^2(\tilde{m})] \cos^2(2\beta)$$

Degrassi, Di Vita, Elias-Miró, Espinosa,Giudice, Isidori, Strumia (13)

Multiverse?

- Or, maybe, that's the consequence of some multiverse dynamics.
- The value of λ could be due to a mechanics that push it to negative value for λ>0, or to positive ones for λ<0.
- Other models have λ and y given by a statistics.

Conclusions

- The SM is in (too) good health.
- If the SM is the only theory the EW vacuum is in a meta-stable state. (Absolute stability is disfavored at more than 2 σs)
- The exact value of the top mass plays the central role between the full stability or metastability (preferred) options.
- λ and β_λ are very close to zero around the Planck mass: deep meaning or coincidence?

Thank you!