



## **BELLE PART 2:**

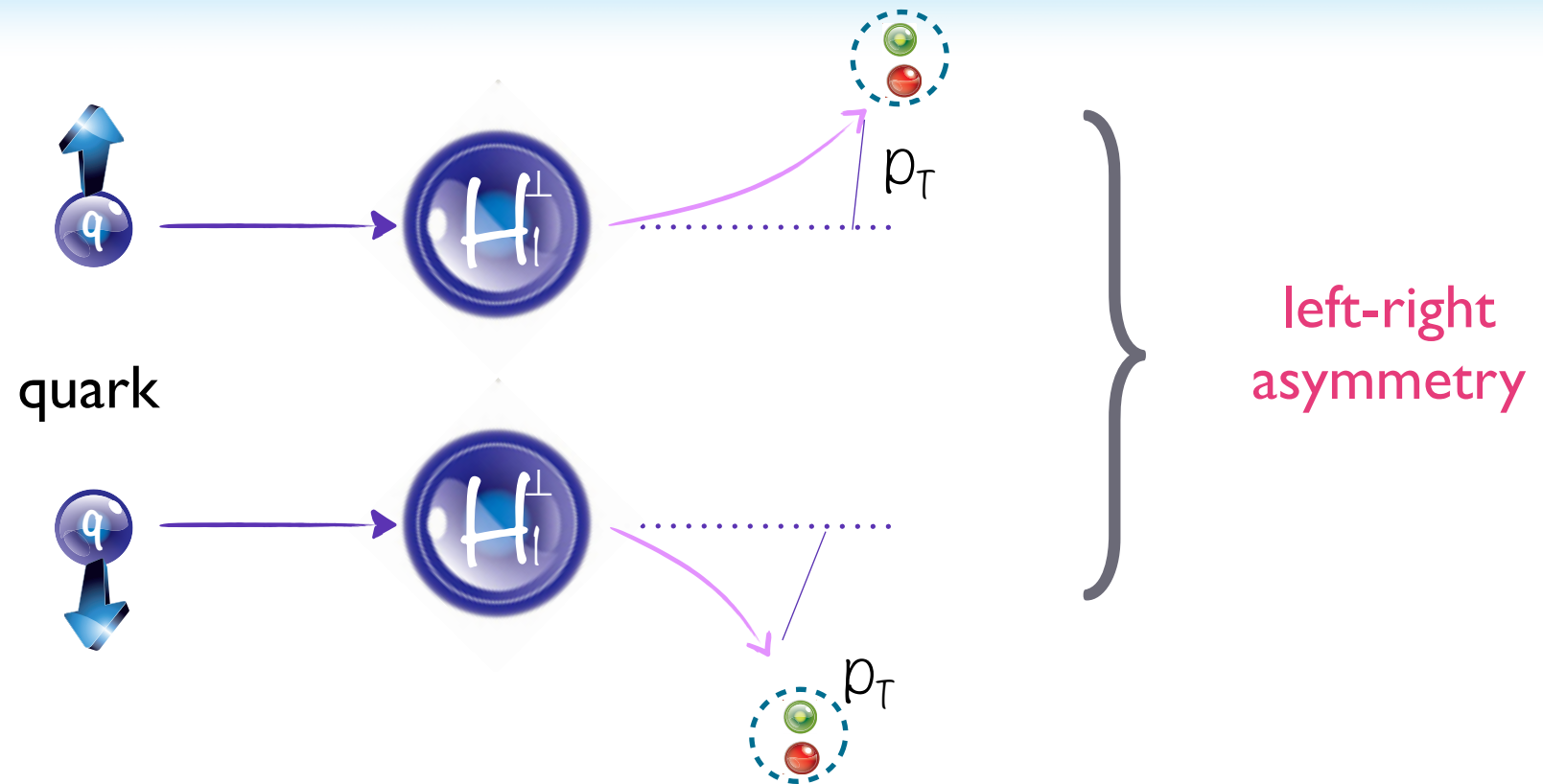
# **KAON COLLINS MEASUREMENTS**

4th International Workshop on Transverse Polarisation Phenomena in Hard Processes  
(Transversity 2014)

Chia, June 9<sup>th</sup>-13<sup>th</sup> 2014

Francesca Giordano, for the BELLE collaboration

# Collins Fragmentation

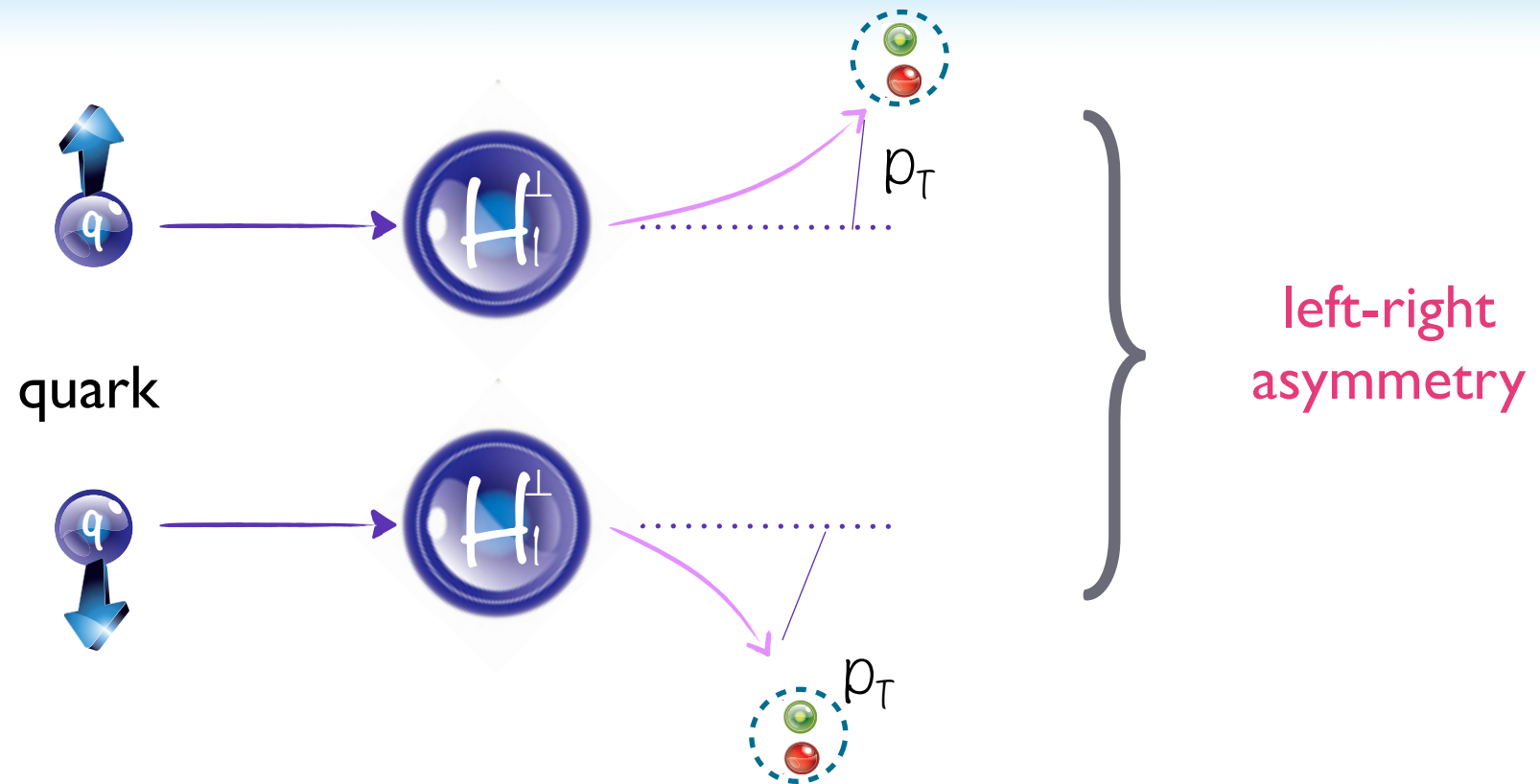


Collins mechanism: correlation between the parton transverse spin and the direction of final hadron





# Collins Fragmentation



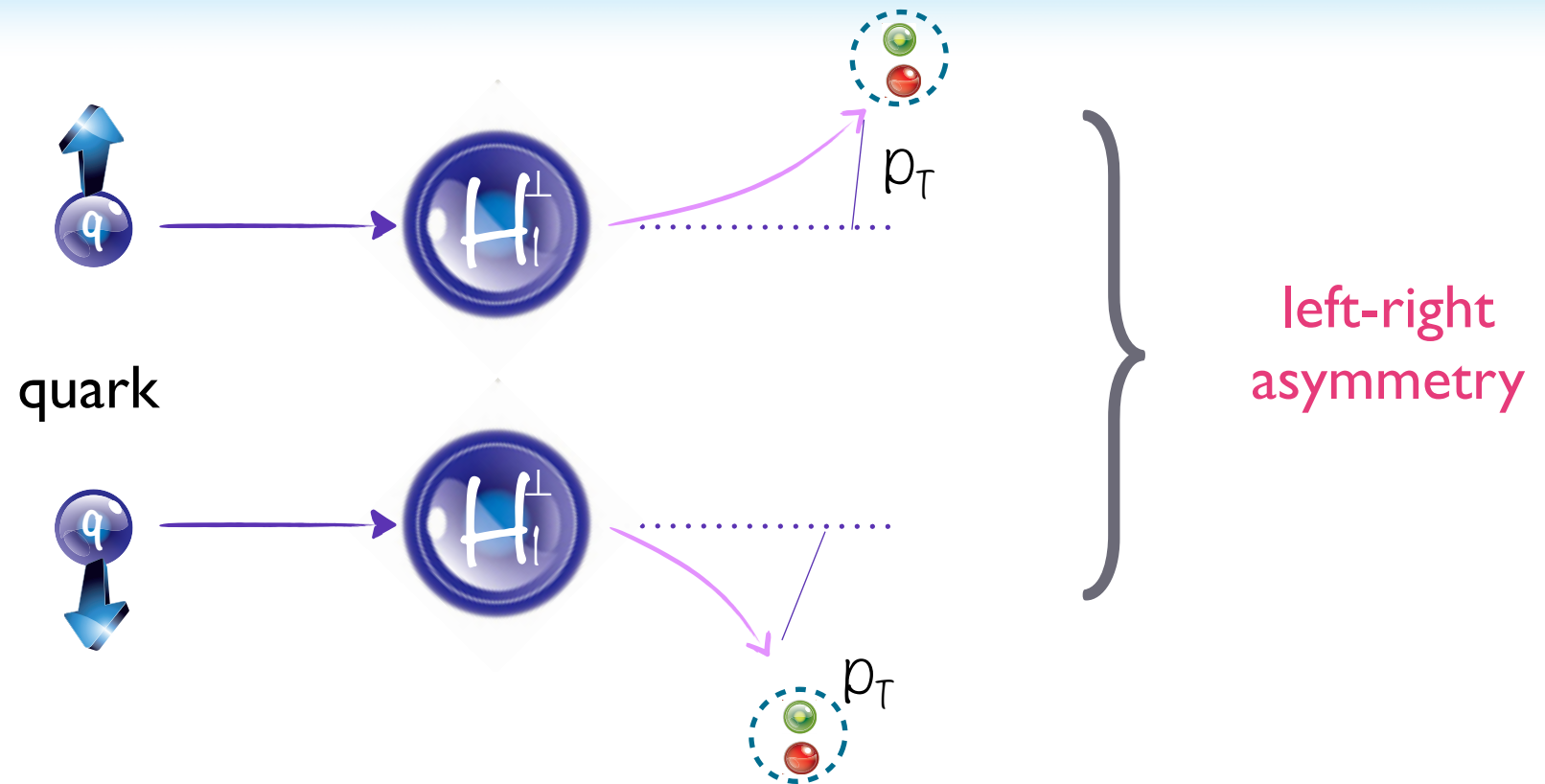
Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

strictly related to the outgoing hadron transverse momentum

**TMD!**



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Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

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**TMD!**

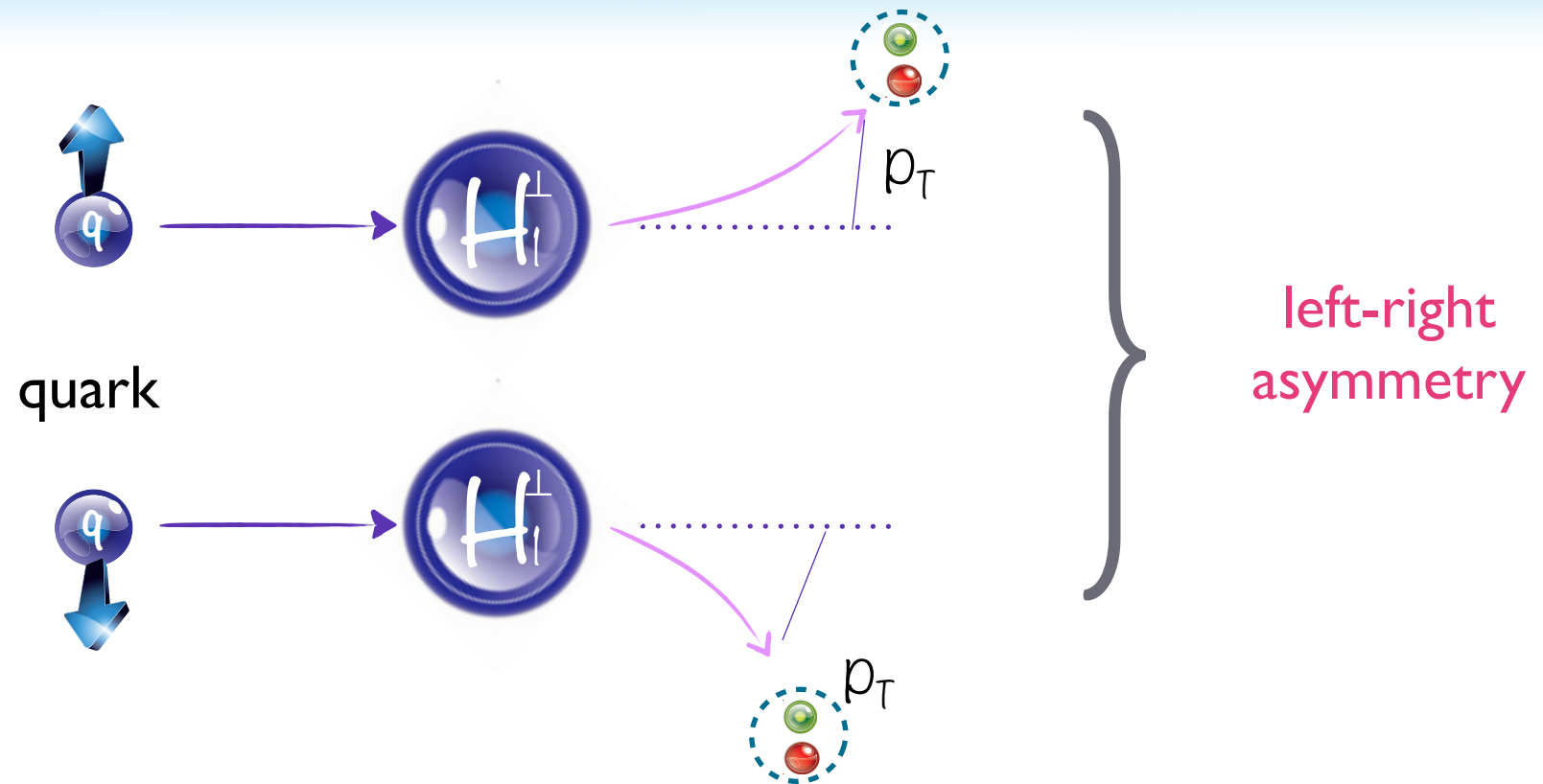
**Chiral odd!**

$$\begin{array}{c}
 X \otimes H_1^\perp \\
 \text{chiral odd} \quad \text{chiral odd} \\
 \underbrace{\hspace{10em}} \\
 \text{chiral even}
 \end{array}$$





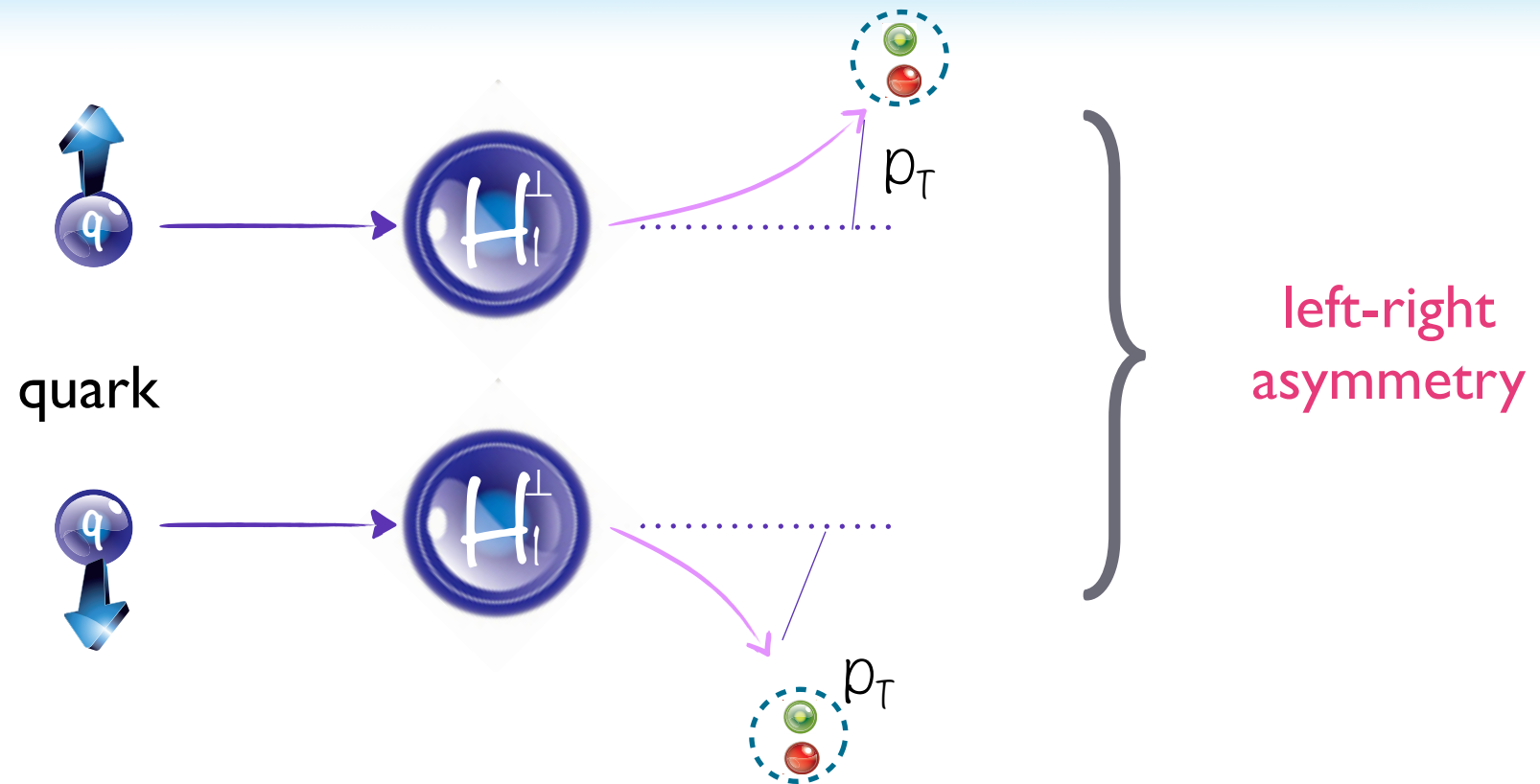
# Collins Fragmentation



In  $e^+e^-$  reaction, there is no fixed transverse axis to define azimuthal angles to, and even if there were one the net quark polarization would be 0



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In  $e^+e^-$  reaction, there is no fixed transverse axis to define azimuthal angles to, and even if there were one the net quark polarization would be 0

But if we look at the whole event, even though the  $q$  and  $\bar{q}$  spin directions are unknown, they must be parallel

$$h = \pi, K \quad e^+ e^- \rightarrow q \bar{q} \rightarrow h_1 h_2 X$$

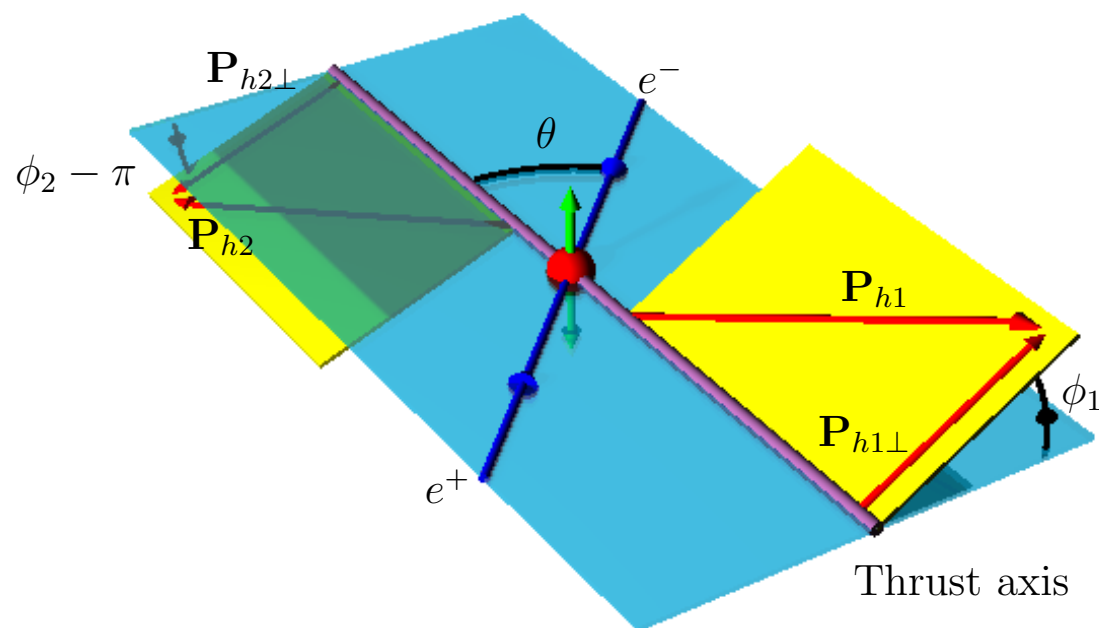


# Reference frames

$$e^+ e^- \rightarrow q \bar{q} \rightarrow h_1 h_2 X \quad h = \pi, K$$

$\phi_1 + \phi_2$  method:

hadron azimuthal angles with respect to the  $q\bar{q}$  axis proxy

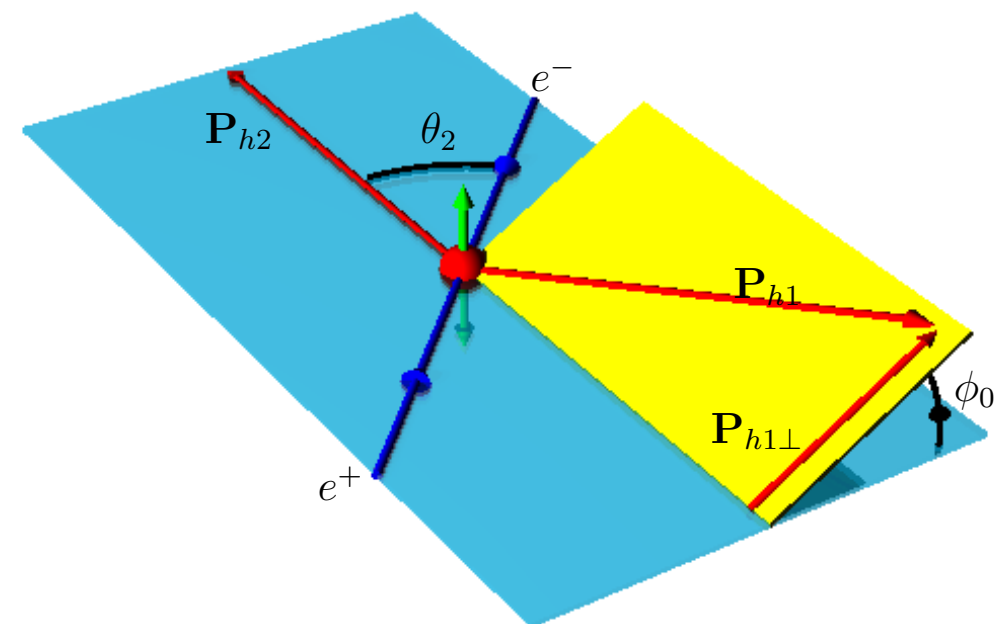


reference plane (in blue) given by the  $e^+e^-$  direction and the  $q\bar{q}$  axis

Thrust axis = proxy for the  $q\bar{q}$  axis

$\phi_0$  method:

hadron 1 azimuthal angle with respect to hadron 2



reference plane (in blue) given by the  $e^+e^-$  direction and one of the hadron

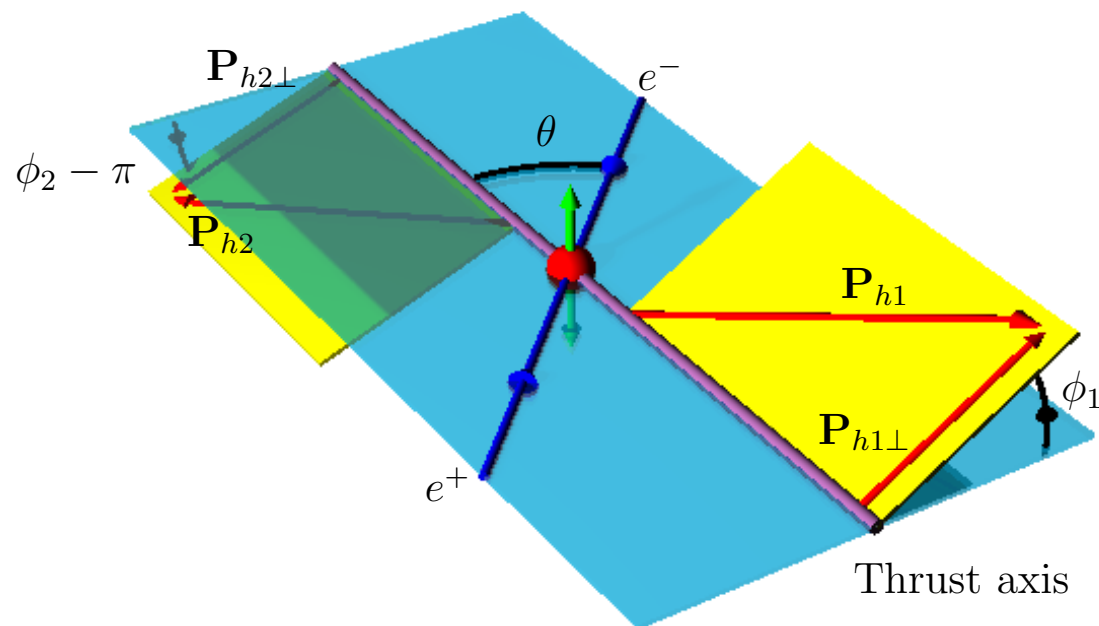




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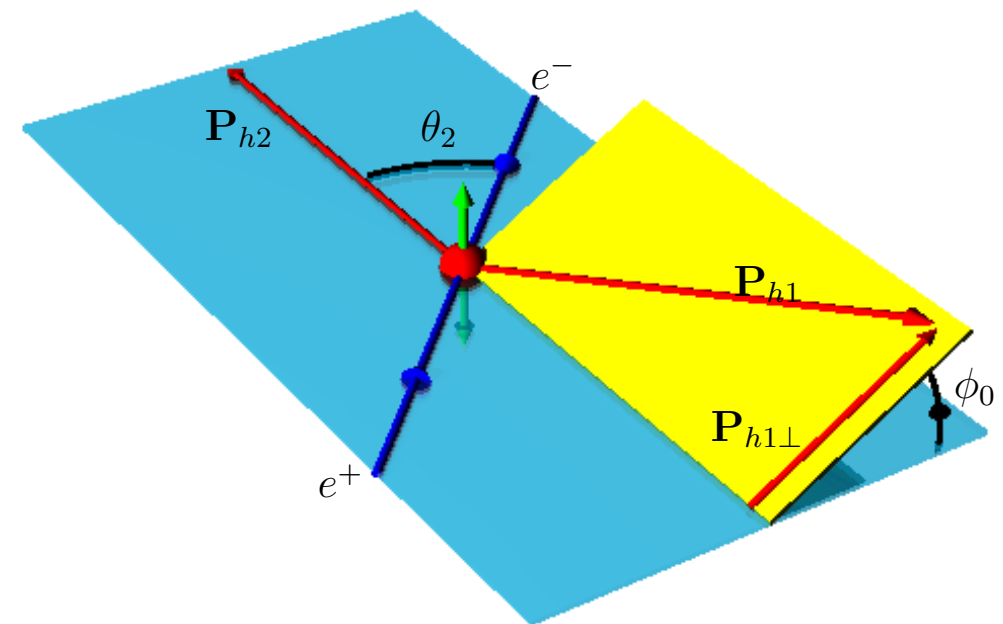
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$$\sigma \sim \mathcal{M}_{12} \left( 1 + \frac{\sin^2 \theta_T}{1 + \cos^2 \theta_T} \cos(\phi_1 + \phi_2) \frac{H_1^{\perp[1]}(z_1) \bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1) \bar{D}_1^{[0]}(z_2)} \right) \quad \sigma \sim \mathcal{M}_0 \left( 1 + \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos(2\phi_0) \mathcal{F} \left[ \frac{H_1^{\perp}(z_1) \bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1) \bar{D}_1^{\perp}(z_2)} \right] \right)$$

$$F^{[n]}(z_i) \equiv \int d|k_T|^2 \left[ \frac{|k_T|}{M_i} \right]^{[n]} F(z_i, |k_T|^2)$$

$$\mathcal{F}[X] = \sum_{q\bar{q}} \int [2\hat{h} \cdot \mathbf{k}_{T1} \hat{h} \cdot \mathbf{k}_{T2} - \mathbf{k}_{T1} \cdot \mathbf{k}_{T2}] d^2\mathbf{k}_{T1} d^2\mathbf{k}_{T2} \delta^2(\mathbf{k}_{T1} + \mathbf{k}_{T2} - \mathbf{q}_T) X$$

D. Boer  
Nucl.Phys.B806:23,2009

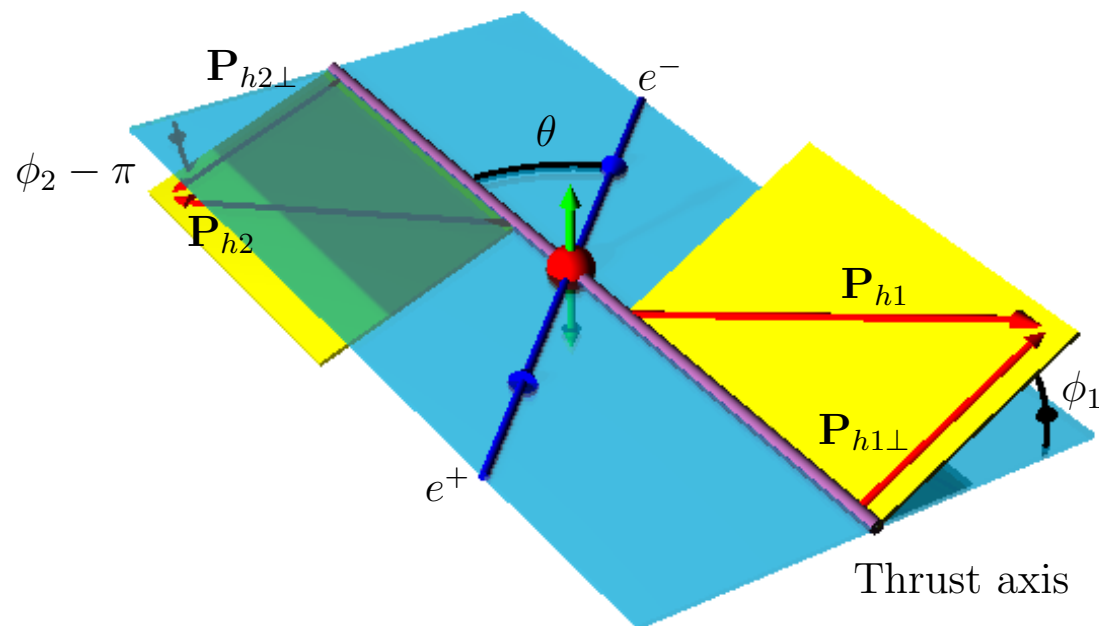
$$k_{Ti} = z_i p_{Ti}$$



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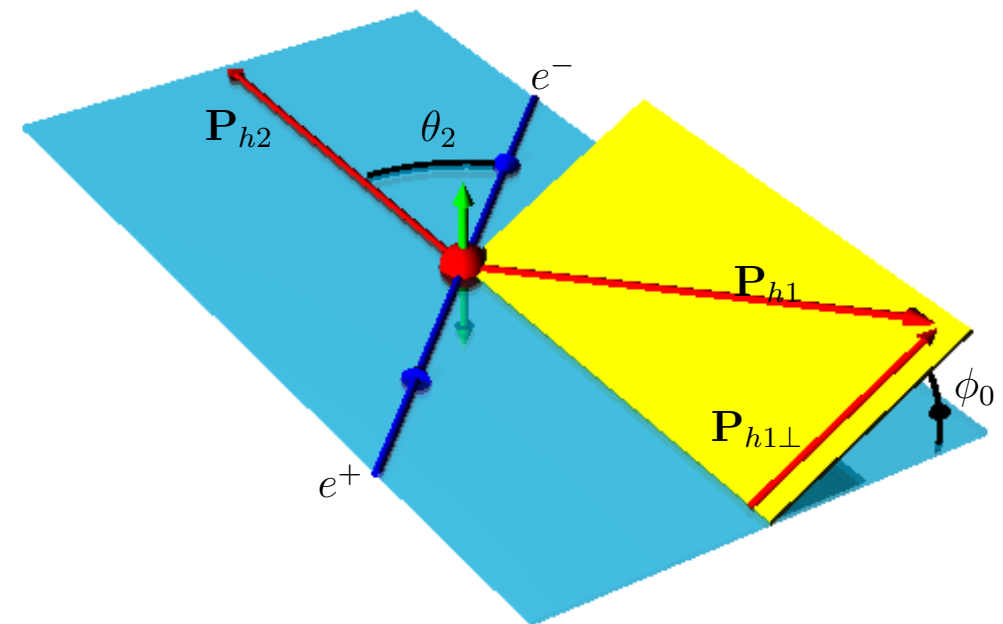
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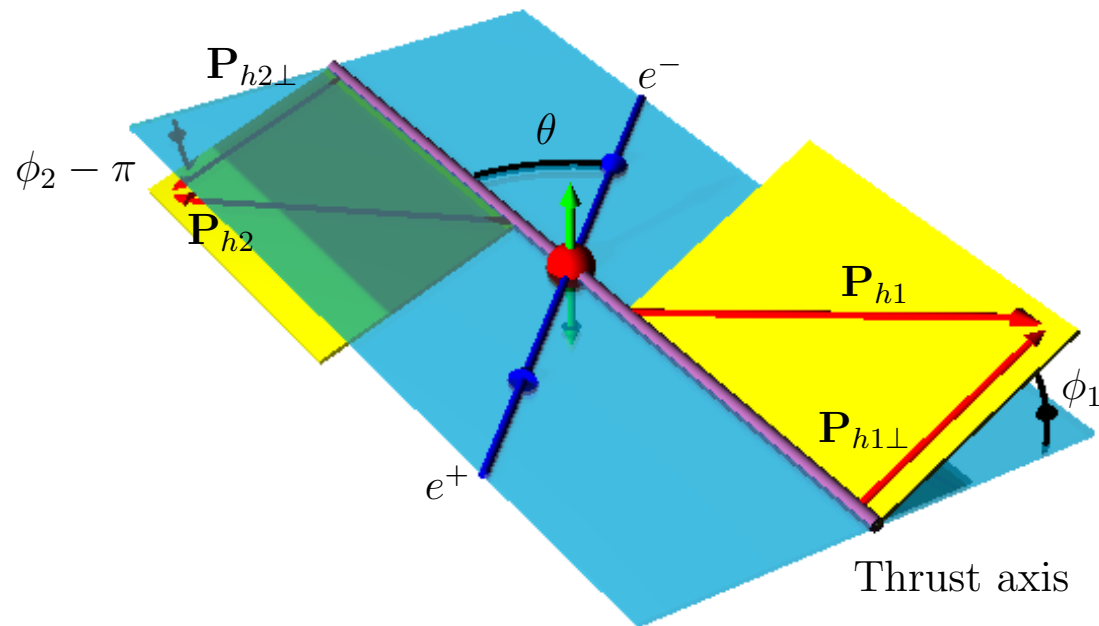
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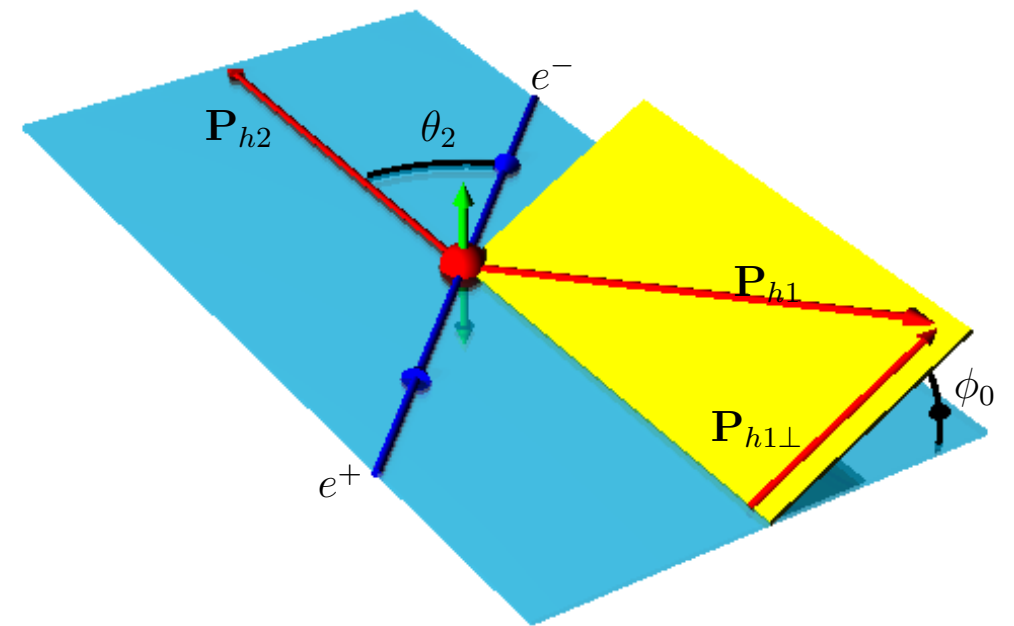
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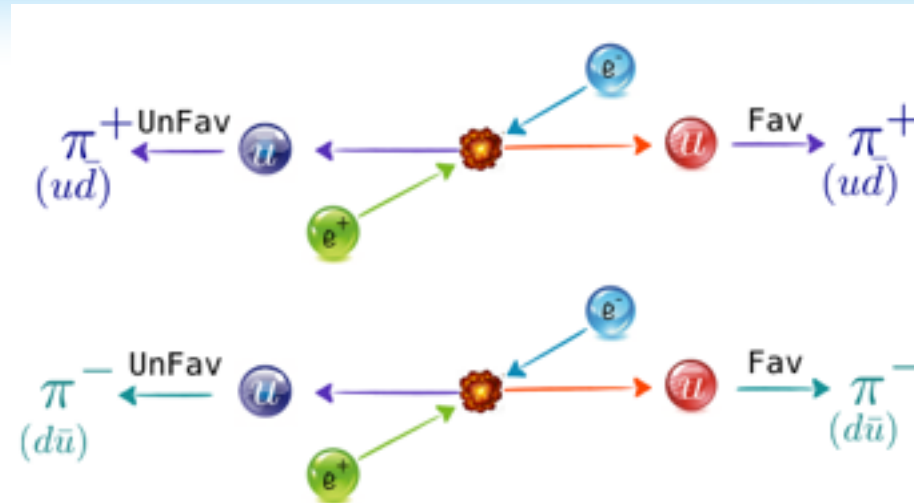
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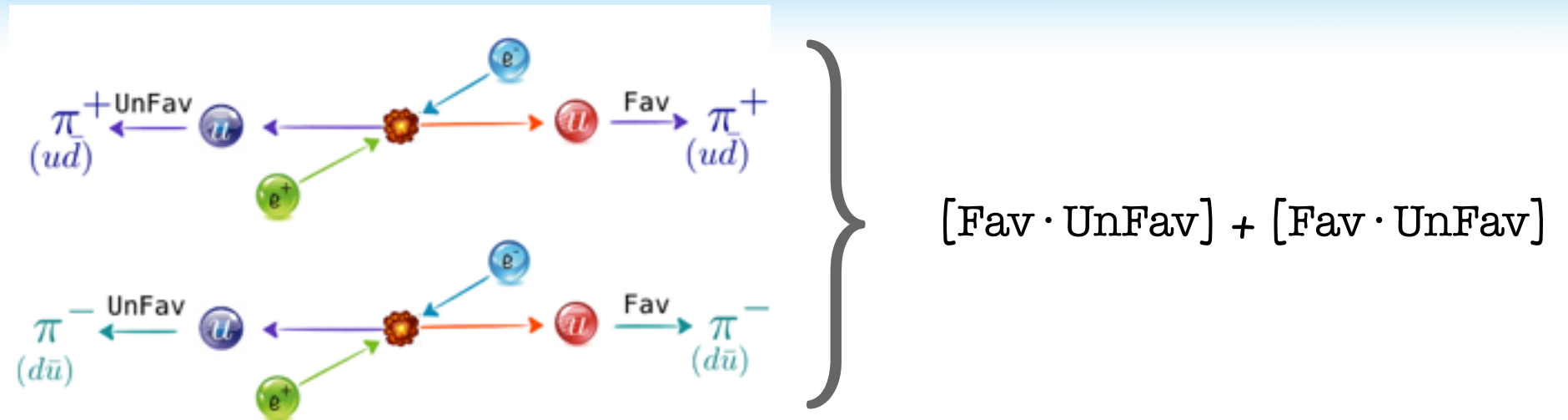
# Product of 2 Collins FFs

Like-sign couples



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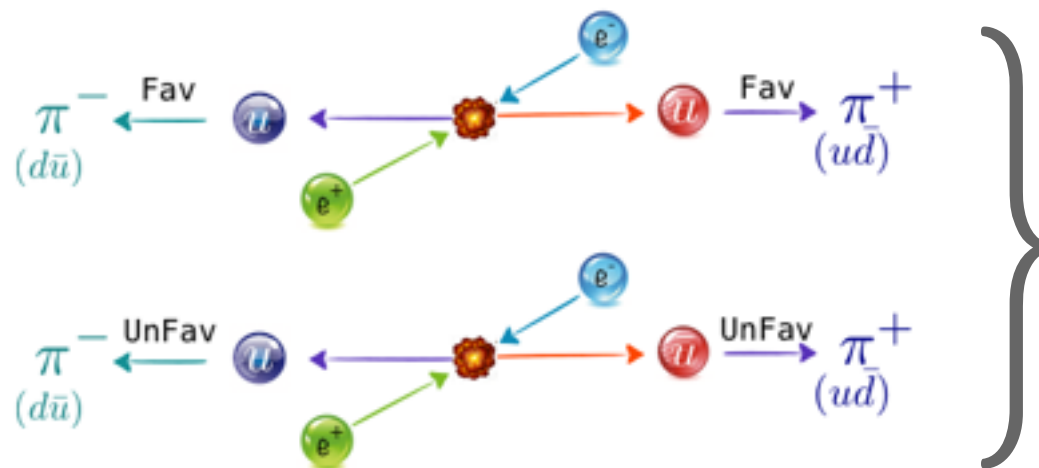
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Like-sign couples



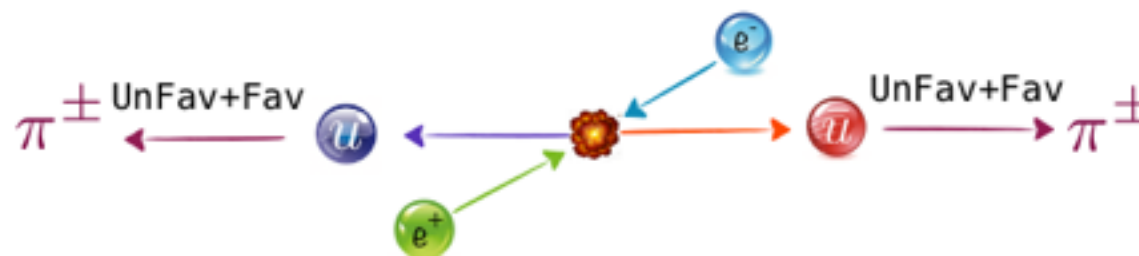
$$[\text{Fav} \cdot \text{UnFav}] + [\text{Fav} \cdot \text{UnFav}]$$

Unlike-sign couples



$$[\text{Fav} \cdot \text{Fav}] + [\text{UnFav} \cdot \text{UnFav}]$$

All charges couples



$$[\text{Fav} + \text{unFav}] \cdot [\text{UnFav} + \text{Fav}]$$



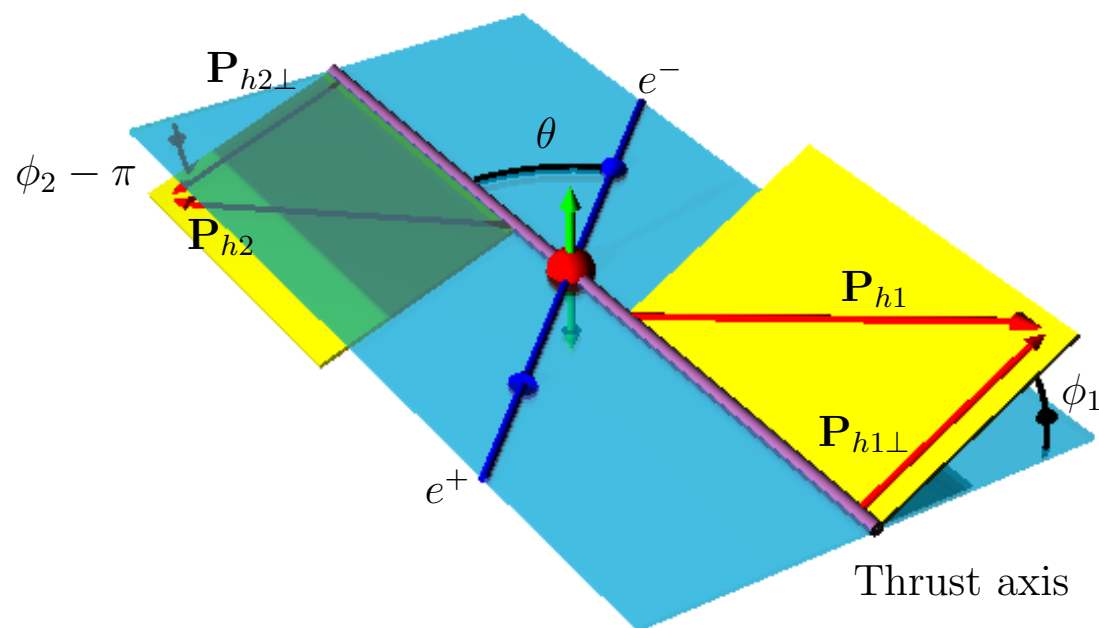


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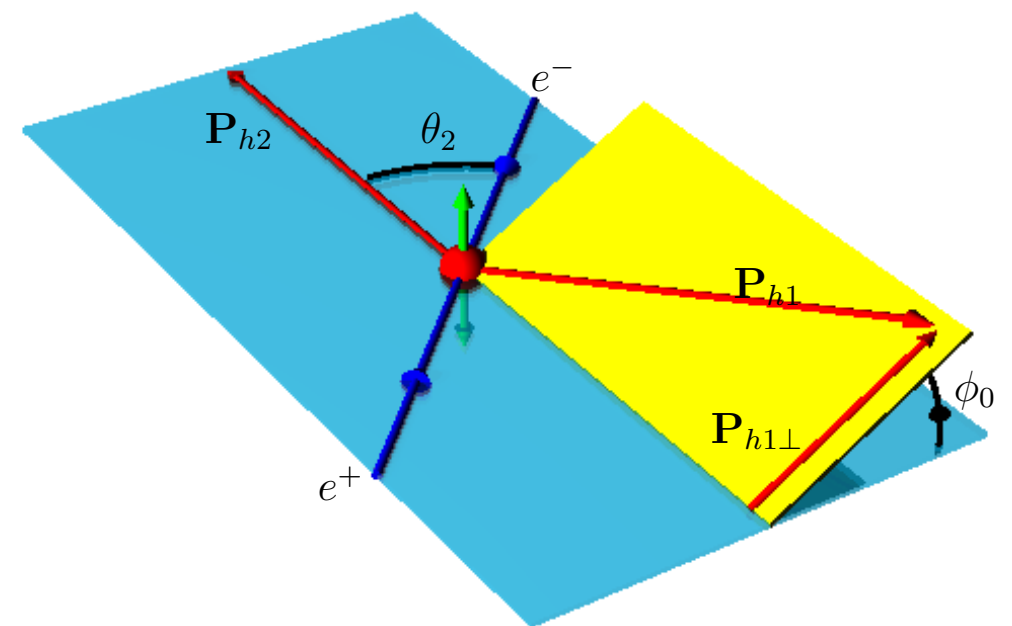
$\phi_1 + \phi_2$  method:

hadron azimuthal angles with respect to the  $q\bar{q}$  axis proxy



$\phi_0$  method:

hadron 1 azimuthal angle with respect to hadron 2



$$\mathcal{R}_{12} = \frac{N_{12}(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$

$$\mathcal{R}_0 = \frac{N_0(\phi_0)}{\langle N_0 \rangle}$$



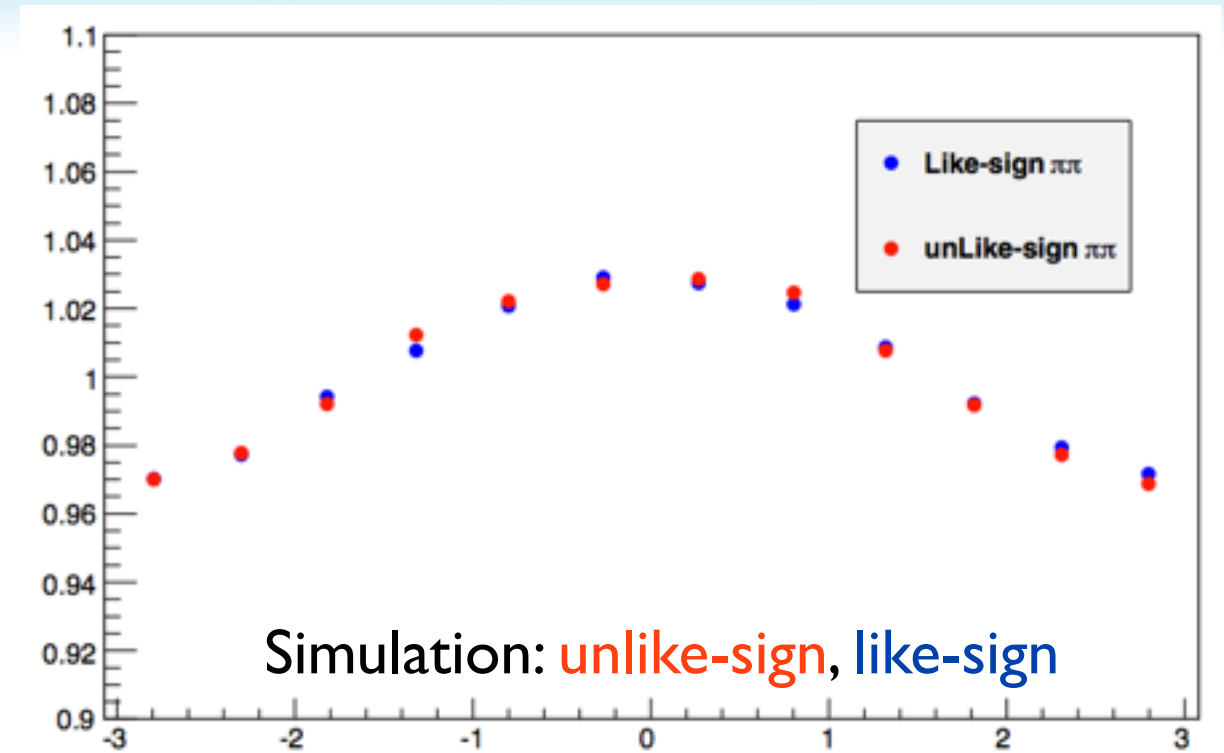
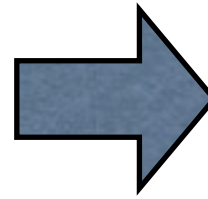
# Double-ratios

But! Acceptance and radiation effects also contribute to azimuthal asymmetries!



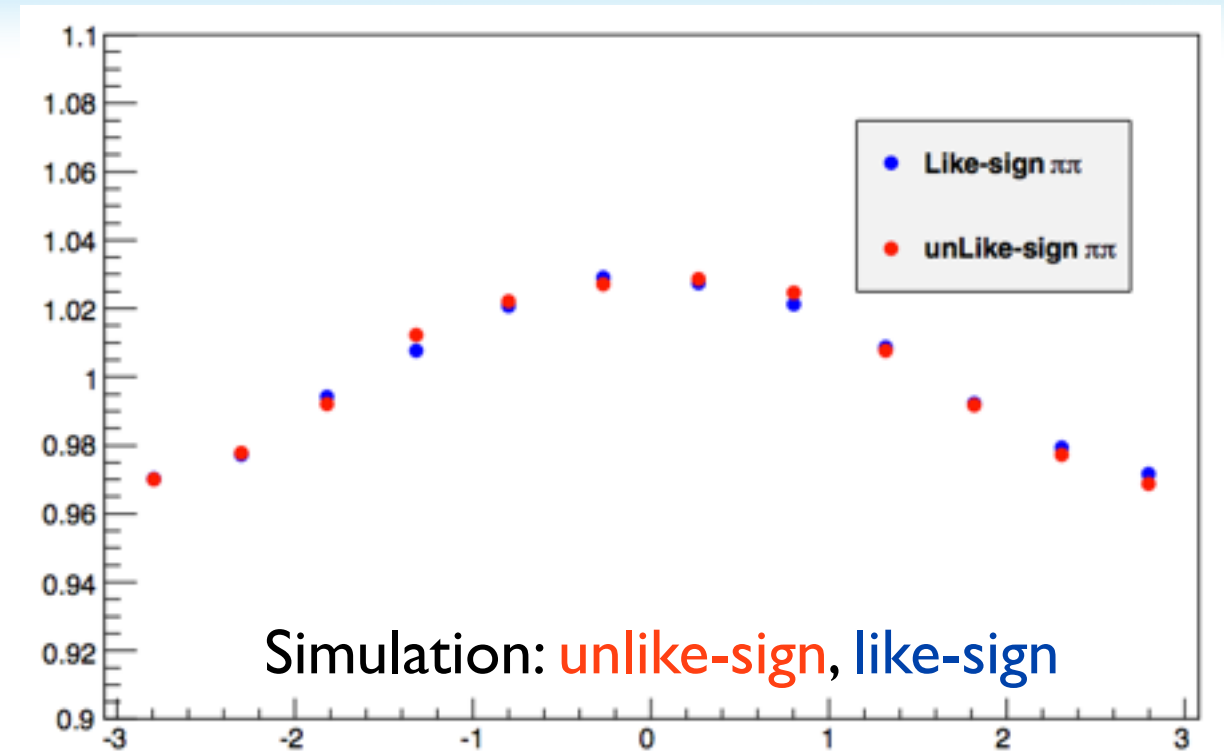
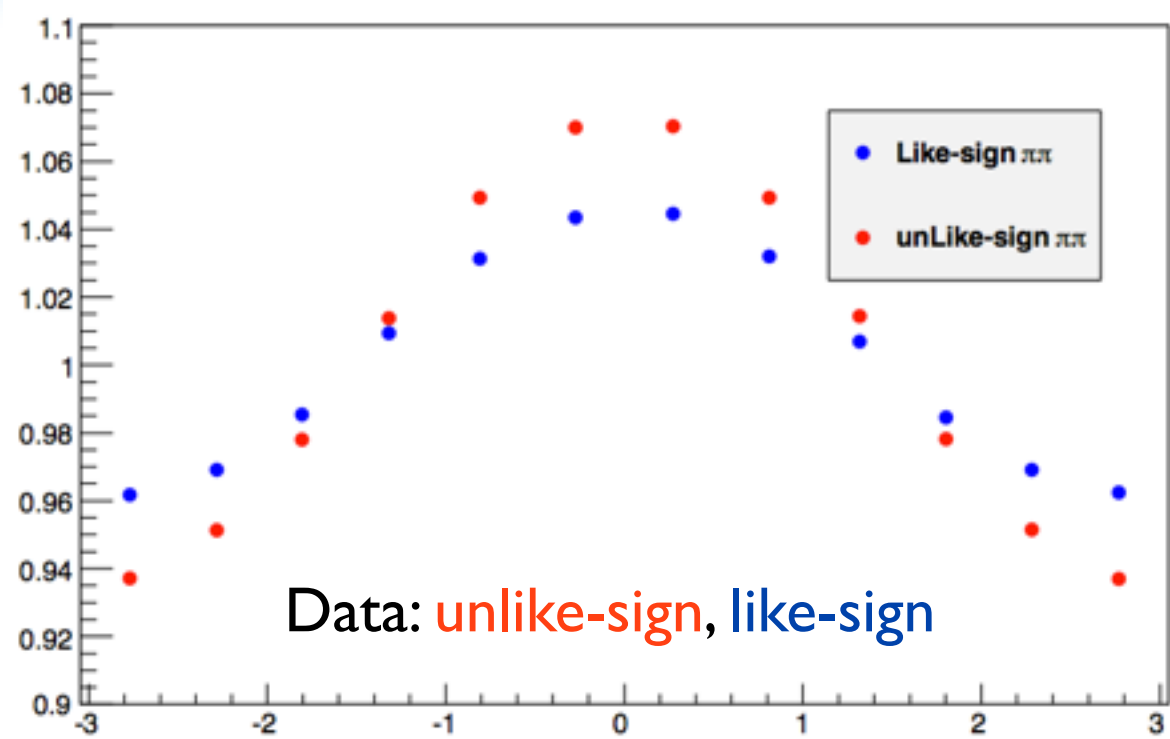
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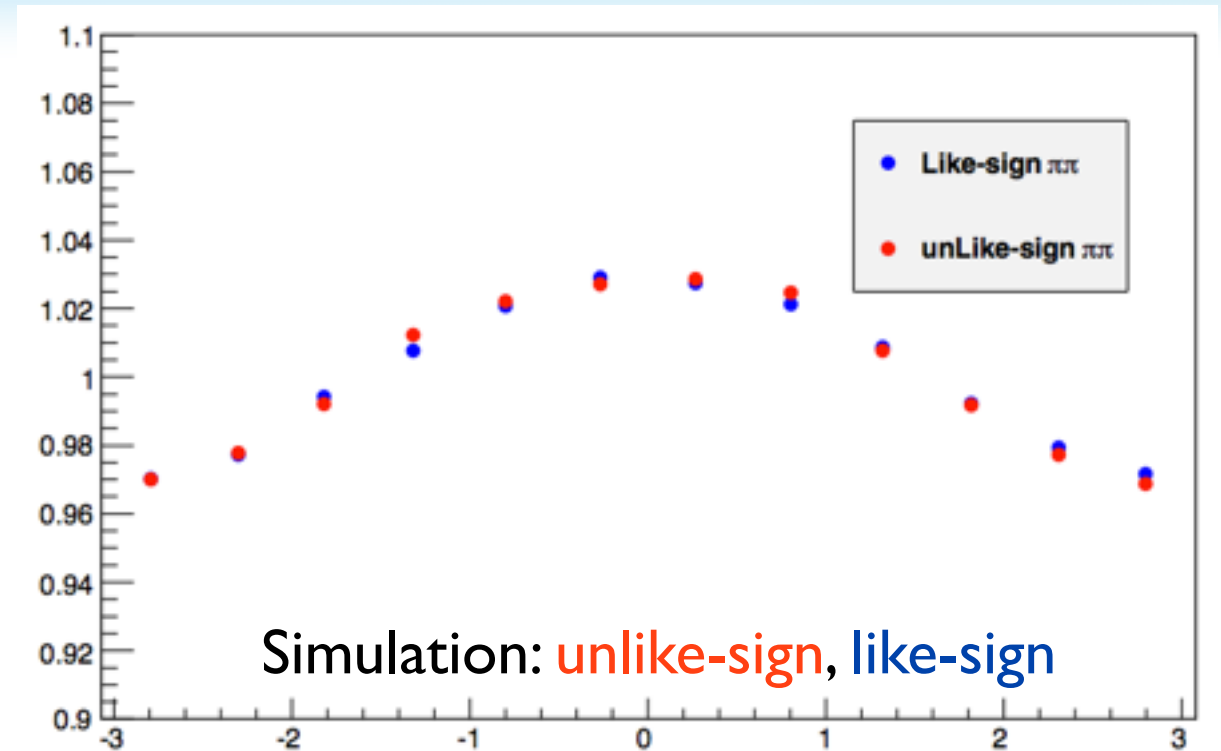
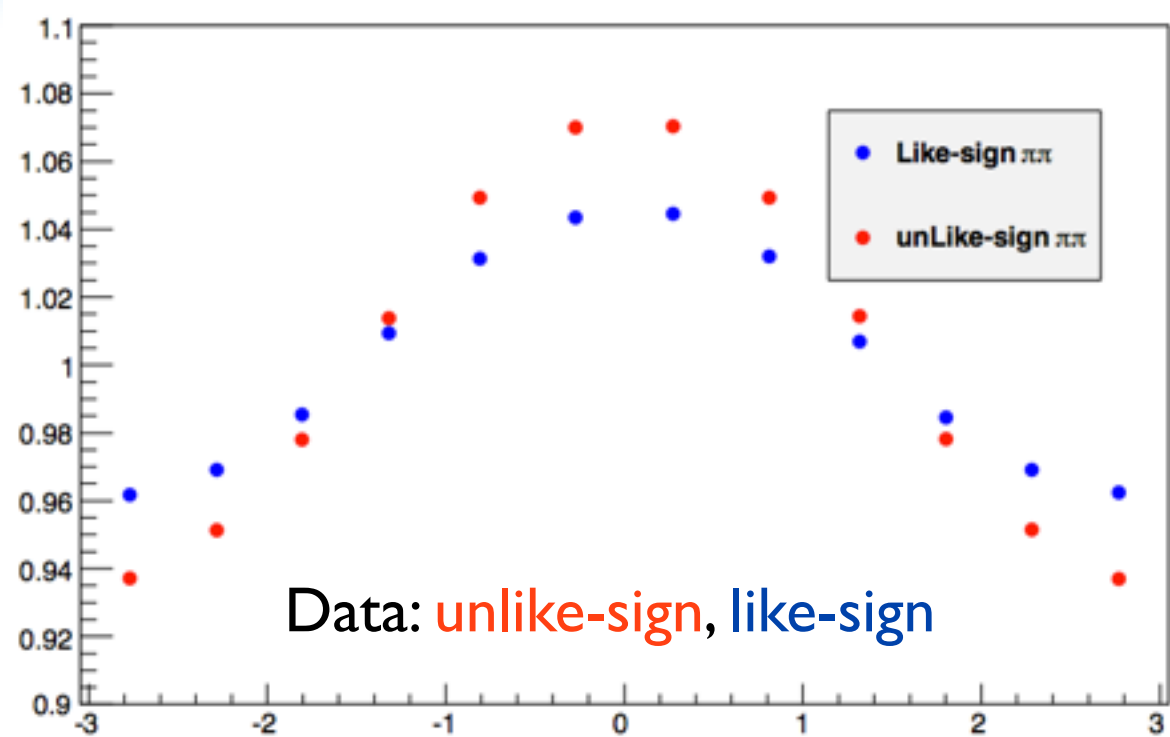




# Double-ratios



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To reduce such non-Collins effects:  
 divide the sample of hadron couples in unlike-sign and like-sign (or All-charges),  
 and extract the asymmetries of the super ratios between these 2 samples:

Unlike-sign couples / Like-sign couples

$$\mathcal{D}_{ul}^{h_1 h_2} = \mathcal{R}^U / \mathcal{R}^L$$

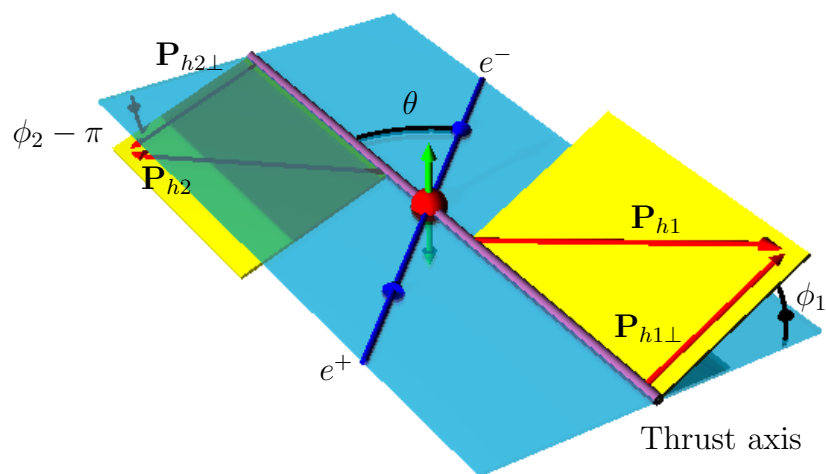
Unlike-sign couples / All charges

$$\mathcal{D}_{uc}^{h_1 h_2} = \mathcal{R}^U / \mathcal{R}^C$$

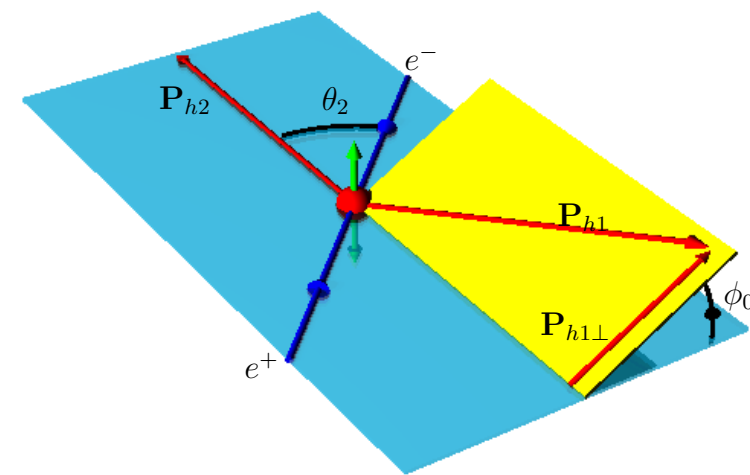


# Double-ratios

$\phi_1 + \phi_2$  method



$\phi_0$  method



$$\sigma \sim \mathcal{M}_{12} \left( 1 + \frac{\sin^2 \theta_T}{1 + \cos^2 \theta_T} \cos(\phi_1 + \phi_2) \frac{H_1^{\perp[1]}(z_1) \bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1) \bar{D}_1^{[0]}(z_2)} \right) \quad \sigma \sim \mathcal{M}_0 \left( 1 + \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos(2\phi_0) \mathcal{F} \left[ \frac{H_1^{\perp}(z_1) \bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1) \bar{D}_1^{\perp}(z_2)} \right] \right)$$

$$\mathcal{D}_{ul}^{h_1 h_2} = \mathcal{R}^U / \mathcal{R}^L$$

Fitted by

$$\mathcal{D}_{uc}^{h_1 h_2} = \mathcal{R}^U / \mathcal{R}^C$$

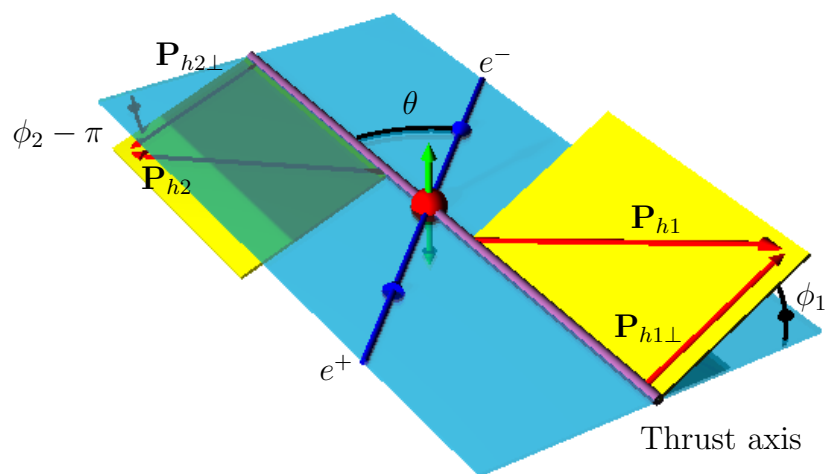
$$\mathcal{B}_{12} (1 + \mathcal{A}_{12} \cos(\phi_1 + \phi_2))$$

$$\mathcal{B}_0 (1 + \mathcal{A}_0 \cos(2\phi_0))$$

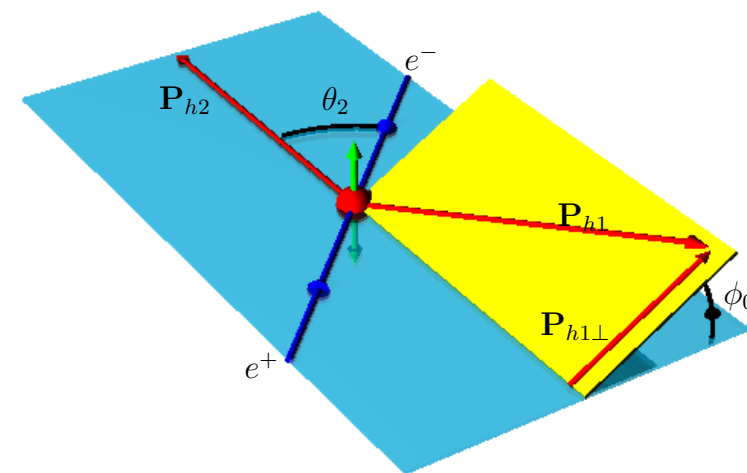


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$$A_{12} = \frac{\sin^2 \theta}{1 + \cos^2 \theta} \frac{H_1^{\perp[1]}(z_1) \bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1) \bar{D}_1^{[0]}(z_2)}$$

Fitted by

$$A_0 = \frac{\sin^2 \theta}{1 + \cos^2 \theta} \mathcal{F} \left[ \frac{H_1^{\perp}(z_1) \bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1) \bar{D}_1^{\perp}(z_2)} \right]$$

$$\mathcal{B}_{12} (1 + \mathcal{A}_{12} \cos(\phi_1 + \phi_2))$$

$$\mathcal{B}_0 (1 + \mathcal{A}_0 \cos(2\phi_0))$$





# Kinematic variables

$$z \equiv \frac{E_h}{E_p}$$

hadron energy fraction  
with respect to parton

$z_1, z_2$

$p_T$  component of hadron momentum transverse  
to reference direction

1.  $\phi_1 + \phi_2$  method: the thrust axis  $p_{T1}, p_{T2}$

2.  $\phi_0$  method: hadron 2  $p_{T0}$

$q_T$  component of virtual photon momentum  
transverse to the  $h_1 h_2$  axis in the frame  
where  $h_1$  and  $h_2$  are back-to-back

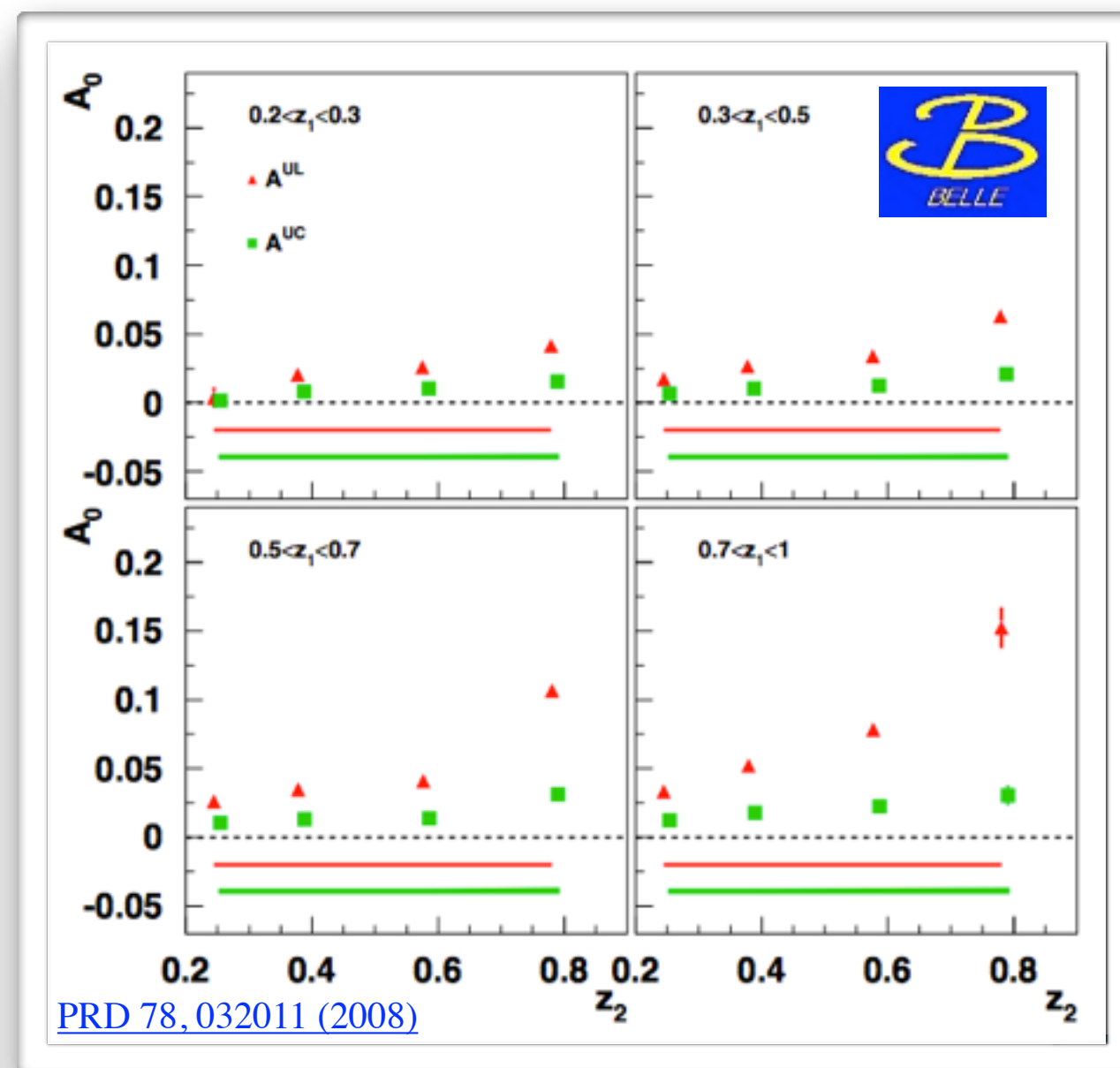
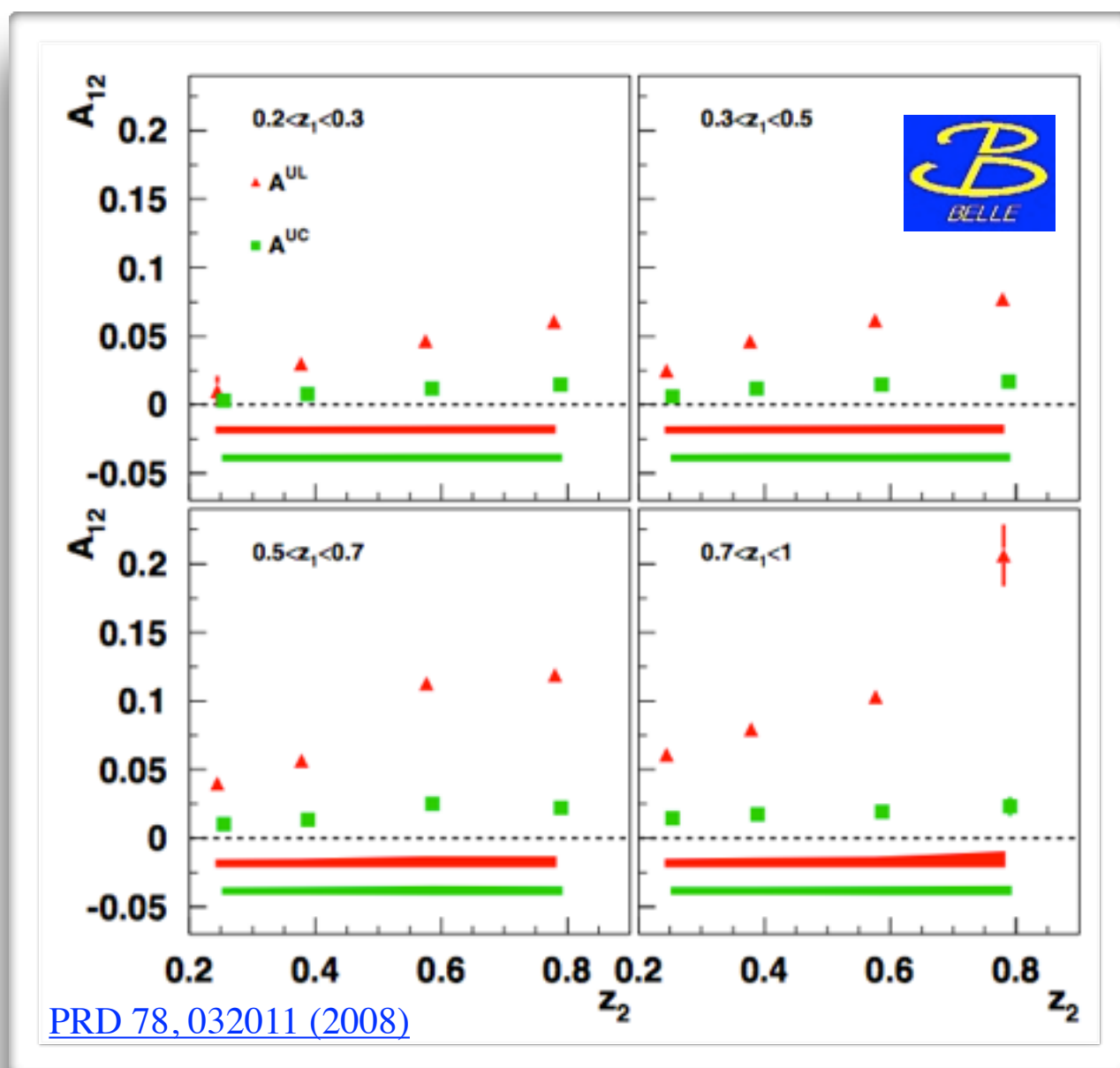
$z$	0.2	0.25	0.3	0.42	1				
$p_{T12}$	0	0.13	0.3	0.5	3				
$p_{T0}$	0	0.13	0.25	0.4	0.5	0.6	0.75	1	3
$q_T$	0	0.5	1	1.25	1.5	1.75	2	2.25	2.5
$\sin^2\theta/(1+\cos^2\theta)$	0.4	0.45	0.5	0.6	0.7	0.8	0.9	0.97	1



# Published results: $\pi\pi$

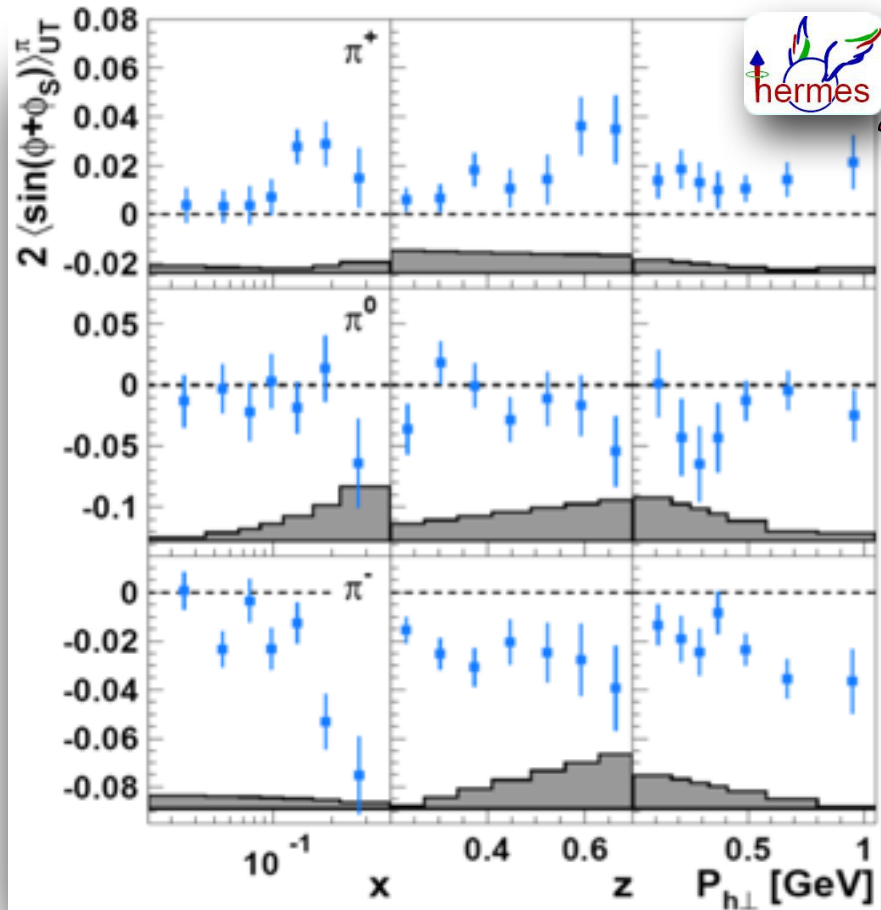
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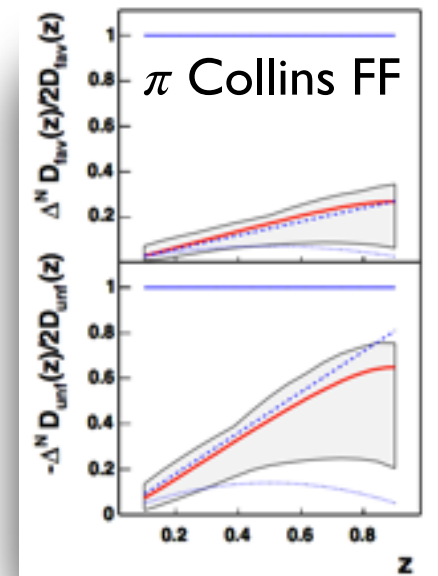


# Collins amplitudes in SIDIS

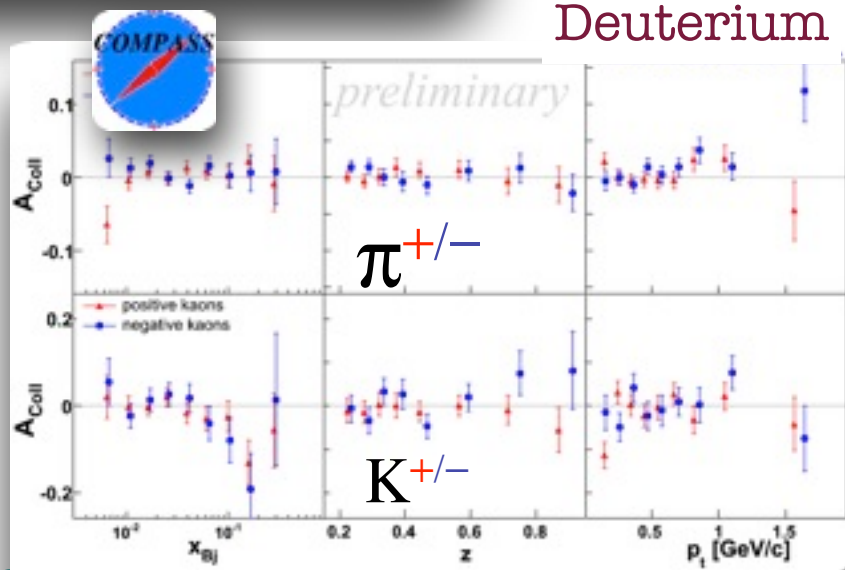
[Airapetian et al., Phys. Lett. B 693 (2010) 11-16]



$$A_{UT} \propto h_1 \otimes H_1^\perp$$

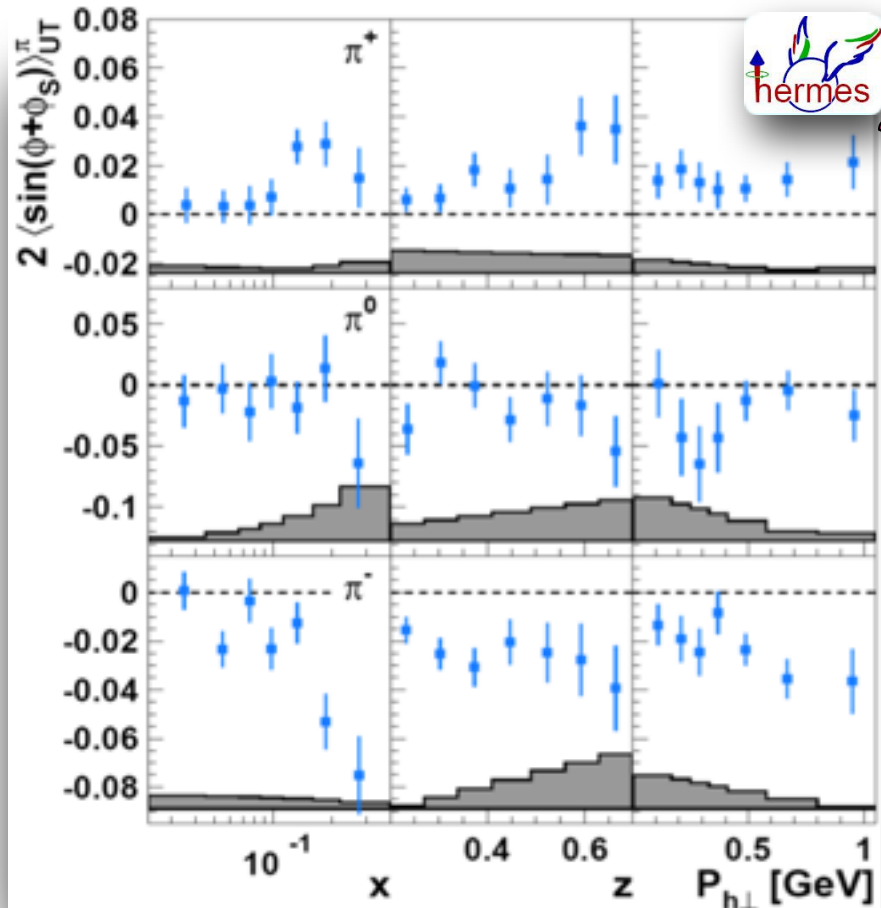


Deuterium



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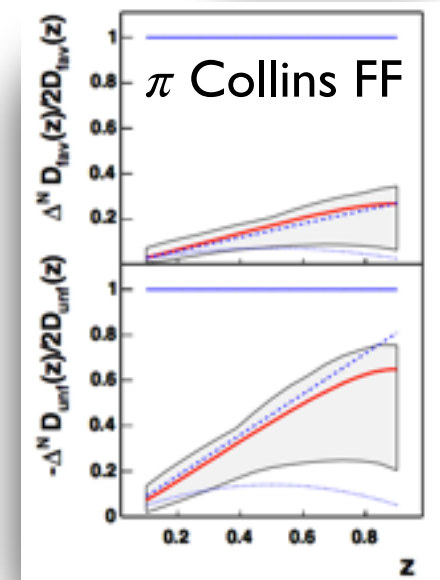
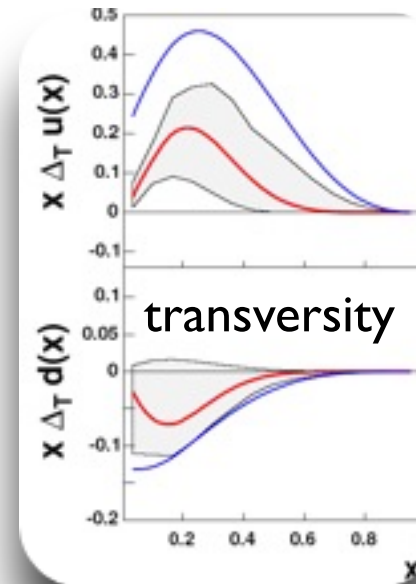
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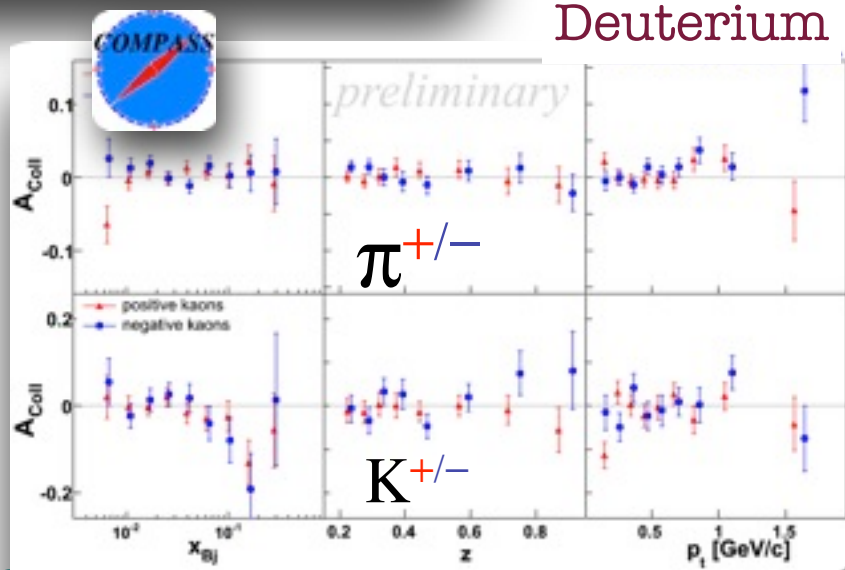
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Anselmino et al.  
Phys.Rev. D75 (2007)



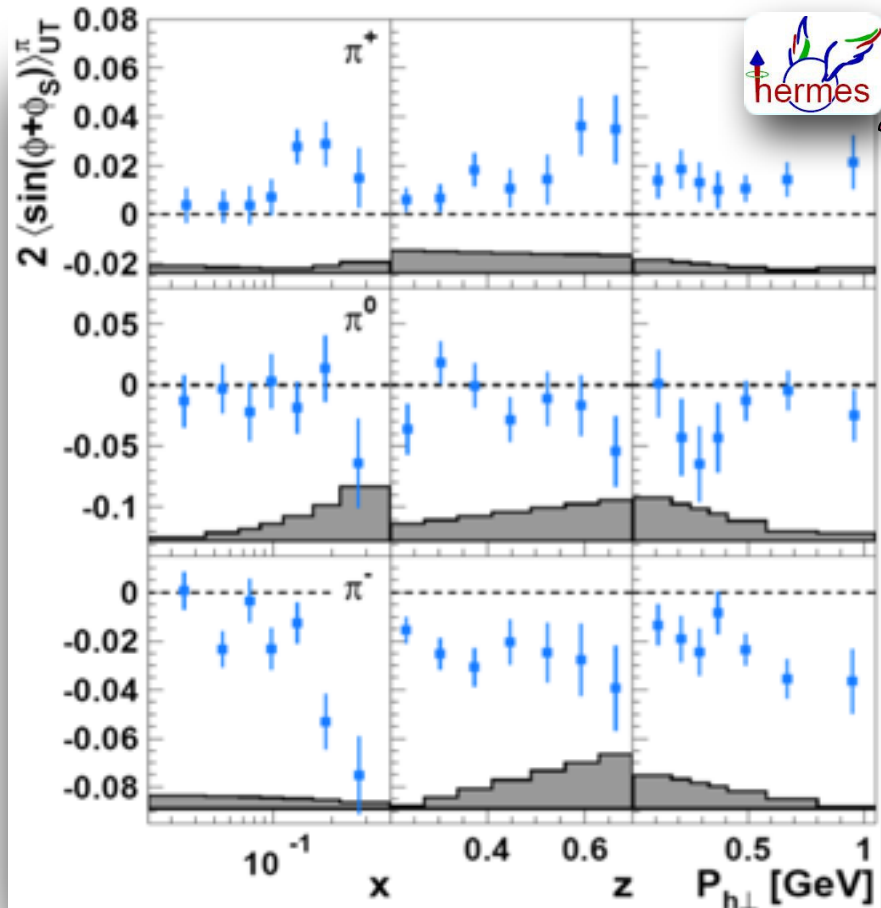
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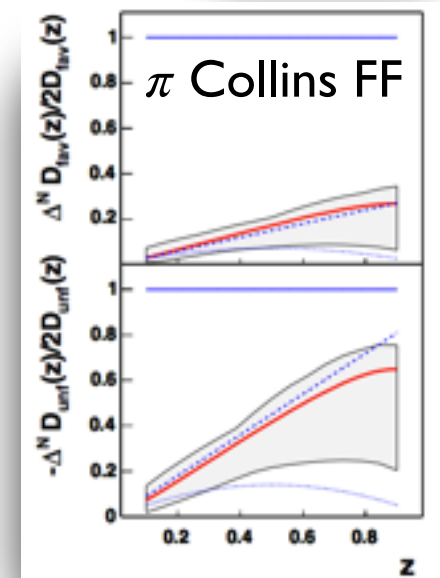
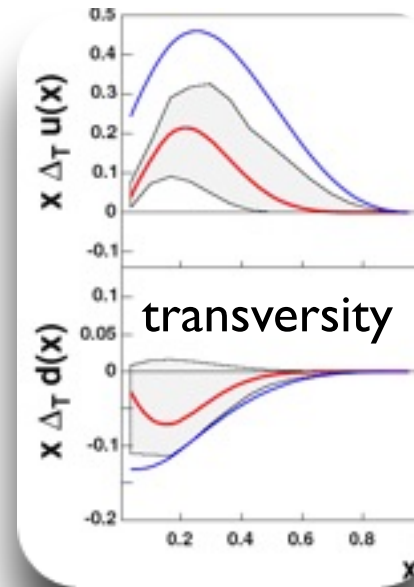
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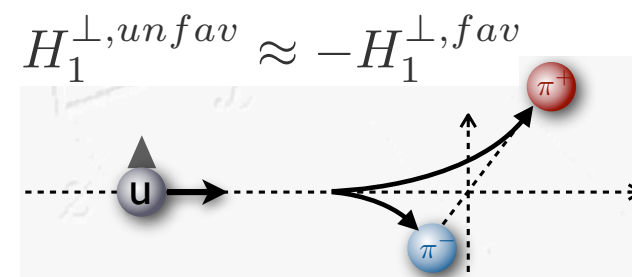
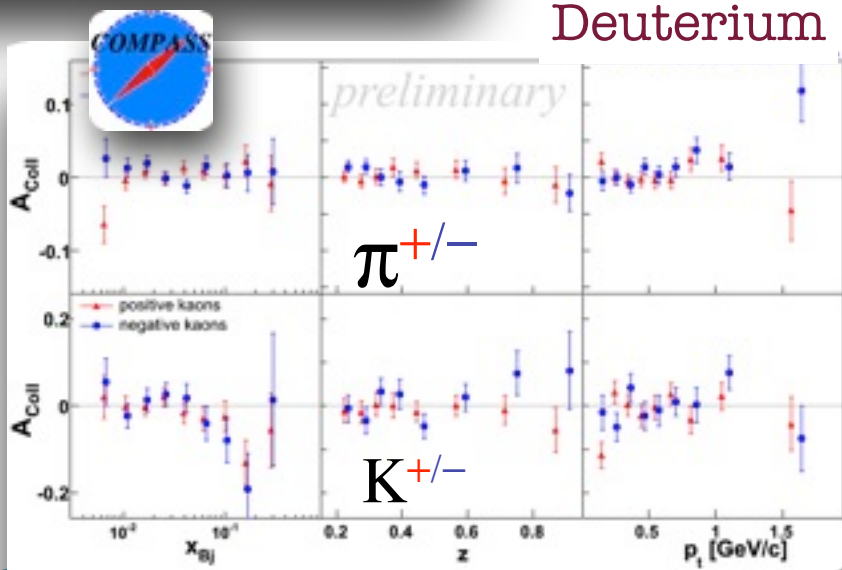
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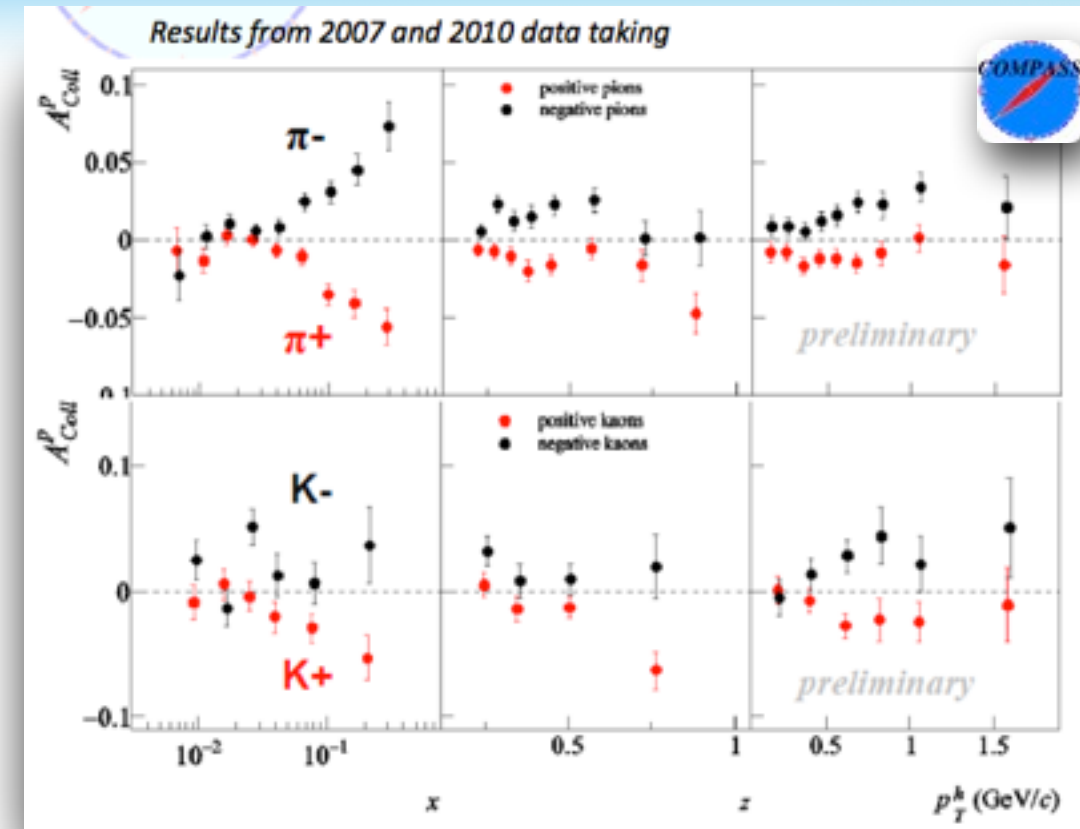
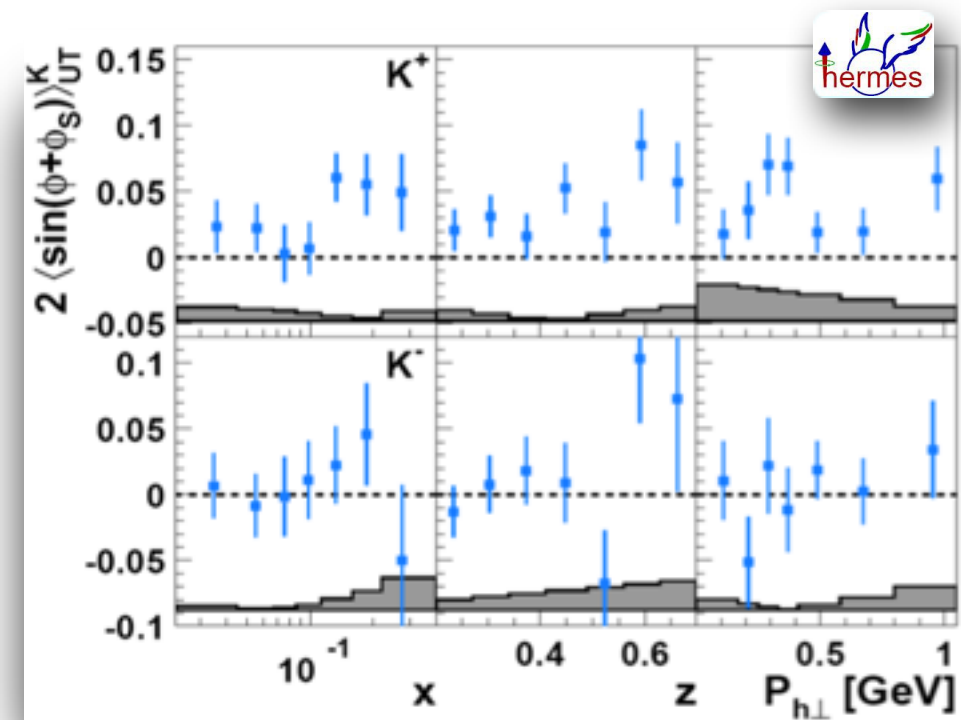
$$H_1^{\perp, unfav} \approx -H_1^{\perp, fav}$$



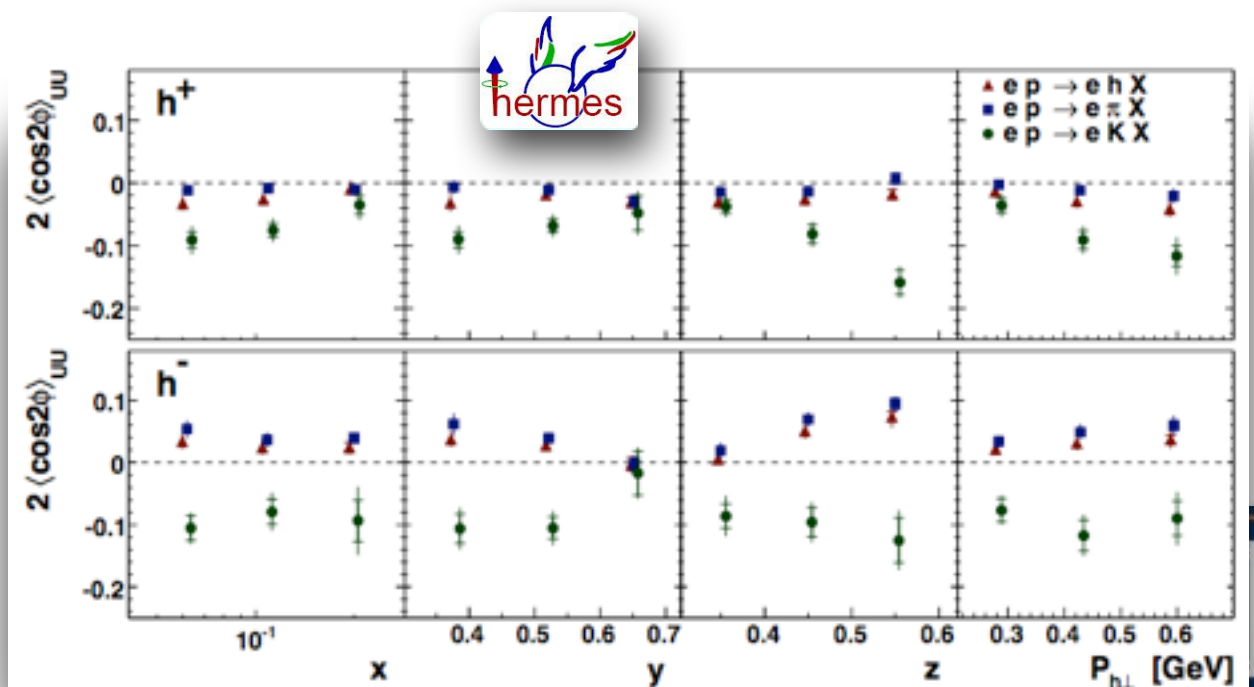


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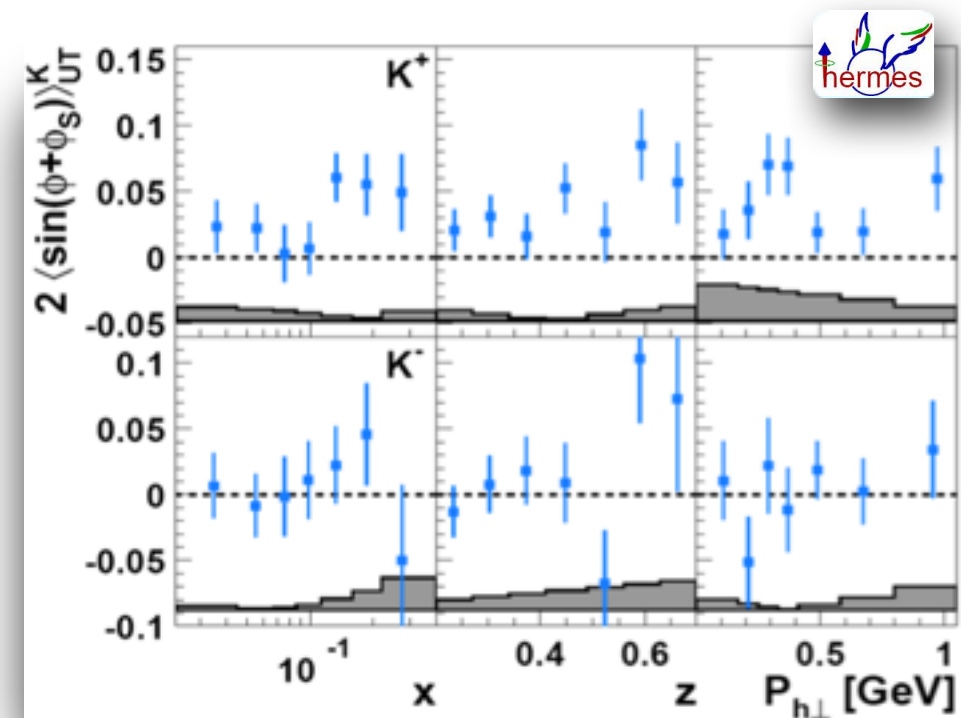


$$A_{UU} \propto h_1^\perp \otimes H_1^\perp$$

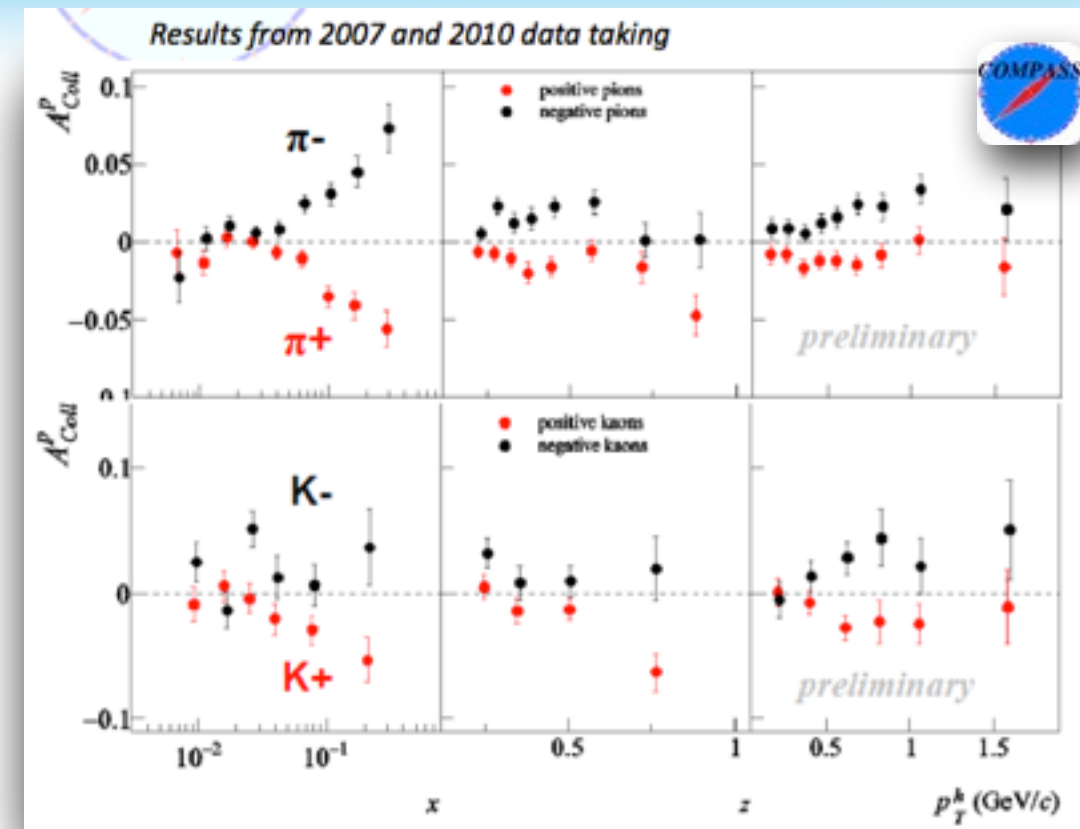


# Collins amplitudes in SIDIS

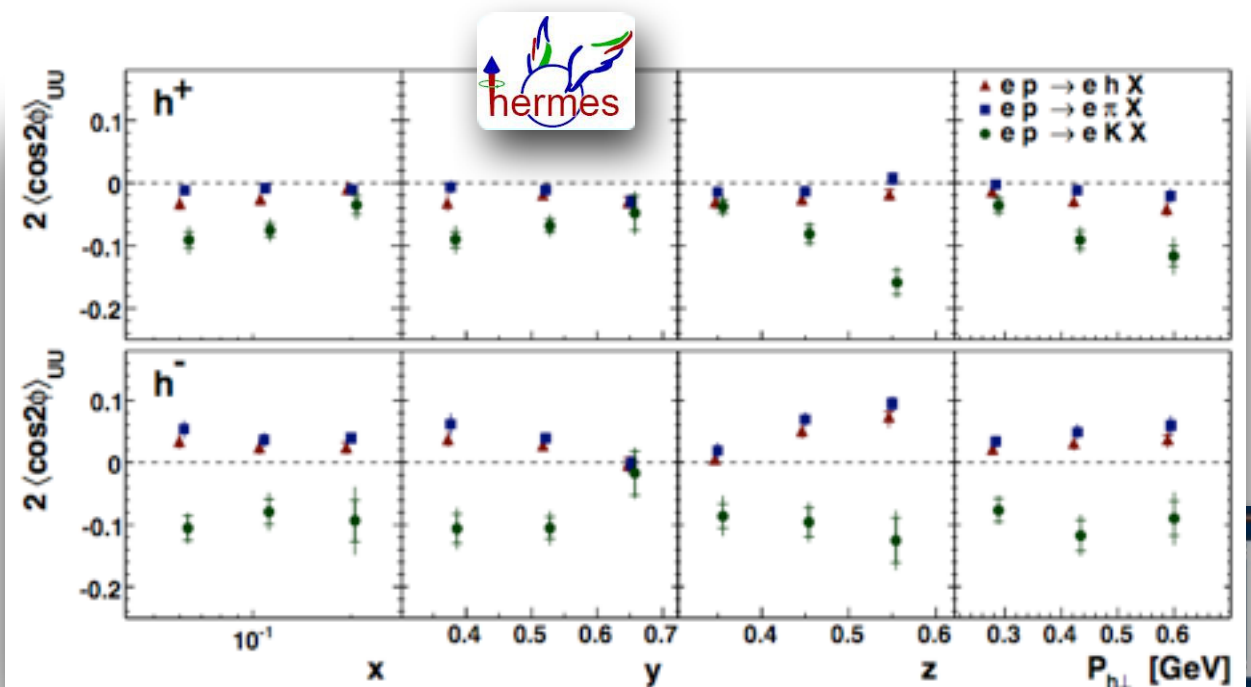
$$A_{UT} \propto h_1 \otimes H_1^\perp$$



$K^+$  amplitudes larger than  $\pi^+$ ?



$$A_{UU} \propto h_1^\perp \otimes H_1^\perp$$



# What's new?

$$D_{ul}^{\pi\pi} = \mathcal{R}^{U\pi\pi} / \mathcal{R}^{L\pi\pi}$$

$$D_{ul}^{\pi k} = \mathcal{R}^{U\pi k} / \mathcal{R}^{L\pi k}$$

$$D_{ul}^{kk} = \mathcal{R}^{Ukk} / \mathcal{R}^{Lkk}$$

z	q $\tau$	$\sin^2\theta/(1+\cos^2\theta)$	p $\tau$
✓	✓	✓	New!
New!	New!	New!	New!
New!	New!	New!	New!

$$D_{uc}^{\pi\pi} = \mathcal{R}^{U\pi\pi} / \mathcal{R}^{C\pi\pi}$$

$$D_{uc}^{\pi k} = \mathcal{R}^{U\pi k} / \mathcal{R}^{C\pi k}$$

$$D_{uc}^{kk} = \mathcal{R}^{Ukk} / \mathcal{R}^{Ckk}$$

z	q $\tau$	$\sin^2\theta/(1+\cos^2\theta)$	p $\tau$
✓	✓	✓	New!
New!	New!	New!	New!
New!	New!	New!	New!



# What's new?

$$\mathcal{D}_{ul}^{\pi\pi} = \mathcal{R}^{U\pi\pi} / \mathcal{R}^{L\pi\pi}$$

$$\mathcal{D}_{ul}^{\pi k} = \mathcal{R}^{U\pi k} / \mathcal{R}^{L\pi k}$$

$$\mathcal{D}_{ul}^{kk} = \mathcal{R}^{Ukk} / \mathcal{R}^{Lkk}$$

z	q $\tau$	$\sin^2\theta/(1+\cos^2\theta)$	p $\tau$
✓	✓	✓	New!
New!	New!	New!	New!
New!	New!	New!	New!

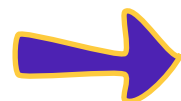
$$\mathcal{D}_{uc}^{\pi\pi} = \mathcal{R}^{U\pi\pi} / \mathcal{R}^{C\pi\pi}$$

$$\mathcal{D}_{uc}^{\pi k} = \mathcal{R}^{U\pi k} / \mathcal{R}^{C\pi k}$$

$$\mathcal{D}_{uc}^{kk} = \mathcal{R}^{Ukk} / \mathcal{R}^{Ckk}$$

z	q $\tau$	$\sin^2\theta/(1+\cos^2\theta)$	p $\tau$
✓	✓	✓	New!
New!	New!	New!	New!
New!	New!	New!	New!

**Word of caution:** this analysis is mainly aimed at kaons, so kinematic cuts and binning are optimized for kaons, and the same values used for pion too.

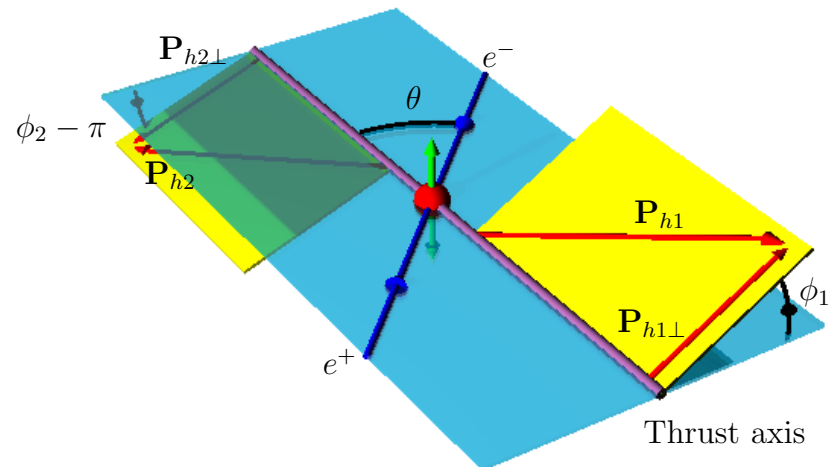


$\pi\pi$  results cannot be compared directly to published results

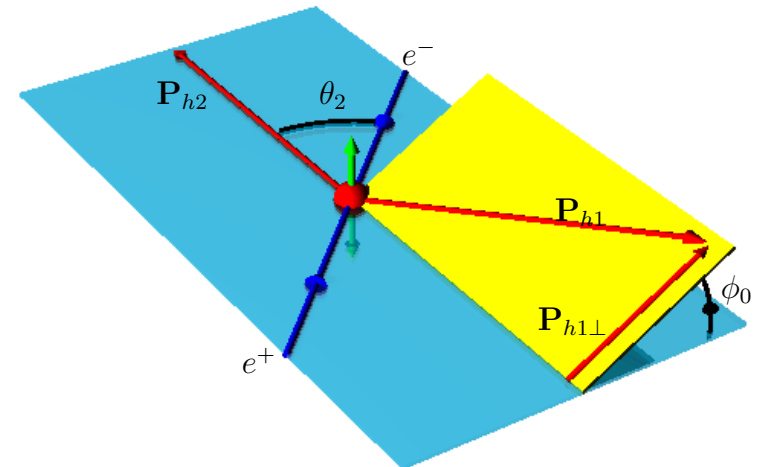


# What's new?

$\phi_1 + \phi_2$  method



$\phi_0$  method



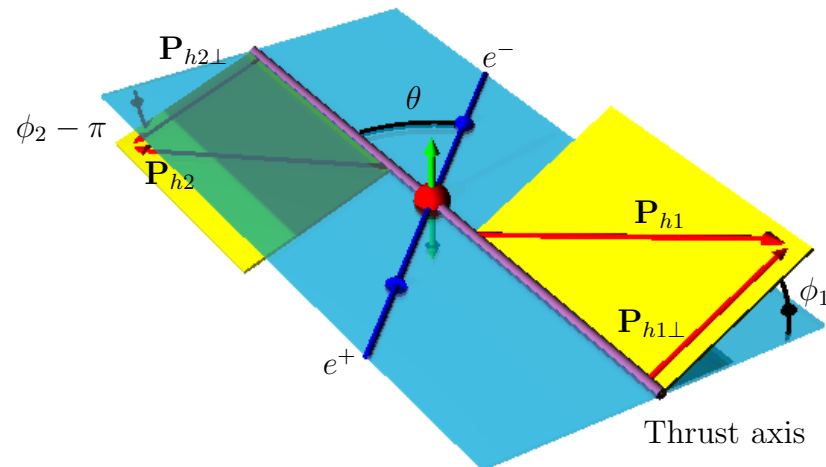
$$\sigma \sim \mathcal{M}_{12} \left( 1 + \frac{\sin^2 \theta_T}{1 + \cos^2 \theta_T} \cos(\phi_1 + \phi_2) \frac{H_1^{\perp[1]}(z_1) \bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1) \bar{D}_1^{[0]}(z_2)} \right) \quad \sigma \sim \mathcal{M}_0 \left( 1 + \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos(2\phi_0) \mathcal{F} \left[ \frac{H_1^{\perp}(z_1) \bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1) \bar{D}_1^{\perp}(z_2)} \right] \right)$$



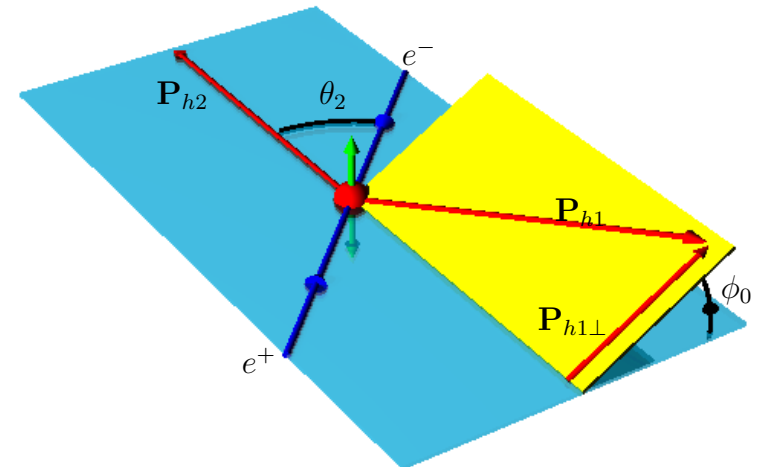


# What's new?

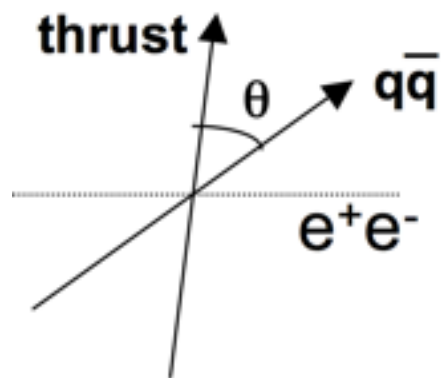
$\phi_1 + \phi_2$  method



$\phi_0$  method



$$\sigma \sim \mathcal{M}_{12} \left( 1 + \frac{\sin^2 \theta_T}{1 + \cos^2 \theta_T} \cos(\phi_1 + \phi_2) \frac{H_1^{\perp[1]}(z_1) \bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1) \bar{D}_1^{[0]}(z_2)} \right) \quad \sigma \sim \mathcal{M}_0 \left( 1 + \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos(2\phi_0) \mathcal{F} \left[ \frac{H_1^{\perp}(z_1) \bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1) \bar{D}_1^{\perp}(z_2)} \right] \right)$$



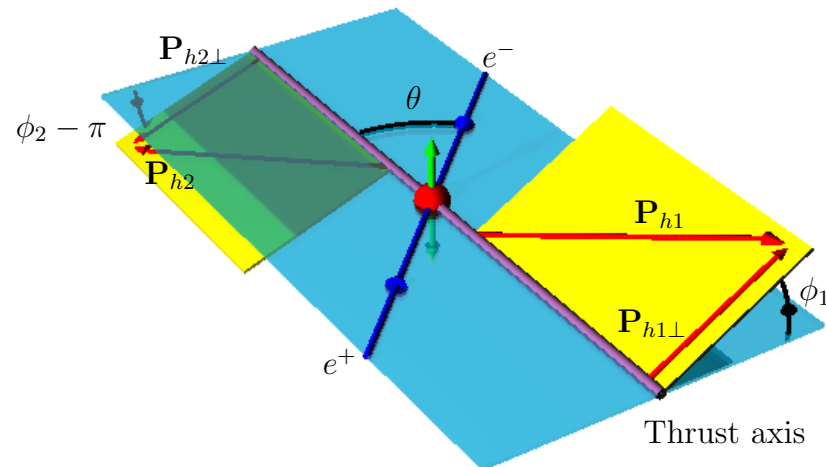
Both interesting: different integration of FF's in  $p_{Ti}$ , might provide information on the Collins  $p_T$  dependence

Technically more complicated: require the determination of a  $q\bar{q}$  proxy (Thrust axis)



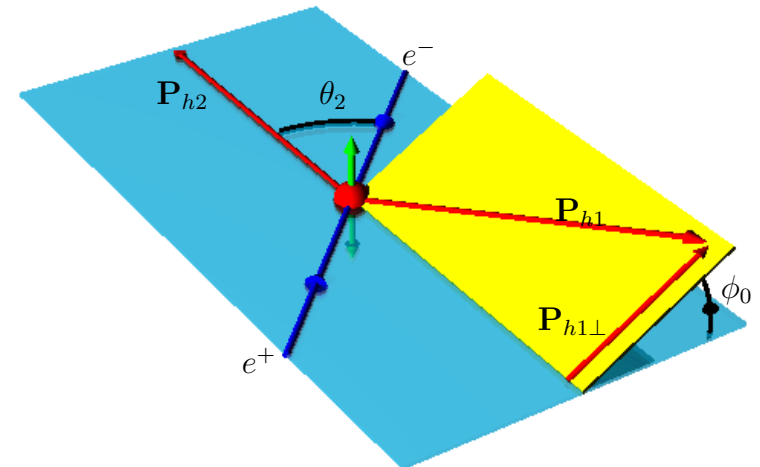
# What's new?

## $\phi_1 + \phi_2$ method

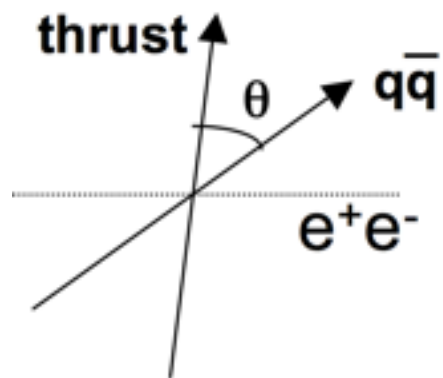


$$\sigma \sim \mathcal{M}_{12} \left( 1 + \frac{\sin^2 \theta_T}{1 + \cos^2 \theta_T} \cos(\phi_1 + \phi_2) \frac{H_1^{\perp[1]}(z_1) \bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1) \bar{D}_1^{[0]}(z_2)} \right)$$

## $\phi_0$ method



$$\sigma \sim \mathcal{M}_0 \left( 1 + \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos(2\phi_0) \mathcal{F} \left[ \frac{H_1^{\perp}(z_1) \bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1) \bar{D}_1^{\perp}(z_2)} \right] \right)$$



Both interesting: different integration of FF's in  $p_{Ti}$ , might provide information on the Collins  $p_T$  dependence

Technically more complicated: require the determination of a  $q\bar{q}$  proxy (Thrust axis)



# Particle ID correction

$$N^{j,raw} = P_{ij} N^i$$

$$i = \pi, K$$

$$j = e, \mu, \pi, K, p$$

Perfect PID  $\Leftrightarrow j = i$



# Particle ID correction

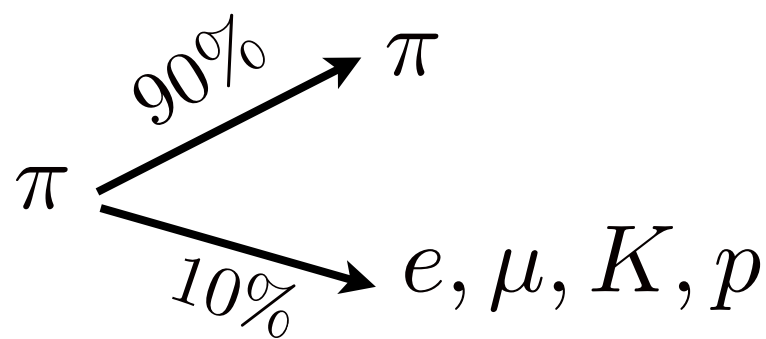
$$N^{j,raw} = P_{ij} N^i$$

$$i = \pi, K$$

$$j = e, \mu, \pi, K, p$$

Perfect PID  $\Leftrightarrow j = i$

$$\varepsilon(\pi) \gtrsim 90\% \quad \varepsilon(K) \gtrsim 85\%$$



$$P_{ij} = \begin{pmatrix} P_{e \rightarrow e} & P_{e \rightarrow \mu} & P_{e \rightarrow \pi} & P_{e \rightarrow K} & P_{e \rightarrow p} \\ P_{\mu \rightarrow e} & P_{\mu \rightarrow \mu} & P_{\mu \rightarrow \pi} & P_{\mu \rightarrow K} & P_{\mu \rightarrow p} \\ P_{\pi \rightarrow e} & P_{\pi \rightarrow \mu} & P_{\pi \rightarrow \pi} & P_{\pi \rightarrow K} & P_{\pi \rightarrow p} \\ P_{K \rightarrow e} & P_{K \rightarrow \mu} & P_{K \rightarrow \pi} & P_{K \rightarrow K} & P_{K \rightarrow p} \\ P_{p \rightarrow e} & P_{p \rightarrow \mu} & P_{p \rightarrow \pi} & P_{p \rightarrow K} & P_{p \rightarrow p} \end{pmatrix}$$

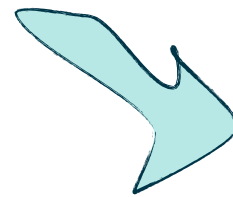


# Particle ID correction

$$N^{j,raw} = P_{ij} N^i$$

$$i = \pi, K$$

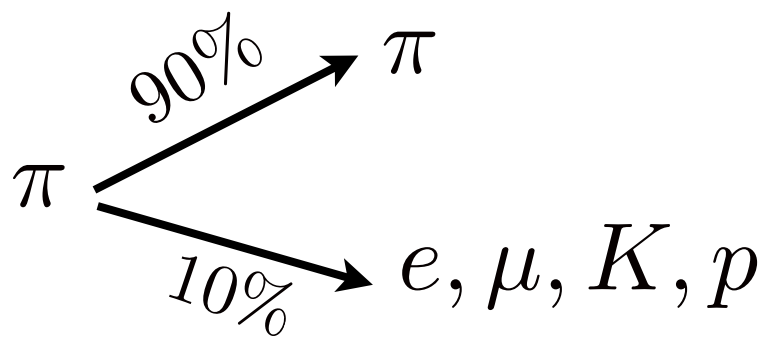
$$j = e, \mu, \pi, K, p$$



$$N^i = P_{ij}^{-1} N^{j,raw}$$

Perfect PID  $\Leftrightarrow j = i$

$$\varepsilon(\pi) \gtrsim 90\% \quad \varepsilon(K) \gtrsim 85\%$$

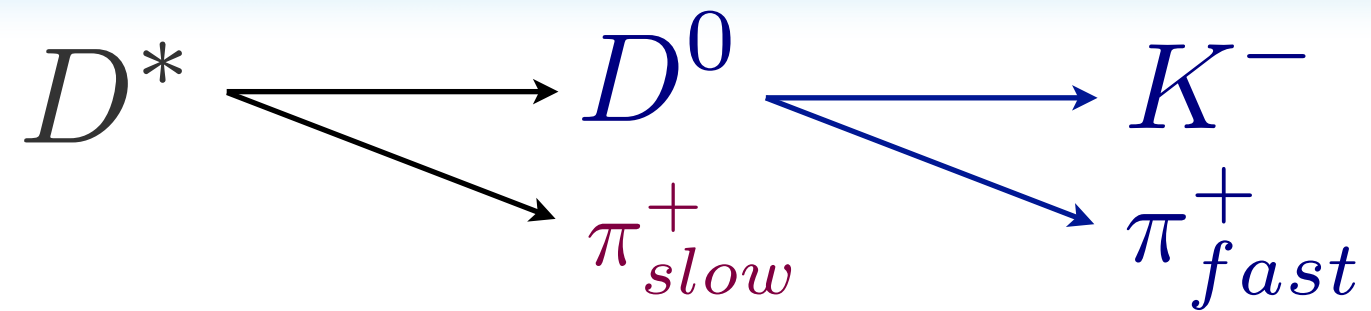


$$P_{ij} = \begin{pmatrix} P_{e \rightarrow e} & P_{e \rightarrow \mu} & P_{e \rightarrow \pi} & P_{e \rightarrow K} & P_{e \rightarrow p} \\ P_{\mu \rightarrow e} & P_{\mu \rightarrow \mu} & P_{\mu \rightarrow \pi} & P_{\mu \rightarrow K} & P_{\mu \rightarrow p} \\ P_{\pi \rightarrow e} & P_{\pi \rightarrow \mu} & P_{\pi \rightarrow \pi} & P_{\pi \rightarrow K} & P_{\pi \rightarrow p} \\ P_{K \rightarrow e} & P_{K \rightarrow \mu} & P_{K \rightarrow \pi} & P_{K \rightarrow K} & P_{K \rightarrow p} \\ P_{p \rightarrow e} & P_{p \rightarrow \mu} & P_{p \rightarrow \pi} & P_{p \rightarrow K} & P_{p \rightarrow p} \end{pmatrix}$$



# How to determine the $P_{ij}$ ?

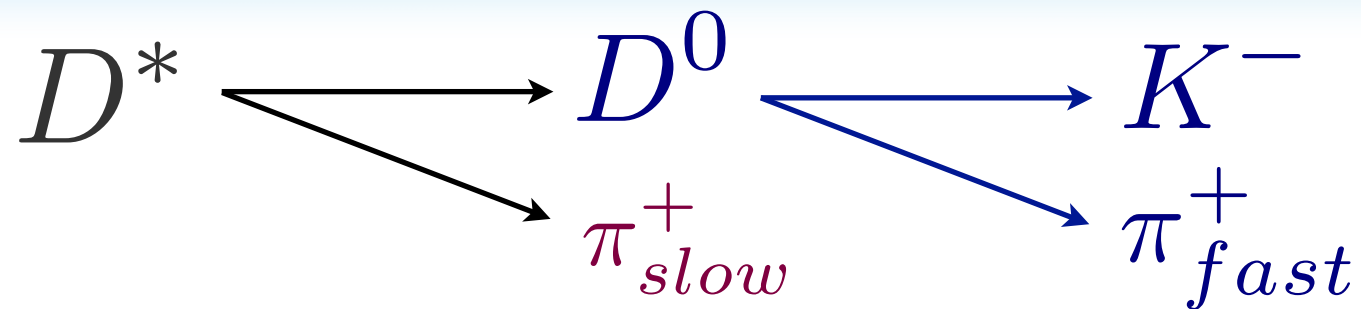
From data!



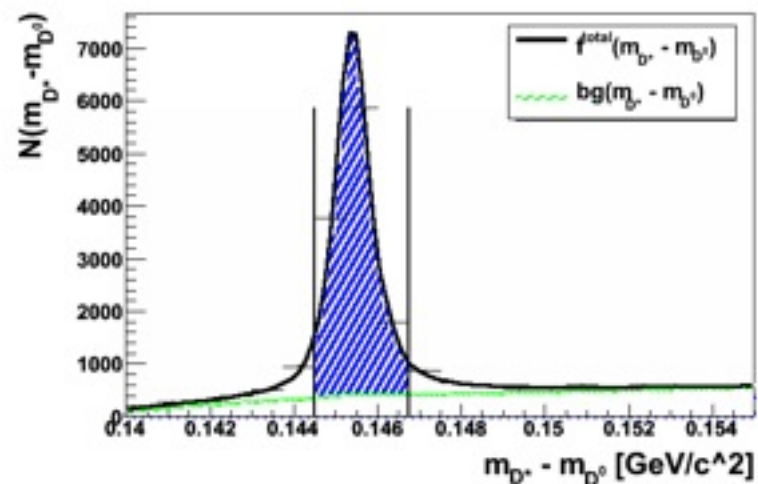


# How to determine the $P_{ij}$ ?

From data!



$$m_{D^*}^* - m_{D^0}^0$$

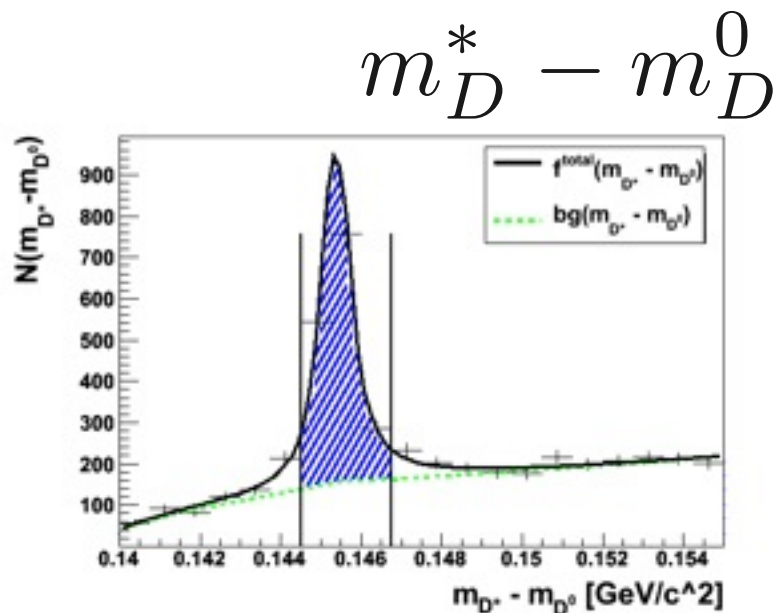
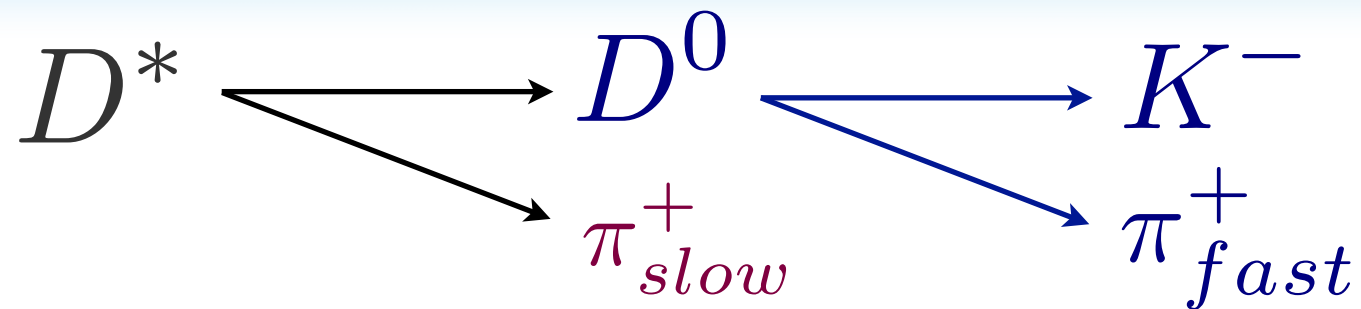


Negative hadron =  $K^-$   
(no PID likelihood used)

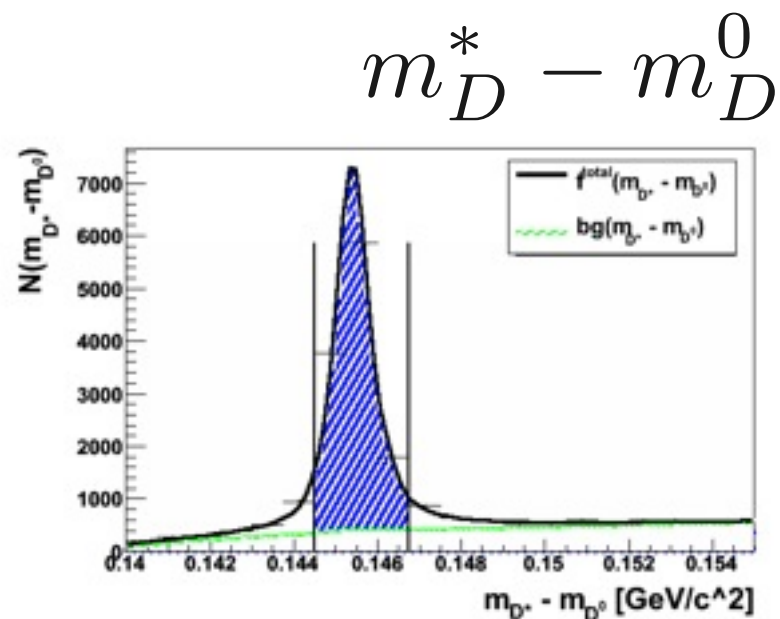


# How to determine the $P_{ij}$ ?

From data!



Negative hadron  
identified as  $\pi^-$

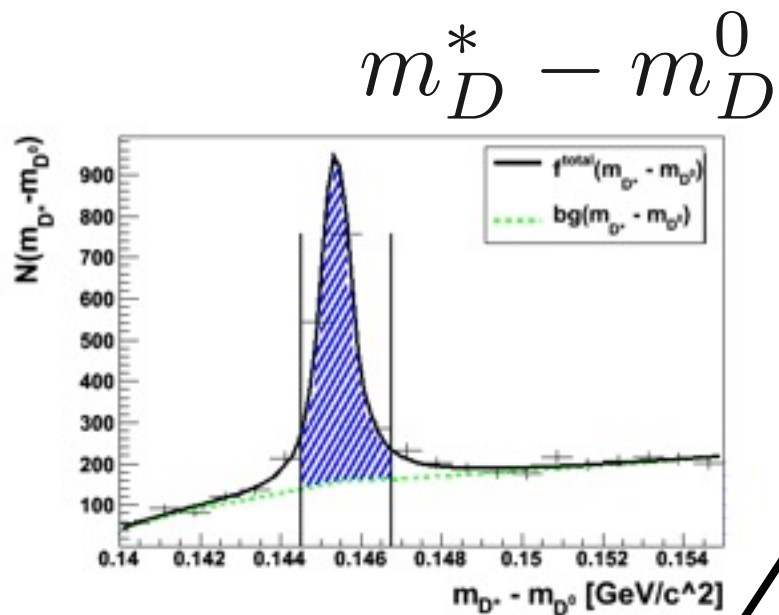
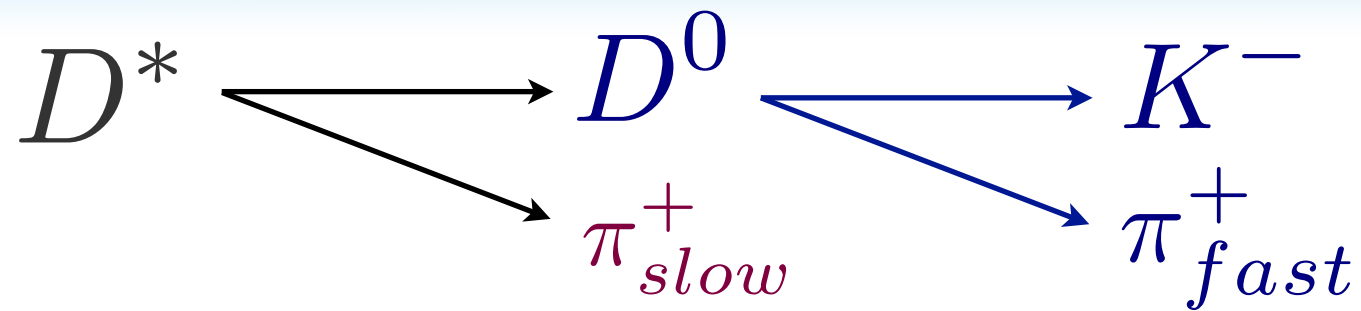


Negative hadron =  $K^-$   
(no PID likelihood used)

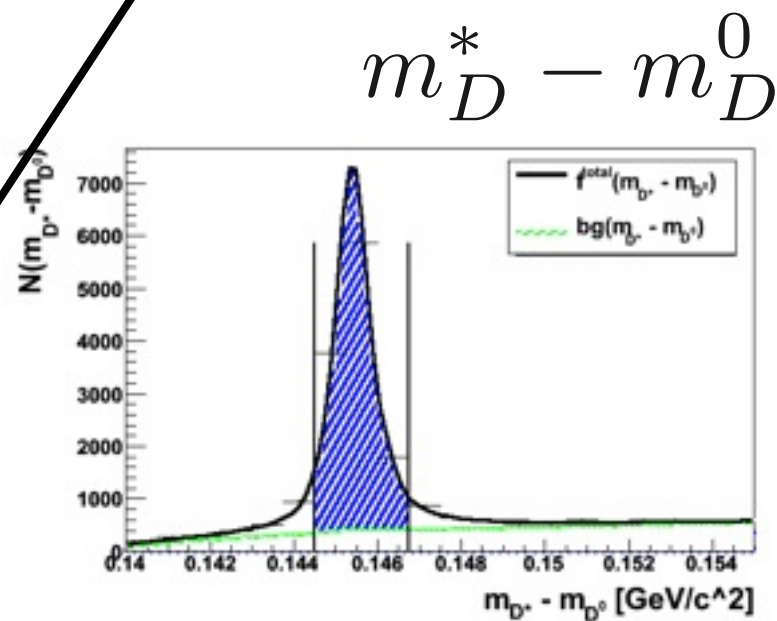


# How to determine the $P_{ij}$ ?

From data!



Negative hadron  
identified as  $\pi^-$



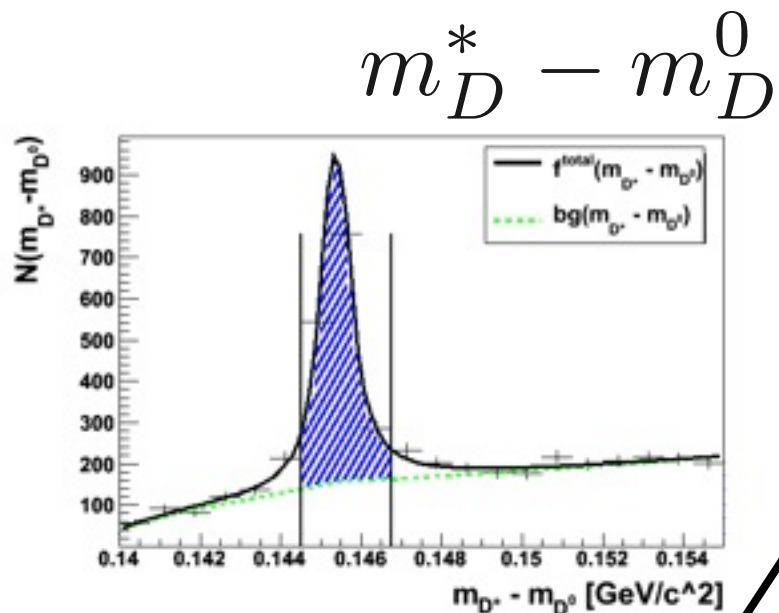
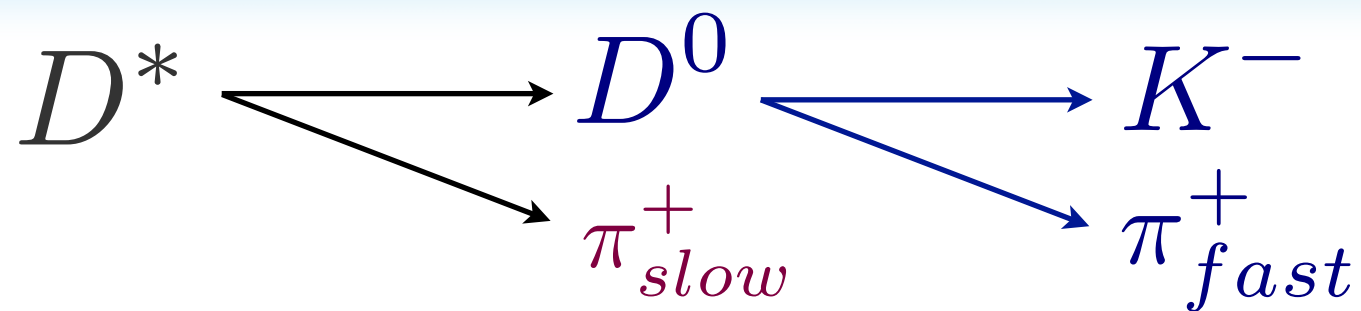
Negative hadron =  $K^-$   
(no PID likelihood used)

$$P_{K^- \rightarrow \pi^-}$$



# How to determine the $P_{ij}$ ?

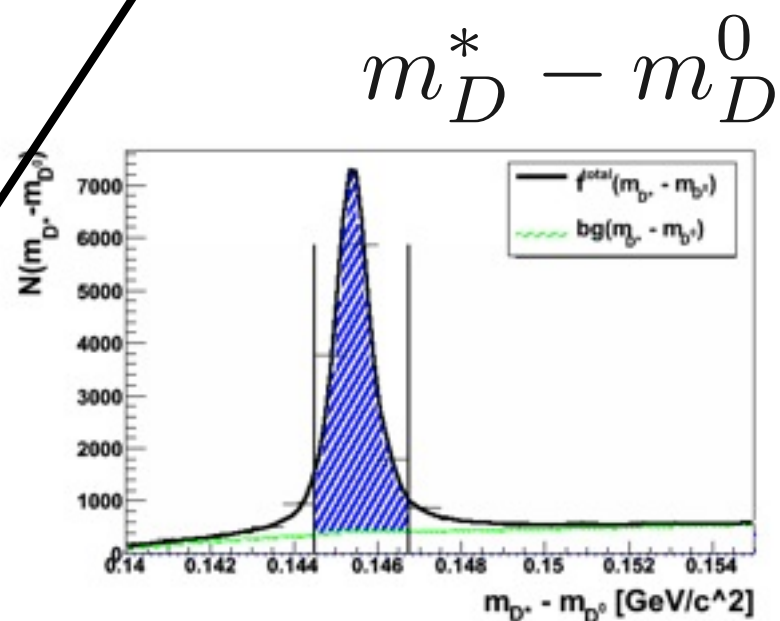
From data!



Negative hadron  
identified as  $\pi^-$

$K^-$

$$P_{K^- \rightarrow \pi^-}$$



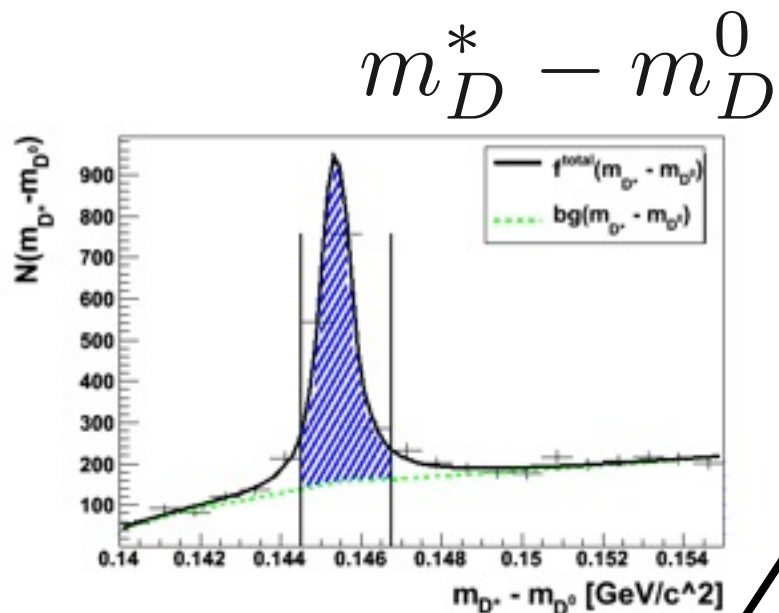
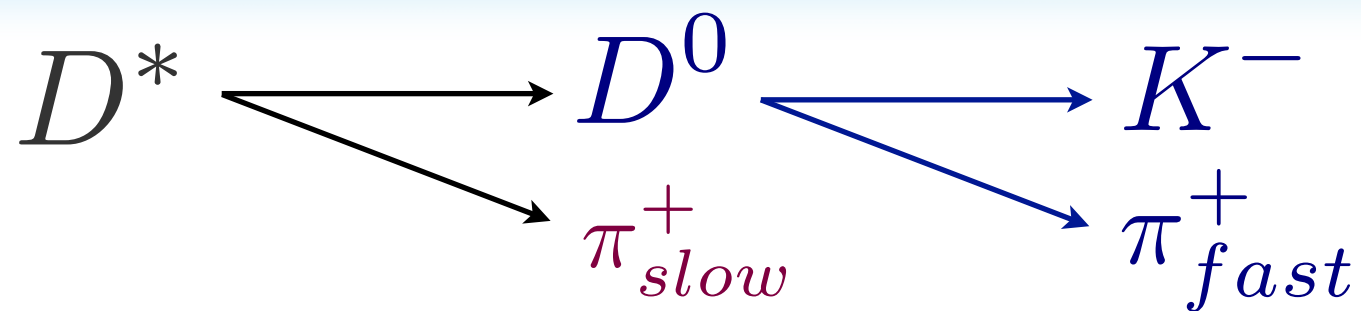
Negative hadron =  $K^-$   
(no PID likelihood used)

$$P_{K^- \rightarrow K^-}$$

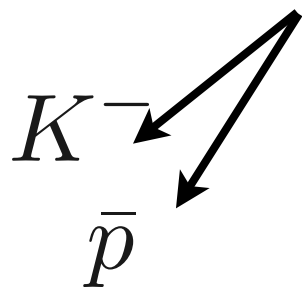


# How to determine the $P_{ij}$ ?

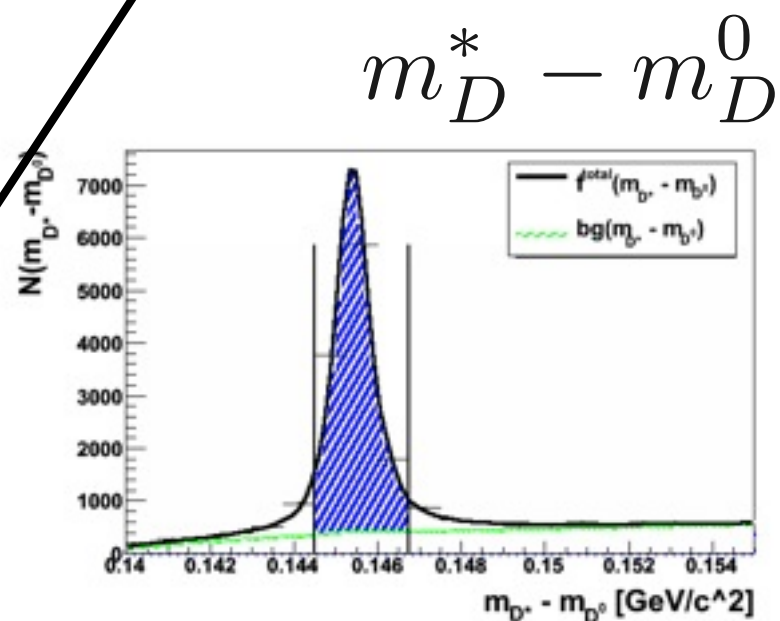
From data!



Negative hadron identified as  $\pi^-$



$$P_{K^- \rightarrow \pi^-}$$



Negative hadron =  $K^-$   
(no PID likelihood used)

$$P_{K^- \rightarrow K^-}$$

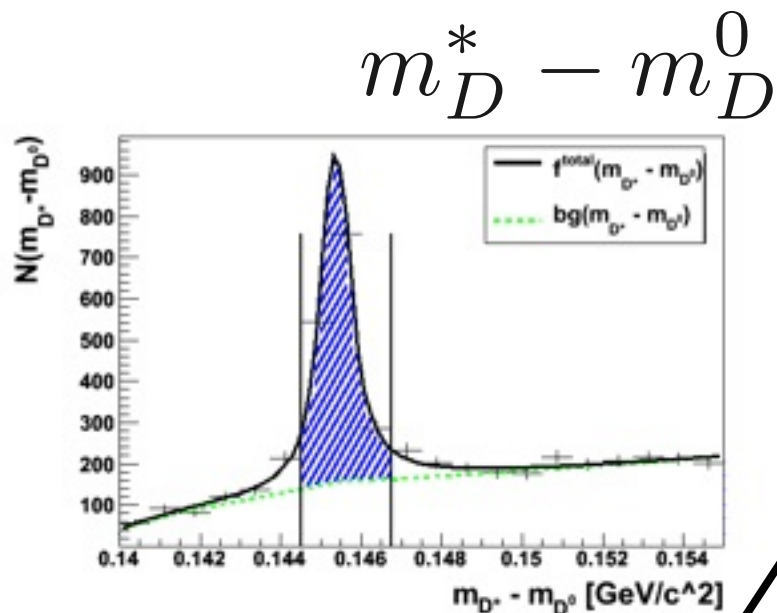
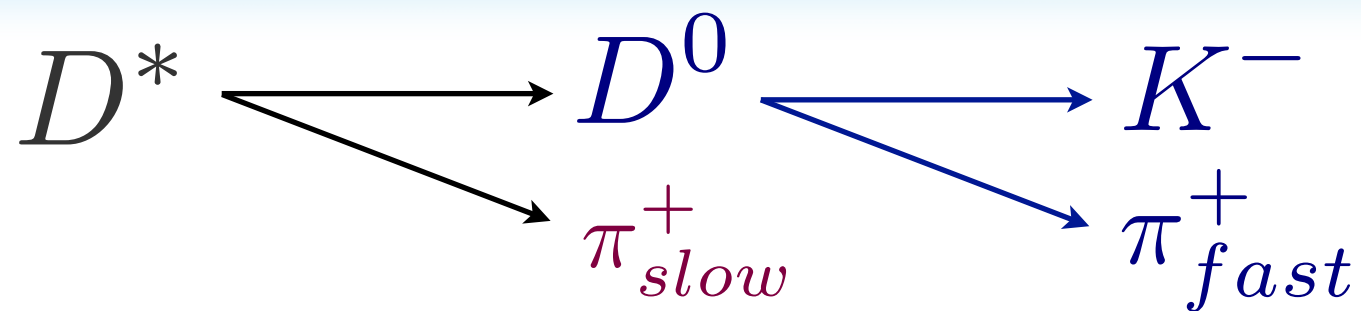
$$P_{K^- \rightarrow \bar{p}}$$



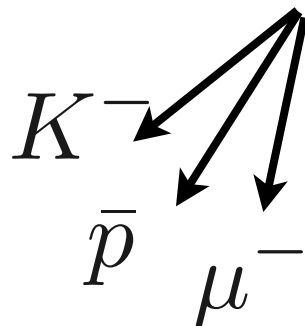


# How to determine the $P_{ij}$ ?

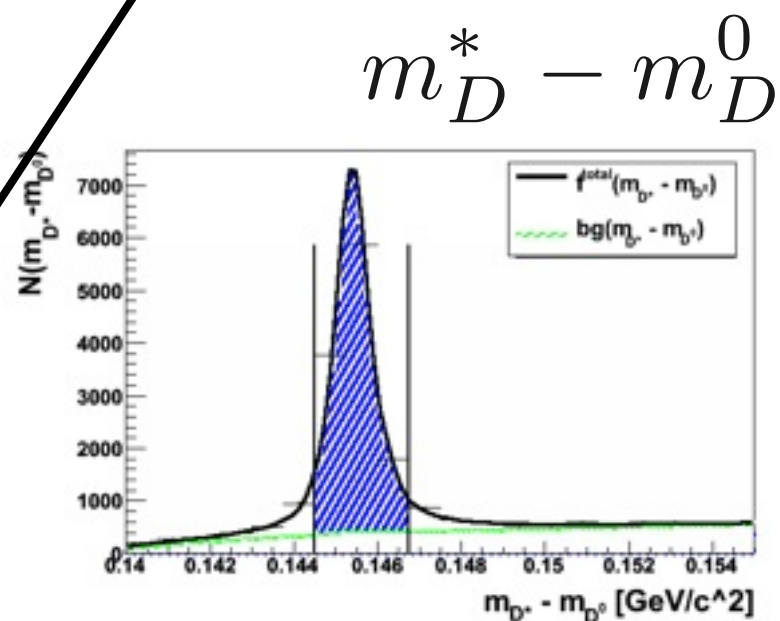
From data!



Negative hadron identified as  $\pi^-$



$$P_{K^- \rightarrow \pi^-}$$



Negative hadron =  $K^-$   
(no PID likelihood used)

$$P_{K^- \rightarrow K^-}$$

$$P_{K^- \rightarrow \bar{p}}$$

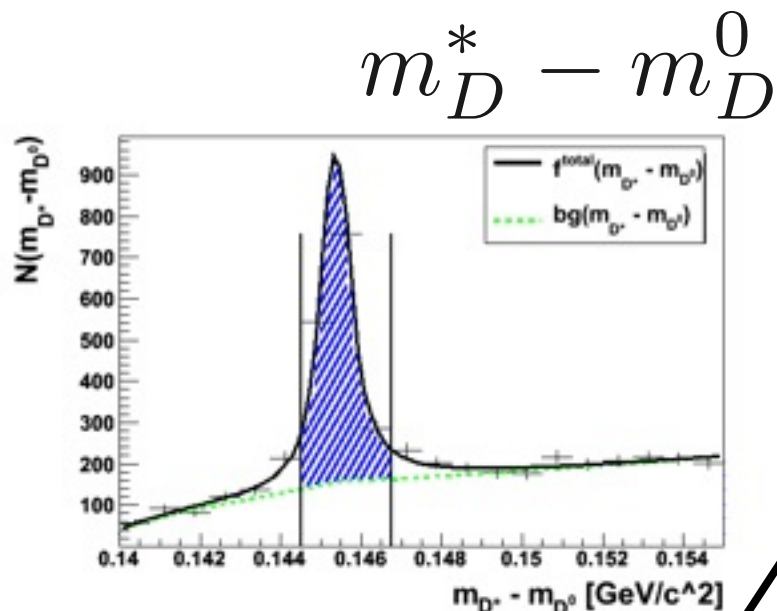
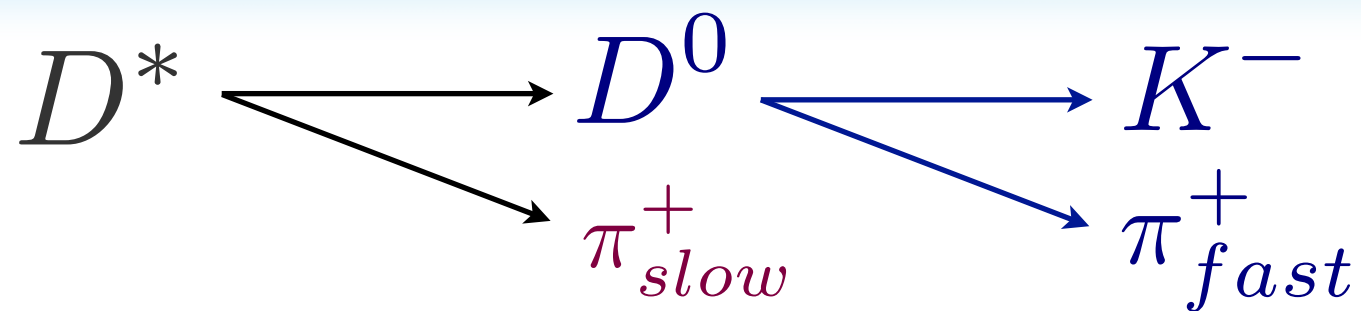
$$P_{K^- \rightarrow \mu^-}$$



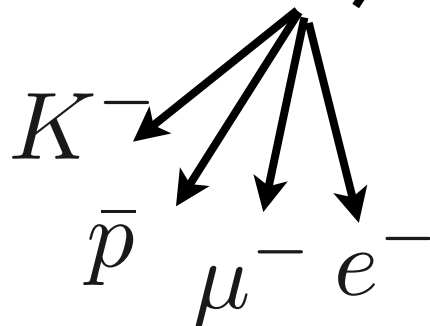


# How to determine the $P_{ij}$ ?

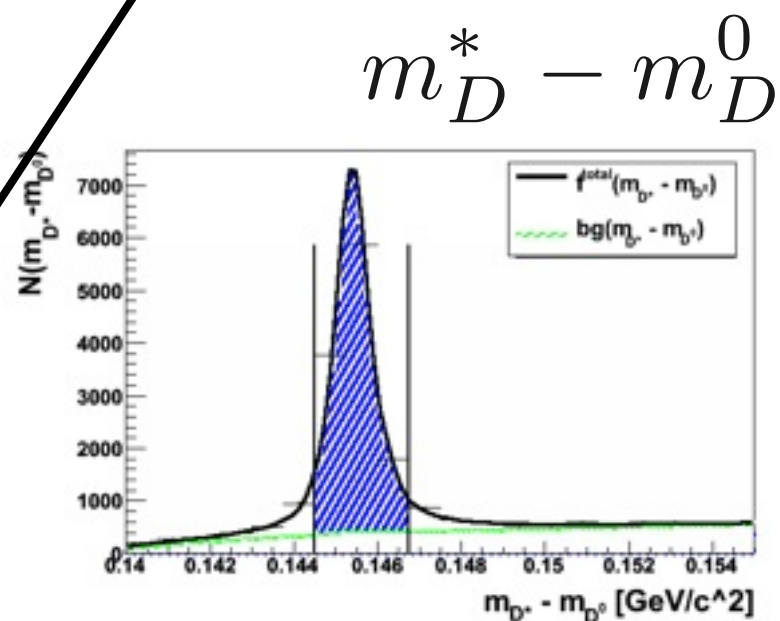
From data!



Negative hadron identified as  $\pi^-$



$$P_{K^- \rightarrow \pi^-}$$



Negative hadron =  $K^-$   
(no PID likelihood used)

$$P_{K^- \rightarrow K^-}$$

$$P_{K^- \rightarrow \bar{p}}$$

$$P_{K^- \rightarrow \mu^-}$$

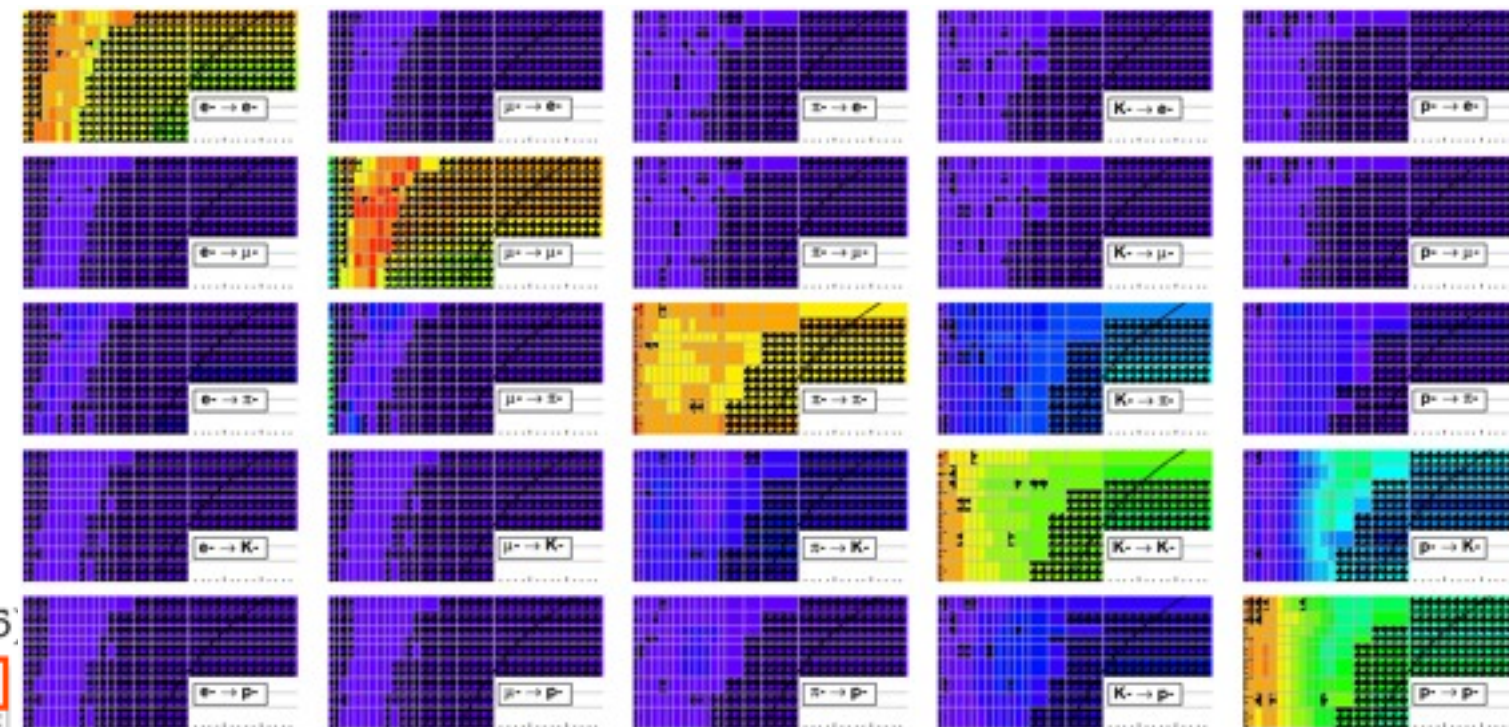
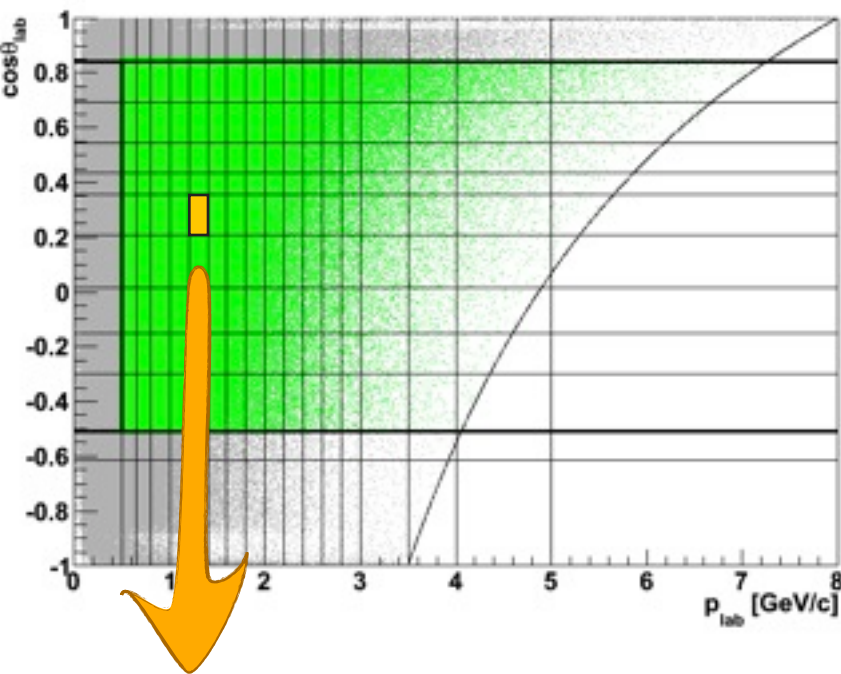
$$P_{K^- \rightarrow e^-}$$



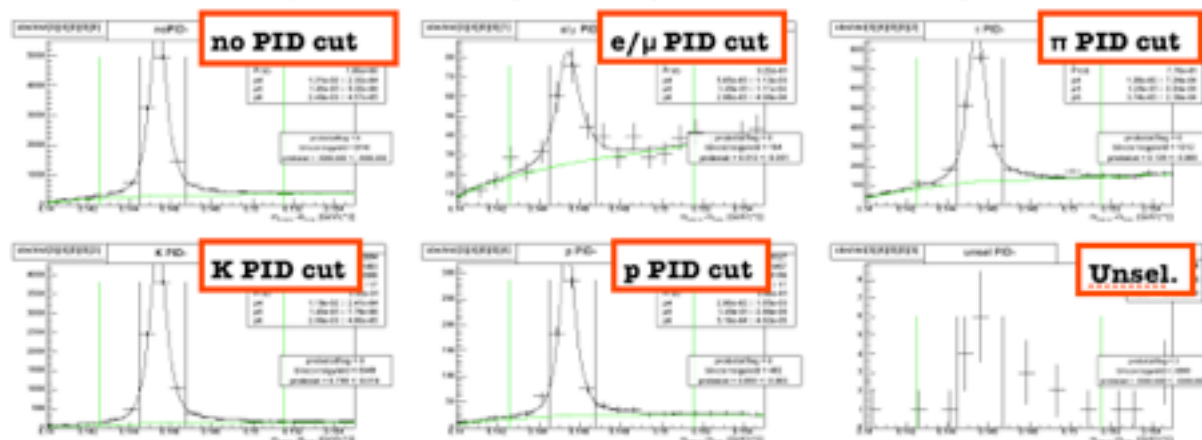
# 2D correction

Detector performance depends on momentum and scattering angle!

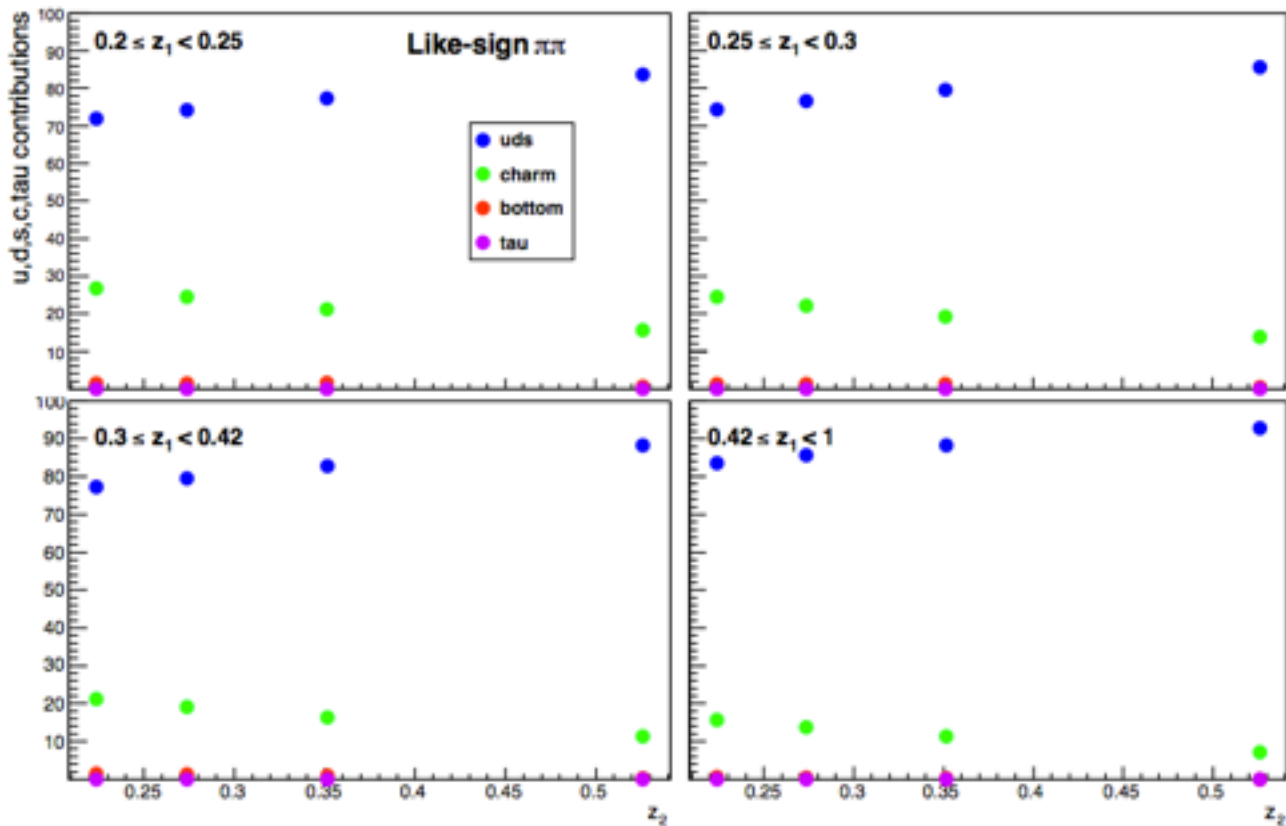
$$P_{ij} \Rightarrow P_{ij}(p, \theta)$$



K from  $D^*$  decay for  $p_{lab}$  in [1.4,1.6) and  $\cos\theta_{lab}$  in [0.209,0.355]

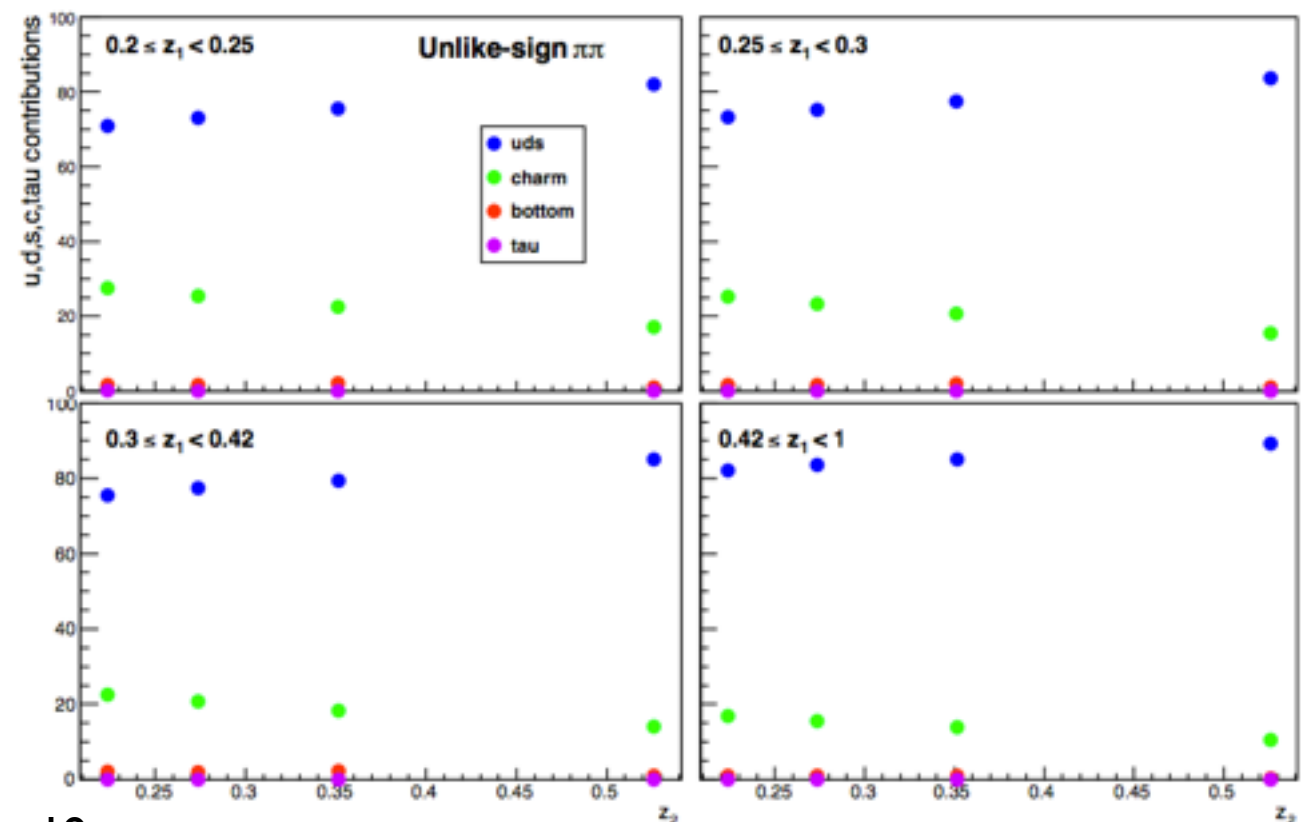


# uds-charm-bottom-tau contributions



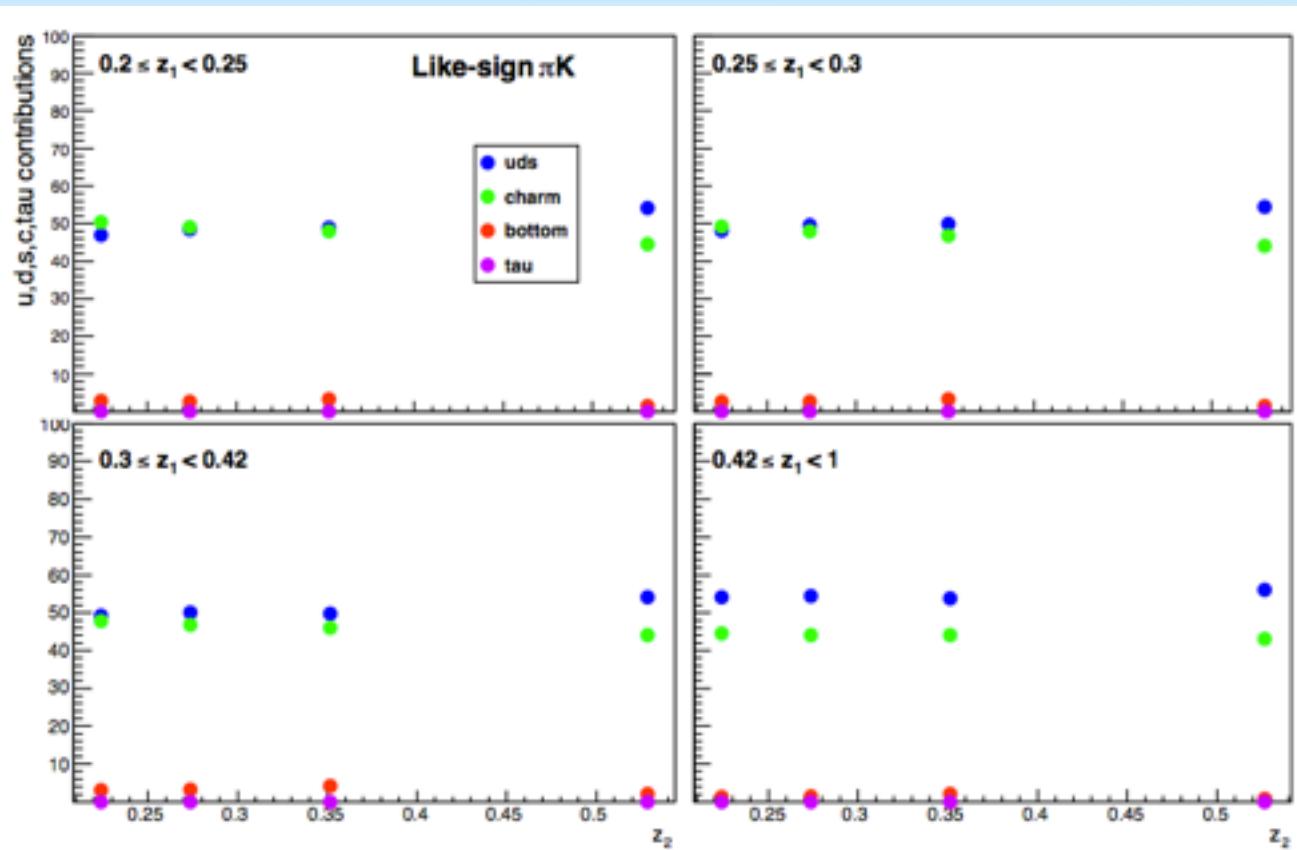
$\pi\pi$  couples

Published  $\pi\pi$  studied a charm enhanced data and found charm contribute only as dilution  
=> charm contribution corrected out

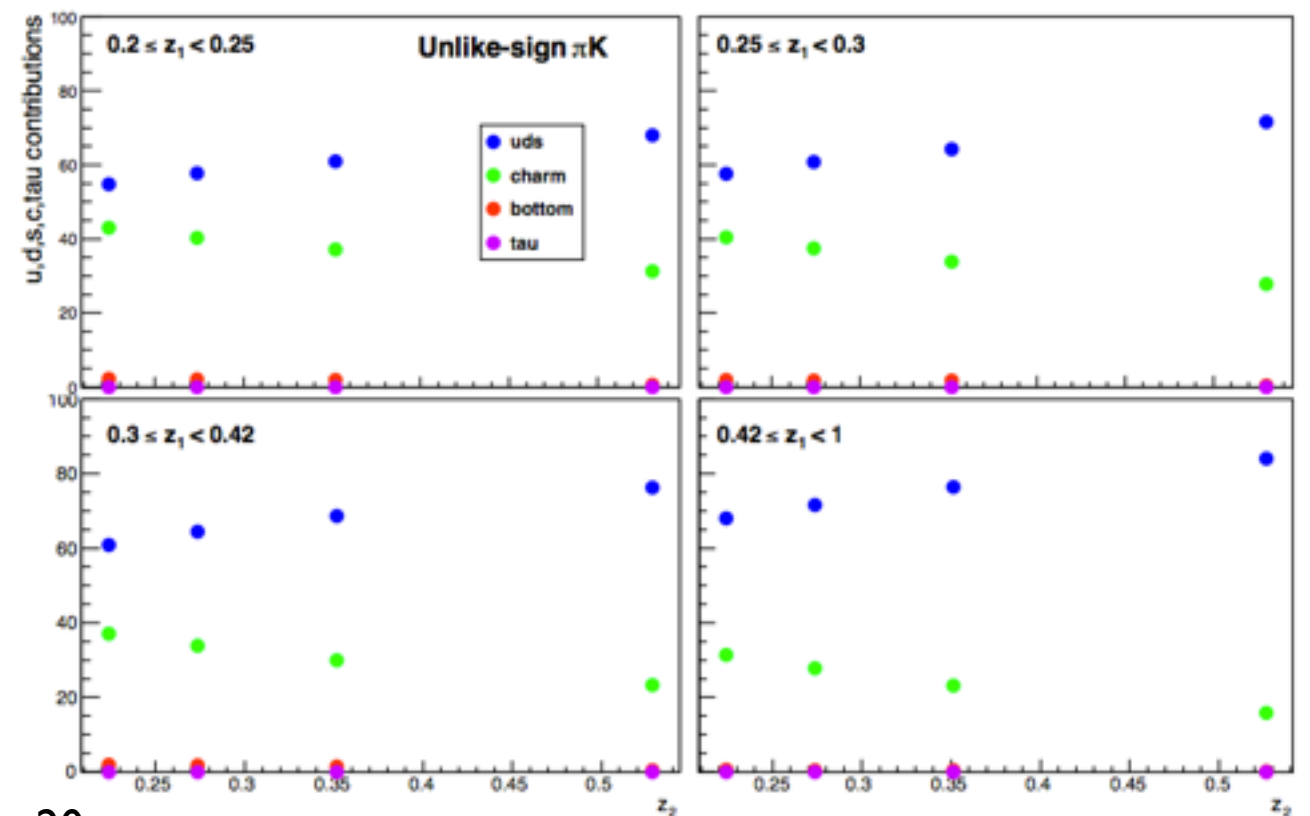




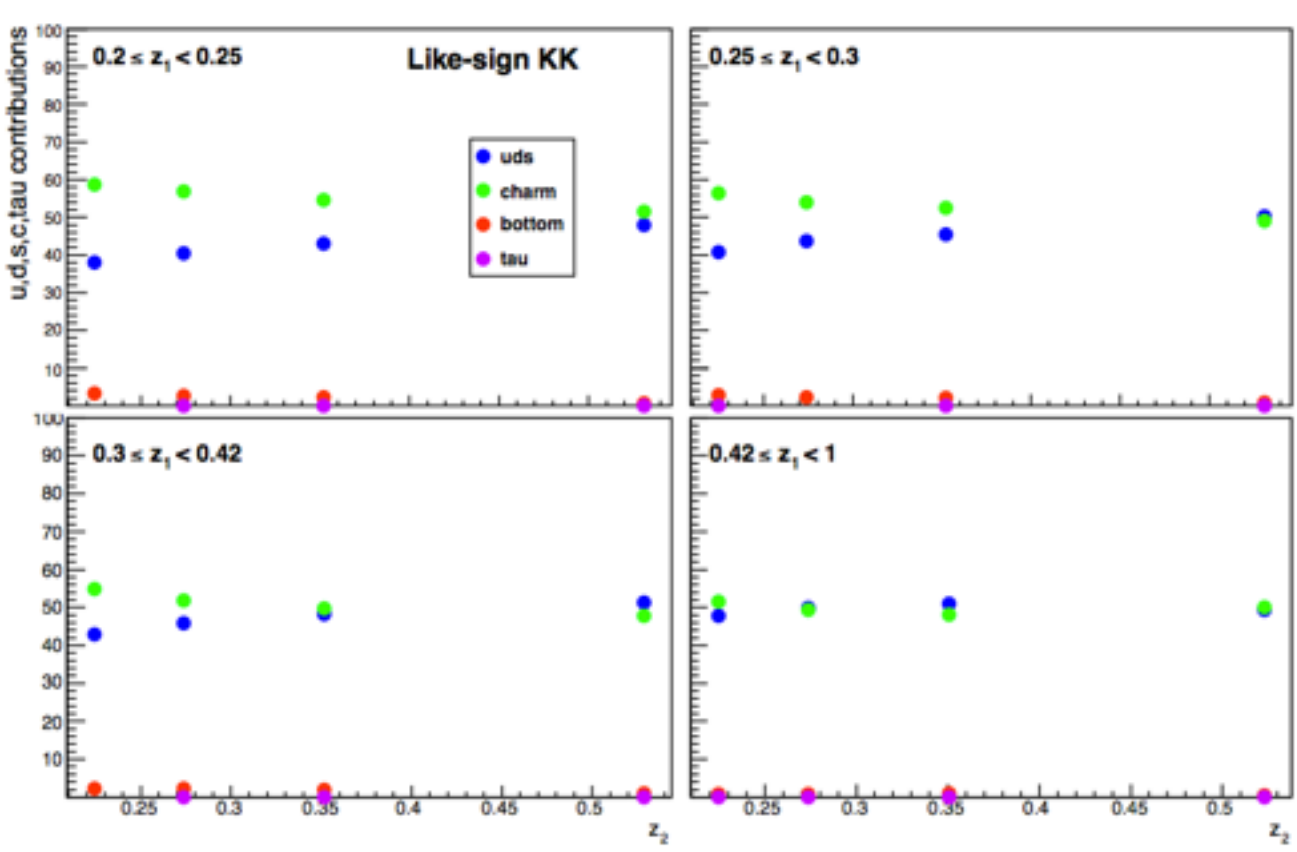
# uds-charm-bottom-tau contributions



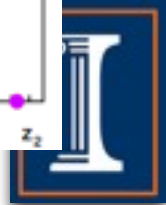
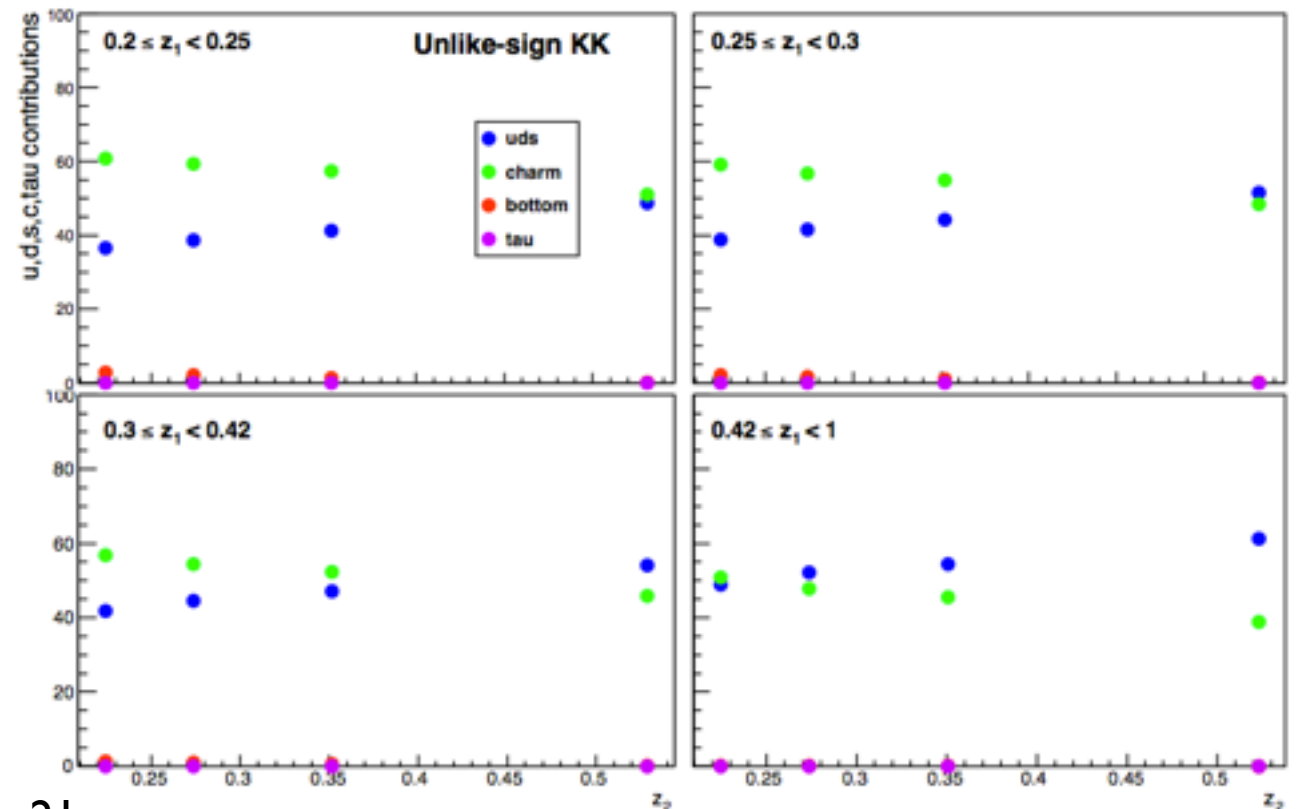
$\pi K$  couples



# uds-charm-bottom-tau contributions



KK couples

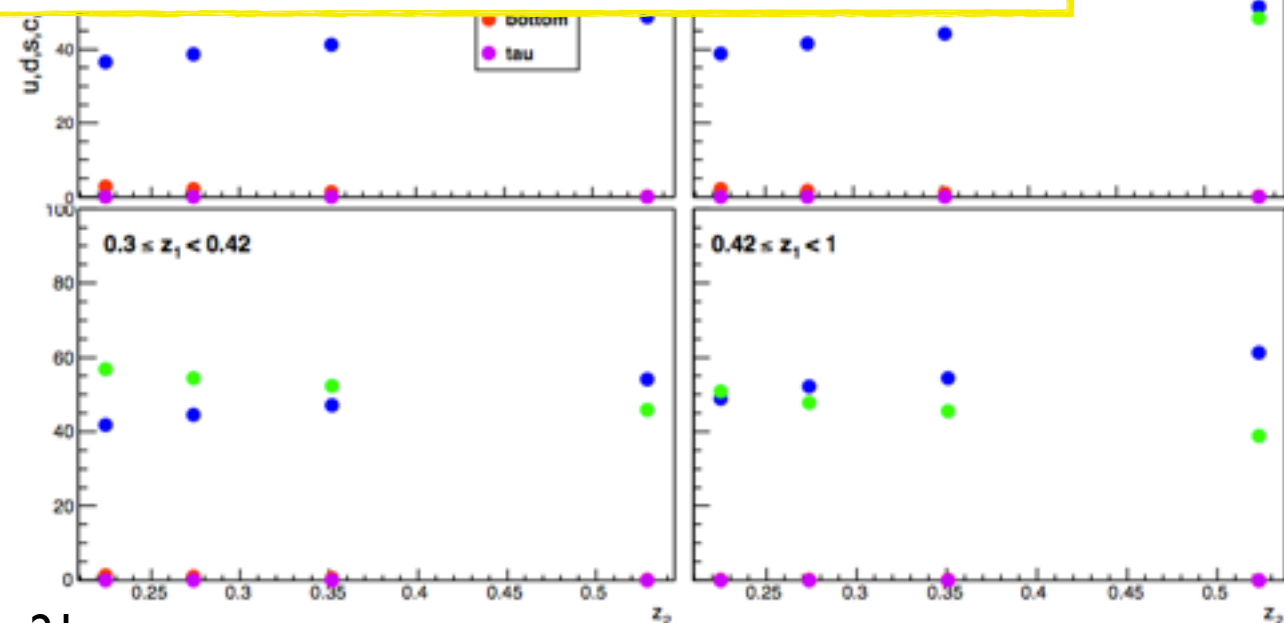


# uds-charm-bottom-tau contributions



KK couples

For the moment charm contribution is not being corrected out in any of the samples ( $\pi\pi$ ,  $\pi K$ ,  $KK$ )

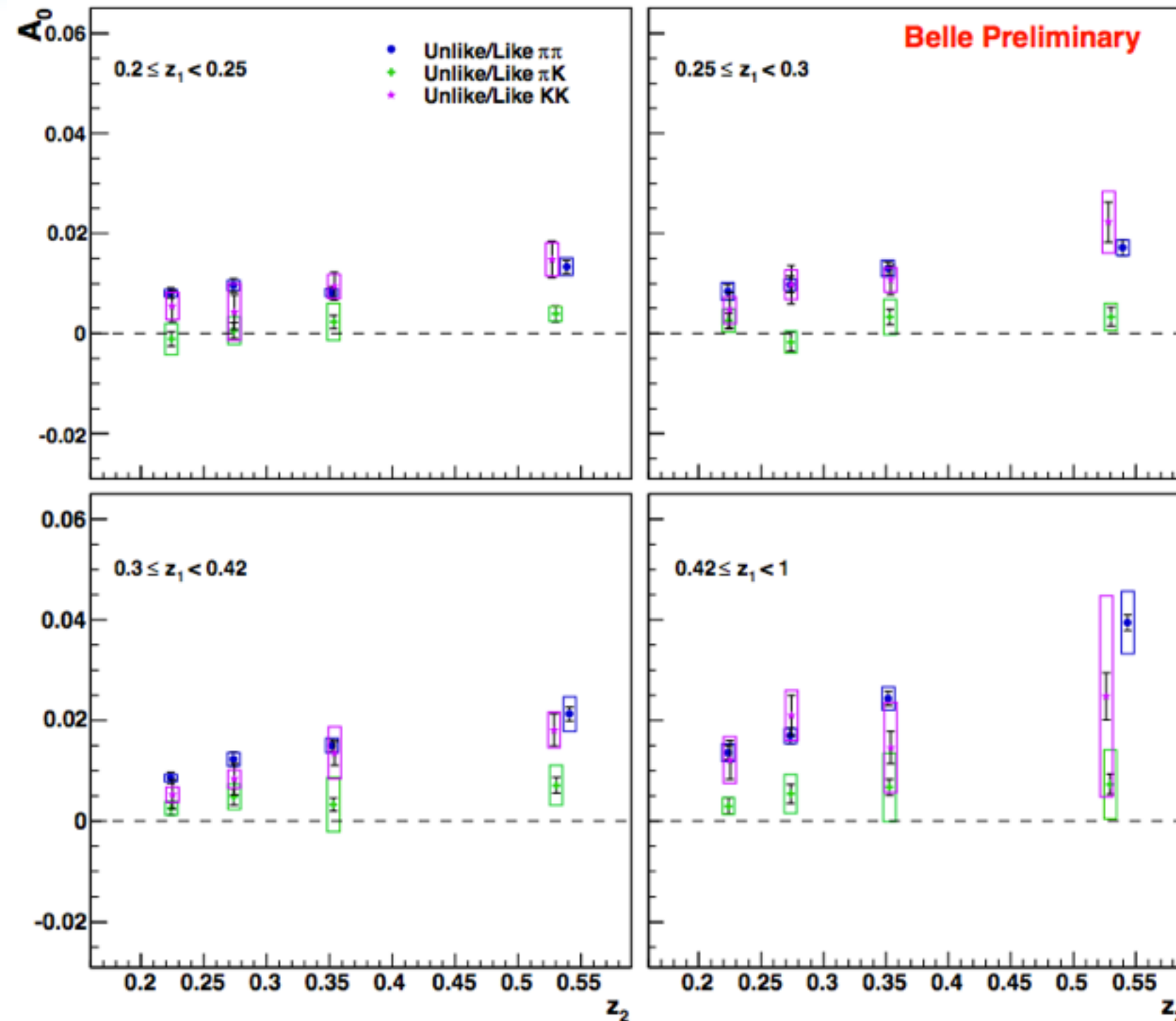




# Collins asymmetries



# $\phi_0$ asymmetries



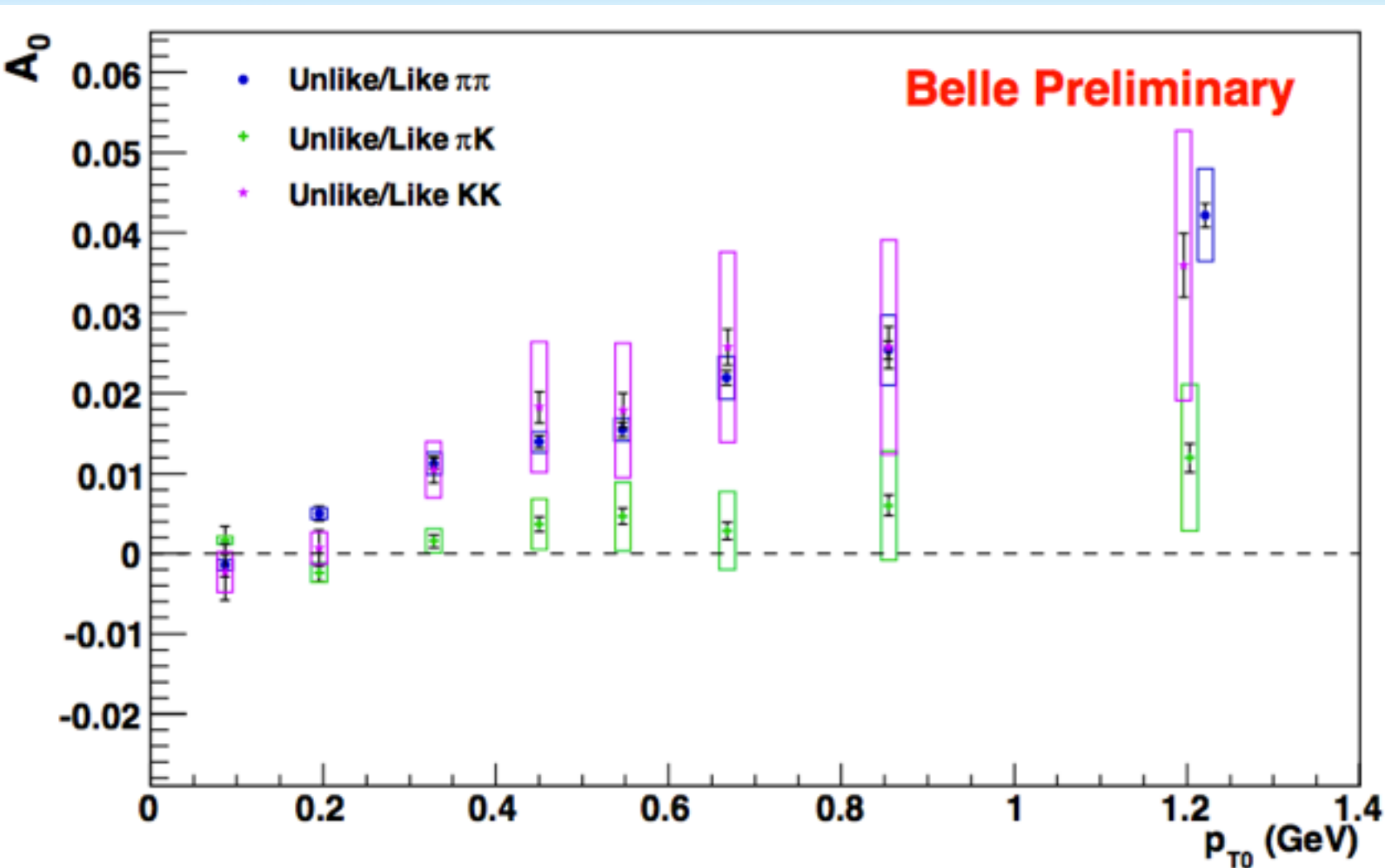
$\pi\pi \Rightarrow$  non-zero asymmetries,  
increase with  $z_1, z_2$

$\pi K \Rightarrow$  asymmetries compatible  
with zero

$KK \Rightarrow$  non-zero asymmetries,  
increase with  $z_1, z_2$   
similar size of pion-pion



# $\phi_0$ asymmetries



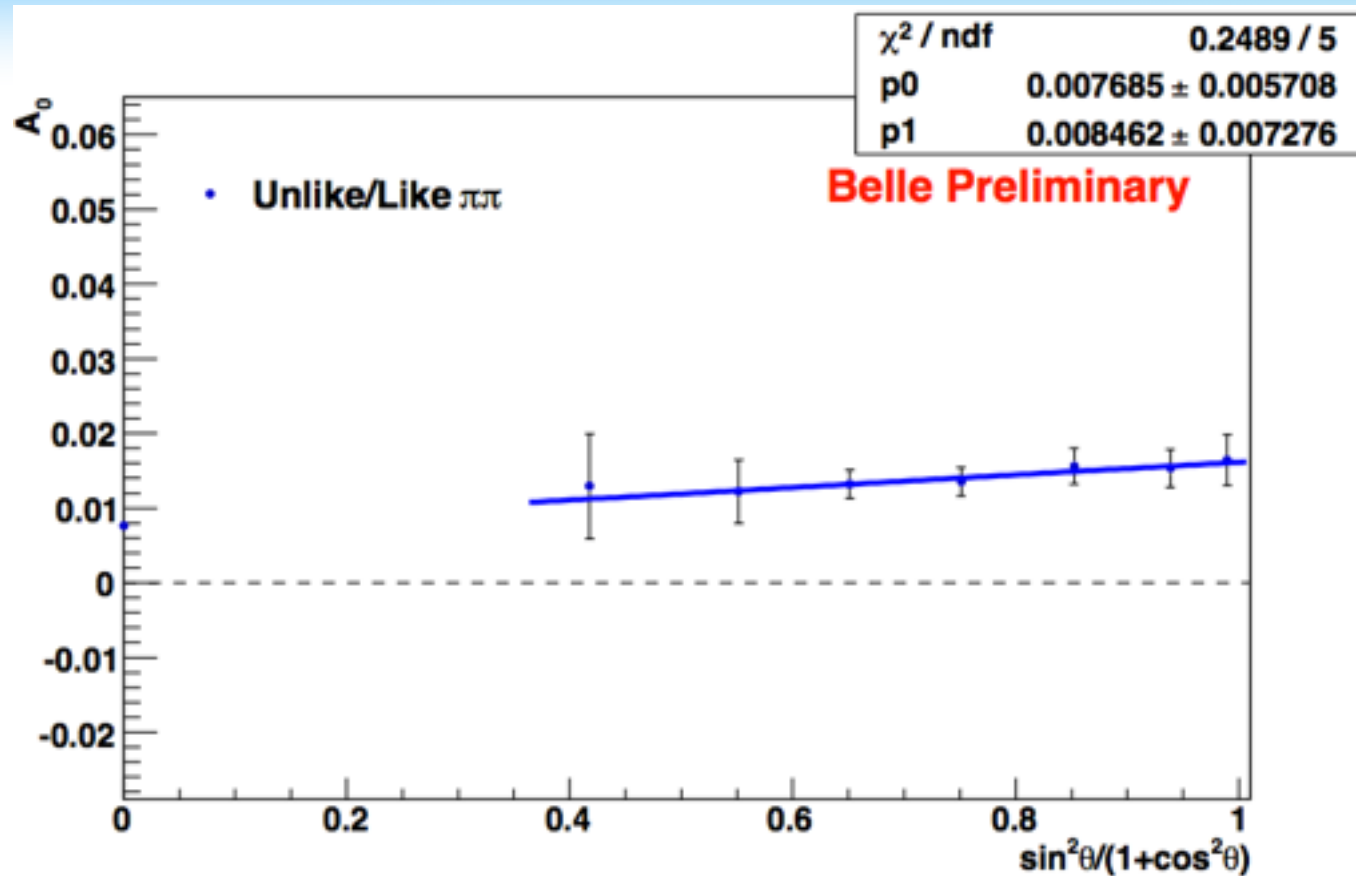
$\pi\pi \Rightarrow$  non-zero asymmetries,  
increase with  $z_1, z_2$

$\pi K \Rightarrow$  asymmetries compatible  
with zero

$KK \Rightarrow$  non-zero asymmetries,  
increase with  $z_1, z_2$   
similar size of pion-pion



# $\pi\pi$ versus $\sin^2\theta/(1+\cos^2\theta)$



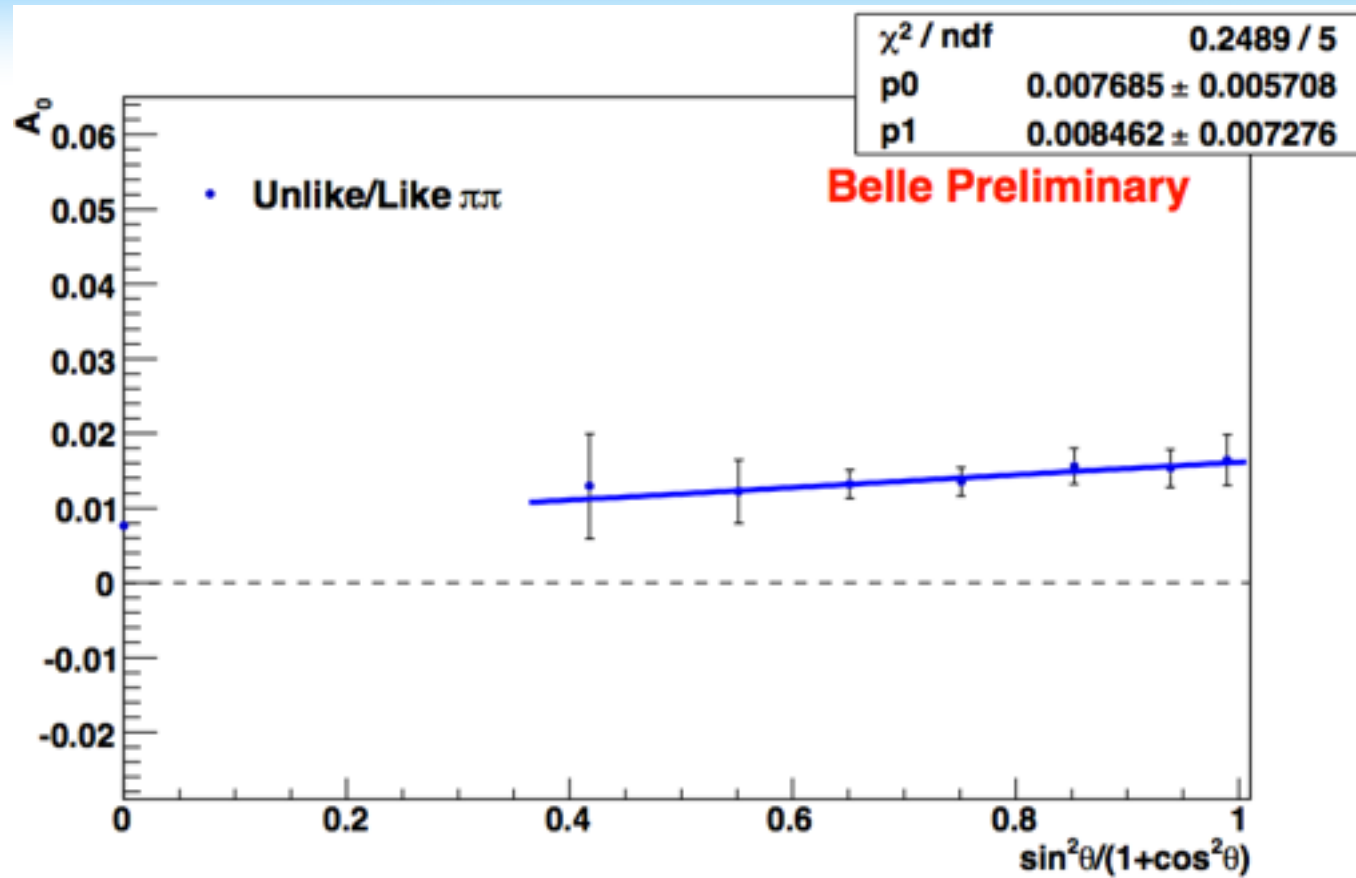
$$A_0 = \frac{\sin^2 \theta}{1 + \cos^2 \theta} \mathcal{F} \left[ \frac{H_1^\perp(z_1) \bar{H}_1^\perp(z_2)}{D_1^\perp(z_1) \bar{D}_1^\perp(z_2)} \right]$$

linear in  $\sin^2\theta/(1+\cos^2\theta)$ ,  
go to 0 for  $\sin^2\theta/(1+\cos^2\theta) \rightarrow 0$

fit form:  $p_0 + p_1 \sin^2\theta/(1+\cos^2\theta)$



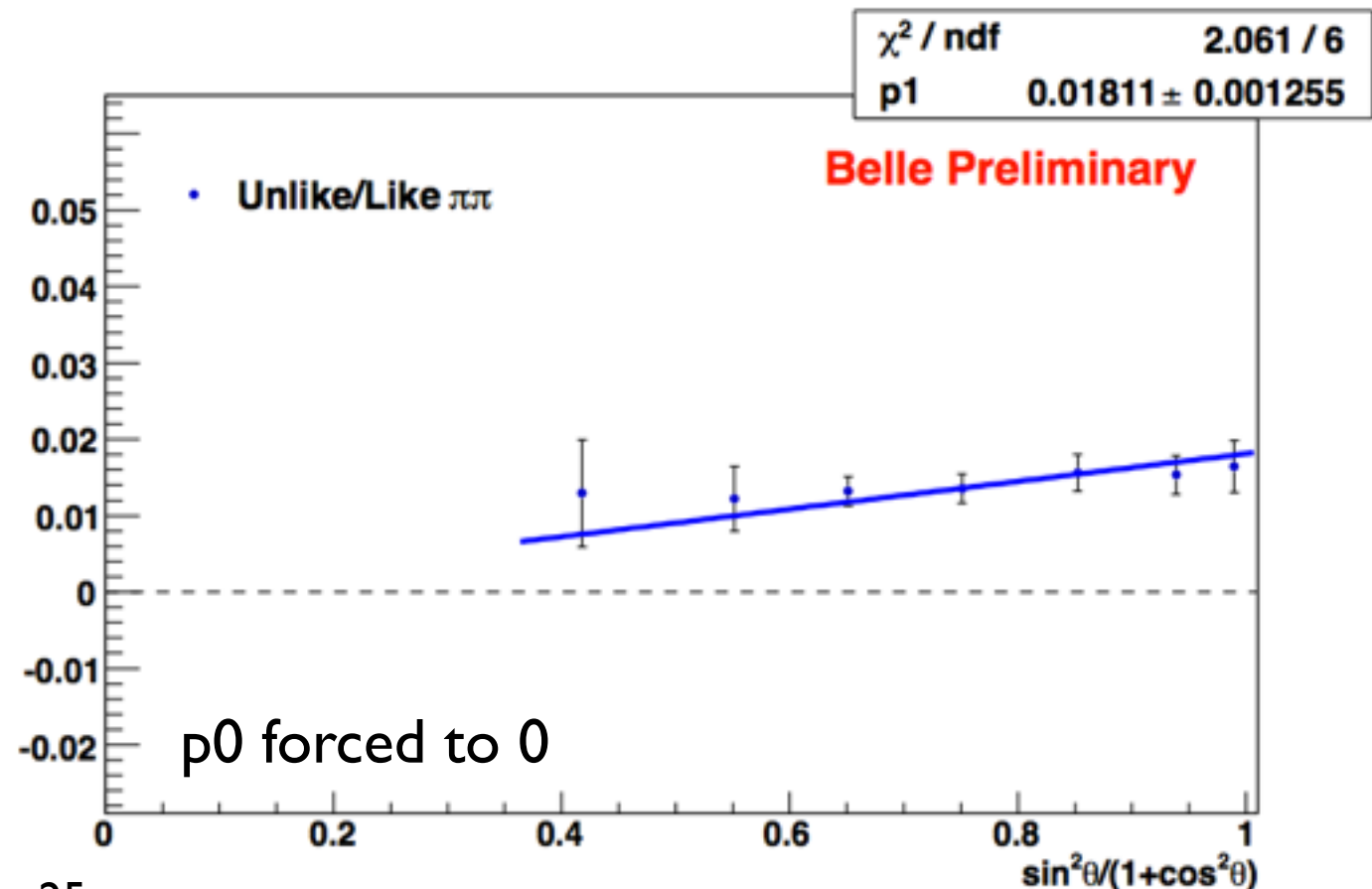
# $\pi\pi$ versus $\sin^2\theta/(1+\cos^2\theta)$



$$A_0 = \frac{\sin^2 \theta}{1 + \cos^2 \theta} \mathcal{F} \left[ \frac{H_1^\perp(z_1) \bar{H}_1^\perp(z_2)}{D_1^\perp(z_1) \bar{D}_1^\perp(z_2)} \right]$$

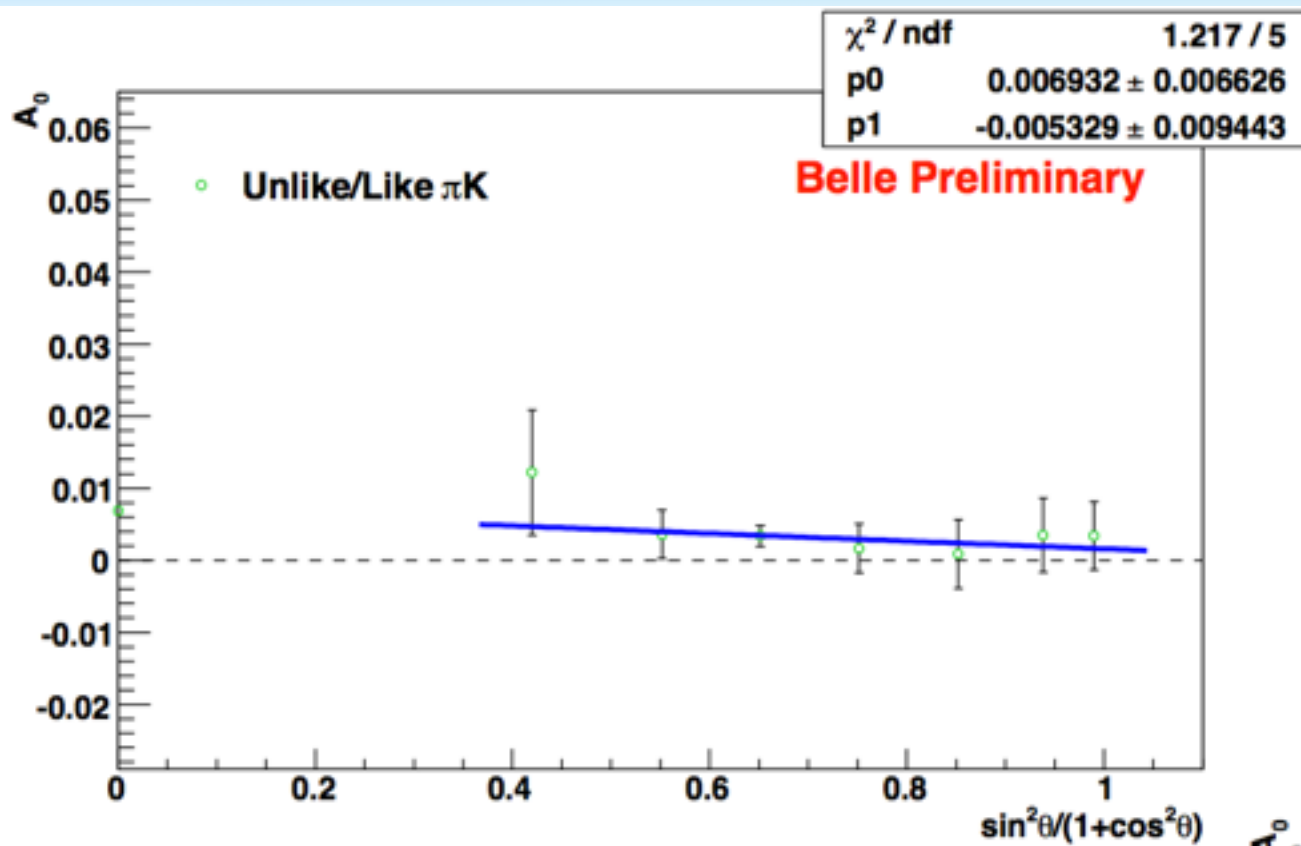
linear in  $\sin^2\theta/(1+\cos^2\theta)$ ,  
go to 0 for  $\sin^2\theta/(1+\cos^2\theta) \rightarrow 0$

fit form:  $p_0 + p_1 \sin^2\theta/(1+\cos^2\theta)$





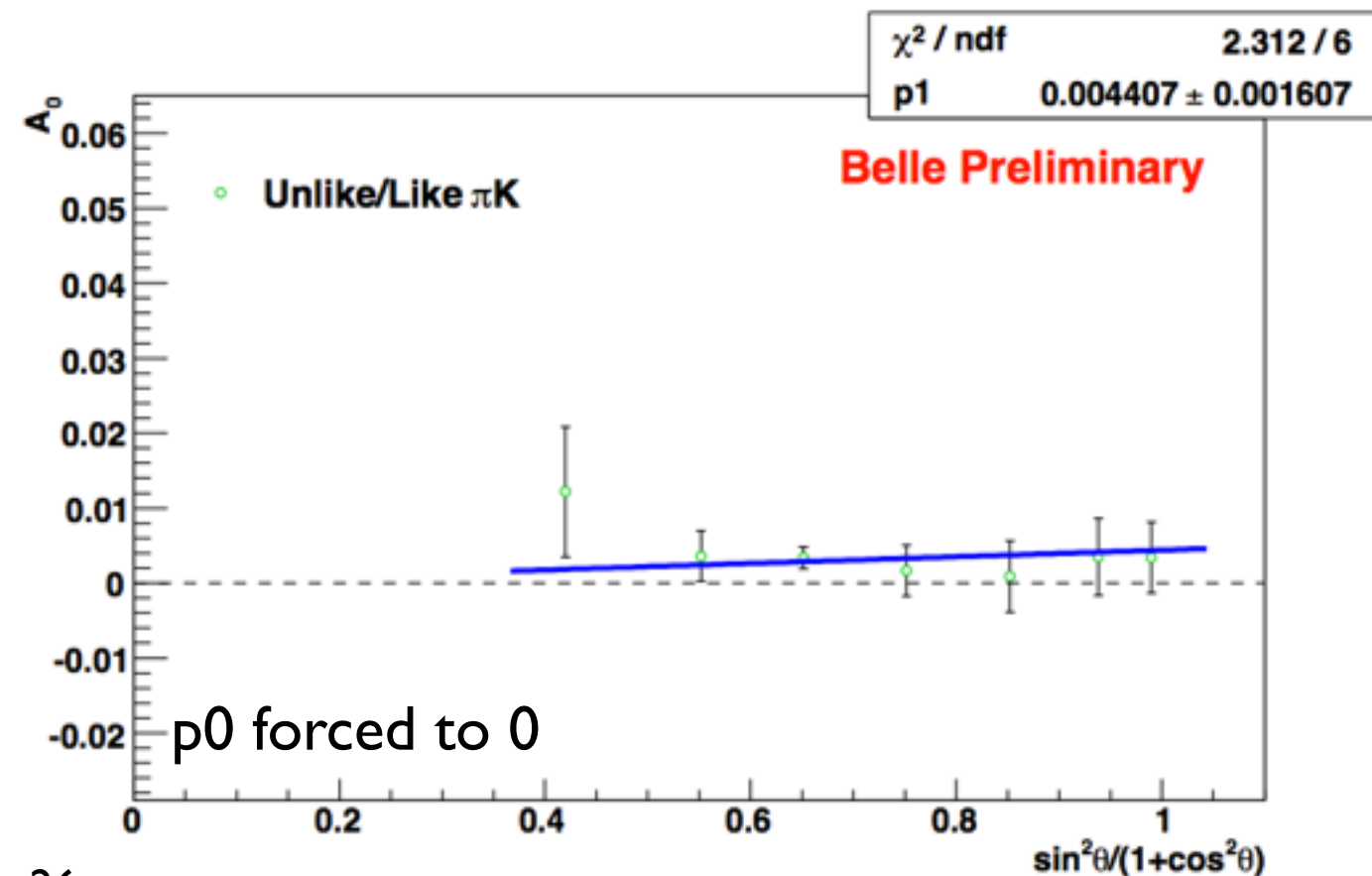
# $\pi K$ versus $\sin^2\theta/(1+\cos^2\theta)$



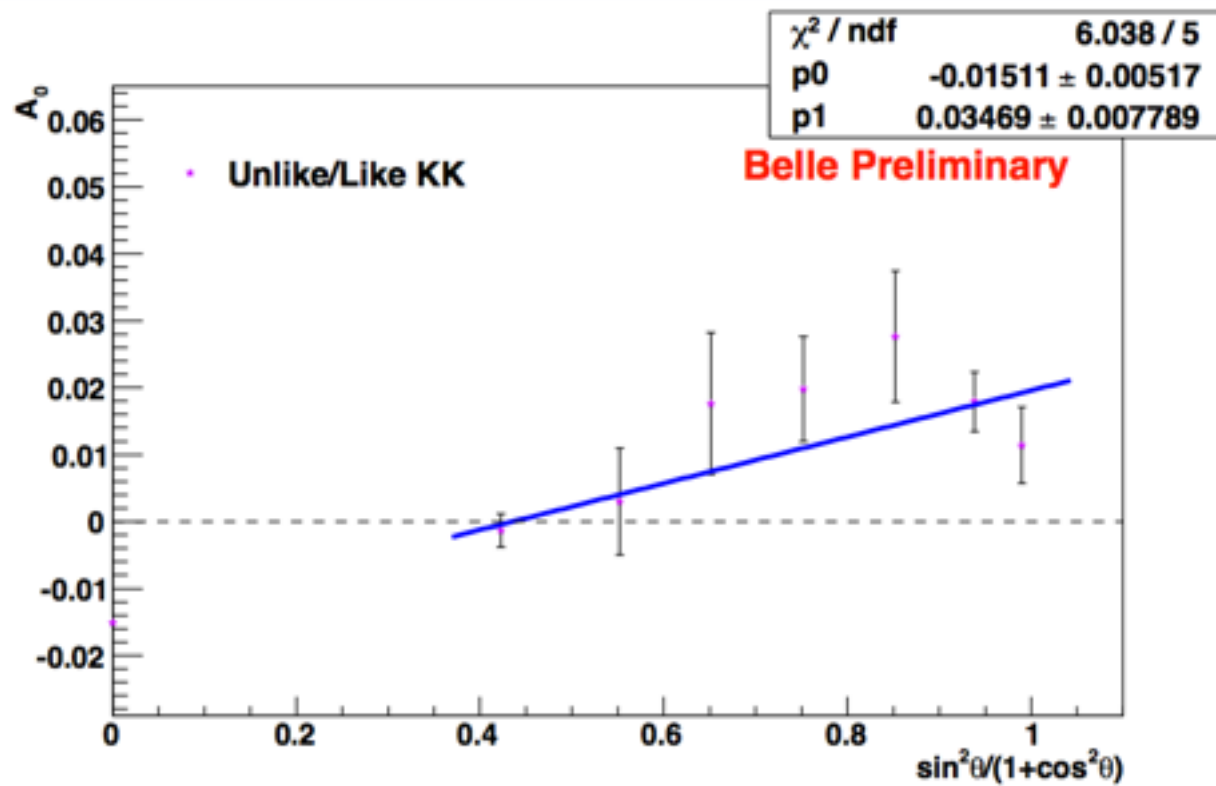
$$A_0 = \frac{\sin^2 \theta}{1 + \cos^2 \theta} \mathcal{F} \left[ \frac{H_1^\perp(z_1) \bar{H}_1^\perp(z_2)}{D_1^\perp(z_1) \bar{D}_1^\perp(z_2)} \right]$$

linear in  $\sin^2\theta/(1+\cos^2\theta)$ ,  
go to 0 for  $\sin^2\theta/(1+\cos^2\theta) \rightarrow 0$

fit form:  $p_0 + p_1 \sin^2\theta/(1+\cos^2\theta)$



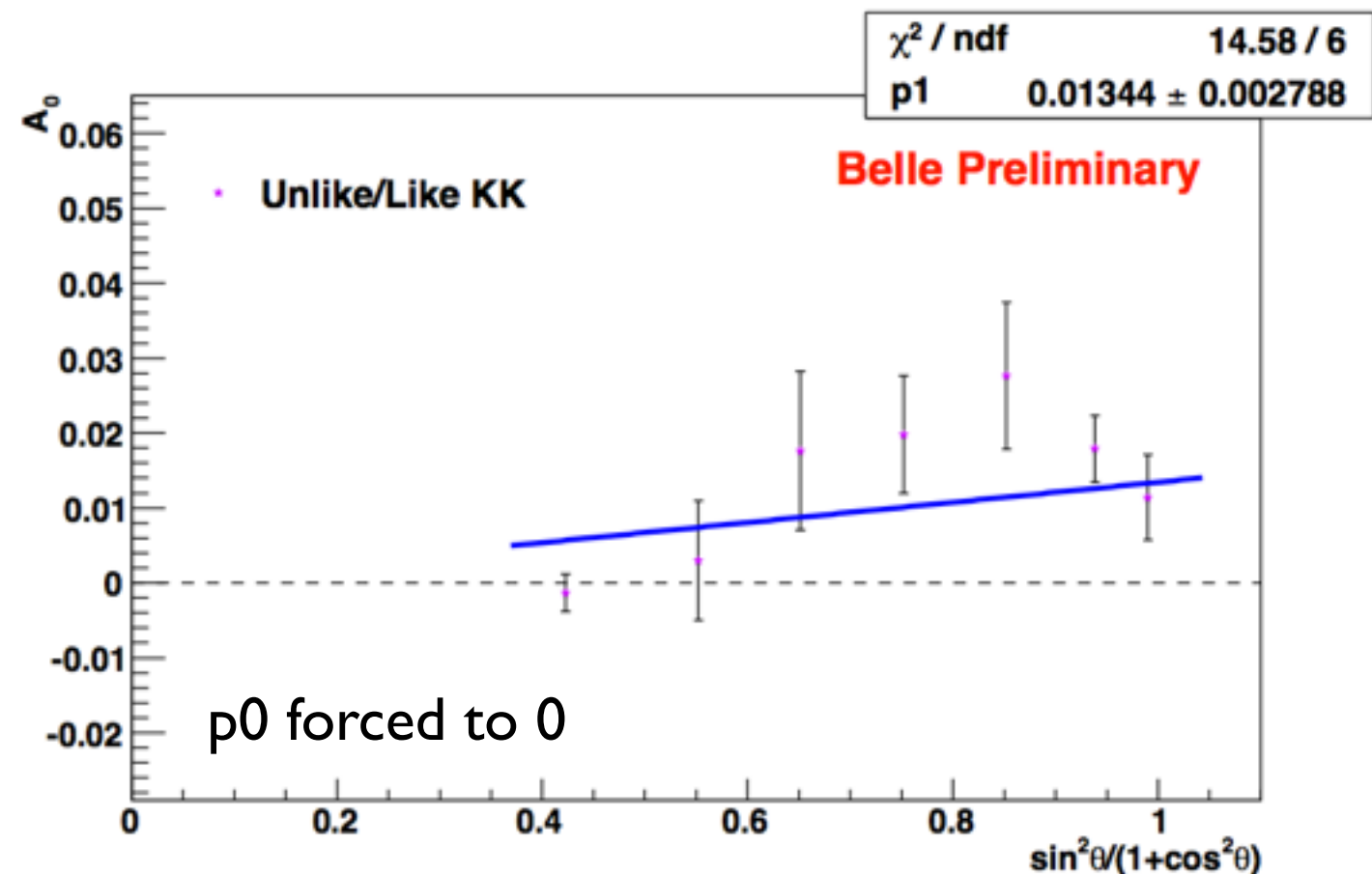
# KK versus $\sin^2\theta/(1+\cos^2\theta)$



$$A_0 = \frac{\sin^2 \theta}{1 + \cos^2 \theta} \mathcal{F} \left[ \frac{H_1^\perp(z_1) \bar{H}_1^\perp(z_2)}{D_1^\perp(z_1) \bar{D}_1^\perp(z_2)} \right]$$

linear in  $\sin^2\theta/(1+\cos^2\theta)$ ,  
go to 0 for  $\sin^2\theta/(1+\cos^2\theta) \rightarrow 0$

fit form:  $p_0 + p_1 \sin^2\theta/(1+\cos^2\theta)$



# Fragmentation contributions

$u, d \rightarrow \pi (u\bar{d}, \bar{u}d)$

$$D^{fav} = D_u^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+}$$

$$D^{dis} = D_u^{\pi^-} = D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_{\bar{d}}^{\pi^-}$$

$s \rightarrow \pi (u\bar{d}, \bar{u}d)$

$$D_{s \rightarrow \pi}^{dis} = D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-}$$

$u, d \rightarrow K (u\bar{s}, \bar{u}s)$

$$D_{u \rightarrow K}^{fav} = D_u^{K^+} = D_{\bar{u}}^{K^-}$$

$$D_{u,d \rightarrow K}^{dis} = D_u^{K^-} = D_{\bar{u}}^{K^+} = D_d^{K^+} = D_{\bar{d}}^{K^-} = D_d^{K^-} = D_{\bar{d}}^{K^+}$$

$s \rightarrow K (u\bar{s}, \bar{u}s)$

$$D_{s \rightarrow K}^{fav} = D_s^{K^-} = D_{\bar{s}}^{K^+}$$

$$D_{s \rightarrow K}^{dis} = D_s^{K^+} = D_{\bar{s}}^{K^-}$$

In the end we are left with 7 possible fragmentation functions:

$$D^{fav}, D^{dis}, D_{s \rightarrow \pi}^{dis}, D_{u \rightarrow K}^{fav}, D_{u,d \rightarrow K}^{dis}, D_{s \rightarrow K}^{fav}, D_{s \rightarrow K}^{dis}$$

Assuming charm contribute  
only as a dilution



# Fragmentation contributions

**For pion-pion couples:**

$$D \frac{U_{\pi\pi}}{L_{\pi\pi}} \propto 1 + \cos 2\phi_0 \frac{\sin^2 \theta}{1+\cos^2 \theta} \left( \frac{5H_1^{fav} H_2^{fav} + 5H_1^{dis} H_2^{dis} + 2H_{1s \rightarrow \pi}^{dis} H_{2s \rightarrow \pi}^{dis}}{5D_1^{fav} D_2^{fav} + 5D_1^{dis} D_2^{dis} + 2D_{1s \rightarrow \pi}^{dis} D_{2s \rightarrow \pi}^{dis}} - \frac{5H_1^{fav} H_2^{dis} + 5H_1^{dis} H_2^{fav} + 2H_{1s \rightarrow \pi}^{dis} H_{2s \rightarrow \pi}^{dis}}{5D_1^{fav} D_2^{dis} + 5D_1^{dis} D_2^{fav} + 2D_{1s \rightarrow \pi}^{dis} D_{2s \rightarrow \pi}^{dis}} \right)$$

**For pion-Kaon couples:**

$$D \frac{U_{\pi K}}{L_{\pi K}} \propto 1 + \cos 2\phi_0 \frac{\sin^2 \theta}{1+\cos^2 \theta} \times$$

$$\left( \frac{4H_{1K}^{fav} H_{2K}^{fav} + H_{1K}^{dis} (5H_2^{dis} + H_2^{fav}) + H_{2K}^{dis} (5H_1^{dis} + H_1^{fav}) + 4H_{1K}^{fav} H_2^{fav} + H_{1s \rightarrow \pi}^{dis} (H_{2s \rightarrow K}^{dis} + H_{2s \rightarrow K}^{fav}) + H_{2s \rightarrow \pi}^{dis} (H_{1s \rightarrow K}^{fav} + H_{1s \rightarrow K}^{dis})}{4D_{1K}^{fav} D_{2K}^{fav} + D_{1K}^{dis} (5D_2^{dis} + D_2^{fav}) + D_{2K}^{dis} (5D_1^{dis} + D_1^{fav}) + 4D_{1K}^{fav} D_2^{fav} + D_{1s \rightarrow \pi}^{dis} (D_{2s \rightarrow K}^{dis} + D_{2s \rightarrow K}^{fav}) + D_{2s \rightarrow \pi}^{dis} (D_{1s \rightarrow K}^{fav} + D_{1s \rightarrow K}^{dis})} \right.$$

$$\left. - \frac{H_{2K}^{dis} (5H_1^{fav} + H_1^{dis}) + 4H_{1K}^{fav} H_2^{dis} + 4H_1^{dis} H_{2K}^{fav} + H_{1K}^{dis} (5H_2^{fav} + H_2^{dis}) + H_{1s \rightarrow \pi}^{dis} (H_{2s \rightarrow K}^{fav} + H_{2s \rightarrow K}^{dis}) + (H_{1s \rightarrow K}^{dis} + H_{1s \rightarrow K}^{fav}) H_{2s \rightarrow \pi}^{dis}}{D_{2K}^{dis} (5D_1^{fav} + D_1^{dis}) + 4D_{1K}^{fav} D_2^{dis} + 4D_1^{dis} D_{2K}^{fav} + D_{1K}^{dis} (5D_2^{fav} + D_2^{dis}) + D_{1s \rightarrow \pi}^{dis} (D_{2s \rightarrow K}^{fav} + D_{2s \rightarrow K}^{dis}) + (D_{1s \rightarrow K}^{dis} + D_{1s \rightarrow K}^{fav}) D_{2s \rightarrow \pi}^{dis}} \right)$$

**For Kaon-Kaon couples:**

$$D \frac{U_{KK}}{L_{KK}} \propto 1 + \cos 2\phi_0 \frac{\sin^2 \theta}{1+\cos^2 \theta} \left( \frac{4H_{1K}^{fav} H_{2K}^{fav} + 6H_{1K}^{dis} H_{2K}^{dis} + H_{1s \rightarrow K}^{dis} H_{2s \rightarrow K}^{dis} + H_{1s \rightarrow K}^{fav} H_{2s \rightarrow K}^{fav}}{4D_{1K}^{fav} D_{2K}^{fav} + 6D_{1K}^{dis} D_{2K}^{dis} + D_{1s \rightarrow K}^{dis} D_{2s \rightarrow K}^{dis} + D_{1s \rightarrow K}^{fav} D_{2s \rightarrow K}^{fav}} - \frac{4H_{1K}^{fav} H_{2K}^{dis} + 4H_{1K}^{dis} H_{2K}^{fav} + 2H_{1K}^{dis} H_{2K}^{dis} + H_{1s \rightarrow K}^{dis} H_{2s \rightarrow K}^{fav} + H_{1s \rightarrow K}^{fav} H_{2s \rightarrow K}^{dis}}{4D_{1K}^{fav} D_{2K}^{dis} + 4D_{1K}^{dis} D_{2K}^{fav} + 2D_{1K}^{dis} D_{2K}^{dis} + D_{1s \rightarrow K}^{dis} D_{2s \rightarrow K}^{fav} + D_{1s \rightarrow K}^{fav} D_{2s \rightarrow K}^{dis}} \right)$$





# Fragmentation contributions

**For pion-pion couples:**

$$D \frac{U_{\pi\pi}}{L_{\pi\pi}} \propto 1 + \cos 2\phi_0 \frac{\sin^2 \theta}{1+\cos^2 \theta} \left( \frac{5H_1^{fav} H_2^{fav} + 5H_1^{dis} H_2^{dis} + 2H_{1s \rightarrow \pi}^{dis} H_{2s \rightarrow \pi}^{dis}}{5D_1^{fav} D_2^{fav} + 5D_1^{dis} D_2^{dis} + 2D_{1s \rightarrow \pi}^{dis} D_{2s \rightarrow \pi}^{dis}} - \frac{5H_1^{fav} H_2^{dis} + 5H_1^{dis} H_2^{fav} + 2H_{1s \rightarrow \pi}^{dis} H_{2s \rightarrow \pi}^{dis}}{5D_1^{fav} D_2^{dis} + 5D_1^{dis} D_2^{fav} + 2D_{1s \rightarrow \pi}^{dis} D_{2s \rightarrow \pi}^{dis}} \right)$$

**For pion-Kaon couples:**

$$D \frac{U_{\pi K}}{L_{\pi K}} \propto 1 + \cos 2\phi_0 \frac{\sin^2 \theta}{1+\cos^2 \theta} \times$$

$$\left( \frac{4H_{1K}^{fav} H_{2K}^{fav} + H_{1K}^{dis} (5H_2^{dis} + H_2^{fav}) + H_{2K}^{dis} (5H_1^{dis} + H_1^{fav}) + 4H_{1K}^{fav} H_2^{fav} + H_{1s \rightarrow \pi}^{dis} (H_{2s \rightarrow K}^{dis} + H_{2s \rightarrow K}^{fav}) + H_{2s \rightarrow \pi}^{dis} (H_{1s \rightarrow K}^{fav} + H_{1s \rightarrow K}^{dis})}{4D_{1K}^{fav} D_{2K}^{fav} + D_{1K}^{dis} (5D_2^{dis} + D_2^{fav}) + D_{2K}^{dis} (5D_1^{dis} + D_1^{fav}) + 4D_{1K}^{fav} D_2^{fav} + D_{1s \rightarrow \pi}^{dis} (D_{2s \rightarrow K}^{dis} + D_{2s \rightarrow K}^{fav}) + D_{2s \rightarrow \pi}^{dis} (D_{1s \rightarrow K}^{fav} + D_{1s \rightarrow K}^{dis})} \right.$$

$$\left. \frac{H_{2K}^{dis} (5H_1^{fav} + H_1^{dis}) + 4H_{1K}^{fav} H_2^{dis} + 4H_1^{dis} H_{2K}^{fav} + H_{1K}^{dis} (5H_2^{fav} + H_2^{dis}) + H_{1s \rightarrow \pi}^{dis} (H_{2s \rightarrow K}^{fav} + H_{2s \rightarrow K}^{dis}) + (H_{1s \rightarrow K}^{dis} + H_{1s \rightarrow K}^{fav}) H_{2s \rightarrow \pi}^{dis}}{D_{2K}^{dis} (5D_1^{fav} + D_1^{dis}) + 4D_{1K}^{fav} D_2^{dis} + 4D_1^{dis} D_{2K}^{fav} + D_{1K}^{dis} (5D_2^{fav} + D_2^{dis}) + D_{1s \rightarrow \pi}^{dis} (D_{2s \rightarrow K}^{fav} + D_{2s \rightarrow K}^{dis}) + (D_{1s \rightarrow K}^{dis} + D_{1s \rightarrow K}^{fav}) D_{2s \rightarrow \pi}^{dis}} \right)$$

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**Not so easy! A full phenomenological study needed!**



# Summary & outlook

- $\phi_0$  asymmetries
  - present similar features for  $\pi\pi$  and KK couples
  - very small/compatible with zero for  $\pi K$  couples
  - for  $\pi\pi$  and  $\pi K$  the  $\sin^2\theta/(1+\cos^2\theta)$  dependence of asymmetries are not inconsistent with a linear dependence going to zero
  - KK show a more convoluted  $\sin^2\theta/(1+\cos^2\theta)$  dependence





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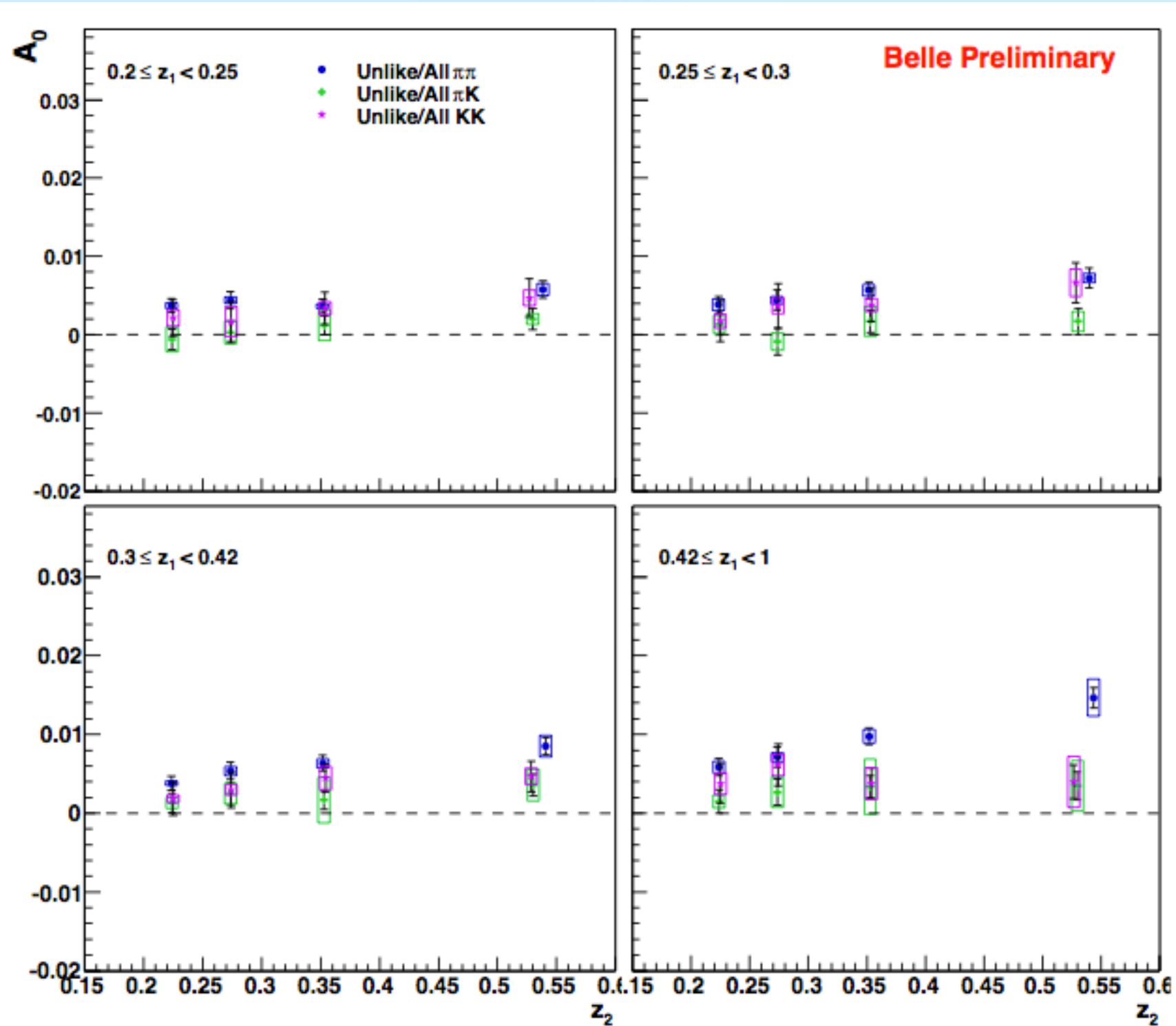
**Stay tuned!**



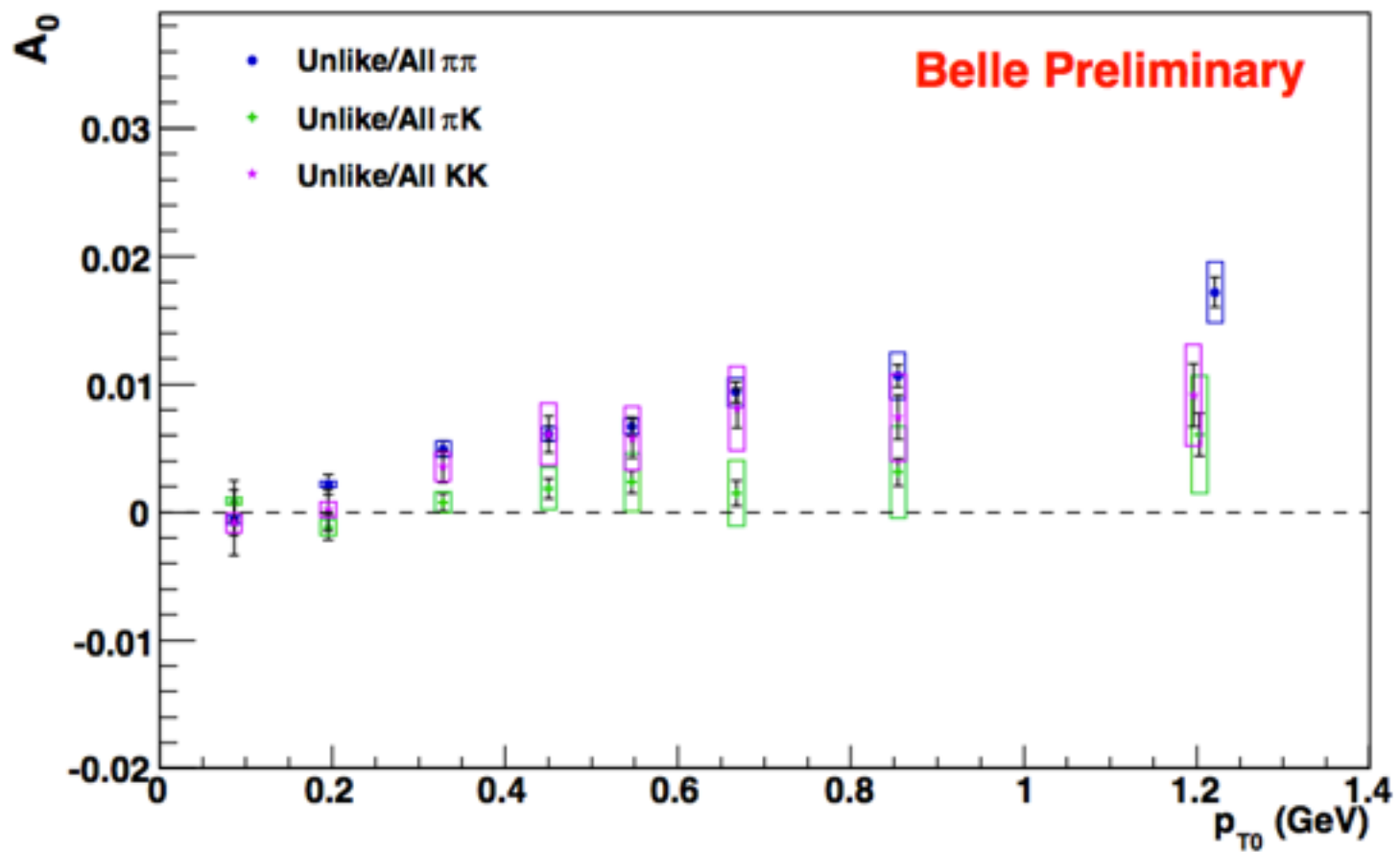
# Backups



# $\phi_0$ asymmetries



# $\phi_0$ asymmetries



# More $\phi_0$ asymmetries

