Quark-hadron duality and large-x PDFs

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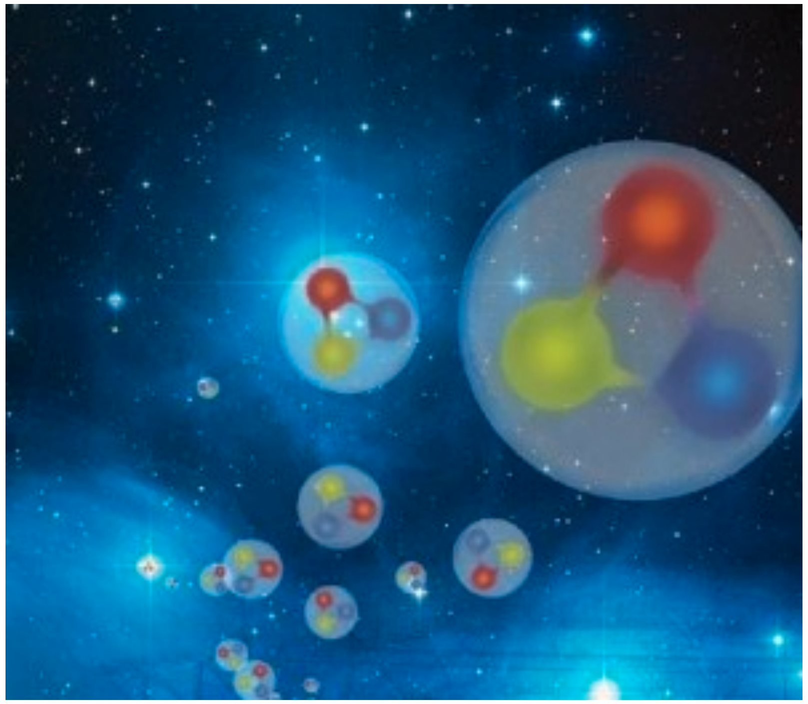
Interactions Fondamentales en Physique et Astrophysique

Outline

- Quark-Hadron Duality
  - Intersection of pQCD and nonperturbative QCD
  - Strong coupling constant at low energy
- Hadronic scale
  - Initial conditions from nonperturbative QCD
- Large-x PDFs matter!



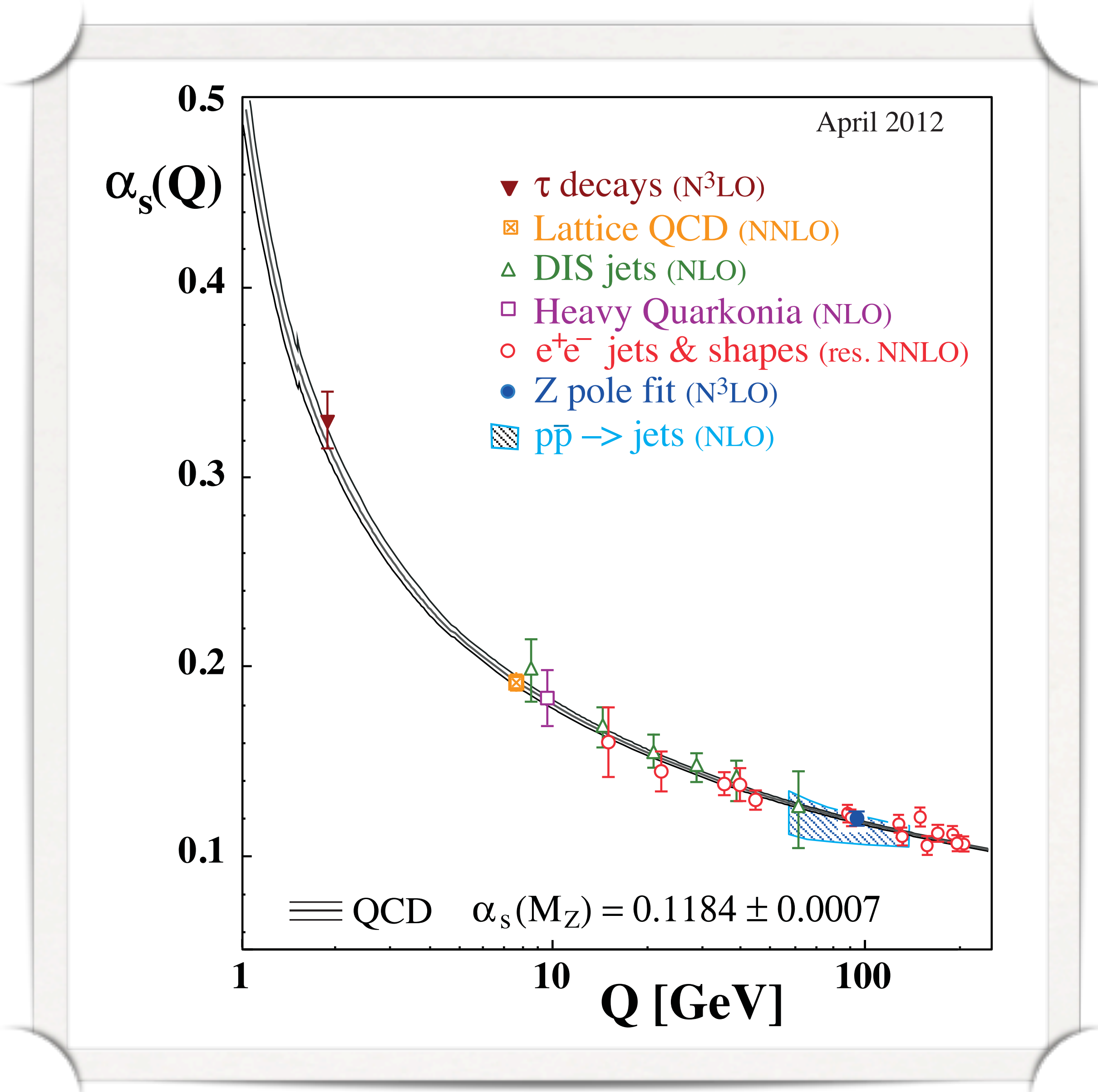
Based on collaboration with S. Liuti

Phys.Lett. B726 (2013)

and work(s) in progress

Quantum ChromoDynamics

Running of α_s

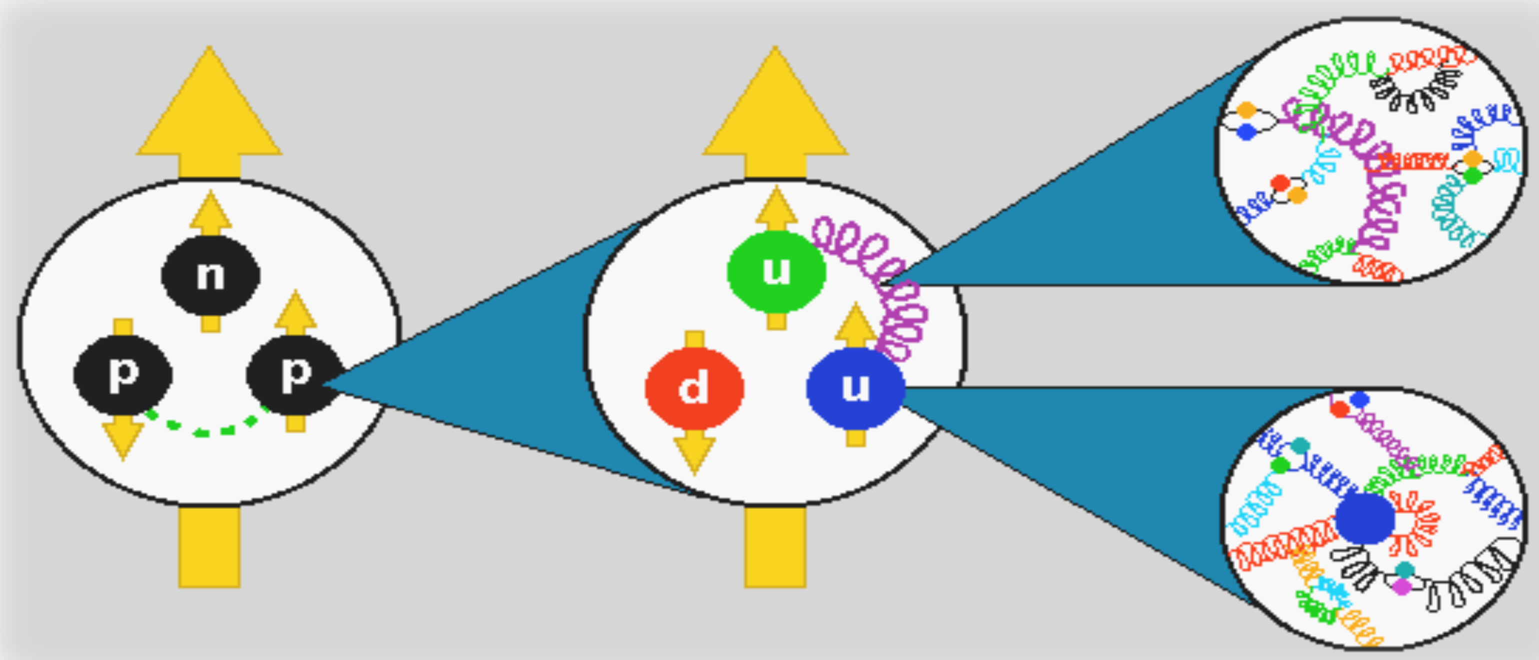


Confinement

Asymptotic Freedom

Quantum ChromoDynamics

Hadron ⇔ ``Constituent” quarks ⇔ Current quarks

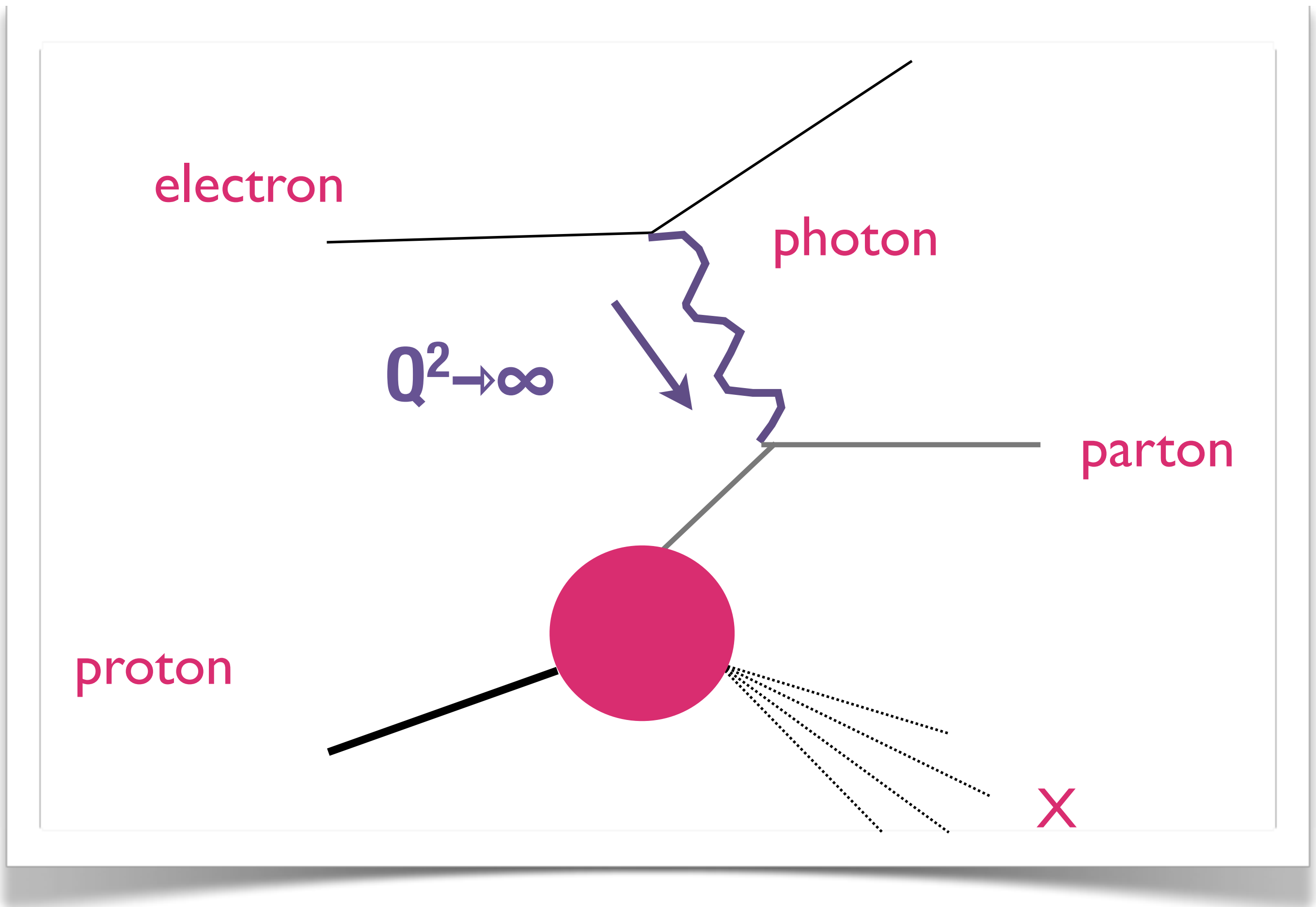


Nonperturbative vs. Perturbative QCD

Evolution in Q^2

Deeper in the structure

Hard probes



$$\frac{d\sigma}{d\nu dQ^2} \propto l_{\mu\nu} W^{\mu\nu}$$

Parton Model

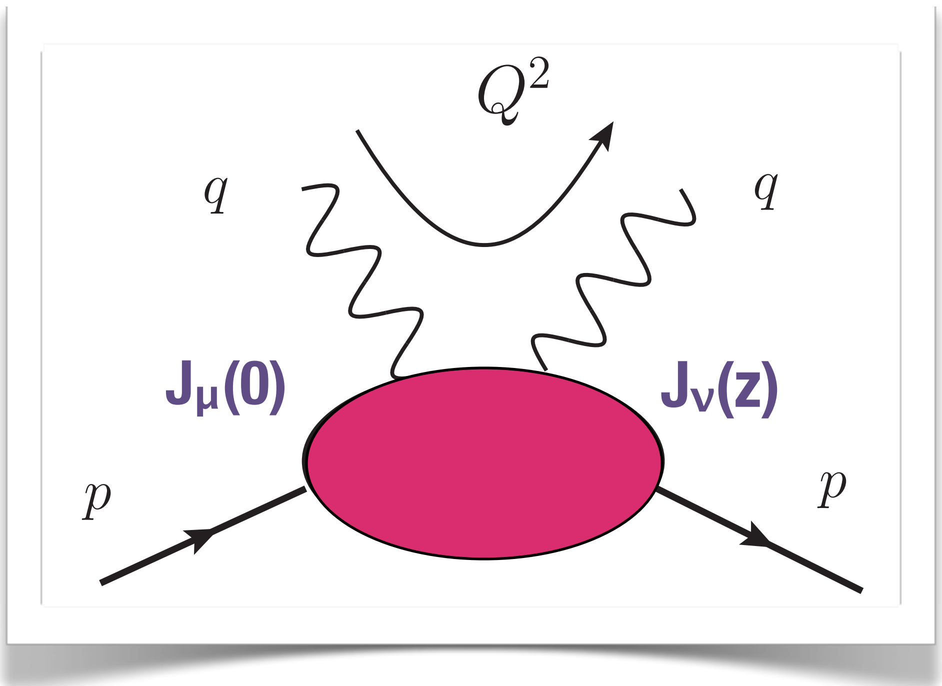
Kinematics of the Bjorken scaling

$Q^2 \rightarrow \infty$

$p.q \rightarrow \infty$

$Q^2/2p.q \equiv x = \text{finite}$

Parton Distribution Functions



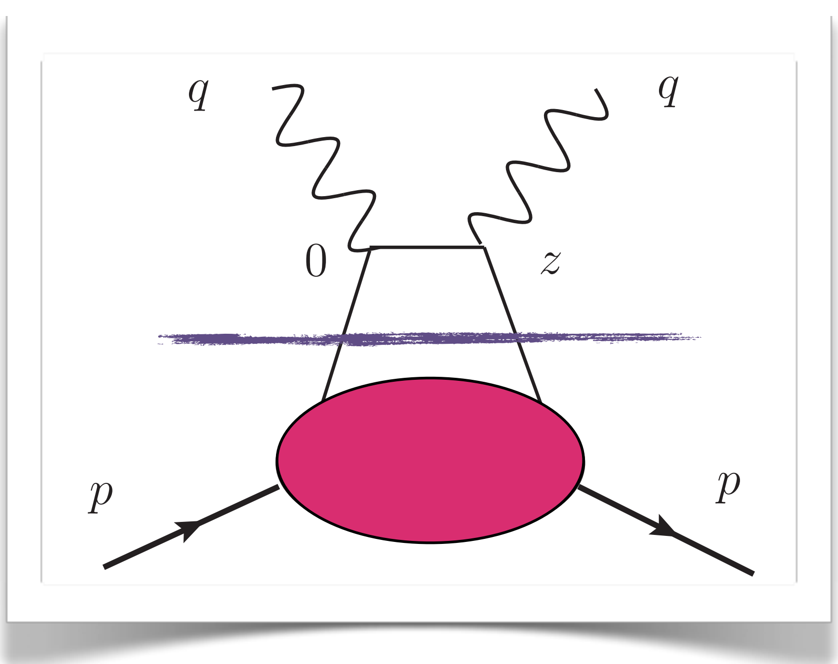
In the Bjorken limit,

hadronic tensor dominated by the light-cone $z^2 \sim 0$

→ can be "ordered" by OPE

$$\frac{d\sigma}{d\nu dQ^2} \propto l_{\mu\nu} W^{\mu\nu} \propto F_2(x)$$

Factorization



Perturbative part

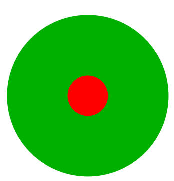
$$F_2(x) \overset{\text{LO}}{=} \sum_{q\bar{q}} \int_0^1 dy\, y\, e_q^2\, \delta(y-x) f_1(y, Q^2)$$

Non-perturbative part=PDF

Leading order structure

Selection of the QCD operator

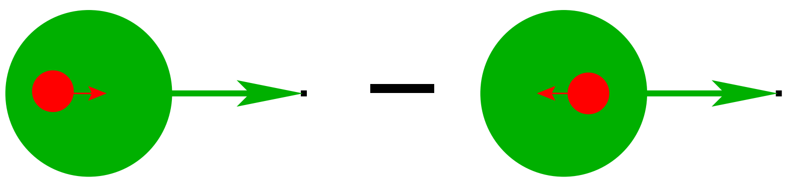
$f_1(x)$



Vector

U target

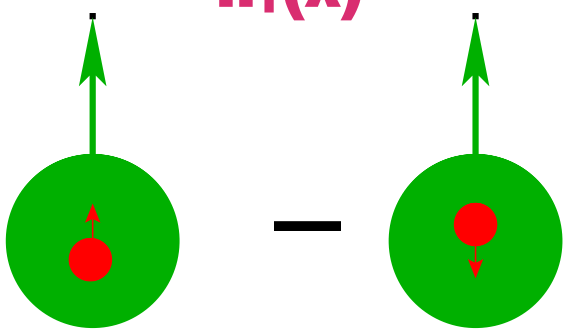
$g_1(x)$



Axial vector

LP target

$h_1(x)$



Tensor

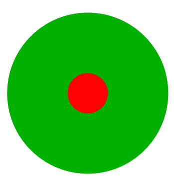
TP target

PDFs are universal

Leading order structure

Selection of the QCD operator

$f_1(x)$



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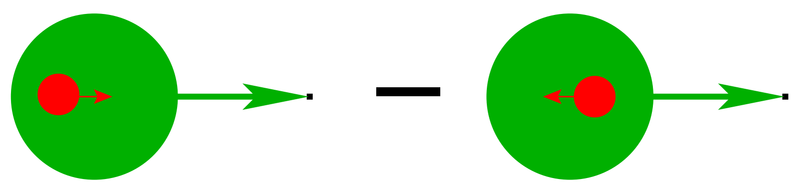

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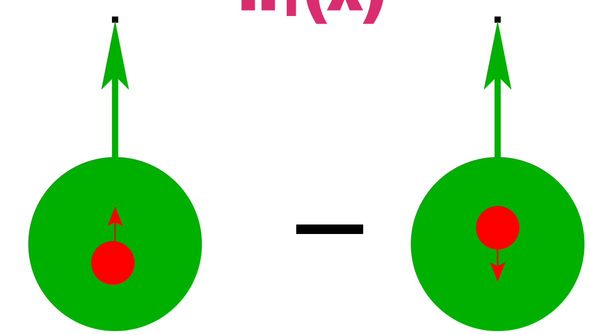


Tensor

TP target

That's the well-known ``PDF"

PDFs are universal

Scaling violations

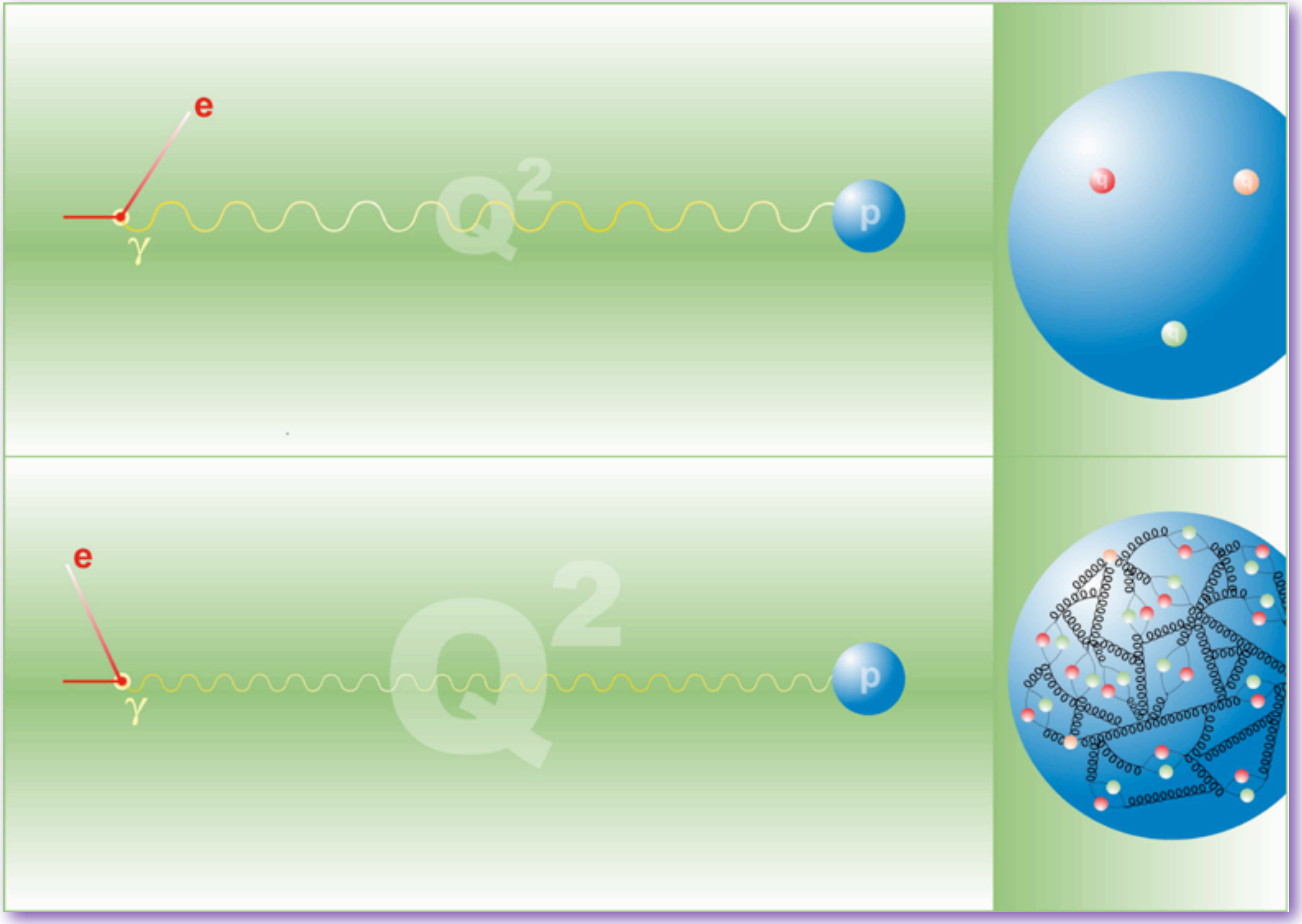


Image credit: DESY Hamburg

Structure Functions and DIS

Parton Model
Bjorken scaling

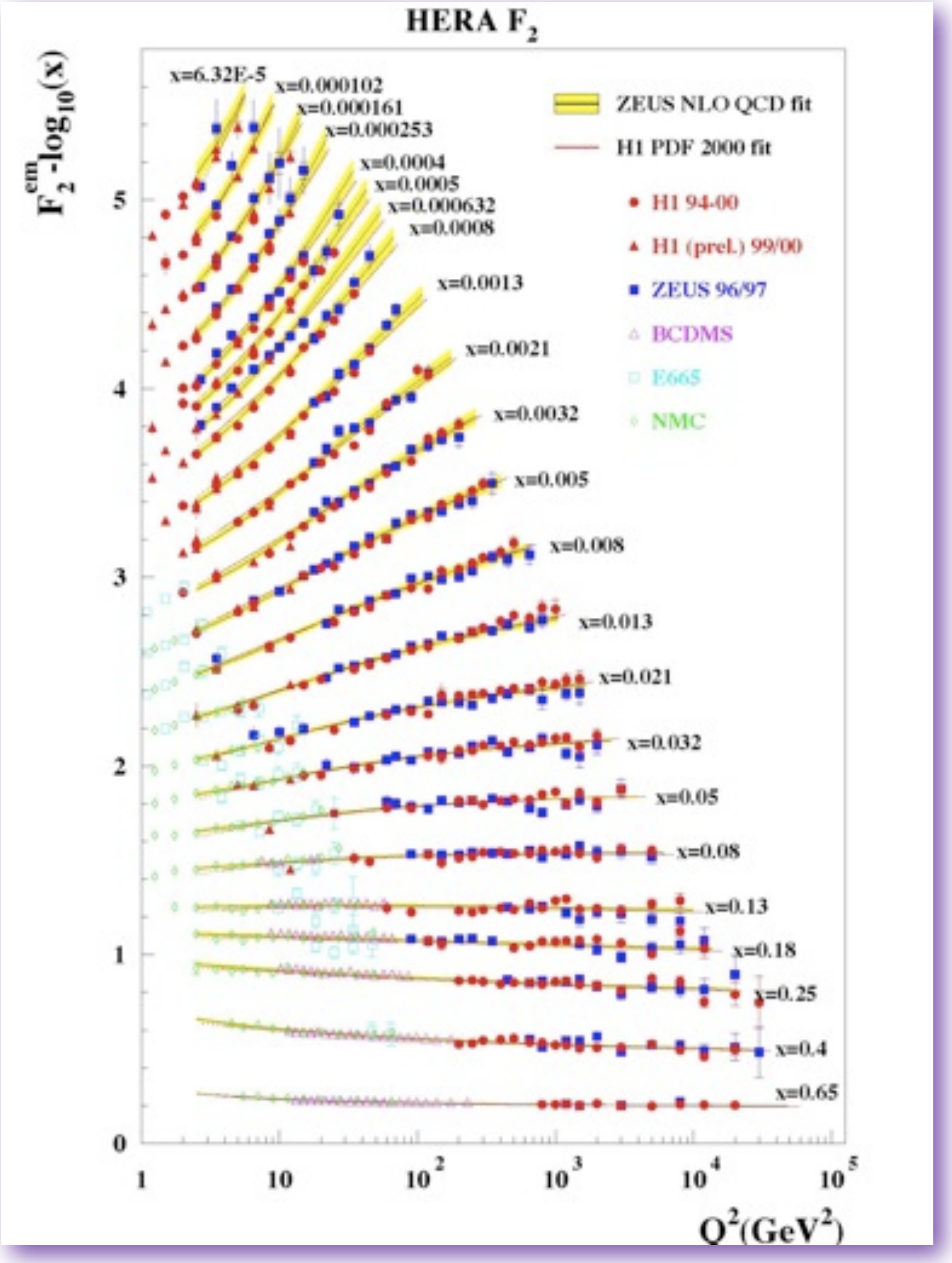
$$F_2(x,Q^2) = \sum_{q\bar{q}} \int_0^1 d\xi\, f_1(x,Q^2)\, x e_q^2\, \delta(x-\xi)$$

$$F_2(x) \equiv F_2(x,Q^2)$$

Scaling violations lead to

Q^2-dependence of the Structure Functions

- DGLAP equations
  [Dokshitzer–Gribov–Lipatov Altarelli-Parisi]
- Jargon: “Q^2 or QCD evolution”

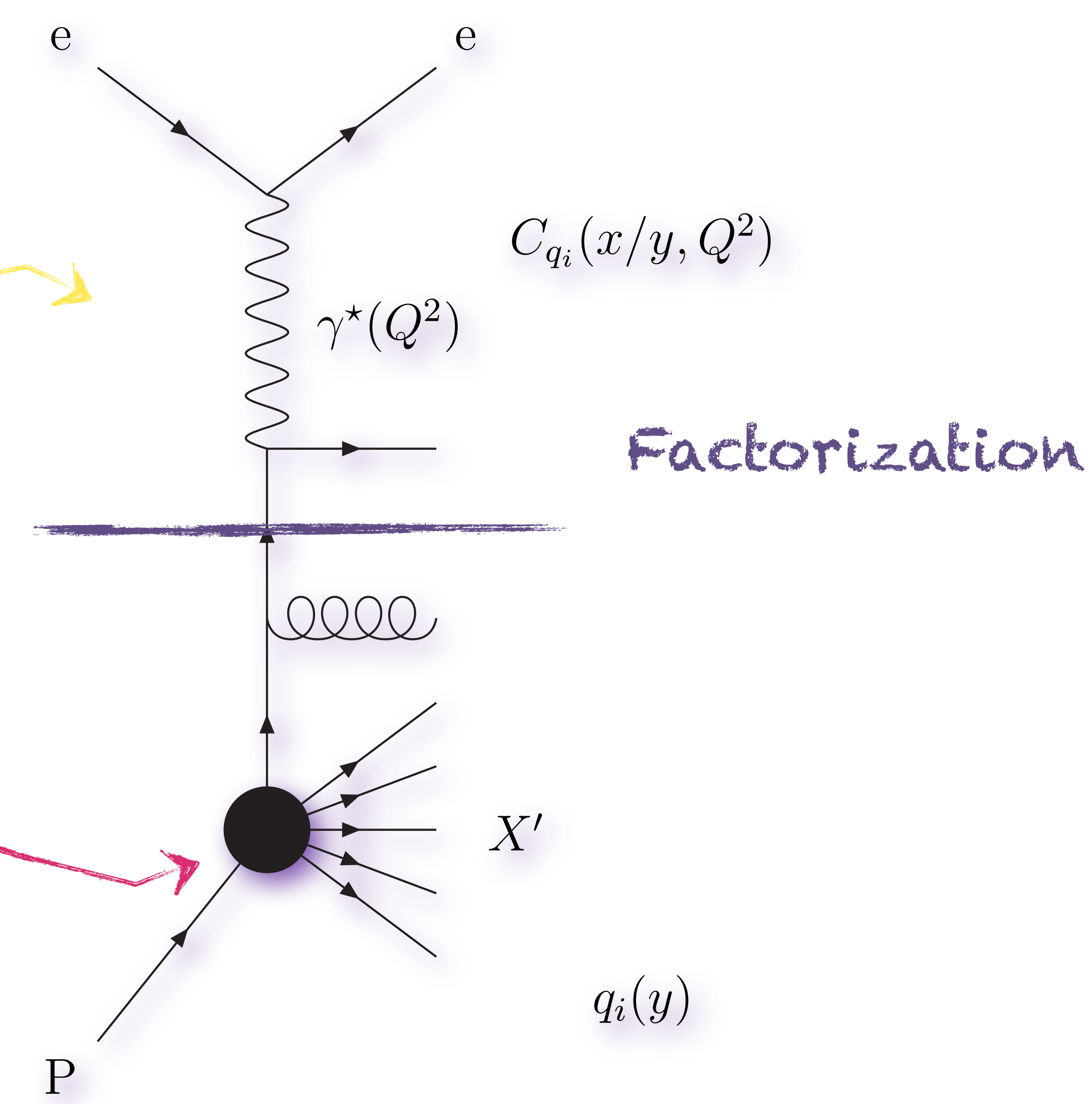


Perturbative QCD

Q^2 evolution → DGLAP equations

$$F(x,Q^2) \sim \sum_{i=q_f,\bar{q}_f,g} \int_x^1 \frac{\mathrm{d}y}{y}\, C_i\Big(\frac{x}{y}, 1; \alpha_S(Q^2)\Big)\, f_i(y,Q^2)$$

$$\mu_F \frac{\mathrm{d}}{\mathrm{d}\mu_F} f_i(y,\mu_F^2) \;=\; \sum_{j=q_f,\bar{q}_f,g} \int_y^1 \frac{\mathrm{d}z}{z}\, P_{ij}\Big(\frac{y}{z};\alpha_S\Big)\, f_j(z,\mu_F^2)$$



Choice of factorization scheme !

PDFs are non-perturbative objects

- related to confinement and chiral symmetry
- transition of degrees of freedom
- related to angular momentum (of quarks and gluons)
- ...

- little first principles based constraints
  - QCD sum rules, symmetries, ...
- evaluated in models for hadron structure
- fitted from data (Q^2 behavior = pQCD)

Resolution matters ⟶ NonPerturbative scales

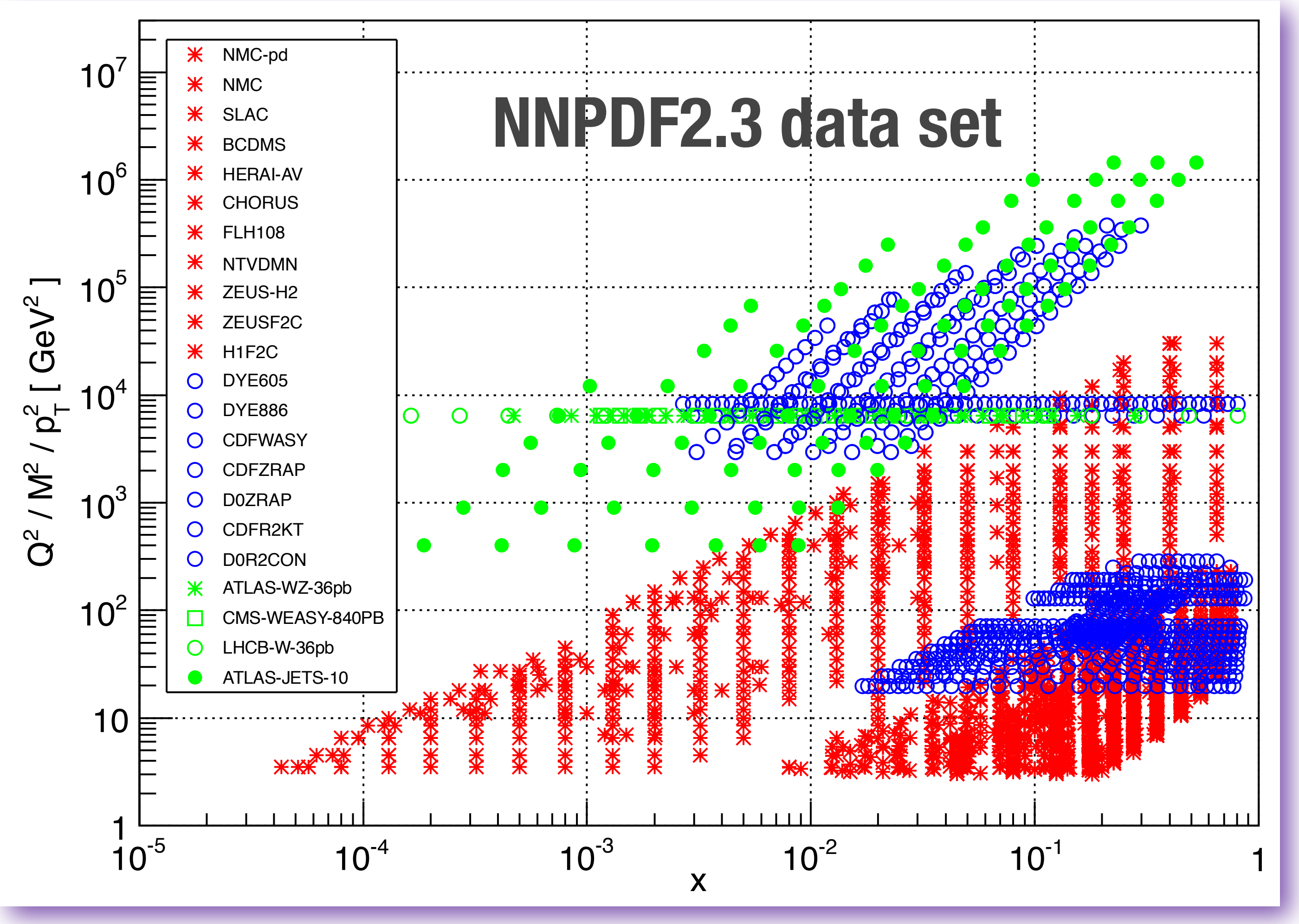
Parton Distributions from Experiments

- **Where?**
  - **DIS: eP→eX**
  - **Drell-Yan: PP→l^+l^-X**
  - **jets ...**
- **How?**
  - **fit from scale Q_0^2**
  - **functional form**

$$f_i(x, Q_0^2) = x^{\alpha_i}(1-x)^{\beta_i} g_i(x)$$

  - **treatment of error**

$$\Delta\chi^2 = T^2$$

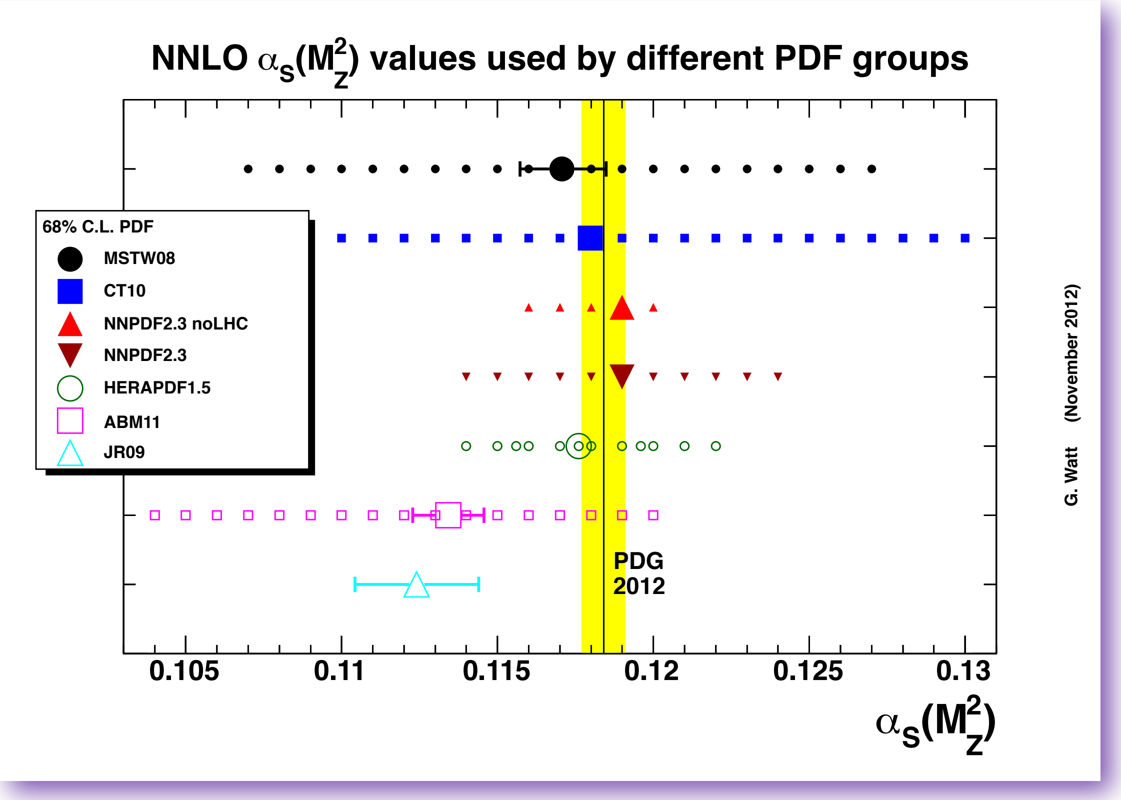


S.Forte & G.Watt
Ann.Rev.Nucl.Part.Sci. 63 (2013)

- **Who?**
  - ✱ ABM
  - ✱ HERAfitter
  - ✱ CTEQ
  - ✱ MSTW
  - ✱ NNPDF
  - ✱ SOMPDF

Uncertainties for PDF from Low Energy

- Standard approach for fitting PDF: arbitrary Q_0^2>1GeV^2
- Value of $\alpha_s(M_Z^2)$ differs for each set



Improvements?

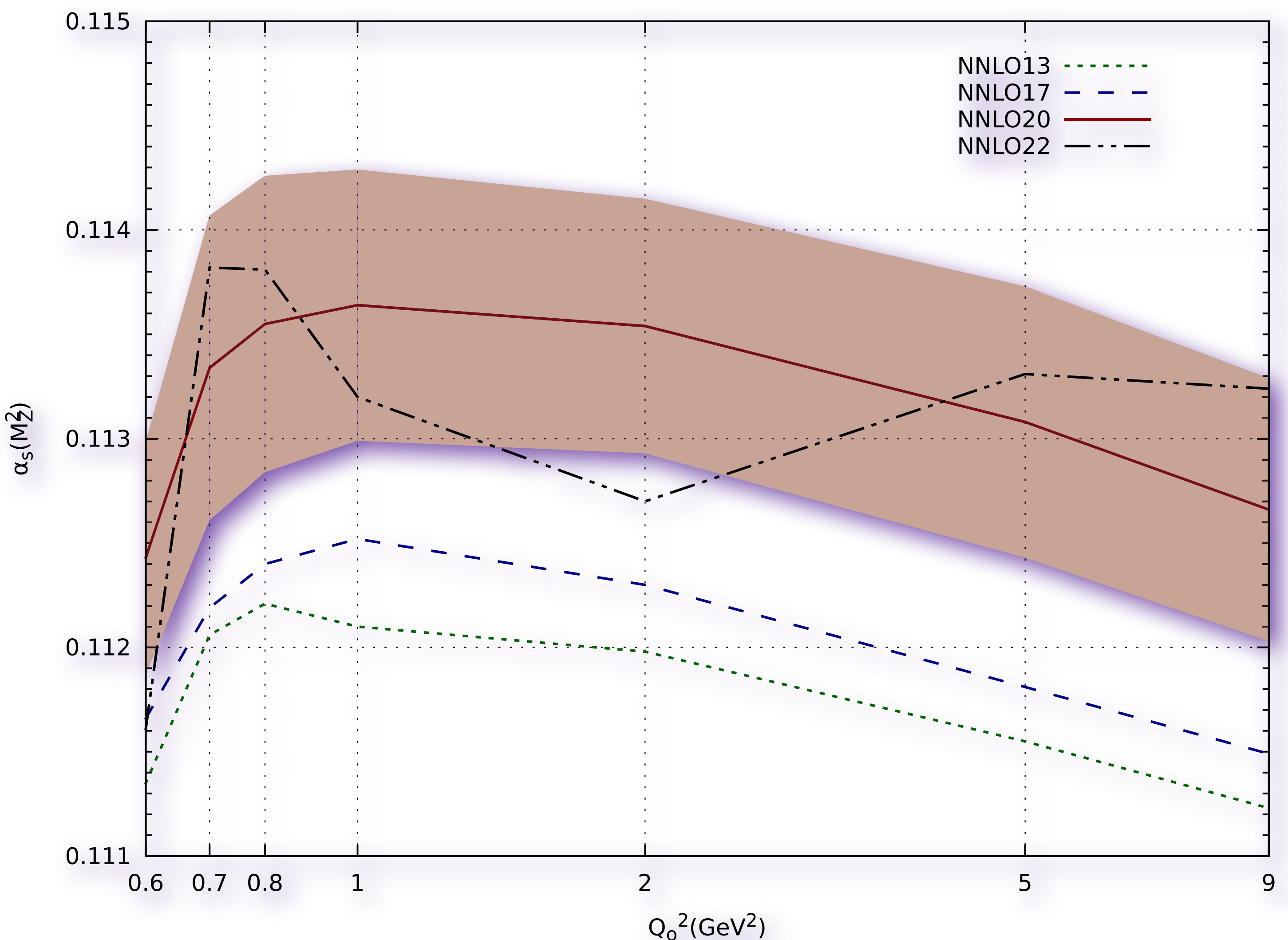
- Dynamical GJR parameterization: Q_0^2 as a guideline !
  - ➡ Valence vs. radiative behaviour
  - ➡ Q_0^2 turns out to be of the order of 0.5GeV^2 (with $\Lambda_{NLO}^{n_f=3}$~303MeV)
- Non-perturbative input needed!

Input vs. Hadronic scale

- input scale uncertainty studied in GJR/JR
- *procedural* bias
- **red band**: experimental uncertainty
- uncertainty from scale

~ order of magnitude as exp. unc.

P. Jimenez-Delgado
Physics Letters B 714 (2012)



Hadronic Scale from models

Standard method:

- use RGE
- one first principle based assumption
- set partonic scenarios

Parisi & Petronzio, Phys. Lett. B 62 (1976) 331
Traini et al, Nucl. Phys. A 614, 472 (1997)
Stratmann, Z.Phys. C60 (1993)

Say there exists a scale at which there is no sea and no gluon, then

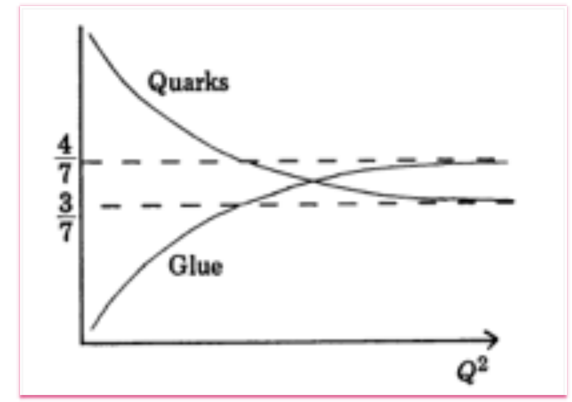
$$\langle (u_v + d_v)\,(\mu_0^2) \rangle_{n=2} = 1$$

QCD evolution introduces gluons and sea quarks:

$$\langle q_v(Q^2) \rangle_n = \langle q_v(\mu_0^2) \rangle_n \left(\frac{\alpha(Q^2)}{\alpha(\mu_0^2)} \right)^{d^n_{NS}}$$

$$\langle (u_v + d_v)\,(Q^2 = 10\,\mathrm{GeV}^2) \rangle_{n=2} = 0.36$$

DATA= PDFs parameterization



R.G.Roberts
"The Structure of the Proton"

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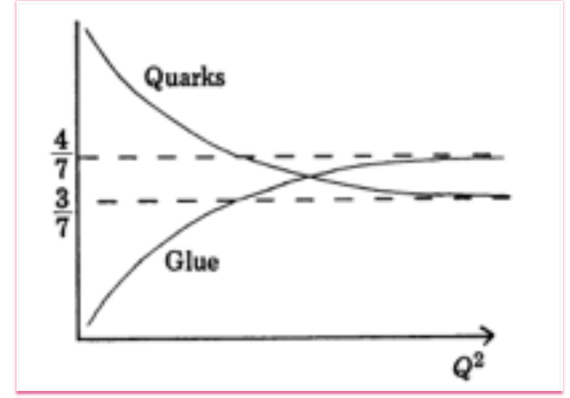

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DATA= PDFs parameterization

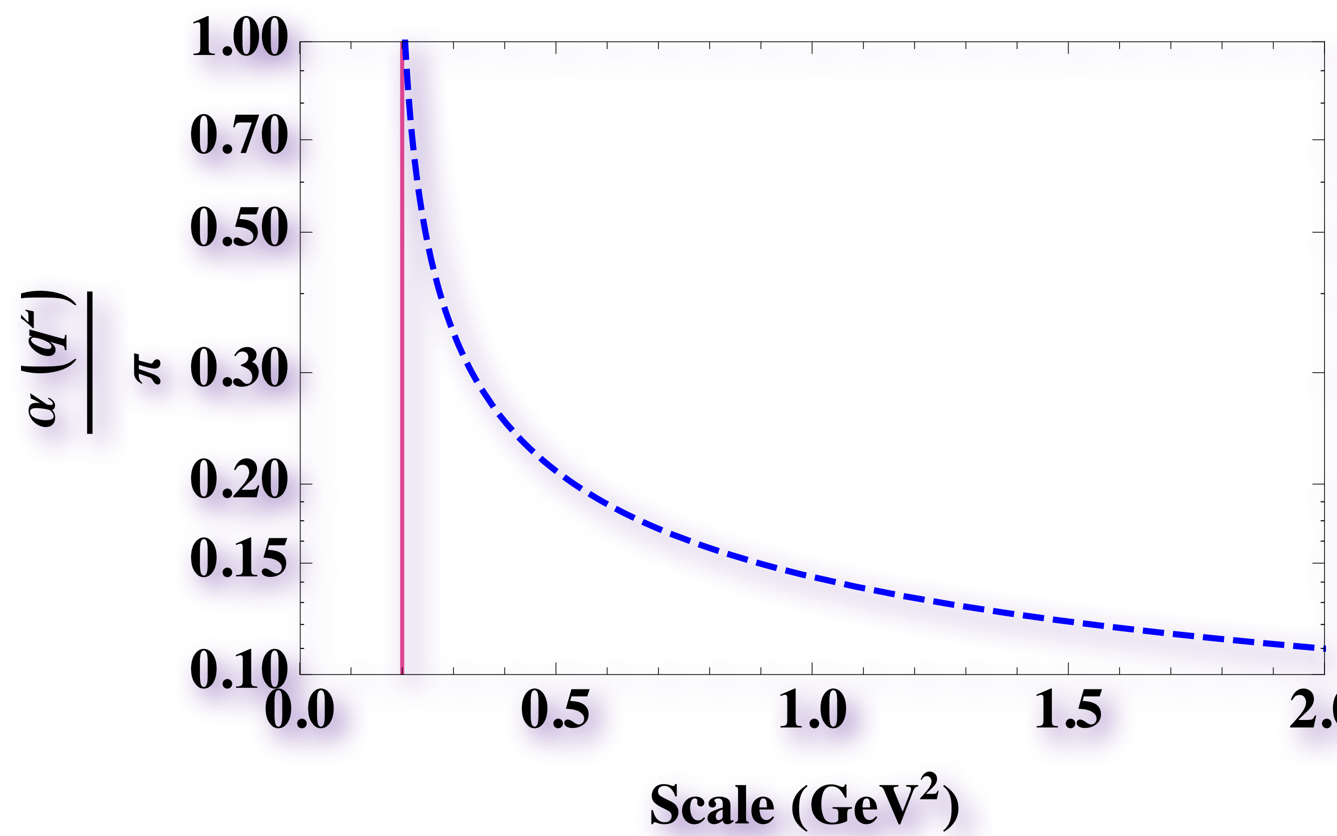
Evolve in energy until 2nd moment=1
Find μ_0^2~0.1GeV^2 +$\Delta\mu_0^2$

Hadronic scale

- What does a low $\mu_0^2 \sim 0.2 GeV^2$ $+\Delta\mu_0^2$ means?

guess for MSTW08NLO

NLO α exact solution (Λ=0.402GeV)

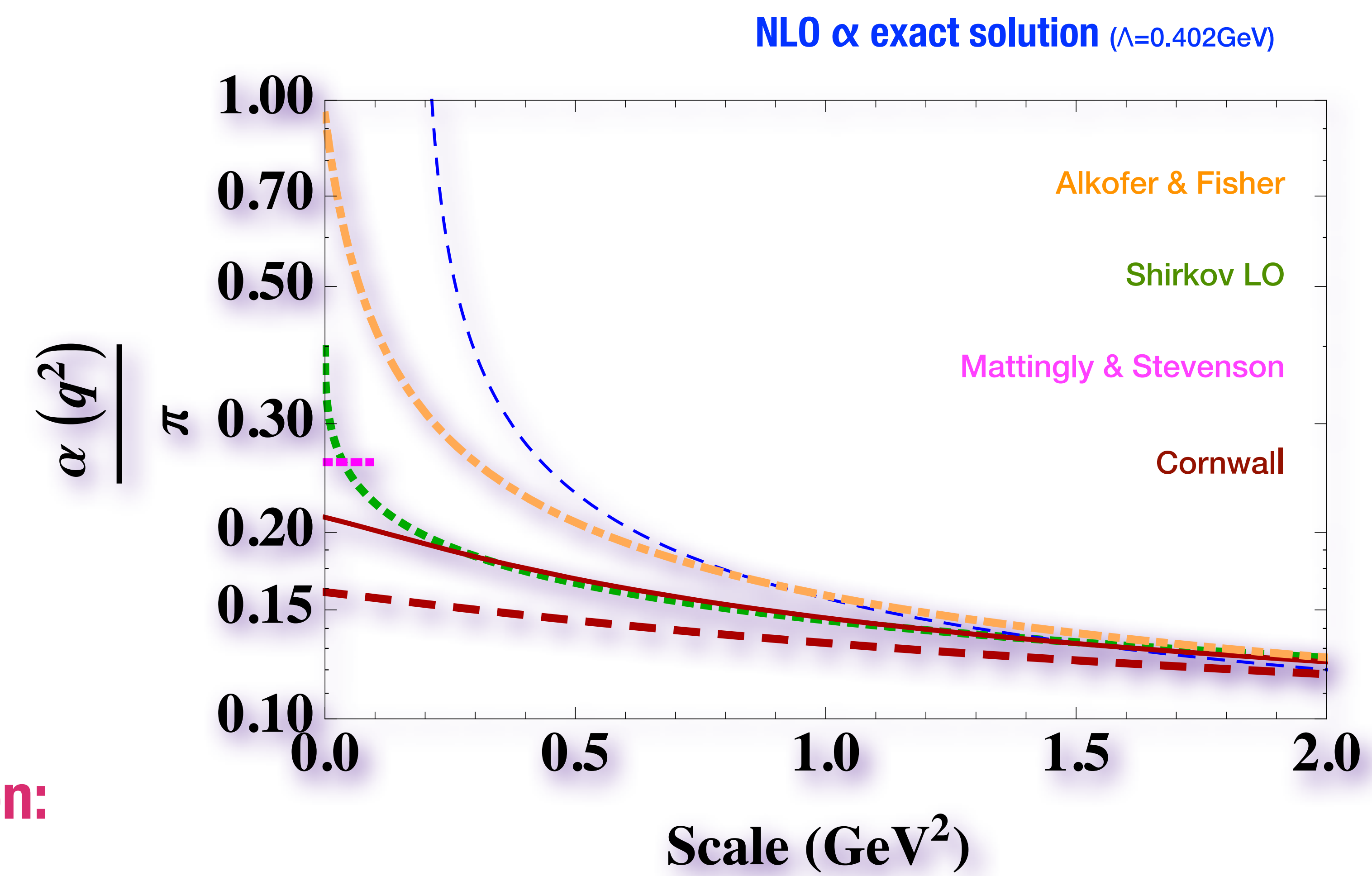


- Approaching the Landau pole...

Effective charges

The non-perturbative approach:

- Importance of finite couplings
- Taming the Landau pole



The non-perturbative interpretation:

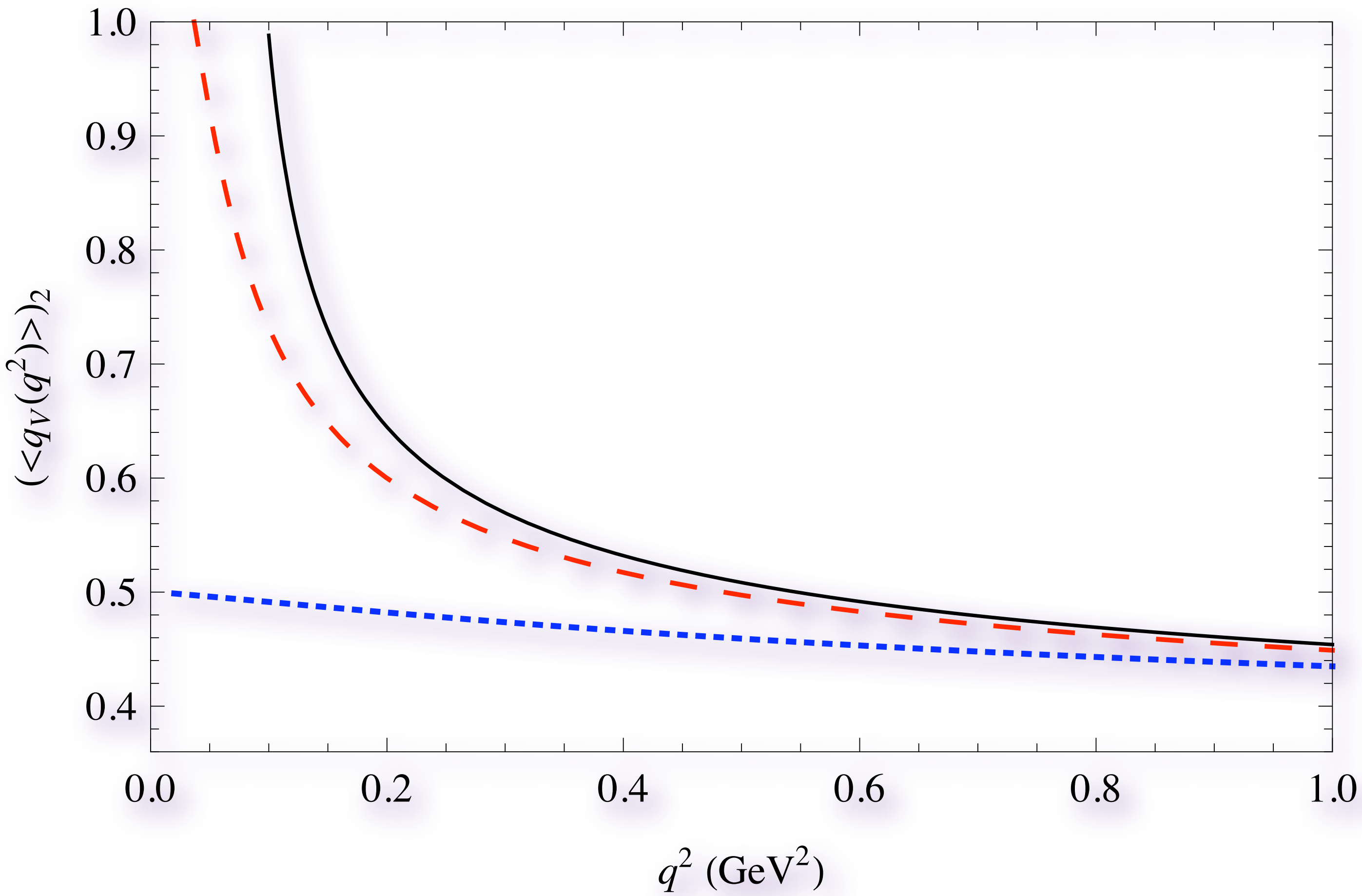
- Effective couplings from phenomenology
- Dimensional transmutation (RG-improved)
  - from RS dependence to Observable dependence (à la Grunberg)

Uncertainty on the hadronic scale

A.C., Vento & Scopetta,
Eur.Phys.J.A47

We can find a scale for which the sum rule is OK

$$0.1 < \frac{\alpha_s(\mu_0^2)}{4\pi} < 0.3$$



Cornwall's massive gluon approach

- LO perturbative evolution Λ=250 MeV ; $\overline{MS}$ scheme
- Low gluon mass scenario NP coupling constant

  (m_0=250 MeV ; Λ=250 MeV ; ρ=1.5)
- High gluon mass scenario NP coupling constant

  (m_0=500 MeV ; Λ=250 MeV ; ρ=2)

J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

Need to better constrain Q_0^2!

Strong correlation with dof

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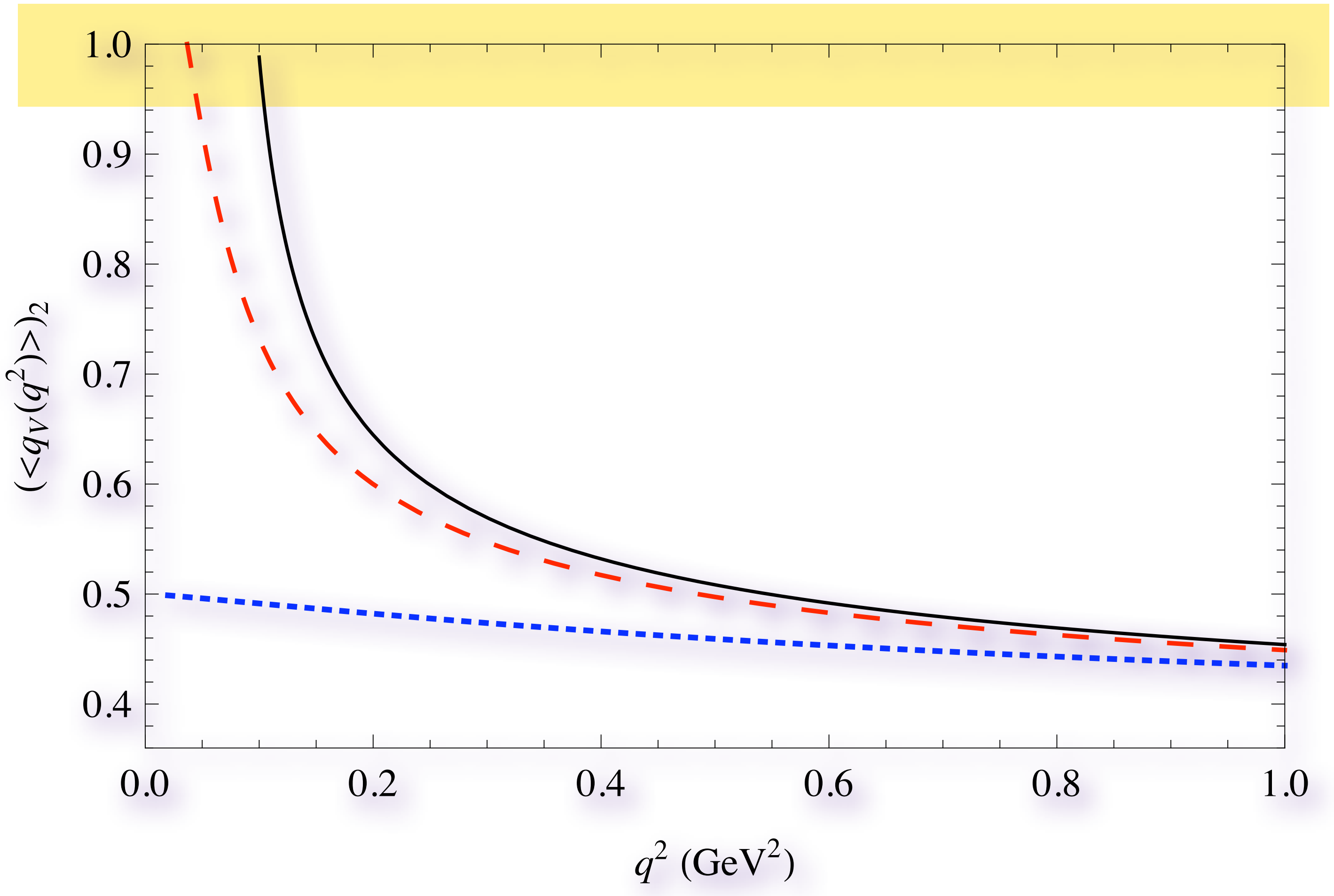

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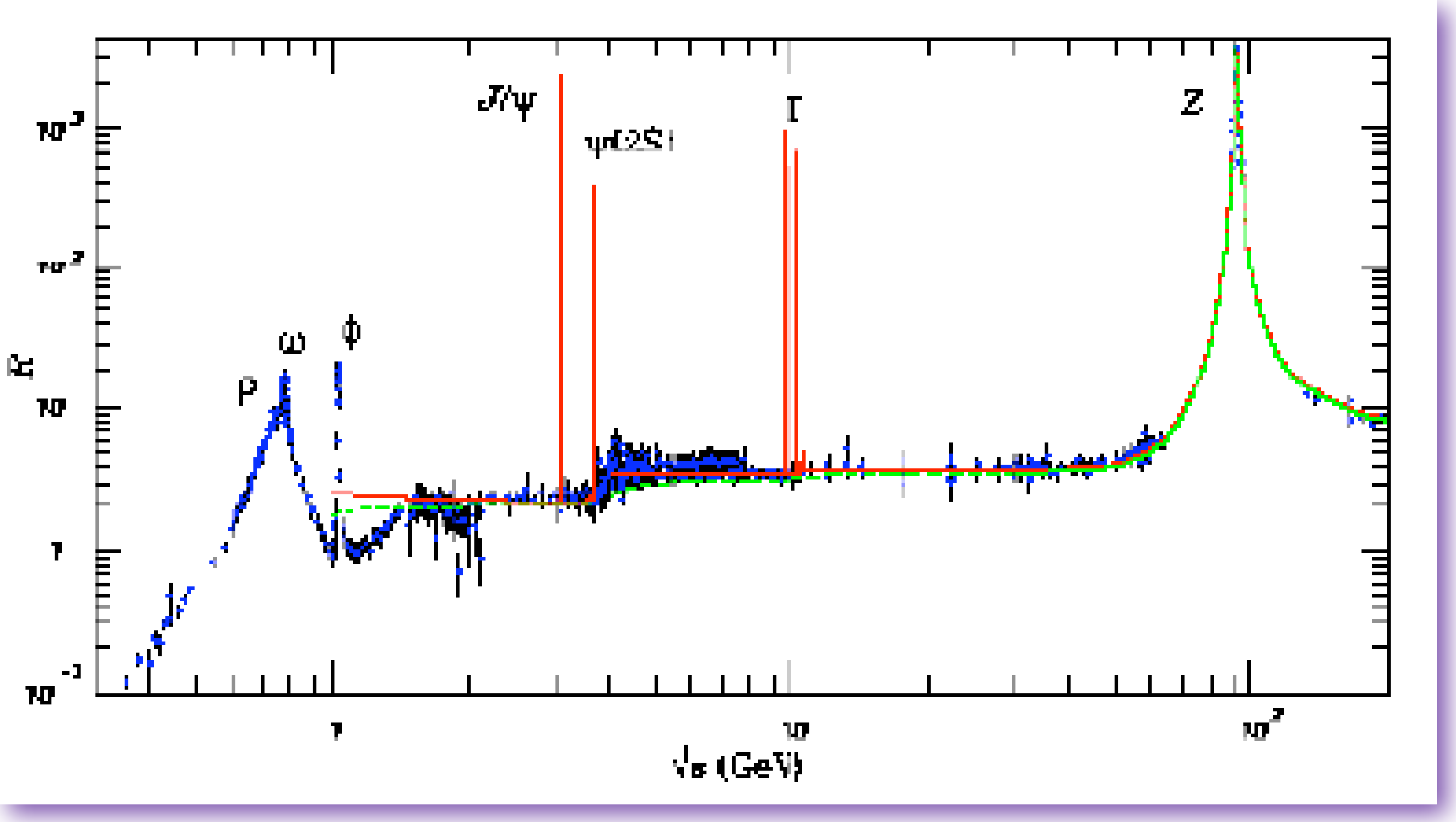
Quark-hadron duality

[Poggio, Quinn & Weinberg, Phys Rev D13]

e^+ - e^- $\longrightarrow$ hadrons $\equiv \sum_q (e^+e^- \to q\bar{q}) \Rightarrow$ $\sigma_{hadrons} \equiv \sum_q \hat{\sigma}_q$



averaged hadronic cross section ⇔ averaged quark cross section

⇒ **Smearing techniques**

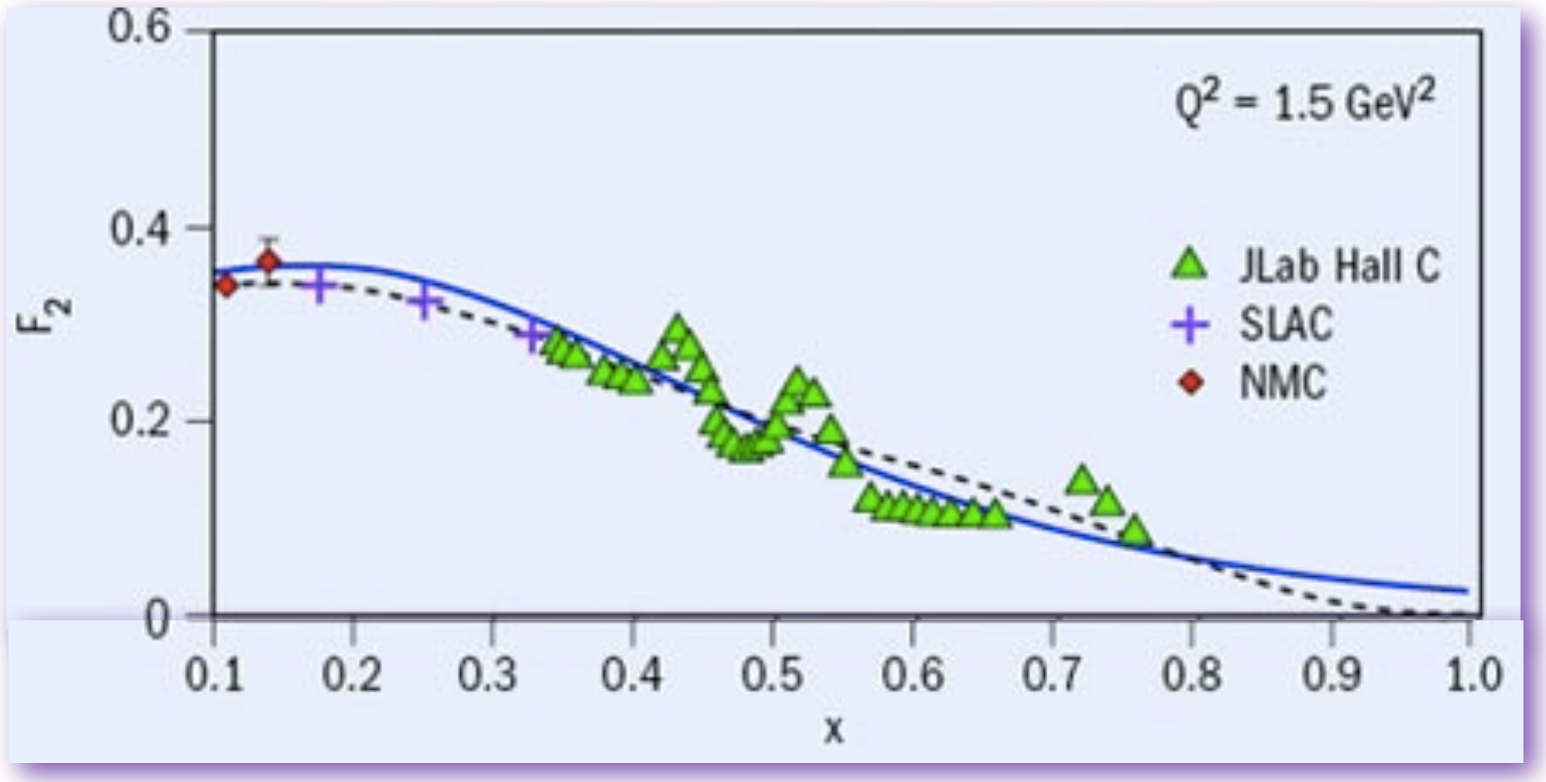
Complementarity between Parton and Hadron descriptions of observable

Bllom-Gilman duality

Structure functions

Resonance region ⇔ Scaling region

- When $x \to 1$, $\to$ elastic scattering
- Exclusive scattering
- Intertwine with resonance region



$x_{Bj} > 0.5$, Q^2 multi-GeV region ⇒ $1.2 < W^2 \leq 4$GeV^2

[Bloom & Gilman, Phys.Rev.Lett.25]

Bloom-Gilman Duality

Resonances created in electroproduction
are a substantial part of the observed
scaling behaviour of inelastic electron-proton scattering

Duality and QCD

- ``Finite Energy Sum Rule "

$$\int_{x_{\rm min}}^{x_{\rm max}} dx\, F_2^{\rm resonances}(x,Q^2) = \int_{x_{\rm min}}^{x_{\rm max}} dx\, F_2^{\rm scaling}(x,Q^2)$$

Global duality:
at fixed Q^2

$$\mathbf{x_M : x_m \Leftrightarrow W_m^2 : W_M^2 \Rightarrow 1.2 : 4\,GeV^2}$$

$$W^2 = Q^2\left(\frac{1}{x} - 1\right) + M^2$$

For review see [Melnitchouk et al, Phys.Rep 406]

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experiment ⟷ **theory**

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Recipe for a perturbative analysis

- Target Mass Corrections (TMC)
- NLO in α_s in pQCD

For review see [Melnitchouk et al, Phys.Rep 406]

Violation of Bloom-Gilman Duality

- **low-Q^2 SF have strong Q^2 dependence**

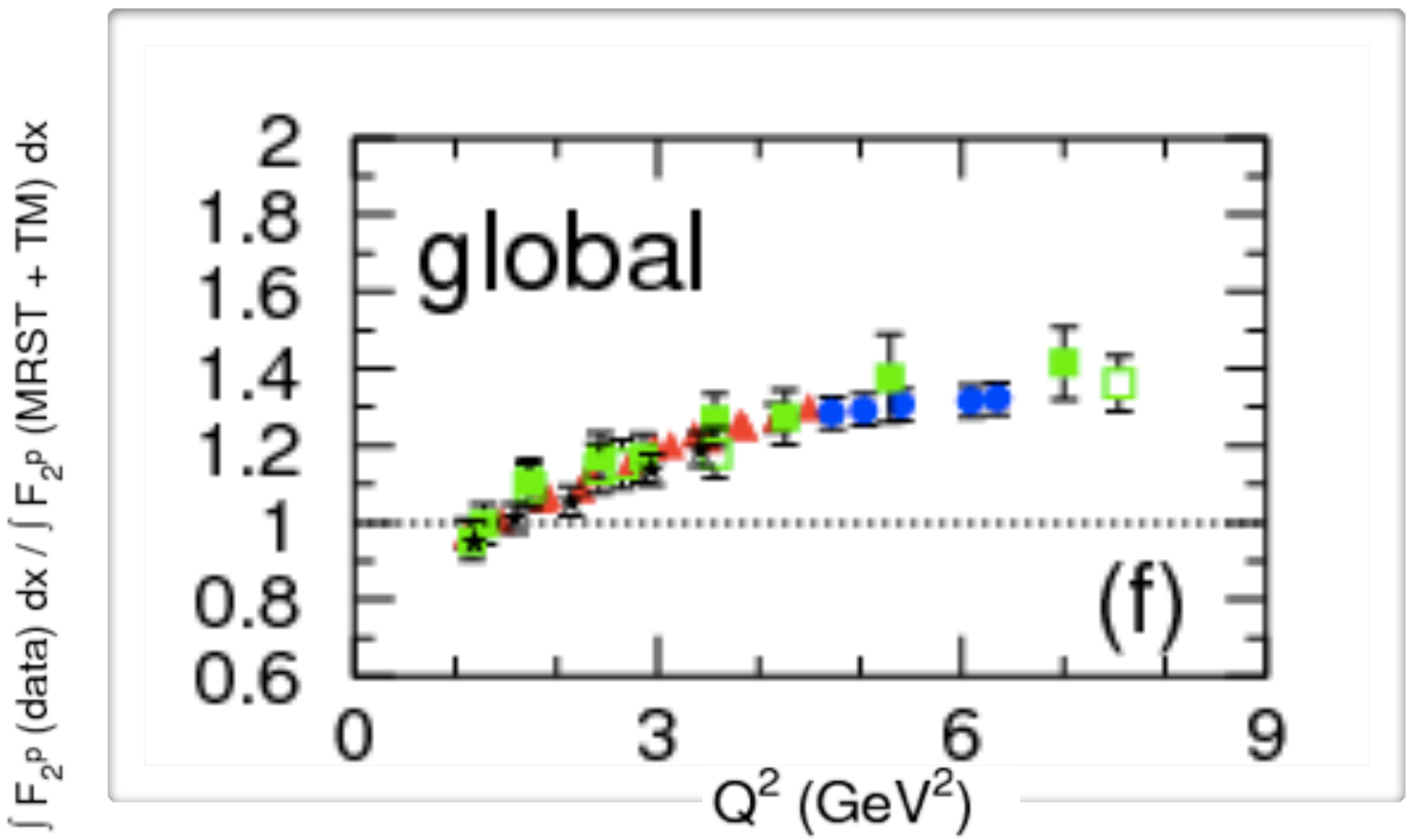
[Malace et al, PRC80]

  - **violates scaling & duality**

- $$F(x,Q^2) = F^{(2)}(x,Q^2) + \frac{F^{(4)}(x,Q^2)}{Q^2} + \dots$$

- **duality implies leading-twist only !**

- **duality gives info on size of nonperturbative corrections**



Intersection of pQCD & non-perturbative QCD

$$\int_{\mathrm{Res.reg}} dx F_2^{\mathrm{Res}}(x,Q^2) \Leftrightarrow \int_{\mathrm{Res.reg}} dx F_2^{\mathrm{scaling}}(x,Q^2)$$

experiment ⟷ **theory**

Recipe for a perturbative analysis

- Target Mass Corrections (TMC)
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Non perturbative info ?

- Higher-Twists
- LxR in definition of α_s

[Bianchi, Fantoni & Liuti, PRD69]

Intersection of pQCD & non-perturbative QCD

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experiment ⟷ **theory**

Recipe for a perturbative analysis

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⟶ **Ok**

Non perturbative info ?

- Higher-Twists ⟶ **?**
- LxR in definition of α_s ⟶ **HERE**

[Bianchi, Fantoni & Liuti, PRD69]

F_2 in perturbative QCD

$$F_2(x,Q^2) = x\sum_{q,\bar{q}} e_q^2\left[q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q_0(\xi)\left\{P\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\kappa^2} + C\left(\frac{x}{\xi}\right)\right\} + \ldots\right]$$

$\overline{\text{MS}}$ scheme →

$$q(x,\mu^2) = q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q_0(\xi)\left\{P\left(\frac{x}{\xi}\right)\ln\frac{\mu^2}{\kappa^2} + C\left(\frac{x}{\xi}\right)\right\}$$

$$F_2(x,Q^2) = x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi}\, q(\xi,Q^2)\left\{\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_S}{2\pi}C_{\overline{\text{MS}}}\left(\frac{x}{\xi}\right) + \ldots\right\}$$

1. q_0→ leading-twist PDFs here MSTW08NLO
2. q_0→ evolved to $q(x, Q^2)$ via DGLAP with P→ splitting functions, to NLO
3. C→ coefficient functions, to NLO

In practice:

1. DGLAP
2. convolution with coefficient functions

F_2 in perturbative QCD

$$F_2(x,Q^2) = x\sum_{q,\bar{q}} e_q^2 \left[q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} q_0(\xi) \left\{ P\left(\frac{x}{\xi}\right) \ln\frac{Q^2}{\kappa^2} + C\left(\frac{x}{\xi}\right) \right\} + \ldots \right]$$

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In practice:

1. DGLAP
2. convolution with coefficient functions

Is it still true at large-x ?

Target Mass Corrections

- **Effects associated with the mass of the target**
- **infinite vs. finite target mass ⇒ Bjorken vs. Nachtmann variable**

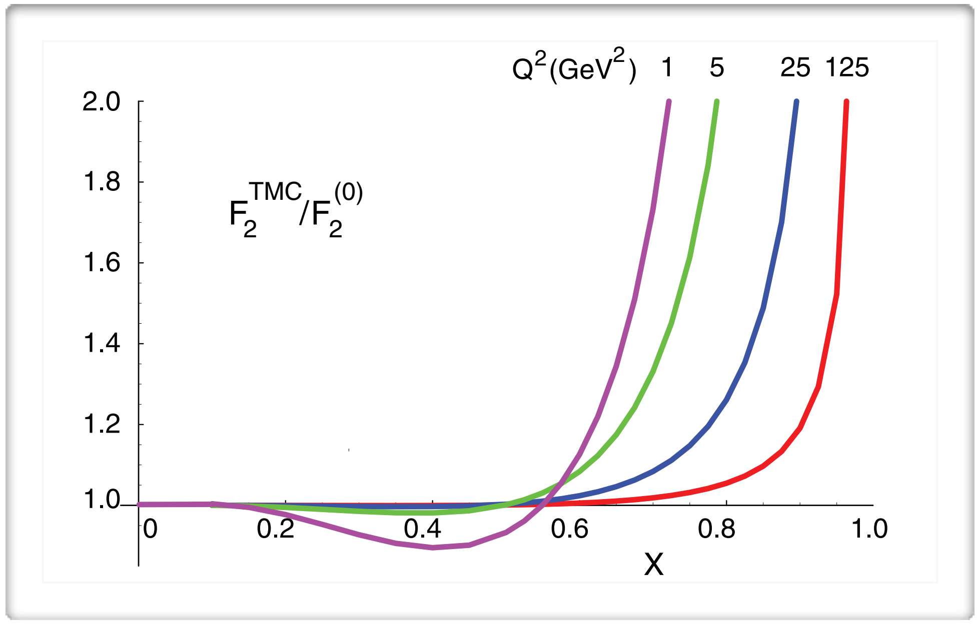
$$x = \frac{Q^2}{2P.q} \Leftrightarrow \xi = \frac{2x}{1+\sqrt{1+4x^2M^2/Q^2}}$$

$$F_2^{NS(TMC)}(x,Q^2) \; = \; \frac{x^2}{\xi^2\gamma^3}F_2^{\infty}(\xi,Q^2) + 6\frac{x^3M^2}{Q^2\gamma^4}\int_{\xi}^{1}\frac{d\xi'}{\xi'^2}F_2^{\infty}(\xi',Q^2)$$

Georgi & Politzer (1976)

$$F(x,Q^2,M^2) \propto \int_{\xi}^{\xi/x}\frac{dz}{z}H(\xi/z,Q^2)q(z,Q^2)$$

, ..., Accardi & Qiu (2008)



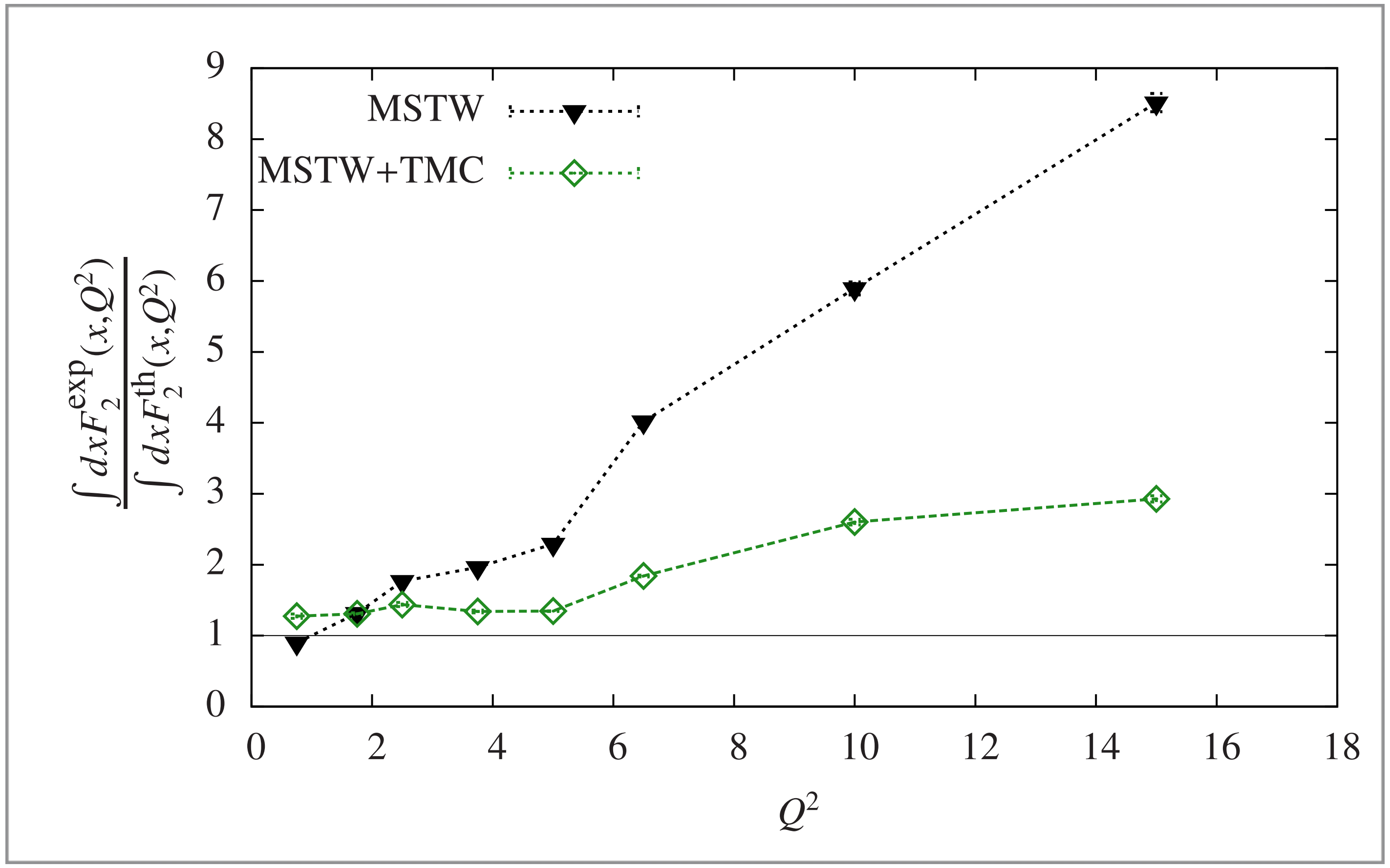
[J. Phys. G: Nucl. Part. Phys. 35]

Data analysis: F2 at JLab

$$R^{\mathrm{exp/th}}(Q^2) = \frac{\int_{x_{\min}(W^2=4\mathrm{GeV}^2)}^{x_{\max}(W^2=1.2\mathrm{GeV}^2)} dx\, F_2^{\mathrm{exp}}(x,Q^2)}{\int_{x_{\min}(W^2=4\mathrm{GeV}^2)}^{x_{\max}(W^2=1.2\mathrm{GeV}^2)} dx\, F_2^{\mathrm{th}}(x,Q^2)}$$

Hall C E94-110 reanalyzed by Monaghan [1209.4542]

=1 if duality fulfilled



Data analysis: F2 at JLab

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$$R^{\mathrm{exp/th}}(Q^2) = \frac{\int_{x_{\min}(W^2=4\mathrm{GeV}^2)}^{x_{\max}(W^2=1.2\mathrm{GeV}^2)} dx\, F_2^{\mathrm{exp}}(x,Q^2)}{\int_{x_{\min}(W^2=4\mathrm{GeV}^2)}^{x_{\max}(W^2=1.2\mathrm{GeV}^2)} dx\, F_2^{\mathrm{th}}(x,Q^2)}$$

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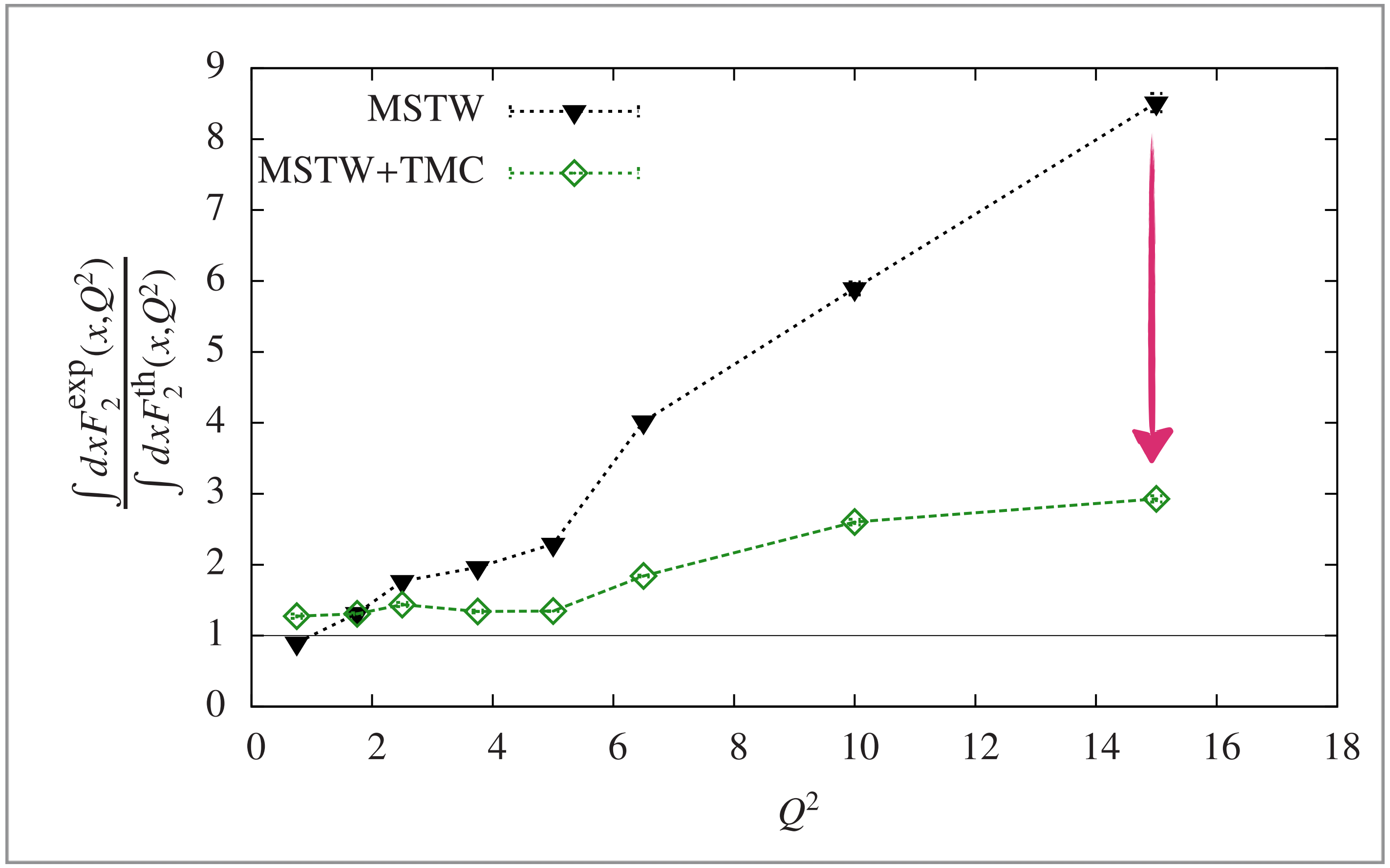

TMC effect

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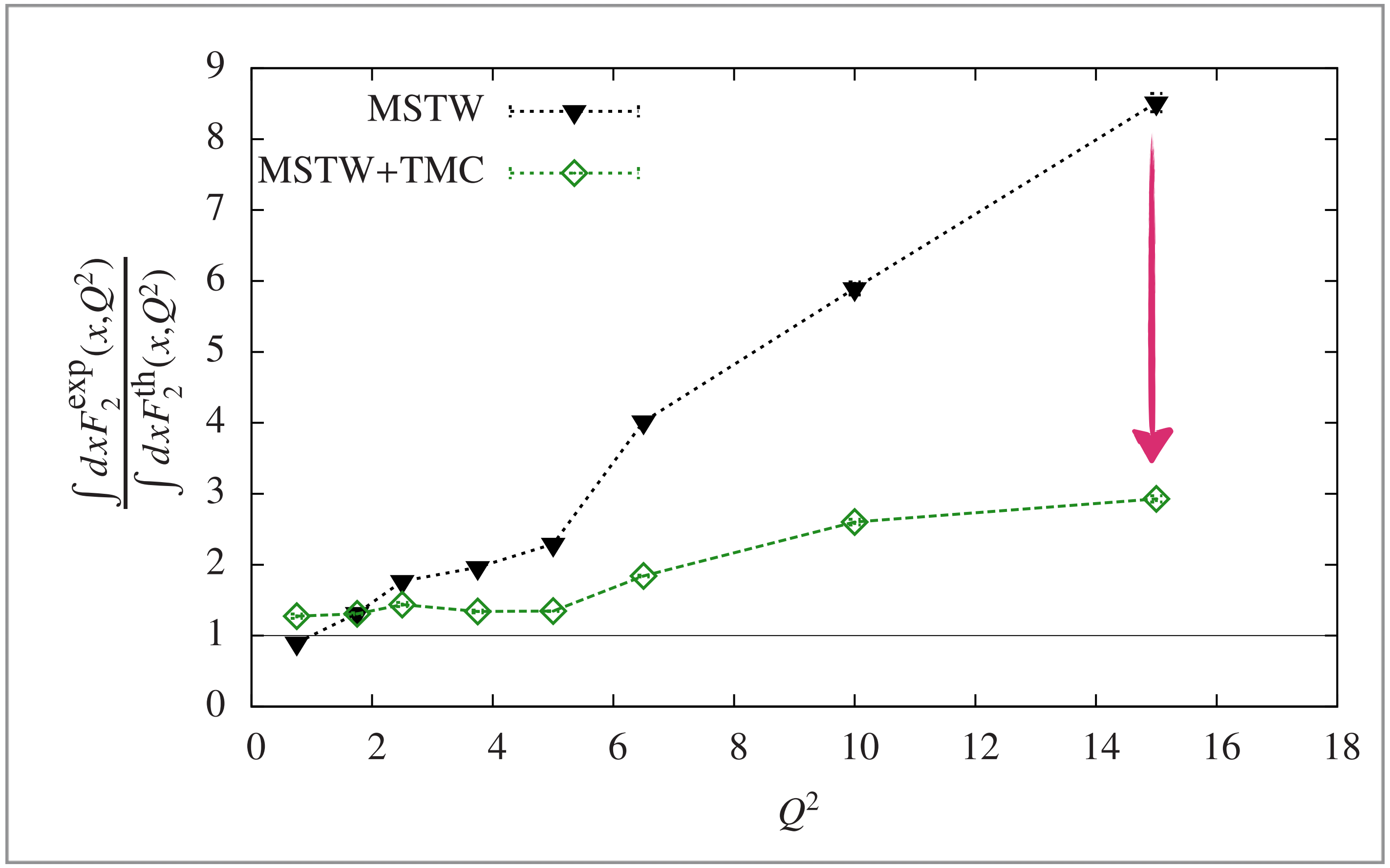


TMC effect

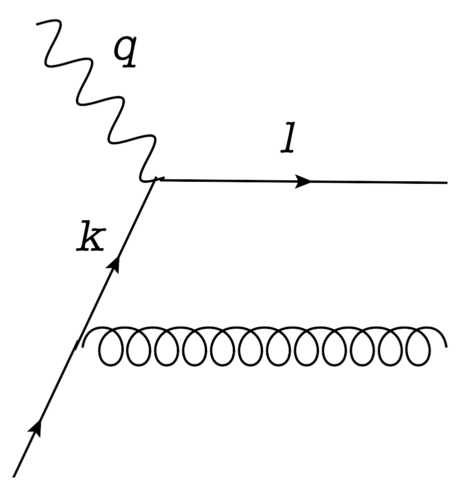
Still missing something...

Large-x resummation

Amati et al., Nucl.Phys. B173 (1980) 429

- Large invariants: $\Lambda^2 \ll W^2 \sim Q^2$
- Argument for α_s is s, mass square of final state of γ^* parton collision

$$\hat{s} = (q+k)^2 = Q^2 \frac{1-z}{z}$$



Without LxR, upper limit $=Q^2$

DGLAP

$$q(x,Q^2) = \int_x^1 \frac{dz}{z} \int_{\mu^2}^{Q^2 \frac{1-z}{z}} dk_T^2\, \alpha_s(k_T^2) P_{qq}\left(z, \alpha_s(k_T^2)\right)\, q\left(\frac{x}{z}, k_T^2\right)$$

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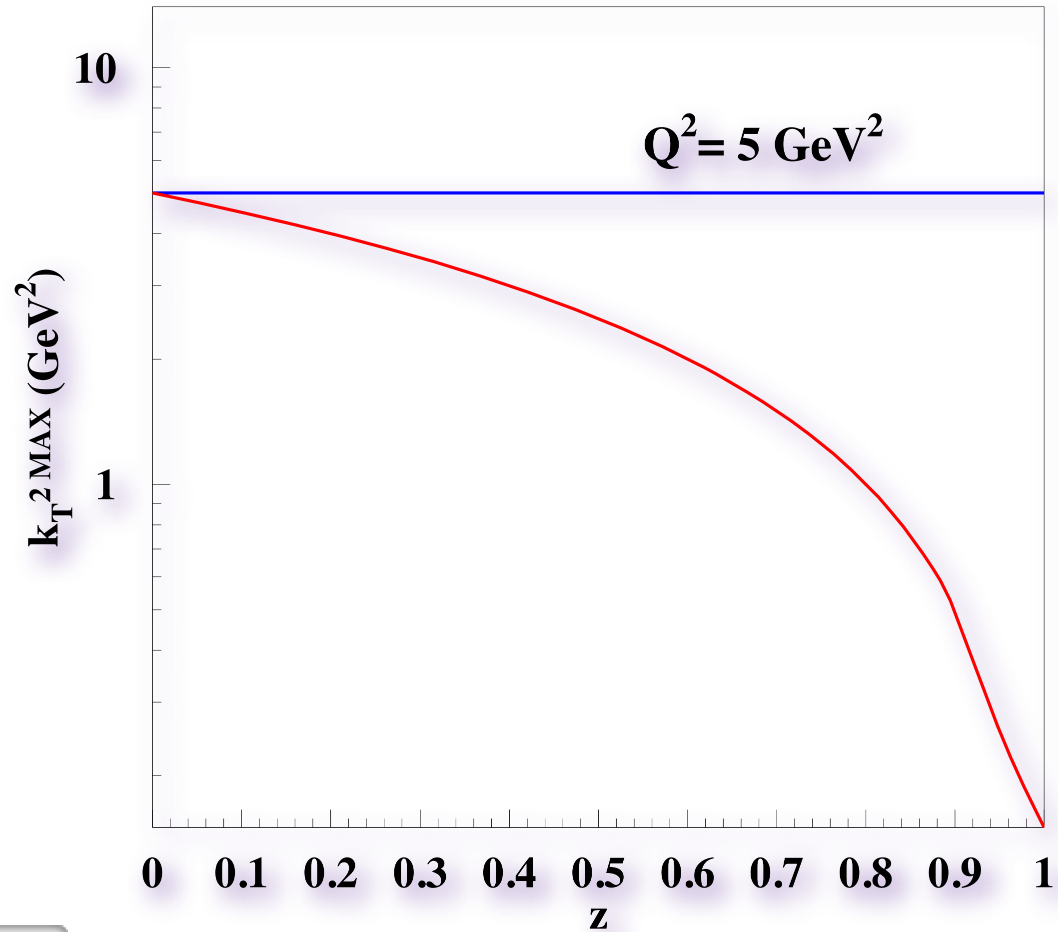

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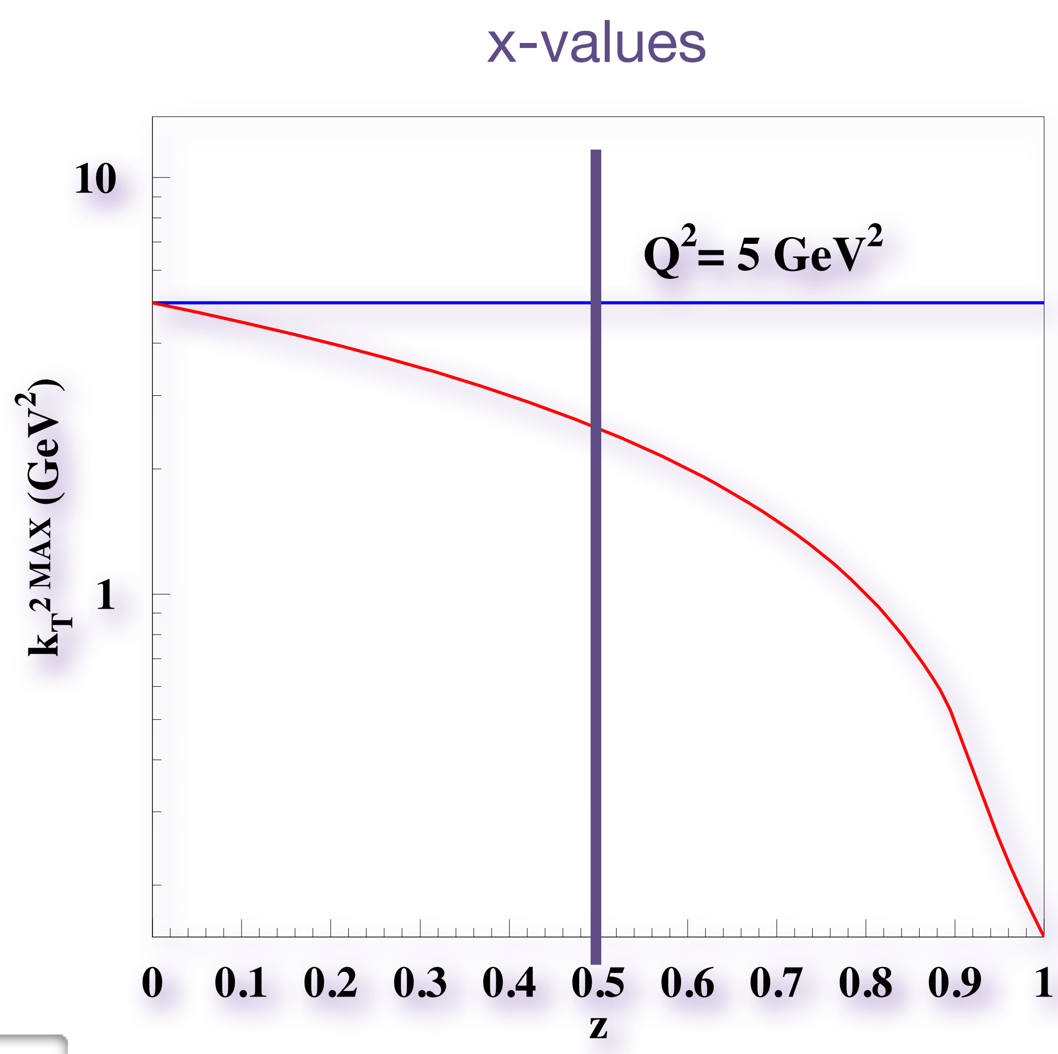

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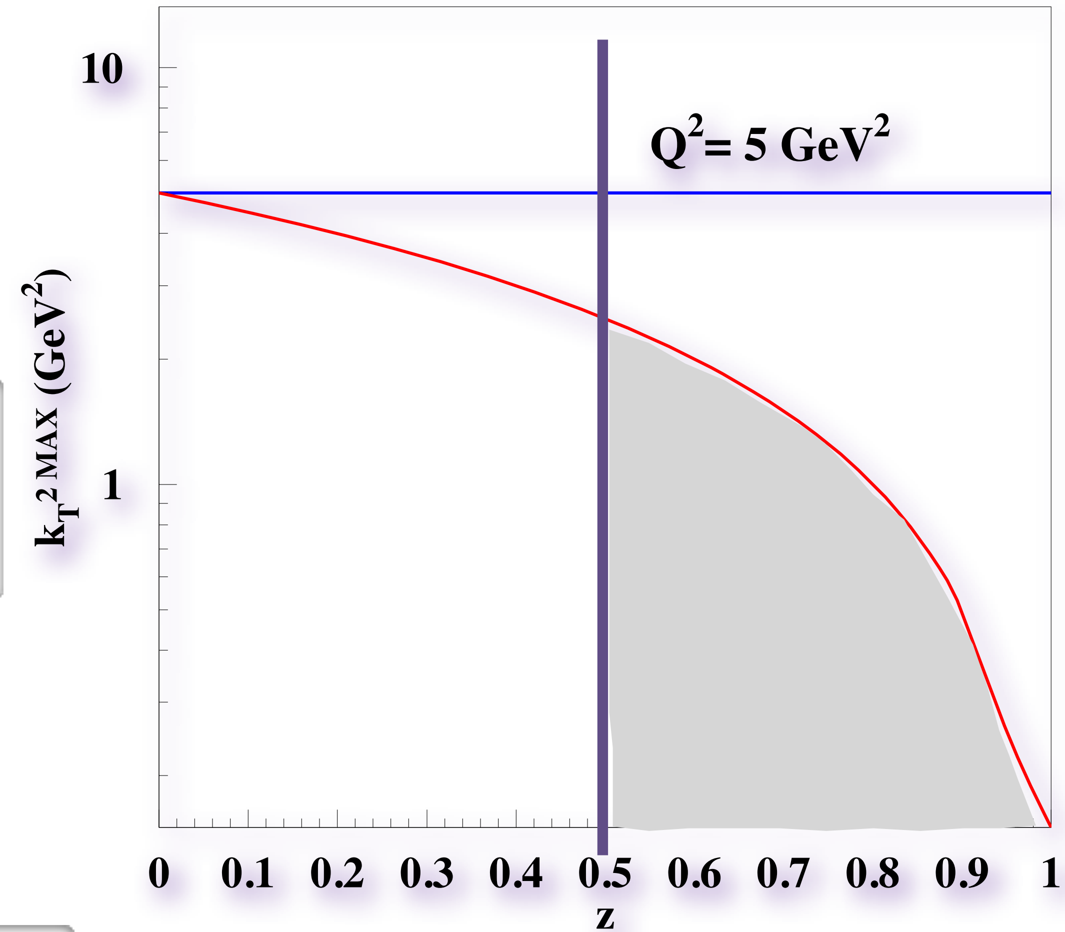
x-values



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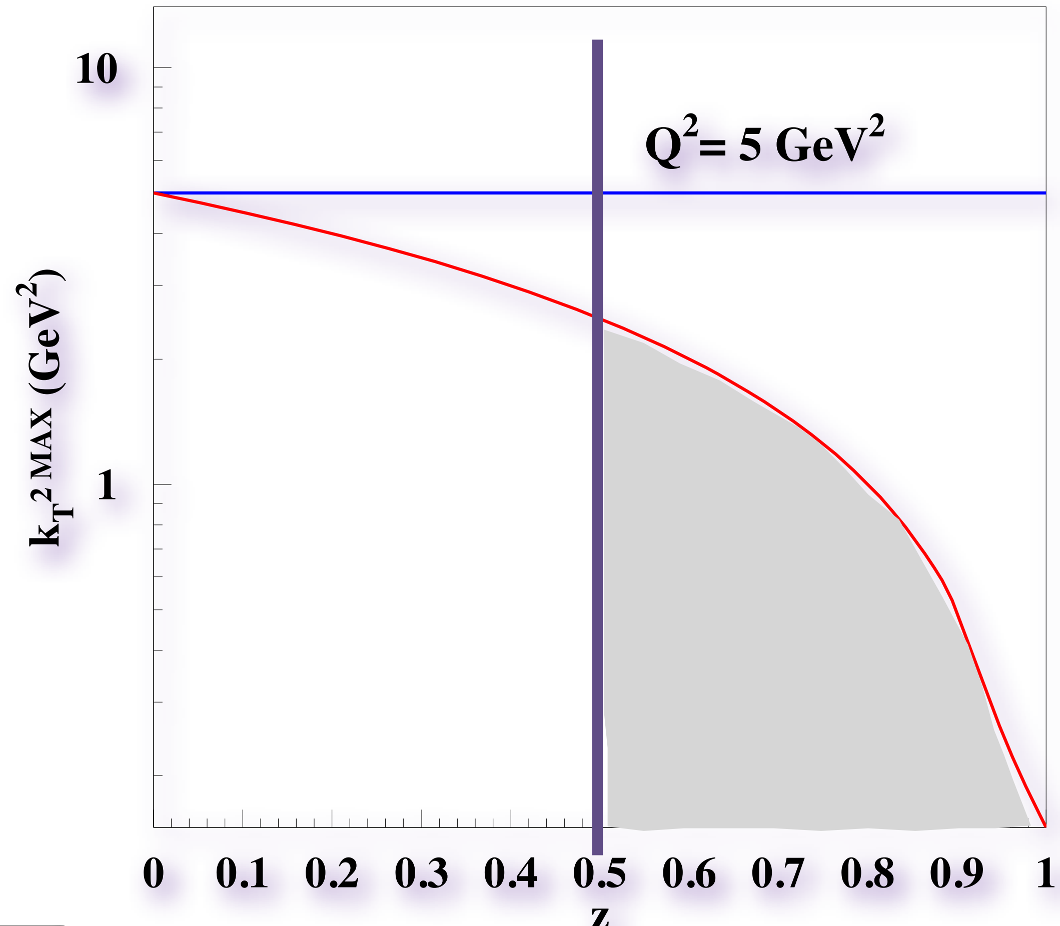

restricted phase space for real gluon emission

Many ways to implement LxR

- Our strategy
  - We don't touch the DGLAP part
  - Resummation at the coefficient function level :

$$F_2^{NS}(x,Q^2) = xq(x,Q^2) + \frac{\alpha_s}{4\pi}\sum_q \int_x^1 dz\, B^q_{\rm NS}(z)\,\frac{x}{z}\,q\left(\frac{x}{z},Q^2\right)$$

  - Divergent term at x→1,

$$B^q_{\rm NS}(z) = \left[\hat{P}^{(0)}_{qq}(z)\left\{\ln\left(\frac{1-z}{z}\right) - \frac{3}{2}\right\} + \text{E.P.}\right]_+$$

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- Need to be resummed to all logs in the argument of α_s

  - defining the correct kinematics $\alpha_s(Q^2) \to \alpha_s\left(Q^2\frac{(1-z)}{z}\right)$

[A.C. & Liuti, Phys.Lett. B726 (2013)]

- Resummed as (contains all logs):

$$\ln(1-z) \;=\; \frac{1}{\alpha_{s,\rm LO}(Q^2)}\int^{Q^2} d\ln Q^2\;\left[\alpha_{s,\rm LO}(Q^2(1-z)) - \alpha_{s,\rm LO}(Q^2)\right] \equiv \ln_{\rm LxR}$$

Behaviour of the coupling constant

- LO exact solution

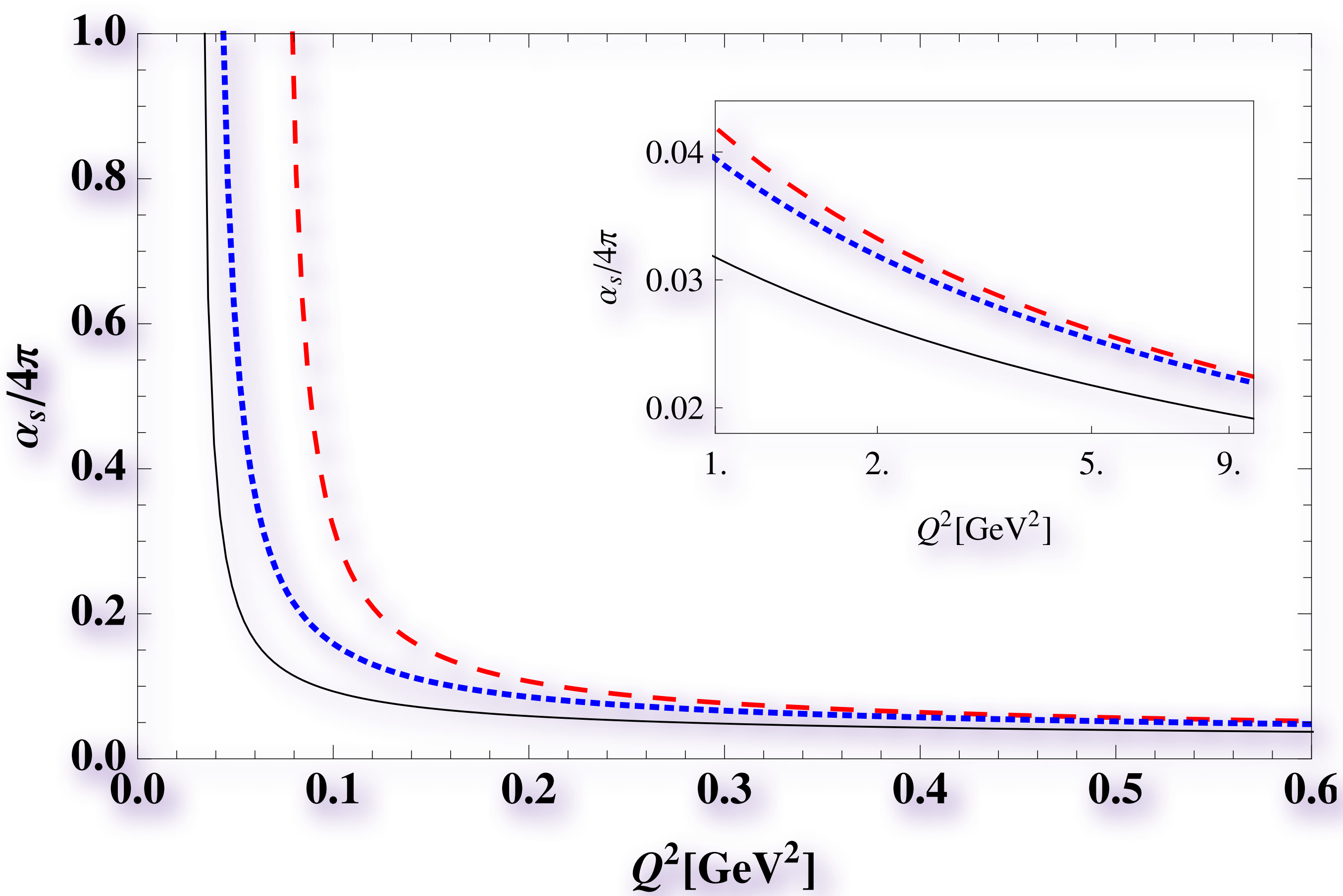
  Λ=174MeV → reaches Landau pole at Q=174MeV

- expansion in α_s :

$$\alpha_s(\tilde{W}^2) = \alpha_s(Q^2) - \frac{\beta_0}{4\pi} \ln\left(\frac{1-z}{z}\right) \alpha_s^2(Q^2)$$

- full dependence in z

  Λ=174MeV → reaches Landau pole at Q >174MeV



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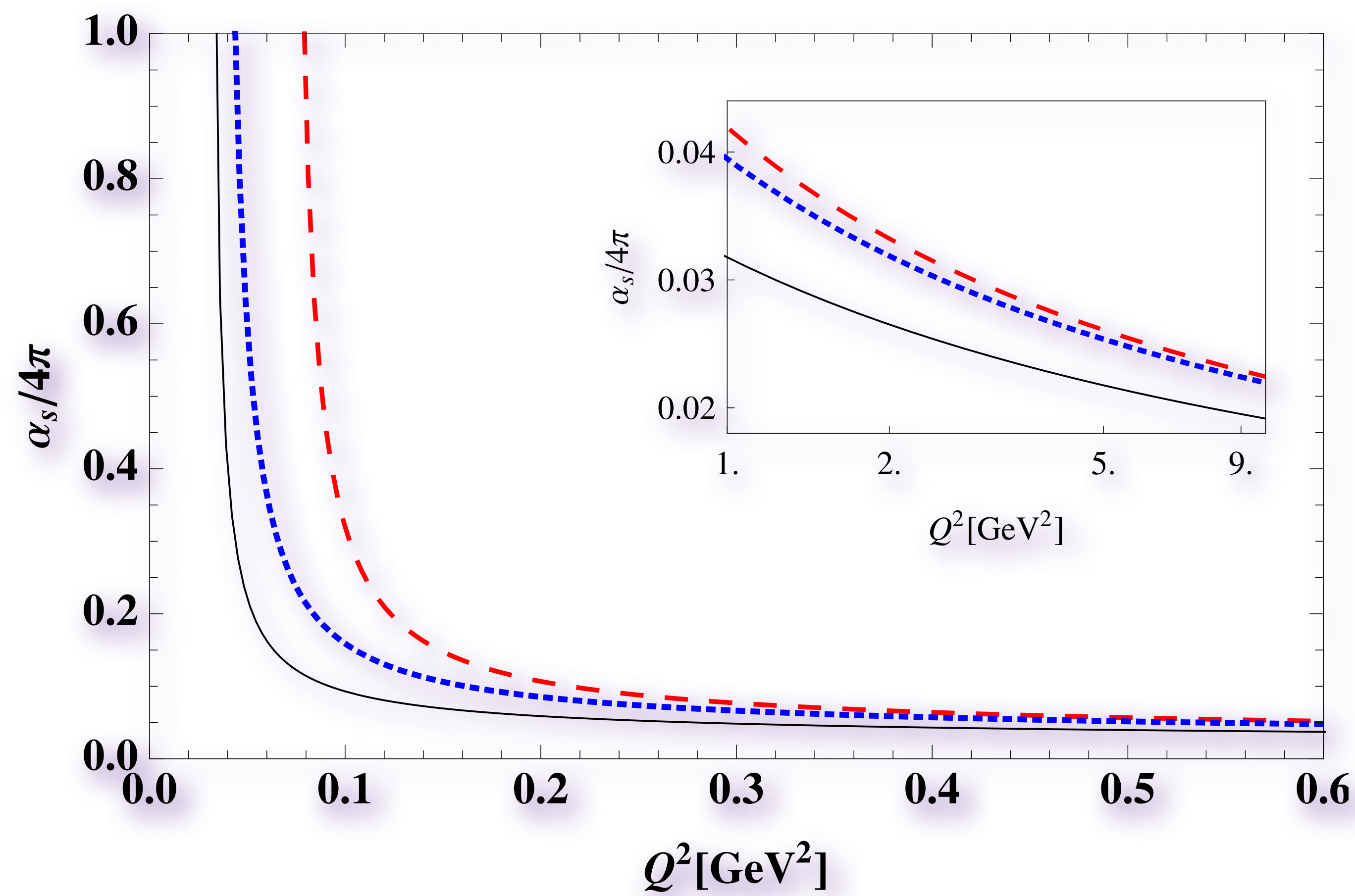
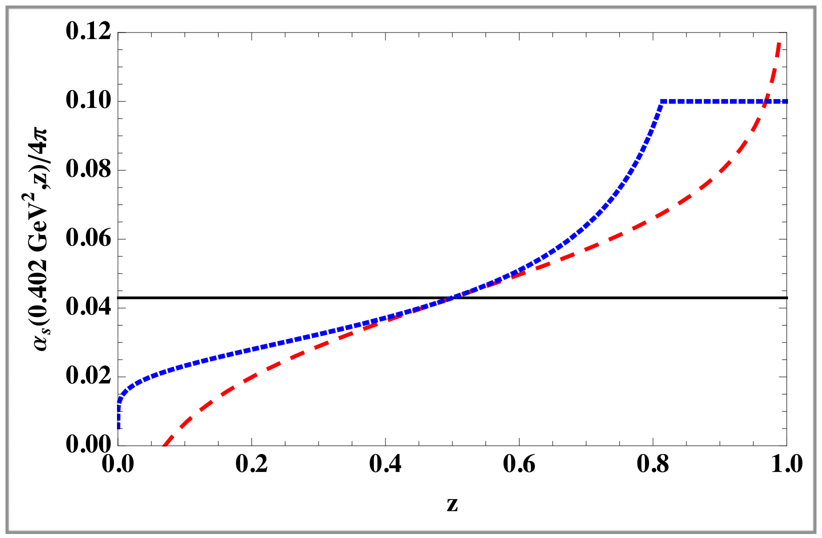
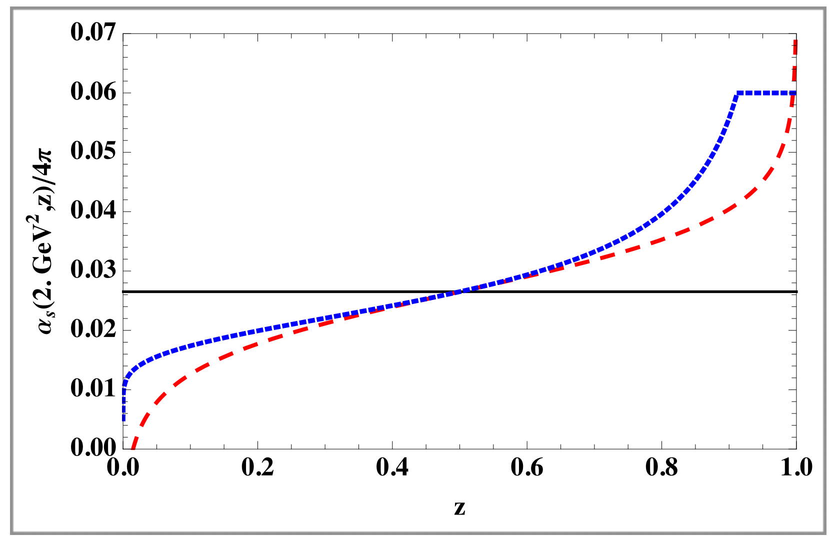


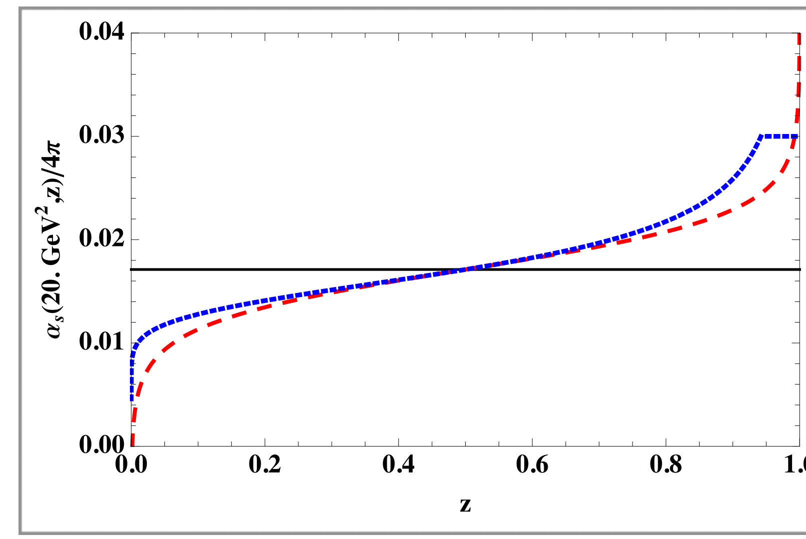
α_s might blow up!

Large-x Resummation: α_s as free parameter

[Courtoy & Liuti, 1208.5636]







- $\alpha_s(Q^2)$;
- an expansion of $\alpha_s(\tilde{W}^2)$ in $\ln((1-z)/z)$, to NLO,

$$\alpha_s(\tilde{W}^2) = \alpha_s(Q^2) - \frac{\beta_0}{4\pi} \ln\left(\frac{1-z}{z}\right) \alpha_s^2(Q^2)$$

- the complete z dependence of $\alpha_s(\tilde{W}^2)$ **cut**

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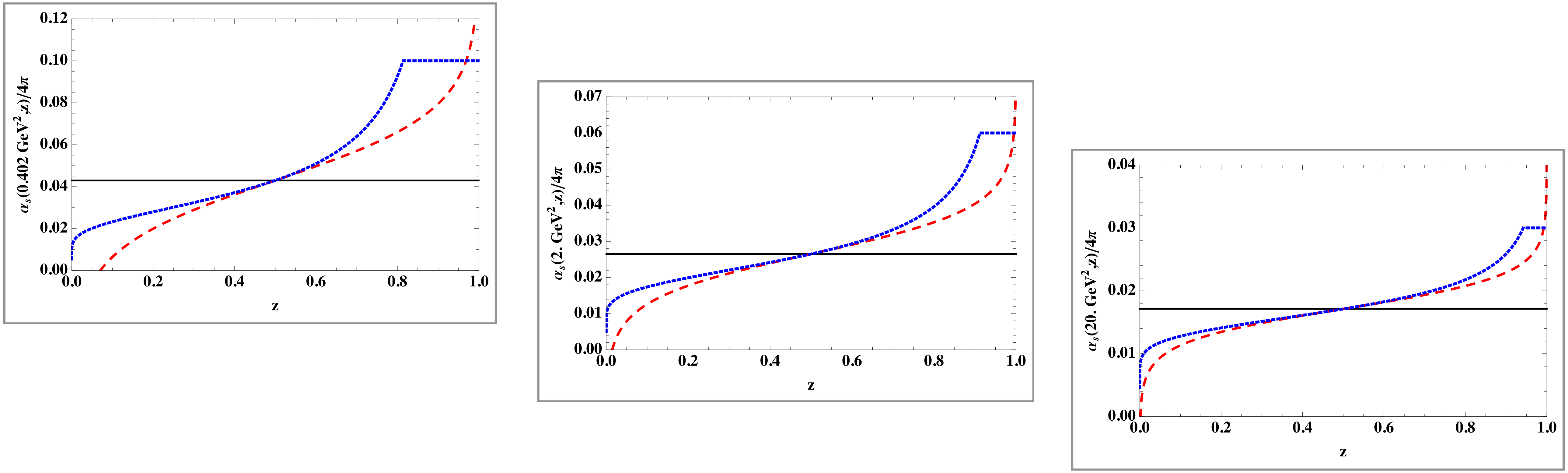

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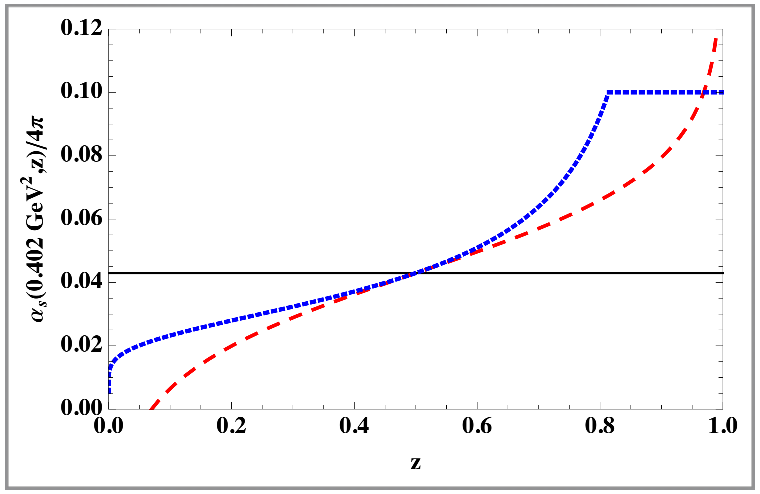
What does a cut in α_s means?

Back to duality

Parametrize the nonperturbative effects from realization of duality

- **Freeze α_s by imposing a z_{max}:** $\widetilde{W}^2(z_{\max}) = Q^2(1 - z_{\max})/z_{\max}$

- **Changes the behavior of the coefficient function $x \rightarrow 1$**



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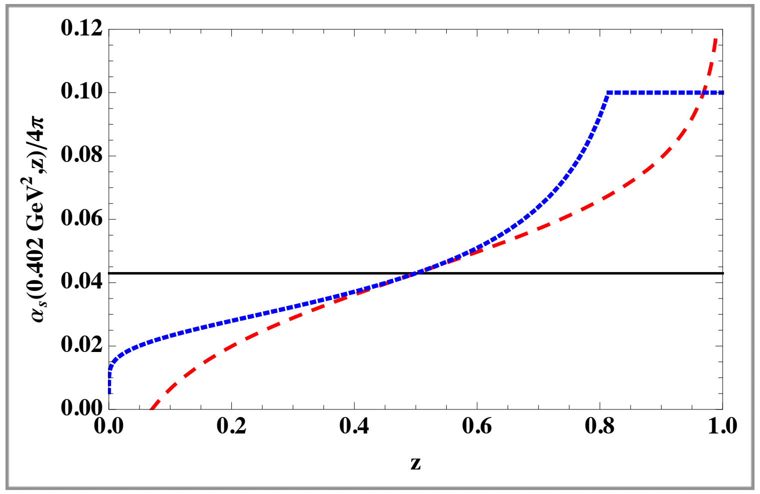


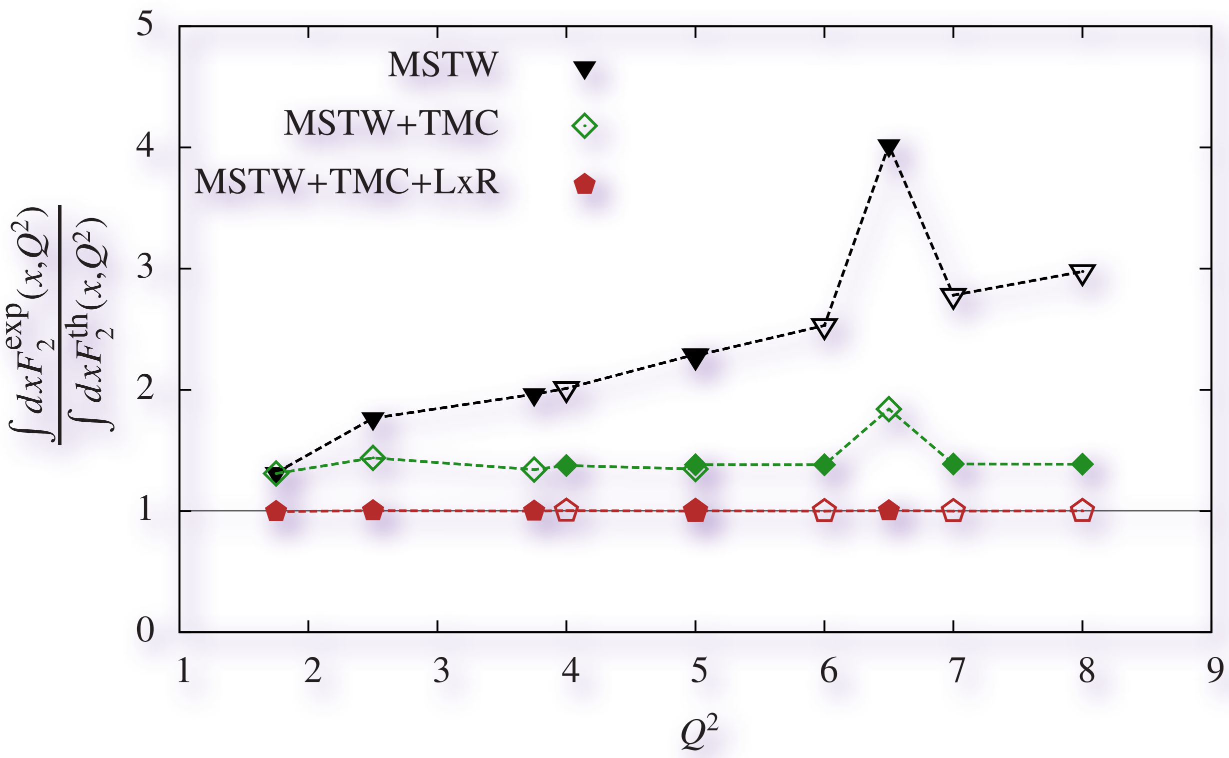
- Changes the behavior of the coefficient function x→1

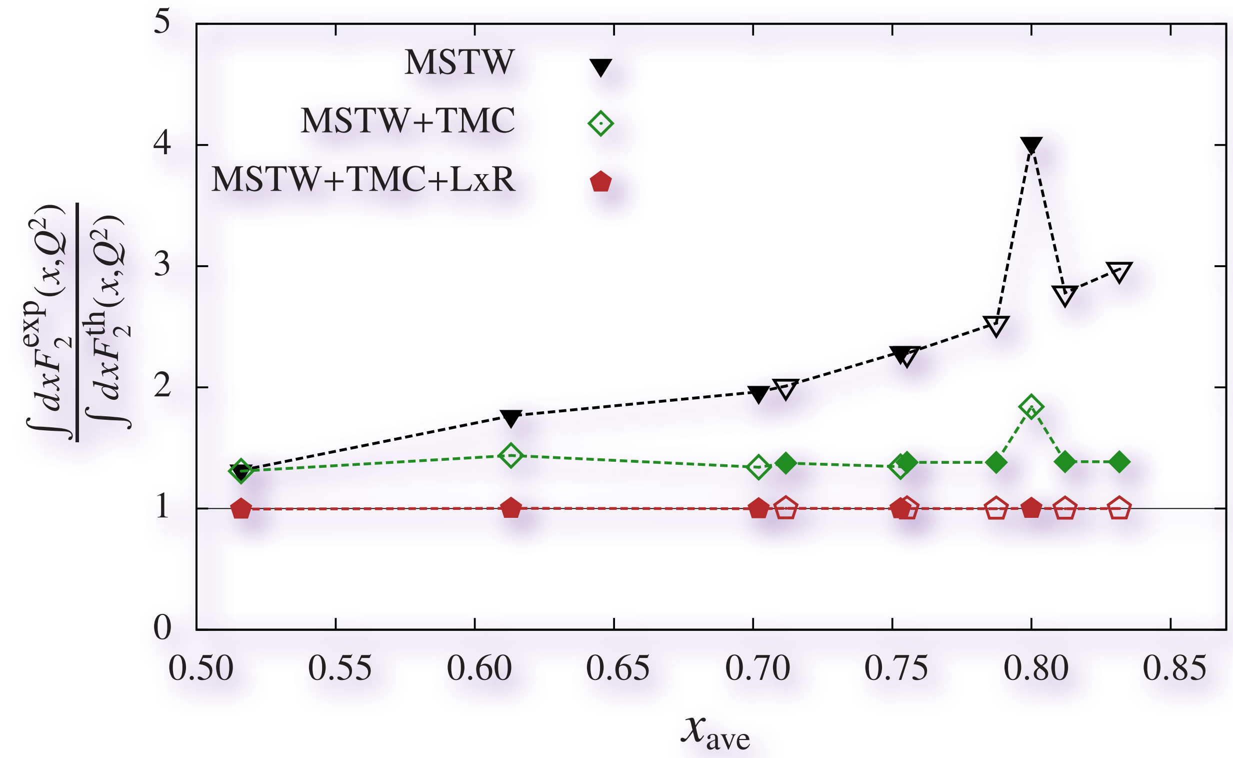
- Realization of duality depends on z_{max} :

$$R^{\exp/\mathrm{th}}(z_{\max}, Q^2) = \frac{\displaystyle\int_{x_{\min}}^{x_{\max}} dx\, F_2^{\exp}(x, Q^2)}{\displaystyle\int_{x_{\min}}^{x_{\max}} dx\, F_2^{NS,\mathrm{Resum}}(x, z_{\max}, Q^2)} = \frac{I^{\exp}}{I^{\mathrm{Resum}}} = 1$$

- Adjust z_{max} according to the data

Results





JLab data

SLAC data

Phys.Lett. B282

Q^2 [GeV^2]	$I^{\rm exp}(Q^2)$	$I^{(0),\rm DIS}(Q^2)$	$I^{(0),\rm DIS+TMC}(Q^2)$	$I^{\rm Resum}(z_{\rm max}, Q^2)$	$z_{\rm max}$
1.75	6.994×10^{-2}	5.316×10^{-2}	5.345×10^{-2}	7.025×10^{-2}	0.63
2.5	4.881×10^{-2}	2.765×10^{-2}	3.393×10^{-2}	4.872×10^{-2}	0.745
3.75	2.356×10^{-2}	1.201×10^{-2}	1.756×10^{-2}	2.359×10^{-2}	0.76
5.	1.267×10^{-2}	0.553×10^{-2}	0.942×10^{-2}	1.270×10^{-2}	0.79
6.5	0.685×10^{-2}	0.170×10^{-2}	0.372×10^{-2}	0.683×10^{-2}	0.9
4.	2.045×10^{-2}	1.017×10^{-2}	1.487×10^{-2}	2.041×10^{-2}	0.79
5.	1.255×10^{-2}	0.550×10^{-2}	0.909×10^{-2}	1.255×10^{-2}	0.811
6.	0.802×10^{-2}	0.317×10^{-2}	0.581×10^{-2}	0.803×10^{-2}	0.825
7.	0.531×10^{-2}	0.191×10^{-2}	0.383×10^{-2}	0.532×10^{-2}	0.837
8.	0.363×10^{-2}	0.122×10^{-2}	0.262×10^{-2}	0.363×10^{-2}	0.845

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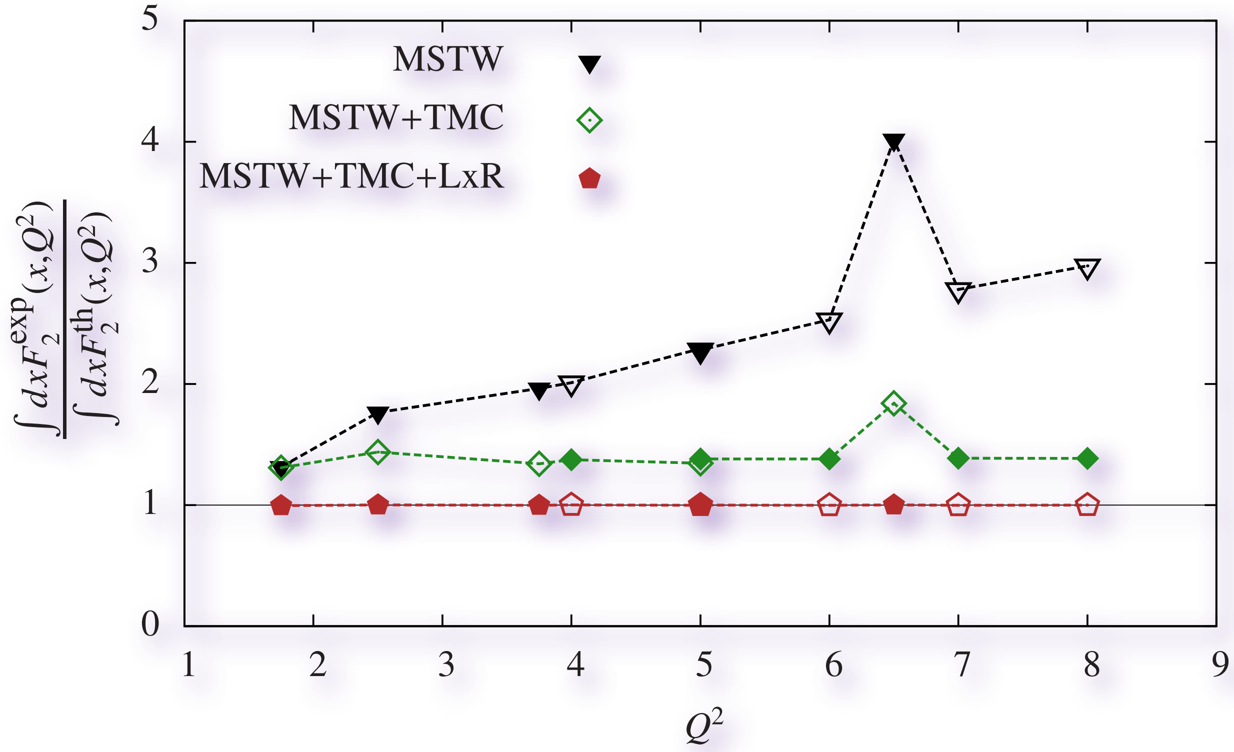

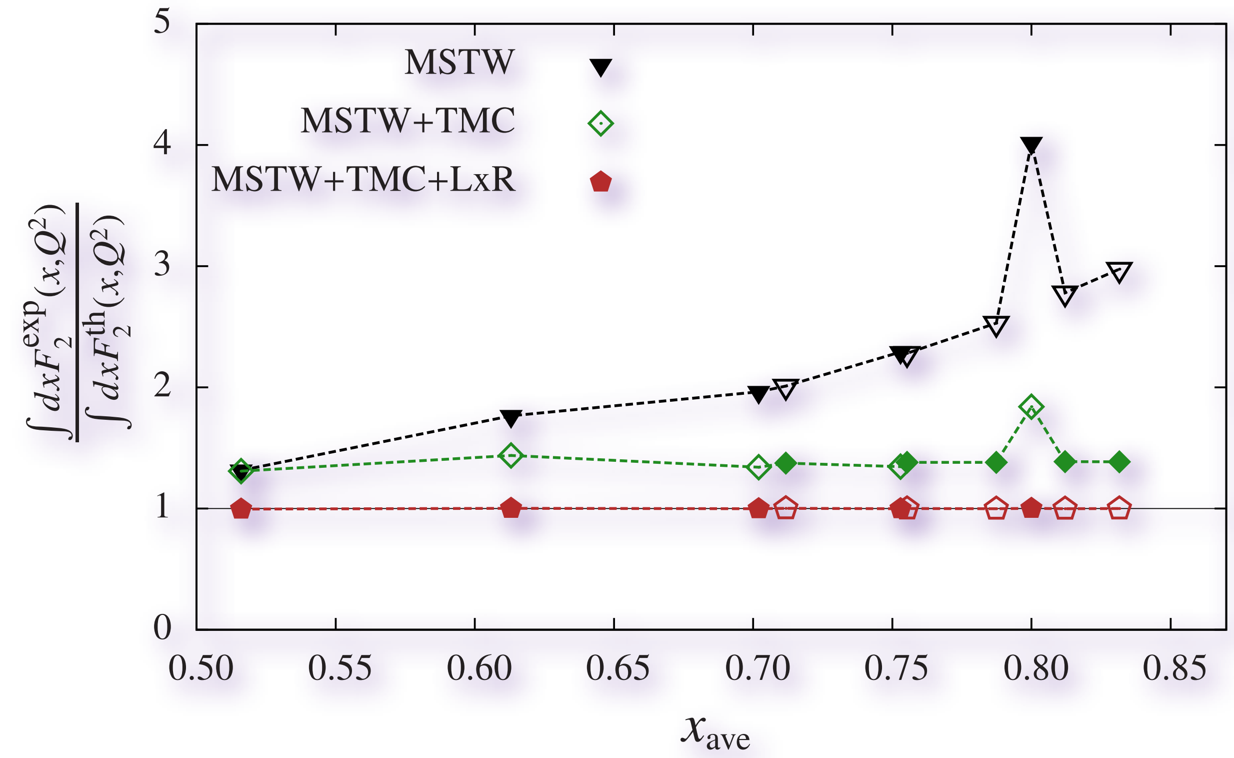

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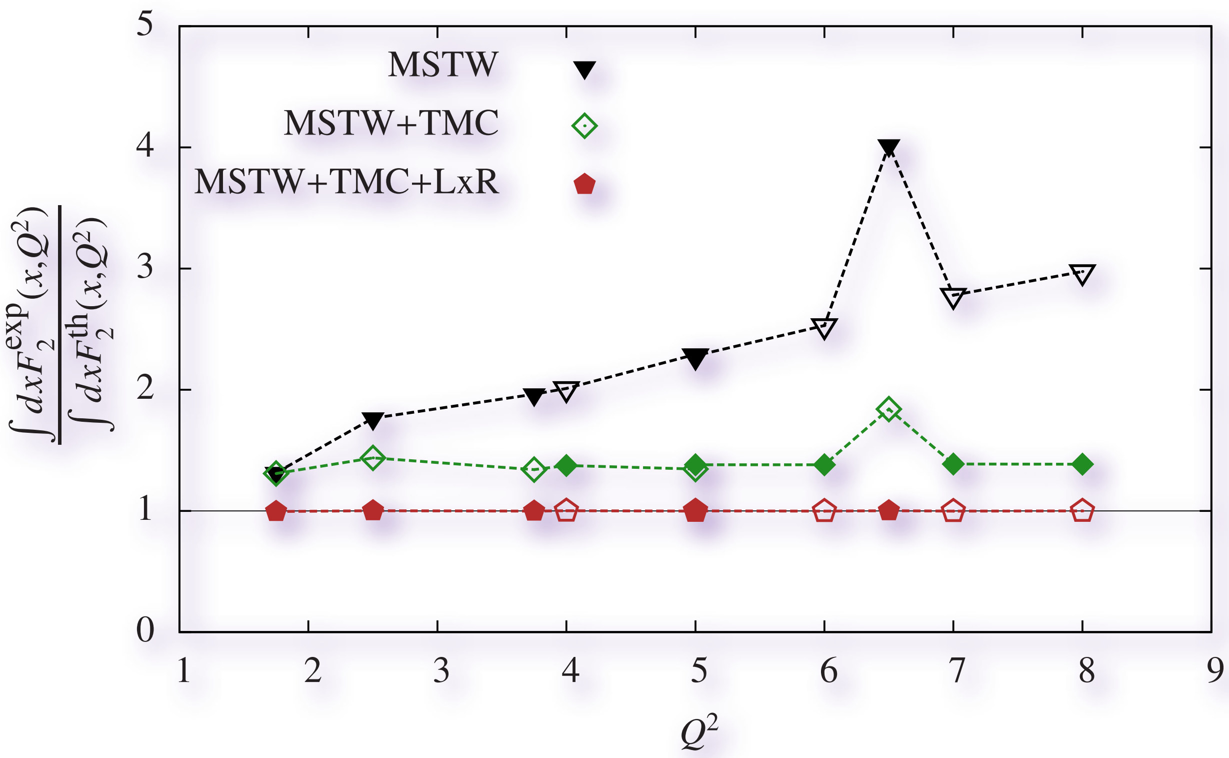

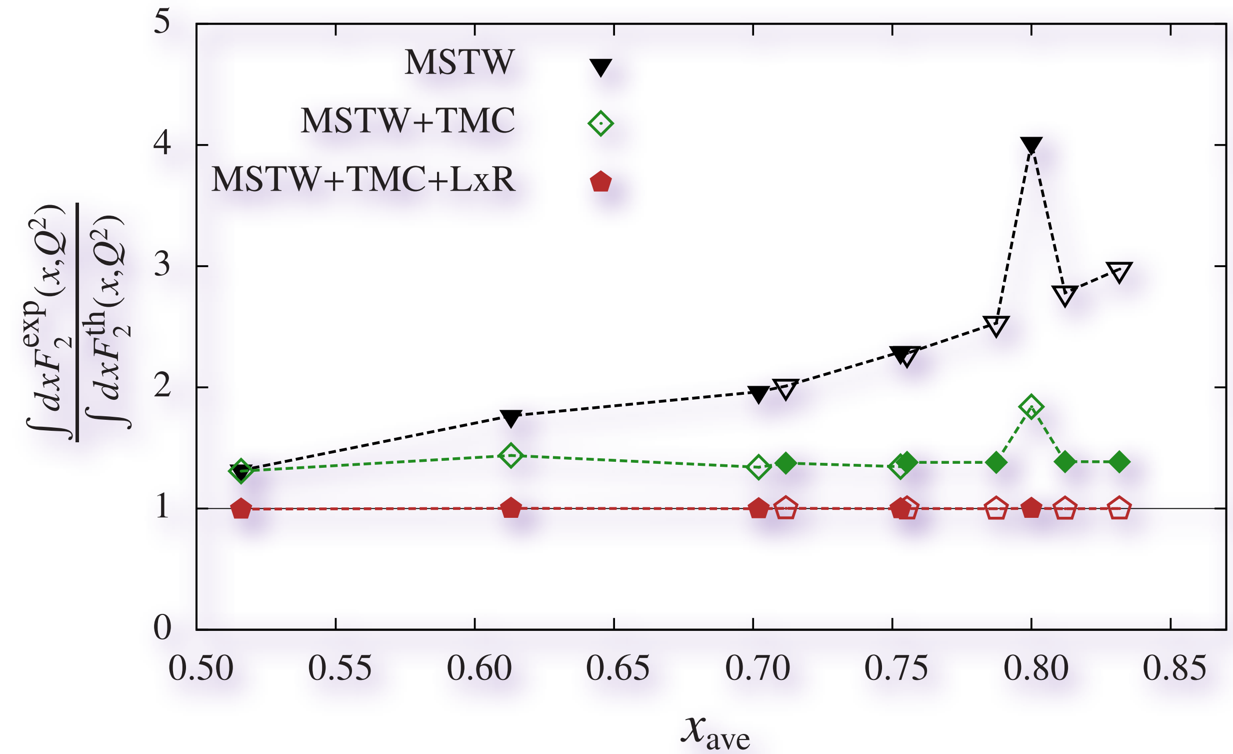

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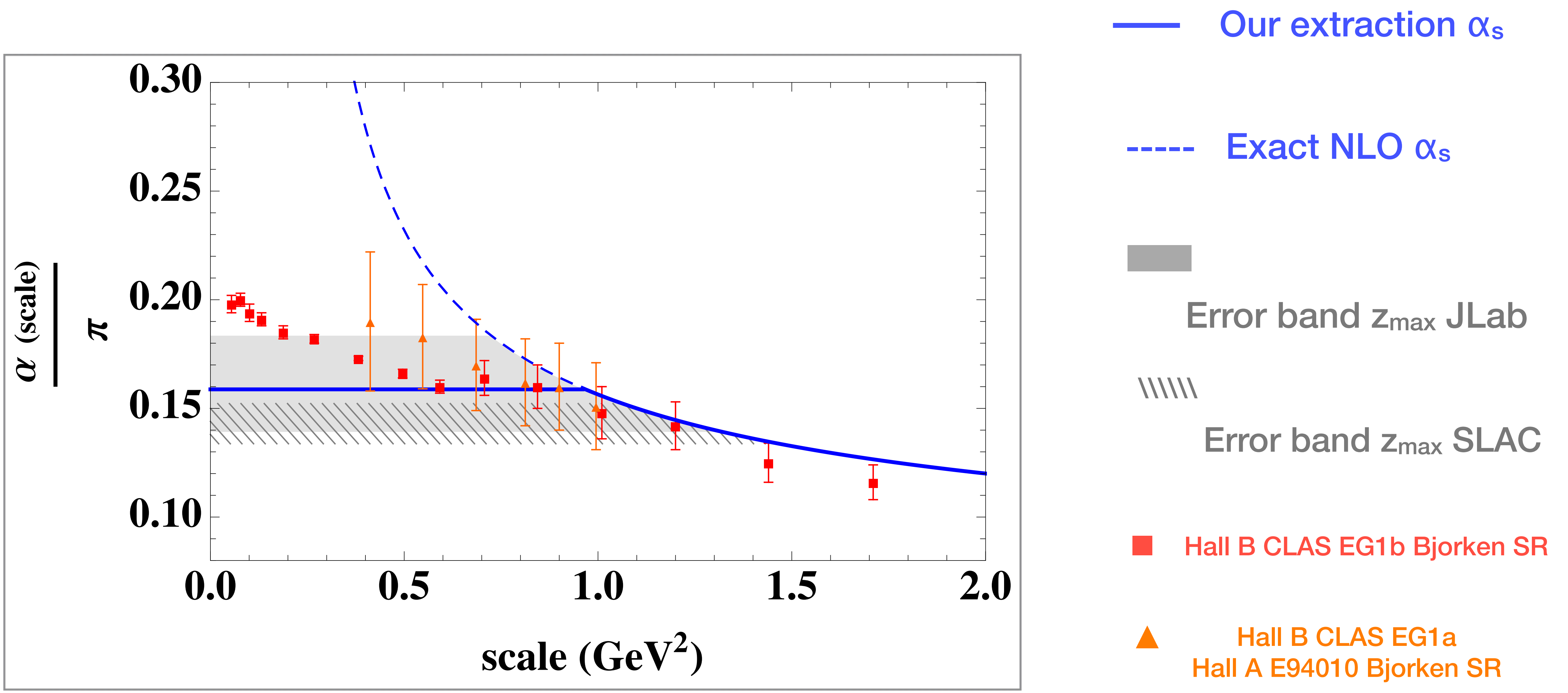
?

$$\alpha_s\left(Q^2\frac{(1-z)}{z}\right)$$

α_s at low energy from duality



A.C. & Liuti, Phys Lett B726 (2013)

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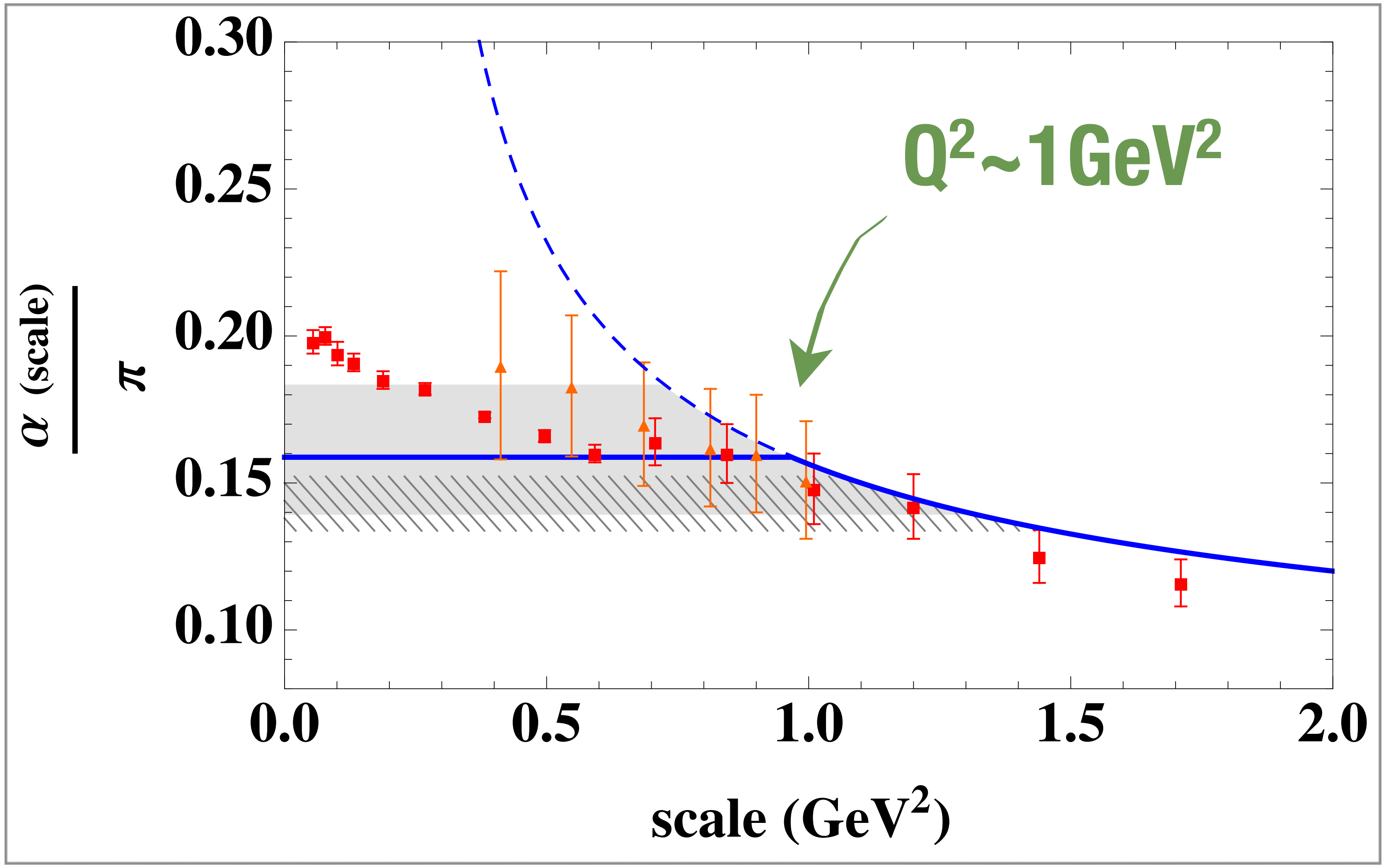


Our extraction α_s

Exact NLO α_s

Error band z_{max} JLab

Error band z_{max} SLAC

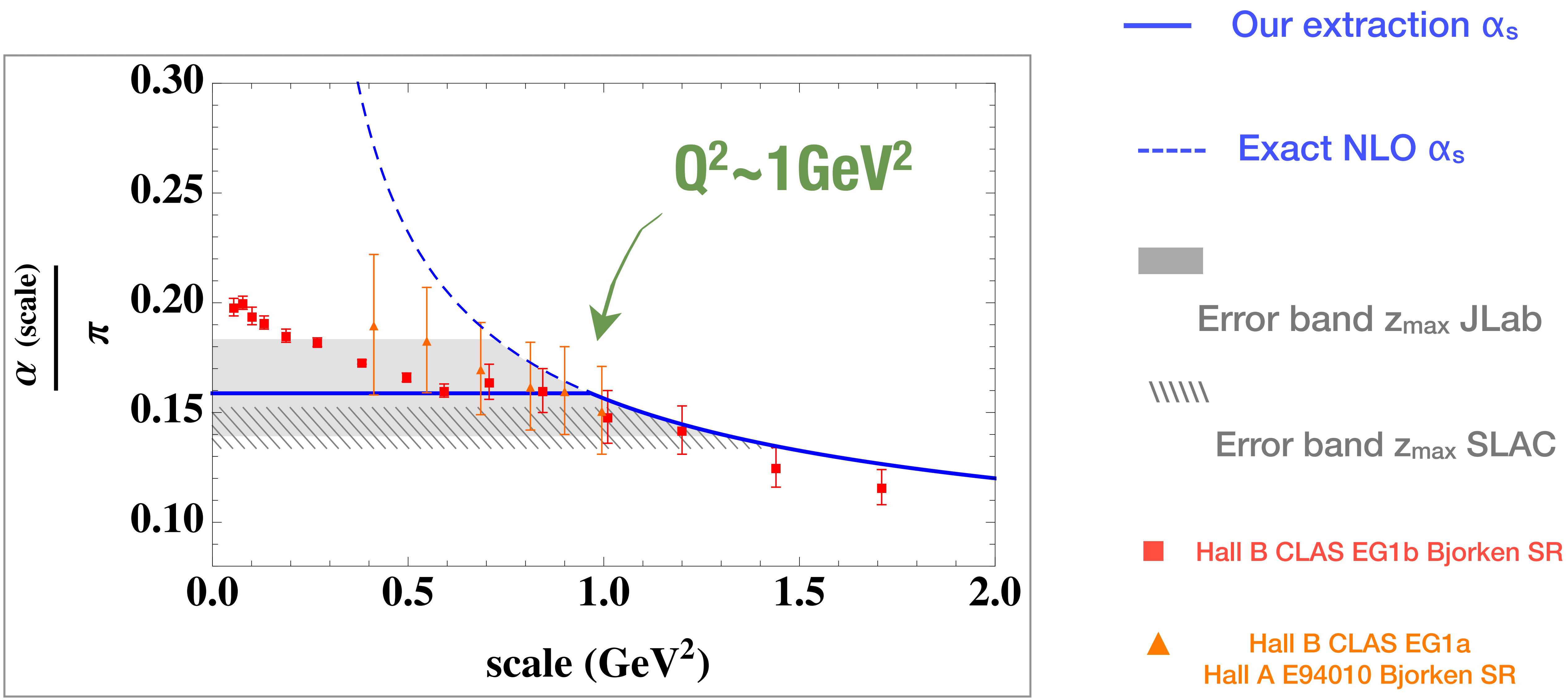
Hall B CLAS EG1b Bjorken SR

Hall B CLAS EG1a
Hall A E94010 Bjorken SR

Jefferson Lab EXPLORING THE NATURE OF MATTER

A.C. & Liuti, Phys Lett B726 (2013)

α_s at low energy from duality



$\alpha_s\ (Q^2 < 1\text{GeV}^2)/\pi = 0.16$

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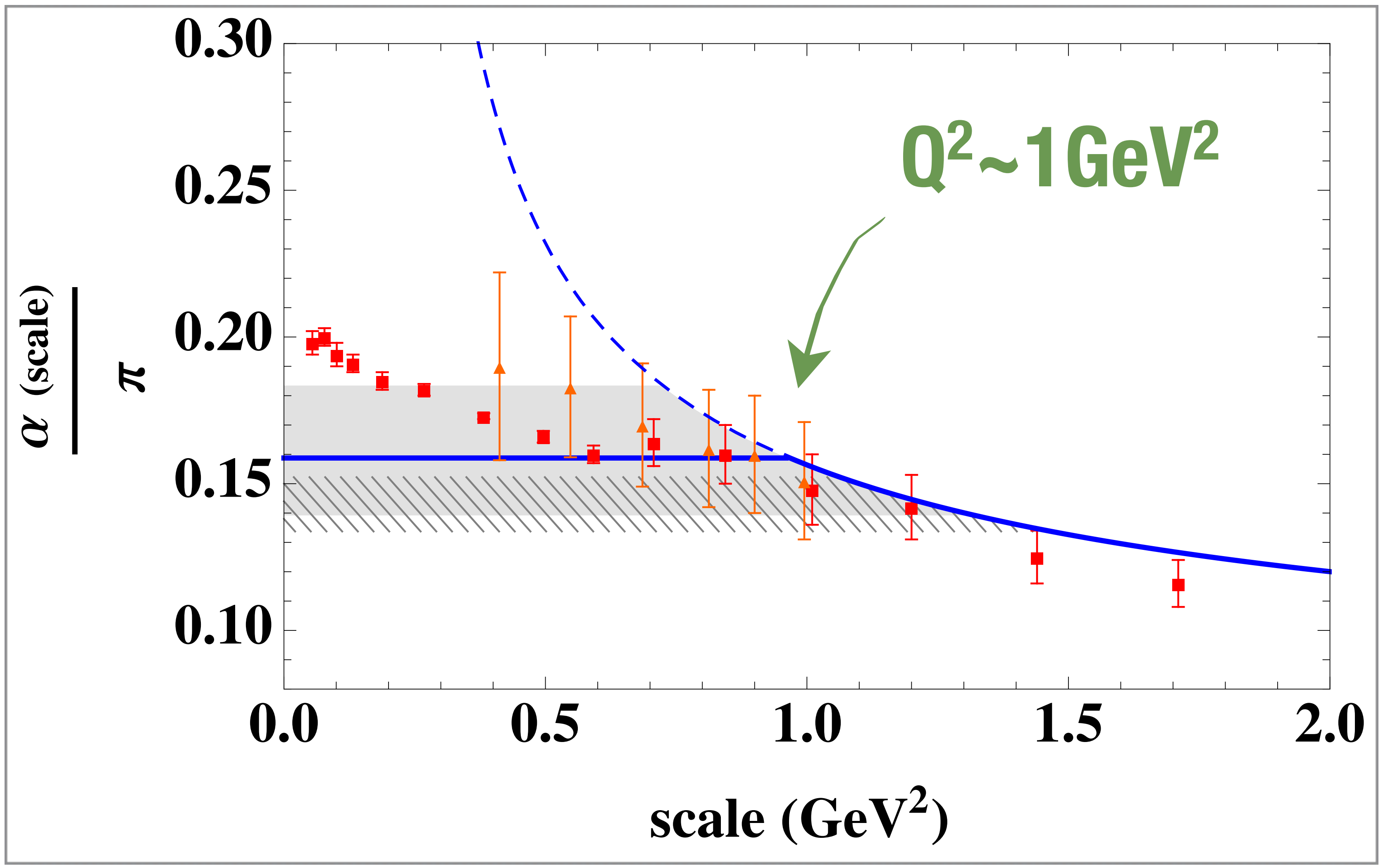


Our extraction α_s

Exact NLO α_s

Error band z_{max} JLab

\\\\\\\\\\\\

Error band z_{max} SLAC

Hall B CLAS EG1b Bjorken SR

Hall B CLAS EG1a
Hall A E94010 Bjorken SR

α_s (Q^2<1GeV^2)/π=0.16

~optimal parametrization scale of the dynamical PDF fit GJR

A.C. & Liuti, Phys Lett B726 (2013)

Comparison with nonperturbative approaches

What if I try to BEST compare the NP approaches to our extraction?

Exact NLO α_s

Error band z_{max} JLab

Error band z_{max} SLAC

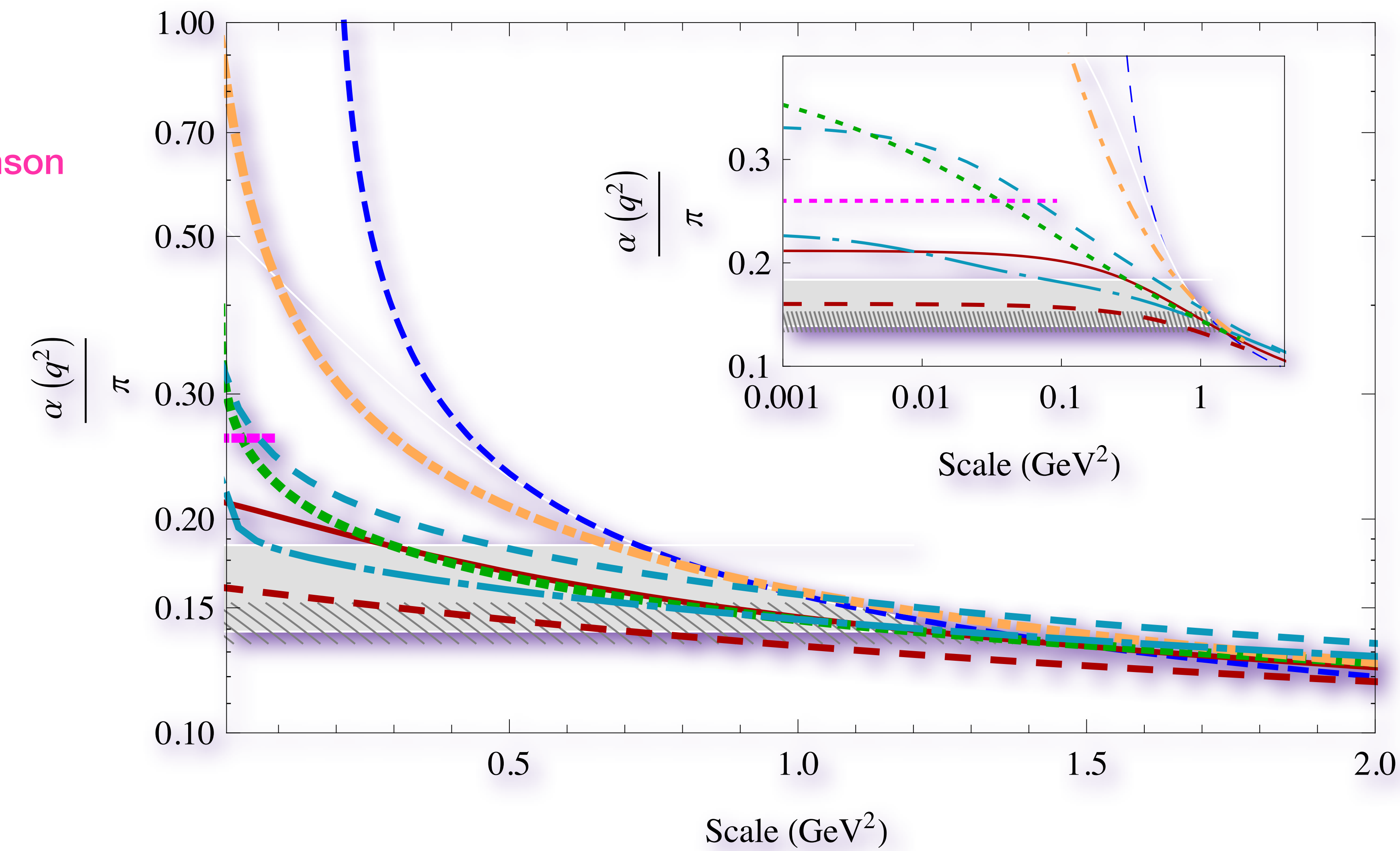
Mattingly & Stevenson
PRD49

Shirkov LO
PRL79

Alkofer & Fischer
PRD67

Cornwall
PRD26

Aguilar, et al
PRD80



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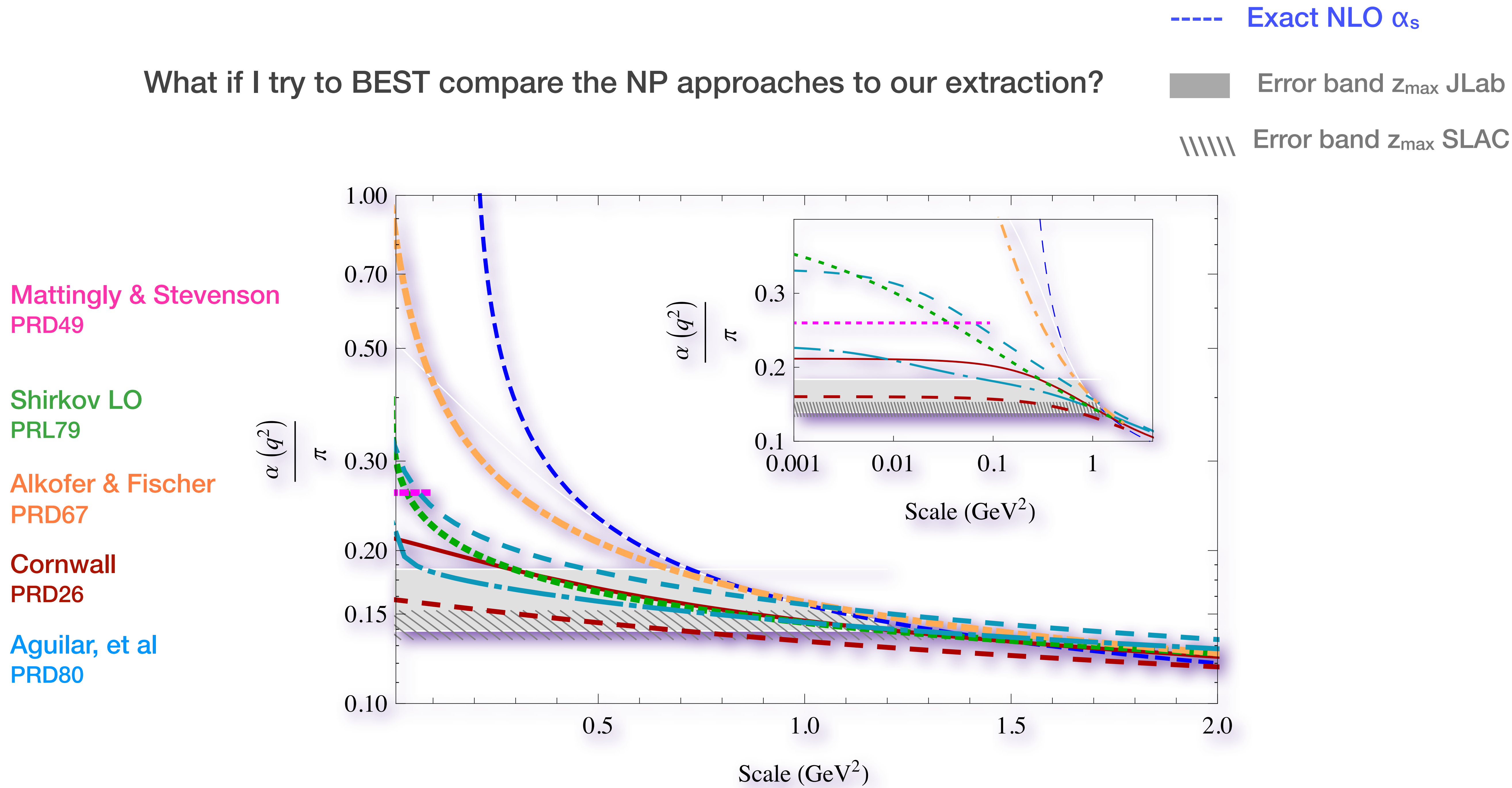
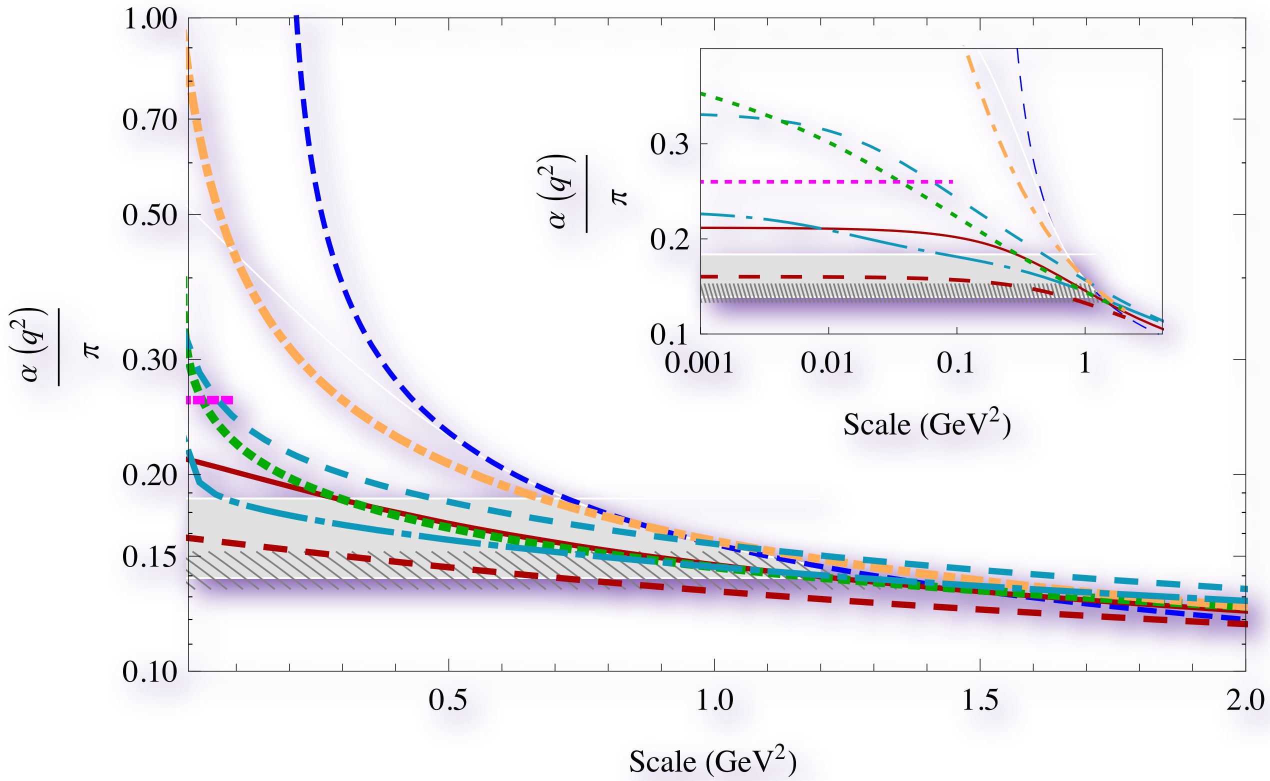


Free parameters of the theories can be fitted

Here Λ is free BUT it has to be adapted

Effective charges & schemes

Can we still understand the **relation** between
schemes (and their physical content)
in the NP regime ?



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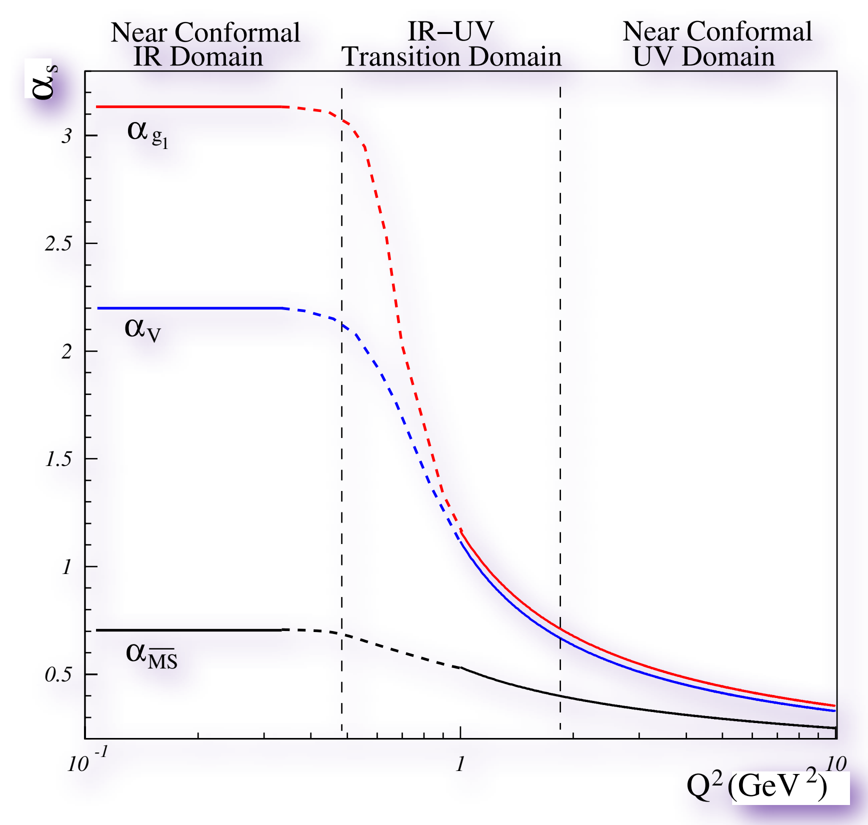

[Brodsky et al., Phys.Rev.D81]

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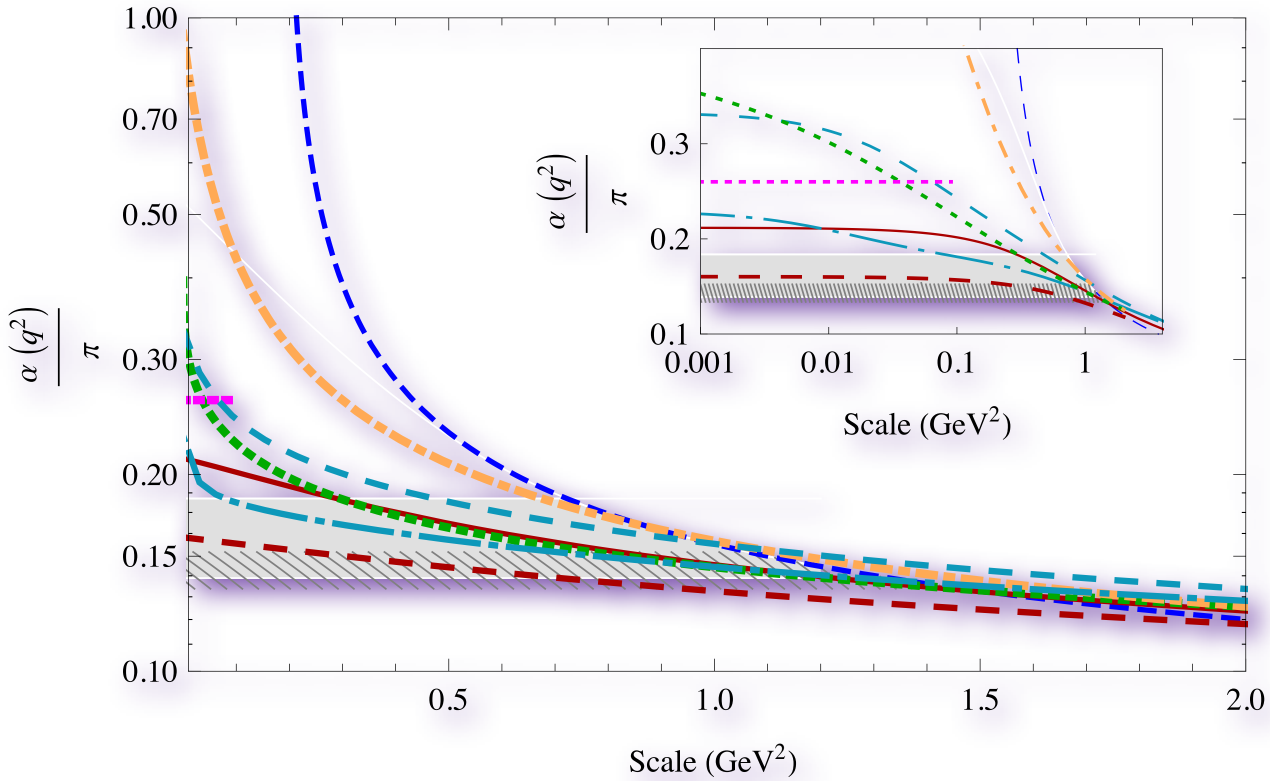

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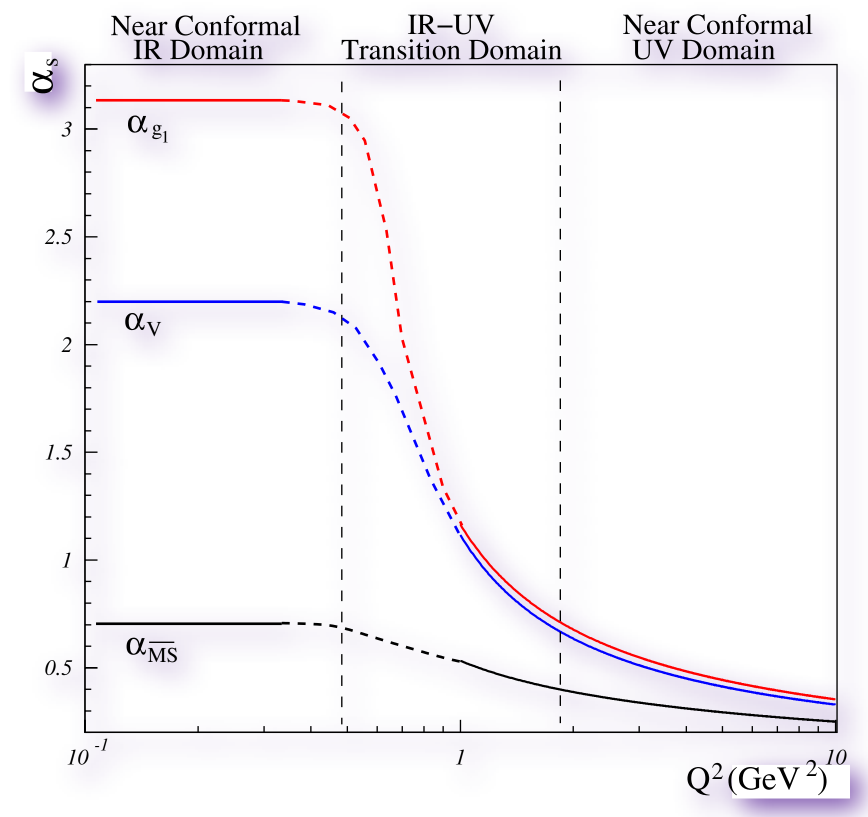

[Brodsky et al., Phys.Rev.D81]

- **How to relate the effective couplings?**
  - **Commensurate Scale Relations?** [Brodsky & Lu, Phys. Rev. D251]
  - **RG-improved perturbation theory?** [Grunberg, Phys. Rev. D29]

Possible higher-twist effects

Note: Ambiguities at the pQCD analysis level

- Quark-gluon interaction is expected to dominate at $x \to 1$
- Resonances $= \infty$ number of twists
- Intricate rôle of higher-twist at the frontier with NP QCD

  → compatibility with confinement?
- Here: all the nonperturbative effects into α_s

  → smooth transition from perturbative to nonperturbative physics

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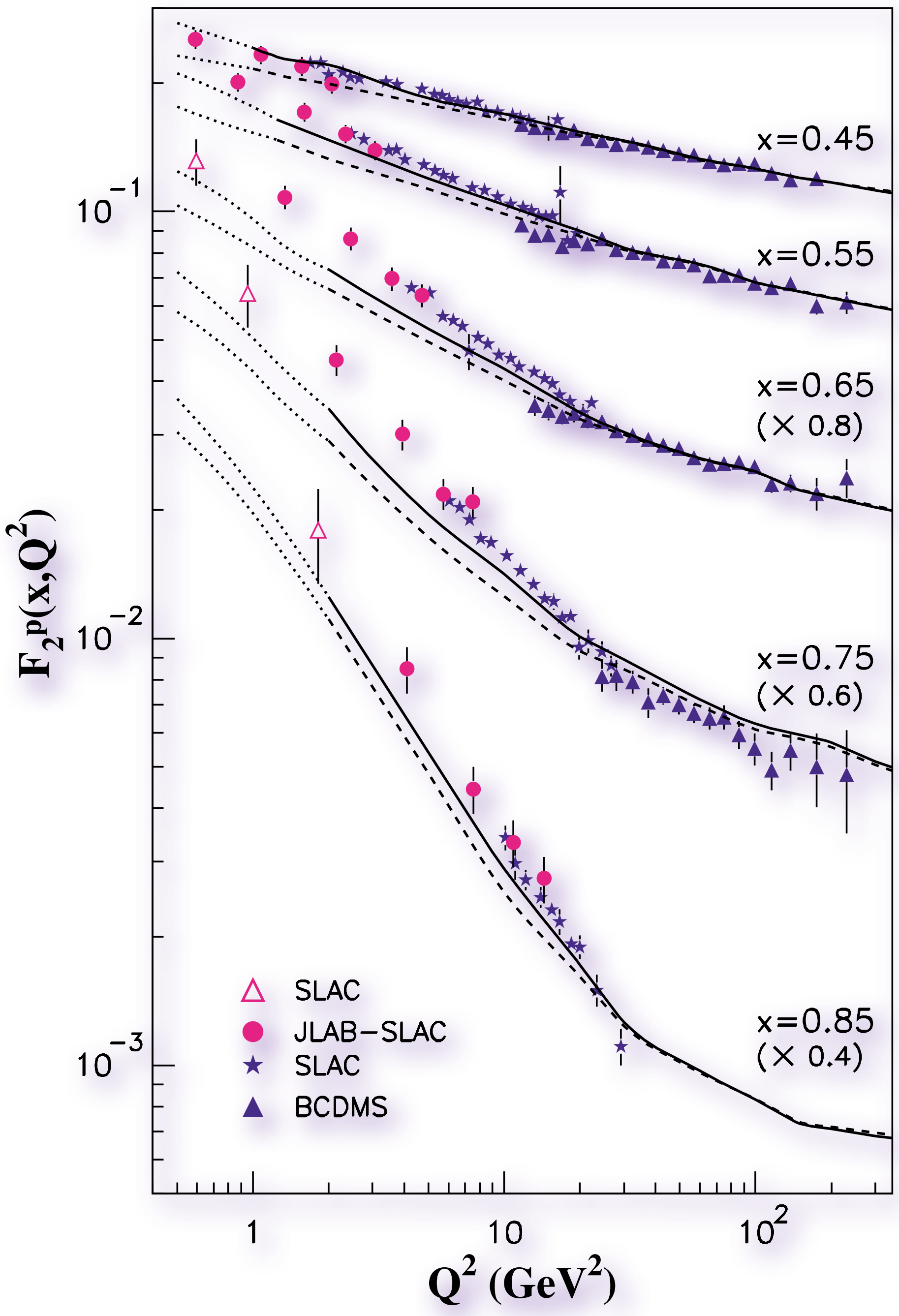
  → smooth transition from perturbative to nonperturbative physics
- Possibly 'double' counting due to uncertainty on PDFs at large-x

Large-x, the other way round

- When x→1, →elastic scattering
- Exclusive scattering
- Intertwine with resonance region
- How to obtain clean PDFs?

➡ using DUALITY !



[Liuti et al., PRL89]

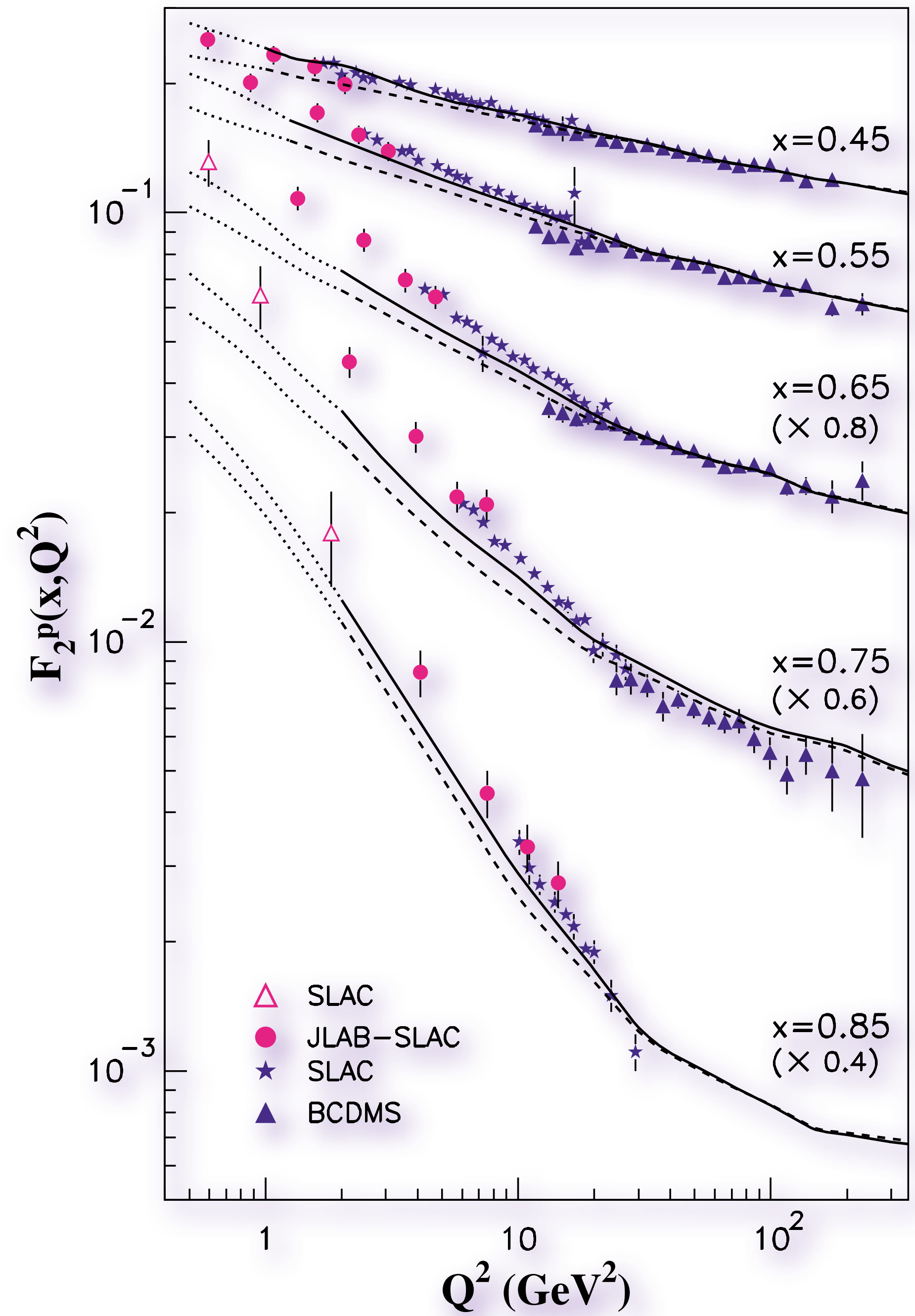
Large-x, the other way round

- When x→1, →elastic scattering
- Exclusive scattering
- Intertwine with resonance region
- How to obtain clean PDFs?

  ➡ using DUALITY !

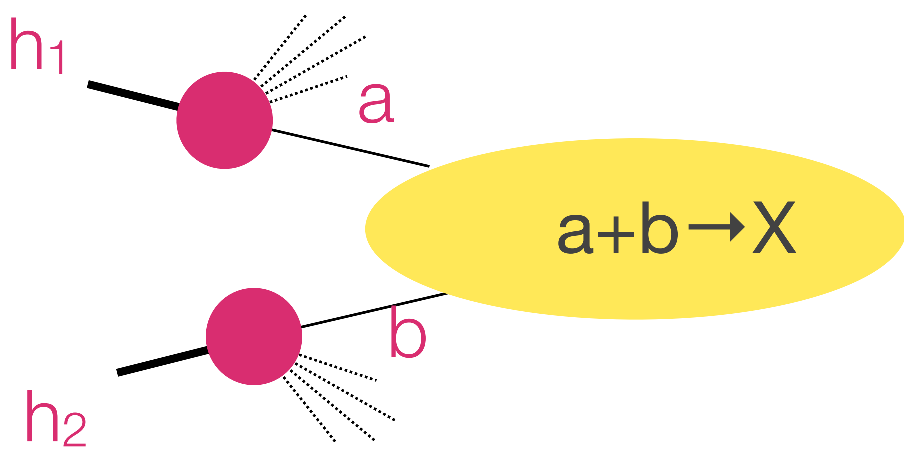
- usually thru a Higher-twist term $\mathcal{O}(1/Q^2)$
- correction of the same $\mathcal{O}$ as cut in α_s

➡ Only solution: properly fit large-x PDFs !



[Liuti et al., PRL89]

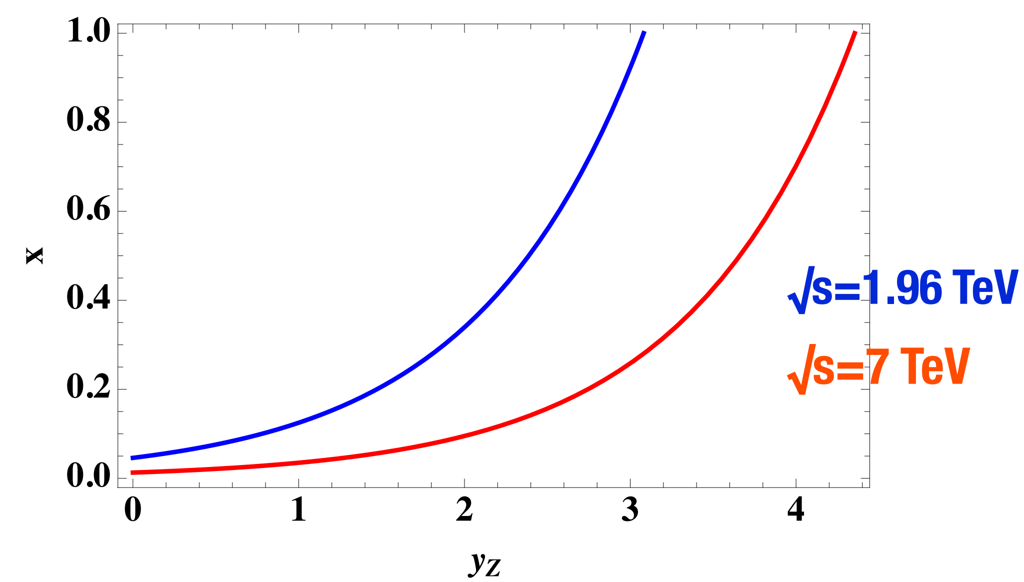
Large-x matters



- hadron-hadron collisions
- 2 partons with scaling variables: x_1 and x_2
- fixed $\sqrt{s}$
- boson rapidity y, mass M

$$x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}$$

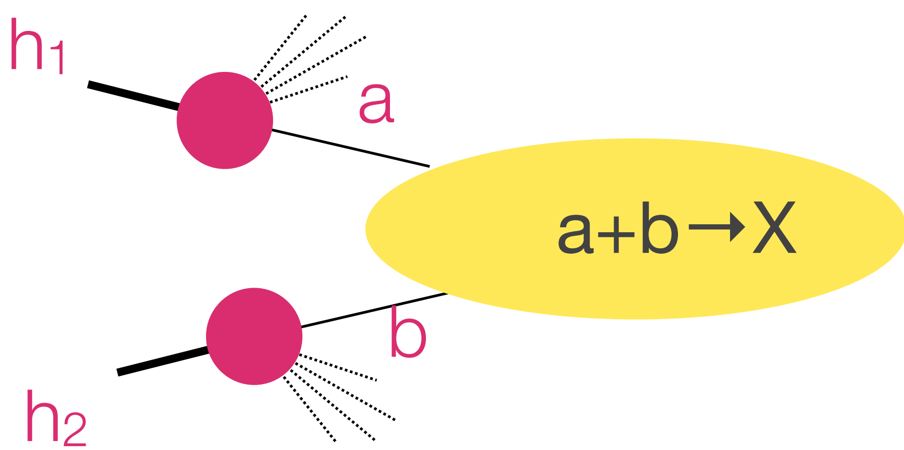
M_Z →



$$\sigma_X(s, M_X^2) = \sum_{a,b} \int_{x_{\min}}^{1} dx_1\, dx_2\, f_{a/h_1}(x_1, M_X^2)\, f_{b/h_2}(x_2, M_X^2)\, \hat{\sigma}_{ab\to X}\left(x_1 x_2 s, M_X^2\right)$$

Brady et al, JHEP (2012) 1206

Large-x matters



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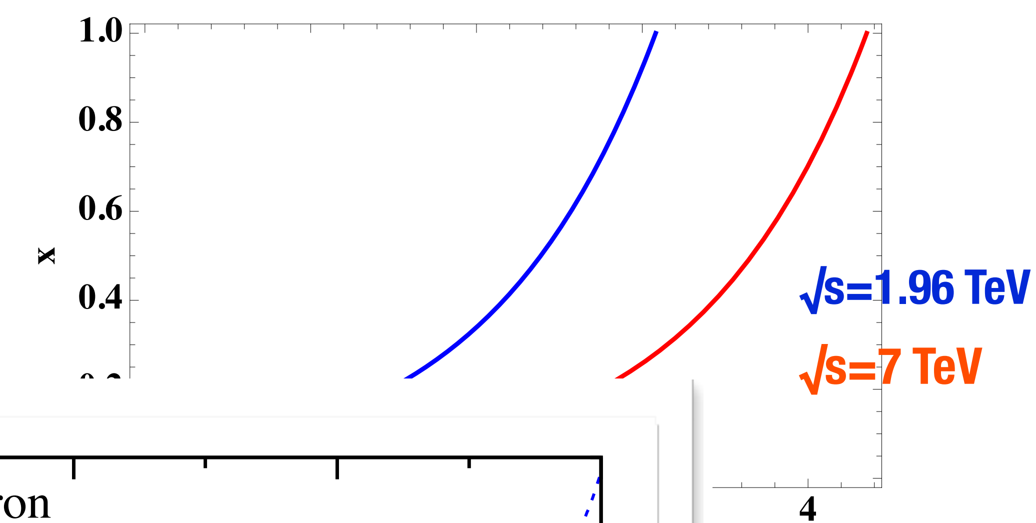
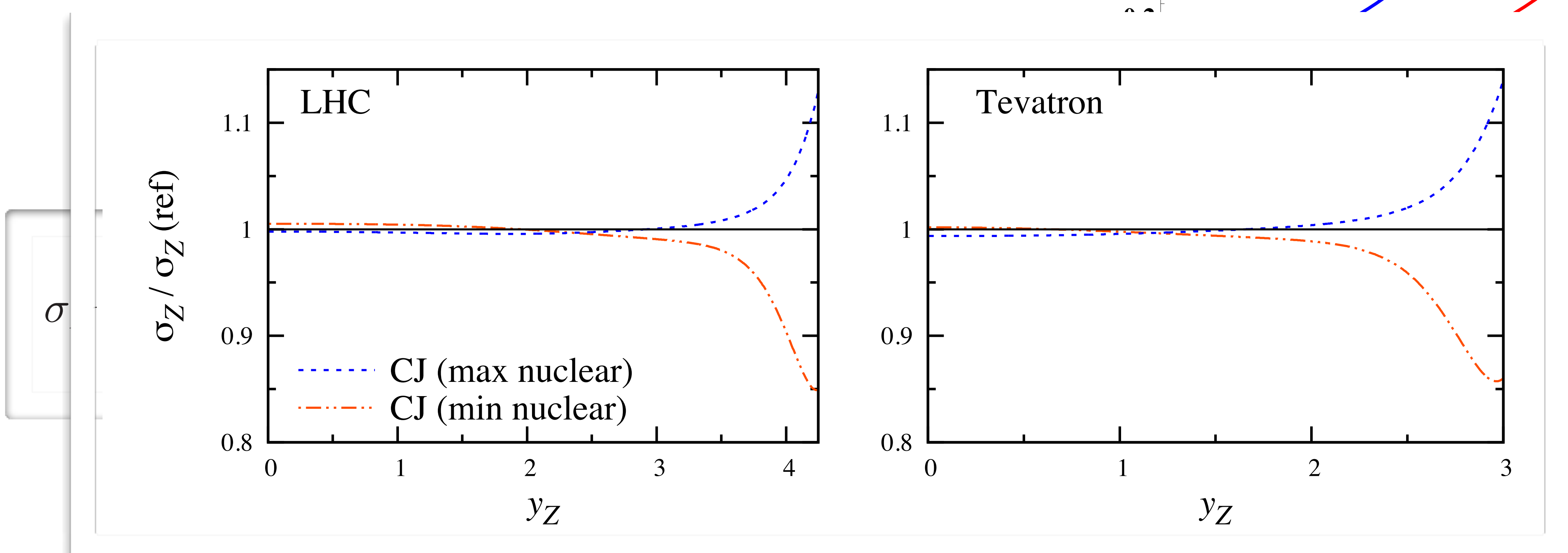


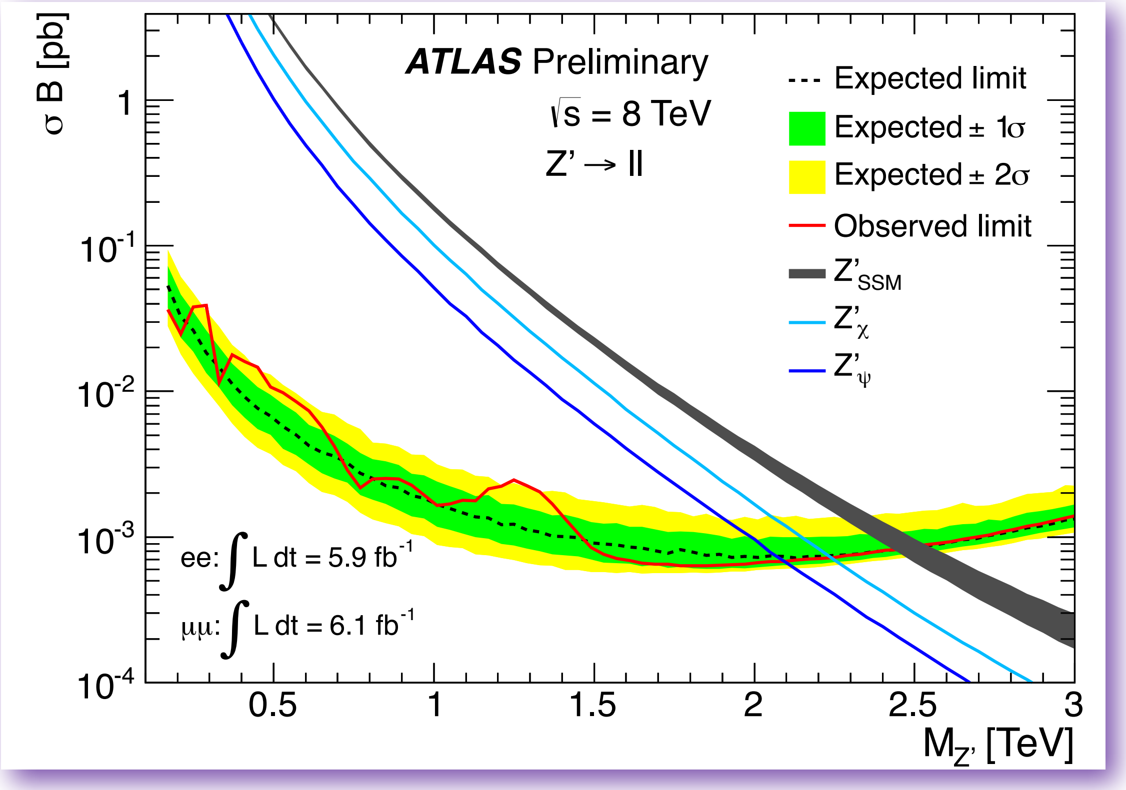


Large-x matters

Z' and W' searches: *e.g. ATLAS-CONF-2012-129*

- $x_{min}=M_X^2/s$~large
- PDF uncertainties+ α_s ...at 2TeV~20%
- Large-x DOES matter

talk by S.Forte at PDF4LHC meeting



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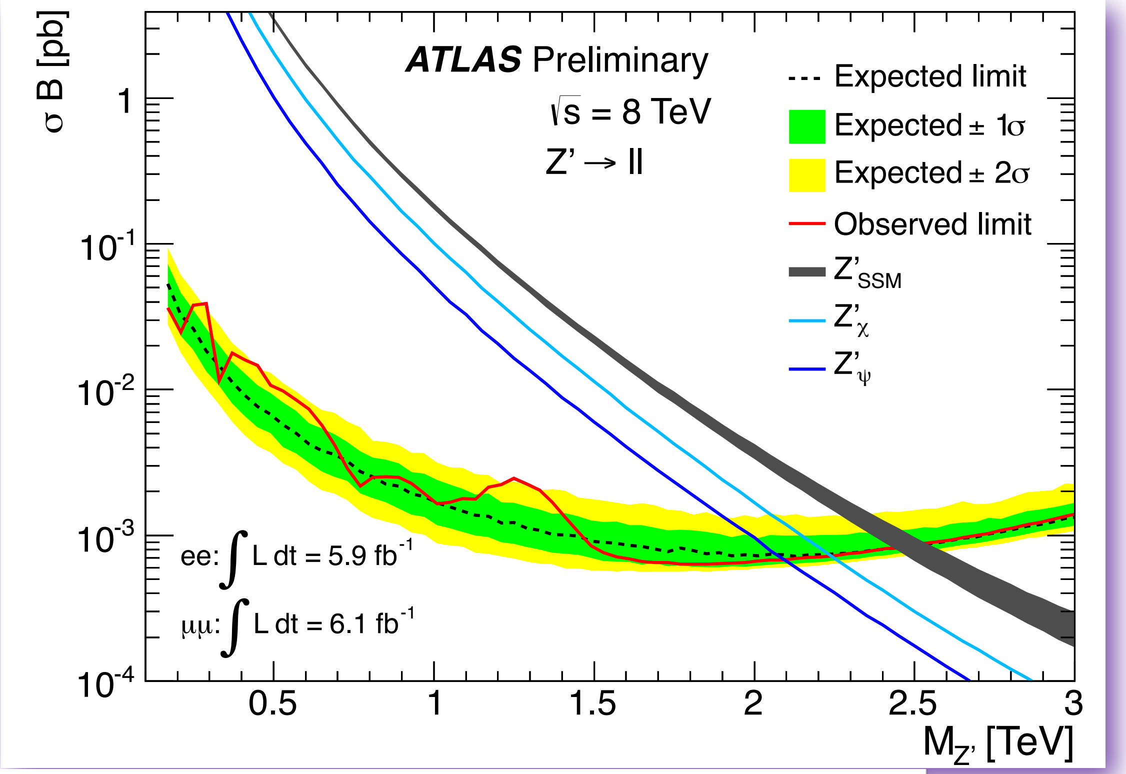
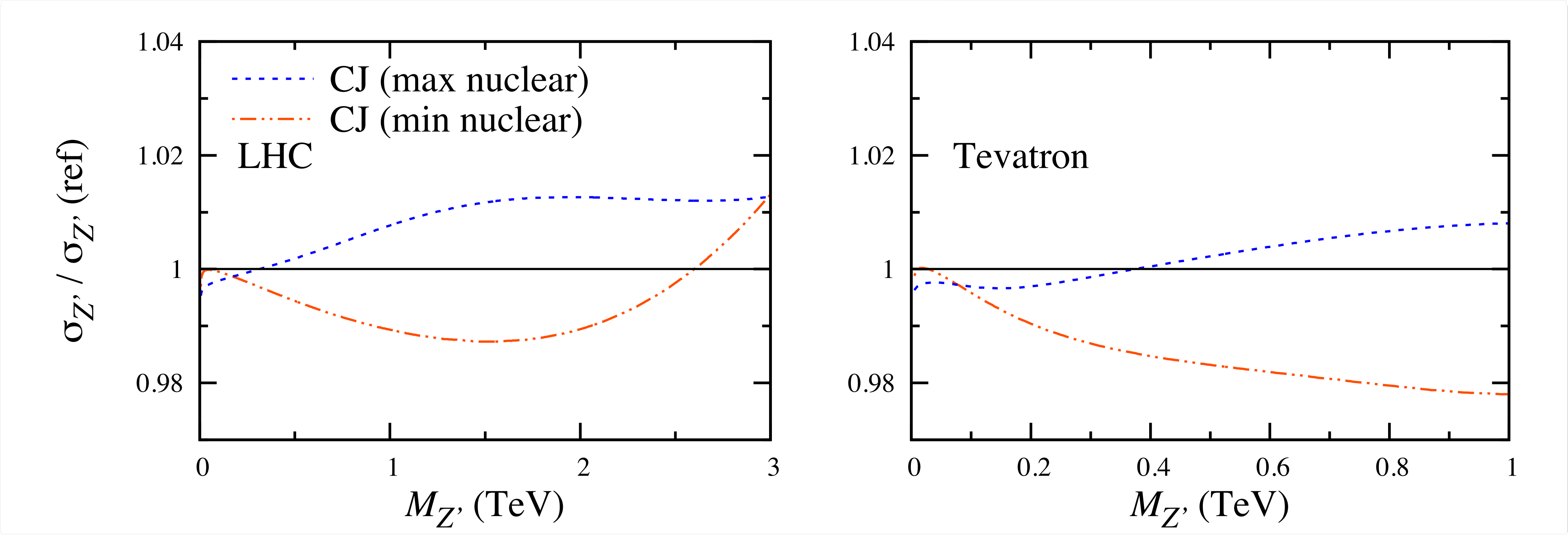


talk b



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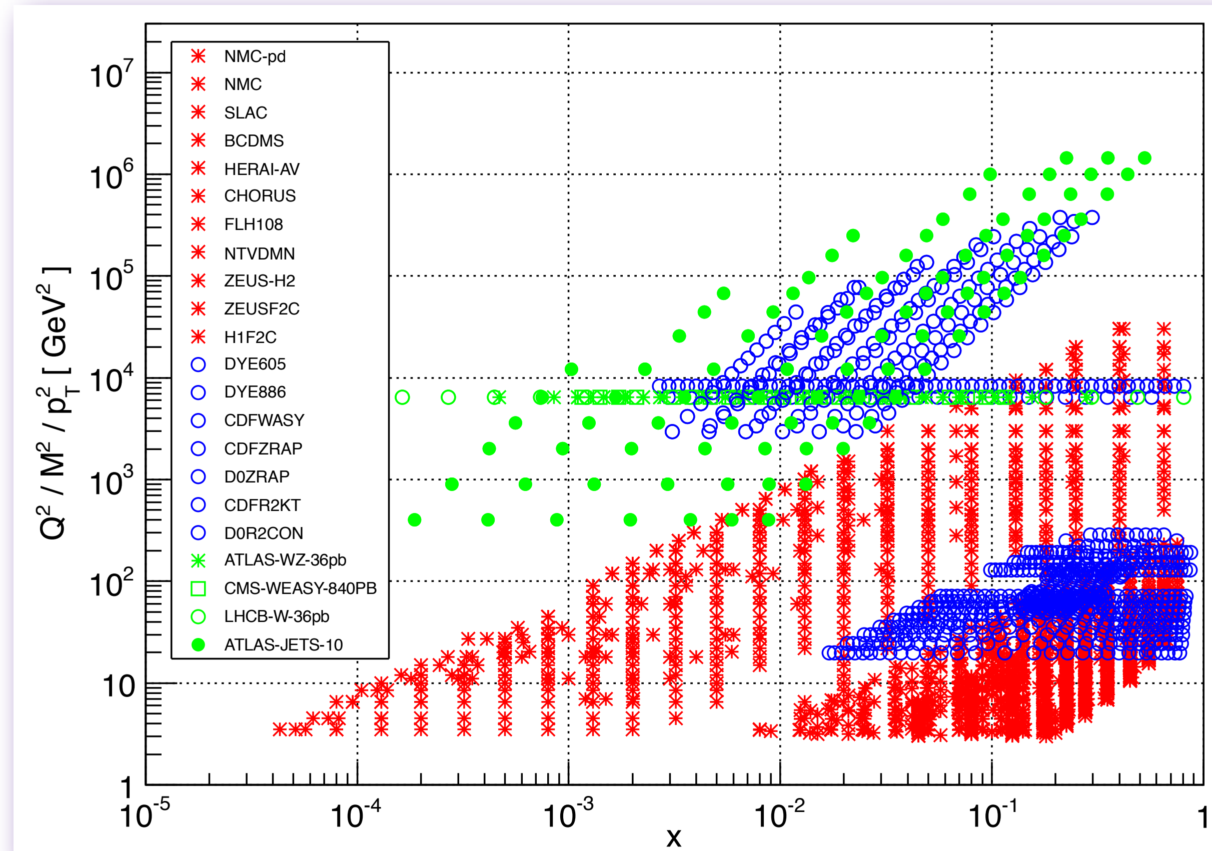
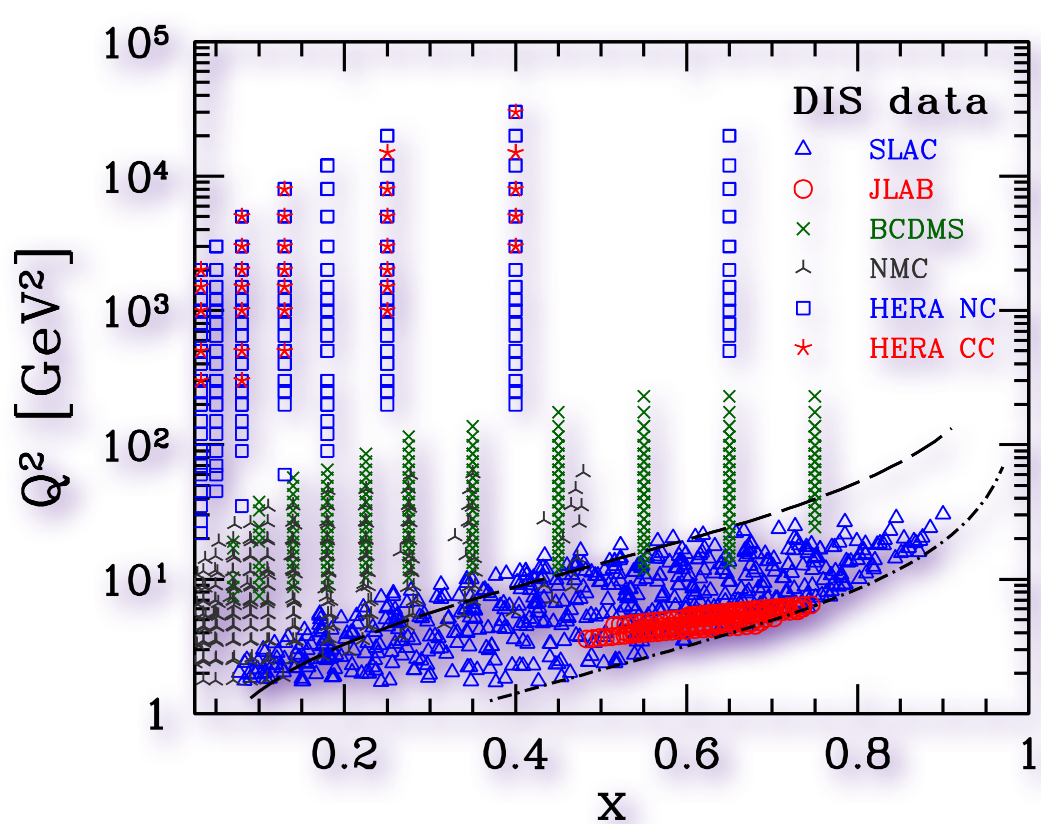
Data set for PDF fits

NNPDF2.3

- DIS data: x<0.6
- TEVATRON data extend to slightly higher-x,

  BUT large uncertainties
- LHC data might cover high-x region





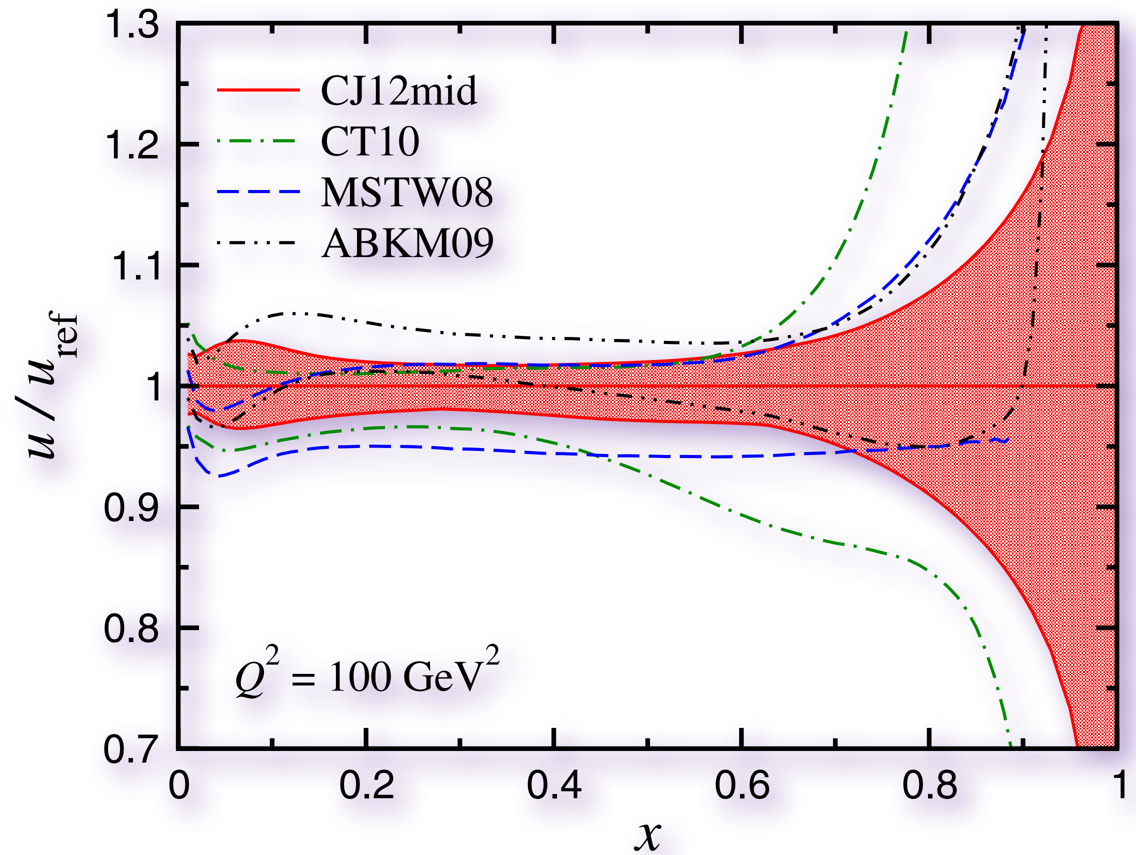
CJ (CTEQ/JLab) DIS data

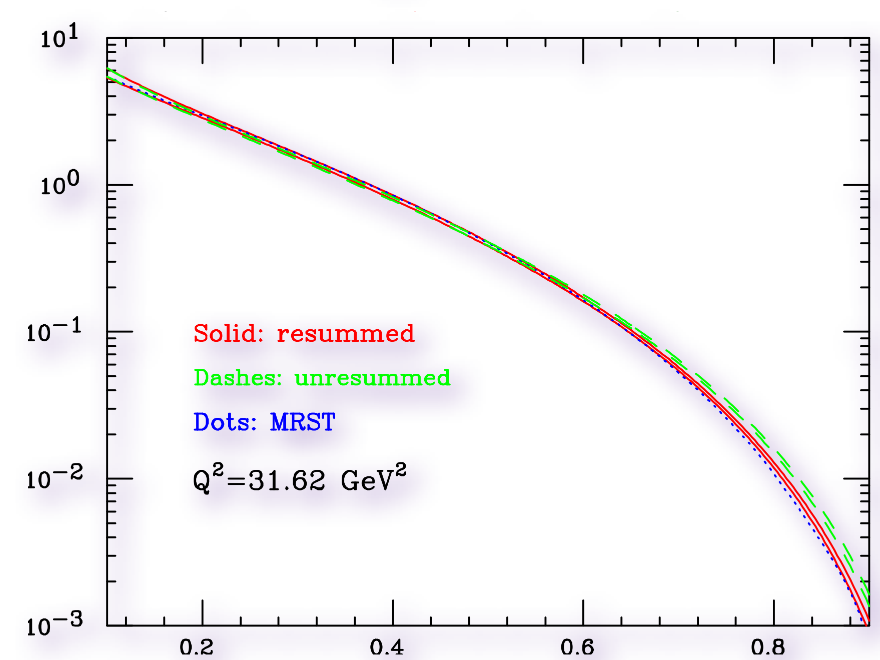
----- $W^2 \gtrsim 14$ GeV^2 cut typically used in global PDF fits

- · - · - $W^2 \gtrsim 3$ GeV^2 cut typically used in CJ fits

Large-x: present and future

- Nuclear corrections: CJ (CTEQ/JLab)
- Non-perturbative inputs needed
  - add & fit a term $\mathcal{O}(1/Q^2)$: CJ, ABKM, MSTW
  - resum: Corcella & Magnea [Phys.Rev. D72 (2005)]





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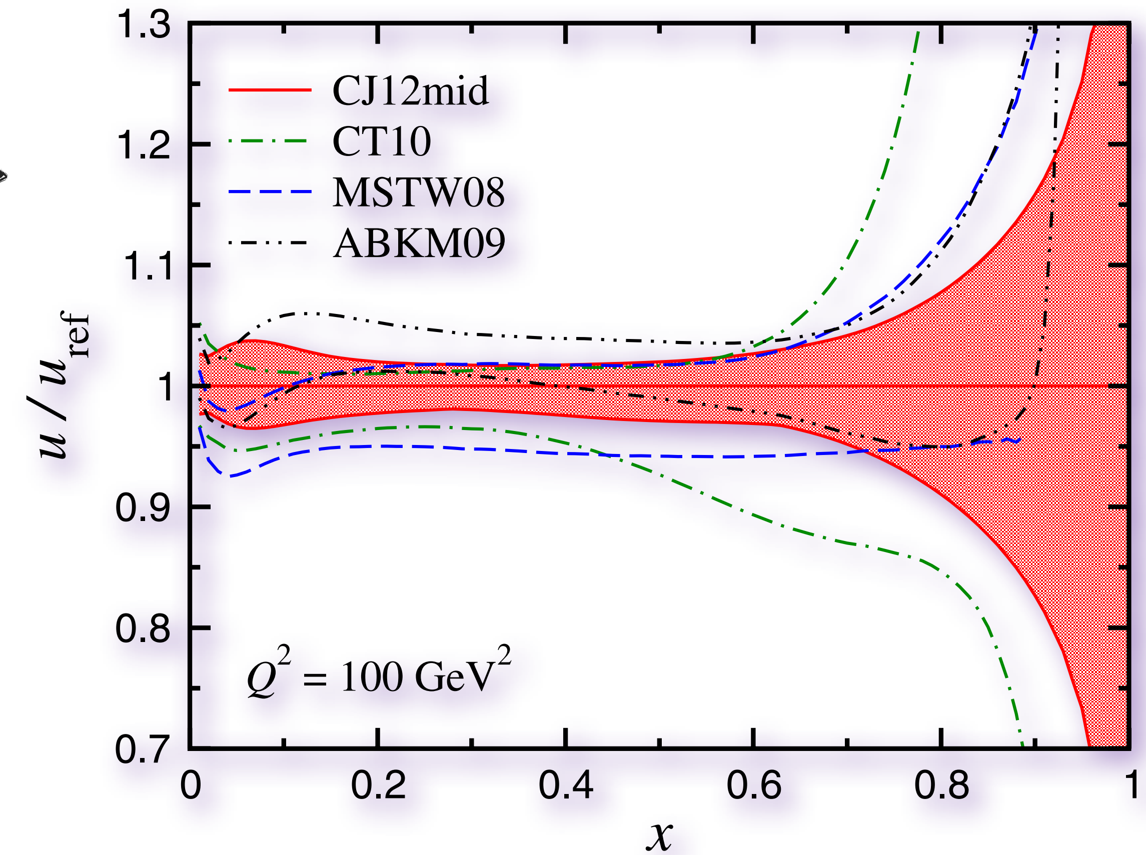

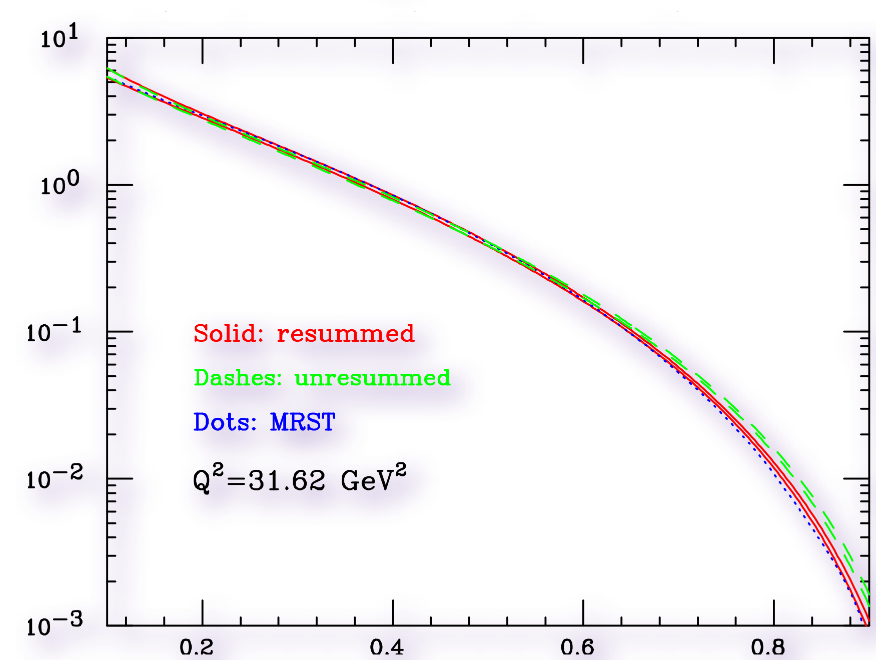

Future:

new fit of PDFs including large-x resummation with full data sets

Extension of the SOMPDF [Askanazi et al, 1309.7085] in preparation [Askanazi et al.]

Conclusions

- Analyzis of the Bloom-Gilman quark-hadron duality in perturbative QCD
- Its realization is parametrized by the freezing of the running coupling constant
- Our approach:

  ➡ *All the NP effects are embedded in the effective charge at the hadronic scale*

- The hadronic scale turns out to be Q_0^2=1GeV^2
- $\alpha_s\ (Q^2 < 1\text{GeV}^2)/\pi = 0.16$
- Doesn't disagree with NP approaches.
- Comparison of perturbative & NP schemes has to be understood!

Hadronic physics:

- Intersection between perturbative and non-perturbative QCD
- Transition of degrees of freedom
- Non-perturbative QCD provides (or shall provide) for INPUTS to pQCD
- QCD as a whole
- Impact on high energy phenomenology
  - hadronic matrix elements (spin also matters)
  - QCD evolution (Q_0^2, scale, α_s)
  - fits/extractions

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- **Rich phenomenology in itself:**
  - confinement, chiral symmetry, duality
  - knowledge on the proton!