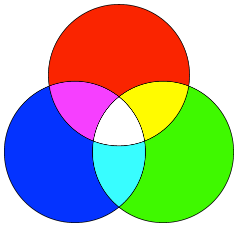


Universality of TMD correlators

Maarten Buffing

Transversity 2014

June 10, 2014

Content

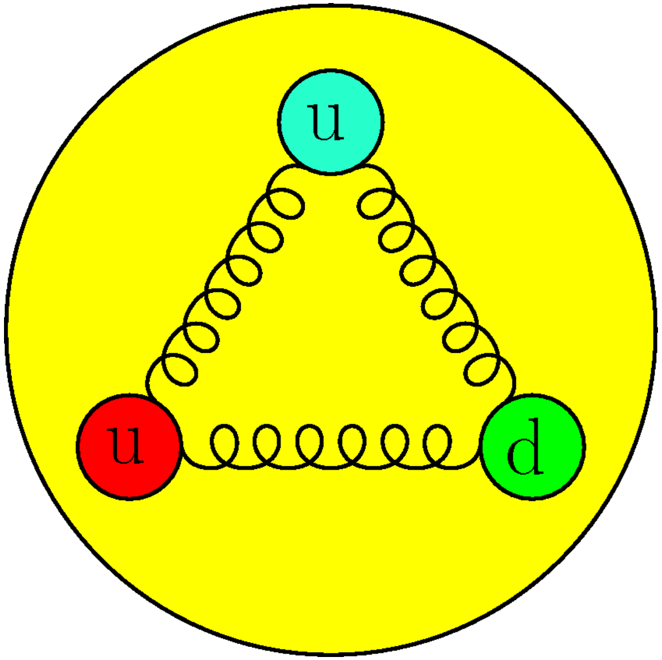
- Part I: universality of quark correlators
  - TMDs
  - Transverse momentum weighting for quarks
  - Universality of quark correlators

- Part II: universality of gluon correlators
  - Universality of gluon correlators

- Summary and conclusions

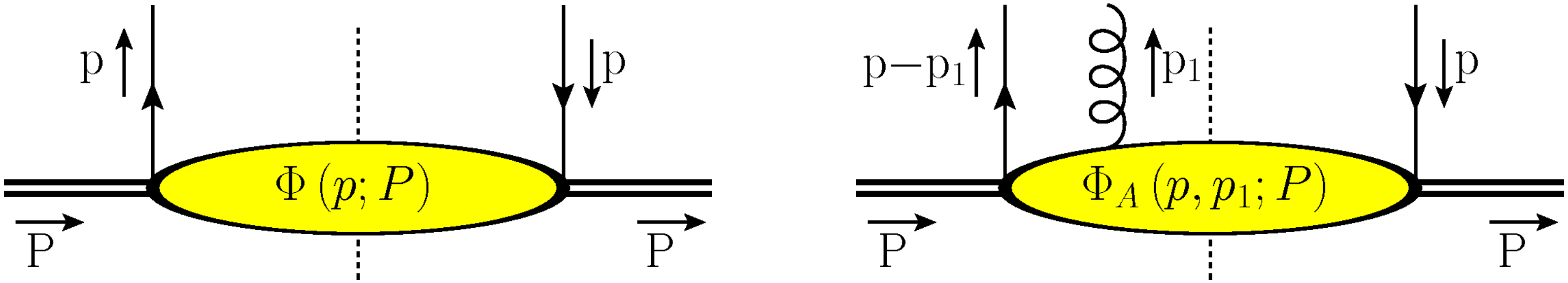
Work in collaboration with Piet Mulders and Asmita Mukherjee

Part I: quark correlators



Quark correlators

- Quark correlators can be written as matrix elements



$$\Phi_{ij}(p;P) = \Phi_{ij}(p|p) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip\cdot\xi} \langle P|\overline{\psi}_j(0)\psi_i(\xi)|P\rangle$$

$$\Phi^{\alpha}_{A;ij}(p-p_1|p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8} e^{i(p-p_1)\cdot\xi + ip_1\cdot\eta} \langle P|\overline{\psi}_j(0)A^{\alpha}(\eta)\psi_i(\xi)|P\rangle$$

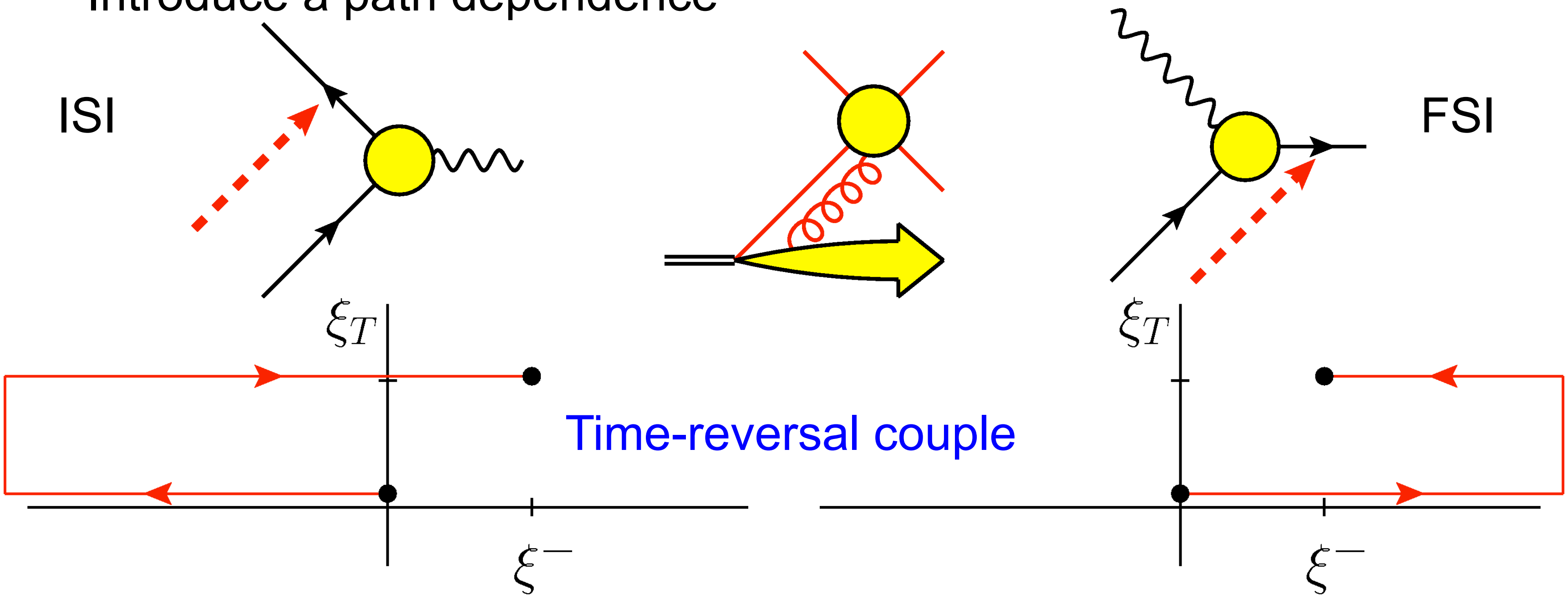
- The field combination is non-local

Gauge invariance for quark TMDs

- Gauge links make the nonlocal combinations of fields gauge invariant

$$U_{[0,\xi]} = \mathcal{P} \exp\left(-ig \int_0^{\xi} ds^{\mu} A_{\mu}\right)$$

- Introduce a path dependence



A.V. Belitsky, X. Ji, F. Yuan, NP B 656, (2003) 165
D. Boer, P.J. Mulders & F. Pijlman, NP B 667 (2003) 2012

Sivers, PRD 41 (1990) 83;
Collins, PLB 536, 43 (2002);
Brodsky et al, NPB642, 344 (2002)

TMD PDFs

- Distribution functions could be described in terms of parton distribution functions (TMDs)

- Depending on polarization(s), differerent contributions are required

$$\Phi^{[U]}(x, p_T) = \left(f_1^{[U]}(x, p_T^2) + i h_1^{\perp [U]}(x, p_T^2) \frac{\not{p}_T}{M} + \dots \right) \frac{\not{P}}{2}$$

quark polarization

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_{1T}, h_{1T}^\perp$

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T	$f_{1T}^{\perp}$	g_{1T}	$h_{1T}, h_{1T}^{\perp}$

T-odd

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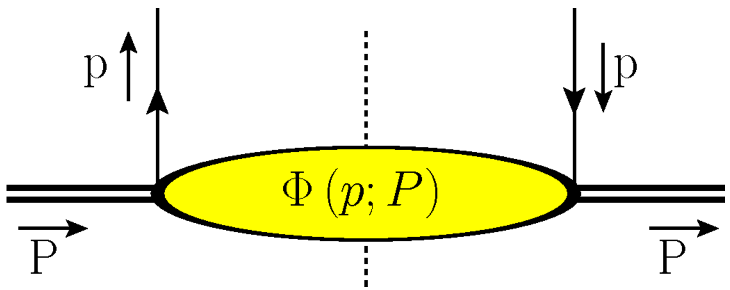
Process dependent

Pretzelocity is T-even and process dependent

Reflection: comparing descriptions



- Differences

Matrix elements | | TMDs

$$\text{F.T.}\langle P|\overline{\psi}_j(0)U_{[0,\xi]}\psi_i(\xi)|P\rangle \quad \leftrightarrow \quad f_1^{[U]}(x,p_T^2),\ ih_1^{\perp[U]}(x,p_T^2)\frac{\not{p}_T}{M},\ \ldots$$

$$\text{F.T.}\langle P|\overline{\psi}_j(0)U_{[0,\xi]}A_T(\xi)\psi_i(\xi)|P\rangle$$

- Similarities

Rank in A_T, iD_T, … $\leftrightarrow$ Rank in p_T, p_Tp_T, …

Time reversal symmetry $\leftrightarrow$ Time reversal symmetry

Mellin moments

- Collinear functions

$$\Phi^q(x) = \int \frac{d(\xi\cdot P)}{2\pi} e^{ip\cdot\xi} \langle P|\overline{\psi}(0) U^{[n]}_{[0,\xi]} \psi(\xi)|P\rangle_{\xi\cdot n=\xi_T=0}$$

$$x^{N-1}\Phi^q(x) = \int \frac{d(\xi\cdot P)}{2\pi} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)(i\partial^n)^{N-1} U^{[n]}_{[0,\xi]} \psi(\xi)|P\rangle_{\xi\cdot n=\xi_T=0}$$

$$= \int \frac{d(\xi\cdot P)}{2\pi} e^{ip\cdot\xi} \langle P|\overline{\psi}(0) U^{[n]}_{[0,\xi]} (iD^n)^{N-1} \psi(\xi)|P\rangle_{\xi\cdot n=\xi_T=0}$$

- Moments correspond to local matrix elements with calculable anomalous dimensions, that can be Mellin transformed to splitting functions

$$\Phi^{(N)} = \langle P|\overline{\psi}(0)(iD^n)^{N-1}\psi(\xi)|P\rangle_{\xi\cdot n=\xi_T=0}$$

Transverse moments

- For TMD functions one can consider transverse moments

$$p_T^\alpha \Phi^{[U]}(x, p_T; n) = \int \frac{d(\xi \cdot P) d^2\xi_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0) i\partial_T^\alpha U_{[0,\xi]} \psi(\xi)|P\rangle_{\xi\cdot n=0}$$

- Due to transverse directions, partonic operators show up. For the simplest gauge links one finds

$$i\partial_T^\alpha U_{[0,\xi]}^{[\pm]} = U_{[0,\xi]}^{[\pm]} \Big(iD_T^\alpha(\xi) - gA_T^\alpha(\xi) \pm G^{n\alpha}(\xi) \Big)$$

Bomhof, Mulders, (2007); Dominguez, Xiao, Yuan (2011); MGAB, Mukherjee, Mulders (2012)

Transverse moments

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$$p_T^\alpha \Phi^{[U]}(x, p_T; n) = \int \frac{d(\xi \cdot P) d^2\xi_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P | \overline{\psi}(0) i\partial_T^\alpha U_{[0,\xi]} \psi(\xi) | P \rangle_{\xi\cdot n = 0}$$

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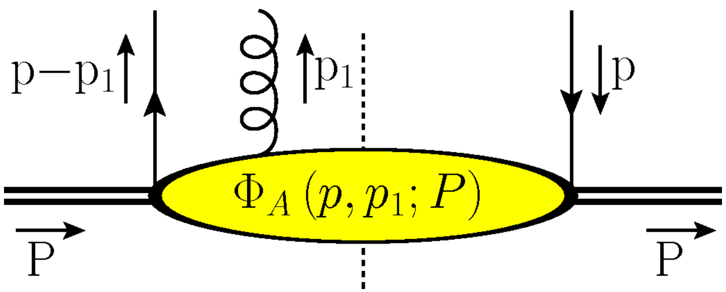

$$\Phi_D(x) = \int dx_1 \Phi_D^\alpha(x - x_1, x_1 | x)$$

$$\Phi_A(x) = \mathrm{PV} \int dx_1 \frac{1}{x_1} \Phi_F^{n\alpha}(x - x_1, x_1 | x)$$

$$\widetilde{\Phi}_\partial(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)$$

$$\Phi_G(x) = \pi\, \Phi_F^\alpha(x, 0 | x)$$

Bomhof, Mulders, (2007); Dominguez, Xiao, Yuan (2011); MGAB, Mukherjee, Mulders (2012)

Transverse moments

- For a general gauge link

$$\int d^2p_T\, p_T^\alpha \Phi^{[U]}(x,p_T;n) = \widetilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\Phi_G^\alpha(x)$$

T-even

$$\widetilde{\Phi}_\partial(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)$$

T-odd (gluonic pole or ETQS m.e.)

$$\Phi_G(x) = \pi\,\Phi_F^{n\alpha}(x,0|x)$$

Efremov & Teryaev (1982); Qiu and Sterman (1991); Boer, Mulders & Pijlman, NP B667 (2003) 201; Bomhof, Mulders and Pijlman, EPJ C 47 (2006) 147

Transverse moments

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$$\int d^2p_T\, p_T^\alpha \Phi^{[U]}(x,p_T;n) = \widetilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\,\Phi_G^\alpha(x)$$

T-even — T-odd (gluonic pole or ETQS m.e.)

$$\widetilde{\Phi}_\partial(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x) \qquad \Phi_G(x) = \pi\,\Phi_F^{n\alpha}(x,0|x)$$

- Behavior of TMDs under time reversal symmetry and rank can be used to identify their corresponding matrix element

$i\frac{\not p_T}{M} h_1^{\perp[U]}(x,p_T^2)$ ⟶ T-odd & rank 1 ⟶ part of $\Phi_G(x)$

$-\frac{p_T\cdot S_T}{M}\gamma_5\, g_{1T}^{[U]}(x,p_T^2)$ ⟶ T-even & rank 1 ⟶ part of $\widetilde{\Phi}_\partial(x)$

Efremov & Teryaev (1982); Qiu and Sterman (1991); Boer, Mulders & Pijlman, NP B667 (2003) 201; Bomhof, Mulders and Pijlman, EPJ C 47 (2006) 147

Transverse moments

- Generalization for quarks

# GPs	RANK 0	1	2
0	$\Phi(x, p_T^2)$	$\widetilde{\Phi}_\partial$	$\widetilde{\Phi}_{\partial\partial}$
1		$C_G^{[U]}\Phi_G$	$C_G^{[U]}\widetilde{\Phi}_{\{\partial G\}}$
2			$C_{GG,c}^{[U]}\Phi_{GG,c}$

# GPs	RANK OF TMD PDFs FOR QUARKS 0	1	2
0	$f_1^q,\, g_1^q,\, h_1^q$	$g_{1T}^q,\, h_{1L}^{\perp q}$	$h_{1T}^{\perp q(A)}$
1		$f_{1T}^{\perp q},\, h_1^{\perp q}$	
2			$h_{1T}^{\perp q(Bc)}$

MGAB, Mukherjee, Mulders, PRD 86 (2012) 074030

Universality

- Some PDFs are non-universal, but they can always be written in the form

$$f_{1T}^{\perp[U]} = C_G^{[U]} f_{1T}^{\perp}$$

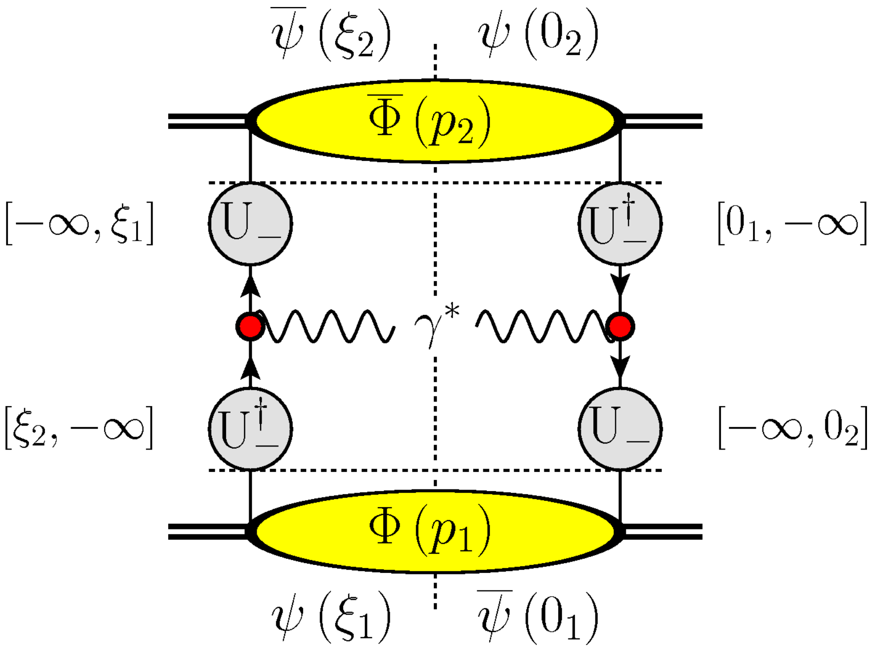
$$h_1^{\perp[U]} = C_G^{[U]} h_1^{\perp}$$

$$h_{1T}^{\perp[U]} = h_{1T}^{\perp(A)} + C_{GG,1}^{[U]} h_{1T}^{\perp(B1)} + C_{GG,2}^{[U]} h_{1T}^{\perp(B2)}$$

- Non-universality is isolated in numerical coefficients

Examples of universality

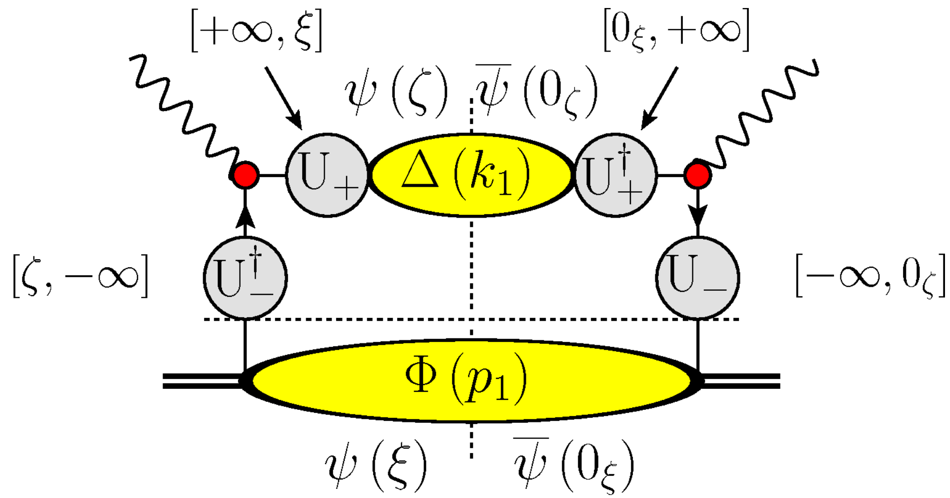
- Different processes give different gauge link structures



Drell-Yan

$$f_{1T}^{\perp[DY]} = -f_{1T}^{\perp}$$

$$h_1^{\perp[DY]} = -h_1^{\perp}$$



SIDIS

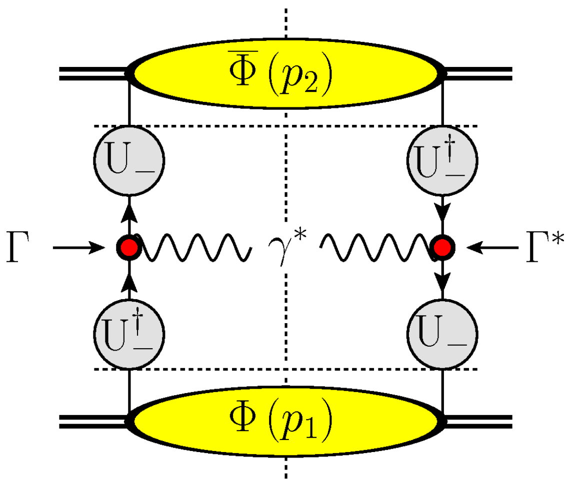
$$f_{1T}^{\perp[SIDIS]} = +f_{1T}^{\perp}$$

$$h_1^{\perp[SIDIS]} = +h_1^{\perp}$$

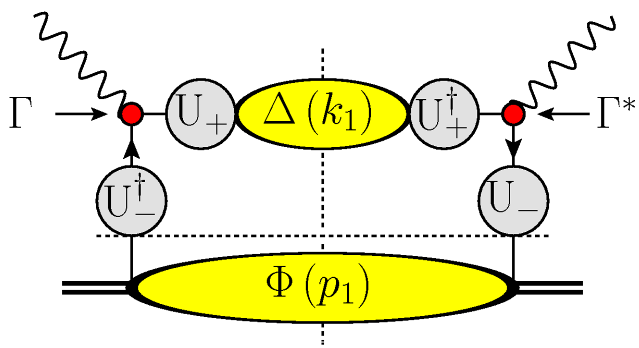
Sivers, PRD 41 (1990) 83; Collins, PLB 536, 43 (2002); Brodsky et al, NPB642, 344 (2002)

Examples of universality

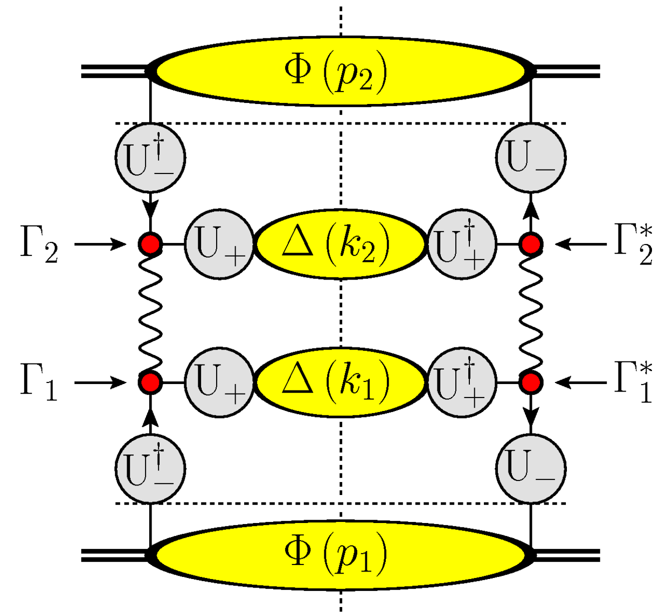
- Different processes give different gauge link structures



Drell-Yan



SIDIS



qq → qq contribution

$$h_{1T}^{\perp[DY]} = h_{1T}^{\perp(A)} + h_{1T}^{\perp(B_1)}$$

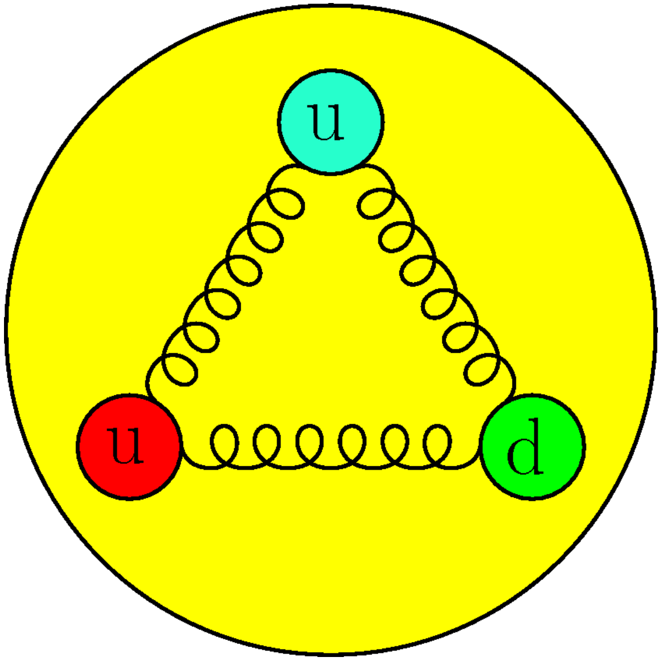
$$h_{1T}^{\perp[SIDIS]} = h_{1T}^{\perp(A)} + h_{1T}^{\perp(B_1)}$$

$$h_{1T}^{\perp[process]} = h_{1T}^{\perp(A)} + 9\, h_{1T}^{\perp(B_1)}$$

- More complicated processes also give $h_{1T}^{\perp(B_2)}$

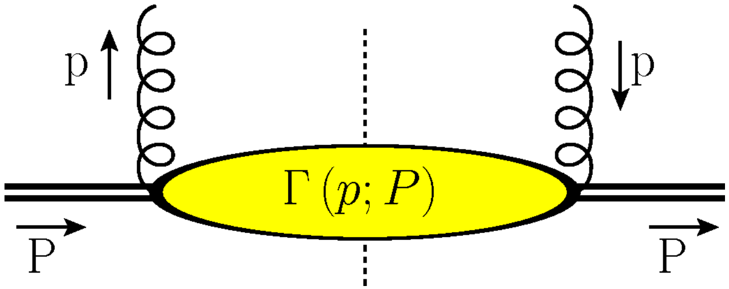
MGAB, Mukherjee, Mulders, PRD 86 (2012) 074030

Part II: gluon correlators



Gluon correlators

- Similar to quarks, the gluon correlator can be written as a matrix element.



$$\Gamma^{\mu\nu}(x, p_T; n) = \int \frac{d(\xi\cdot P)d^2\xi_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P | F^{n\mu}(0) F^{n\nu}(\xi) | P \rangle$$

- Depending on polarization(s), differerent contributions are required

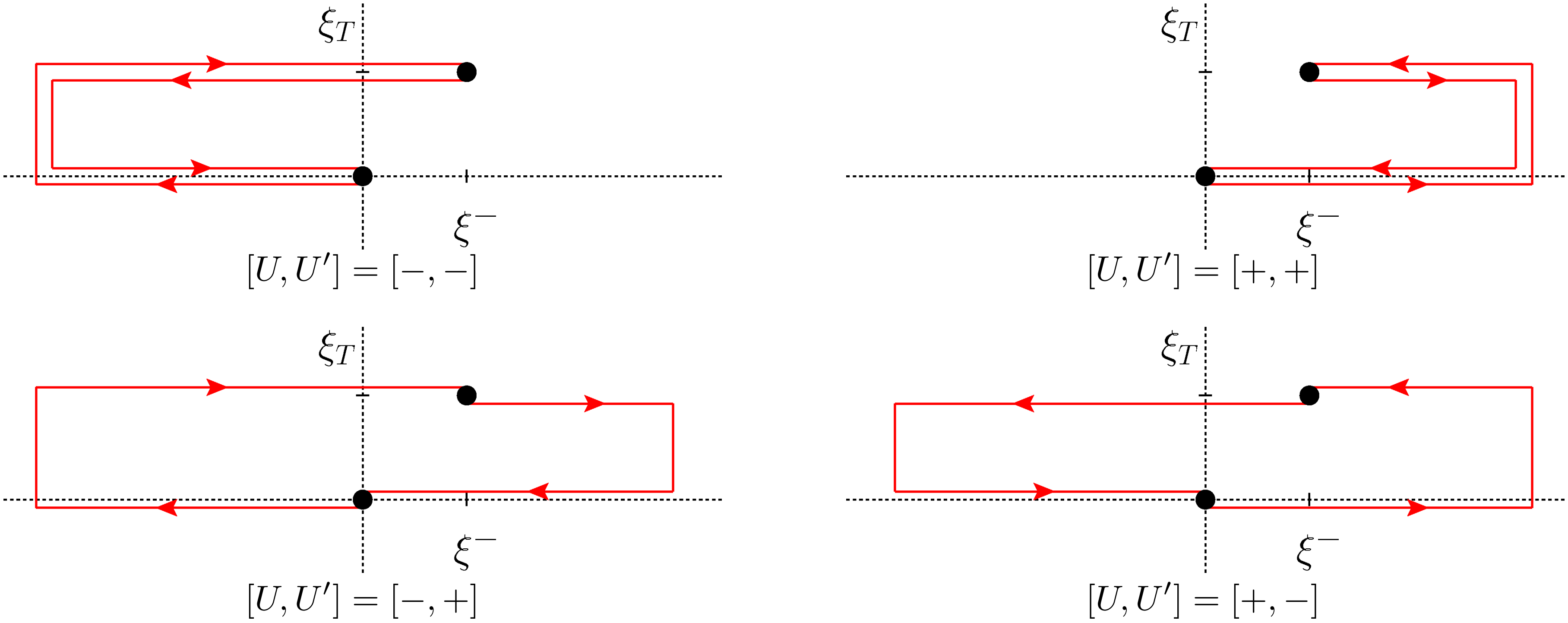
gluon polarization

nucleon pol.	U	L	Lin.
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

Gauge links

- For gluons more gauge link structures exist

$$\Gamma^{\alpha\beta[U,U']}(x,p_T;n) = \int \frac{d(\xi\cdot P)\,d^2\xi_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P|F^{n\alpha}(0)U_{[0,\xi]}F^{n\beta}(\xi)U_{[\xi,0]}|P\rangle_{\xi\cdot n=0}$$

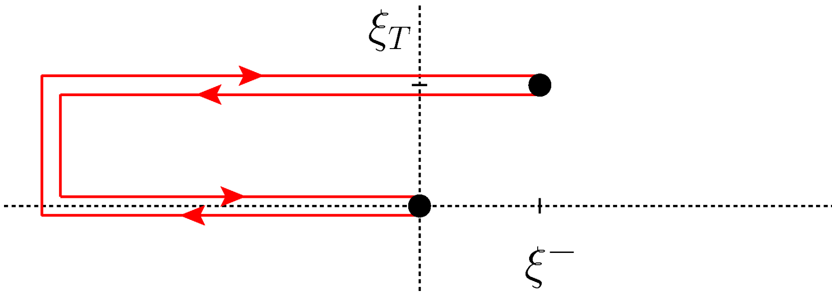


- More complicated structures arise for complicated processes

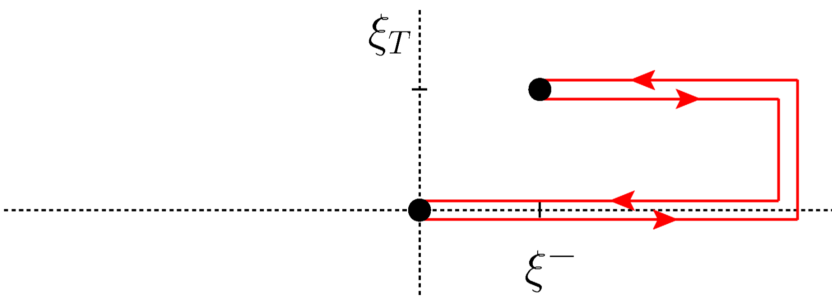
C.J. Bomhof, P.J. Mulders and F. Pijlman, PLB 596 (2004) 277
C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur.Phys.J. C47 (2006) 147

Gauge links

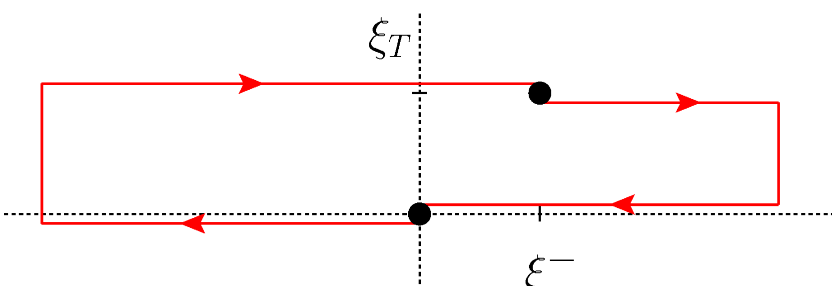
- The simplest gauge links have simple interpretations



$[U, U'] = [-, -]$

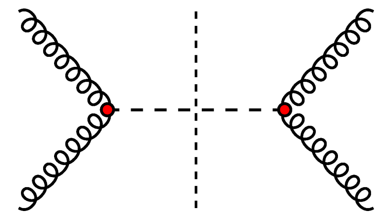


$[U, U'] = [+, +]$

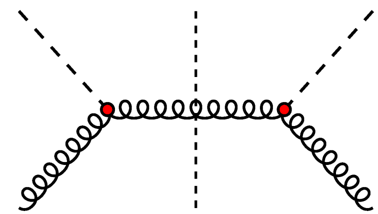


$[U, U'] = [-, +]$

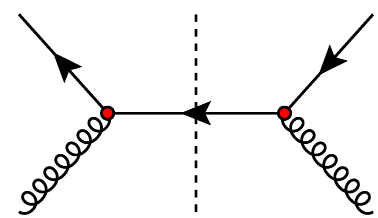
- All color in the initial state, e.g.



- All color to the final state, e.g.

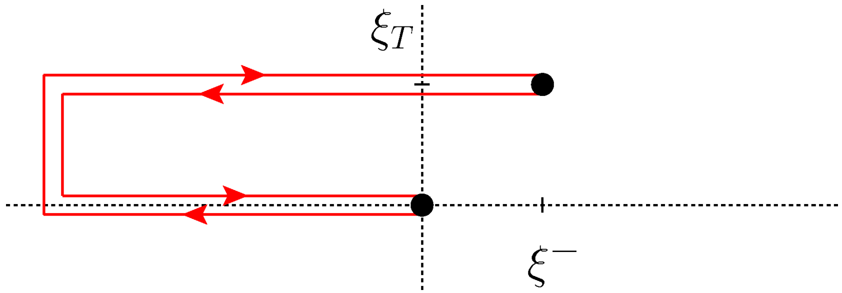


- Color splitting: some in the initial state, some to the final state

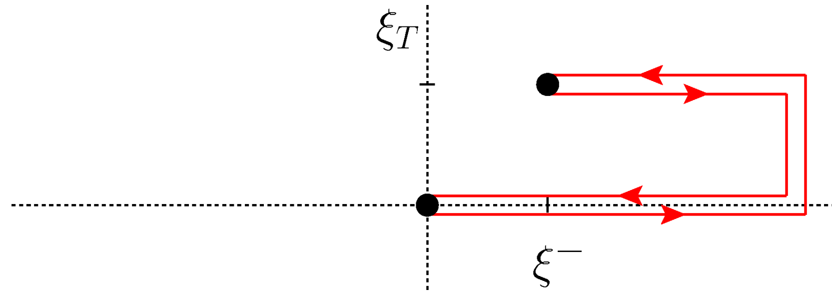


Color structures

- This gives the operator combinations

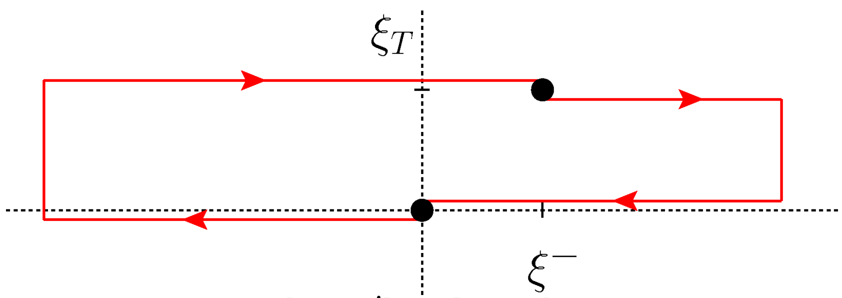


$[U,U'] = [-,-]$

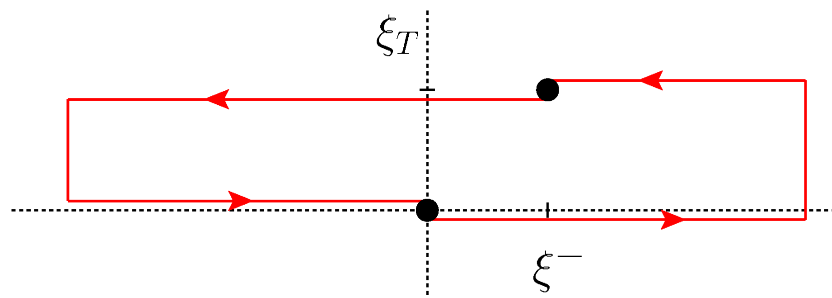


$[U,U'] = [+,+]$

$$\Gamma^{\mu\nu,\alpha}_{G,c=1} = \ldots \langle P|\mathrm{Tr}_c\left(F^{n\mu}(0)U_{[0,\xi]}\left[G_T^{n\alpha}, F^{n\nu}(\xi)\right]U_{[\xi,0]}\right)|P\rangle_{\xi\cdot n=0}$$



$[U,U'] = [-,+]$



$[U,U'] = [+,-]$

$$\Gamma^{\mu\nu,\alpha}_{G,c=2} = \ldots \langle P|\mathrm{Tr}_c\left(F^{n\mu}(0)U_{[0,\xi]}\left\{G_T^{n\alpha}, F^{n\nu}(\xi)\right\}U_{[\xi,0]}\right)|P\rangle_{\xi\cdot n=0}$$

Bomhof, PJM, 2007; Dominguez, Xiao, Yuan (2011); MGAB, A Mukherjee, Mulders, 2013

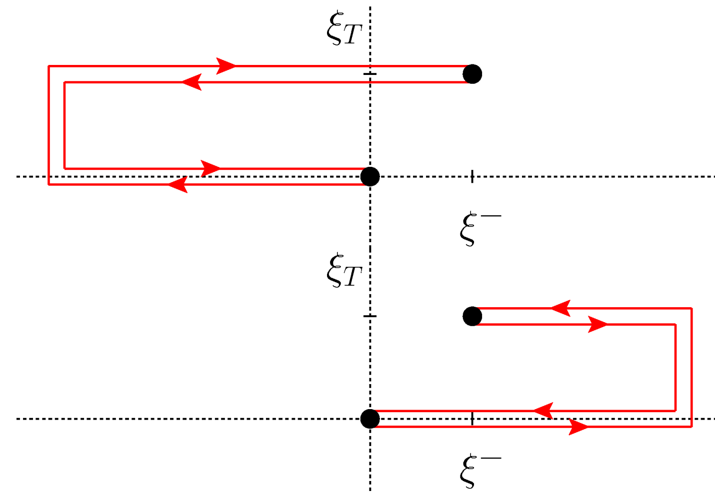
Color structures

- This gives the operator combinations
  - Commutators

$$\Gamma_{G,c=1} \rightarrow \mathrm{Tr}_c\Big(F(0)\,[G, F(\xi)]\Big)$$

$$\Gamma_{GG,c=1} \rightarrow \mathrm{Tr}_c\Big(F(0)\,[G,[G, F(\xi)]]\Big)$$

$$\Gamma_{GGG,c=1} \rightarrow \text{etc.}$$

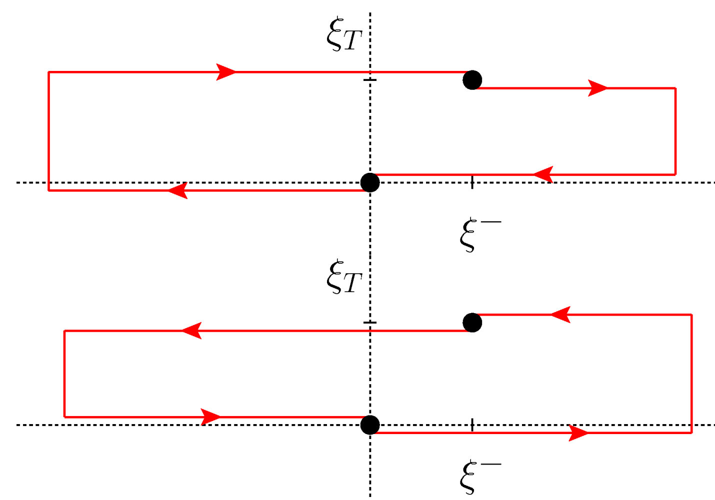


  - Anti-commutators

$$\Gamma_{G,c=2} \rightarrow \mathrm{Tr}_c\Big(F(0)\,\{G, F(\xi)\}\Big)$$

$$\Gamma_{GG,c=2} \rightarrow \mathrm{Tr}_c\Big(F(0)\,\{G,\{G, F(\xi)\}\}\Big)$$

$$\Gamma_{GGG,c=2} \rightarrow \text{etc.}$$

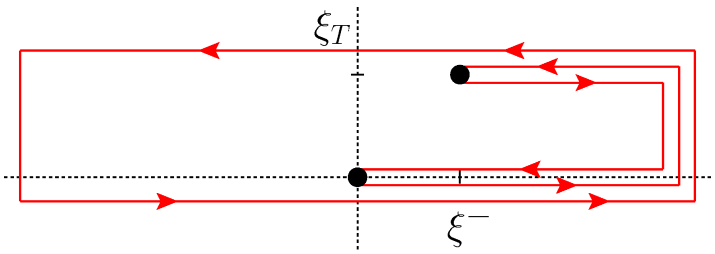


MGAB, Mukherjee, Mulders, PRD 88 (2013) 054027

Color structures

- For higher transverse moments, operator structures contain
  - Color traces

$$\Gamma_{GG,c=3} \rightarrow \mathrm{Tr}_c\Big(F(0)F(\xi)\Big)\mathrm{Tr}_c\Big(\,\{G,G\}\,\Big)$$



  - And combinations thereof

- One obtains a finite number of operator structures

MGAB, Mukherjee, Mulders, PRD 88 (2013) 054027

Universality

- Depending on the process, TMDs contain multiple contributions

$$f_{1T}^{\perp g[U]} = \sum_{c=1}^{2} C_{G,c}^{[U]} \, f_{1T}^{\perp g(Ac)}$$

$$h_{1T}^{g[U]} = \sum_{c=1}^{2} C_{G,c}^{[U]} \, h_{1T}^{g(Ac)}$$

$$h_{1L}^{\perp g[U]} = \sum_{c=1}^{2} C_{G,c}^{[U]} \, h_{1L}^{\perp g(Ac)}$$

$$h_{1}^{\perp g[U]} = h_{1}^{\perp g(A)} + \sum_{c=1}^{4} C_{GG,c}^{[U]} \, h_{1}^{\perp g(Bc)}$$

$$h_{1T}^{\perp g[U]} = \sum_{c=1}^{2} C_{G,c}^{[U]} \, h_{1T}^{\perp g(Ac)} + \sum_{c=1}^{7} C_{GGG,c}^{[U]} \, h_{1T}^{\perp g(Bc)}$$

gluon polarization

nucleon pol.	U	L	Lin.
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

Bomhof, Mulders, (2007)
MGAB, Mukherjee, Mulders, (2013)

Summary and conclusions

- Transverse directions are required
  - TMDs
  - Polarizations
- Gauge links are required
  - TMDs become process dependent
  - Universality broken?
- Solving the puzzles
  - Linear combination of finite number of TMDs
  - Quarks: for $h_1^{\perp}$, $f_{1T}^{\perp}$ and $h_{1T}^{\perp}$
  - Gluons: for $h_1^{\perp g}$, $h_{1L}^{\perp g}$, $f_{1T}^{\perp g}$, h_{1T}^{g} and $h_{1T}^{\perp g}$