Quark Orbital Angular Momentum

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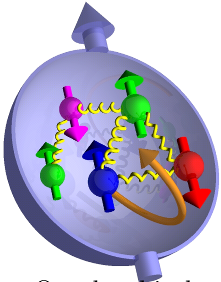
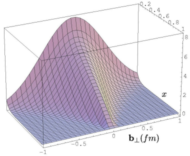
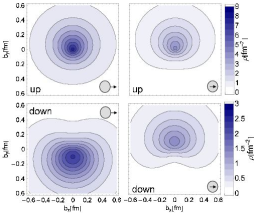
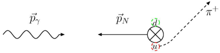
Outline

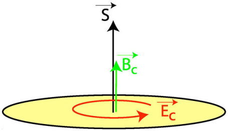


- Quark orbital angular momentum
- angular momentum decompositions (Jaffe v. Ji)
- quark-gluon correlations
- Summary

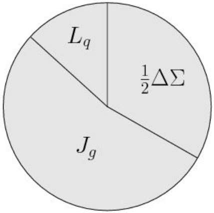








Ji decomposition



'pizza tre stagioni'

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

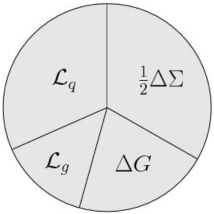
$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S|\, q^\dagger(\vec{x})\Sigma^3 q(\vec{x})\, |P,S\rangle$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S|\, \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe-Manohar decomposition



'pizza quattro stagioni'

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S|\, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) |P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \, \langle P,S|\, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S| \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j |P,S\rangle$$

manifestly gauge invariant definition for each term exists ($\rightarrow$ Hatta)

QED with electrons

$$\begin{aligned}\vec{J}_\gamma &= \int d^3r\, \vec{r}\times\left(\vec{E}\times\vec{B}\right) = \int d^3r\, \vec{r}\times\left[\vec{E}\times\left(\vec{\nabla}\times\vec{A}\right)\right] \\ &= \int d^3r \left[E^j\left(\vec{r}\times\vec{\nabla}\right)A^j - \vec{r}\times(\vec{E}\cdot\vec{\nabla})\vec{A}\right] \\ &= \int d^3r \left[E^j\left(\vec{r}\times\vec{\nabla}\right)A^j + \left(\vec{r}\times\vec{A}\right)\vec{\nabla}\cdot\vec{E} + \vec{E}\times\vec{A}\right]\end{aligned}$$

- replace 2^{nd} term (eq. of motion $\vec{\nabla}\cdot\vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger\vec{r}\times e\vec{A}\psi + E^j\left(\vec{r}\times\vec{\nabla}\right)A^j + \vec{E}\times\vec{A}\right]$$

- $\psi^\dagger\vec{r}\times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger\vec{r}\times(\vec{p}-e\vec{A})\psi$

$\hookrightarrow$ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

- can also be done for only part of $\vec{A}\to$ Chen/Goldman, Wakamatsu

- L_q matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q = \bar{q}\gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q}\gamma^+ \left[\vec{r} \times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- (for $\vec{p} = 0$) matrix element of $\bar{q}\gamma^z \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$ vanishes (parity!)

↪ L_q identical to matrix element of $\bar{q}\gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$ (nucleon at rest)

↪ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger \left(\vec{r} \times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger \left(r^x g A^y - r^y g A^x\right) q\Big|_{A^+=0}$

- $L_x = yp_z - zp_y$

$\hookrightarrow$ $J_x = \int d^3r \left[yT^{0z} - zT^{0y} \right]$

- if state invariant under rotations about $\hat{x}$ axis then $\langle yp_z \rangle = -\langle zp_y \rangle$

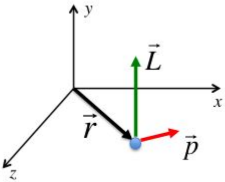
$\hookrightarrow$ $\langle L_x \rangle = 2\langle yp_z \rangle \rightarrow J_z = 2\int d^3r\, yT^{0z}$

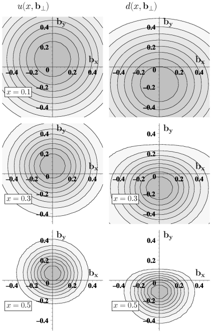
- GPDs provide simultaneous information about p_z & $\mathbf{b}_\perp$

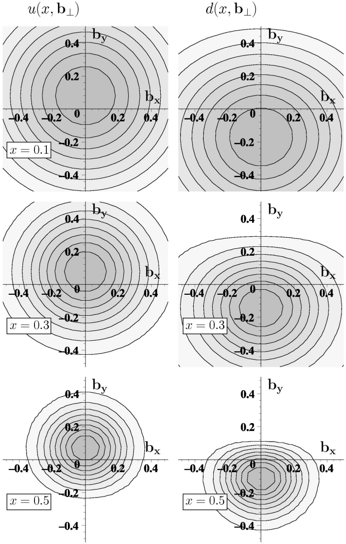
$\hookrightarrow$ use quark GPDs to determine angular momentum carried by quarks

$\hookrightarrow$ $J_q^x = \frac{1}{2}\int dx\, x\left[H(x,0,0) + E(x,0,0)\right]$ (X.Ji, 1996)

- partonic interpretation in terms of 3D distribution (MB,2001,2005)







proton polarized in $+\hat{x}$ direction

no axial symmetry!

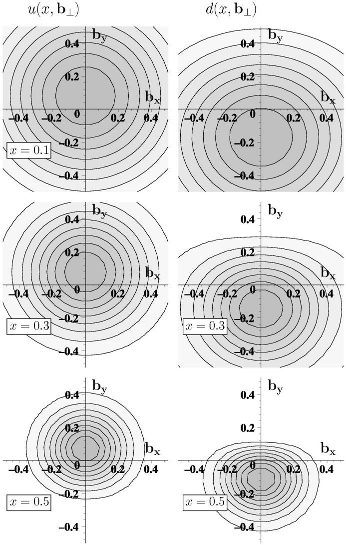
$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$
$$-\frac{1}{2M}\frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

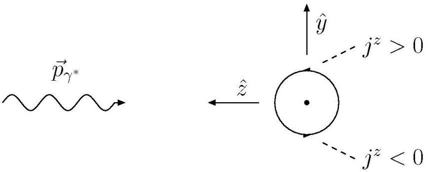
Physics: relevant density in DIS is $j^+ \equiv j^0 + j^z$ and left-right asymmetry from j^z

intuitive explanation

- moving Dirac particle with anomalous magnetic moment has electric dipole moment $\perp$ to $\vec{p}$ and $\perp$ magnetic moment

$\hookrightarrow$ γ^* 'sees' flavor dipole moment of oncoming nucleon





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Physics: relevant density in DIS is $j^+ \equiv j^0 + j^z$ and left-right asymmetry from j^z

Total (Spin+Orbital) Quark Angular Momentum

$$J_q^x = L_q^x + S_q^x = \int d^3r \left[y T_q^{0z}(\vec{r}) - z T_q^{0y}(\vec{r}) \right]$$

- $T_q^{\mu\nu}(\vec{r})$ energy momentum tensor $(T_q^{\mu\nu}(\vec{r}) = T_q^{\nu\mu}(\vec{r}))$
- $T_q^{0i}(\vec{r})$ momentum density $[P_q^i = \int d^3r T_q^{0i}(\vec{r})\ ]$
- think: $(\vec{r} \times \vec{p})^x = yp^z - zp^y$

relate to impact parameter dependent quark distributions $q(x, \mathbf{r}_\perp)$:

Consider spherically symmetric wave packet with nucleon polarized in $+\hat{x}$ direction

- eigenstate under rotations about x-axis

$\hookrightarrow$ both terms in J_q^x equal:
$J_q^x = 2\int d^3r\, y T_q^{0z}(\vec{r}) = \int d^3r\, y \left[T_q^{0z}(\vec{r}) + T_q^{z0}(\vec{r}) \right]$

- $\int d^3r\, y T_q^{00}(\vec{r}) = 0 = \int d^3r\, y T_q^{zz}(\vec{r})$

$$\Rightarrow \quad J_q^x = \int d^3r\, y T_q^{++}(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

relate to impact parameter dependent quark distributions $q(x, \mathbf{r}_\perp)$:

Consider spherically symmetric wave packet with nucleon polarized in $+\hat{x}$ direction

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$$\Rightarrow \quad J_q^x = \int d^3r\, yT_q^{++}(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

- $\int dx\, xq(x, \mathbf{r}_\perp) = \frac{1}{2m_N}\int dz T^{++}(\vec{r})$
(note: here x is momentum fraction and not r^x)

$\hookrightarrow$ $J_q^x = m_N \int dx\, xr^y q(x, \mathbf{r}_\perp)$

- before applying this result to $\perp$ shifted PDFs, need to consider 'overall $\perp$ shift' of CoM for $\perp$ polarized target...

relate to impact parameter dependent quark distributions $q(x,\mathbf{b}_\perp)$:

- Thus $J_q^x = m_N \int dx\, x r^y q(x,\mathbf{r}_\perp)$ with $b^y = r^y - \frac{1}{2m_N}$, where $q(x,\mathbf{r}_\perp)$ is distribution relative to CoM of whole nucleon
- recall: $q(x,\mathbf{b}_\perp)$ for nucleon polarized in $+\hat{x}$ direction

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} - \frac{1}{2M_N}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

$$\Rightarrow J_q^x = M_N \int dx\, x r^y q(x,\mathbf{r}_\perp) = \int dx\, x \left(m_N b^y + \frac{1}{2}\right) q(x,\mathbf{r}_\perp) = \frac{1}{2}\int dx\, x\left[H(x,0,0) + E(x,0,0)\right]$$

relate to impact parameter dependent quark distributions $q(x,\mathbf{b}_\perp)$:

- Thus $J_q^x = m_N \int dx\, x r^y q(x,\mathbf{r}_\perp)$ with $b^y = r^y - \frac{1}{2m_N}$, where $q(x,\mathbf{r}_\perp)$ is distribution relative to CoM of whole nucleon
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$$
\begin{aligned}
q(x,\mathbf{b}_\perp) \quad &= \quad \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} \\
&\quad - \frac{1}{2M_N}\frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow J_q^x \quad &= \quad M_N \int dx\, x r^y q(x,\mathbf{r}_\perp) = \int dx\, x \left(m_N b^y + \frac{1}{2} \right) q(x,\mathbf{r}_\perp) \\
&= \quad \frac{1}{2} \int dx\, x \left[H(x,0,0) + E(x,0,0) \right]
\end{aligned}
$$

- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- partonic interpretation exists only for $\perp$ components!

crucial ingredients used:

- rotational invariance (around $\perp$ axis) to relate $\langle T^{0z}y\rangle$ to $\langle T^{0y}z\rangle$
- rotational invariance (around $\perp$ axis) to relate $\langle T^{++}z\rangle$ to $\langle T^{0z}y\rangle$
- parity to argue that $\langle \bar{q}\gamma^z \left[\vec{r}\times\vec{\partial}\right]^z q\rangle = 0$

Note: these symmetries are implied when writing down tensors!

consequences

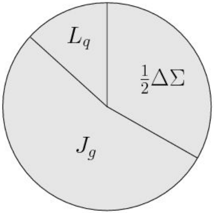
- $\langle \bar{q}\gamma^z \left[\vec{r}\times\vec{\partial}\right]^0 q\rangle = 0 = \langle \bar{q}\gamma^+ \left[\vec{r}\times\vec{\partial}\right]^z q\rangle = 0$ (irrelevant whether quantized at equal time or on light front
- $L_q = \mathcal{L}_q$ in absence of $\vec{A}$
- none of the above valid in light-front constituent models

E. Leader's talk:

Model	LFCQM			LFχQSM		
q	u	d	Total	u	d	Total
$\ell^q_{\text{kin},z}$	0.071	0.055	0.126	−0.008	0.077	0.069
$\ell^q_{\text{can},z}$	0.131	−0.005	0.126	0.073	−0.004	0.069
$\mathcal{L}^q_{\text{can},z}$	0.169	−0.042	0.126	0.093	−0.023	0.069

$\sum_q l_q = \sum_q L_q = \frac{1}{2} - \frac{1}{2}\sum_q \Delta_q$ due to conservation of L_z in models

Ji decomposition



'pizza tre stagioni'

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

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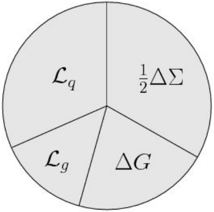
$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe-Manohar decomposition



'pizza quattro stagioni'

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S| \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) |P,S\rangle$$

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manifestly gauge invariant definition for each term exists ($\rightarrow$ Hatta)

scalar diquark model

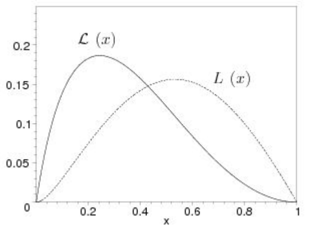
- 'mother functions' $\psi_s^S(x, \mathbf{k}_\perp)$

$\hookrightarrow$ $\mathcal{L}_q$ from $|\psi_s^S(x, \mathbf{k}_\perp)|^2$

- GPDs from overlap integrals of $\psi^\dagger\psi$

$\hookrightarrow$ L_q from Ji

- $L_q = \mathcal{L}_q$.
  No surprise since $L_q - \mathcal{L}_q \sim \langle q^\dagger \vec{r} \times \vec{A} q \rangle$ and no $\vec{A}$ in scalar diquark model
- $L_q(x) \neq \mathcal{L}_q(x)$



M.B. + Hikmat BC,
PRD **79**, 071501 (2009)

QED for dressed e^- in QED

- 'mother functions' $\psi_{sh}^S(x, \mathbf{k}_\perp)$

$\hookrightarrow$ $\mathcal{L}_q$ from $|\psi_{sh}^S(x, \mathbf{k}_\perp)|^2$

- GPDs from overlap integrals of $\psi^\dagger\psi$

$\hookrightarrow$ L_q from Ji

- $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$

Ji decomposition

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Jaffe-Manohar decomposition

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

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manifestly gauge invariant definitions for each term exist ($\rightarrow$ Hatta)

- GPDs $\longrightarrow L^q$
- $\vec{p}\,\overleftarrow{p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i \in q,g} \mathcal{L}^i$
- QED: $\mathcal{L}^e \neq L^e$ [M.B. + Hikmat BC, PRD **79**, 071501 (2009)]
- $\mathcal{L}^q - L^q = ?$
  - can we calculate/predict/measure the difference?
  - what does it represent?

Wigner Functions (Belitsky, Ji, Yuan; Metz et al.)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ q(\xi)|PS\rangle .$$

- (quasi) probabilty distribution for $\mathbf{b}_\perp$ and $\mathbf{k}_\perp$
- $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- $q(x,\mathbf{b}_\perp) = \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$

OAM from Wigner (Lorcé, Pasquini, ...)

$$L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$$

$$= \int d^3r \langle P,S|\,\bar{q}(\vec{r})\gamma^+ \left(\vec{r}\times i\vec{\partial}\right)^z q(\vec{r})|P,S\rangle = \mathcal{L}^q$$

Gauge Invariance?

need to include Wilson-line gauge link to connect 0 and ξ (Ji, Yuan; Hatta; Lorcé;...)

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+\mathcal{U}_{0\xi}q(\xi)|PS\rangle.$$

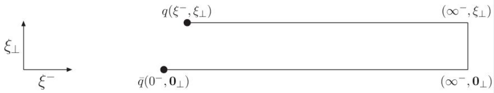
$\langle\vec{k}_\perp\rangle \equiv \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)\vec{k}_\perp$ depends on choice of path!

straight-line gauge link

$$\langle\vec{k}_\perp\rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{D}q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$
- $\langle\vec{k}_\perp\rangle = 0$ (T-odd !)

light-cone staple



- correct choice for $\mathbf{k}_\perp$ distributions relevant for SIDIS

$$\langle\vec{\mathcal{K}}_\perp\rangle = \langle P,S|\bar{q}(\vec{x})\gamma^+ i\vec{\mathcal{D}}q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty,\mathbf{x}_\perp)$ $\quad A^+ = 0$
- $\langle\vec{\mathcal{K}}_\perp\rangle \neq 0$ (FSI! Brodsky,Hwang,Schmidt)

Quark $\perp$ momentum from Wigner Distributions

difference $\langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle$

$$\langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle = -g \int d^3x \langle P,S | \bar{q}(\vec{x}) \gamma^+ \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x}) | P,S \rangle$$

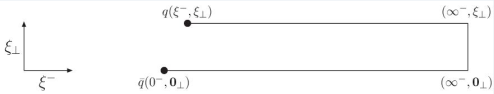
$$\sqrt{2} F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

straight-line gauge link

$$\langle \vec{k}_\perp \rangle = \langle P,S | \bar{q}(\vec{x}) \gamma^+ i\vec{D} q(\vec{x}) | P,S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$
- $\langle \vec{k}_\perp \rangle = 0$ (T-odd !)

light-cone staple



- correct choice for $\mathbf{k}_\perp$ distributions relevant for SIDIS

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- $i\mathcal{D}^j = i\partial^j - gA^j(x^-, \mathbf{x}_\perp) - g \int_{x^-}^{\infty} dr^- F^{+j}$

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color Lorentz Force acting on ejected quark (Qiu, Sterman)

$$\sqrt{2} F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B} \right)^y \text{ for } \vec{v} = (0,0,-1)$$

Impulse due to FSI

$\Delta \vec{k}^q_\perp \equiv \langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle$
$=$ (average) change in $\perp$ momentum due to FSI!

straight-line gauge link

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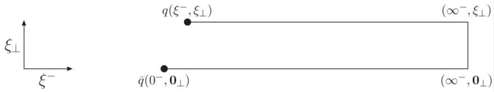

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Corollary: $d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS: MB, PRD 88 (2013) 114502

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ g F^{+y}(0) q(0) \right| P,S \right\rangle$$

matrix element defining d_2 $\leftrightarrow$ 1^{st} integration point in QS-integral

light cone staple

Quark $\perp$ momentum from Wigner Distributions

difference $\langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle$

$$\langle \vec{\mathcal{K}}^q_\perp \rangle - \langle \vec{k}^q_\perp \rangle = -g \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+ \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x})|P,S\rangle$$

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sign of d_2^q opposite Sivers $f_{1T}^{\perp q}$ $\leftrightarrow$ $\perp$ deformation of quark distributions

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+\mathcal{U}_{0\xi}q(\xi)|PS\rangle.$$

W and thus $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$ may depend on choice of path!

straight line (Ji et al.)

straigth Wilson line from 0 to ξ yields

$$L^q = \int d^3x \langle P,S|q^\dagger(\vec{x})\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$
- same as Ji-OAM
- $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$ not the TMDs relevant for SIDIS (missing FSI!)

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ \mathcal{U}_{0\xi} q(\xi)|PS\rangle.$$

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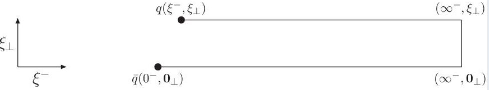
Light-Cone Staple for $\mathcal{U}_{0\xi}^{\pm LC}$ (Hatta)

- want Wigner function that yields TMDs relevant for SIDIS when integrated $d^2\mathbf{b}_\perp$

$\hookrightarrow$ path for gauge link $\longrightarrow$ 'light-cone staple' $\longrightarrow \mathcal{U}_{0\xi}^{+LC}$



$$\mathcal{L}^q_+ = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(\infty,\mathbf{x}_\perp) \qquad (A^+ = 0)$$

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ \mathcal{U}_{0\xi} q(\xi)|PS\rangle.$$

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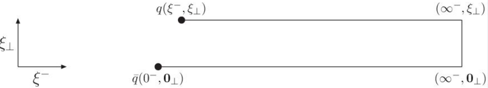

$$\mathcal{L}_+^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(\infty,\mathbf{x}_\perp) + g\int_{x^-}^\infty dr^- \vec{\partial}A^+$$

$$i\mathcal{D}^j = i\partial^j - gA^j(x^-,\mathbf{x}_\perp) - g\int_{x^-}^\infty dr^- F^{+j}$$

straight line ($\rightarrow$Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple ($\rightarrow$ Jaffe-Manohar)

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

Quark OAM from Wigner Distributions

straight line ($\to$Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

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color Lorentz Force acting on ejected quark (MB: arXiv:08103589)

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Torque along the trajectory of q

$$T^z = \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

Change in OAM

$$\Delta L^z = \int_{x^-}^{\infty} dr^- \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

straight line ($\to$Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$

light-cone staple ($\to$ Jaffe-Manohar)

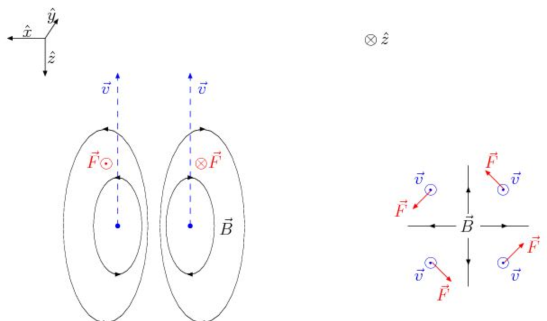
$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

difference $\mathcal{L}^q - L^q$ ($\to$ Wakamatsu: L^q_{pot})

$\mathcal{L}^q - L^q = \Delta L^q_{FSI}$ = change in OAM as quark leaves nucleon

example: torque in magnetic dipole field



antisymm. boundary condition

- $A^+ = 0$
- fix residual gauge inv. $A^\mu \to A^\mu + \partial^\mu \phi(\vec{x}_\perp)$ by imposing $\vec{A}_\perp(\infty, \vec{x}_\perp) = -\vec{A}_\perp(-\infty, \vec{x}_\perp)$
- $\vec{A}_\perp(\infty, \vec{x}_\perp) - \vec{A}_\perp(-\infty, \vec{x}_\perp) = \int dx^- F^{+\perp}$ gauge inv.
- $\mathcal{L}_+$ involves $i\vec{\mathcal{D}}_+ = i\vec{\partial} - g\vec{A}(\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_-$ involves $i\vec{\mathcal{D}}_- = i\vec{\partial} - g\vec{A}(-\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_+ = \mathcal{L}_- \to$ no contribution from $\vec{A}(\infty, \mathbf{x}_\perp)$

$\hookrightarrow$ 'naive' JM OAM $\mathcal{L}_{JM} = \mathcal{L}_+ = \mathcal{L}_-$

PT (Hatta)

- PT$\longrightarrow \mathcal{L}_+ = \mathcal{L}_-$

(different from SSAs due to factor $\vec{x}$ in OAM)

alternative: Bashinsky-Jaffe

- $A^+ = 0$
- $\vec{x} \times i\vec{\partial} \quad \to \quad \vec{x} \times \left[i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp) \right]$
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$\hookrightarrow \mathcal{L}_{JB} = \frac{1}{2}\left(\mathcal{L}_+ + \mathcal{L}_-\right) = \mathcal{L}_+ = \mathcal{L}_-$

The Difference $\mathcal{L}_q - L_q$ [MB, PRD88, 056009 (2013)]

$$\mathcal{L}^q - L^q = -\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- gF^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{0})|P,S\rangle /\mathcal{N}$$

- in QCD, additional Wilson lines (along r^-)

compare $\langle \mathbf{k}_\perp\rangle \equiv \int dx \int d^2\mathbf{k}_\perp f(x,\mathbf{k}_\perp)\mathbf{k}_\perp$

$$f(x,\vec{k}_\perp) \equiv \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi}\langle P'S'|\bar{q}(0)\gamma^+\mathcal{U}_{0\xi}q(\xi)|PS\rangle$$

- Wilson line $\mathcal{U}_{0\xi}$ along x^- to properly account for FSI acting on ejected quark, i.e. $f(x,\mathbf{k}_\perp)$ momentum distribution incl. FSI
- relevant for SIDIS (JLab, EIC) and DY (RHIC)

$$\langle \mathbf{k}_\perp\rangle = \langle P,S|\bar{q}(0)\gamma^+\int_0^{\infty} dr^- gF^{+\perp}(r^-,\mathbf{0}_\perp)q(\vec{0})|P,S\rangle$$

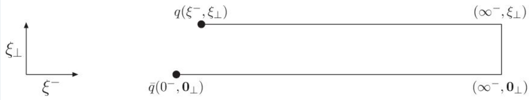
$\langle \mathbf{k}_\perp\rangle$ = (average) change in $\perp$ momentum due to FSI as quark leaves target (Qiu, Sterman)

Color Lorentz Force

$$\begin{aligned}\sqrt{2}F^{+y} &= F^{0y}+F^{zy}\\ &= -E^y+B^x\\ &= -\left(\vec{E}+\vec{v}\times\vec{B}\right)^y\end{aligned}$$

for $\vec{v}=(0,0,-1)$

Light-Cone Staple for $\mathcal{U}_{0\xi}^{\pm LC}$



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$$\mathcal{L}^q - L^q = -\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- gF^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{0})|P,S\rangle /\mathcal{N}$$

$\mathcal{L}_q - L_q =$ (average) change in OAM due to FSI as quark leaves target

compare $\langle \mathbf{k}_\perp \rangle \equiv \int dx \int d^2\mathbf{k}_\perp f(x,\mathbf{k}_\perp)\mathbf{k}_\perp$

$$f(x,\vec{k}_\perp) \equiv \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi}\langle P'S'|\bar{q}(0)\gamma^+\mathcal{U}_{0\xi}q(\xi)|PS\rangle$$

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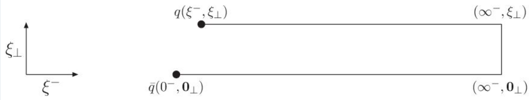
Light-Cone Staple for $\mathcal{U}_{0\xi}^{\pm LC}$



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for $\vec{v} = (0,0,-1)$

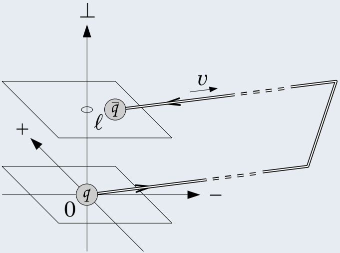
M.Burkardt+A.Miller+W.-D.Nowak, Rept.Prog.Phys. 73 (2010) 016201

In decomposition (103), each term has a partonic interpretation. The gluon spin contribution ΔG appears explicitly. It is experimentally accessible (see section 2.8) and can be defined as the expectation value of a (nonlocal) manifestly gauge invariant operator. In light-cone gauge, this operator collapses to a local operator (and its expectation value has a partonic interpretation). No direct experimental access to the parton orbital angular momentum $\mathcal{L}$ has been identified. Its value can be obtained only by subtracting the quark and gluon spin contributions from the nucleon spin. Both ΔG and $\mathcal{L}$ can be defined through matrix elements of local operators only in light-cone gauge $A^+ = 0$. Explicit definitions for the operators appearing in both decompositions can be found in [242]. Since neither one can be represented as the matrix element of a manifestly gauge invariant local operator, they cannot be analytically continued to Euclidean space and are thus inaccessible for lattice QCD.

Jaffe-Manohar Decomposition

$$\frac{1}{2} = J_z = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}. \tag{103}$$

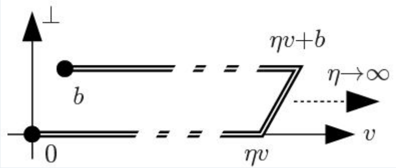
challenge



- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

TMDs in lattice QCD

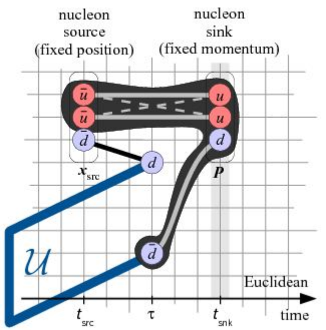
B. Musch, P. Hägler, M. Engelhardt

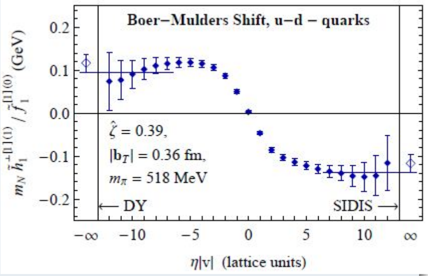


- calculate space-like staple-shaped Wilson line pointing in $\hat{z}$ direction; length $L \to \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios

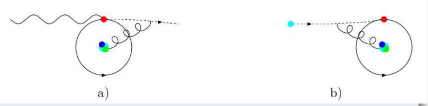
$\hookrightarrow$ extrapolate/evolve to $P_z \to \infty$

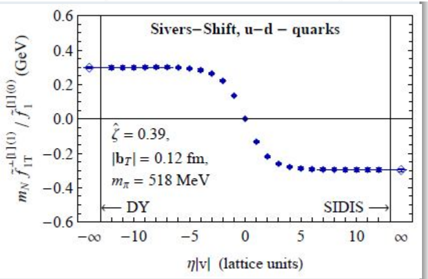
Quasi Light-Like Wilson Lines from Lattice QCD



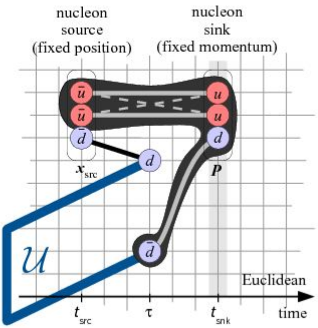


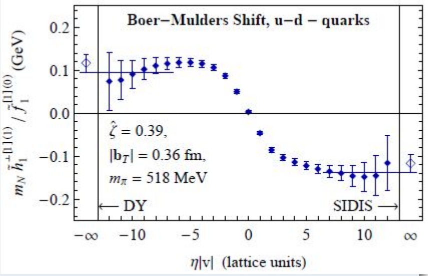
$f_{1T,SIDIS}^{\perp} = -f_{1T,DY}^{\perp}$ (Collins)





$f_{1T}^{\perp}(x, \mathbf{k}_\perp)$ is $\mathbf{k}_\perp$-odd term in quark-spin averaged momentum distribution in $\perp$ polarized target





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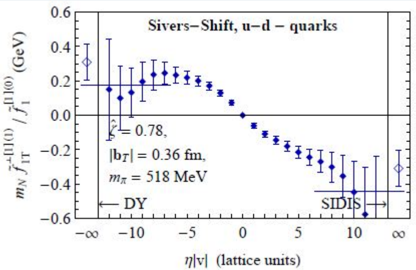

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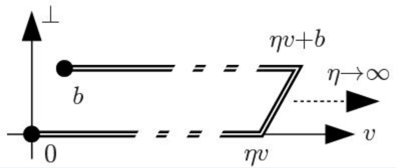

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next: Orbital Angular Momentum

- same operator as for TMDs, only nonforward matrix elements:
  - momentum transfer provides position space information ($\to \mathbf{r}_\perp \times \mathbf{k}_\perp$)
  - staple with long side in $\hat{z}$ direction
  - (large) nucleon momentum in $\hat{z}$ direction
  - small momentum transfer in $\hat{y}$ direction

$\hookrightarrow$ generalized TMD F_{14} (Metz et al.)

- quark OAM
- renormalization same as $f_{1T}^{\perp}$

$\hookrightarrow$ study ratios...

Summary

- GPDs and OAM
- TMDs and OAM from Wigner distributions
- straight-(Wilson)line gauge link $\longrightarrow L^q$ ('Ji-OAM')
- light-cone staple- gauge link $\longrightarrow \mathcal{L}^q_+$ ('JM-OAM')
- $\mathcal{L}^q_+ - L^q$ = change in OAM as quark leaves nucleon (torque due to FSI)
- $A^+ = 0$ gauge (with anti-symmetric boundary condition) $\mathcal{L}^q_+ \to$ canonical OAM (Jaffe-Manohar-OAM)
- JM-OAM from lattice QCD