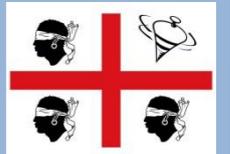




Transversity 2014

Fourth International Workshop on
Transverse Polarisation Phenomena
in Hard Processes

9-13 June, 2014 - Chia, Cagliari, Italy



Di-hadron Fragmentation Functions and Transversity

Marco Radici
INFN - Pavia



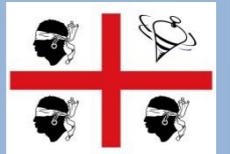
in collaboration with:
A. Bacchetta (Univ. Pavia)
A. Courtoy (Univ. Liege)



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Di-hadron Fragmentation Functions and Transversity

NEW **FIT !**

Marco Radici
INFN - Pavia



in collaboration with:
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Outline

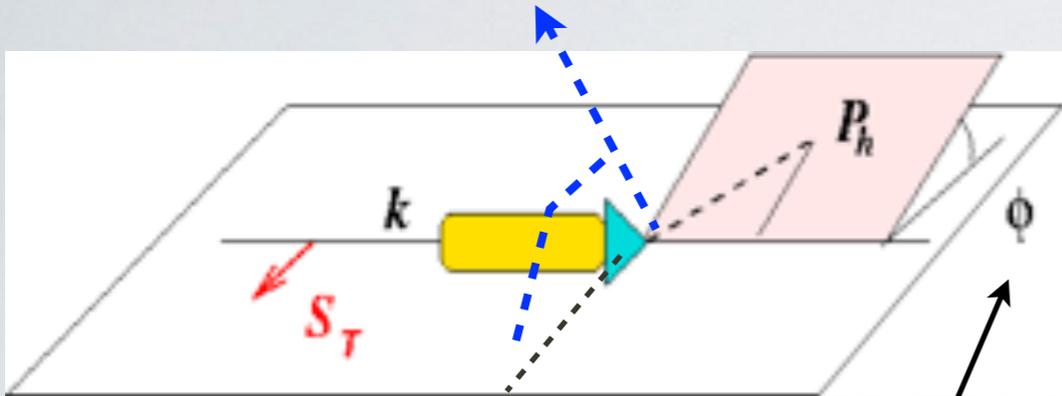
- why di-hadron semi-inclusive production?
brief review of advantages w.r.t. Collins effect
- review of existing results about extraction of transversity
including recent Compass analysis of their new proton data
(see C. Braun's talk)
- new fit : what's new ?
- conclusions and outlooks

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- why di-hadron semi-inclusive production?
brief review of advantages w.r.t. Collins effect
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- **new fit** : what's new ?
- conclusions and outlooks

Collins effect vs. DiFF

J. Collins, NPB396 (93)

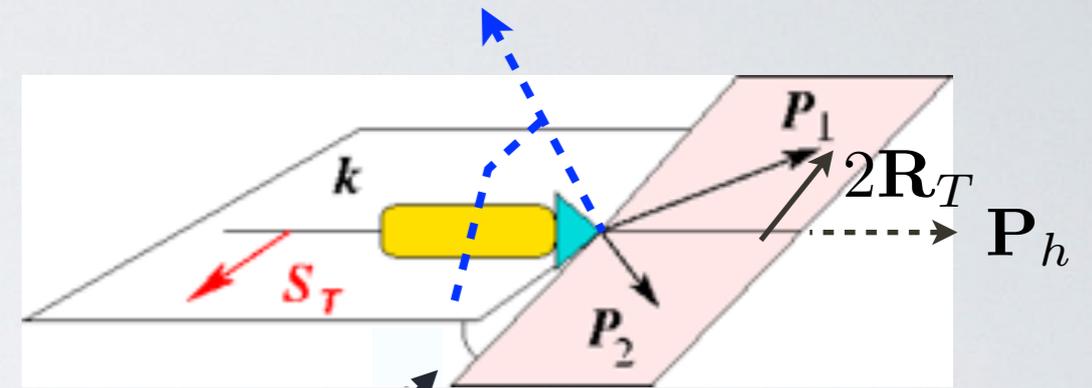


Collins angle

$$\mathbf{k} \times \mathbf{P}_h \cdot \mathbf{S}_T \propto \sin(\phi + \phi_S)$$

$\mathbf{P}_{hT} \neq 0$ transverse momentum of hadron required

Collins, Heppelman, Ladinsky, NP B420 (94)



ϕ_{RT}

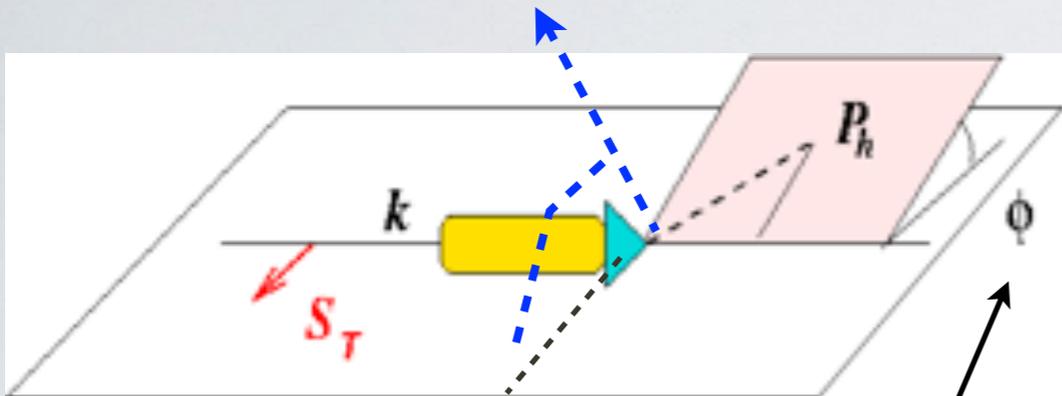
$$\begin{aligned} \mathbf{P}_h &= \mathbf{P}_1 + \mathbf{P}_2 \\ 2\mathbf{R} &= \mathbf{P}_1 - \mathbf{P}_2 \end{aligned}$$

$$\mathbf{P}_h \times \mathbf{R}_T \cdot \mathbf{S}_T \propto \sin(\phi_{RT} + \phi_S)$$

effect relies on $\mathbf{R}_T \neq 0$
 $\mathbf{P}_{hT} = 0$ the pair is collinear

Collins effect vs. DiFF

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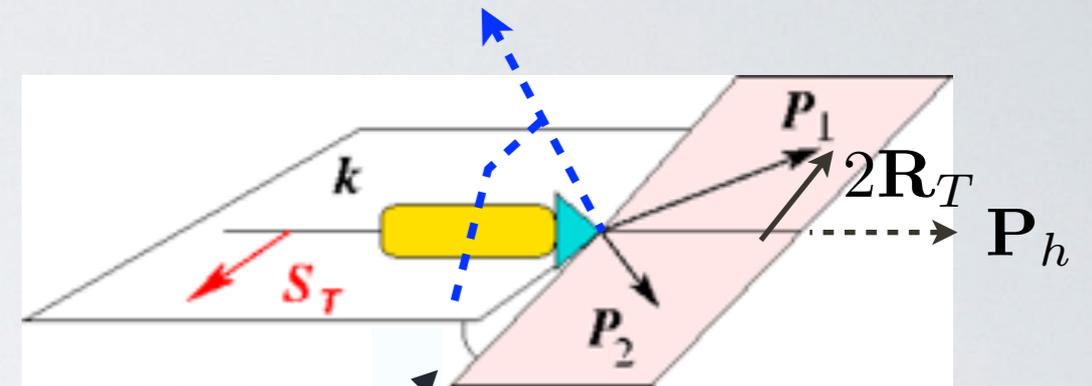
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framework of
TMD factorization

Collins, Heppelman, Ladinsky, NP B420 (94)



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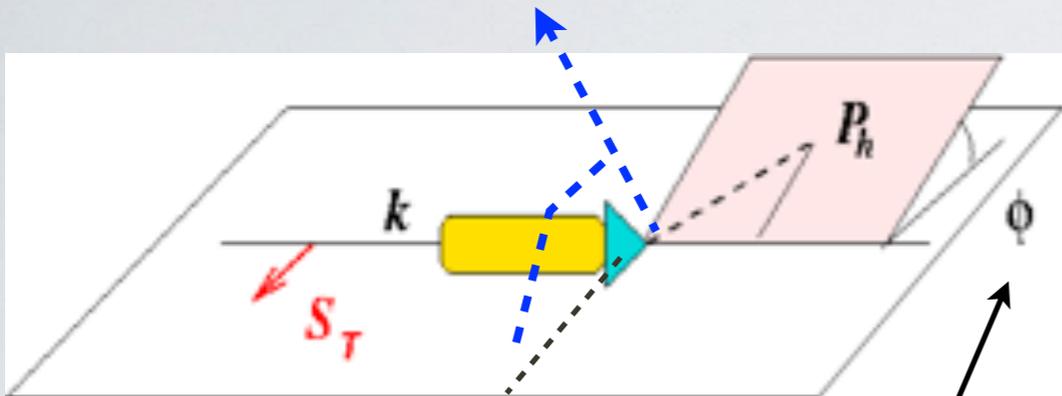
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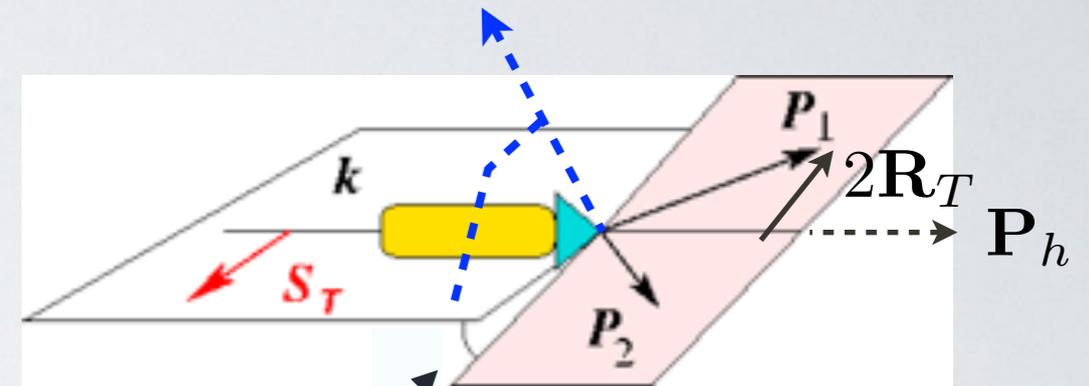
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effect relies on $\mathbf{R}_T \neq 0$
 $\mathbf{P}_{hT} = 0$ the pair is collinear

framework of collinear factorization

Collins effect vs. DiFF

Collins effect TMD factorization

1h - SIDIS single-spin asymmetry

$$A_{UT}^{\sin(\phi+\phi_S)} \propto \frac{\sum_q e_q^2 h_1^q \otimes_w H_1^{\perp q}}{\sum_q e_q^2 f_1^q \otimes D_1^q}$$

DiFF collinear factorization

2h - SIDIS single-spin asymmetry

$$A_{UT}^{\sin(\phi_R+\phi_S)} \propto -\frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^q(x) H_1^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$

*Radici, Jakob, Bianconi
PR D65 (02)
Bacchetta, Radici,
PR D67 (03)*

Collins effect vs. DiFF

Collins effect TMD factorization

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convolution on
⊥-moment dependence
of h_1 and H_1^{\perp}

TMD evolution eq.
required

DiFF collinear factorization

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simple product
of h_1 and H_1^{\triangleleft}

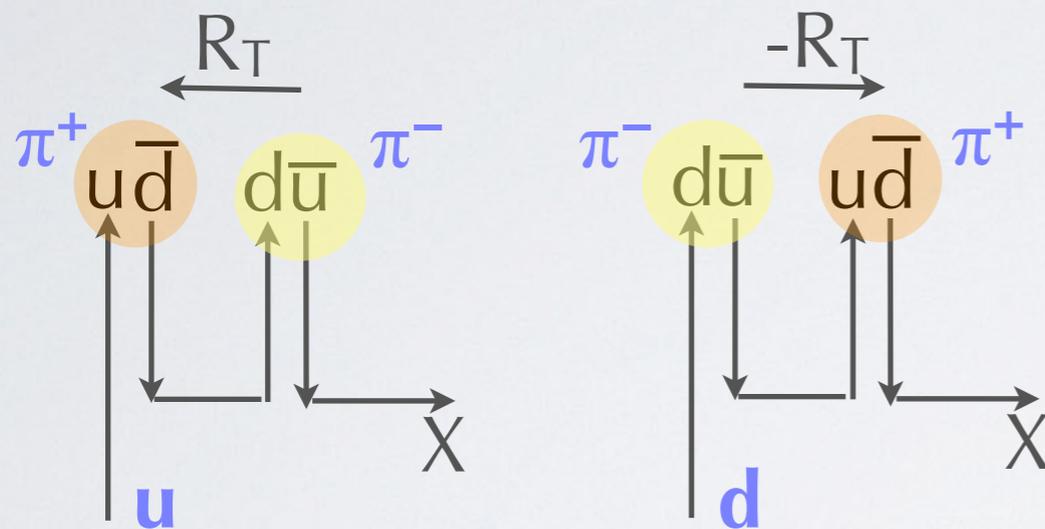
DGLAP evolution eq.
(well known)

*Radici, Jakob, Bianconi
PR D65 (02)
Bacchetta, Radici,
PR D67 (03)*

*Ceccopieri, Radici, Bacchetta,
P.L. B650 (07)*

first extraction of $x h_1^{u_v} - x h_1^{d_v}/4$

$$A_{UT}^{\sin(\phi_R + \phi_S)} \propto -\frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^q(x) H_1^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$



isospin symmetry + charge conjugation

$$H_1^{\triangleleft u} = -H_1^{\triangleleft d}$$

$$H_1^{\triangleleft q} = -H_1^{\triangleleft \bar{q}}$$

$$D_1^q = D_1^{\bar{q}}$$

consistent with
sign of exp. A_{UT}

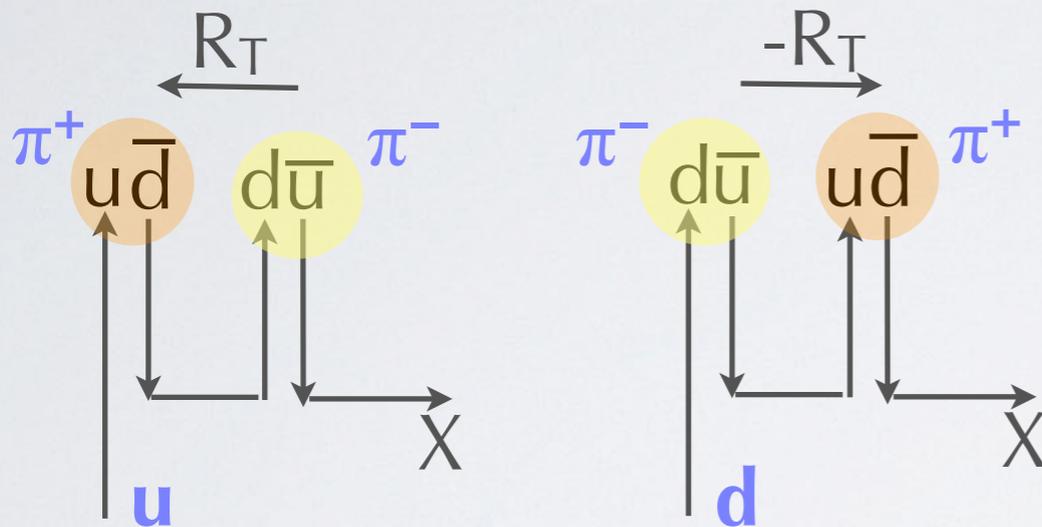
proton target

$$x h_1^p(x) \equiv x h_1^{u_v}(x) - \frac{1}{4} x h_1^{d_v}(x)$$

$$\propto -\frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 H_1^{\triangleleft u}} \left[\sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_1^{q+\bar{q}}(x) \int dz dM_h^2 D_1^q \right]$$

first extraction of $x h_1^{u_v} - x h_1^{d_v}/4$

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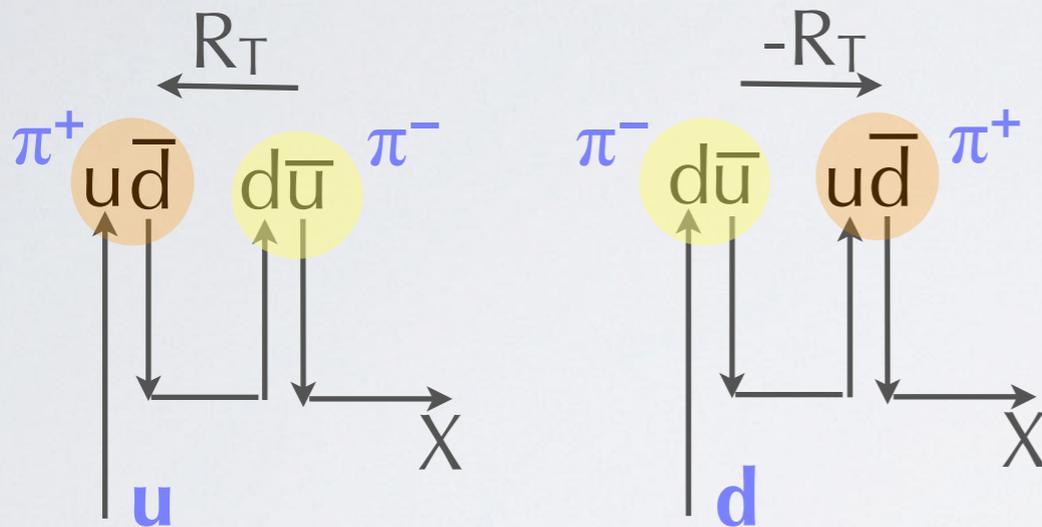
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MSTW08

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isospin symmetry + charge conjugation

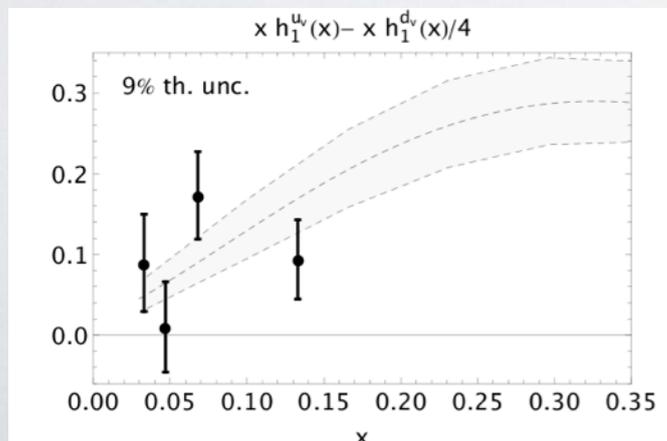
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Bacchetta, Courtoy, Radici,
P.R.L. **107** (11) 012001

extracting from
 $e+e^- \rightarrow (\pi+\pi^-)(\pi+\pi^-)$



first extraction of $x h_1^{u_v}$, $x h_1^{d_v}$

repeat for
deuteron target

$$\begin{aligned}
 x h_1^D(x) &\equiv x h_1^{u_v}(x) + x h_1^{d_v}(x) \\
 &\propto -\frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2} H_1^{\leq u} \frac{4}{3} \\
 &\quad \times \left[\left(\sum_{q=u,d} x f_1^{q+\bar{q}}(x) \right) \left(\sum_{q=u,d} \frac{e_q^2}{e_u^2} \int dz dM_h^2 D_1^q \right) + x f_1^{s+\bar{s}}(x) \frac{1}{2} \int dz dM_h^2 D_1^s \right]
 \end{aligned}$$

proton data



access to $x h_1^{u-\bar{u}}(x) - \frac{1}{4} x h_1^{d-\bar{d}}(x)$

deuteron data



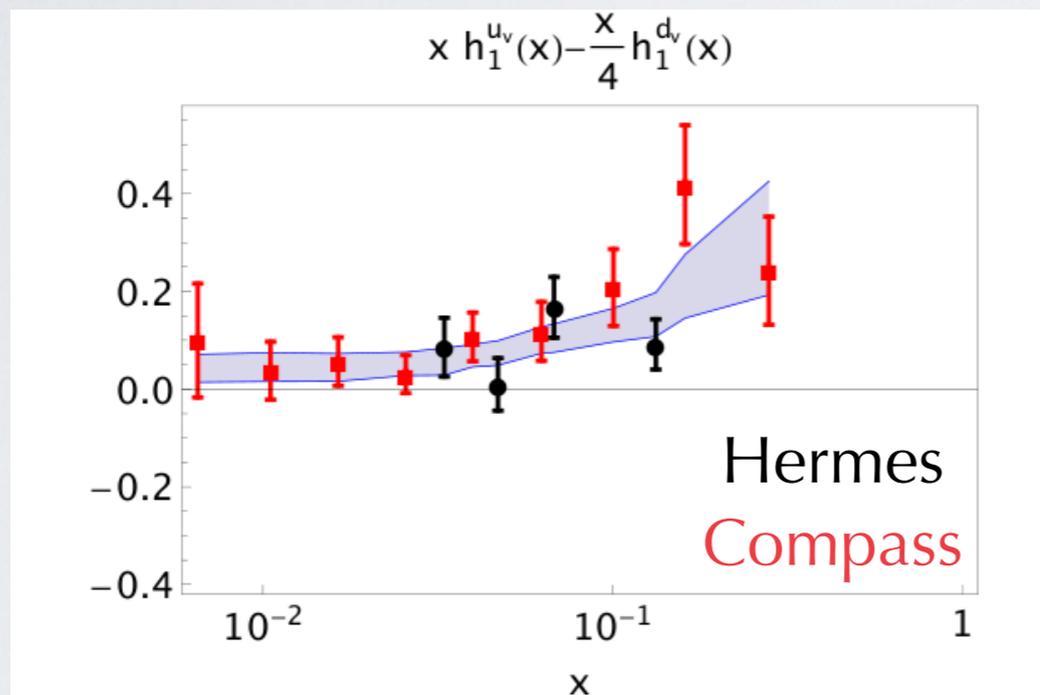
access to $x h_1^{u-\bar{u}}(x) + x h_1^{d-\bar{d}}(x)$

combination of both sets \rightarrow access to valence transversities
separately

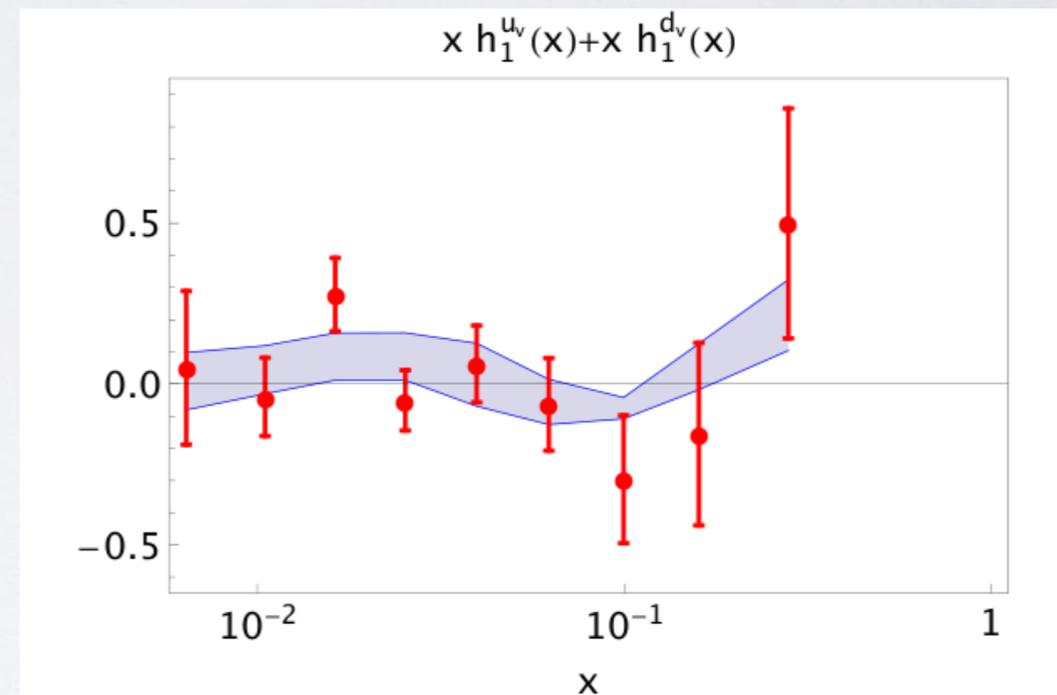
first extraction of $x h_1^{u_v}$, $x h_1^{d_v}$

Bacchetta, Courtoy, Radici, JHEP 1303 (13) 119

proton data
 $x h_1^p(x)$



deuteron data
 $x h_1^D(x)$

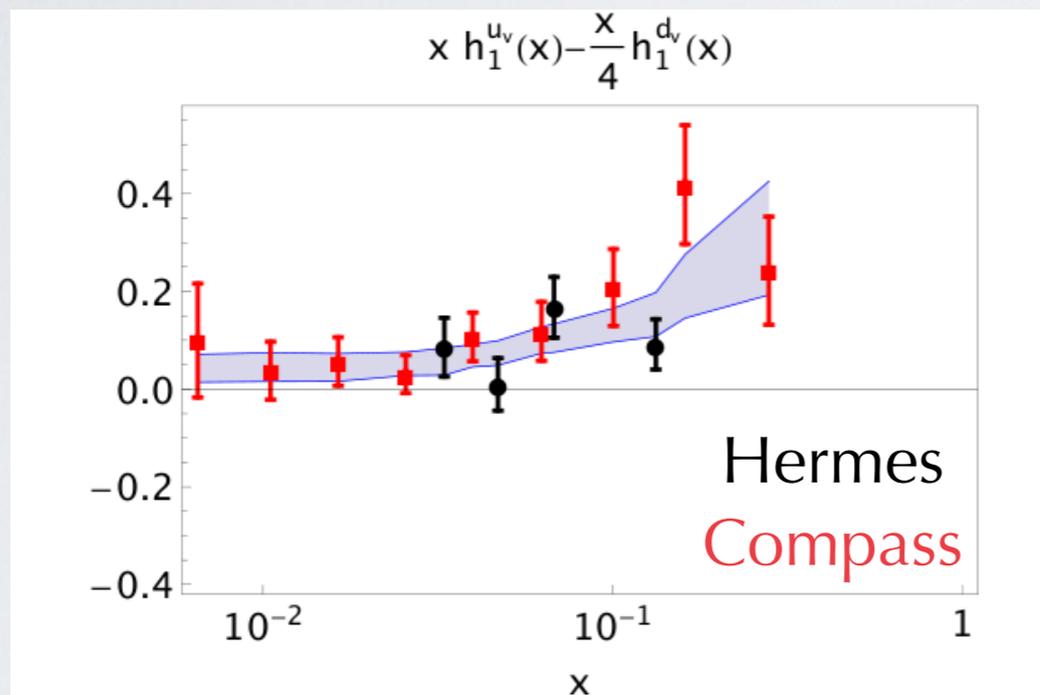


extraction point by point

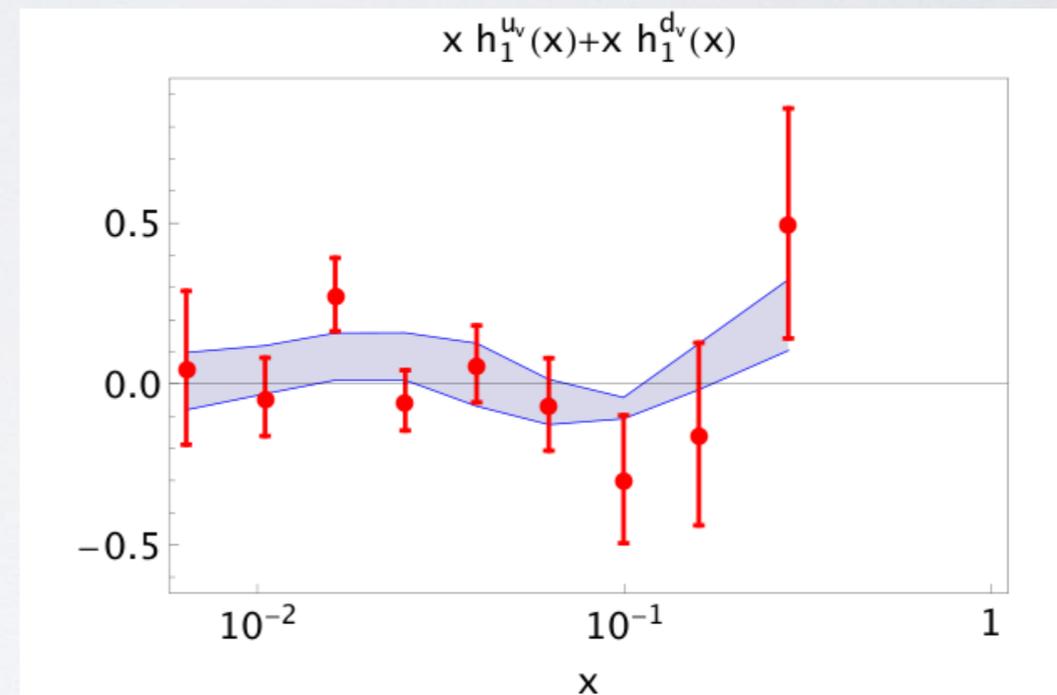
first extraction of $x h_1^{u_v}$, $x h_1^{d_v}$

Bacchetta, Courtoy, Radici, JHEP 1303 (13) 119

proton data
 $x h_1^p(x)$



deuteron data
 $x h_1^D(x)$



extraction point by point

next step: true fit of $x h_1^p(x)$ and $x h_1^D(x)$

two ways: standard Hessian method
replica method

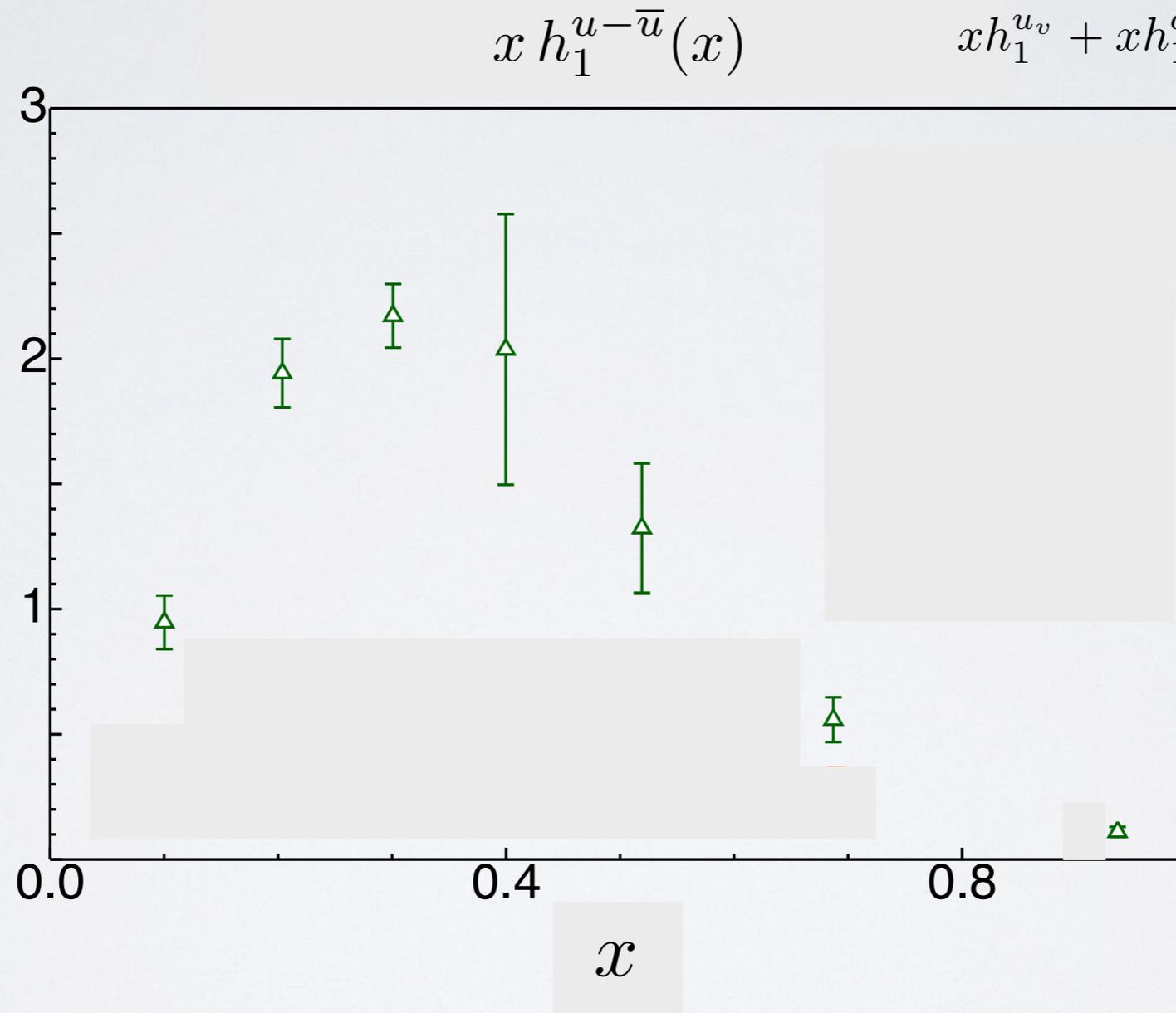


our fitting procedure

inspired by NNPDF

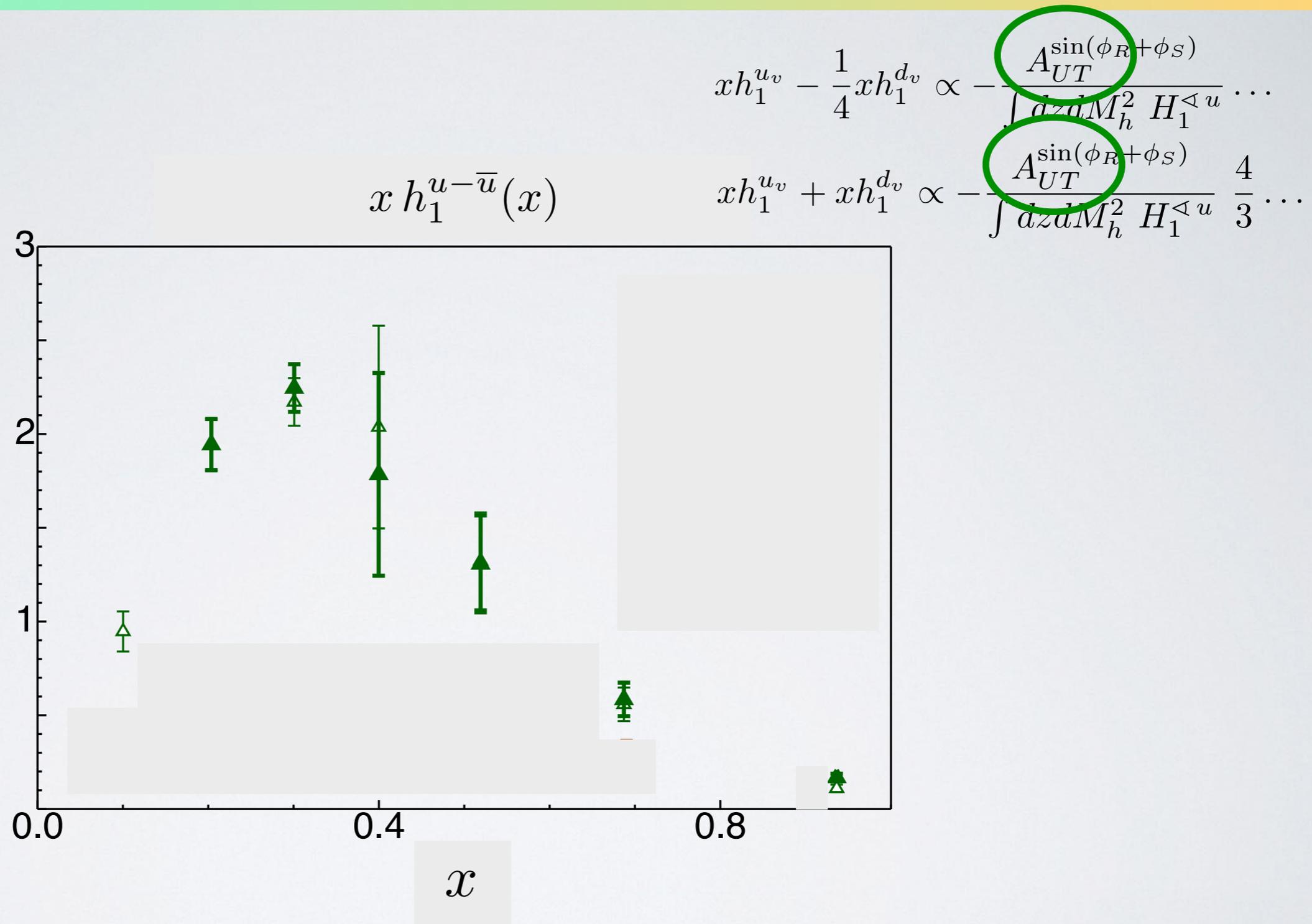
$$xh_1^{u_v} - \frac{1}{4}xh_1^{d_v} \propto -\frac{A_{UT}^{\sin(\phi_R+\phi_S)}}{\int dzdM_h^2 H_1^{\triangleleft u}} \dots$$

$$xh_1^{u_v} + xh_1^{d_v} \propto -\frac{A_{UT}^{\sin(\phi_R+\phi_S)}}{\int dzdM_h^2 H_1^{\triangleleft u}} \frac{4}{3} \dots$$



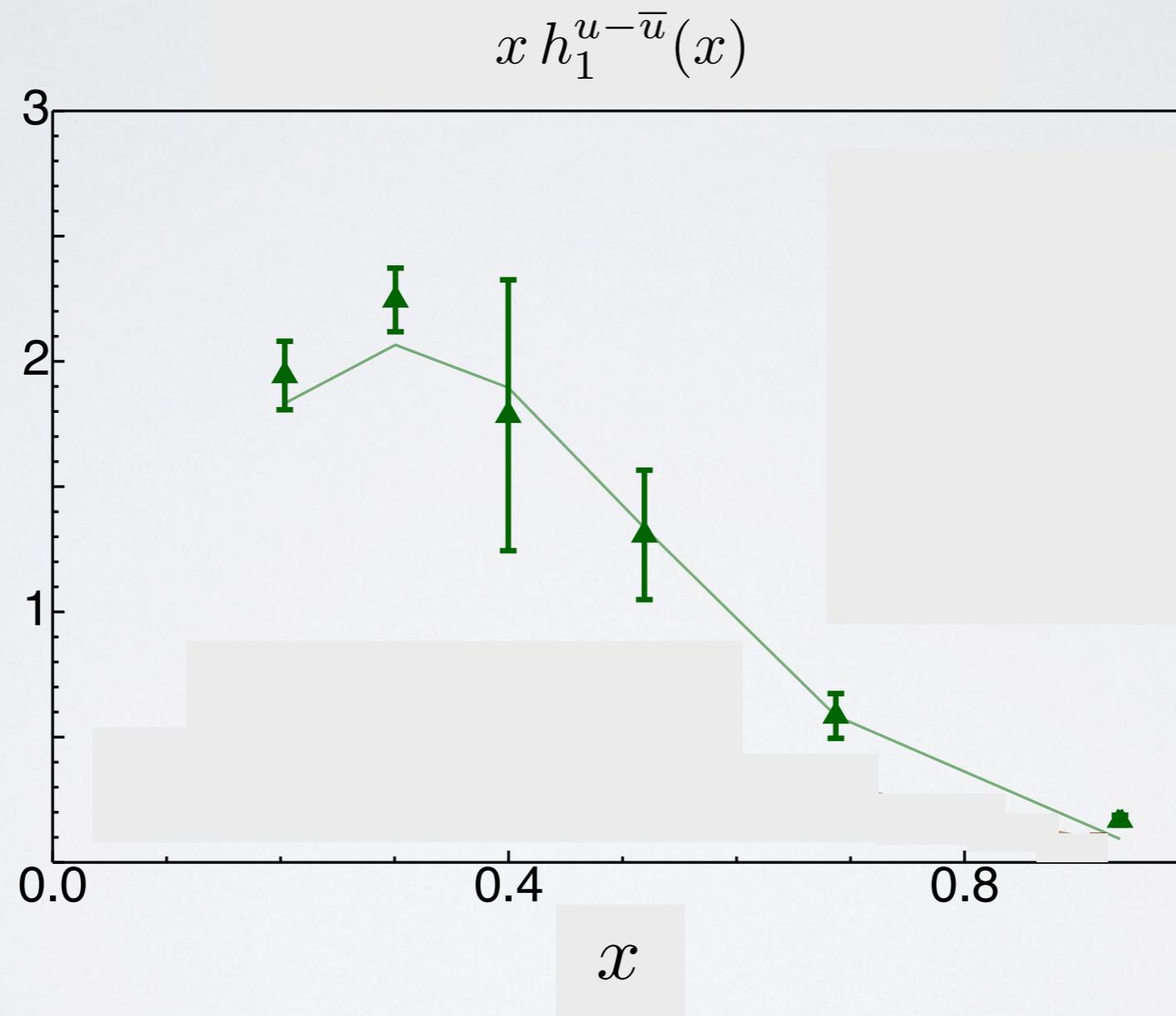
sample of original data

our fitting procedure



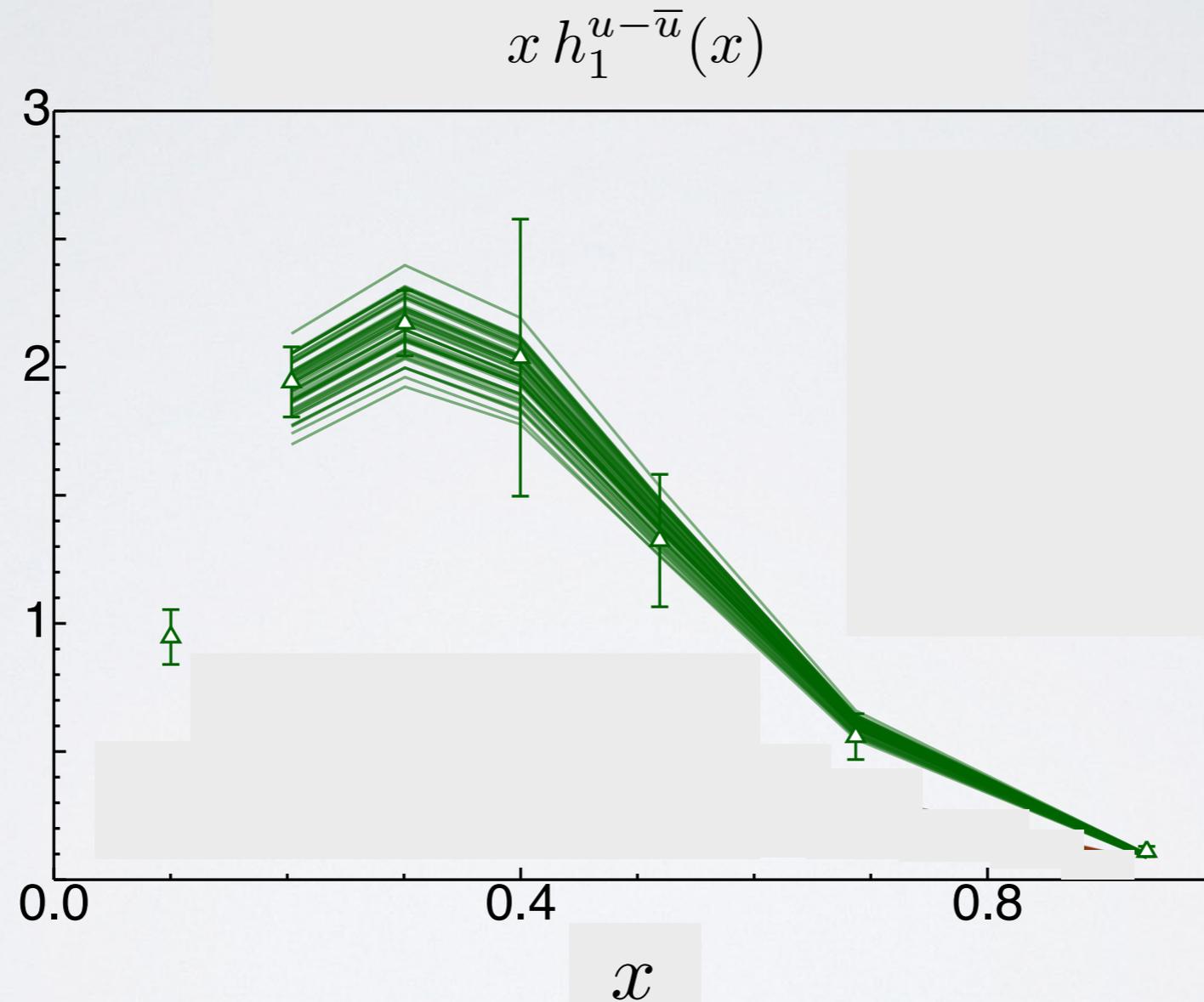
data are replicated with Gaussian noise
within exp. variance

our fitting procedure



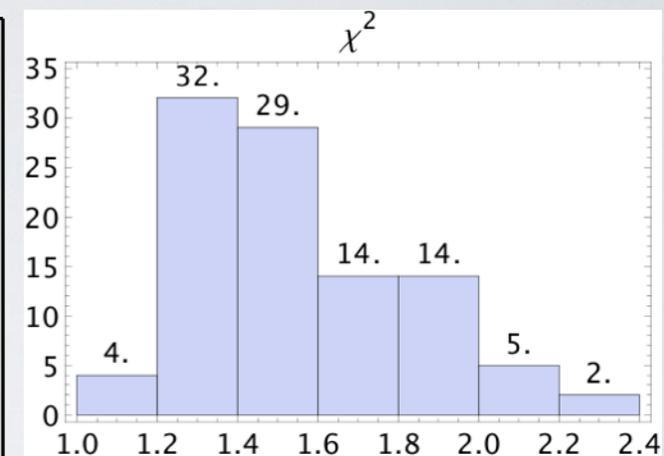
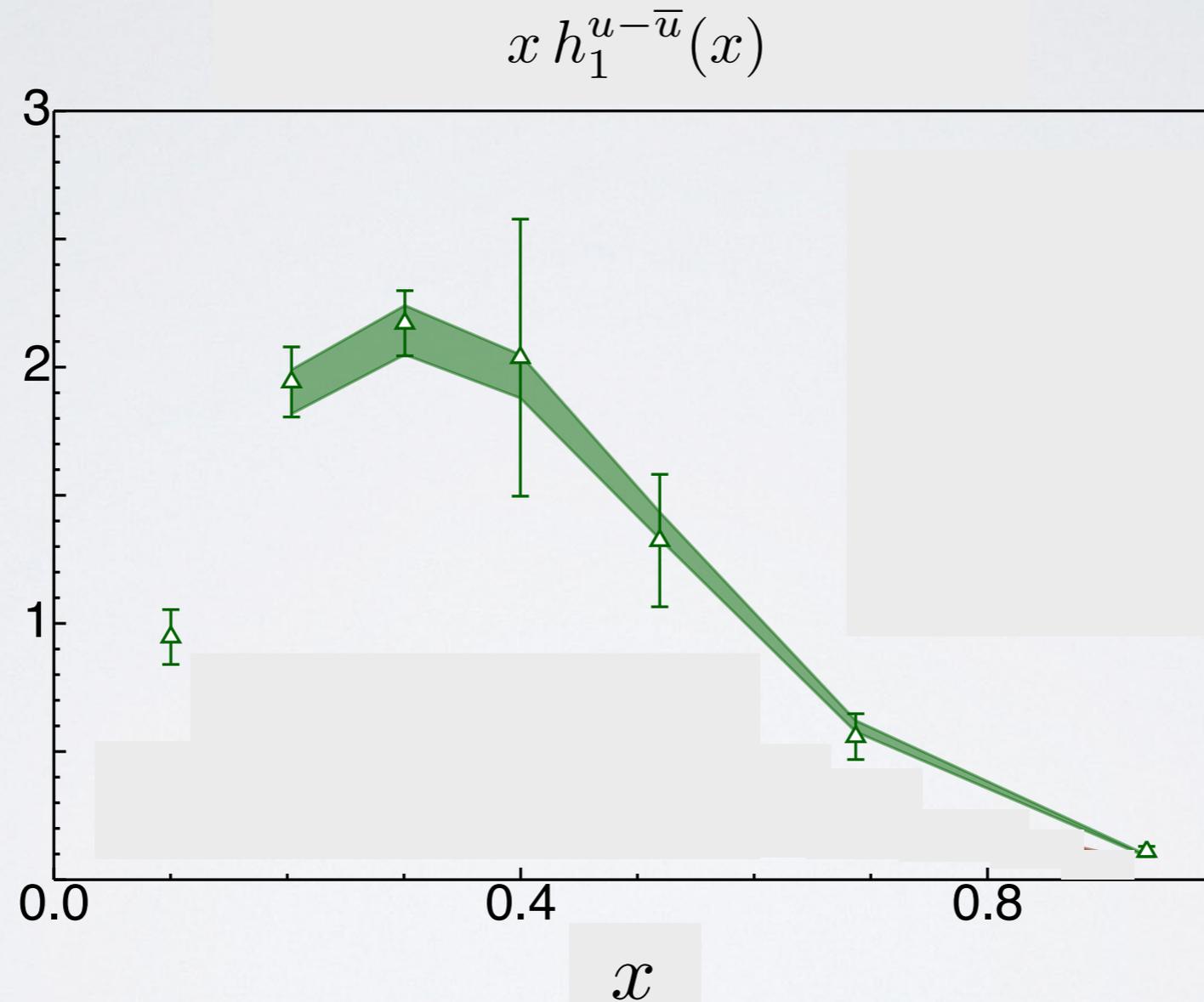
fit the replicated data

our fitting procedure



procedure repeated 100 times
(until reproduce mean and std. deviation of original data)

our fitting procedure



distribution of χ^2
should be peaked
around $\chi^2 \sim 1$

for each point, a central 68% confidence interval is identified
(distribution is not necessarily Gaussian)

Fitting : the functional form

at starting scale $Q_0^2 = 1 \text{ GeV}^2$

$$xh_1^{qv}(x) = \tanh \left[\sqrt{x} (A_q + B_q x + C_q x^2 + D_q x^3) \right] [x \text{SB}_q(x) + x \overline{\text{SB}}_{\bar{q}}(x)]$$

$\text{SB}_q + \overline{\text{SB}}_{\bar{q}} \rightarrow \infty \quad x \rightarrow 0$
grants finite and stable
tensor charge

automatically satisfies
Soffer bound
at any Q^2

$$2|h_1^q(x, Q^2)| \leq 2 \text{SB}_q(x) = |f_1^q(x) + g_1^q(x)|$$

MSTW08LO

DSSV

error on SB negligible
w.r.t. exp. error and
uncertainty on DiFF fit

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MSTW08LO

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rigid



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 MSTW08LO DSSV

error on SB negligible
w.r.t. exp. error and
uncertainty on DiFF fit

rigid



flexible



Fitting : the functional form

at starting scale $Q_0^2 = 1 \text{ GeV}^2$

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$\text{SB}_q + \overline{\text{SB}}_{\bar{q}} \rightarrow \infty \quad x \rightarrow 0$
grants finite and stable
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Soffer bound
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$$2|h_1^q(x, Q^2)| \leq 2 \text{SB}_q(x) = |f_1^q(x) + g_1^q(x)|$$

rigid



flexible



extra-flexible



$$\chi^2/\text{dof} \sim 1.2$$

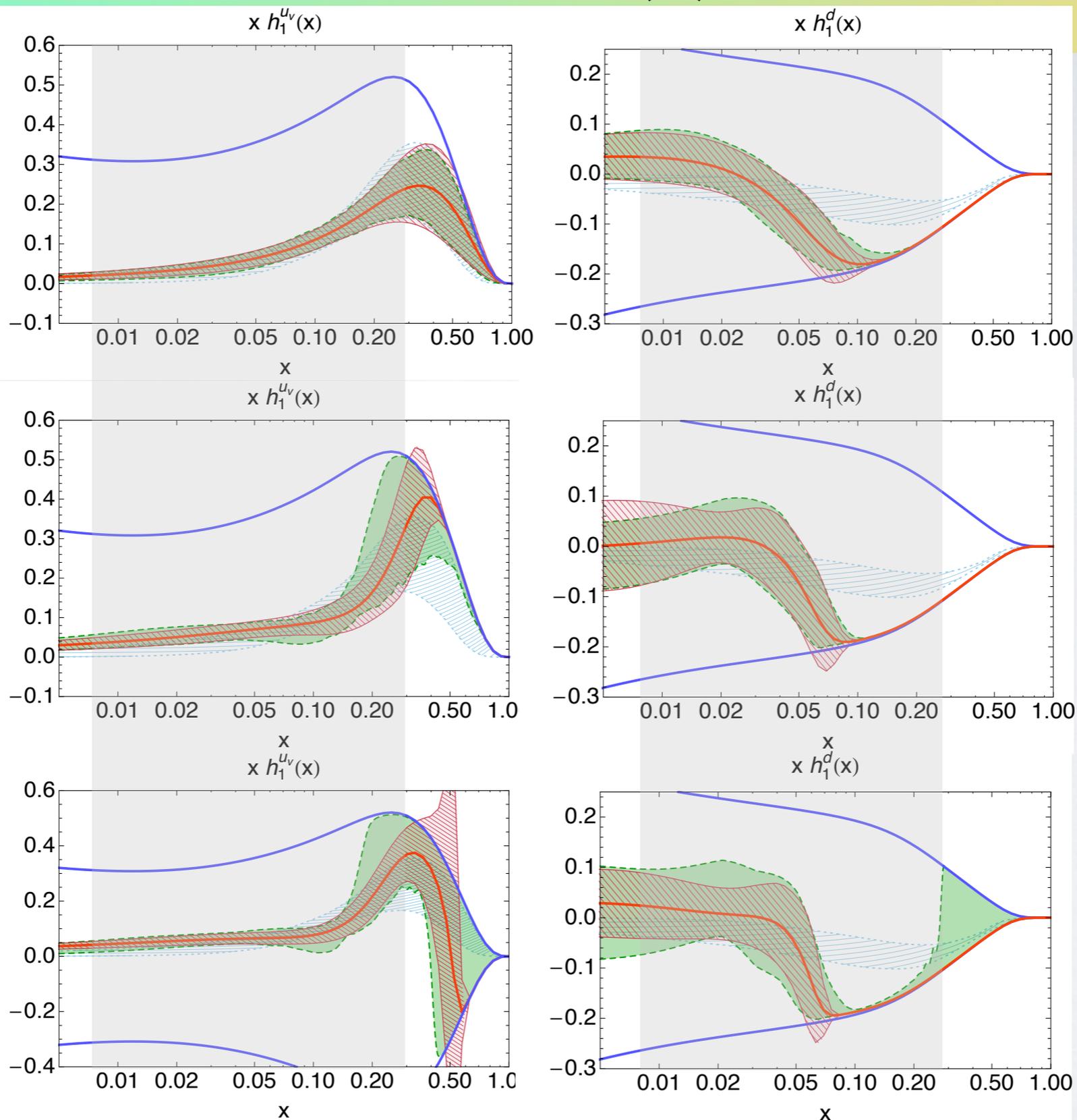
MSTW08LO

DSSV

error on SB negligible
w.r.t. exp. error and
uncertainty on DiFF fit

$Q^2 = 2.4 \text{ GeV}^2$

$u - \bar{u} \quad x h_1^{q-\bar{q}}(x) \quad d - \bar{d}$



rigid



flexible

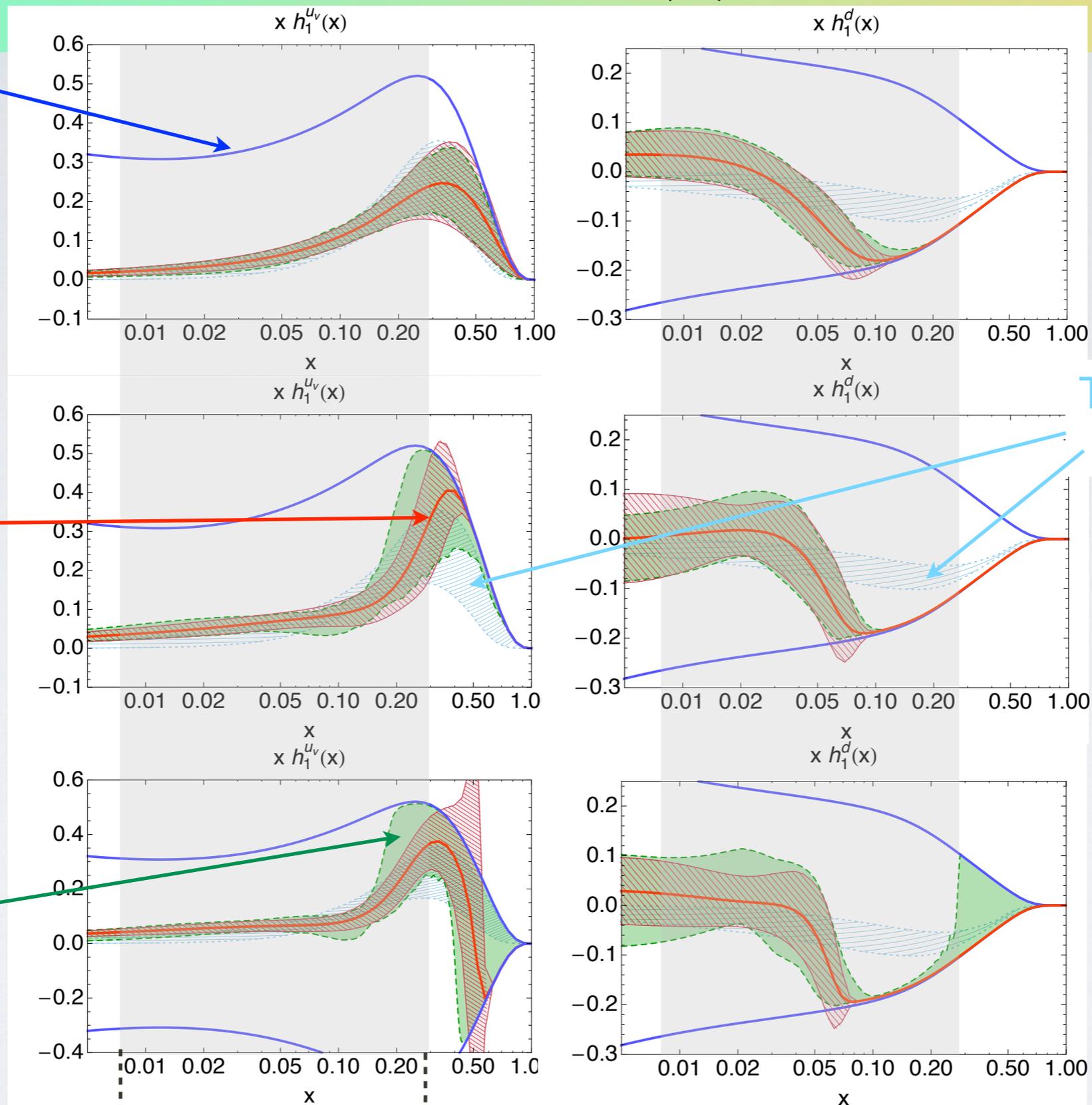


extra flexible

$Q^2 = 2.4 \text{ GeV}^2$

$u - \bar{u} \quad x h_1^{q-\bar{q}}(x) \quad d - \bar{d}$

Soffer bound



central value
for standard fit
with 1σ band

68% band of
replicas

Bacchetta, Courtoy, Radici,
JHEP **1303** (13) 119

data



rigid

Torino param.
2009



flexible

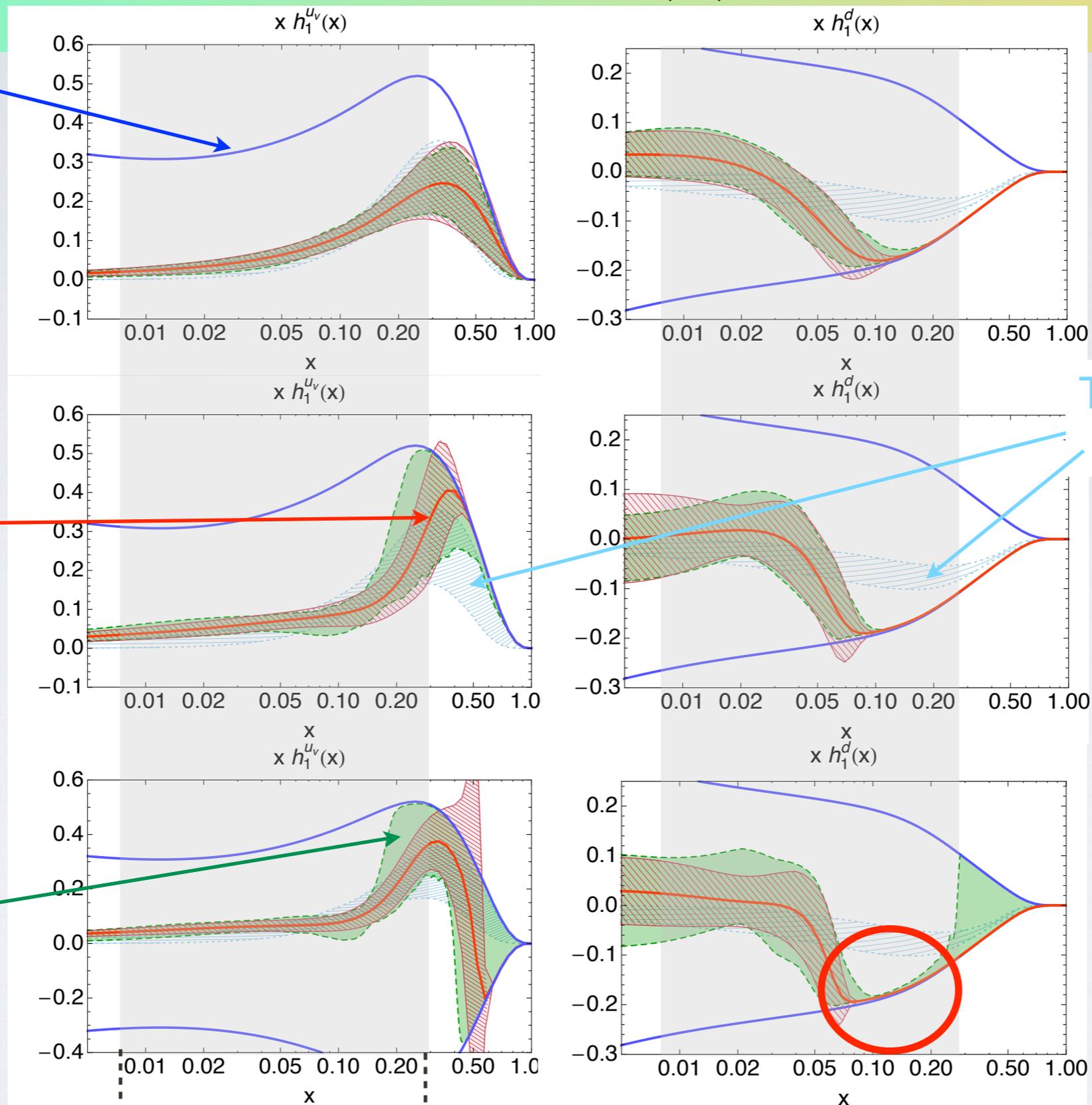


extra
flexible

$Q^2 = 2.4 \text{ GeV}^2$

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Soffer bound



central value
for standard fit
with 1σ band

68% band of
replicas

Bacchetta, Courtoy, Radici,
JHEP **1303** (13) 119

← data →

driven by COMPASS deuteron data



rigid

Torino param.
2009



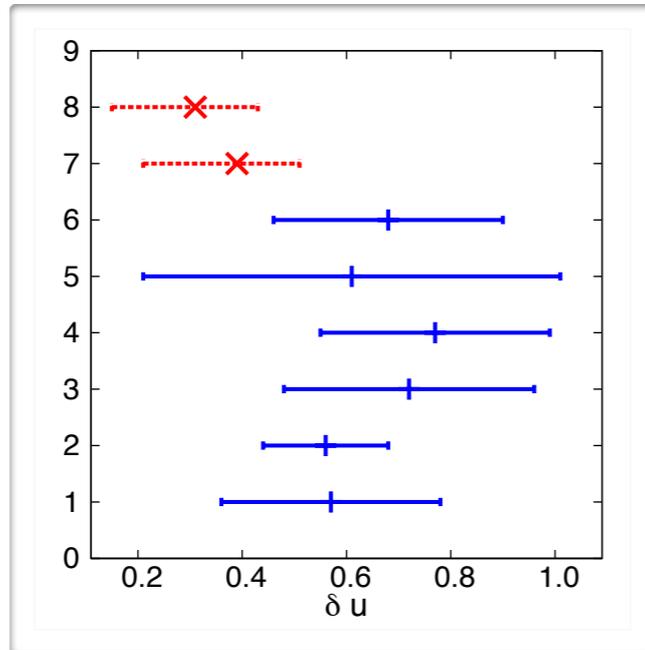
flexible



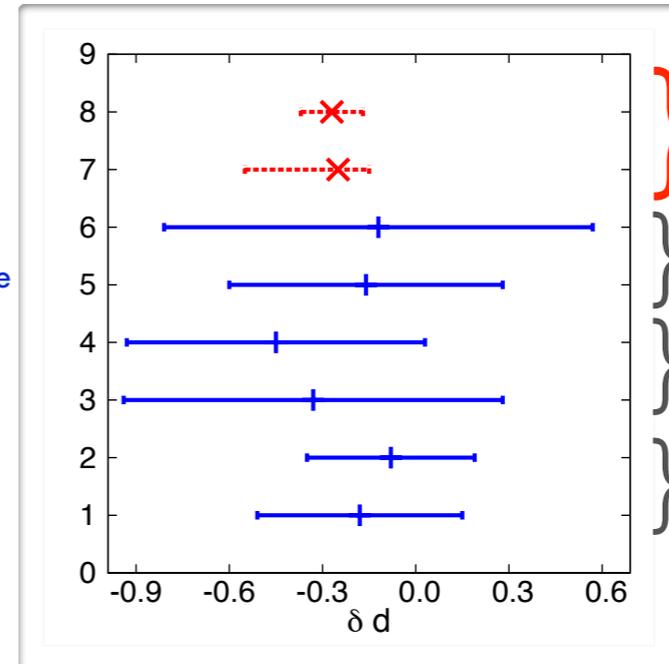
extra
flexible

tensor charges

up



down



8. fit of A₀
7. fit of A₁₂
6. MC extra flexible
5. standard extra flexible
4. MC flexible
3. standard flexible
2. MC rigid
1. standard rigid

Collins effect



$$Q_0^2 = 1 \text{ GeV}^2$$

$$\delta q = \int_{\sim 0}^1 dx h_1^{qv}(x)$$

full range

tensor charges

$$g_T = \delta u - \delta d$$

LHPC

$$g_T = 1.038(20)$$

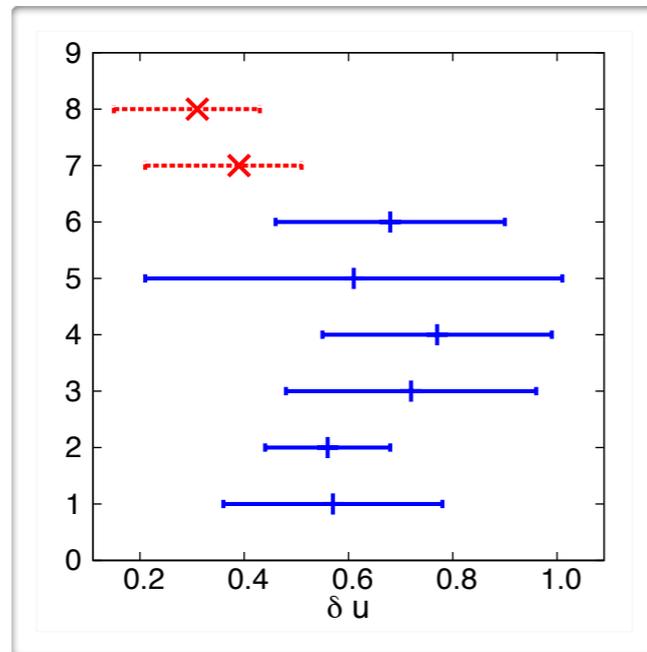
*Green et al.,
P.R. D86 (12)*

MILC

$$g_T = 1.083(48)$$

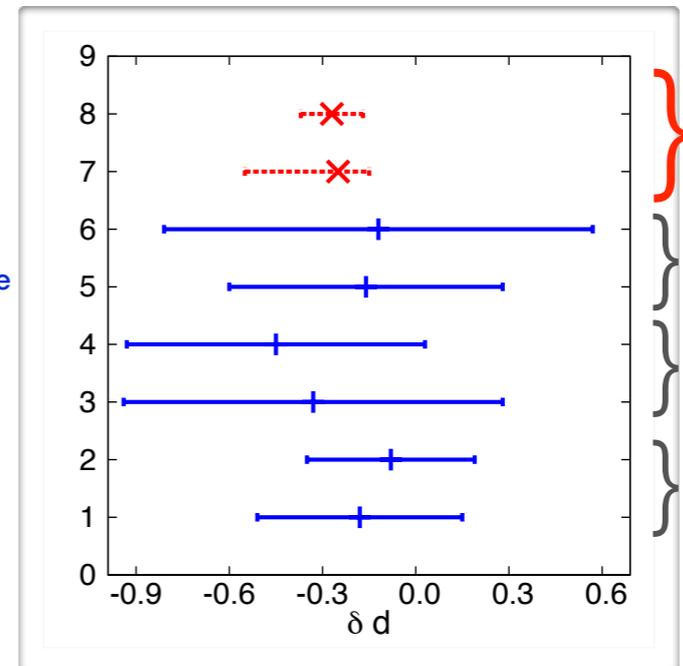
*Bhattacharya et al.,
arXiv: 1306.5435*

up



8. fit of A_0
7. fit of A_{12}
6. MC extra flexible
5. standard extra flexible
4. MC flexible
3. standard flexible
2. MC rigid
1. standard rigid

down



Collins effect



$$Q_0^2 = 1 \text{ GeV}^2$$

$$\delta q = \int_{\sim 0}^1 dx h_1^{qv}(x)$$

full range

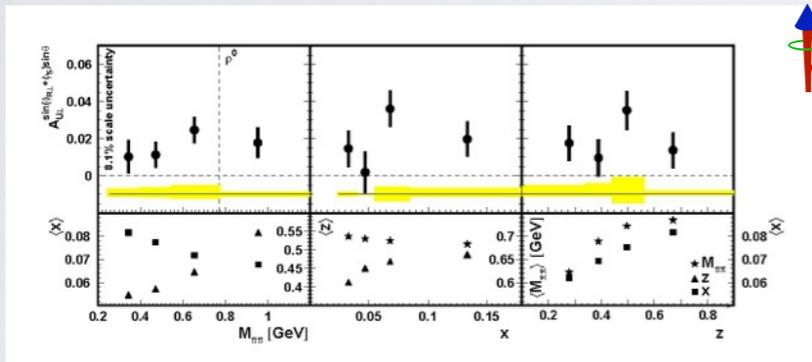
extrapolation of data
large uncertainty!

interlude with partial summary

our analysis

*Bacchetta, Courtoy, Radici,
JHEP 1303 (13) 119*

has used data from



proton

Airapetian et al., JHEP 0806 (08) 017

2002-4 Deuteron Data

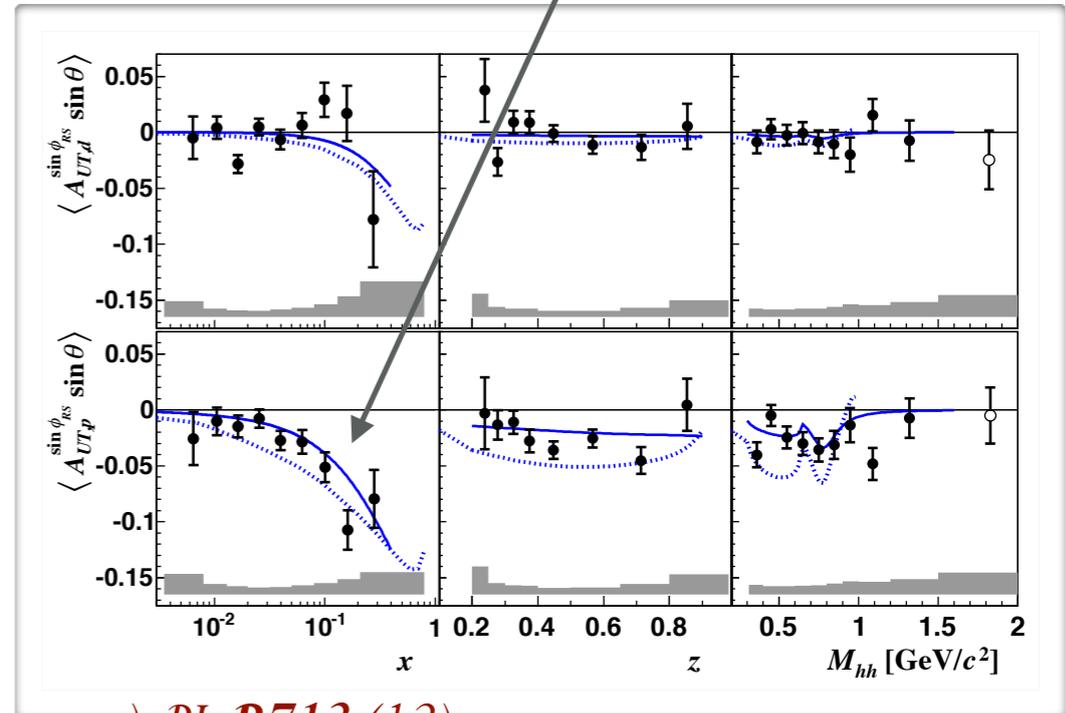


2007 Proton Data

C.Adolph et al. (Compass), PL B713 (12)

*Bacchetta & Radici,
P.R. D74 (06) 114007*

— prediction

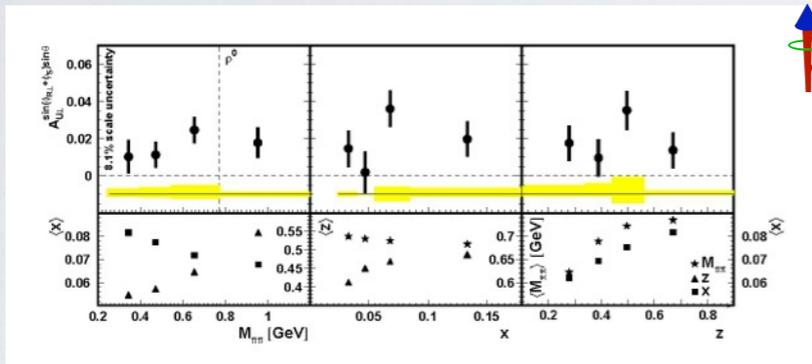


interlude with partial summary

our analysis

Bacchetta, Courtoy, Radici,
JHEP **1303** (13) 119

has used data from



proton

Airapetian et al., *JHEP* **0806** (08) 017

2002-4 Deuteron Data

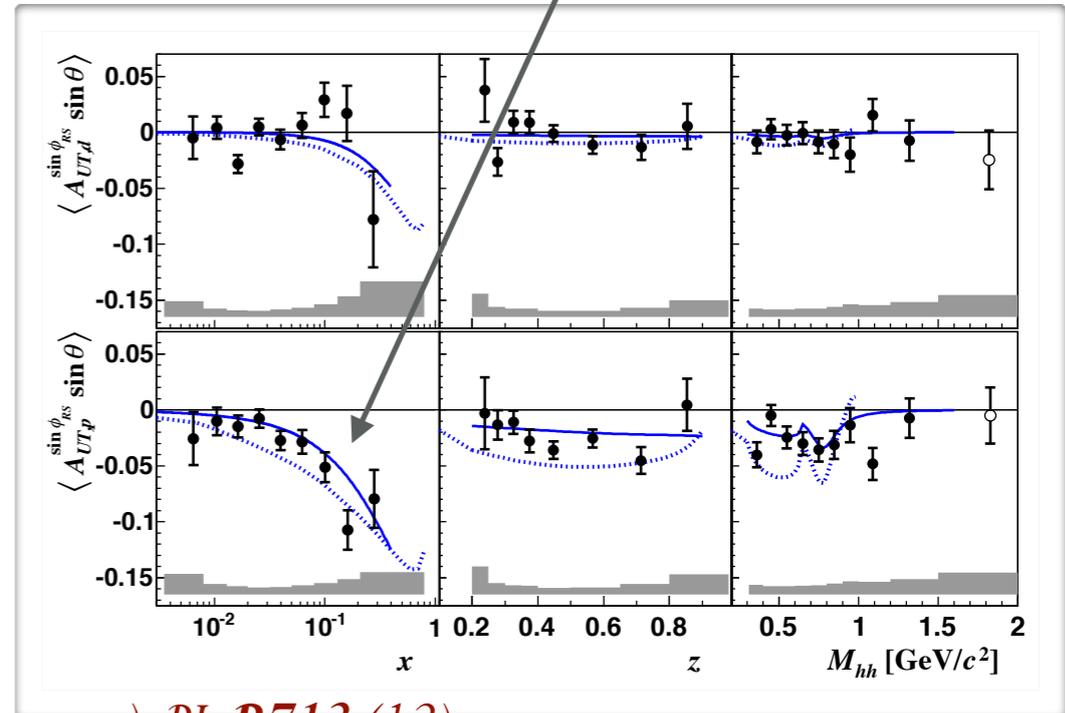


2007 Proton Data

C. Adolph et al. (Compass), *PL* **B713** (12)

Bacchetta & Radici,
P.R. D74 (06) 114007

— prediction



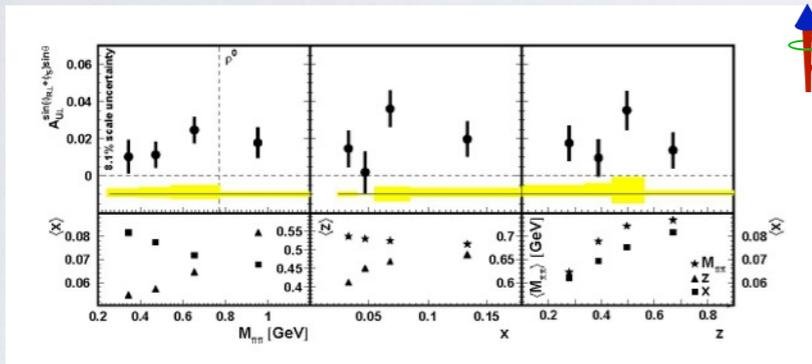
interlude with partial summary

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Bacchetta, Courtoy, Radici,
JHEP **1303** (13) 119

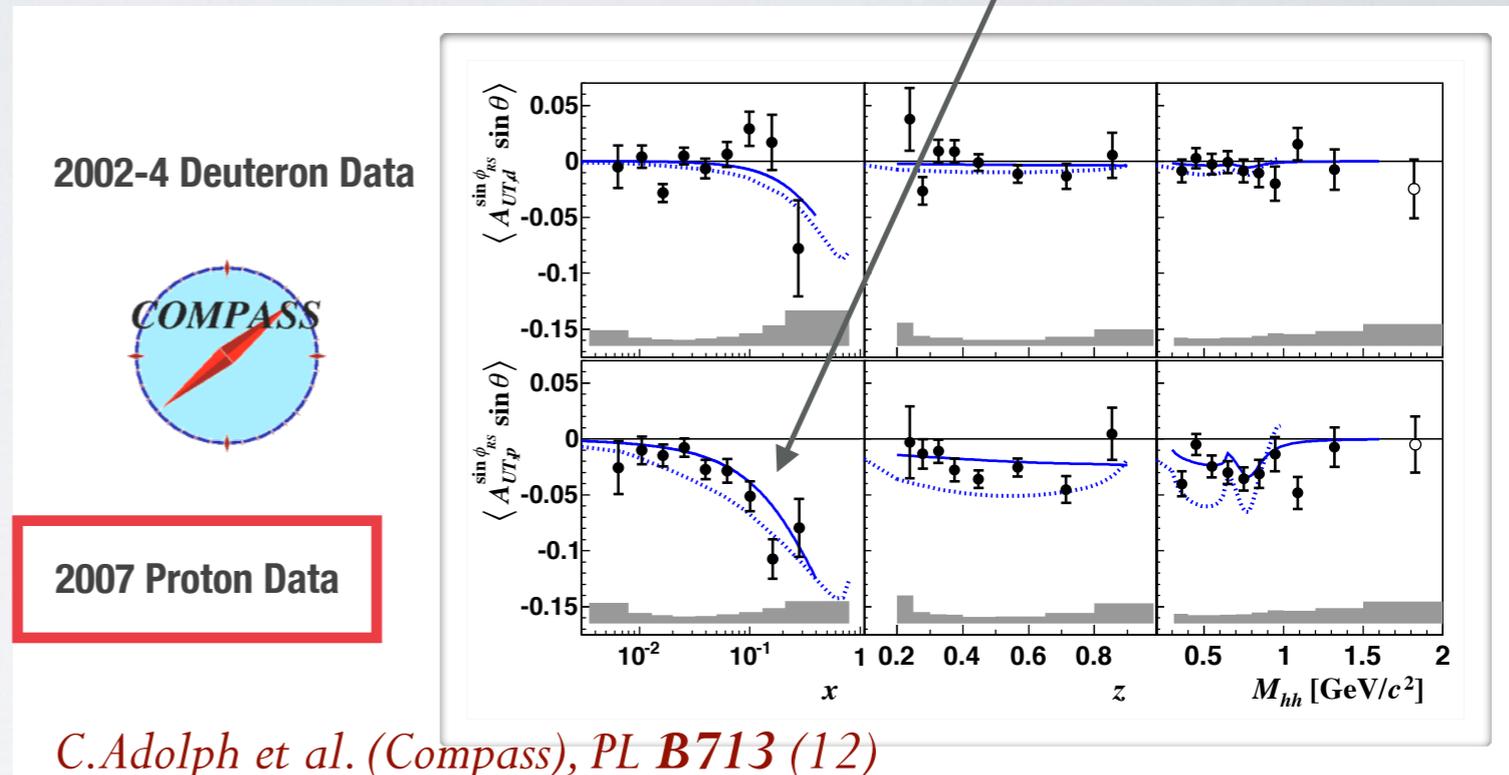
Bacchetta & Radici,
P.R. D74 (06) 114007

has used data from



proton

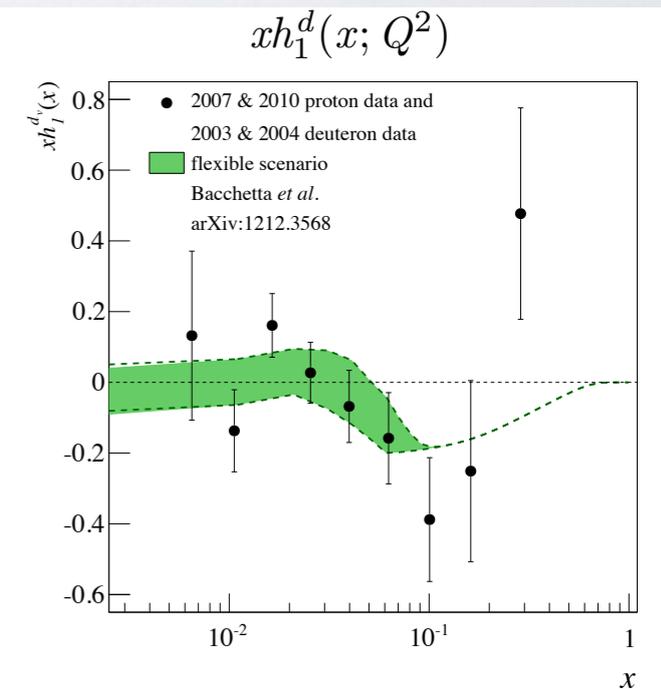
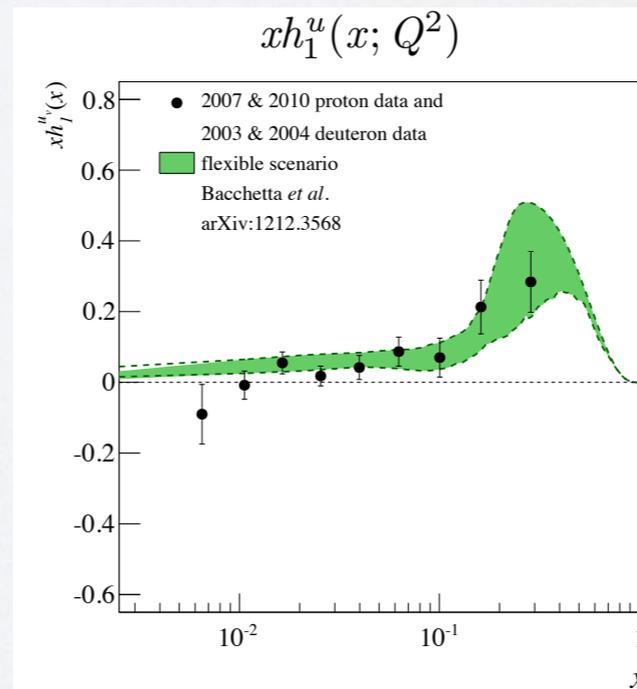
Airapetian et al., *JHEP* **0806** (08) 017



C. Adolph et al. (Compass), *PL* **B713** (12)

C. Braun's talk
point-by-point extraction
but uses new 2010 proton
data for $\pi^+\pi^-$

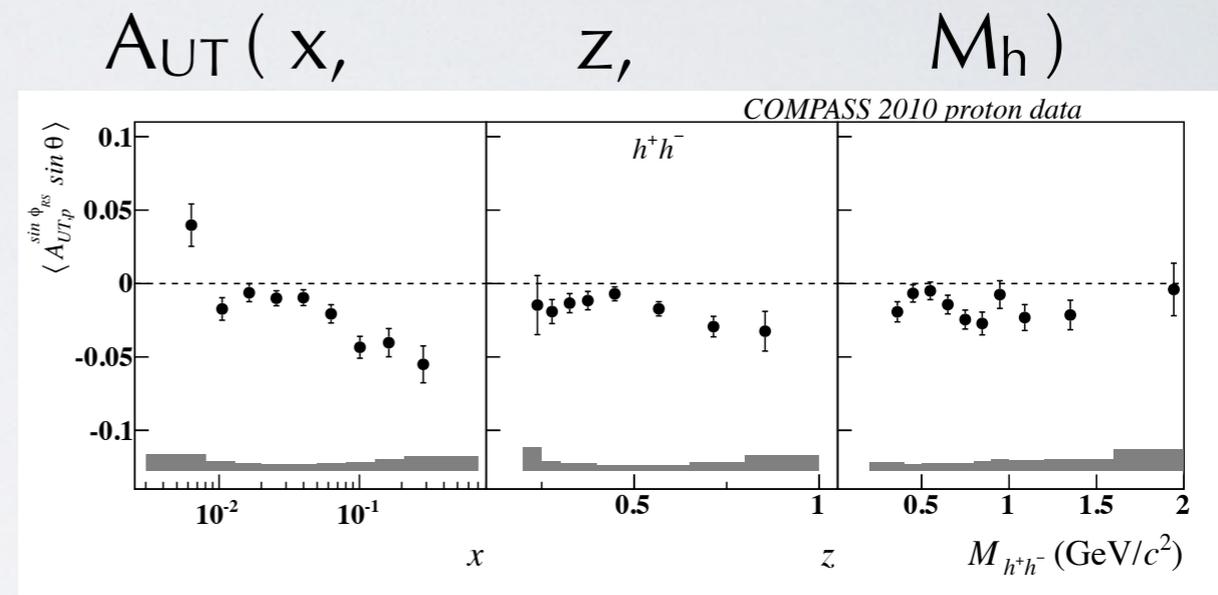
⇒ agreement



new fit

1. use new 

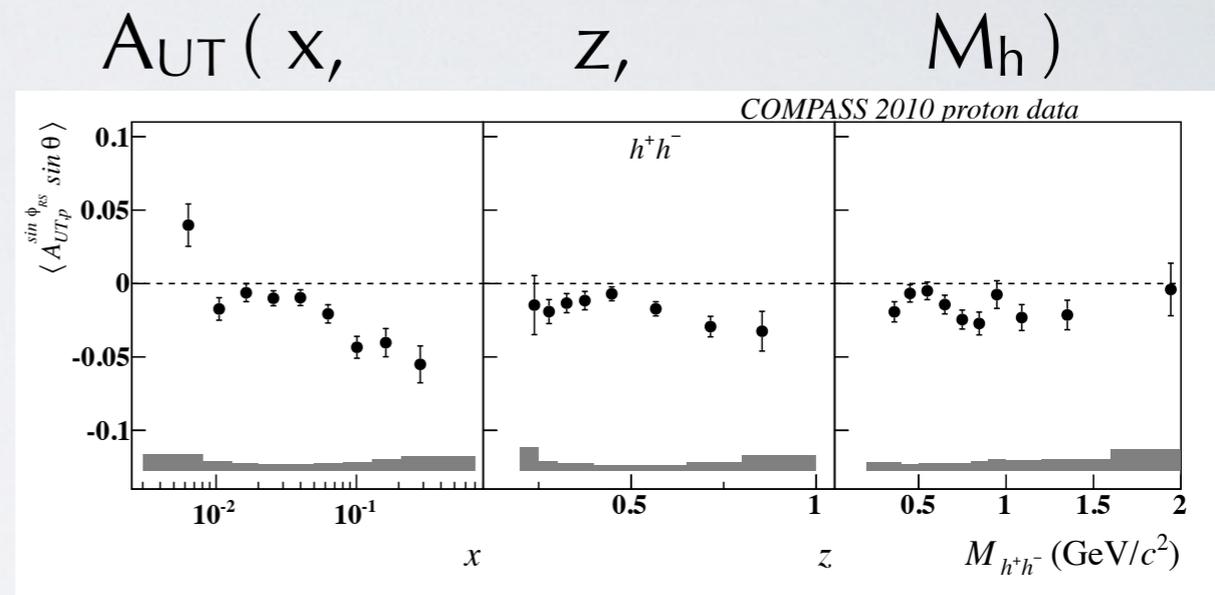
2010 proton data for h^+h^-



C. Adolph et al. (Compass), arXiv:1401.7873

new fit

1. use new  2010 proton data for h^+h^-

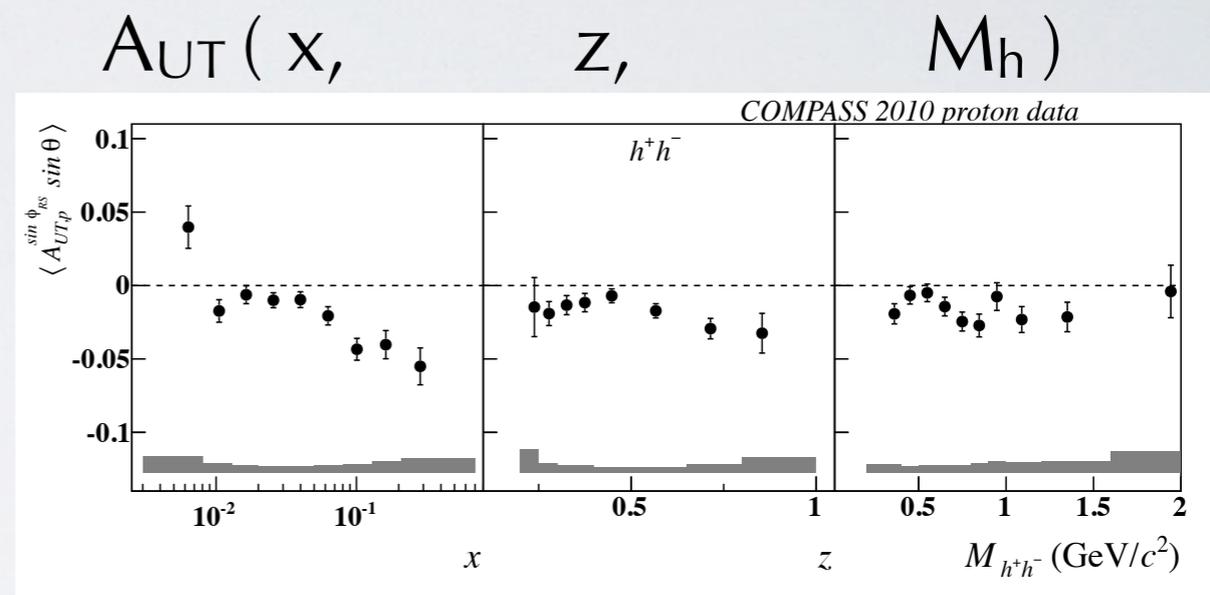


C. Adolph et al. (Compass), arXiv:1401.7873

2. use replica method to extract DiFF from  data

new fit

1. use new  2010 proton data for $h+h^-$



C. Adolph et al. (Compass), arXiv:1401.7873

2. use replica method to extract DiFF from  data

current most realistic estimate of
uncertainty on transversity

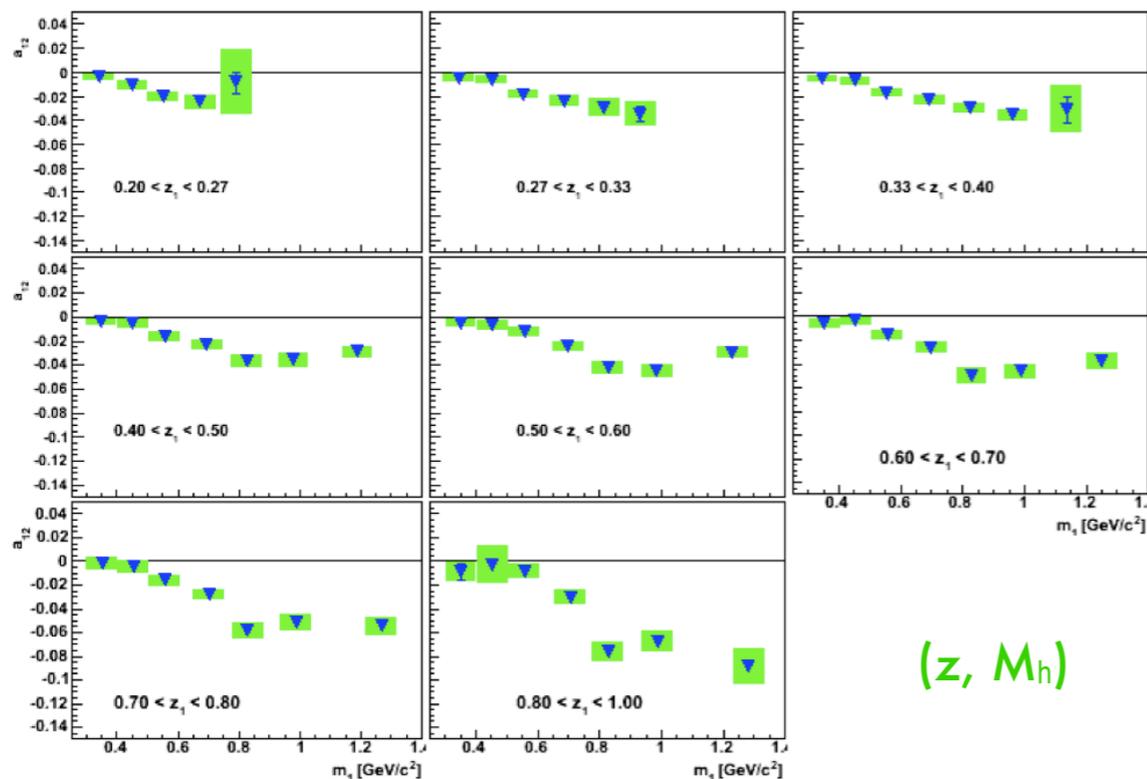
fit Belle data → extract DiFF

$$A^{\cos(\phi_R + \bar{\phi}_R)} = \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \frac{|\mathbf{R}_T|}{M_h} \frac{|\bar{\mathbf{R}}_T|}{\bar{M}_h} \times \frac{\sum_q e_q^2 H_{1,sp}^{\triangleleft q}(z, M_h^2) \bar{H}_{1,sp}^{\triangleleft \bar{q}}(\bar{z}, \bar{M}_h^2)}{\sum_q e_q^2 D_1^q(z, M_h^2) \bar{D}_1^{\bar{q}}(\bar{z}, \bar{M}_h^2)}$$

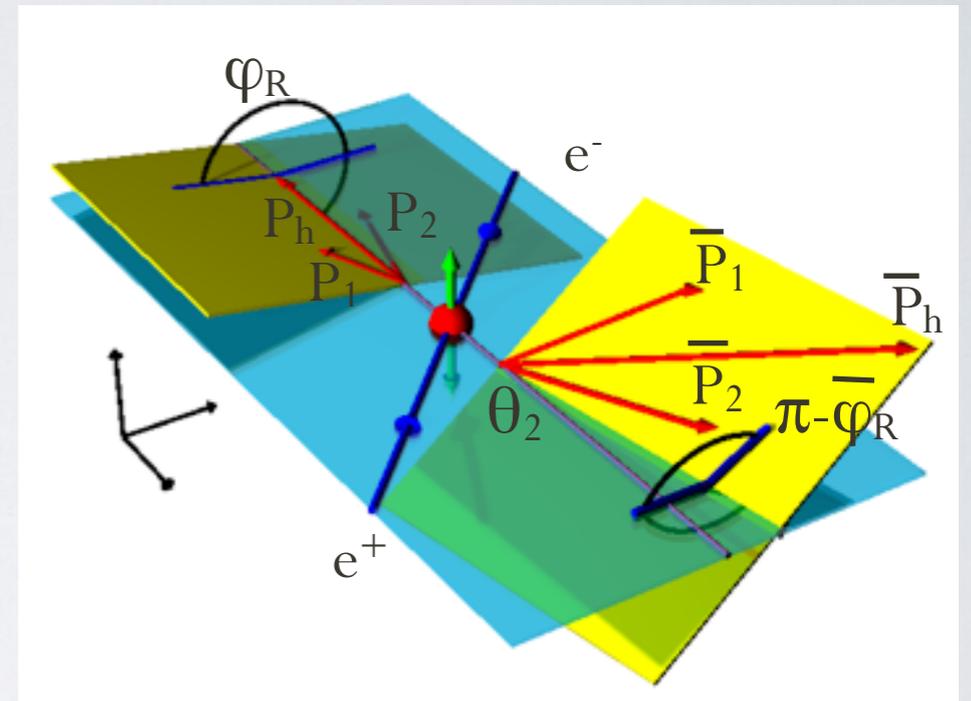
Boer, Jakob, Radici, P.R. D67 (03) 094003

Artru & Collins, Z.Ph. C69 (96) 277

(integrating on one hemisphere)



$$e^+e^- \rightarrow (\pi^+\pi^-)(\pi^+\pi^-) + X$$



unpol. D_1 extracted from
PYTHIA adapted to Belle
(very large statistics)
pol. H_1^{\triangleleft} extracted from fitting A^{\cos}



Vossen et al., P.R.L. 107 (11) 072004

first ever extraction of DiFF

Courtoy, Bacchetta, Radici, Bianconi, P.R. D85 (12) 114023

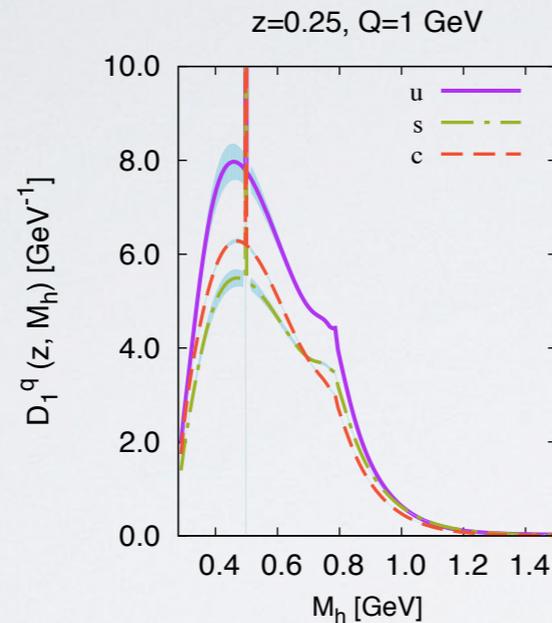
D_1^q

M_h behaviour

z behaviour

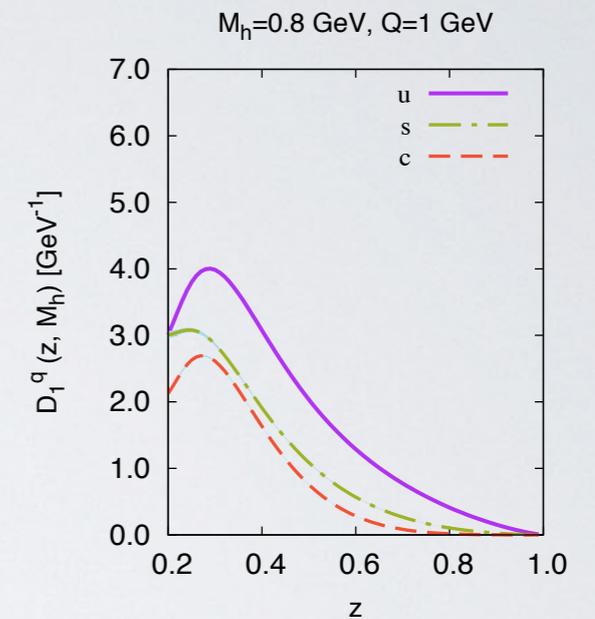
$D_1^q(z=0.25, M_h)$

$Q_0^2 = 1 \text{ GeV}^2$



$D_1^q(z, M_h=0.8)$

$Q_0^2 = 1 \text{ GeV}^2$



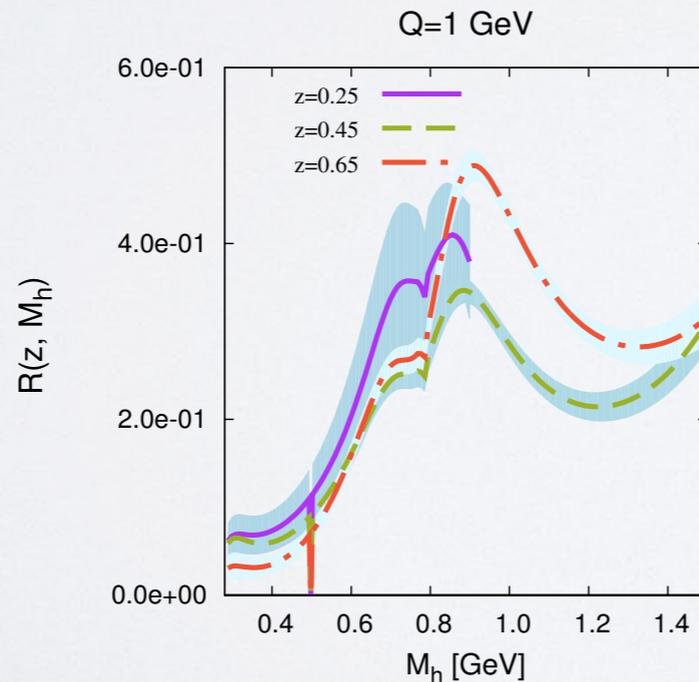
$\frac{|\mathbf{R}|}{M_h} \frac{H_1^{\leftarrow u}}{D_1^u}$

$z=0.25$

$z=0.45, M_h$

$z=0.65$

$Q_0^2 = 1 \text{ GeV}^2$

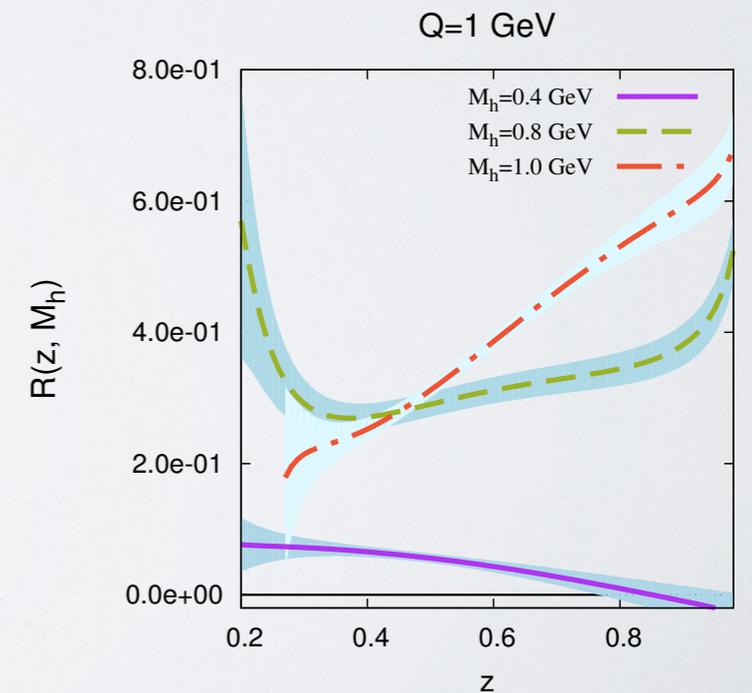


$M_h=0.4$

$M_h=0.8, z$

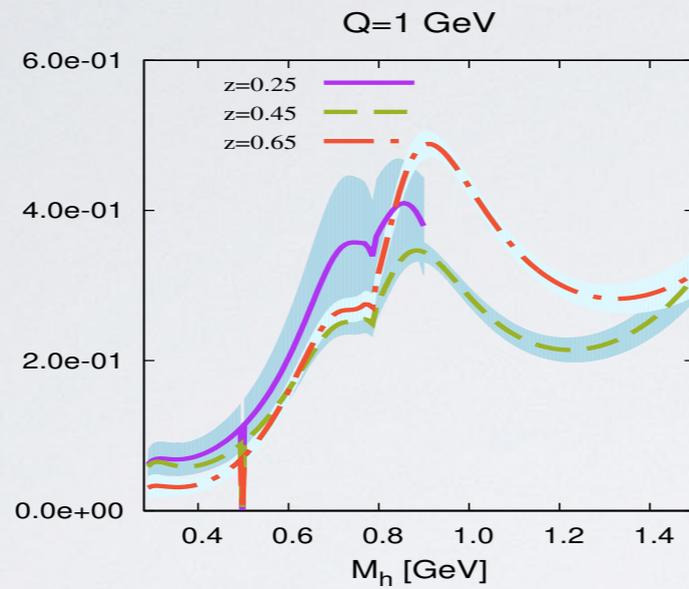
$M_h=1.$

$Q_0^2 = 1 \text{ GeV}^2$

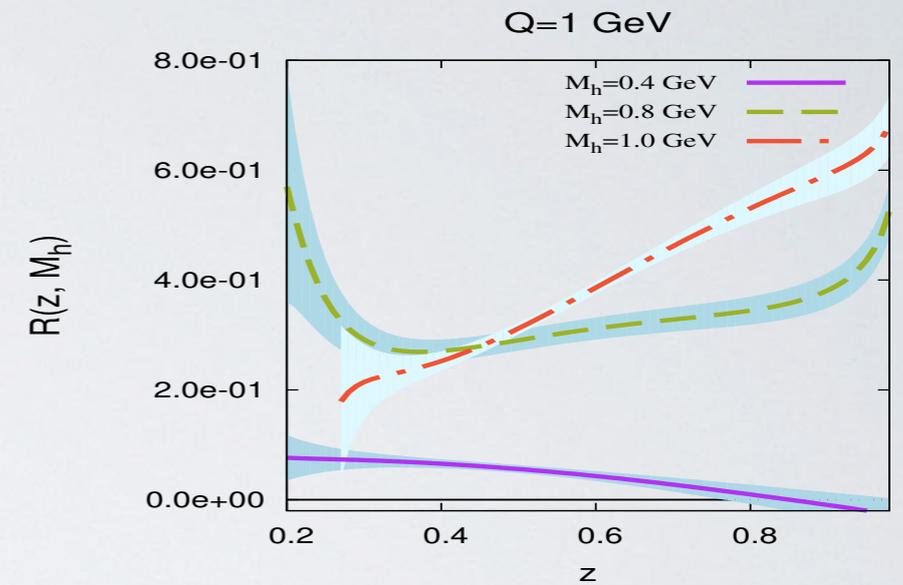


re-fit $H_1 \langle q \rightarrow \pi^+\pi^- \rangle$ using replica method

M_h behaviour



z behaviour



$$u \rightarrow \pi^+\pi^-$$

$i(z, M_h)$

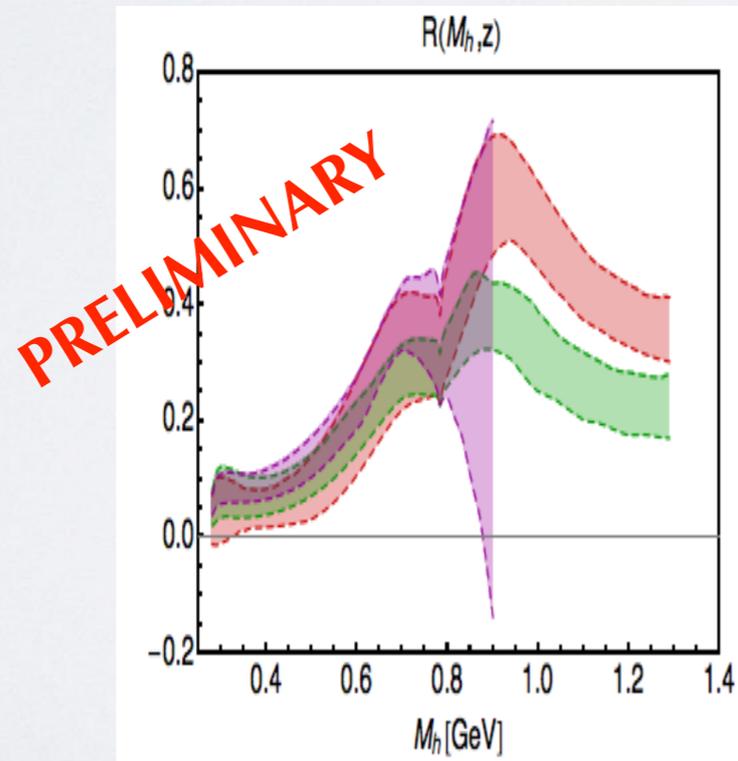
$$\frac{|\mathbf{R}|}{M_h} \frac{H_1^{\langle q \rangle u}}{D_1^u}$$

$z=0.25$

$z=0.45, M_h$

$z=0.65$

$$Q_0^2 = 1 \text{ GeV}^2$$

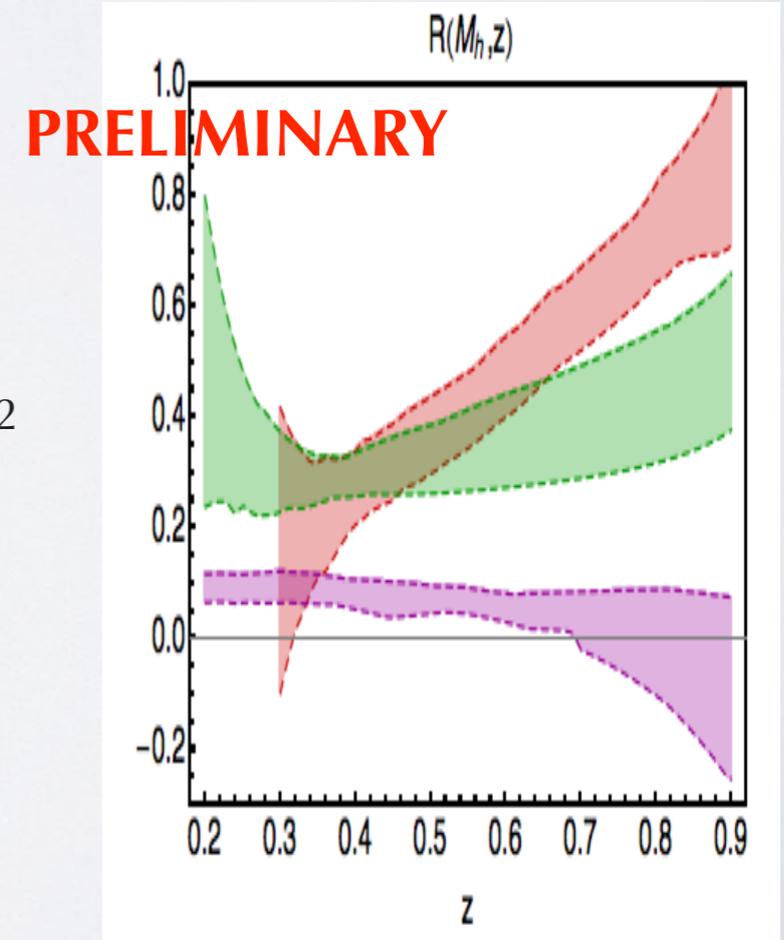


$M_h=0.4$

$M_h=0.8, z$

$M_h=1.$

$$Q_0^2 = 1 \text{ GeV}^2$$



impact on transversity extraction

Ex: proton data

$$\begin{aligned} xh_1^p(x) &\equiv xh_1^{u_v}(x) - \frac{1}{4}xh_1^{d_v}(x) \\ &\propto -\frac{A_{UT}^{\sin(\phi_R+\phi_S)}}{\int dzdM_h^2 H_1^{\triangleleft u}} \left[\sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_1^{q+\bar{q}}(x) \int dzdM_h^2 D_1^q \right] \end{aligned}$$

impact on transversity extraction

Ex: proton data

$$xh_1^p(x) \equiv xh_1^{u_v}(x) - \frac{1}{4}xh_1^{d_v}(x)$$

more precise data points

more realistic error on H_1^*

$$\propto \frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 H_1^{\leq u}} \left[\sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_1^{q+\bar{q}}(x) \int dz dM_h^2 D_1^q \right]$$

impact on transversity extraction

Ex: proton data

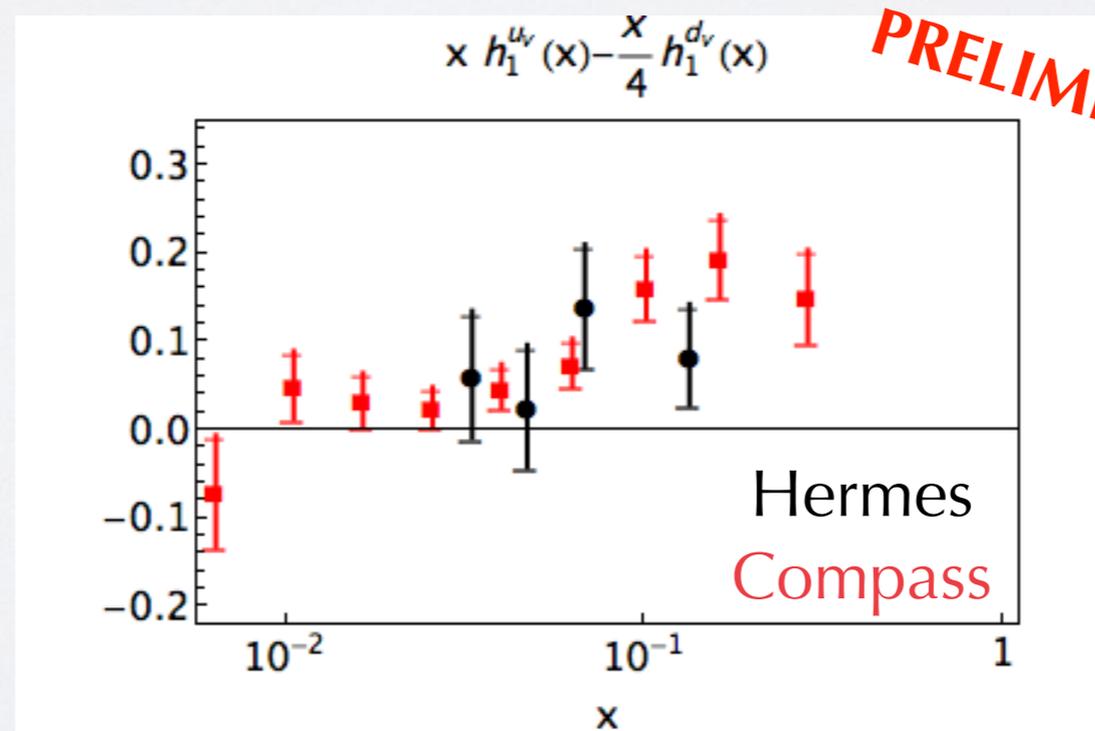
$$xh_1^p(x) \equiv xh_1^{u_v}(x) - \frac{1}{4}xh_1^{d_v}(x)$$

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extraction
point by point



impact on transversity extraction

Ex: proton data

$$xh_1^p(x) \equiv xh_1^{uv}(x) - \frac{1}{4}xh_1^{dv}(x)$$

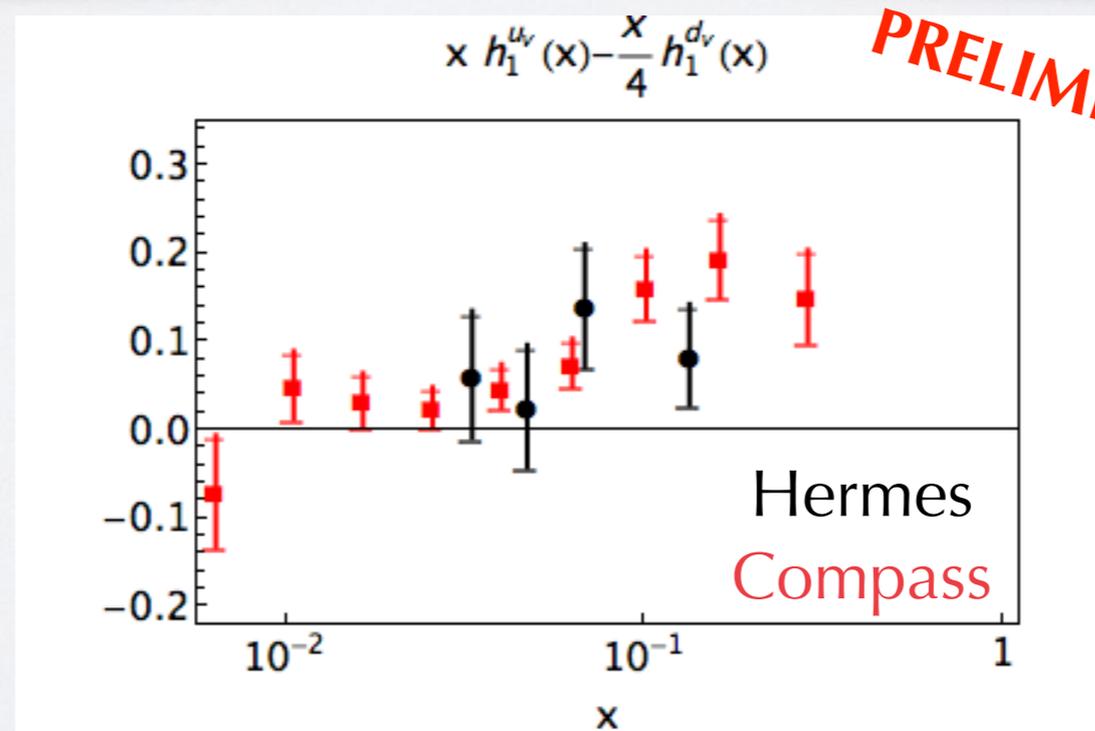
more precise data points

more realistic error on H_1^*

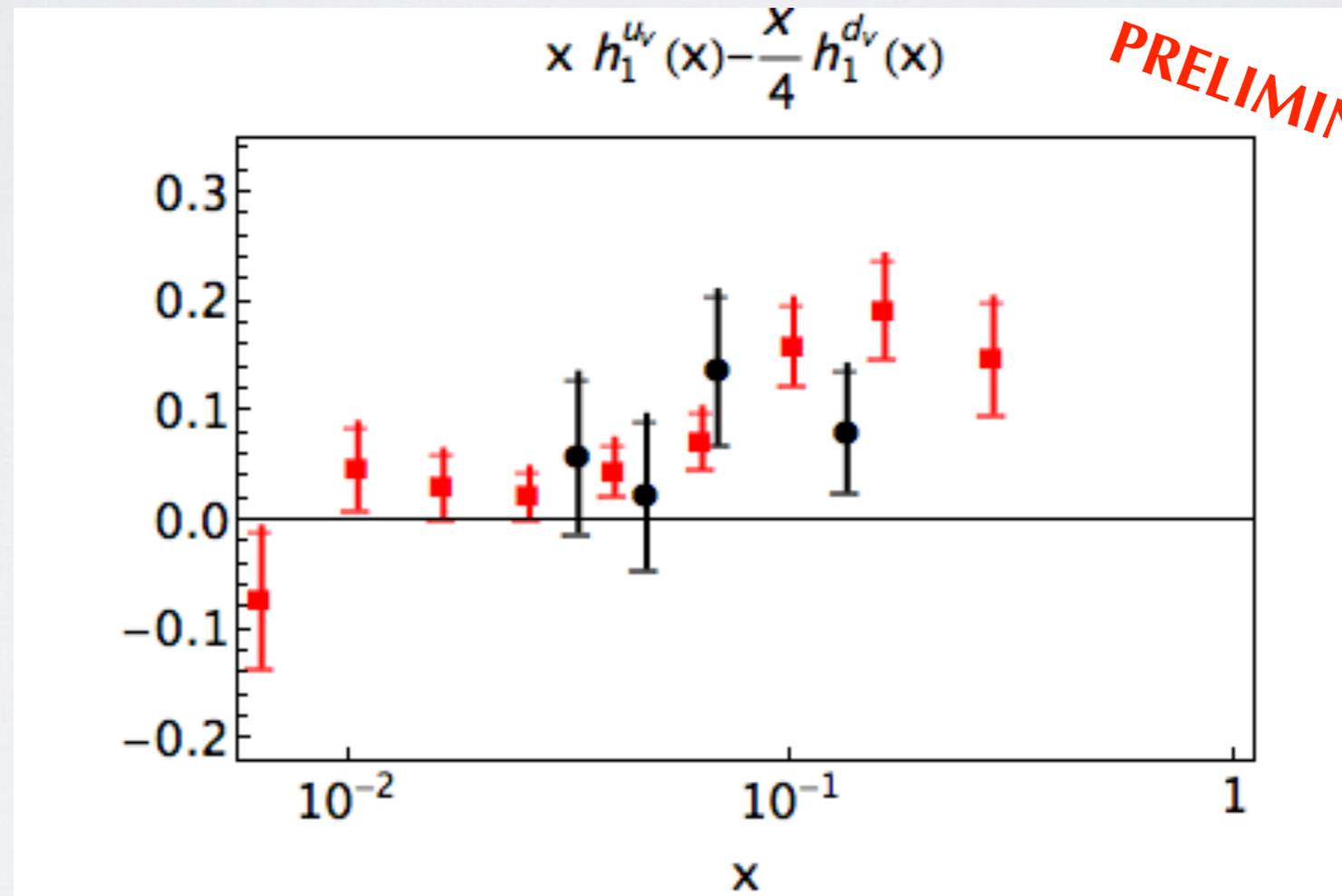
$$\propto \frac{A_{UT}^{\sin(\phi_R+\phi_S)}}{\int dz dM_h^2 H_1^{\triangleleft u}} \left[\sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_1^{q+\bar{q}}(x) \int dz dM_h^2 D_1^q \right]$$

replica method: alter data with Gaussian noise and randomly pick up corresponding H_1^*

extraction point by point

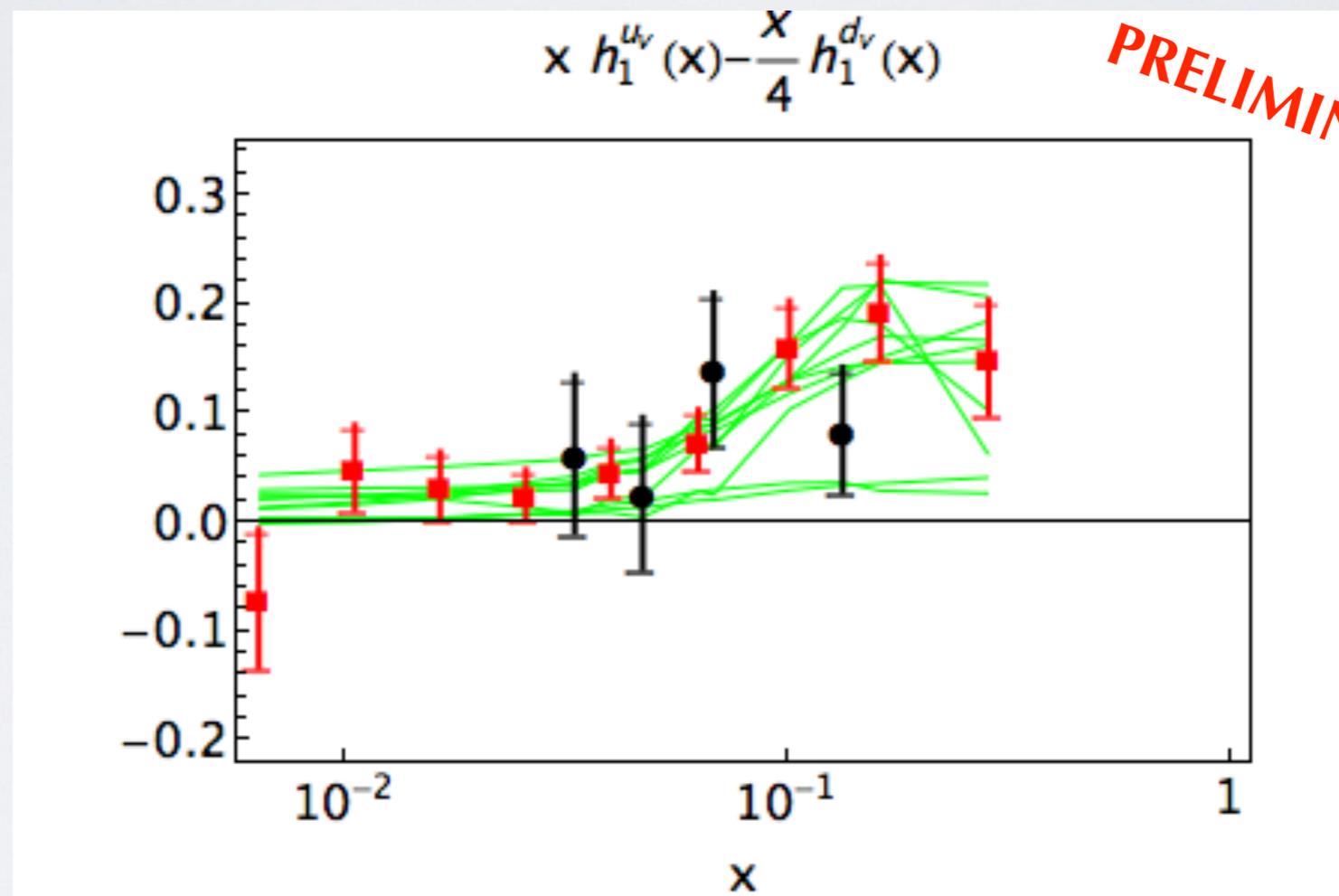


results of **new** fit



results of **new** fit

with **10** replica



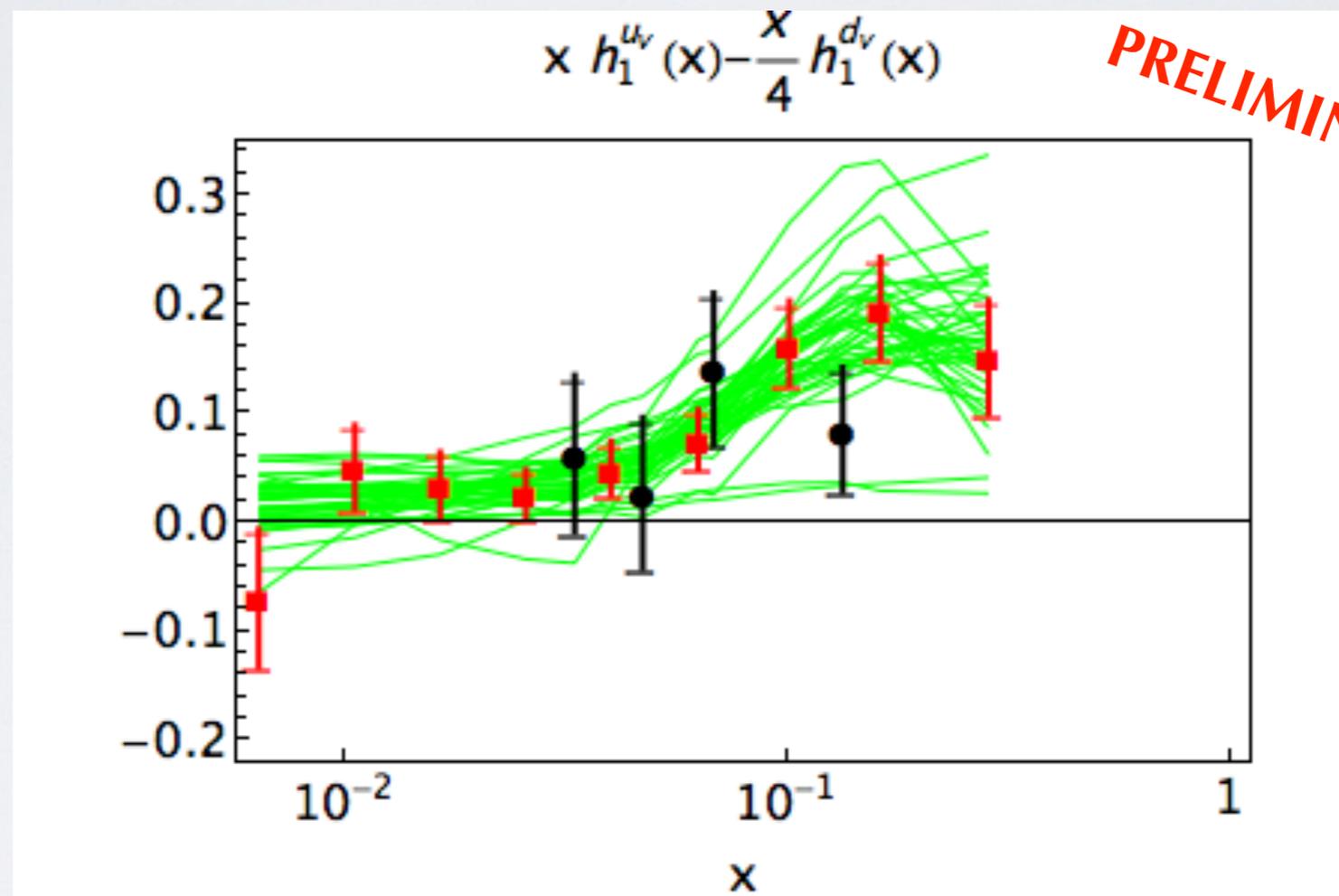
PRELIMINARY



flexible

results of **new** fit

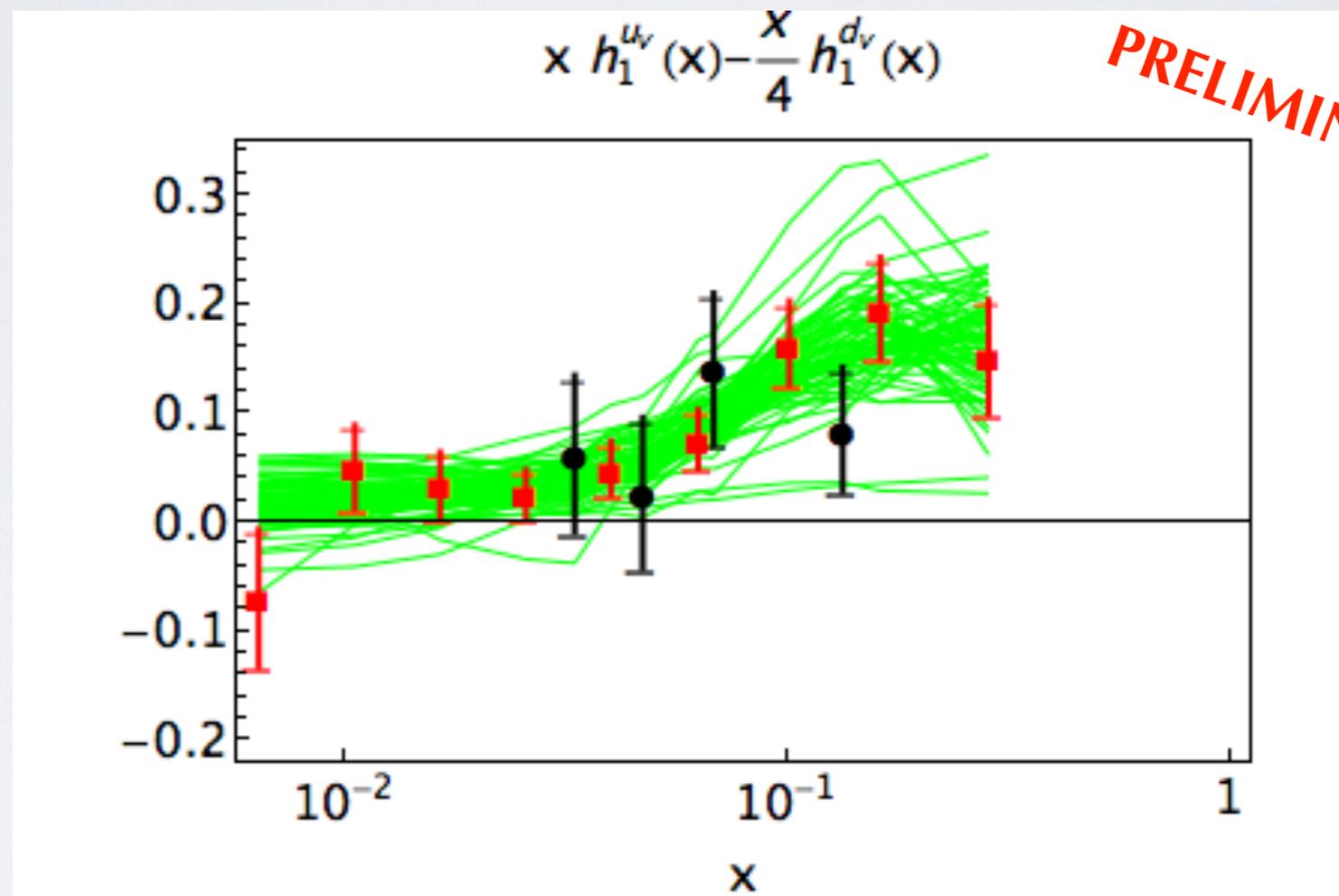
with **40** replica



flexible

results of **new** fit

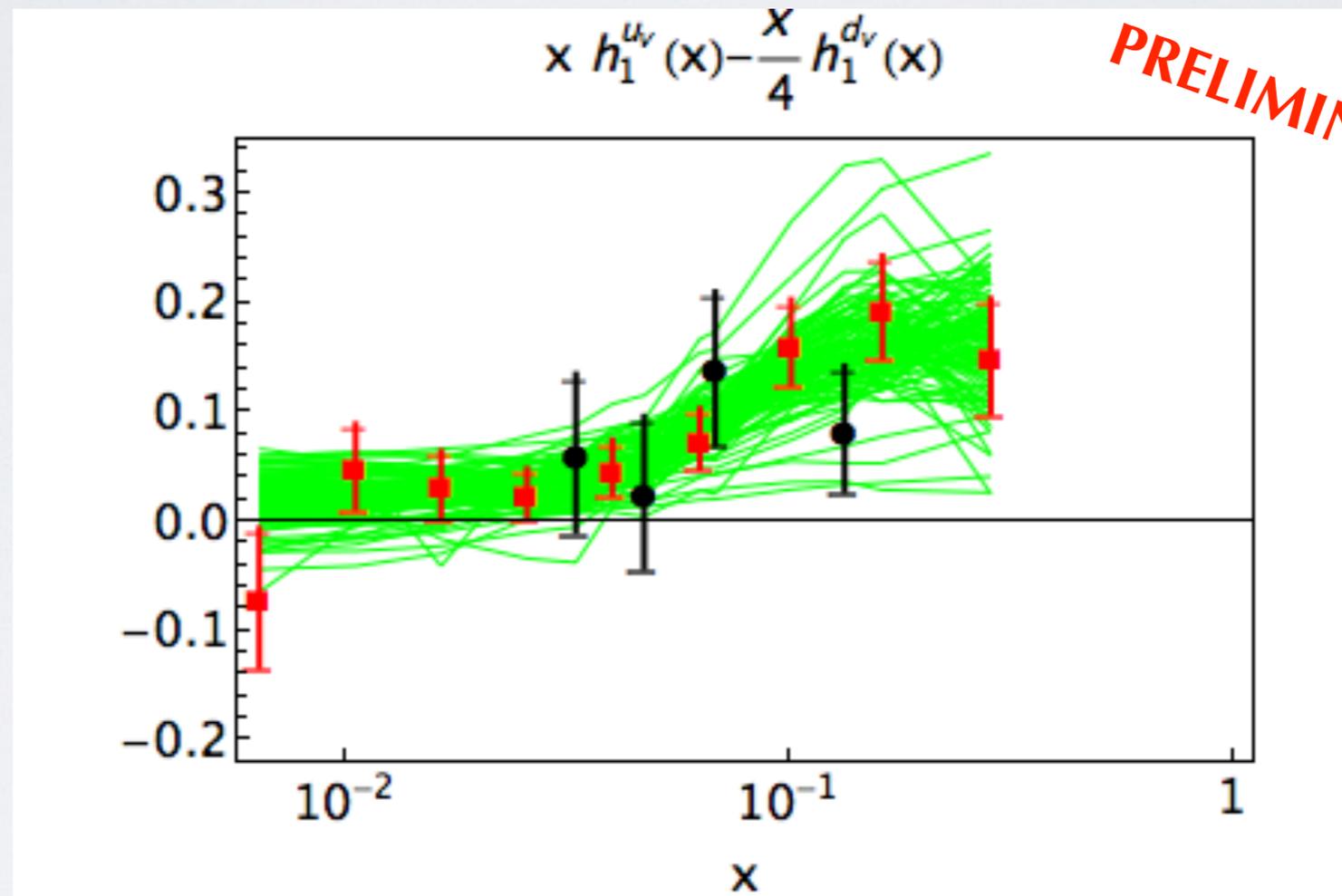
with **70** replica



flexible

results of **new** fit

with **100** replica



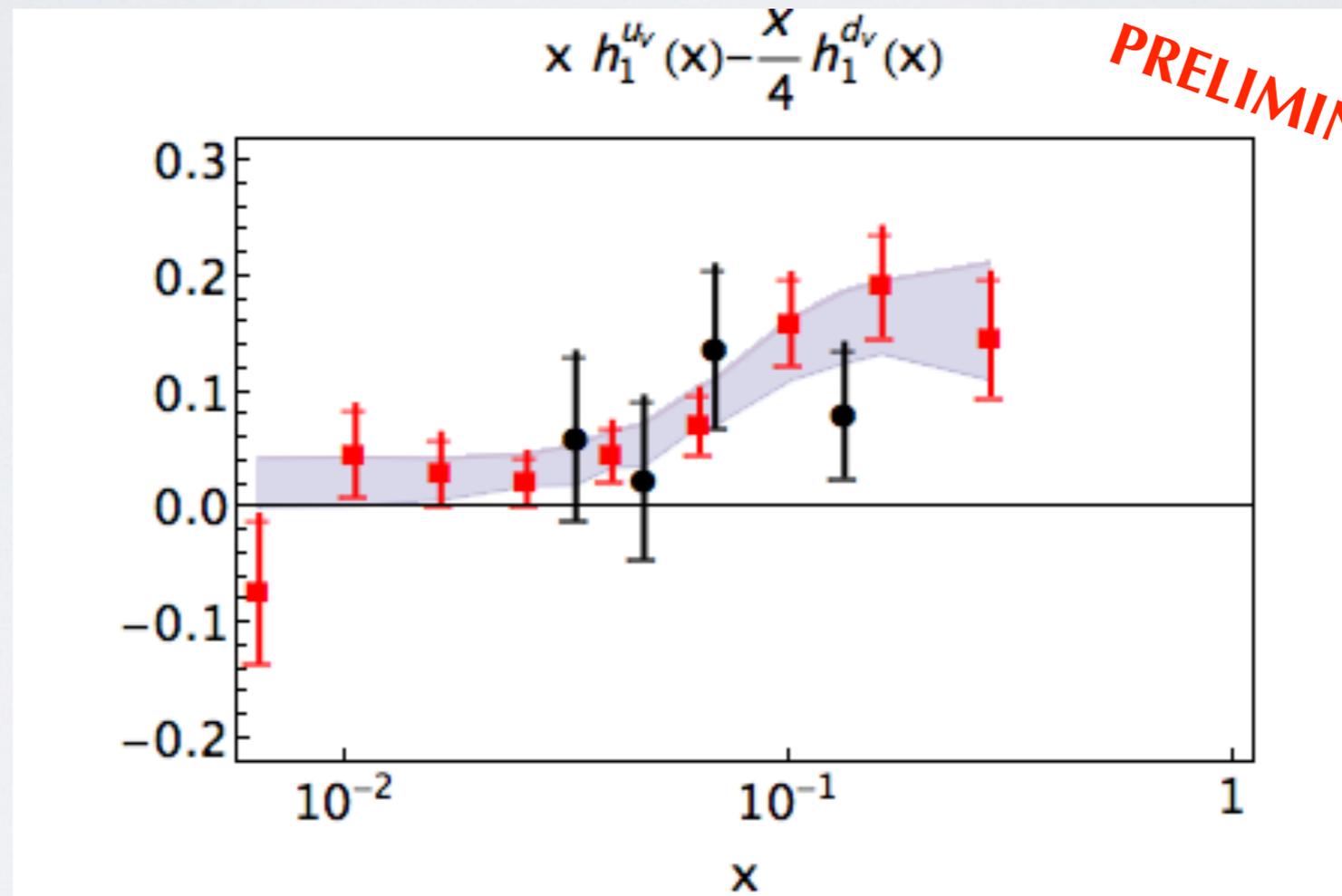
PRELIMINARY



flexible

results of **new** fit

taking the **68%** band

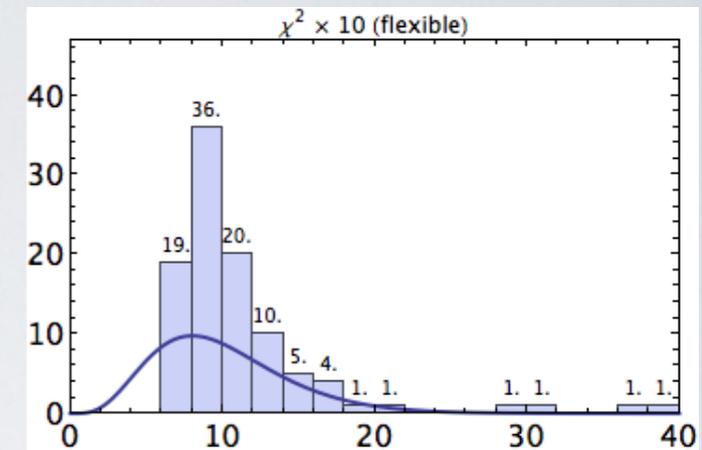
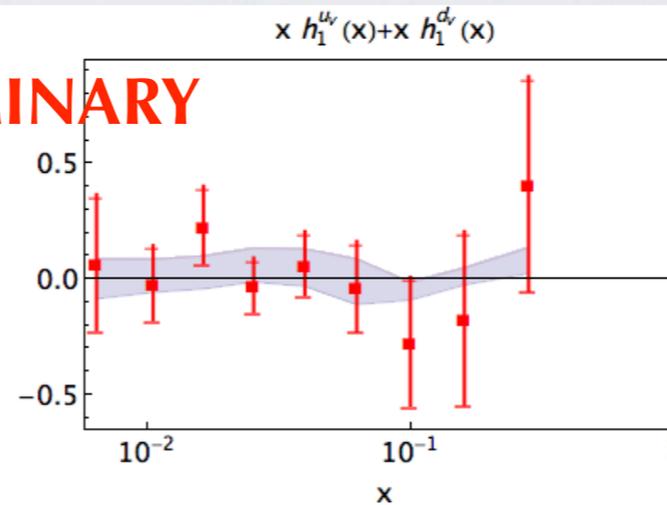
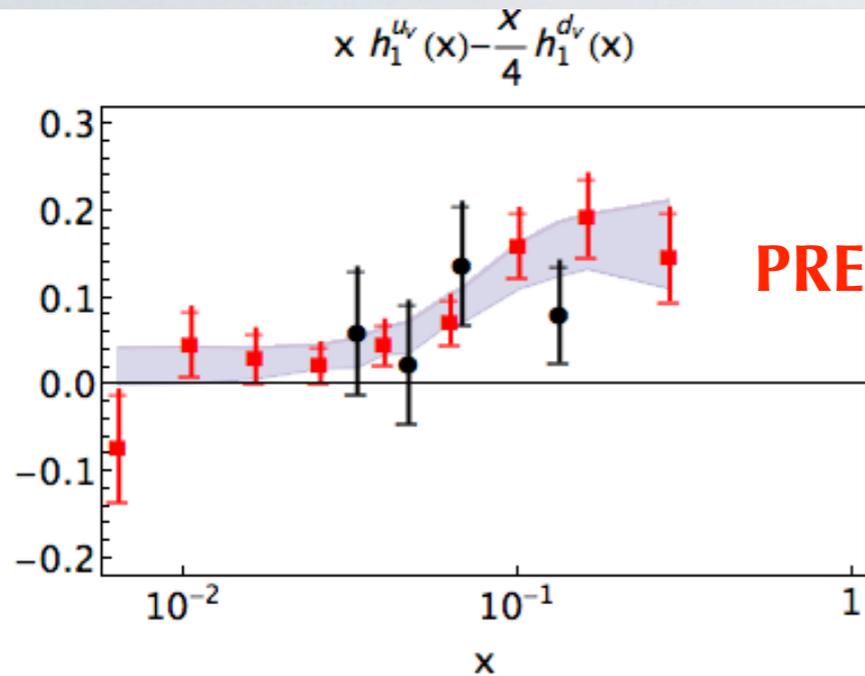


flexible

comparison new with previous fit

new

$\chi^2/\text{dof} \times 10$

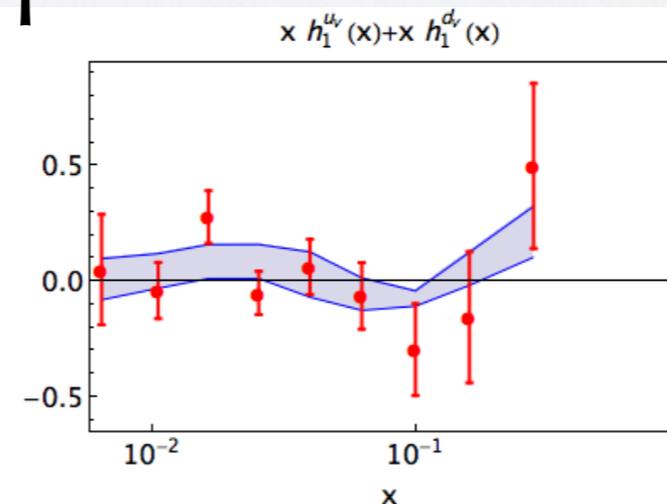
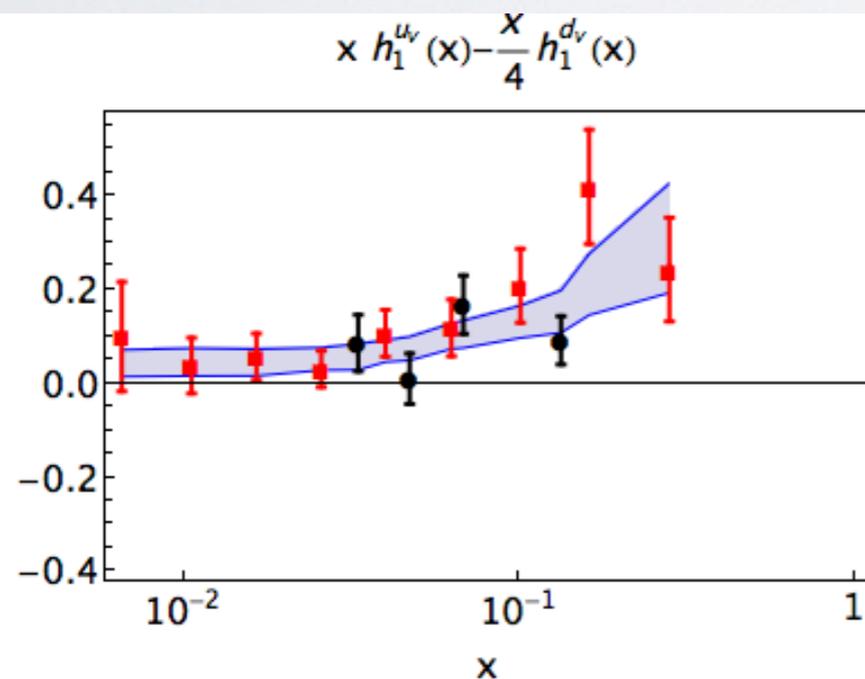


$$x h_1^{uv} - \frac{1}{4} x h_1^{dv} \propto - \frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 H_1^{\leftarrow u} \dots}$$



flexible

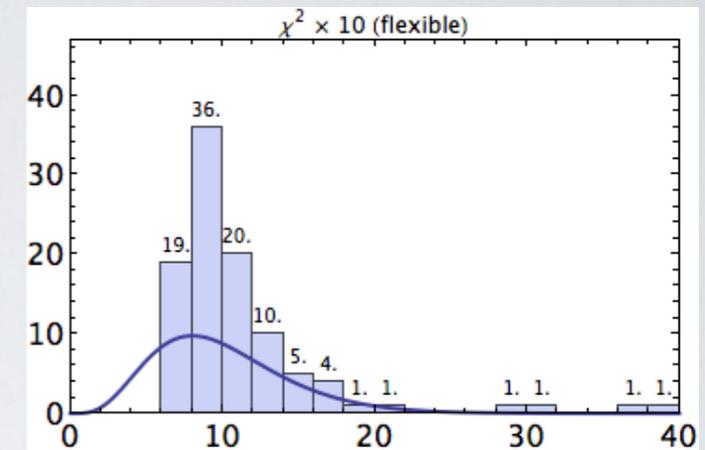
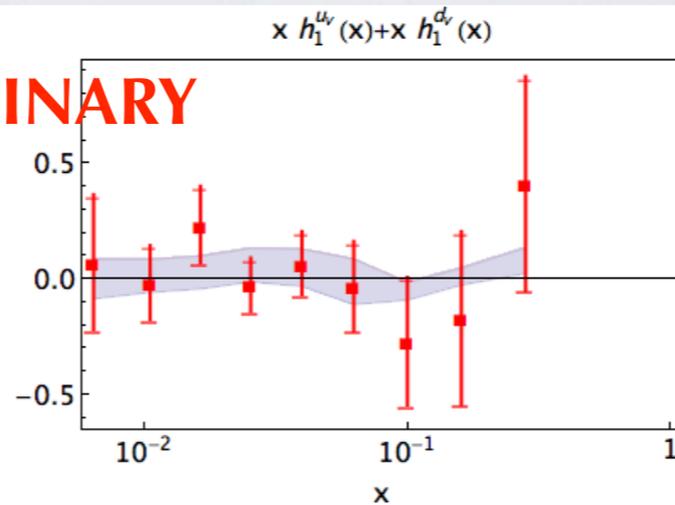
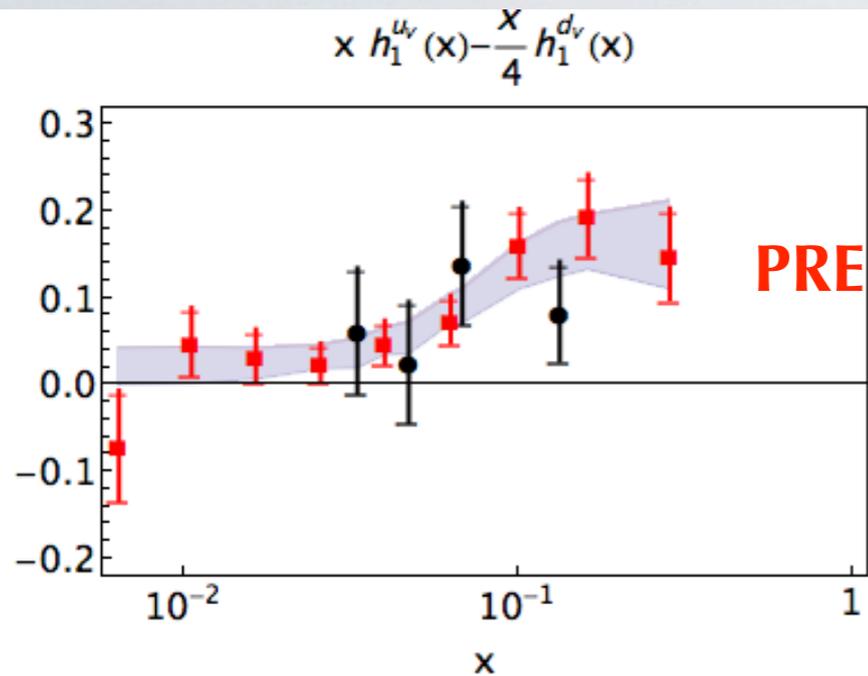
previous



comparison new with previous fit

new

$\chi^2/\text{dof} \times 10$



$$x h_1^{u_v} - \frac{1}{4} x h_1^{d_v} \propto - \frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 H_1^{\leftarrow u} \dots}$$

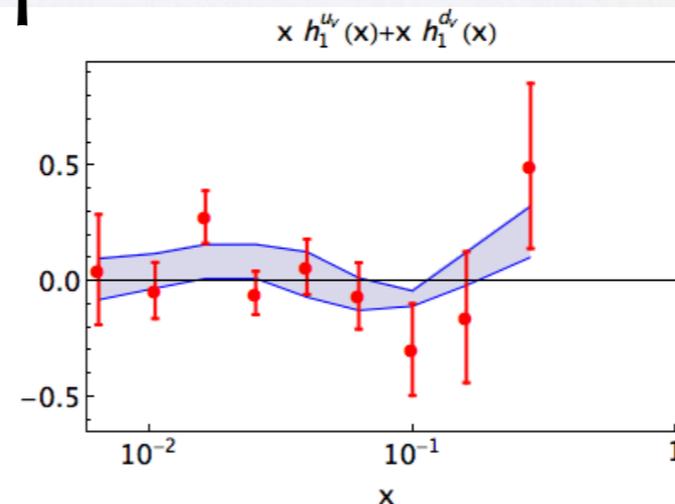
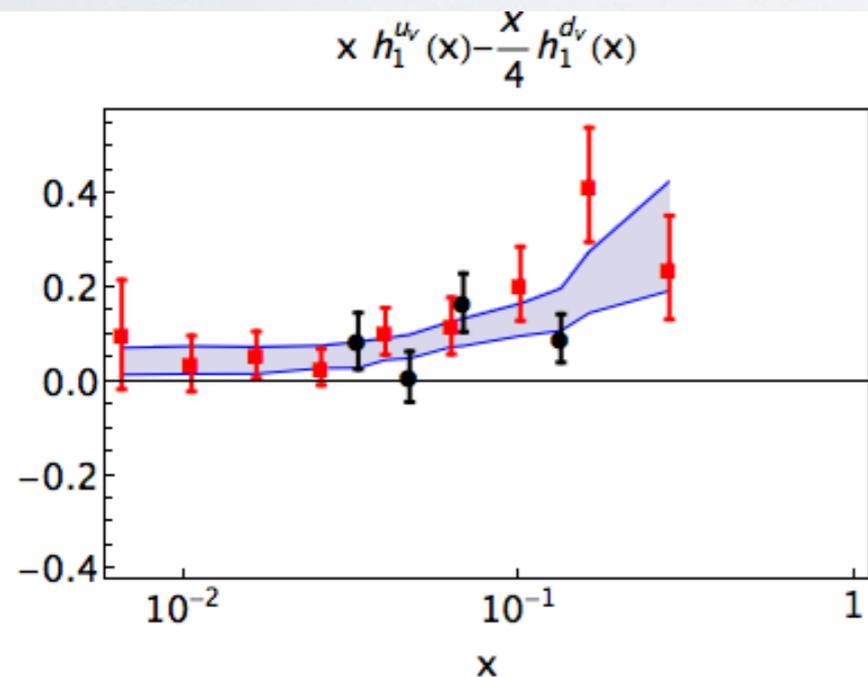
more precise data points

more realistic error on H_1^*



flexible

previous



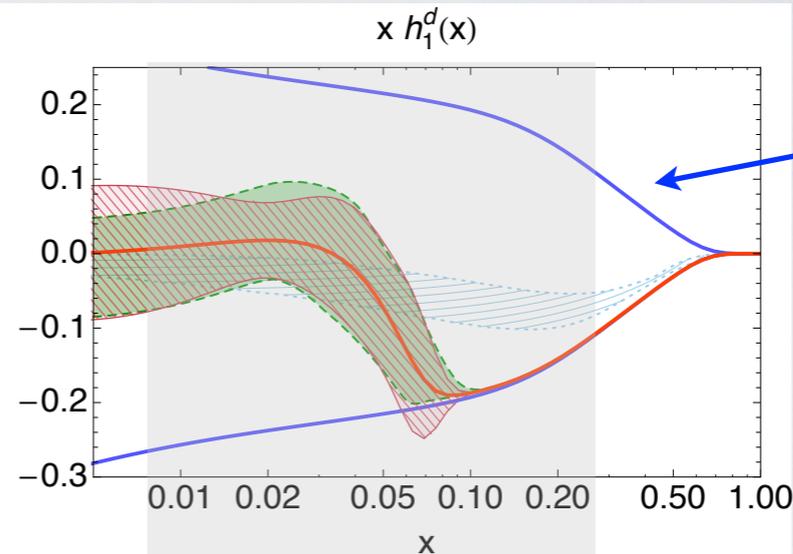
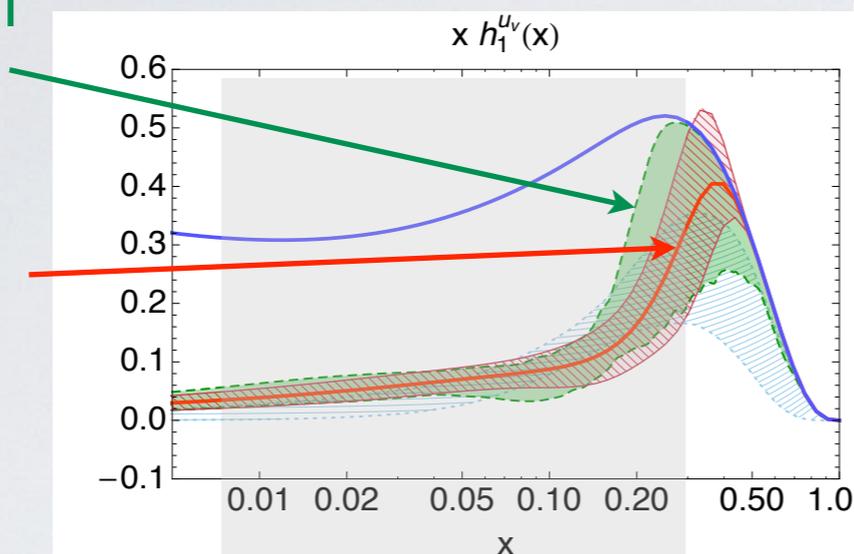
$Q^2 = 2.4 \text{ GeV}^2$

$$u - \bar{u} \quad x h_1^{q-\bar{q}}(x) \quad d - \bar{d}$$

Bacchetta, Courtoy, Radici, JHEP 1303 (13) 119

68% band of replicas

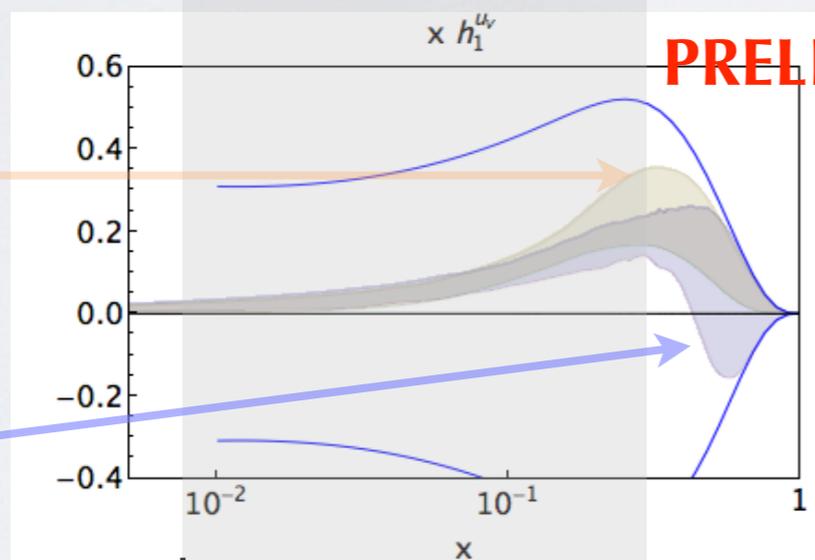
central value for standard fit with 1σ band



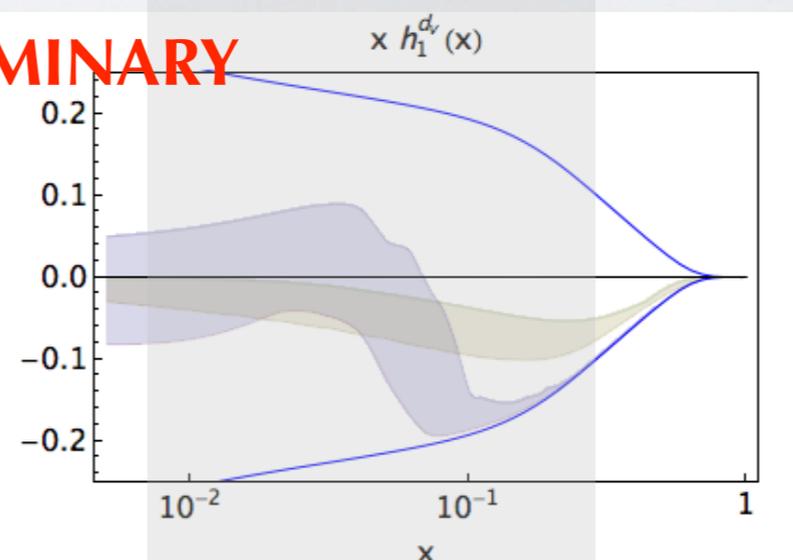
Soffer bound

Torino param.

new 68% band of replicas



PRELIMINARY



← data →



flexible

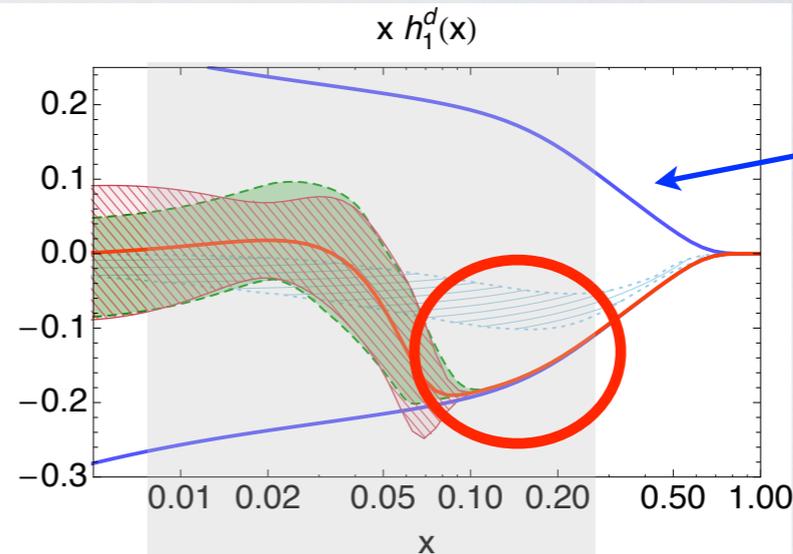
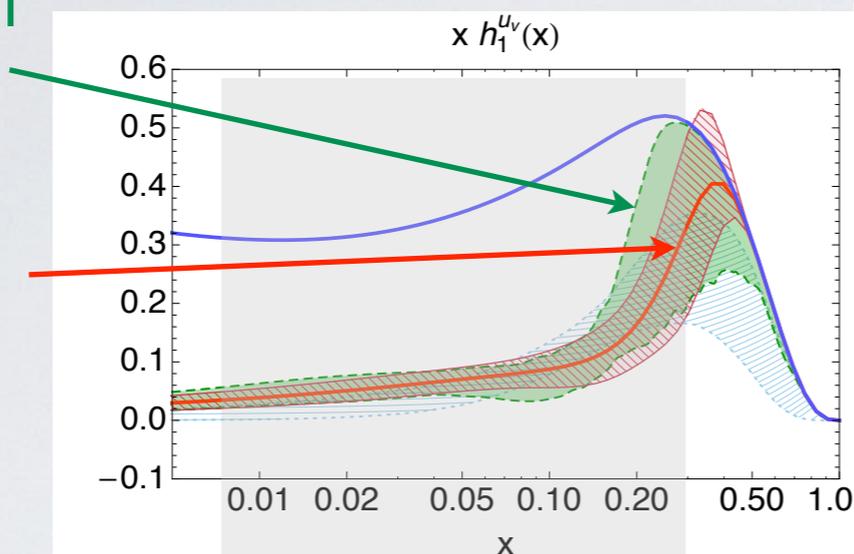
$Q^2 = 2.4 \text{ GeV}^2$

$$u - \bar{u} \quad x h_1^{q-\bar{q}}(x) \quad d - \bar{d}$$

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68% band of replicas

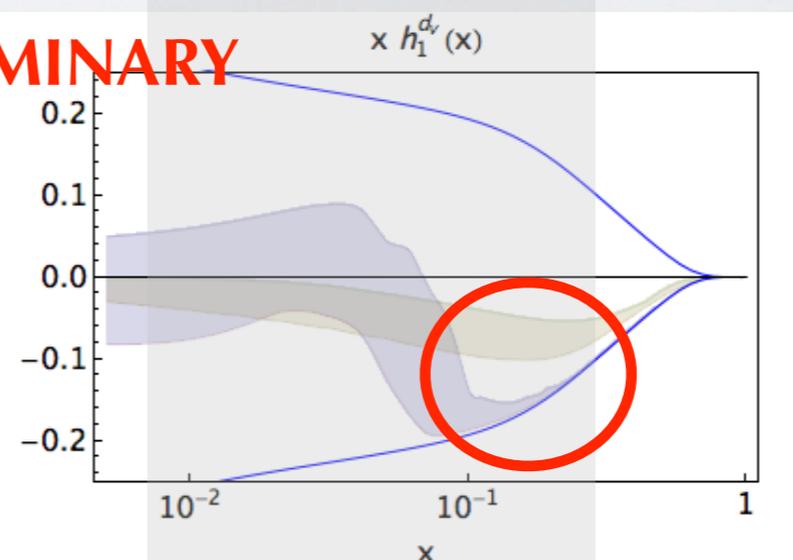
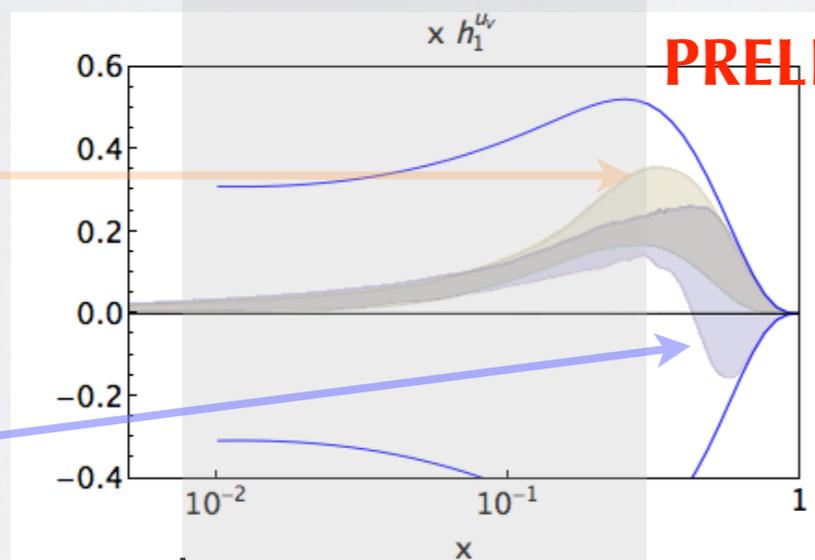
central value for standard fit with 1σ band



Soffer bound

Torino param.

new 68% band of replicas



flexible

← data →

tension driven by COMPASS deuteron data

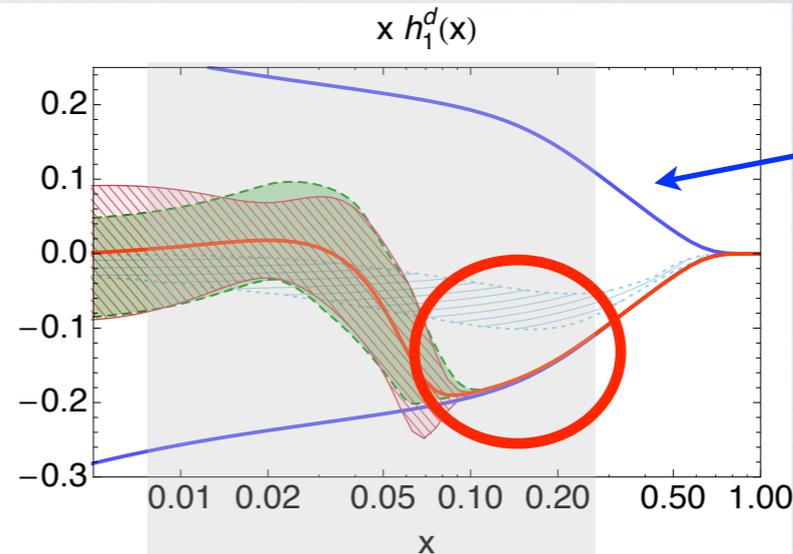
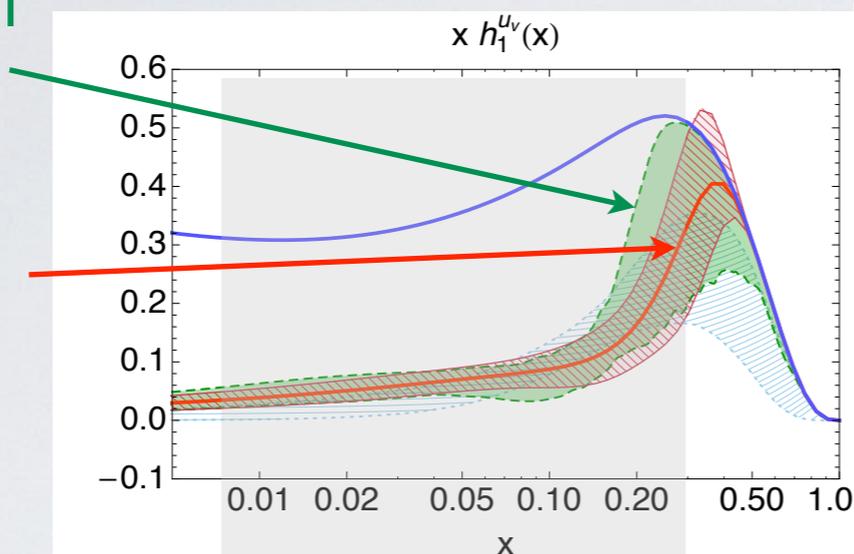
$Q^2 = 2.4 \text{ GeV}^2$

$$u - \bar{u} \quad x h_1^{q-\bar{q}}(x) \quad d - \bar{d}$$

Bacchetta, Courtoy, Radici, JHEP 1303 (13) 119

68% band of replicas

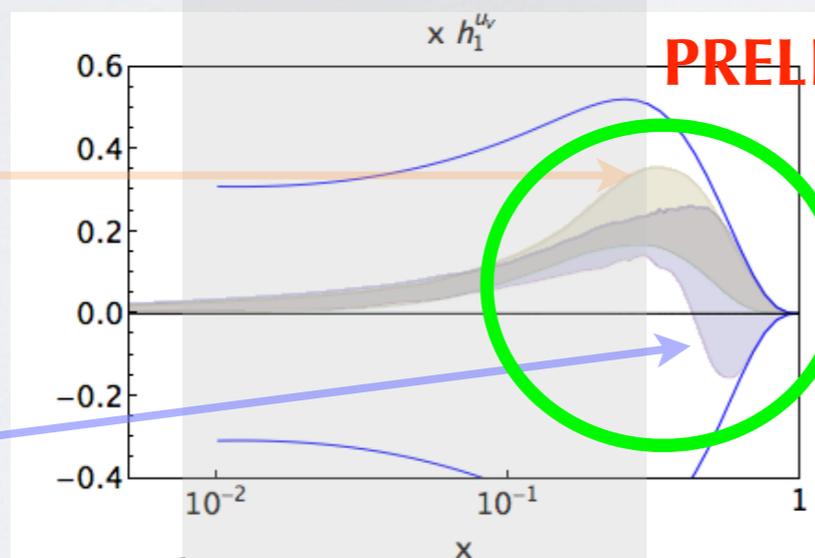
central value for standard fit with 1σ band



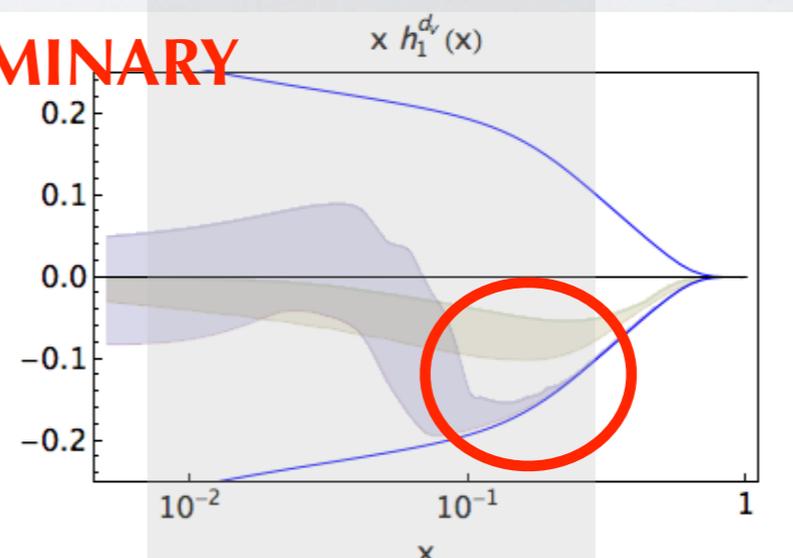
Soffer bound

Torino param.

new 68% band of replicas



PRELIMINARY



flexible

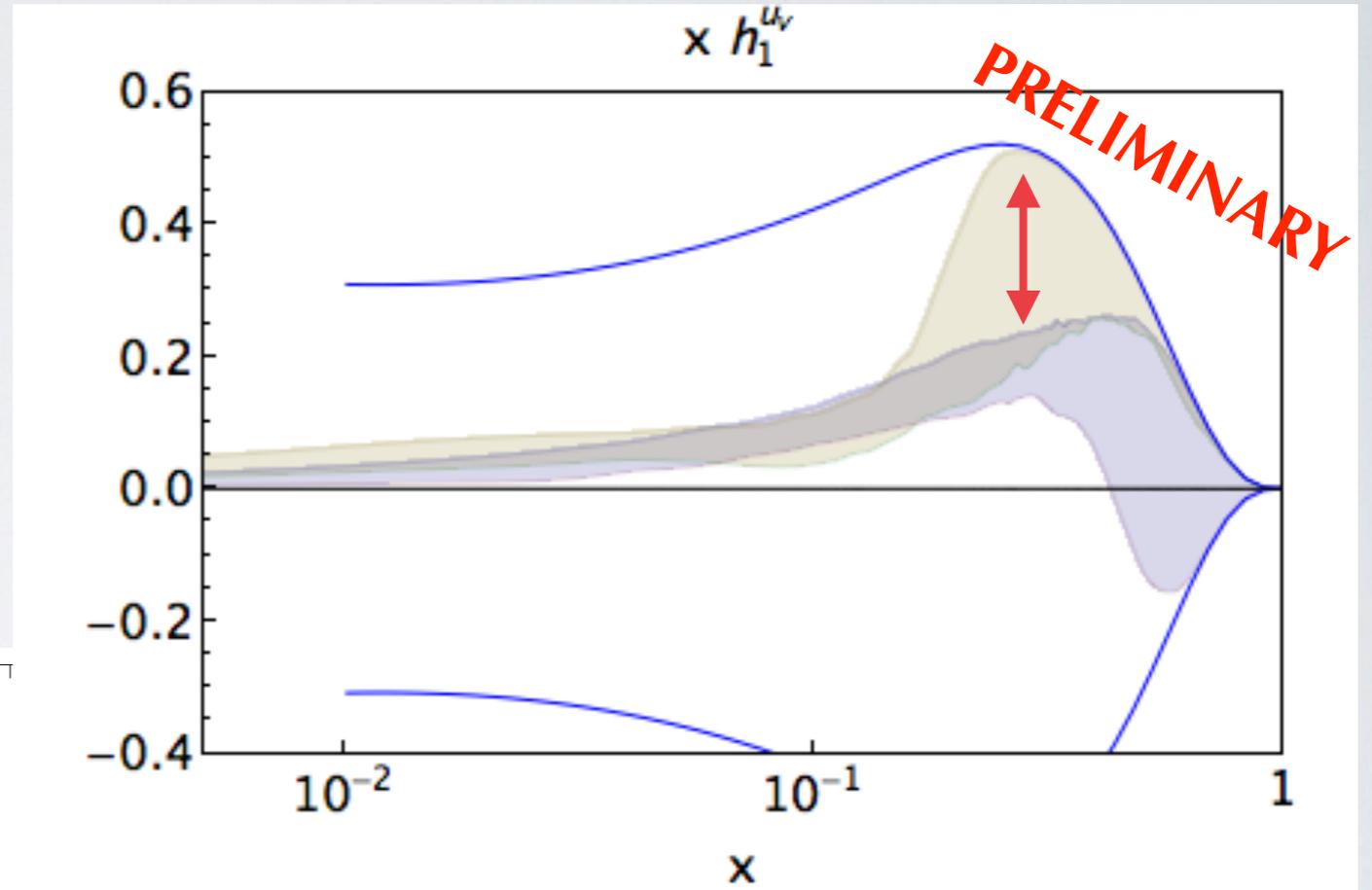
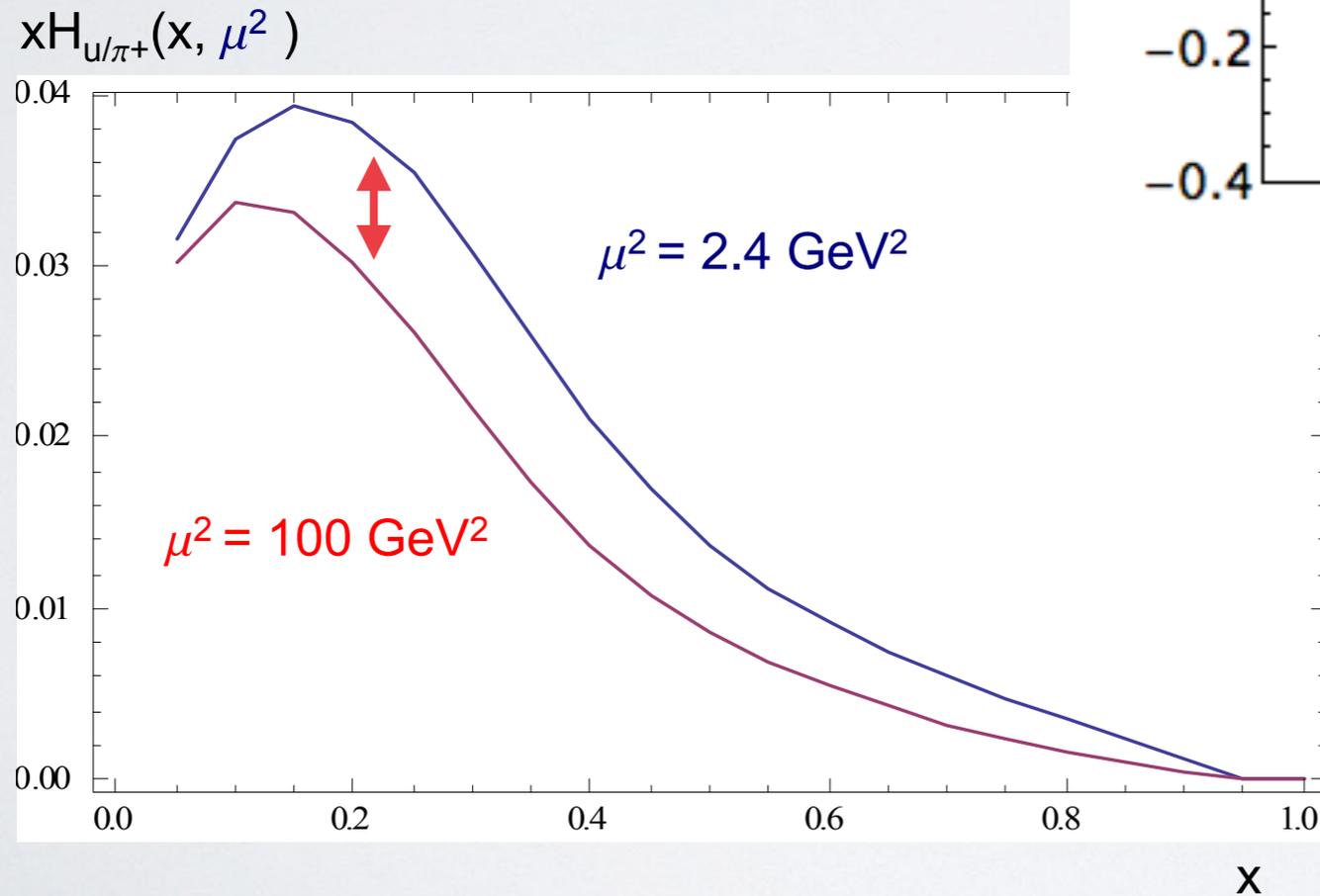
tension driven by COMPASS deuteron data

new 68% band for h_1 up is narrower (where there are data) and "smaller"

smaller transversity ??

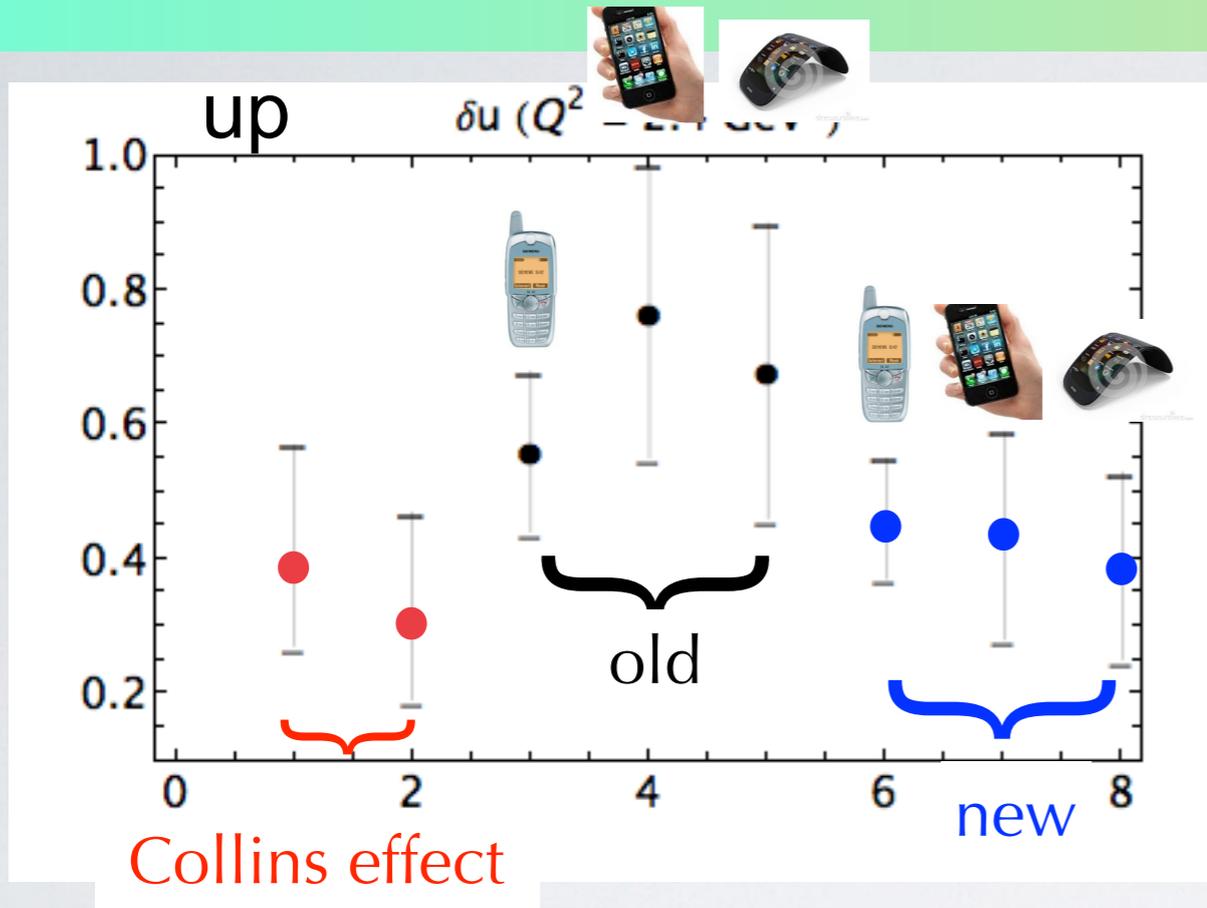
smaller transversity

larger Collins



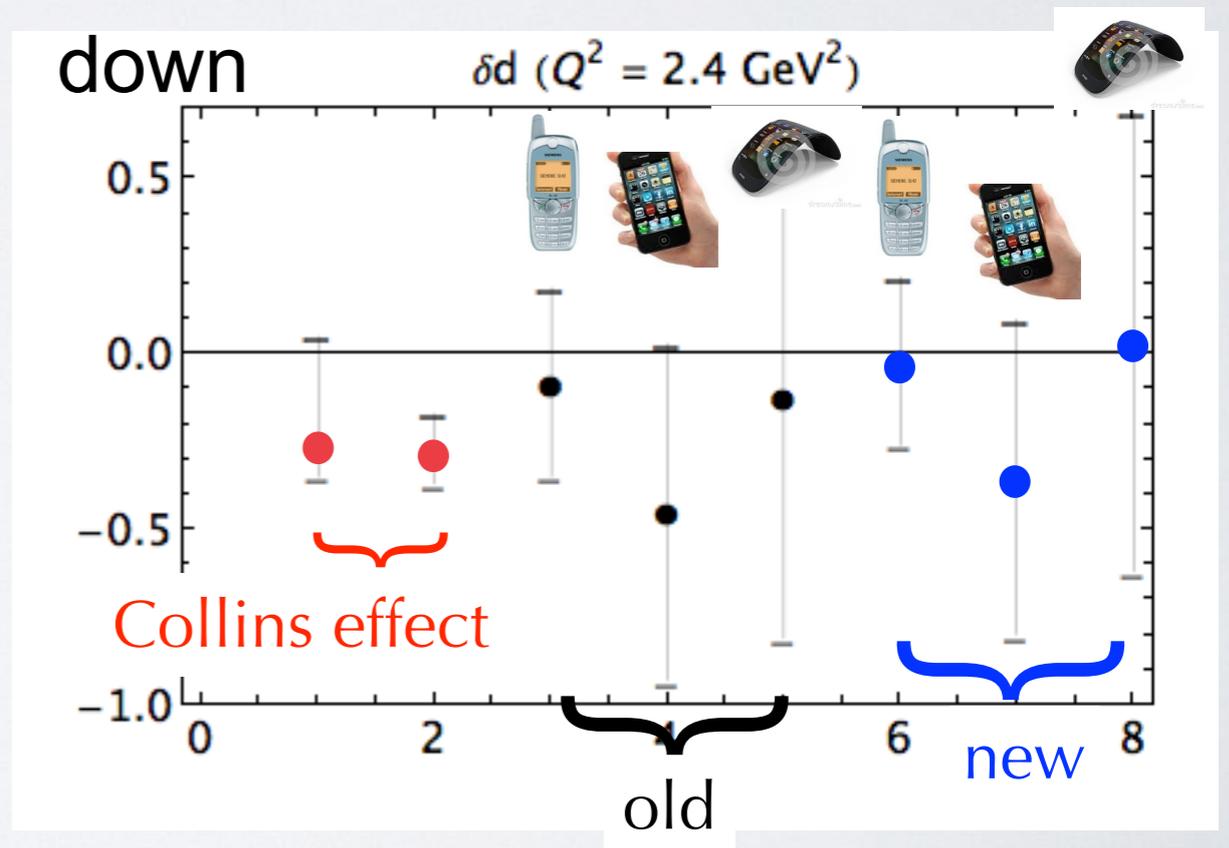
is it a consistent picture ??

tensor charges

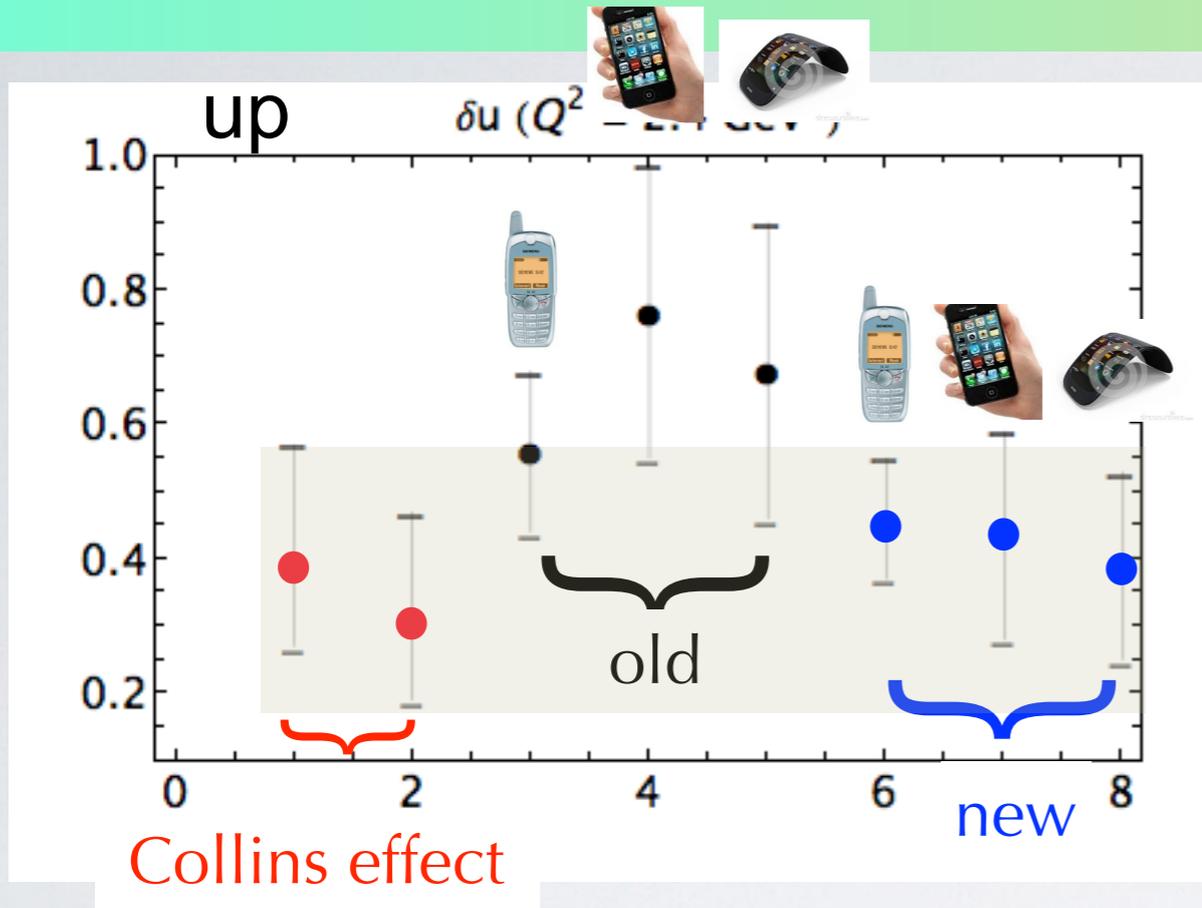


$$\delta q = \int_{\sim 0}^1 dx h_1^{qv}(x)$$

$$Q_0^2 = 1 \text{ GeV}^2$$

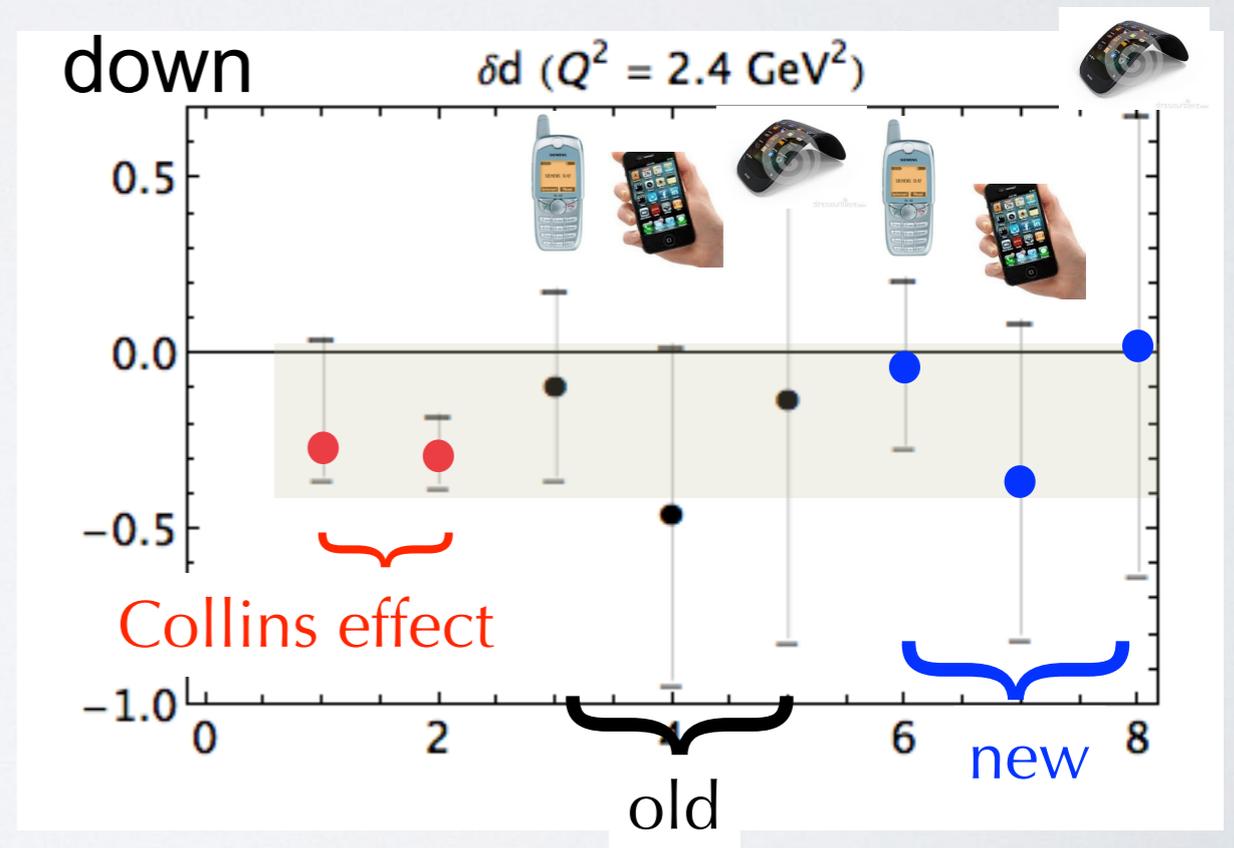


tensor charges

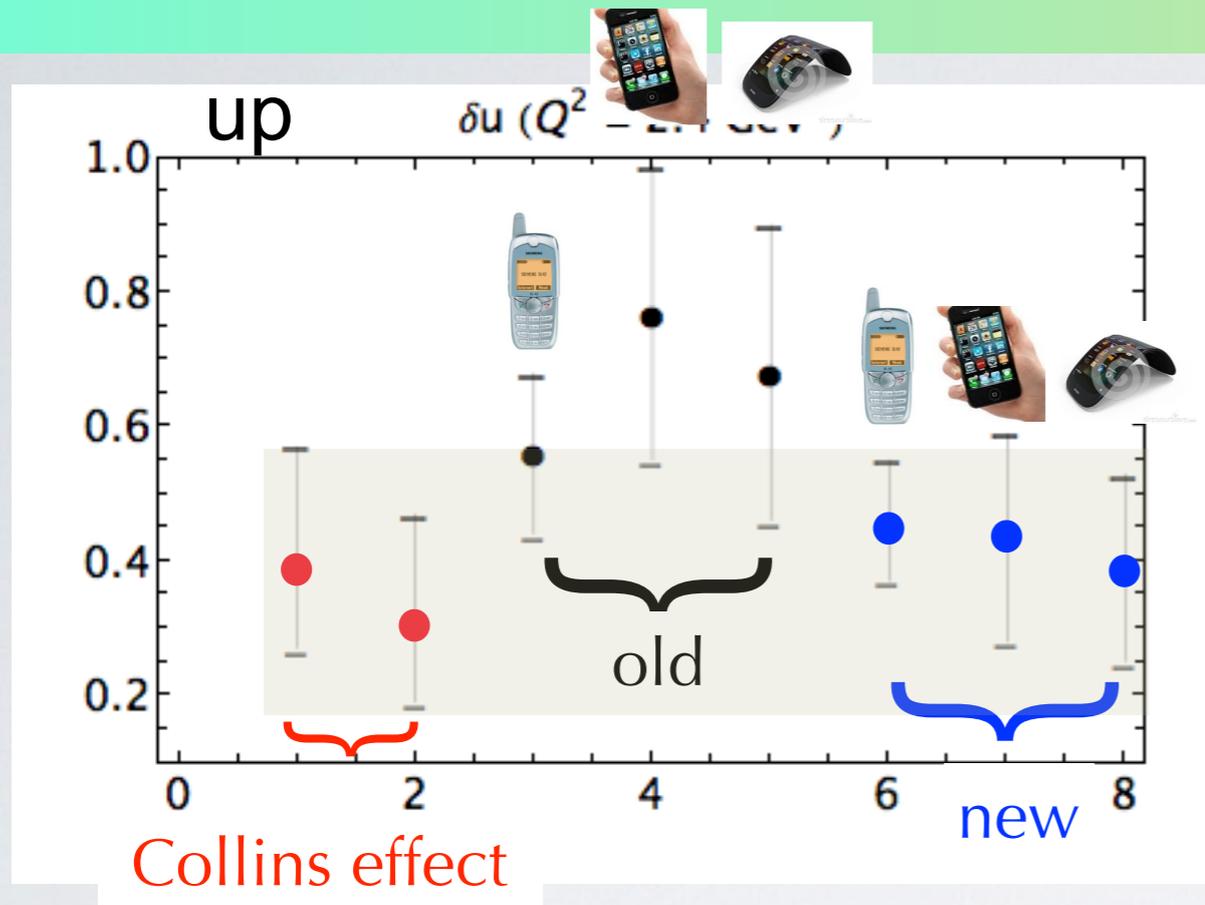


$$\delta q = \int_{\sim 0}^1 dx h_1^{qv}(x)$$

$$Q_0^2 = 1 \text{ GeV}^2$$

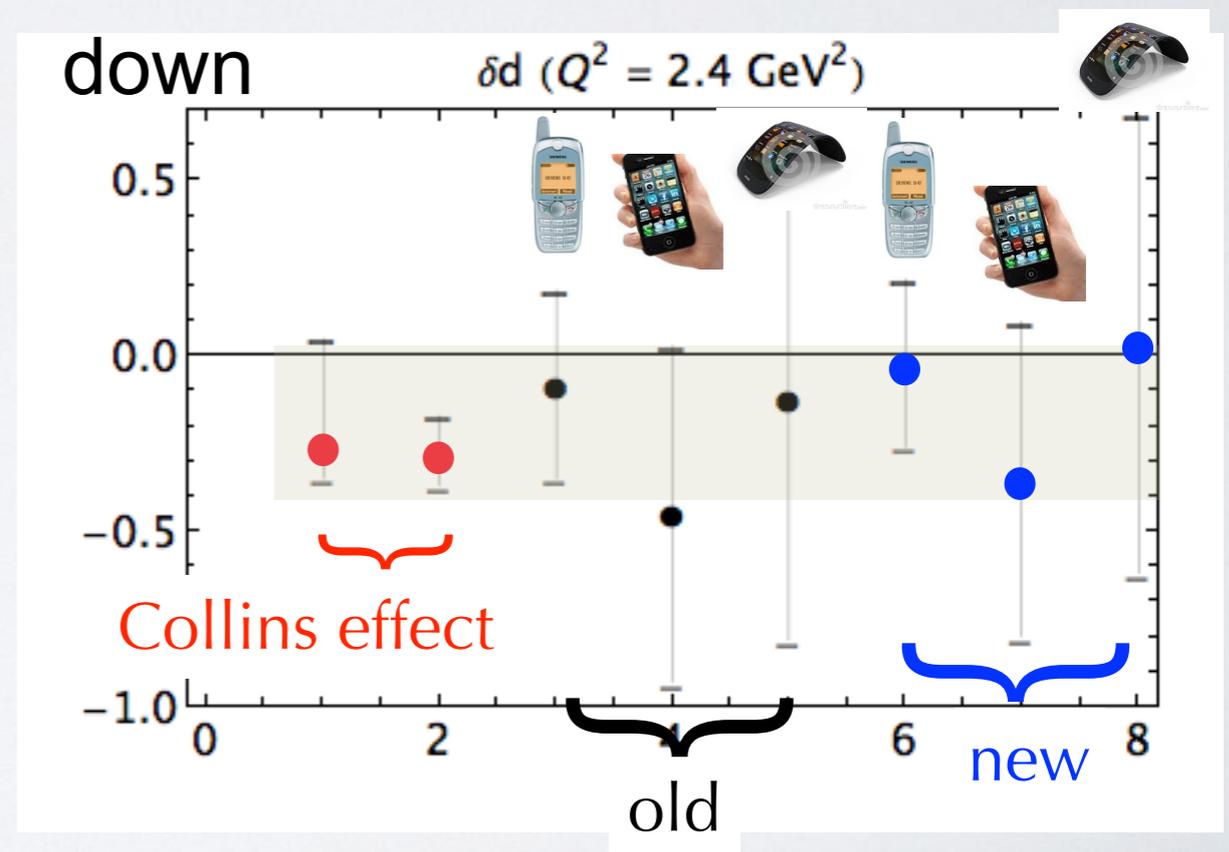


tensor charges



$$\delta q = \int_{\sim 0}^1 dx h_1^{qv}(x) \quad Q_0^2 = 1 \text{ GeV}^2$$

extrapolation \rightarrow large uncertainty!
better \int where we have data



Conclusions and outlook

- di-hadron semi-inclusive production allows to consider single-spin asymmetries in collinear factorization framework
⇒ easier manipulation / known DGLAP evolution
- better quality of data → improving fit
at present, new  proton data induce narrower uncertainty band for h_1^u
- new fit based also on more realistic errors on extraction of DiFF
⇒ current most realistic estimate of errors on h_1
- h_1^d unchanged, h_1^u seems smaller
compatible with larger H_1^\perp from TMD evolution ?

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compatible with larger H_1^\perp from TMD evolution ?
- need D_1 from data, not from PYTHIA..
- beyond fitting functional form: Neural Network analysis ?



thank you

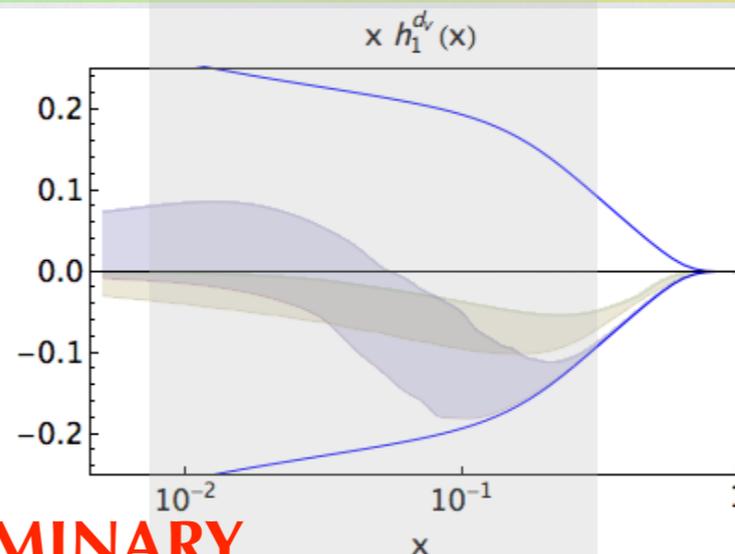
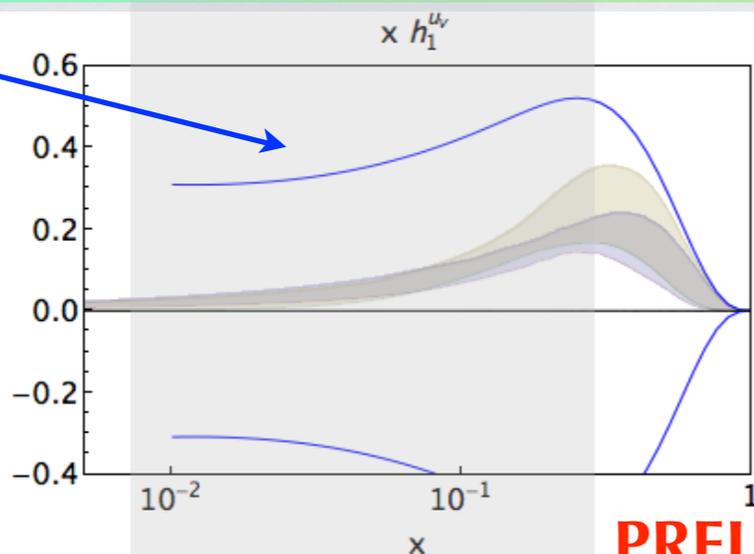
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Backup

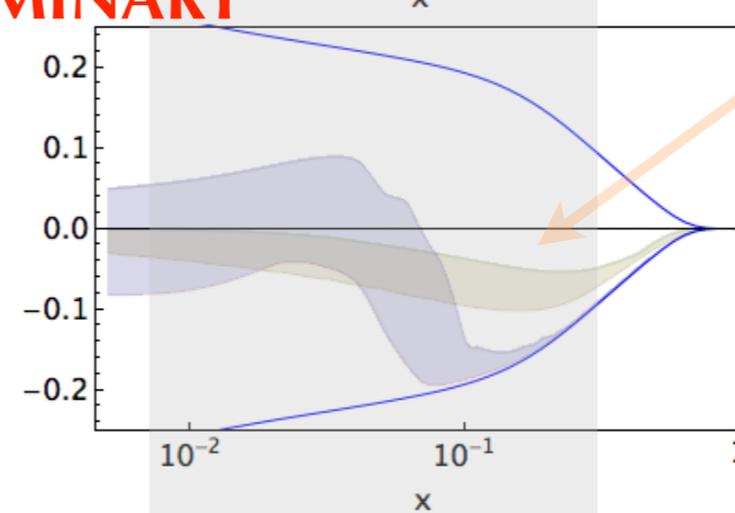
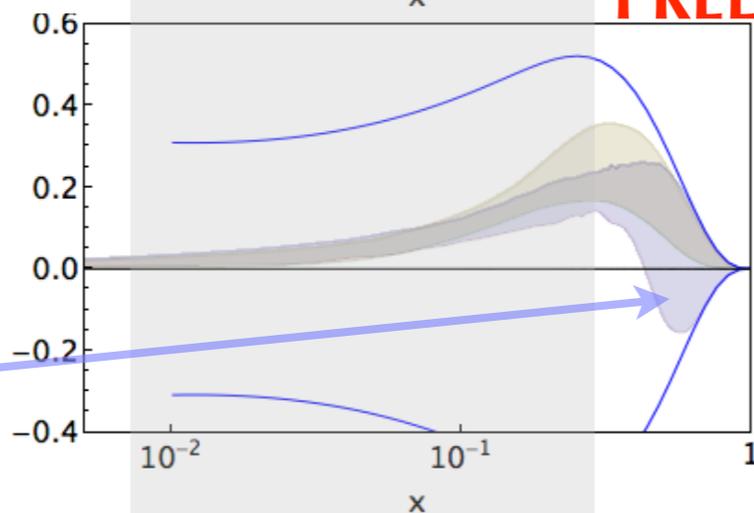
$Q^2 = 2.4 \text{ GeV}^2$

$$u - \bar{u} \quad \times \quad h_1(x) \quad d - \bar{d}$$

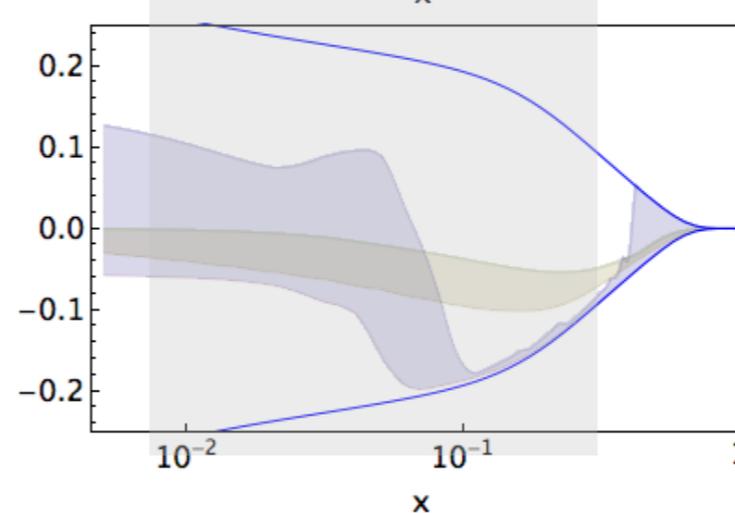
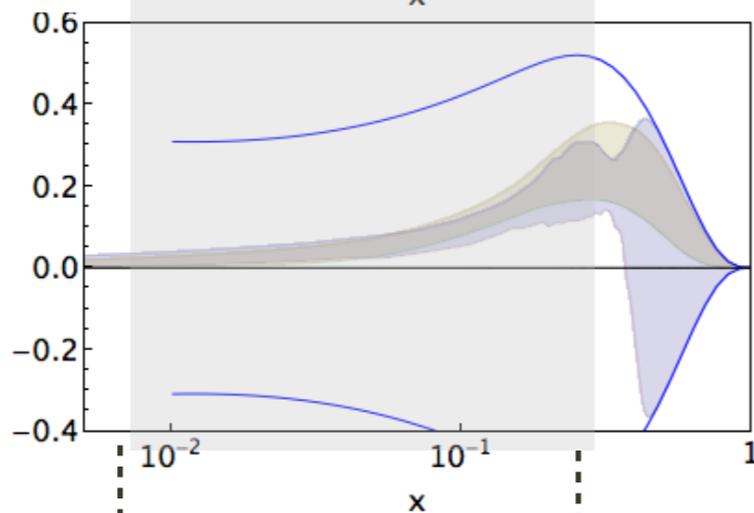
Soffer bound



PRELIMINARY



68% band of replicas



data



rigid

Torino param.
2009



flexible

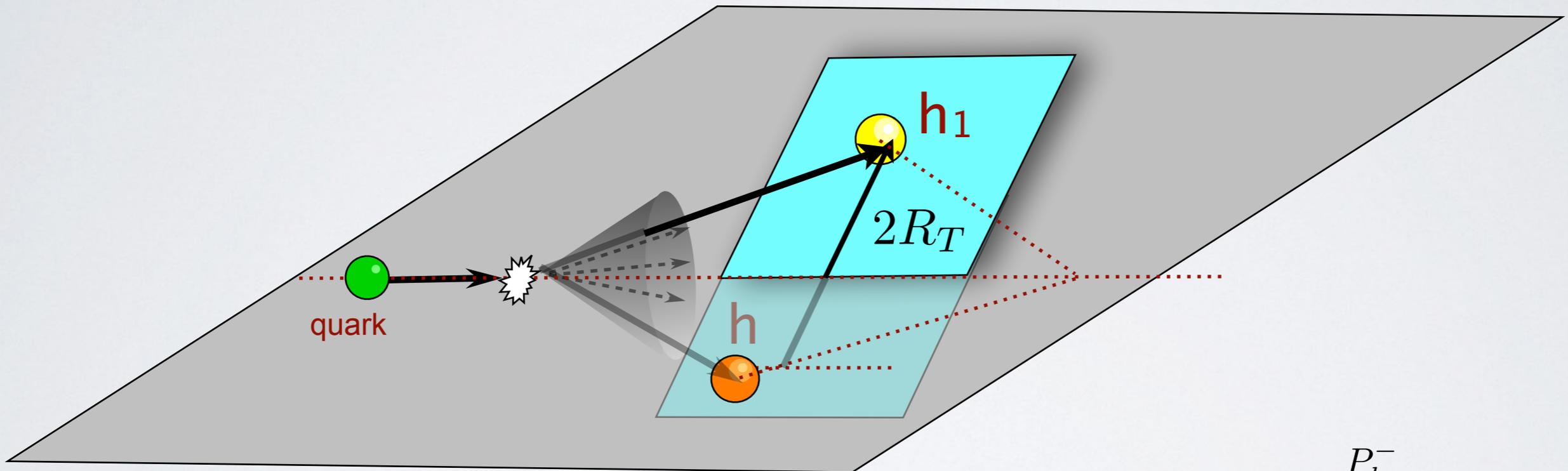


extra flexible

collinear pairs

$$\int d\mathbf{P}_{hT}$$

the total momentum of the pair
is collinear with
the fragmenting quark momentum



Δ depends on 4 variables :

$$z = \frac{P_h^-}{k^-} = z_1 + z_2$$

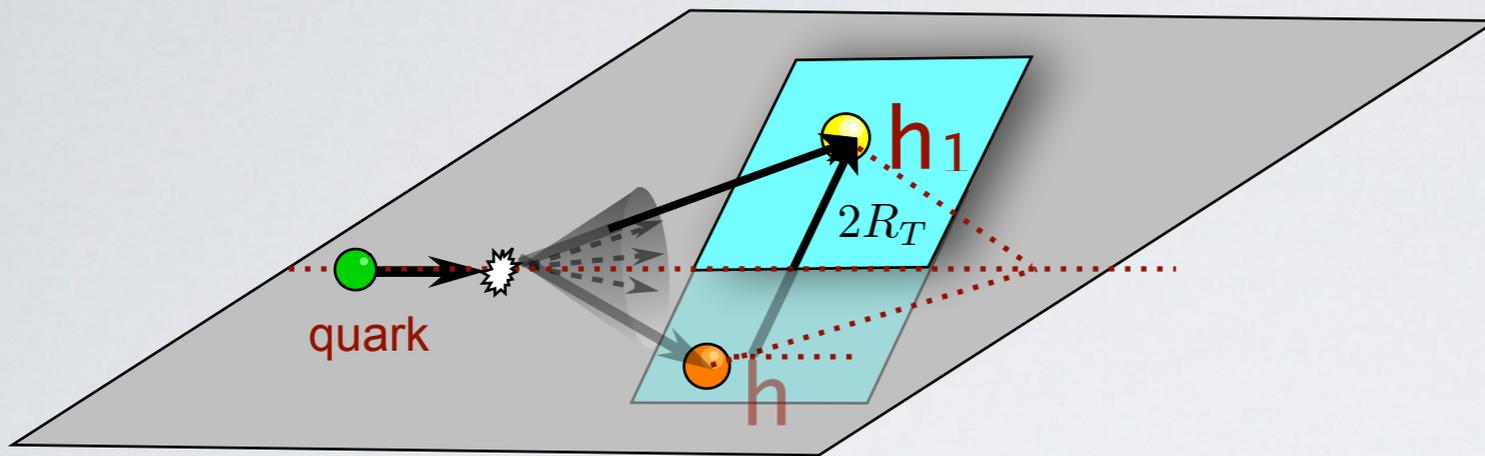
$$\zeta = \frac{2R^-}{P_h^-} = \frac{z_1 - z_2}{z}$$

$$\mathbf{R}_T^2 \rightarrow M_h^2 = P_h^2$$

$$\phi_R \rightarrow \mathbf{P}_{hT} \cdot \mathbf{R}_T$$

expansion in partial waves

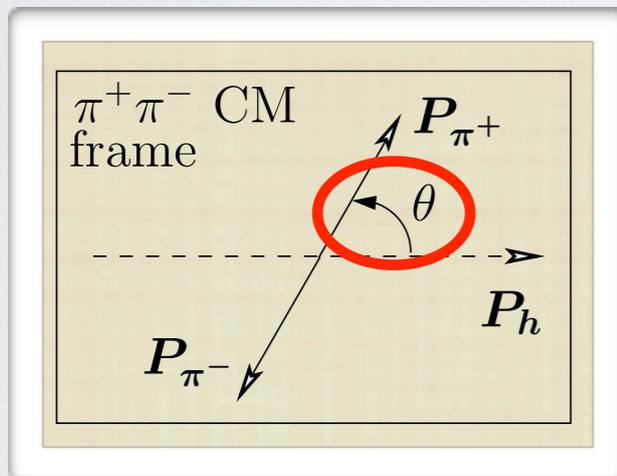
Bacchetta & Radici, P.R. D67 (03) 094002



for $M_h \lesssim 1$ GeV,
the system $(h_1, h_2)_L$
can be in $L = 0$ (s) or 1 (p)
relative partial wave

for (h_1, h_2) system in its c.m. frame

change of variable



$$\zeta = \frac{z_1 - z_2}{z} \quad \longleftrightarrow \quad \cos \theta$$

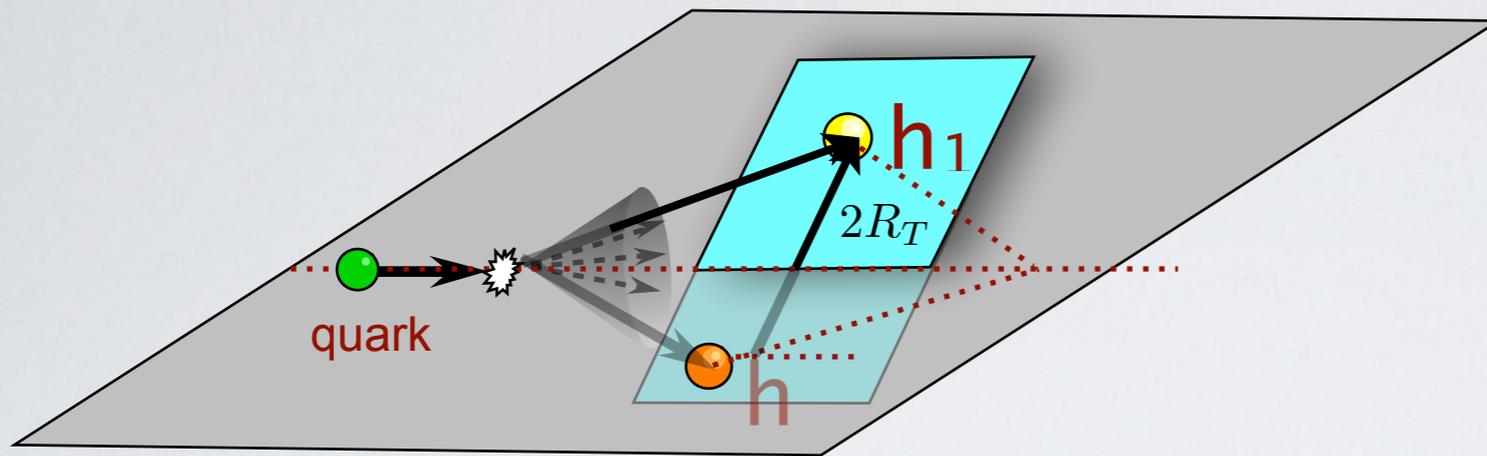
expansion in Legendre polynomials of $\cos \theta$

$$D_1^q(z, \zeta, M_h^2) \approx D_1^q(z, M_h^2) + D_{1,sp}^q(z, M_h^2) \cos \theta + \dots$$

$$H_1^{\triangleleft q}(z, \zeta, M_h^2) \approx H_{1,sp}^{\triangleleft q}(z, M_h^2) + H_{1,pp}^{\triangleleft q}(z, M_h^2) \cos \theta + \dots$$

expansion in partial waves

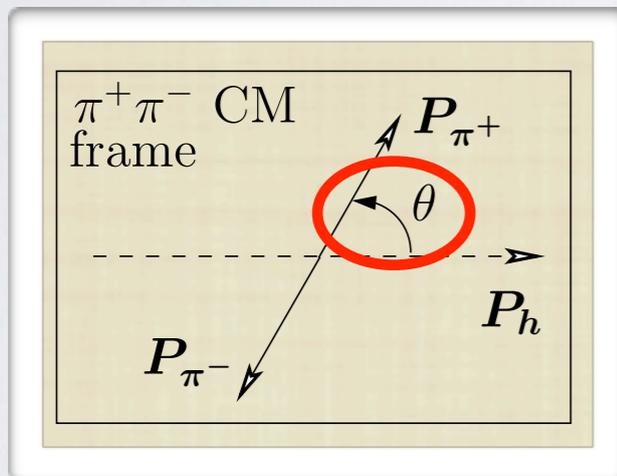
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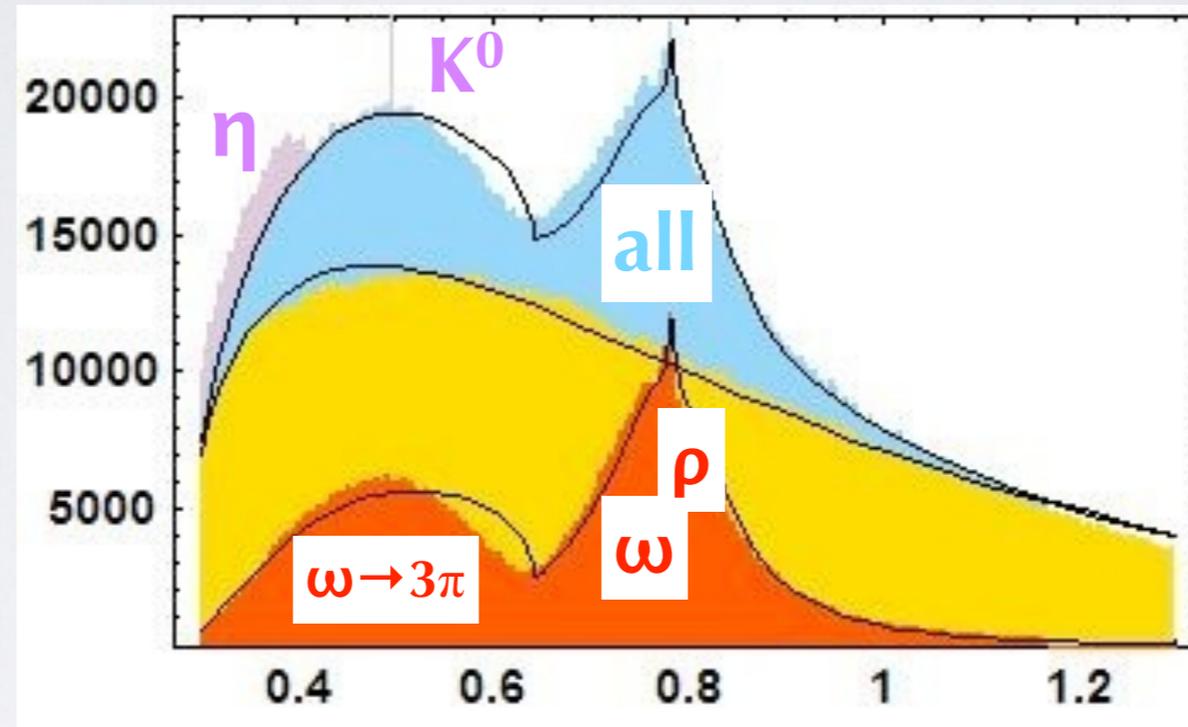
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involved in recent measured asymmetries

extract DiFFs : warning #2

invariant mass M_h dependence very complicated
model-inspired fitting functional form

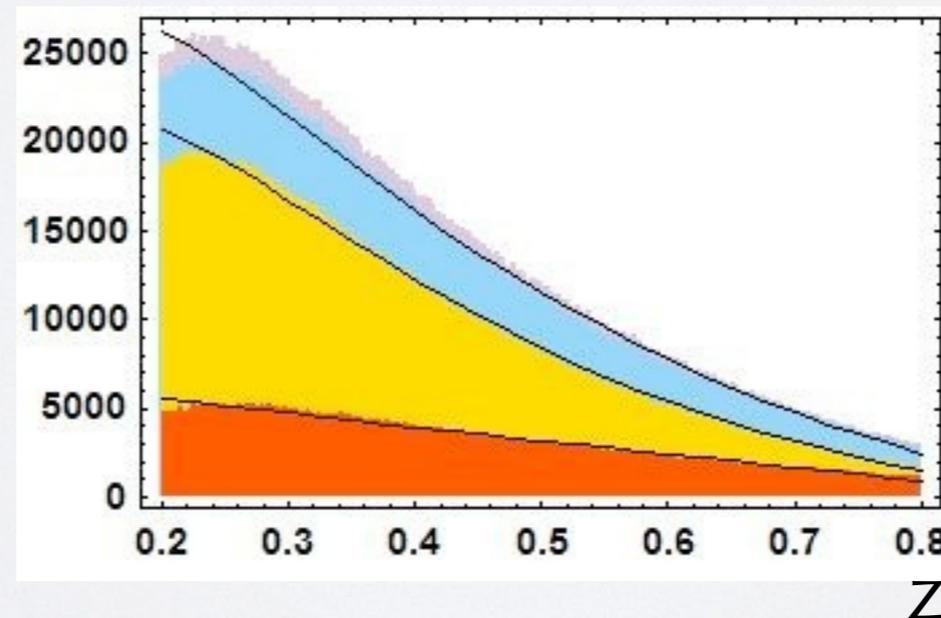
counts



*Bacchetta & Radici,
P.R. D74 (06) 114007*

background
=
all - (resonances)

assumed
 $(\pi^+\pi^-)_{L=0}$



M_h

Z