Scale Evolution of Gluon TMDPDFs

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- Gluon TMDPDFs for the Higgs transverse-momentum distribution
MGE, Tomas Kasemets, Piet Mulders, Cristian Pisano [arXiv: 1407.XXXX]

Introduction/Outline

• **Goal**: derive the evolution of gluon TMDPDFs

• For that we need to properly define individual TMDs in the context of a factorization theorem!!

• We take the Higgs transverse momentum distribution as a benchmark where gluon TMDPDFs are relevant.

• **Steps:**

1.- Factorize the Higgs transverse momentum distribution

2.- Define gluon TMDPDFs by combining soft and collinear matrix elements.

   * Motivation behind? Cancellation of Rapidity Divergences!!

3.- Unpolarized gluon TMDPDF at NLO (free from RDs)

4.- For an unpolarized proton: OPE coefficients for unpolarized and Boer-Mulders TMDPDFs.

5.- Derive the evolution kernel for all (un-)polarized gluon TMDPDFs

6.- Discuss non-perturbative inputs for TMDs

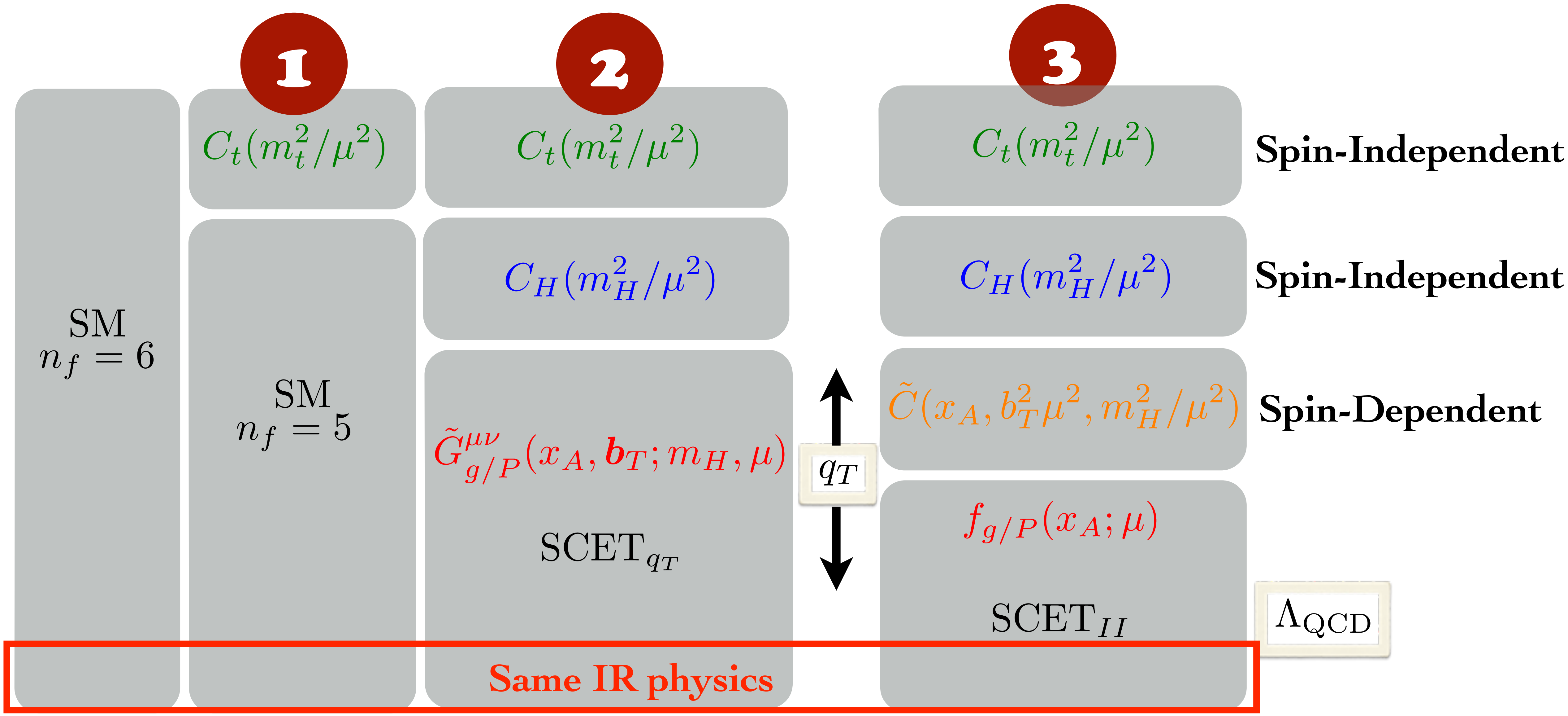
7.- Show a preliminary application of their evolution.

Factorization: Overview

- I will show the derivation of the factorization theorem for

$$h_A(P, S_A) + h_B(\bar{P}, S_B) \to H(q_T) + X$$

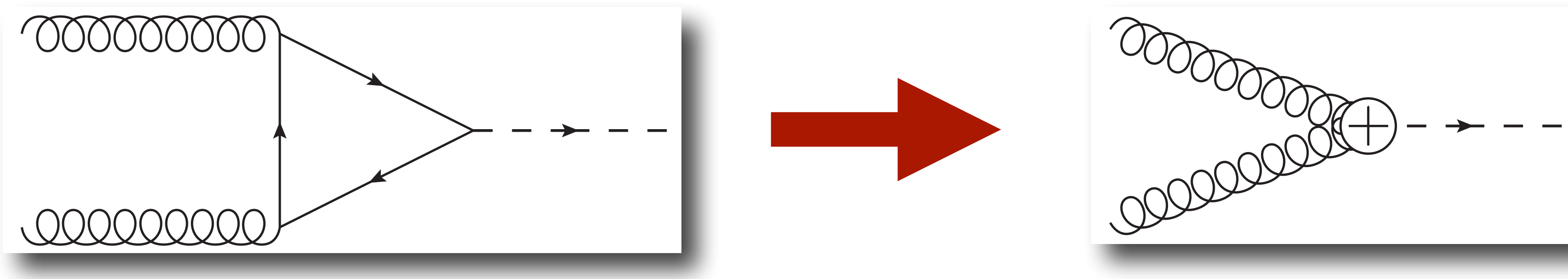
- Problem with different relevant scales:



Factorization Theorem = Multistep Matching Procedure

1 Factorization of the Top Quark

- The glue-glue fusion process is well approximated by an effective lagrangian:



Top quark dominates!!

$$\mathcal{L}_{\rm eff} = \frac{\alpha_s(\mu^2)}{12\pi} C_t(m_t^2,\mu^2)\, \frac{H}{v} F^{\mu\nu,a} F^a_{\mu\nu}$$

- The coefficient (and thus its anomalous dimension) is known up to 3-loops:

$$\frac{d\ln C_t(m_t^2,\mu^2)}{d\ln\mu} = \gamma^t\left(\alpha_s(\mu)\right)$$

$$\gamma^t\left(\alpha_s(\mu)\right) = \alpha_s^2 \frac{d}{d\alpha_s}\frac{\beta(\alpha_s)}{\alpha_s^2}$$

Not surprising, since the effective lagrangian has pure Yang-Mills piece…

- The cross-section can then be written as:

$$d\sigma = \frac{1}{2s}\left(\frac{\alpha_s(\mu)}{12\pi v}\right)^2 C_t^2(m_t^2,\mu^2)\frac{d^3q}{(2\pi)^2 2E_q}\int d^4y e^{-iq\cdot y}$$
$$\times \sum_X \langle PS_A, \bar{P}S_B|\, F^a_{\mu\nu}F^{\mu\nu,a}(y)\,|X\rangle\,\langle X|\, F^b_{\alpha\beta}F^{\alpha\beta,b}(0)\,|PS_A,\bar{P}S_B\rangle$$

Hard Part & Relevant Modes

- Now we integrate out the Higgs mass:

$$F^{\mu\nu,a}\,F^{a}_{\mu\nu} \longrightarrow -2q^2 C_H(-q^2,\mu^2)\, g^{\perp}_{\mu\nu} B^{\mu,a}_{n\perp}\left(\mathcal{S}^{\dagger}_{n}\mathcal{S}_{\bar n}\right)^{ab} B^{\nu,b}_{\bar n\perp}$$

Known at 3-loops!! Its anomalous dimension as well

$$B^{\mu}_{n\perp} = \frac{1}{g}[\bar n\cdot\mathcal{P}W^{\dagger}_{n} iD^{\perp\mu}_{n} W_n]$$

$$W_n(x) = \bar P \exp\left[\int_{-\infty}^{0} ds\,\bar n\cdot A^a_n(x+\bar n s)t^a\right]$$

$$S_n(x) = P \exp\left[\int_{-\infty}^{0} ds\, n\cdot A^a_s(x+ n s)t^a\right]$$

Adjoint representation: $(t^a)^{bc} = -if^{abc}$

- And the cross section is given by:

$$\sigma_0(\mu) = \frac{m_H^2\,\alpha_s^2(\mu)}{72\pi(N_c^2-1)sv^2}$$

$$d\sigma = \sigma_0(\mu)\, C_t^2(m_t^2,\mu)\, H(m_H,\mu)\, \frac{m_H^2}{\tau s}\, dy\, \frac{d^2q_\perp}{(2\pi)^2}\int d^2y_\perp\, e^{-iq_\perp\cdot y_\perp}$$
$$\times\, 2J_n^{\mu\nu}(x_A,y_\perp,\mu)\, J_{\bar n\,\mu\nu}(x_B,y_\perp,\mu)\, S(y_\perp,\mu)$$

$$H = |C_H|^2$$

- Collinear and soft matrix elements are defined by:

$$J_n^{\mu\nu}(x_A,y_\perp,\mu) = -\frac{x_A P^+}{2}\int\frac{dy^-}{2\pi}\, e^{-i\frac{1}{2}x_A y^- P^+}\sum_{X_n}\langle PS_A|\, B^{\mu,a}_{n\perp}(y^-,y_\perp)\,|X_n\rangle\,\langle X_n|\, B^{\nu,a}_{n\perp}(0)\,|PS_A\rangle$$

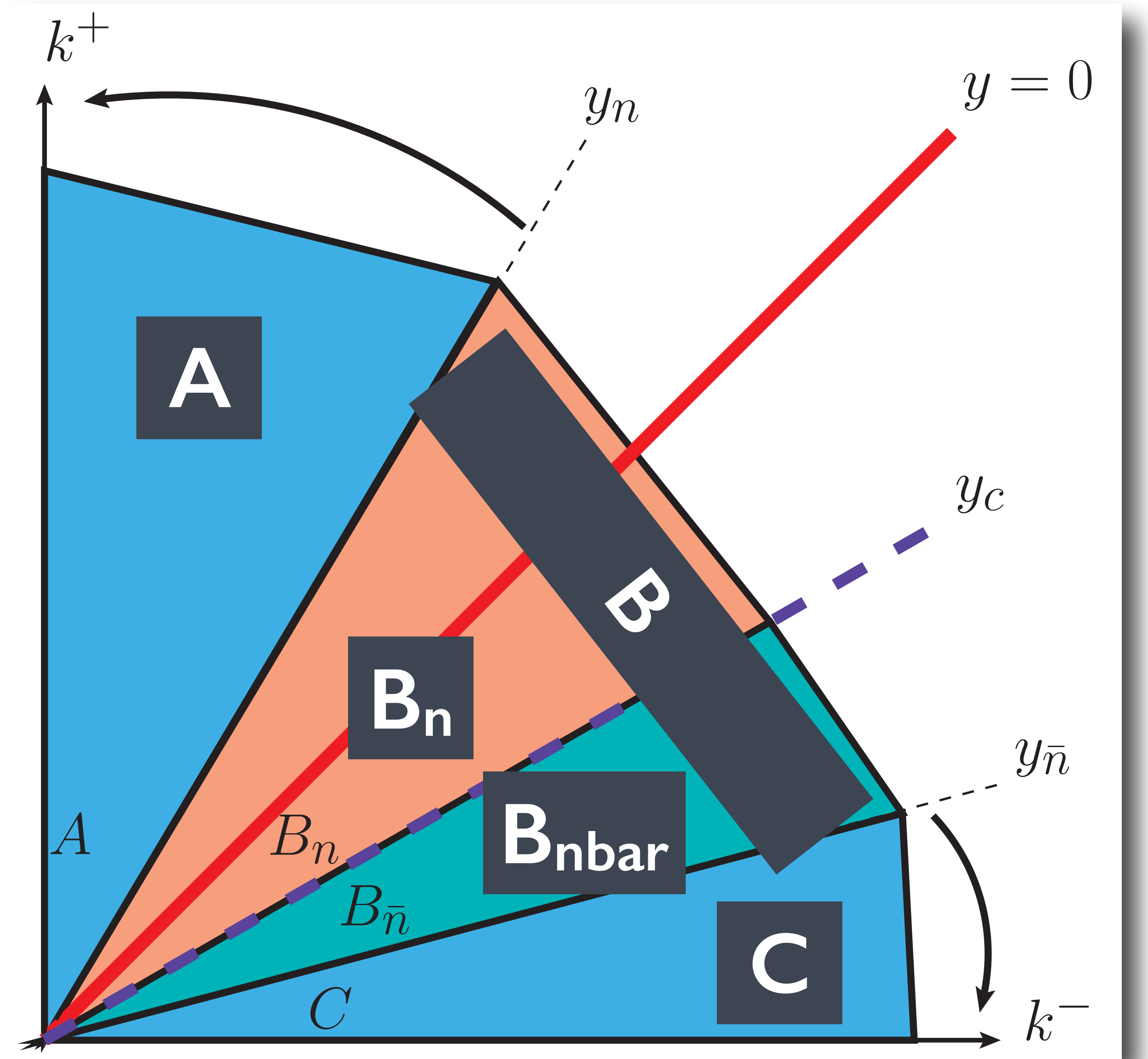
$$S(y_\perp,\mu) = \frac{1}{N_c^2-1}\sum_{X_s}\langle 0|\left(\mathcal{S}^{\dagger}_{n}\mathcal{S}_{\bar n}\right)^{ab}(y_\perp)\,|X_s\rangle\,\langle X_s|\left(\mathcal{S}^{\dagger}_{\bar n}\mathcal{S}_{n}\right)^{ba}(0)\,|0\rangle$$

- **Individually they are ill-defined!! They contain rapidity divergences (RDs)…**

Definition of TMDPDFs: Cancellation of RDs

MGE, Idilbi, Scimemi JHEP'12, PLB'13

- Pictorially, the relevant (anti-)collinear and soft modes are represented as:



$$k_n \sim (1, \lambda^2, \lambda)$$
$$k_{\bar{n}} \sim (\lambda^2, 1, \lambda)$$
$$k_s \sim (\lambda, \lambda, \lambda)$$

$$y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$

- **Naive collinear = A+B**
- **Soft = B**
- **Naive anticollinear = C+B**
- (Pure collinear = A)
- (Pure anticollinear = C)
- Each piece is boost invariant and depends on the difference of rapidities at the borders.
- **x-section = (A+B) + (C+B) - B = A+B+C**

- **Divergences at yn and ynbar as spurius…**
- **(Anti-)Collinear and Soft are ill-defined!!!**

So in order to cancel rapidity divergences, we define the TMDPDFs as:

$$G^{\mu\nu}_{g/A}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu^2) = A + B_n$$
$$G^{\mu\nu}_{g/B}(x_B, \boldsymbol{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu^2) = C + B_{\bar{n}}$$

Definition of TMDPDFs: Cancellation of RDs

The goal is to cancel Rapidity Divergences. The particular regulator is irrelevant!!

MGE, Idilbi, Scimemi JHEP'12, PLB'13

- Rapidity regulator I: Δ-regulator (MGE, Idilbi, Scimemi JHEP'12)

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_T, S_A; \zeta_A, \mu^2) = \tilde{J}^{\mu\nu}_n(x_A, \boldsymbol{b}_T, S_A; Q^2, \mu^2; \Delta^+)\, \tilde{S}^{-1}_+(b_T; \zeta_B, \mu^2; \Delta^+)$$

$$\zeta_A = Q^2/\alpha$$
$$\zeta_B = Q^2\alpha$$

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- Rapidity regulator II: rapidity-regulator (eta) (Chiu, Jain, Neill, Rothstein PRL'12)

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_T, S_A; \zeta_A, \mu^2) = \tilde{J}^{\mu\nu\,(0)}_n(x_A, \boldsymbol{b}_T, S_A; Q^2, \mu^2; \nu_-; \eta)\, \tilde{S}_-(b_T; \mu^2; \alpha\nu_-; \eta)$$

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- Rapidity regulator III: "combining integrands" (Collins'11)

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_T, S_A; \zeta_A, \mu^2) = \lim_{\substack{y_n \to +\infty \\ y_{\bar{n}} \to -\infty}} \tilde{J}^{\mu\nu}_n(x_A, \boldsymbol{b}_T, S_A; \mu^2; y_{\bar{n}}) \sqrt{\frac{\tilde{S}(y_n, y_c)}{\tilde{S}(y_c, y_{\bar{n}})\, \tilde{S}(y_n, y_{\bar{n}})}}$$

$\zeta_A = (p^+)^2 e^{-2y_c}$
$\zeta_B = (\bar{p}^-)^2 e^{+2y_c}$

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$$\zeta_A = (p^+)^2 e^{-2y_c} \qquad \zeta_B = (\bar{p}^-)^2 e^{+2y_c}$$

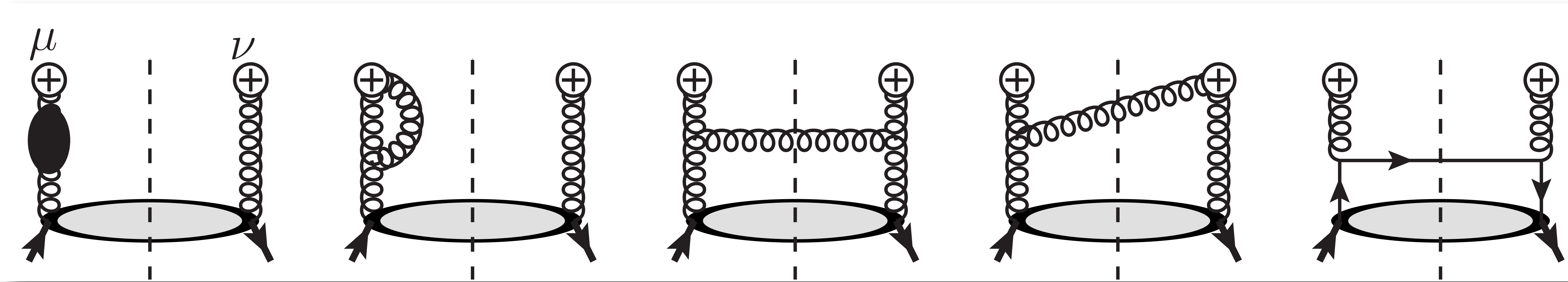
- **One could also use off-shellnesses, masses, “real Δ's”, analytic regulator, etc… Yet they all mean (pictorially):**

$$\tilde{G}^{\mu\nu}_{g/A}(x_A, \boldsymbol{b}_T, S_A; \zeta_A, \mu^2) = A + B_n$$

Previous slide!

- **For partonic calculations we pick up the Δ-regulator (has nothing to do with “SCET” label…)**

Cancellation of RDs at NLO



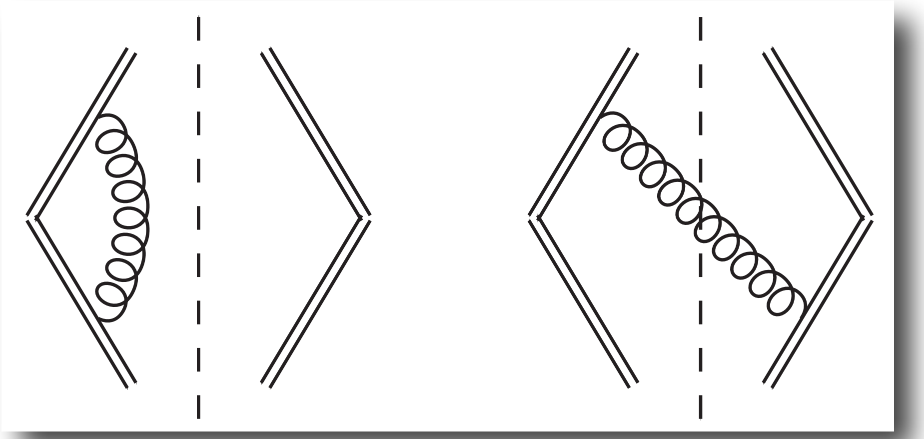
- Naive collinear matrix element for an unpolarized gluon inside unpolarized proton (gg channel):

$$\tilde{J}_{g/g}(x,b_T;Q^2,\mu^2) = \delta(1-x) + \frac{\alpha_s}{2\pi}\Bigg\{ \left[\frac{1}{\varepsilon_{\rm UV}}\left(\frac{\beta_0}{2} + 2C_A \ln\frac{\Delta^+}{Q^2}\right)\right]\delta(1-x)$$
$$+ C_A\left(2L_T \ln\frac{\Delta^+}{Q^2}\right)\delta(1-x) - \mathcal{P}_{gg}L_T + \frac{\beta_0}{2}\delta(1-x)L_T$$
$$-\ln\frac{\Delta^-}{\mu^2}\mathcal{P}_{gg} - 2C_A\ln(1-x)\frac{(1-x)(1+x^2)}{x} - 2C_A\left(\frac{\ln(1-x)}{1-x}\right)_+$$
$$-\delta(1-x)\left(\frac{C_A}{6} - \frac{n_f}{9} - 2C_A + C_A\frac{\pi^2}{2}\right)\Bigg\}$$

$$\mathcal{P}_{gg} = 2C_A\left[\frac{x}{(1-x)_+} + \frac{(1-x)(1+x^2)}{x}\right] + \frac{\beta_0}{2}\delta(1-x)$$

- There are un-cancelled rapidity divergences!! It's ill-defined no matter which regulator we use.
- Only combining it with the ("right piece of the") soft function we get the well-defined TMDPDF

Splitting of the Soft Function



$$L_T = \ln \frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

$$\tilde{S}(b_T; Q^2, \mu) = 1 + \frac{\alpha_s C_A}{2\pi}\left[-\frac{2}{\varepsilon_{\rm UV}^2} + \frac{2}{\varepsilon_{\rm UV}} \ln\frac{\Delta^-\Delta^+}{\mu^2 Q^2} + L_T^2 + 2L_T \ln\frac{\Delta^-\Delta^+}{\mu^2 Q^2} + \frac{\pi^2}{6}\right]$$

Rapidity divergences!!

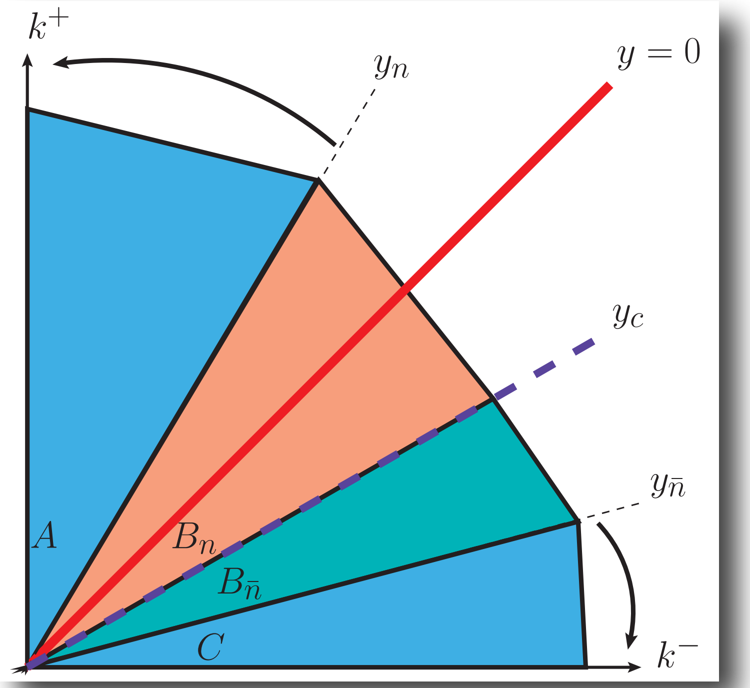
It is linear in the log(Q^2) and thus can be split in two pieces

- With the Δ-regulator, the soft function can be split to all orders as:

$$\tilde{S}(b_T; m_H^2, \mu^2) = \tilde{S}_-\left(b_T; \zeta_A, \mu^2; \Delta^-\right) \tilde{S}_+\left(b_T; \zeta_B, \mu^2; \Delta^+\right)$$

$$\tilde{S}_-\left(b_T; \zeta_A, \mu^2; \Delta^-\right) = \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \alpha\frac{\Delta^-}{\bar{p}^-}\right)}$$ **B_n**

$$\tilde{S}_+\left(b_T; \zeta_B, \mu^2; \Delta^+\right) = \sqrt{\tilde{S}\left(\frac{1}{\alpha}\frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)}$$ **B_{nbar}**



Unpolarized Gluon TMDPDF: Result

- Only combining it with the ("right piece of the") soft function we get a well-defined TMDPDF:

$$
\begin{aligned}
\tilde{f}^{g}_{1,g/g}(x,b_T;\zeta_A,\mu^2) = \delta(1-x) + \frac{\alpha_s}{2\pi}\Bigg\{ & \left[\frac{C_A}{\varepsilon_{\rm UV}^2} + \frac{1}{\varepsilon_{\rm UV}}\left(\frac{\beta_0}{2} - C_A \ln\frac{\zeta_A}{\mu^2}\right)\right]\delta(1-x) \\
& + C_A\left(-\frac{1}{2}L_T^2 - L_T \ln\frac{\zeta_A}{\mu^2} - \frac{\pi^2}{12}\right)\delta(1-x) - \mathcal{P}_{gg}L_T + \frac{\beta_0}{2}\delta(1-x)L_T \\
& - \ln\frac{\Delta^-}{\mu^2}\mathcal{P}_{gg} - 2C_A \ln(1-x)\frac{(1-x)(1+x^2)}{x} - 2C_A\left(\frac{\ln(1-x)}{1-x}\right)_+ \\
& - \delta(1-x)\left(\frac{C_A}{6} - \frac{n_f}{9} - 2C_A + C_A\frac{\pi^2}{2}\right)\Bigg\}
\end{aligned}
$$

Rapidity divergences have been cancelled!!

- zeta_A is NOT a rapidity regulator, it is actually the hard scale (a "fraction" of it)

- Now let me continue with the **re-factorization** of the TMDPDFs (for an unpolarized proton)…

3 Unpolarized Proton: Re-Factorization 1/4

- Inside an unpolarized proton we can have unpolarized or linearly polarized gluons:

$$G_{g/A}^{\mu\nu[O]}(x, \boldsymbol{k}_{nT}) = -g_\perp^{\mu\nu} f_1^g(x, k_{nT}^2) + \frac{1}{2}\left(g_\perp^{\mu\nu} - \frac{2k_{n\perp}^\mu k_{n\perp}^\nu}{k_{n\perp}^2}\right) h_1^{\perp g}(x, k_{nT}^2)$$

$$\tilde{G}_{g/A}^{\mu\nu[O]}(x, \boldsymbol{b}_T) = -g_\perp^{\mu\nu} \tilde{f}_1^g(x, b_T^2) + \frac{1}{2}\left(g_\perp^{\mu\nu} - \frac{2b_\perp^\mu b_\perp^\nu}{b_\perp^2}\right) \tilde{h}_1^{\perp g}(x, b_T^2)$$

$$\tilde{f}_1^g(x, b_T^2) = \int \frac{d^2\boldsymbol{k}_{n\perp}}{(2\pi)^2}\, e^{-i\boldsymbol{k}_{n\perp}\cdot\boldsymbol{b}_\perp}\, f_1^g(x, k_{nT}^2)$$

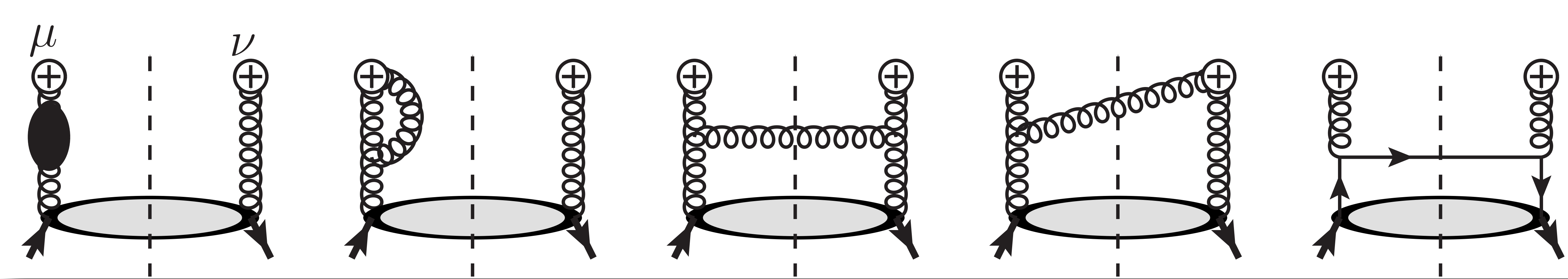
$$\tilde{h}_1^{\perp g}(x, b_T^2) = -2\pi \int dk_{nT}\, k_{nT}\, J_2(k_{nT} b_T)\, h_1^{\perp g}(x, k_{nT}^2)$$

- The OPEs of (renormalized) unpolarized and Boer-Mulders gluon TMDPDFs are both given in terms of the (renormalized) collinear quark/gluon PDFs:

$$\tilde{f}_{1,g/A}^g(x, b_T; \zeta_A, \mu) = \left(\frac{\zeta_A b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g(b_T;\mu)} \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{I}_{g\leftarrow j}^f(\bar{x}, b_T; \mu)\, f_{1,j/A}(x/\bar{x}; \mu) + \mathcal{O}(b_T\Lambda_{\rm QCD})$$

$$\tilde{h}_{1,g/A}^{\perp g}(x, b_T; \zeta_A, \mu) = \left(\frac{\zeta_A b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g(b_T;\mu)} \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{I}_{g\leftarrow j}^h(\bar{x}, b_T; \mu)\, f_{1,j/A}(x/\bar{x}; \mu) + \mathcal{O}(b_T\Lambda_{\rm QCD})$$

Unpolarized Proton: Re-Factorization 2/4



- First, the unpolarized collinear gluon PDF (gg channel):

$$f^g_{1,g/g}(x,\mu) = \delta(1-x) + \frac{\alpha_s}{2\pi}\left\{\left(\frac{1}{\varepsilon_{\rm UV}} - \ln\frac{\Delta^-}{\mu^2}\right)\mathcal{P}_{gg} - 2C_A \ln(1-x)\frac{(1-x)(1+x^2)}{x} -2C_A\left(\frac{\ln(1-x)}{1-x}\right)_+ - \delta(1-x)\left[\frac{C_A}{6} - \frac{n_f}{9} - 2C_A + C_A\frac{\pi^2}{2}\right]\right\}$$

$$\mathcal{P}_{gg} = 2C_A\left[\frac{x}{(1-x)_+} + \frac{(1-x)(1+x^2)}{x}\right] + \frac{\beta_0}{2}\delta(1-x)$$

- Single UV pole: evolution given by DGLAP
- IR is regulated by Δ-regulator
- Finite terms are regulator dependent. Only matter to obtain matching coefficients
- Rapidity divergences cancel between virtual and real diagrams

Unpolarized Proton: Re-Factorization 3/4

- I already showed you the result for the unpolarized TMDPDF in the gg channel.
- For the gq channel we have (there are no rapidity divergences at this order):

$$\tilde{f}^{g}_{1,g/q}(x,b_T;\zeta_A,\mu^2)=\frac{\alpha_s}{2\pi}\Big[-L_T\mathcal{P}_{gq}+C_F x - \ln\frac{\Delta^-}{\mu^2}\mathcal{P}_{gq}-C_F\ln(1-x)-C_F x\Big]$$

$$L_T=\ln\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

$$\mathcal{P}_{gq}=C_F\frac{1+(1-x)^2}{x}$$

- The collinear quark/gluon PDFs necessary for gq channel are:

$$f^{g}_{1,g/q}(x,\mu)=\frac{\alpha_s}{2\pi}\left[\left(\frac{1}{\varepsilon_{\rm UV}}-\ln\frac{\Delta^-}{\mu^2}\right)\mathcal{P}_{gq}-C_F\ln(1-x)-C_F x\right]$$

$$f^{q}_{1,q/q}(x,\mu)=\delta(1-x)$$

- The OPE coefficients for unpolarized gluon TMDPDF are then:

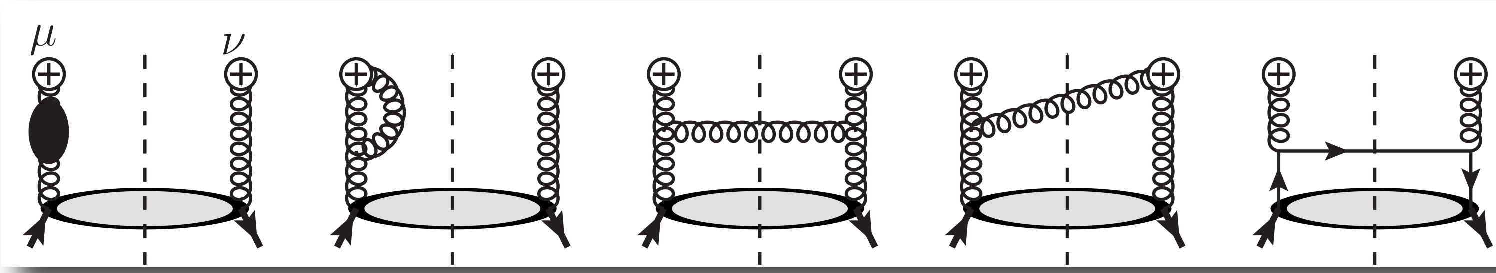
$$\tilde{I}^{f}_{g/g}(x,b_T;\mu)=\delta(1-x)+\frac{\alpha_s}{2\pi}\left[\delta(1-x)\left(C_A\frac{L_T^2}{2}+\beta_0\frac{L_T}{2}\right)-\mathcal{P}_{gg}L_T-C_A\frac{\pi^2}{12}\delta(1-x)\right]$$

$$\tilde{I}^{f}_{g/q}(x,b_T;\mu)=\frac{\alpha_s}{2\pi}\left[-\mathcal{P}_{gq}L_T+C_F x\right]$$

Unpolarized Proton: Re-Factorization 4/4

- For Boer-Mulders function the results are simpler (starts at order alpha and there are no rapidity divergences to cancel):

$$h_1^{\perp g}(x,k_{nT}^2) = \left(g_{\mu\nu}^{\perp} - \frac{2k_{n\mu}^{\perp}k_{n\nu}^{\perp}}{k_{n\perp}^2}\right) G_{g/A}^{\mu\nu[O]}(x,\boldsymbol{k}_{nT})$$



Gluon to gluon channel:

$$\tilde{h}_{1,g/g}^{\perp g}(x,b_T;\zeta_A,\mu^2) = \frac{\alpha_s}{2\pi} 2C_A \frac{1-x}{x}$$

Quark to gluon channel:

$$\tilde{h}_{1,g/q}^{\perp g}(x,b_T;\zeta_A,\mu^2) = \frac{\alpha_s}{2\pi} 2C_F \frac{1-x}{x}$$

- Using the previous results of collinear quark/gluon PDFs, the OPE coefficients for gluon Boer-Mulders function are then:

$$\tilde{I}_{g/g}^{h}(x,b_T;\mu) = \frac{\alpha_s}{2\pi} 2C_A \frac{1-x}{x}$$

$$\tilde{I}_{g/q}^{h}(x,b_T;\mu) = \frac{\alpha_s}{2\pi} 2C_F \frac{1-x}{x}$$

- Next, the evolution of gluon TMDPDFs…

Evolution (Resummation) 1/2

- We derive the evolution properties of gluon TMDPDFs from the factorization theorem:

$$d\sigma \sim \alpha_s(\mu)\, C_t^2(m_t^2,\mu) H(m_H^2,\mu)\, \tilde{G}^{\mu\nu}_{g/A}(x_A,\boldsymbol{b}_\perp,S_A;\zeta_A,\mu)\, \tilde{G}_{g/B\,\mu\nu}(x_B,\boldsymbol{b}_\perp,S_B;\zeta_B,\mu)$$

$$\gamma_G\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right) = -\Gamma^A_{\rm cusp}(\alpha_s(\mu))\ln\frac{\zeta_A}{\mu^2} - \gamma^g(\alpha_s(\mu)) - \gamma_t(\alpha_s(\mu)) - \frac{\beta(\alpha_s)}{\alpha_s}$$

$$\gamma_G\left(\alpha_s(\mu),\ln\frac{\zeta_B}{\mu^2}\right) = -\Gamma^A_{\rm cusp}(\alpha_s(\mu))\ln\frac{\zeta_B}{\mu^2} - \gamma^g(\alpha_s(\mu)) - \gamma_t(\alpha_s(\mu)) - \frac{\beta(\alpha_s)}{\alpha_s}$$

$$\frac{d}{d\ln\mu}\ln\mathcal{O} = \gamma_{\mathcal{O}}$$

Known at 3-loops!!

$$\frac{d}{d\ln\zeta_A}\ln\tilde{G}^{[pol]}_{g/A}(x_n,\boldsymbol{b}_\perp,S_A;\zeta_A,\mu) = -D_g(b_T;\mu)$$

$$\frac{dD_g}{d\ln\mu} = \Gamma^A_{\rm cusp}$$

- The evolution of all (un-)polarized gluon TMDPDFs is driven by the **<u>same evolution kernel</u>**:

$$\tilde{G}^{\mu\nu\,[pol]}_{g/A}(x_n,\boldsymbol{b}_\perp,S_A;\zeta_{A,f},\mu_f^2) = \tilde{G}^{\mu\nu\,[pol]}_{g/A}(x_n,\boldsymbol{b}_\perp,S_A;\zeta_{A,i},\mu_i^2)\,\tilde{R}^g\left(b_T;\zeta_{A,i},\mu_i^2,\zeta_{A,f},\mu_f^2\right)$$

$$\tilde{R}^g\left(b_T;\zeta_{A,i},\mu_i^2,\zeta_{A,f},\mu_f^2\right) = \exp\left\{\int_{\mu_i}^{\mu_f}\frac{d\bar{\mu}}{\bar{\mu}}\,\gamma_G\left(\alpha_s(\bar{\mu}),\ln\frac{\zeta_{A,f}}{\bar{\mu}^2}\right)\right\}\left(\frac{\zeta_{A,f}}{\zeta_{A,i}}\right)^{-D_g(b_T;\mu_i)}$$

Evolution (Resummation) 2/2

- Similar to the quark case, the D_g can be resummed:

MGE, Idilbi, Schafer, Scimemi EPJC'13

$$D_g^R(b_T;\mu_i) = D_g(b_T;\mu_b) + \int_{\mu_b}^{\mu_i} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^A_{\text{cusp}}$$

$\mu_b = 2e^{-\gamma_E}/b_T$

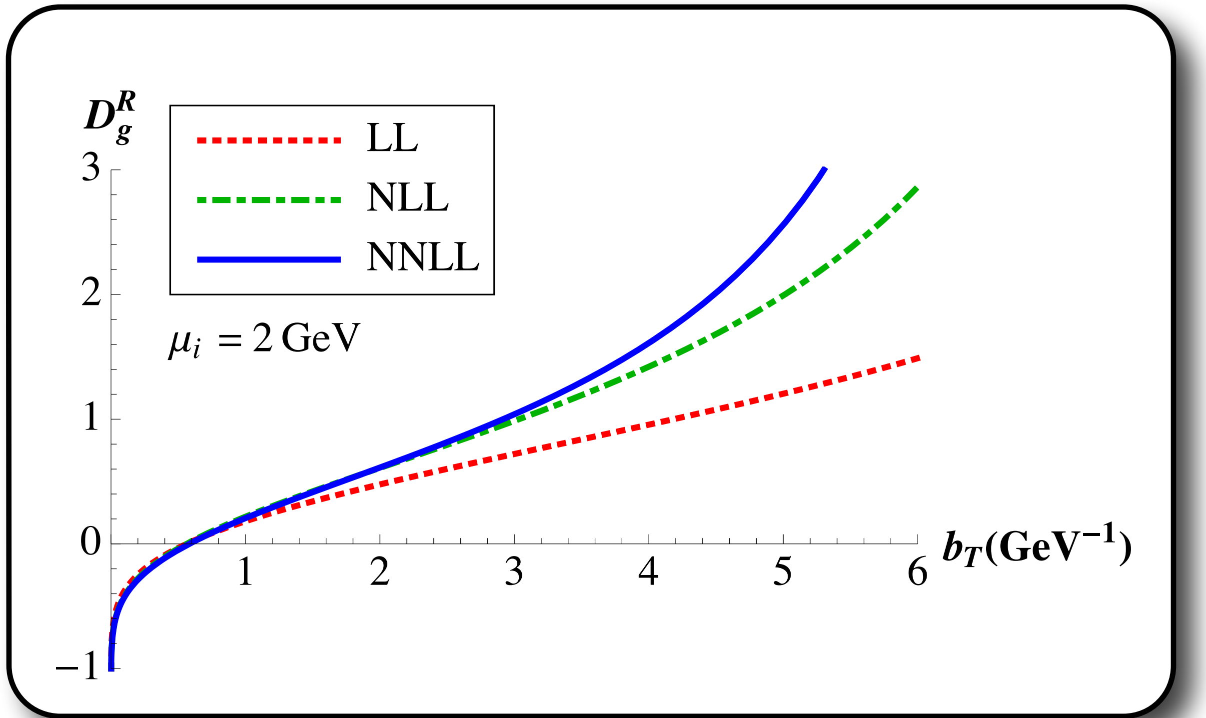
$X = a\beta_0 L_T$

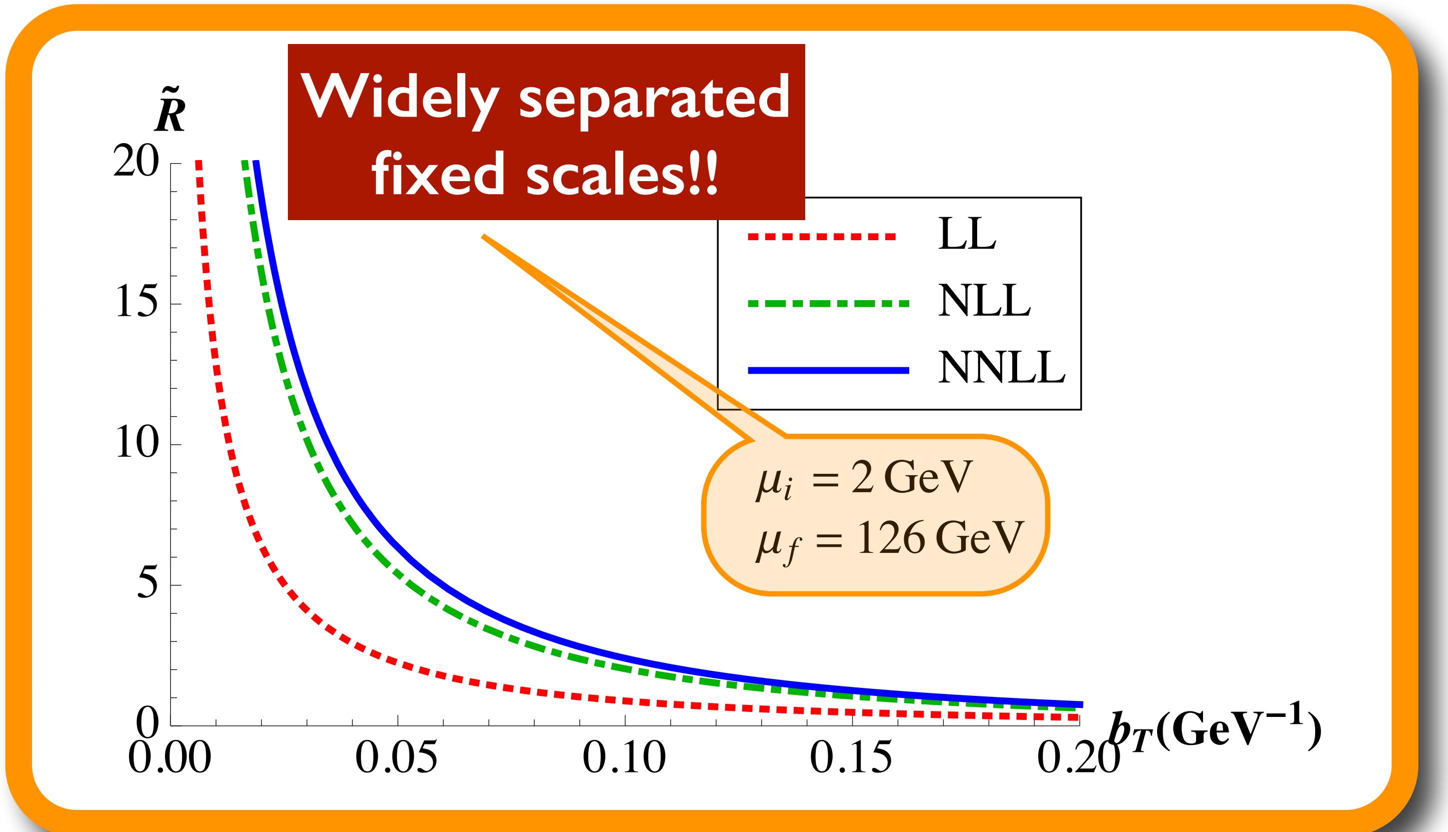
$a = \alpha_s(\mu_i)/(4\pi)$

$$D_g^R(b_T;\mu_i) = -\frac{\Gamma_0^A}{2\beta_0}\ln(1-X) + \frac{1}{2}\left(\frac{a}{1-X}\right)\left[-\frac{\beta_1\Gamma_0^A}{\beta_0^2}(X+\ln(1-X)) + \frac{\Gamma_1^A}{\beta_0}X\right]$$
$$+\frac{1}{2}\left(\frac{a}{1-X}\right)^2\left[2d_2(0) + \frac{\Gamma_2^A}{2\beta_0}(X(2-X)) + \frac{\beta_1\Gamma_1^A}{2\beta_0^2}(X(X-2)-2\ln(1-X)) + \frac{\beta_2\Gamma_0^A}{2\beta_0^2}X^2\right.$$
$$\left.+\frac{\beta_1^2\Gamma_0^A}{2\beta_0^3}(\ln^2(1-X)-X^2)\right] + \ldots$$

- We obtain D_g at 2-loops from the quark case (Casimir scaling):

$$d_2^g(0) = C_A C_A\left(\frac{404}{27} - 14\zeta_3\right) - \left(\frac{112}{27}\right) C_A T_F n_f$$





Non-Perturbative Inputs for TMDs

- Take the unpolarized TMDPDF as an example (this discussion applies to quark/gluon TMDs).

$$\tilde{f}_{1,g/A}^{g,PERT}(x,b_T;\zeta_A,\mu) = \left(\frac{\zeta_{A,0} b_T^2}{4e^{-2\gamma_E}}\right)^{-D_g(b_T;\mu_0)} \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{I}_{g\leftarrow j}^{f}(\bar{x},b_T;\mu_0)\, f_{1,j/A}^{g}(x/\bar{x};\mu_0)$$

$$\times \exp\left\{\int_{\mu_0}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_G\left(\alpha_s(\bar{\mu}), \ln\frac{\zeta_A}{\bar{\mu}^2}\right)\right\} \left(\frac{\zeta_A}{\zeta_{A,0}}\right)^{-D_g(b_T;\mu_0)} + \mathcal{O}(b_T\Lambda_{\rm QCD})$$

- The perturbative and non-perturbative regions are not well-separated:

$$\tilde{f}_{1,g/A}^{g}(x,b_T;\zeta_A,\mu) \overset{b_T\ll\Lambda_{\rm QCD}^{-1}}{=} \tilde{f}_{1,g/A}^{g,PERT}(x,b_T;\zeta_A,\mu) \quad \cdots \quad \overset{b_T\gg\Lambda_{\rm QCD}^{-1}}{\tilde{f}_{1,g/A}^{g,NP}(x,b_T;\zeta_A)}$$

Should go to 1 for small b_T

- Different ways of parameterizing non-perturbative part and matching it with the perturbative:

$$\tilde{f}_{1,g/A}^{g}(x,b_T;\zeta_A,\mu) = \tilde{f}_{1,g/A}^{g,PERT}(x,b_T^*;\zeta_A,\mu)\, \tilde{f}_{1,g/A}^{g,NP}(x,b_T;\zeta_A)$$

$$\tilde{f}_{1,g/A}^{g,NP}(x,b_T;\zeta_A) = g^{NP}(x,b_T)\left(\frac{\zeta_A}{Q_0}\right)^{-D_g^{NP}(b_T)}$$

$$\tilde{f}_{1,g/A}^{g}(x,b_T;\zeta_A,\mu) = \tilde{f}_{1,g/A}^{g,PERT}(x,b_T;\zeta_A,\mu)\, \tilde{f}_{1,g/A}^{g,NP2}(x,b_T;\zeta_A)$$

For quarks: Talk by Ignazio

$$\tilde{f}_{1,g/A}^{g}(x,b_T;\zeta_A,\mu) = \tilde{f}_{1,g/A}^{g,PERT}(x,b_T;\zeta_A,\mu)\Big|_{b<b_c} \quad \tilde{f}_{1,g/A}^{g,NP3}(x,b_T;\zeta_A)\Big|_{b>b_c}$$

Measurement of Gluon BM at LHC

- Ratio of gluon Boer-Mulders and unpolarized TMDPDF contributions:

$$\mathcal{R}(x_A, x_B, q_T; m_H^2, \mu^2) = \frac{\int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\, \tilde{h}_1^{\perp g}(x_A, b_T; \zeta_A, \mu^2)\, \tilde{h}_1^{\perp g}(x_B, b_T; \zeta_B, \mu^2)}{\int d^2\boldsymbol{b}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\, \tilde{f}_1^{g}(x_A, b_T; \zeta_A, \mu^2)\, \tilde{f}_1^{g}(x_B, b_T; \zeta_B, \mu^2)}$$

- To give a quick prediction and a rough estimate, we take the b* prescription and the NP model from Drell-Yan (Konichev-Nadolsky fit):

$$\tilde{f}_1^{g}(x_A, b_T; m_H^2, m_H^2) = \tilde{f}_1^{g}(x_A, b_T; \mu_b^2, \mu_b^2)\tilde{R}(b_*; \mu_b, m_H)F^{NP}(x_A, b_T, m_H)$$
$$\tilde{h}_1^{\perp g}(x_A, b_T; m_H^2, m_H^2) = \tilde{h}_1^{\perp g}(x_A, b_T; \mu_b^2, \mu_b^2)\tilde{R}(b_*; \mu_b, m_H)F^{NP}(x_A, b_T, m_H)$$

$$F^{NP}(x_A, b_T, m_H) = \exp\left\{-\frac{1}{2}\frac{C_A}{C_F}\left[0.184\ln\frac{m_H}{3.2} + 0.332\right] b_T^2\right\}$$

- We integrate the anomalous dimension in the kernel **analytically** and take into account the c- and b-quark thresholds.
- Consistent evolution at NLL (cusp anomalous dimension at 2 loops!)

Measurement of BM at LHC…

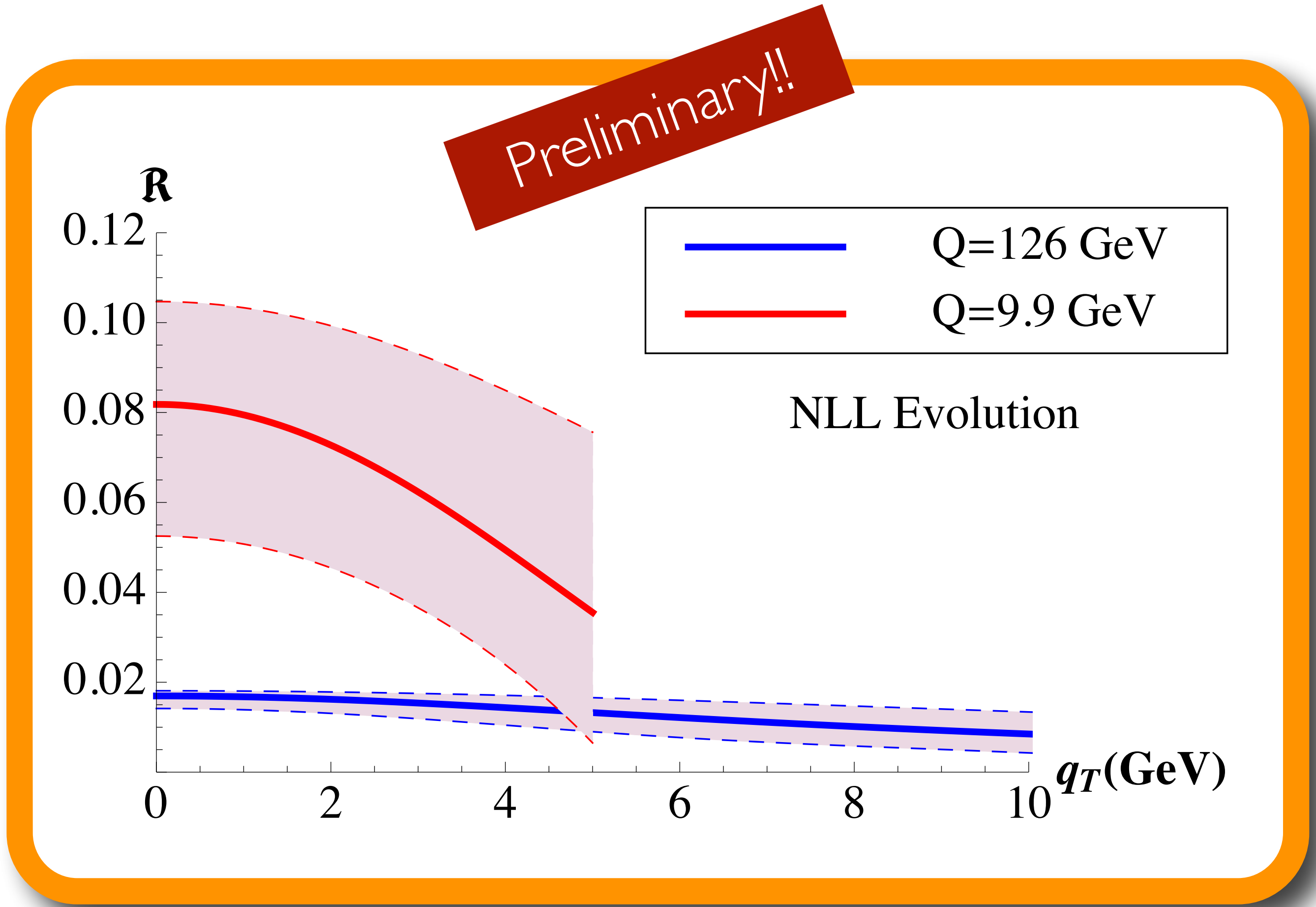
- We plot the ratio R with evolution at NLL

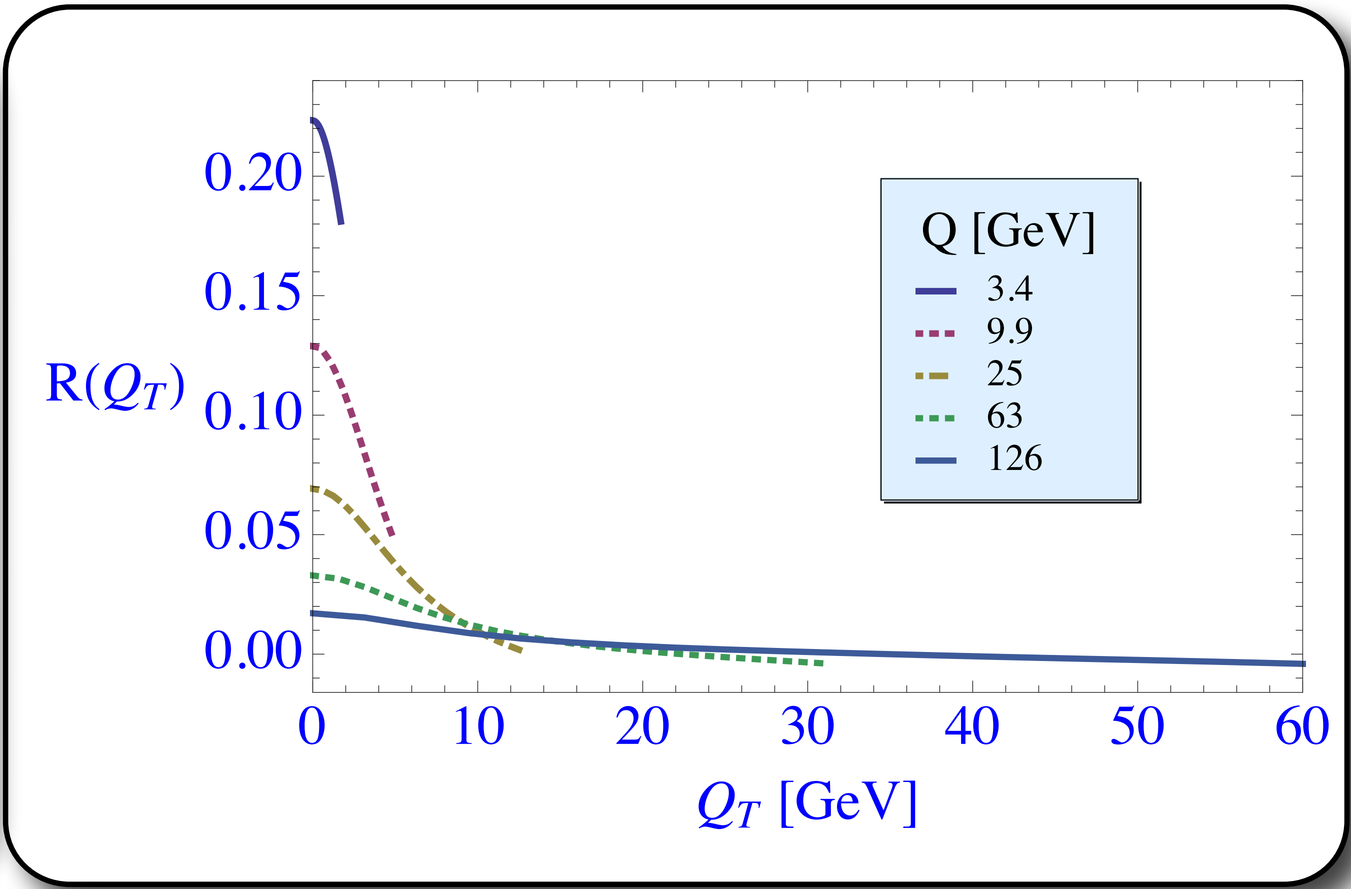
$$\tilde{f}_1^g(x, b_T; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s)\,,$$

$$\tilde{h}_1^{\perp g}(x, b_T; \mu_b^2, \mu_b) = \frac{\alpha_s C_A}{\pi} \int_x^1 \frac{d\bar{x}}{\bar{x}} \left(\frac{\bar{x}}{x} - 1\right) f_{g/P}(\bar{x}; \mu_b) + \frac{\alpha_s C_F}{\pi} \sum_q \int_x^1 \frac{d\bar{x}}{\bar{x}} \left(\frac{\bar{x}}{x} - 1\right) f_{q/P}(\bar{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b^*}$$

$$b_*(b) = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$





Boer, den Dunnen 1404.6753

Conclusions and Outlook

- We derived the factorization theorem for the Higgs transverse-momentum distribution
- The cancellation of rapidity divergences between collinear and soft matrix elements leads us to the proper definition of all (un-)polarized gluon TMDPDFs.
- We showed explicitly at 1-loop for unpolarized and Boer-Mulders TMDPDFs that rapidity divergences cancel.
- We obtained the perturbative tails of the unpolarized and Boer-Mulders functions at NLO.
- We got the evolution for all (un-)polarized gluon TMDPDFs at NNLL (unpolarized TMDPDF, gluon Sivers function, etc) and showed one preliminary application (gluon Boer-Mulders).

- Exploit the evolution at NNLL to make more reliable predictions.
- Write explicitly the OPEs for all gluon TMDPDFs.
- Phenomenology… Waiting for the EIC!!

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Back up slides

Definition of Gluon TMDPDFs

- With the Δ-regulator the gluon TMDPDFs are defined as:

$$G^{\mu\nu}_{g/A}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu^2; \Delta^-) = \int d^2\boldsymbol{b}_\perp\, e^{i\boldsymbol{b}_\perp\cdot\boldsymbol{k}_{n\perp}}\, \tilde{J}^{\mu\nu}_{n}(x_A, \boldsymbol{b}_\perp, S_A; \mu^2; \Delta^-)\, \tilde{S}_-(b_T; \zeta_A, \mu^2; \Delta^-)$$

$$G^{\mu\nu}_{g/B}(x_B, \boldsymbol{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu^2; \Delta^+) = \int d^2\boldsymbol{b}_\perp\, e^{i\boldsymbol{b}_\perp\cdot\boldsymbol{k}_{\bar{n}\perp}}\, \tilde{J}^{\mu\nu}_{\bar{n}}(x_B, \boldsymbol{b}_\perp, S_B; \mu^2; \Delta^+)\, \tilde{S}_+(b_T; \zeta_B, \mu^2; \Delta^+)$$

- At leading twist we have 8 different distributions:

$$G^{\mu\nu[O]}_{g/A}(x, \boldsymbol{k}_{nT}) = g^{\mu\nu}_\perp f_1^g(x, k_{nT}^2) + \left(\frac{k^\mu_{n\perp} k^\nu_{n\perp}}{M_A^2} + g^{\mu\nu}_\perp \frac{k_{nT}^2}{2M_A^2}\right) h_1^{\perp g}(x, k_{nT}^2)\,,$$

$$G^{\mu\nu[L]}_{g/A}(x, \boldsymbol{k}_{nT}) = i\epsilon^{\mu\nu}_\perp \lambda\, g^g_{1L}(x, k_{nT}^2) + \frac{\epsilon_\perp^{k_T\{\mu} k^{\nu\}}_{n\perp}}{2M_A^2} \lambda\, h^{\perp g}_{1L}(x, k_{nT}^2)\,,$$

$$G^{\mu\nu[T]}_{g/A}(x, \boldsymbol{k}_{nT}) = -g^{\mu\nu}_\perp \frac{\epsilon_\perp^{k_T S_T}}{M_A} f^{\perp g}_{1T}(x, k_{nT}^2) - i\epsilon^{\mu\nu}_\perp \frac{\boldsymbol{k}_{nT}\cdot \boldsymbol{S}_T}{M_A} g^g_{1T}(x, k_{nT}^2)$$
$$+ \frac{\epsilon_\perp^{k_T\{\mu} k^{\nu\}}_{n\perp}}{2M_A^2} \frac{\boldsymbol{k}_{nT}\cdot \boldsymbol{S}_T}{M_A} h^{\perp g}_{1T}(x, k_{nT}^2) + \frac{\epsilon_\perp^{k_T\{\mu} S_T^{\nu\}} + \epsilon_\perp^{S_T\{\mu} k^{\nu\}}_{nT}}{4M_A} h^g_{1T}(x, k_{nT}^2)$$

They include the soft function!!