



TMDS: evolution, modeling, precision

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based on:

arXive: 1406., with U. D' Alesio (Cagliari), M.G. Echevarría (NIKHEF), S. Melis (Torino)
arXive: 1402.0869, with M.G. Echevarría (NIKHEF), A. Idilbi (Penn U.), soon PRD
(see Miguel's talk at QCD Evolution '14)

and also PLB726(2013)795,
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Topics and outline

We have finally factorized cross-sections with TMDs (Collins '11, EIS '11-13).
How do we know TMDs?

* Spin dependent observables and transverse momentum dependent observables need factorization theorems with TMD's

* TMD's are the fundamental **non-perturbative** objects to be used in factorization theorems in (un-)polarized Drell-Yan, SIDIS, e^+e^- to 2 jets (multi-jets?). What about LHC?

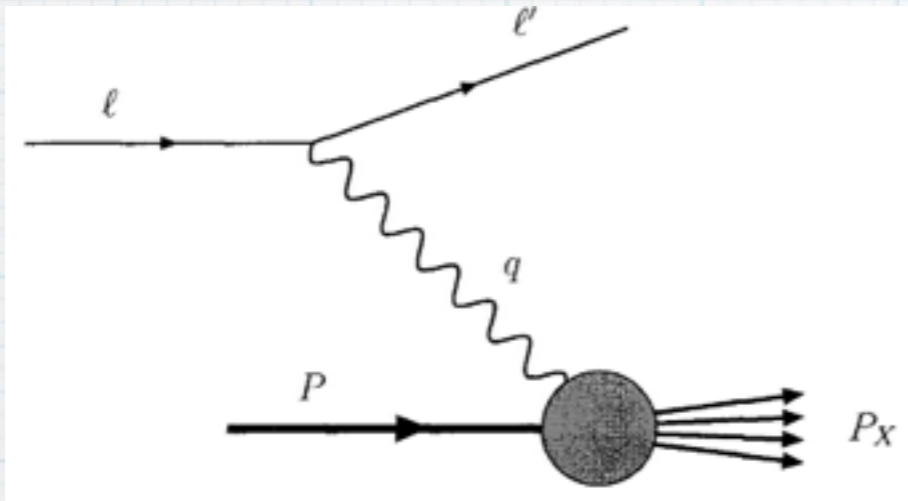
* Properties of TMD's:
1) The evolution of all TMD's is universal (alike PDF and FF it is process independent)
2) The evolution of all TMD's is spin independent and it is the same for TMDPDF and TMDFF

* We know the evolution completely at NNLL...

* Extraction of unpolarized TMDPDF from Drell-Yan and Z-boson production using completely resummed TMD's at NNLL

Preliminary

Outline of Factorization theorem



SIDIS as a study case:
both PDF and FF

$$q^2 \gg q_T^2$$

Fact. scale

$$l(k) + N(P) \rightarrow l'(k') + h(P_h) + X(P_X)$$

$$W^{\mu\nu} = H(Q^2/\mu^2) \frac{2}{N_c} \sum_q e_q \int d^2 k_{n\perp} d^2 k_{\bar{n}\perp} \delta^{(2)}(\mathbf{q}_\perp + \mathbf{k}_{n\perp} - \mathbf{k}_{\bar{n}\perp})$$

$$\times \text{Tr} [F(x, \mathbf{k}_{n\perp}, S; Q^2/\alpha, \mu^2) \gamma^\mu D(z, \hat{P}_{h\perp}, S_h; Q^2/\alpha, \mu^2) \gamma^\nu]$$

Hard coeff.

TMDPDF

TMDFF

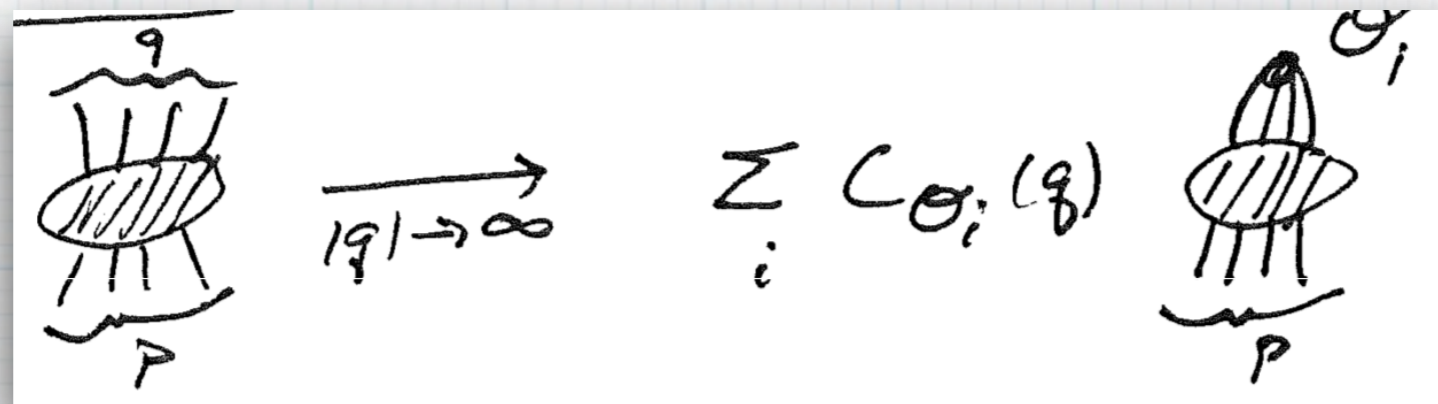
Soft splitting number

$$\mathbf{k}_{\bar{n}\perp} = -\hat{P}_{h\perp}/z$$

$$\zeta_F = Q^2/\alpha$$

$$\zeta_D = \alpha Q^2$$

Origin of factorization and EFT



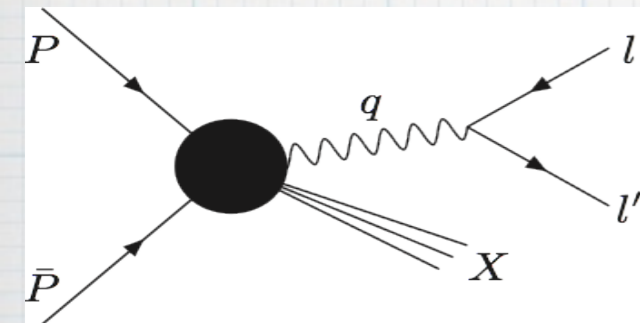
Drawing from J. Preskill lectures

In Green function, when the momenta "q" of some fields are much larger than the others we can "factor out" the hard momenta onto Wilson coefficient and effective matrix elements and use RGE on coefficients to resum large logs (Wilson, Zimmerman, Callan, Symanzyk '70-'72)

Wilson coefficients and hard factors depend only on the high scale "q" and the factorization scale " μ ": no other (IR-sensitive) scale.

Same philosophy in the construction of EFT: identification of modes, effective field theory Lagrangians, factorized cross sections, Wilson coefficients, ... ⁴

Energy scales: DY example



Fundamental condition for factorization

$$q^2 = Q^2 \gg q_T^2$$

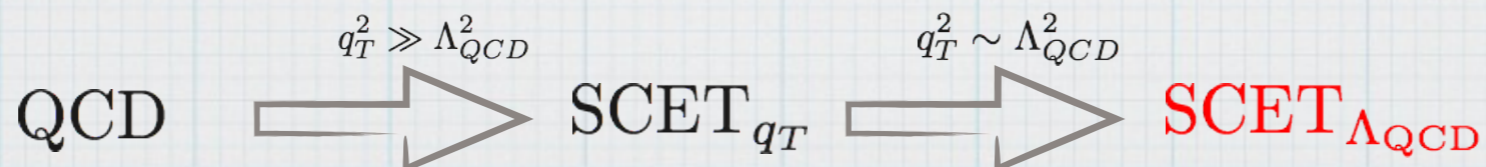
$$q_T^2 \sim \Lambda_{QCD}^2 \quad \longrightarrow \quad \tilde{M} = H(Q^2/\mu^2) \tilde{F}_n(x_n, b; Q^2, \mu^2) \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

$$q_T^2 \gg \Lambda_{QCD}^2 \quad \longrightarrow \quad \tilde{M} = H(Q^2/\mu^2) \tilde{C}_n(b^2\mu^2, Q^2/\mu^2) \tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2) f_n(x_n; \mu^2) f_{\bar{n}}(x_{\bar{n}}; \mu^2)$$

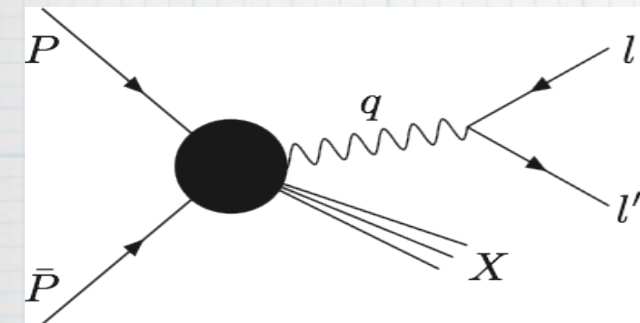
$$\tilde{C}(b^2\mu^2, Q^2/\mu^2)$$

Matching (Wilson) coefficients are extracted regulating consistently two theories above and below the (matching) scale

Processes with different energy scales are more easily treated with EFT: same results with Pert. QCD



Energy scales: DY example



Fundamental condition for factorization

$$q^2 = Q^2 \gg q_T^2$$

$$q_T^2 \sim \Lambda_{QCD}^2 \quad \longrightarrow \quad \tilde{M} = H(Q^2/\mu^2) \tilde{F}_n(x_n, b; Q^2, \mu^2) \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

$$q_T^2 \gg \Lambda_{QCD}^2 \quad \longrightarrow \quad \tilde{M} = H(Q^2/\mu^2) \tilde{C}_n(b^2\mu^2, Q^2/\mu^2) \tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2) f_n(x_n; \mu^2) f_{\bar{n}}(x_{\bar{n}}; \mu^2)$$

In the construction of TMDPDF we take into account these limits

$$\Lambda_{QCD} \sim 1\text{GeV}$$

Evolution kernel for TMD's

$$\tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_{F,f}, \mu_f^2) = \tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_{F,i}, \mu_i^2) \tilde{R}(b_T; \zeta_{F,i}, \mu_i^2, \zeta_{F,f}, \mu_f^2),$$

$$\tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_{D,f}, \mu_f^2) = \tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_{D,i}, \mu_i^2) \tilde{R}(b_T; \zeta_{D,i}, \mu_i^2, \zeta_{D,f}, \mu_f^2),$$

$$\tilde{R}(b; \zeta_i, \mu_i^2, \zeta_f, \mu_f^2) = \exp \left\{ \int_{\mu_i}^{\mu_f} \frac{d\bar{\mu}}{\bar{\mu}} \gamma \left(\alpha_s(\bar{\mu}), \ln \frac{\zeta_f}{\bar{\mu}^2} \right) \right\} \left(\frac{\zeta_f}{\zeta_i} \right)^{-D(b_T; \mu_i)}$$

For consistency the A.D. of the TMD is the opposite of the one of the Hard coefficient

SIDIS:

$$\gamma_H = -\gamma_F \left(\alpha_s(\mu), \ln \frac{\zeta_F}{\mu^2} \right) - \gamma_D \left(\alpha_s(\mu), \ln \frac{\zeta_D}{\mu^2} \right)$$

$$\gamma_{F,D} \left(\alpha_s(\mu), \ln \frac{\zeta_{F,D}}{\mu^2} \right) = -\Gamma_{\text{cusp}}(\alpha_s(\mu)) \ln \frac{\zeta_{F,D}}{\mu^2} - \gamma^V(\alpha_s(\mu))$$

$$\frac{dD}{d \ln \mu} = \Gamma_{\text{cusp}}$$

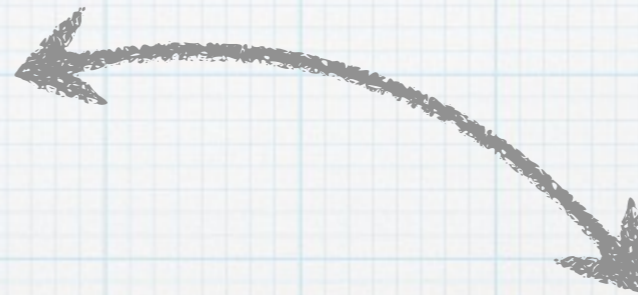
7 Cusp known at order α_s^3

D-resummation

$$\frac{dD(b; \mu)}{d \ln \mu} = \Gamma_{cusp}(\alpha_s)$$

$$D(b; \mu) = \sum_{n=1}^{\infty} d_n(L_{\perp}) \left(\frac{\alpha_s}{4\pi} \right)^n$$

	LL	NLL	NNLL	
$d_1(L_{\perp}) =$	$d_1^{(1)} L_{\perp}$	$+ d_1^{(0)}$		
$d_2(L_{\perp}) =$	$d_2^{(2)} L_{\perp}^2$	$+ d_2^{(1)} L_{\perp}$	$+ d_2^{(0)}$	
$d_3(L_{\perp}) =$	$d_3^{(3)} L_{\perp}^3$	$+ d_3^{(2)} L_{\perp}^2$	$+ d_3^{(1)} L_{\perp}$	$+ d_3^{(0)}$
$d_4(L_{\perp}) =$	$d_4^{(4)} L_{\perp}^4$	$+ d_4^{(3)} L_{\perp}^3$	$+ d_4^{(2)} L_{\perp}^2$	$+ d_4^{(1)} L_{\perp} + d_4^{(0)}$
$d_5(L_{\perp}) =$...			



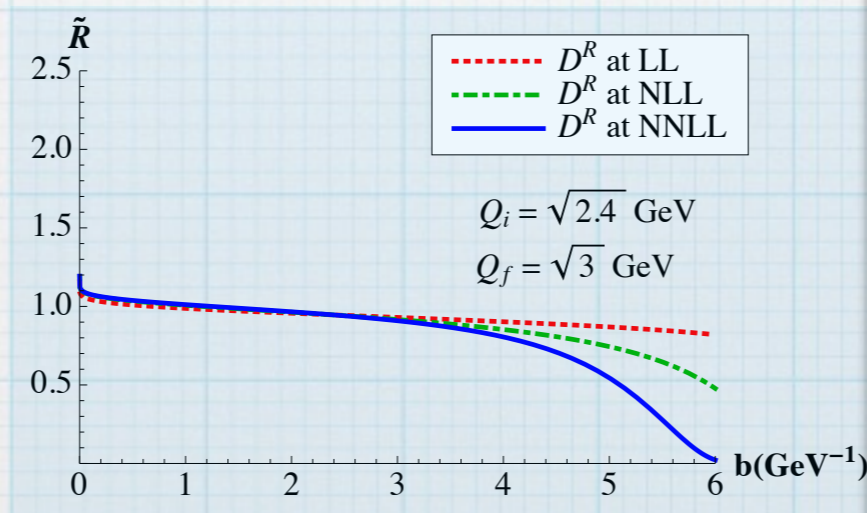
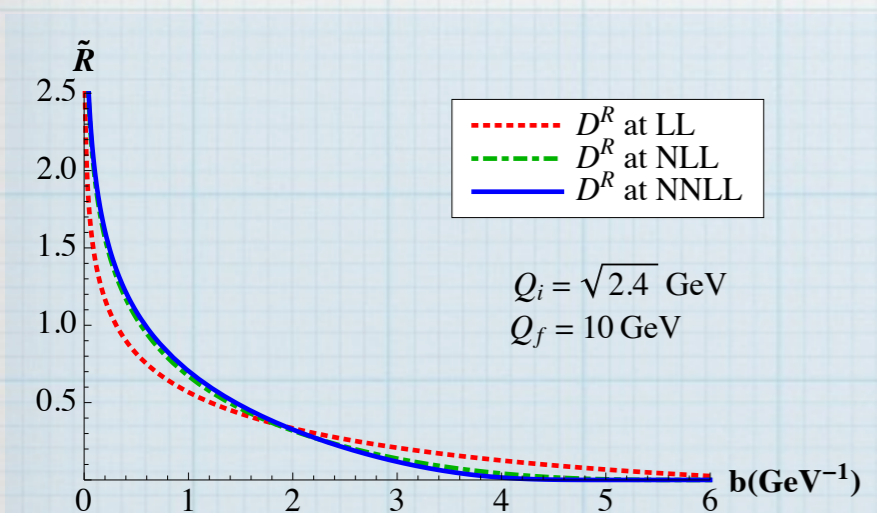
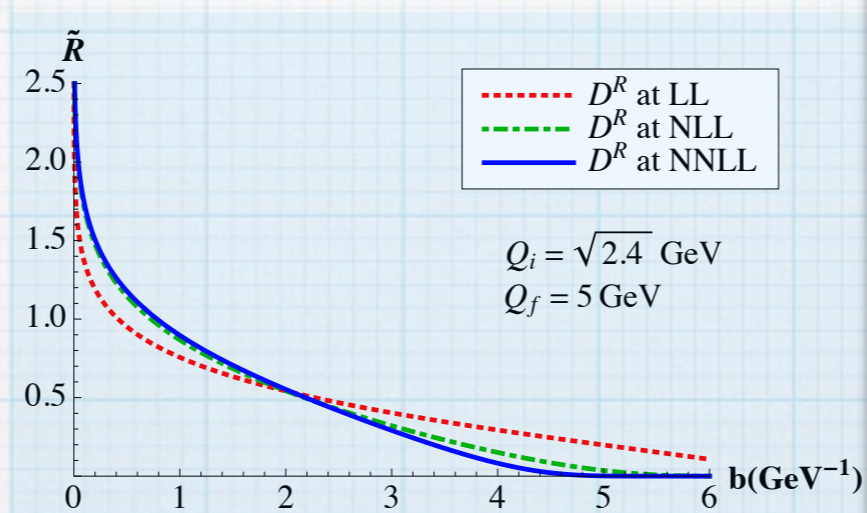
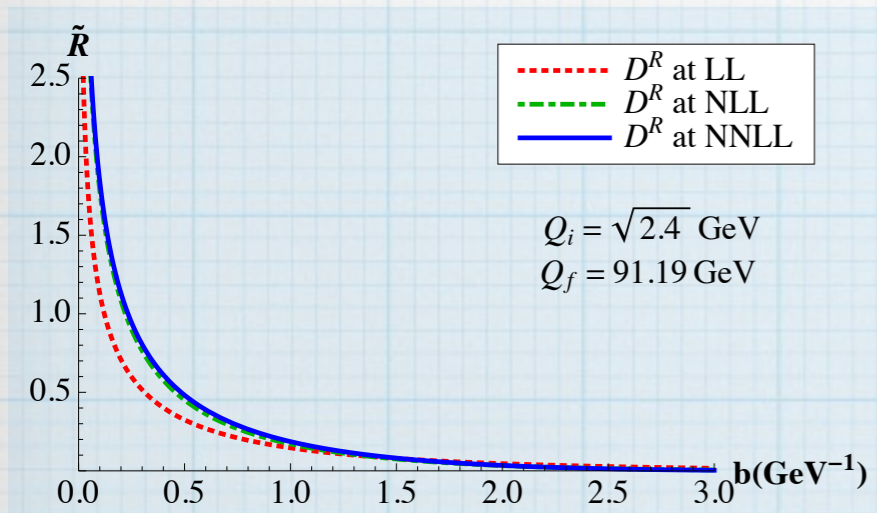
$$D(b; Q_i) = D(b; \mu_b) + \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma_{cusp}; \quad \mu_b = 2e^{-\gamma_E} / b$$

$$D(b; Q_i) = -\frac{\Gamma_0}{2\beta_0} \ln \frac{\alpha_s(Q_i)}{\alpha_s(\mu_b)} \longrightarrow D(b; Q_i) = -\frac{\Gamma_0}{2\beta_0} \ln(1 - X)$$

$$\alpha_s(\mu_b) = \alpha_s(Q) / (1 - X) \quad \text{Landau pole}$$

The perturbative expansion of the D is valid in limited (but large, using **resummation**) portion of Impact Parameter Space

Plots for evolution kernel (fixed scale example)



- * Very good convergence up to $b=4-5/\text{GeV}$ in all cases
- * The region sensitive to the Landau pole is strongly suppressed $b > 5/\text{GeV}$
- * For $M=M_z$ we are sensitive only to $b < 1.5/\text{GeV}$ region
- * For $M=3-5 \text{ GeV}$ we are sensitive only to $b < 4/\text{GeV}$ region
- * For $M < 2 \text{ GeV}$ we can be sensitive to the Landau pole region

- * Studying processes at different energies one explores different regions in IPS
- * The Landau pole problems appear there where also the Factorization fails (or starts to fail). **This is a QCD general problem!**

Building a TMD: Inputs from Pert.QCD

The asymptotic limit of TMDs should be included in the construction of TMDs: The Standard TMD form

$$\tilde{F}_{q/N}(x, \vec{b}, Q_i, \mu) = \left(\frac{Q_i^2 b^2}{4e^{2\gamma_E}} \right)^{-D_R(b, \mu)} \sum_j \tilde{C}_{q \leftarrow j}(x, \vec{b}_\perp, \mu) \otimes f_{j/N}(x; \mu) \otimes M_q(x, \vec{b}, Q_i)$$

OPE to PDF, valid for $q_T \gg \Lambda_{QCD}$

PDF

Process independent
Non-perturbative correction

Common to all analysis:
Calculated at 2-loops

Florence (S. Catani et al.'08), Zurich (T. Gehrmann. et al '13-14)

$$\tilde{C}_{q \leftarrow j}(x, \vec{b}_\perp, \mu) \equiv \exp(h_\Gamma - h_\gamma) \hat{C}_{q \leftarrow j}(x, \vec{b}_\perp, \mu)$$

$$\frac{dh_\Gamma}{d \ln \mu} = \Gamma_{cusp} L_\perp$$

$$\frac{dh_\gamma}{d \ln \mu} = \gamma^V$$

$$h_\Gamma^R(b, \mu) = \int_{\alpha_s(1/\hat{b})}^{\alpha_s(\mu)} d\alpha' \frac{\Gamma_{cusp}^F(\alpha')}{\beta(\alpha')} \int_{\alpha_s(1/\hat{b})}^{\alpha'} \frac{d\alpha}{\beta(\alpha)}$$

- Exponentiation of part of the coefficient (Kodaira, Trentadue 1982, Becher, Neubert Wilhelm 2011)
- Complete resummation of the logs in the exponent, as for the resummed D

Building a TMD: Inputs from Pert.QCD

The asymptotic limit of TMDs should be included in the construction of TMDs: The Standard TMD form

$$\tilde{F}_{q/N}(x, \vec{b}, Q_i, \mu) = \left(\frac{Q_i^2 b^2}{4e^{2\gamma_E}} \right)^{-D_R(b, \mu)} \sum_j \tilde{C}_{q \leftarrow j}(x, \vec{b}_\perp, \mu) \otimes f_{j/N}(x; \mu) \otimes M_q(x, \vec{b}, Q_i)$$

OPE to PDF, valid for $q_T \gg \Lambda_{QCD}$

PDF

Process independent
Non-perturbative correction

M_q expected to correct
resummed \mathcal{D} and OPE

The splitting between coeff. and PDF can generate large logs:
a wise choice of factorization scale can avoid this problem.

Becher, Neubert, Wilhelm '12

Ideally: $\mu \sim q_T$ In practice: $\mu \sim Q_i = Q_0 + q_T$

Q_0 is the scale where PDF are better defined: $Q_0 \sim 2 \text{ GeV}$

Building a TMD: Inputs from Pert.QCD

$$\tilde{M} = H(Q^2 / \mu^2) \tilde{F}_n(x_n, b^2; Q^2, \mu^2) \tilde{F}_{\bar{n}}(x_{\bar{n}}, b^2; Q^2, \mu^2)$$

Hard coeff. using π -resummation (Ahrens et al. '08): evolution of H from $-Q^2$ to Q^2

	$\mu_h^2 = m_Z^2$	$\mu_h^2 = -m_Z^2$
NLL	$1.000^{+0.160}_{-0.060}$	$1.334^{+0.201}_{-0.074}$
NNLL	$1.087^{+0.010}_{-0.001}$	$1.131^{+0.001}_{-0.014}$
N ³ LL	$1.119^{+0.006}_{-0.001}$	$1.130^{+0.001}_{-0.001}$

Better convergence of the perturbative series

Theoretical settings

- * Matching scale of TMDPDF to PDF at $Q_i = 2 \text{ GeV} + q_T$
- * Hard coefficient with π^2 resummation
- * Checked both NLL and NNLL
- * Several sets of PDF checked (MSTW, CTEQ)
- * Checked several form of non-perturbative models: gaussian, exponential, Q-dependence (\mathcal{D}_{NP}), ...
- * Non-perturbative input (***adiabatic model**): corrections from moderate to high values of b ,

$$M_q(x, \vec{b}, Q_i) = \exp[-\lambda_1 b] (1 + b^2 \lambda_2 + \dots)$$

$\alpha_s L_\perp \sim \mathcal{O}(1)$

Order	γ	Γ_{cusp}	C	D
LL	-	α	tree	-
NLL	α	α^2	tree	α
NNLL	α^2	α^3	α	α^2
NNNLL	α^3	α^4	α^2	α^3



Our analysis

* Ahmad suggestion

Experimental Data

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
points	32	16	41	9
\sqrt{s}	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
σ	248 ± 11 pb	221 ± 11.2 pb	256 ± 15.2 pb	255.8 ± 16.7 pb

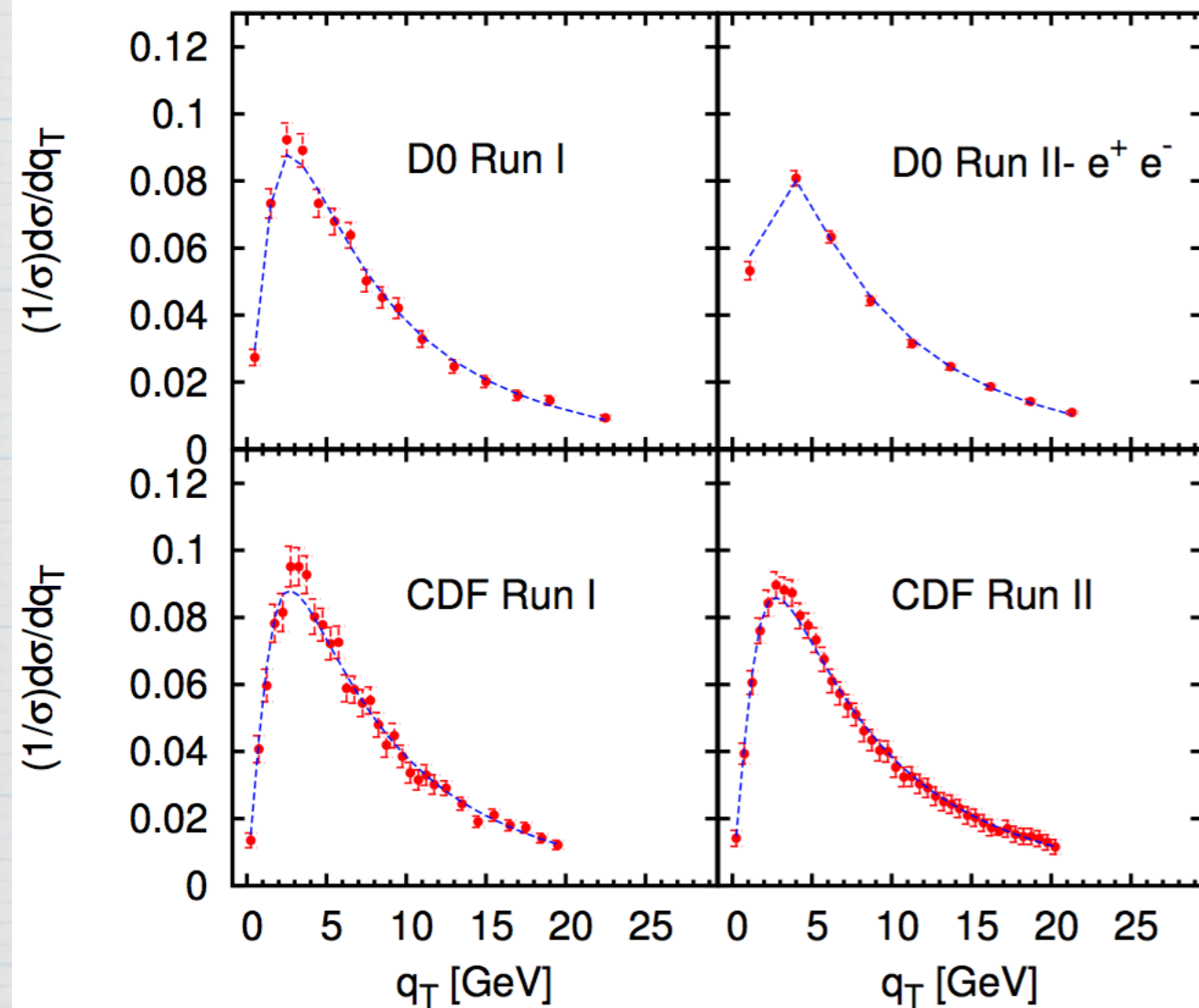
Z, run I: Becher, Neubert, Wilhelm 2011
Catani et al. 2009

Total cross section improved from run I to run II

	E288 200	E288 300	E288 400	R209
points	35	35	49	6
\sqrt{s}	19.4 GeV	23.8 GeV	27.4 GeV	62 GeV
E_{beam}	200 GeV	300 GeV	400 GeV	-
Beam/Target	p Cu	p Cu	p Cu	p p
M range used	4-9 GeV	4-9 GeV	5-9 and 10.5-14 GeV	5-8 and 11-25 GeV
Other kin. var	$y=0.4$	$y=0.21$	$y=0.03$	
Observable	$Ed^3\sigma/d^3p$	$Ed^3\sigma/d^3p$	$Ed^3\sigma/d^3p$	$d\sigma/dq_T^2$

Expected to be insensitive to Landau pole region
Factorization hypothesis hold

Preliminary results at NNLL



Z-boson data are (fairly) sensitive to functional non-perturbative form (gaussian vs. exponential) and (poorly) sensitive just to λ_1 . In order to fix it we need the global fit

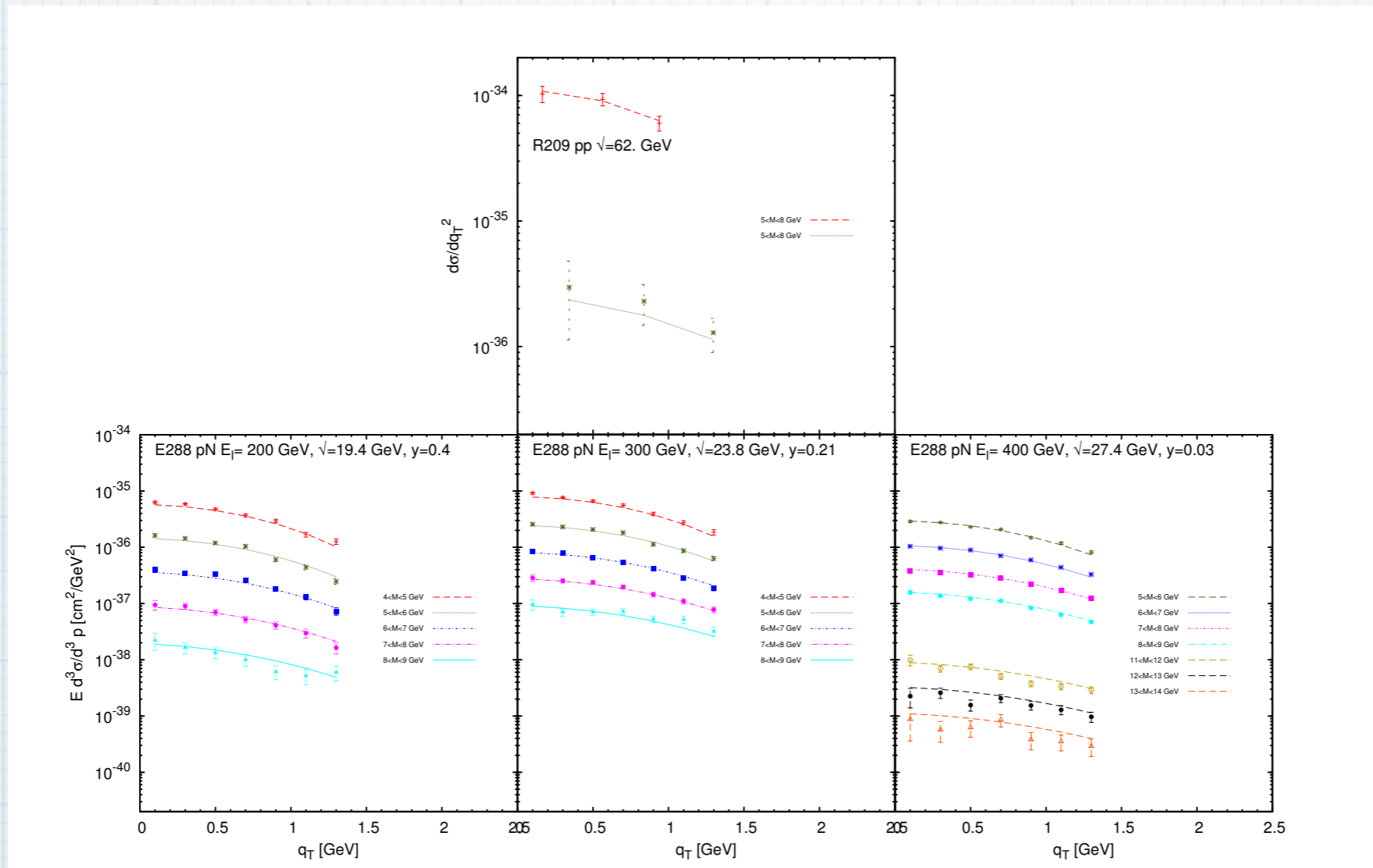
Avoiding systematics

Data: $\frac{1}{\sigma_{exp}} \left(\frac{d\sigma}{dq_T} \right)_{exp}$

Theory: $\frac{1}{\sigma_{th}} \left(\frac{d\sigma}{dq_T} \right)_{th}$

DYNNLO: Catani, Grazzini '07, Catani, Cieri, Ferrera, de Florian, Grazzini '09

Preliminary results at NNLL



Exp. Normalization
NE288, NR209
deduced from the fit.

Total: 4 parameters

$$\lambda_1 = 0.33 \pm 0.05_{stat}$$

$$\lambda_2 = 0.13 \pm 0.03_{stat}$$

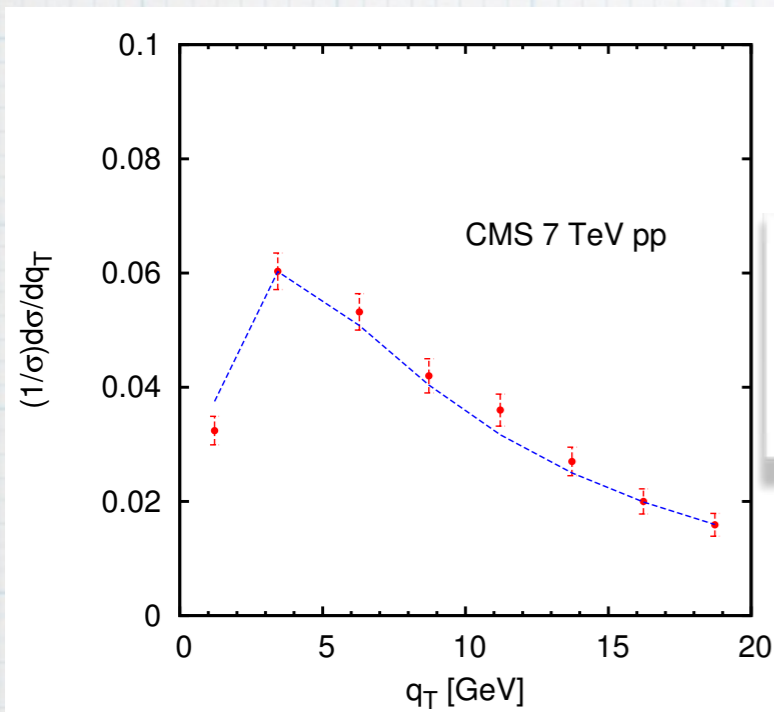
$$N_{E288} = 0.85 \pm 0.04_{stat}$$

$$N_{R209} = 1.5 \pm 0.02_{stat}$$

$$\frac{\chi^2}{\text{points}} \Big|_z \simeq 0.6$$

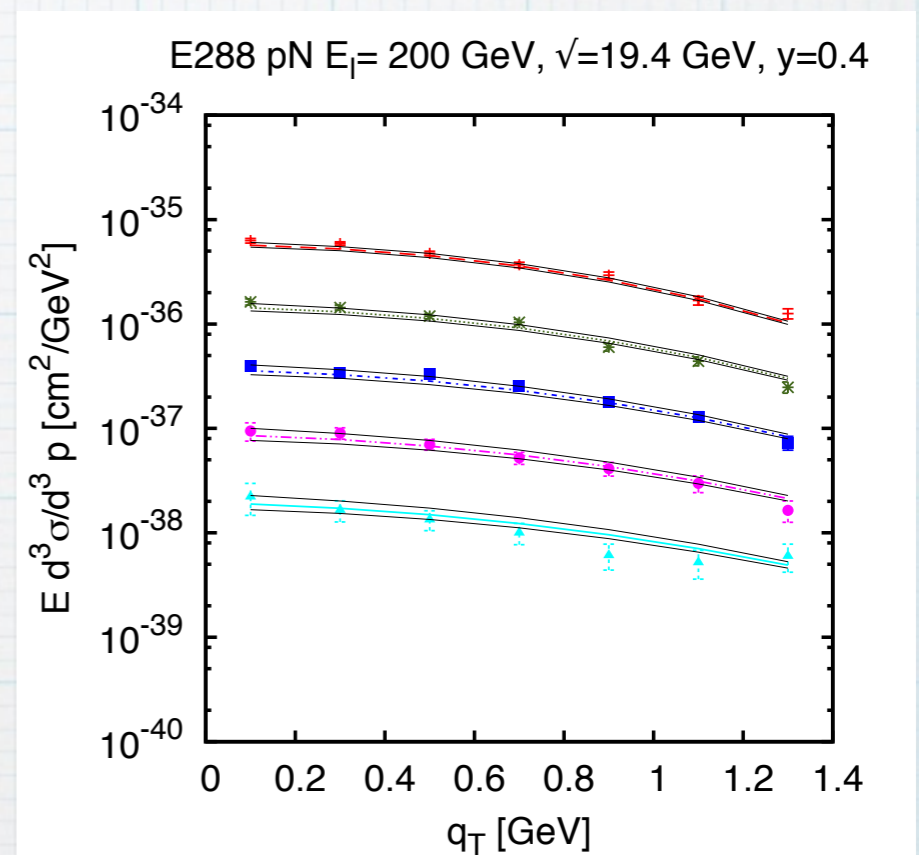
$$\frac{\chi^2}{\text{points}} \Big|_{global} \simeq 1.1$$

Preliminary results at NNLL



Prediction for Z production with **CMS** data.
Bin width not shown

$$\frac{\chi^2}{\text{points}} \simeq 0.9$$



Scale dependence (work in progress)

$$m_c \sim 1.3 \text{ GeV} < Q_0 < 2.7 \text{ GeV}$$

1st bin energy

- * Data are not sensitive to: the Landau pole region in IPS, non-perturbative Q-dependence. The study of flavor dependence needs to include also W-production data....

SIDIS analysis

SIDIS analysis can be performed with the same level of (perturbative) precision:

The evolution kernel is the same for PDF's and FF's to all orders in PT (EIS'14)

Hard coefficient also known at N^3LL

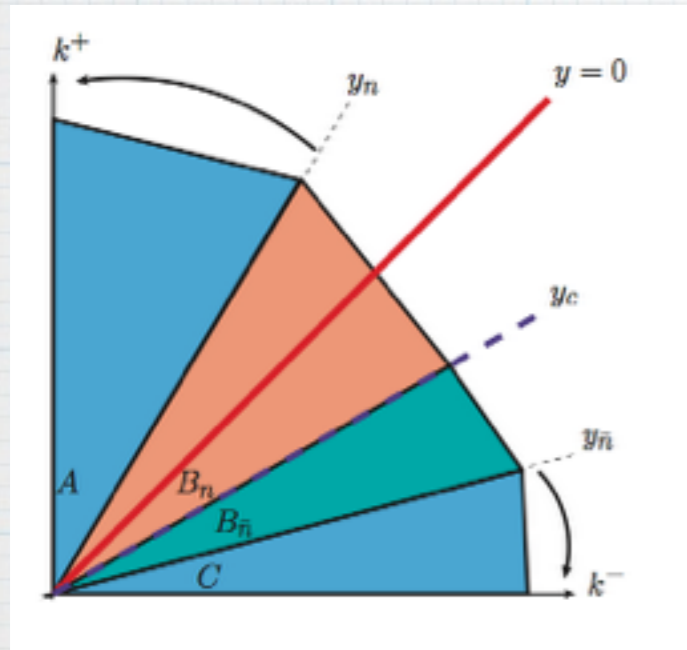
We can rely on integrated FF and matching coefficient also known for NNLL analysis

Input model should have a similar structure

Data: Small photon energy may require higher twist analysis
(see Stefano's talk)

Evolution for TMDFF and TMDPDF: Resume

EIS'14



$$\frac{d}{d \ln \zeta_F} \ln \tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) = -D(b_T; \mu^2),$$

$$\frac{d}{d \ln \zeta_D} \ln \tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) = -D(b_T; \mu^2).$$

$$\zeta_F = Q^2 / \alpha$$

$$\zeta_D = \alpha Q^2$$

- * Each sector is linear in logs of the rapidity cut
- * Each collinear sector depends solely on the corresponding collinear momentum. It is not possible that a Q dependence arise in a pure collinear sector, Q dependence arises only in the soft sector.
- * The soft function is linear in the logs of rapidity regulator to cancel the corresponding logs in collinear sectors (In QCD there are no rapidity divergences)
- * The soft function is Hermitian, so it is the same for DIS, DY and ee to 2 jets
- * The soft function dictates the evolution for all TMDs at all orders in PT

more in Miguel talk ..

Conclusions

The analysis of current data coming from experiments run at different energy require the use of evolution of TMD's and full resummation techniques:

Reduction of model dependence

Recovery of the complete perturbative limit (we have to connect to pQCD results)

Improved Convergence of the perturbative series

We have tested all this in DY and Z-boson qT spectra: **agreement with data with**

$$\chi^2/d.o.f. \sim 1.1$$

- * Fits for unpolarized TMDPDF in DY and Z-production, performed with data which fulfill fundamental conditions for factorization, allow to fix the non-perturbative parameters. More data and more processes are welcome (SIDIS, ee->jj,...)
 - * TMDs can be used to fix the final precision at LHC, fixing the amount of non-perturbative QCD effects: all knowledge from PQCD studies must be used.
 - * Features of TMD's:
 - The evolution for TMDPDF's and TMDFF's is the same and spin independent
 - TMD's are universal (they can be extracted from DY, SIDIS, ee->2 jets,...)
 - * Analysis of spin dependent observables including evolution is starting now: Belle, Compass, LHC, RHIC, JLab,....
- Continues in the next slide.....**

THE BEAUTY OF EVOLUTION

ll... mandatory



THE BEAUTY OF EVOLUTION

LL... mandatory

NLL... great



THE BEAUTY OF EVOLUTION

LL... mandatory

NLL... great

NNLL... fantastic!!



THE BEAUTY OF EVOLUTION

LL... mandatory

NLL... great

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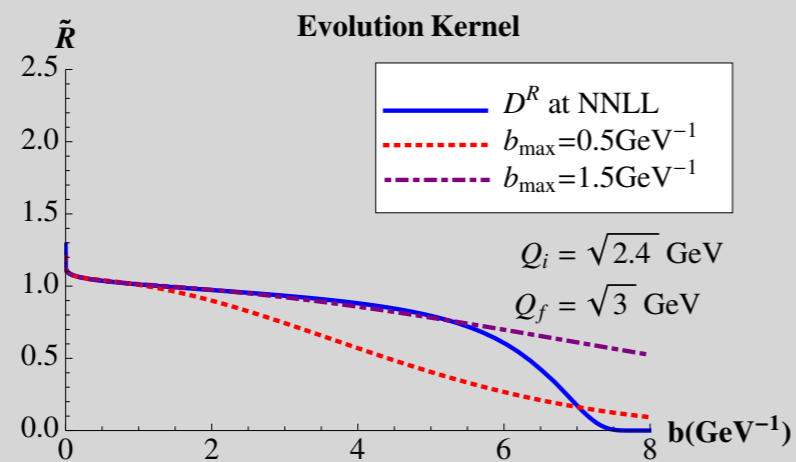
$N^3LL??$

Stay tuned!!

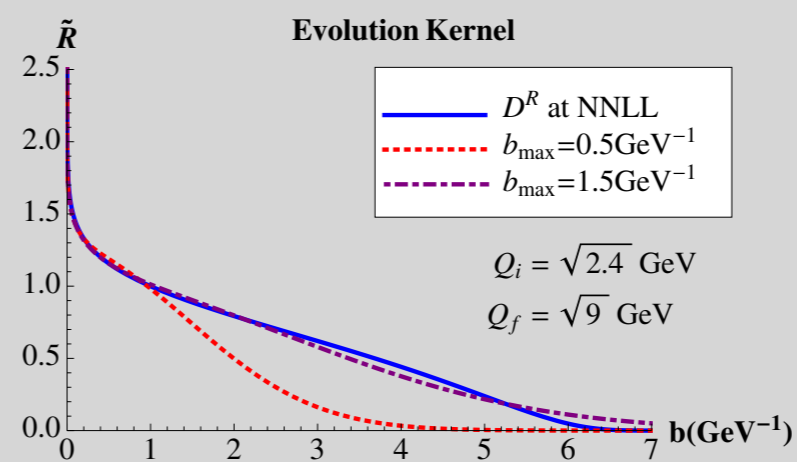


Back up

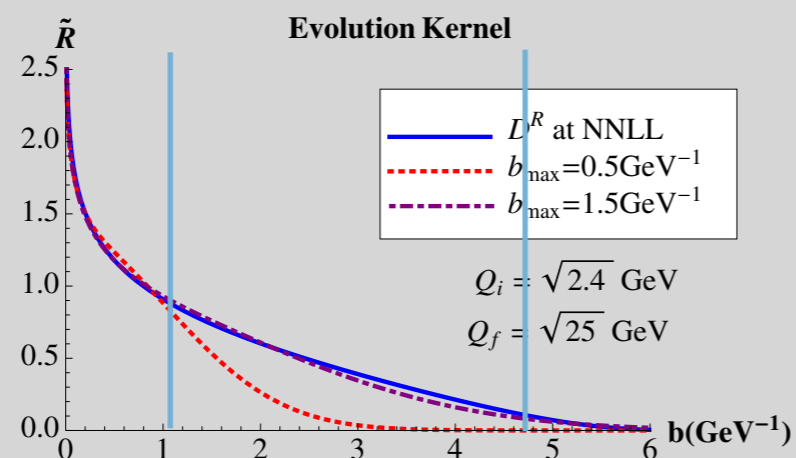
EIS vs CSS



(a)



(b)

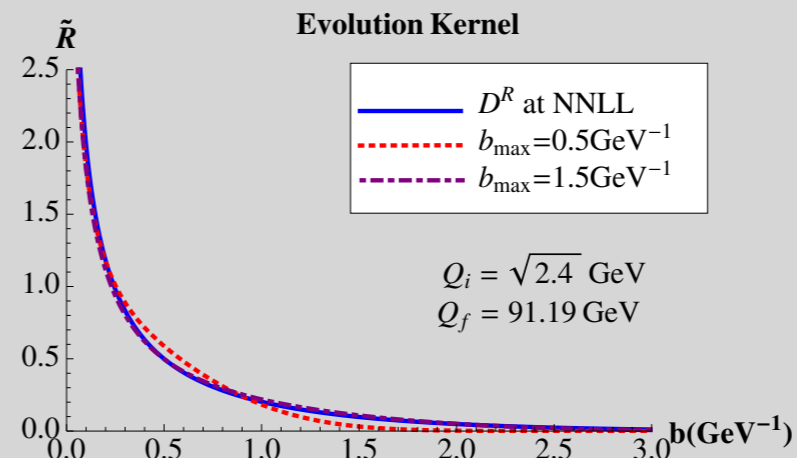


1

2

3

(d)



- 1: EIS and CSS agree, complete perturbative region. Model for TMD bulk
- 2: EIS: Model Independent Evolution (Completely Resummed Kernel). Model for TMD bulk
CSS: Model Dependent Evolution (b_{\max} and g_2). Model for TMD bulk+more "cooking" (see Stefano talk)
- 3: Landau pole region: Modeled both in EIS and CSS

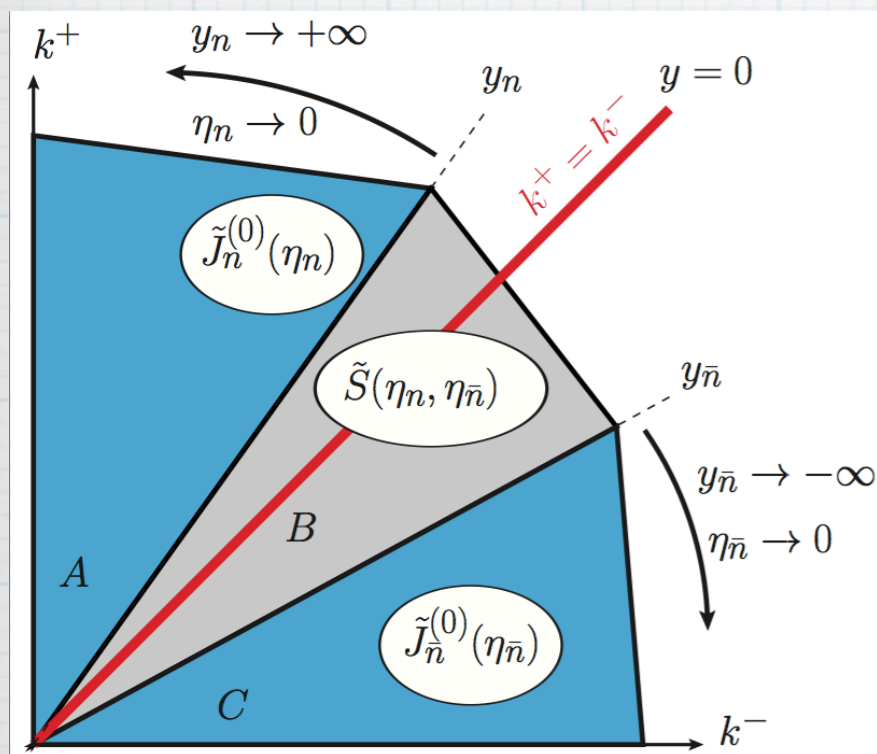
Modes in EFT

Using power counting we have collinear, anti-collinear, and soft sectors

$$H(Q^2) \tilde{J}_n^{(0)}(\eta_n) \tilde{S}(\eta_n, \eta_{\bar{n}}) \tilde{J}_{\bar{n}}^{(0)}(\eta_{\bar{n}})$$

In EFT each mode belongs to a Hilbert space separate from the others.

To each mode correspond a different Lagrangian Boosts mix soft and collinear modes (same invariant mass)



(+, -; perp)

$$k_n \sim Q(1, \lambda^2, \lambda) \rightarrow y \gg 0$$

$$k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda) \rightarrow y \ll 0$$

$$k_s \sim Q(\lambda, \lambda, \lambda) \rightarrow y \approx 0$$

$$\lambda \sim \frac{q_T}{Q}$$

$$J_n^{(0)}(0^+, y^-, y_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_1(P, \sigma_1) | \bar{\chi}_n(0^+, y^-, y_\perp) \frac{\not{h}}{2} \chi_n(0) | N_1(P, \sigma_1) \rangle$$

$$J_{\bar{n}}^{(0)}(y^+, 0^-, y_\perp) = \frac{1}{2} \sum_{\sigma_2} \langle N_2(\bar{P}, \sigma_2) | \bar{\chi}_{\bar{n}}(0) \frac{\not{h}}{2} \chi_{\bar{n}}(y^+, 0^-, y_\perp) | N_2(\bar{P}, \sigma_2) \rangle$$

$$S(0^+, 0^-, y_\perp) = \langle 0 | \text{Tr} \bar{\mathbf{T}}[S_n^{T\dagger} S_n^T](0^+, 0^-, y_\perp) \mathbf{T}[S_{\bar{n}}^{T\dagger} S_{\bar{n}}^T](0) | 0 \rangle, \quad \chi = W^{T\dagger} \xi$$

multipole expansion fixes arguments

$$n = (1, 0, 0, 1)$$

$$\bar{n} = (1, 0, 0, -1)$$

$$\chi_n = W_n^\dagger \xi_n$$

Definition of TMD's

Positive and negative rapidity quanta can be collected into 2 different TMDs because of the splitting of the soft function

$$\tilde{S}(b_T; \frac{Q^2 \mu^2}{\Delta^+ \Delta^-}, \mu^2) = \tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-) \tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^+),$$

$$\tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-) = \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \alpha \frac{\Delta^-}{\bar{p}^-}\right)},$$

$$\tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^+) = \sqrt{\tilde{S}\left(\frac{1}{\alpha} \frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)}$$

$$\zeta_F = Q^2 / \alpha$$

$$\zeta_D = \alpha Q^2$$

Pure collinear

TMDPDF

$$\ln F_{ij}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2; \Delta^-) = \ln \tilde{\Phi}_{ij}^{(0)}(x, \mathbf{b}, S; \mu^2; \Delta^-) + \ln \tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-)$$

TMDFF

$$\ln D_{ij}(x, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2; \Delta^+) = \ln \tilde{\Delta}_{ij}^{(0)}(x, \mathbf{b}, S_h; \mu^2; \Delta^+) + \ln \tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^+)$$

Soft function: structure and properties

In the high q_T limit: $Q \gg q_T \gg \Lambda_{QCD}$ the hadronic tensor is

$$\tilde{M} = H(Q^2/\mu^2) \tilde{C}(x_n, z_{\bar{n}}, L_\perp, Q^2/\mu^2) f_n(x_n, \Delta^-/\mu^2) d_{\bar{n}}(z_{\bar{n}}, \Delta^+/\mu^2)$$

and PDF, f_n , and FF, d_n , have single log dependence on UV/IR cutoff (Korchemsky, Radyushkin 1987)

$$\ln f_n = \mathcal{R}_{f1}(x_n, \alpha_s) + \mathcal{R}_{f2}(x_n, \alpha_s) \ln \frac{\Delta^-}{\mu^2}$$

$$\ln d_{\bar{n}} = \mathcal{R}_{f1}(z_{\bar{n}}, \alpha_s) + \mathcal{R}_{f2}(z_{\bar{n}}, \alpha_s) \ln \frac{\Delta^+}{\mu^2}$$

Splitting of the soft function

$$\begin{aligned}\ln \tilde{S} &= \mathcal{R}_s(b_T, \alpha_s) + 2D(b_T, \alpha_s) \ln \left(\frac{\Delta^+ \Delta^-}{Q^2 \mu^2} \right) \\ \ln \tilde{S}_- &= \frac{1}{2} \mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln \left(\frac{(\Delta^-)^2}{\zeta_F \mu^2} \right), \\ \ln \tilde{S}_+ &= \frac{1}{2} \mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln \left(\frac{(\Delta^+)^2}{\zeta_D \mu^2} \right)\end{aligned}$$

Using single log dependence

Q-dependence of TMD's

$$\begin{aligned}\zeta_F &= Q^2 / \alpha \\ \zeta_D &= \alpha Q^2\end{aligned}$$

$$\begin{aligned}\frac{d}{d \ln \zeta_F} \ln \tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) &= -D(b_T; \mu^2), \\ \frac{d}{d \ln \zeta_D} \ln \tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) &= -D(b_T; \mu^2).\end{aligned}$$

The Q-dependence of the TMD is dictated by the soft function:
spin independent

Soft function: structure and properties

All this implies that each **pure** collinear and soft sectors are of the form

$$\ln \tilde{J}_n^{(0)} = \mathcal{R}_{n1}(x_n, \alpha_s, L_\perp) + \mathcal{R}_{n2}(x_n, \alpha_s, L_\perp) \ln \frac{\Delta^-}{\mu^2},$$

$$\ln \tilde{J}_{\bar{n}}^{(0)} = \mathcal{R}_{\bar{n}1}(x_{\bar{n}}, \alpha_s, L_\perp) + \mathcal{R}_{\bar{n}2}(x_{\bar{n}}, \alpha_s, L_\perp) \ln \frac{\Delta^+}{\mu^2},$$



$$\ln \tilde{S} = \mathcal{R}_{s1}(\alpha_s, L_\perp) + 2D(\alpha_s, L_\perp) \ln \frac{\Delta^- \Delta^+}{Q^2 \mu^2}.$$

- * Each sector is linear in logs of the rapidity cut
- * Each collinear sector depends just on 1 IR/rapidity regulator
- * Each collinear sector depends solely on the corresponding collinear momentum. It is not possible that a Q dependence arise in a pure collinear sector, Q dependence arises only in the soft sector.
- * The soft function is linear in the logs of rapidity regulator to cancel the corresponding logs in collinear sectors (In QCD there are no rapidity divergences)
- * The soft function is Hermitian, so it is the same for DIS, DY and ee to 2 jets