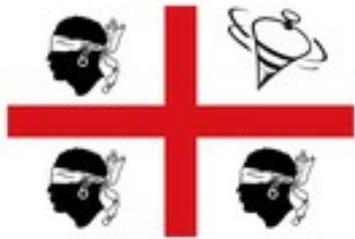
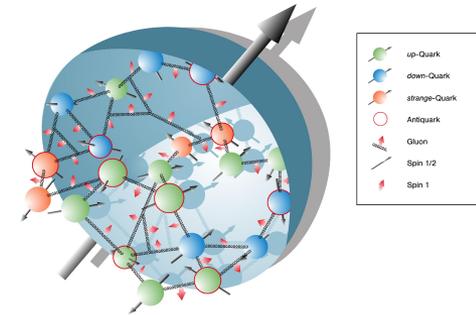


# Multiplicities and phenomenology



*Transversity 2014, Chia (Cagliari)*



**Andrea Signori**

In collaboration with:

*A. Bacchetta  
M. Echevarria  
M. Radici  
G. Schnell*



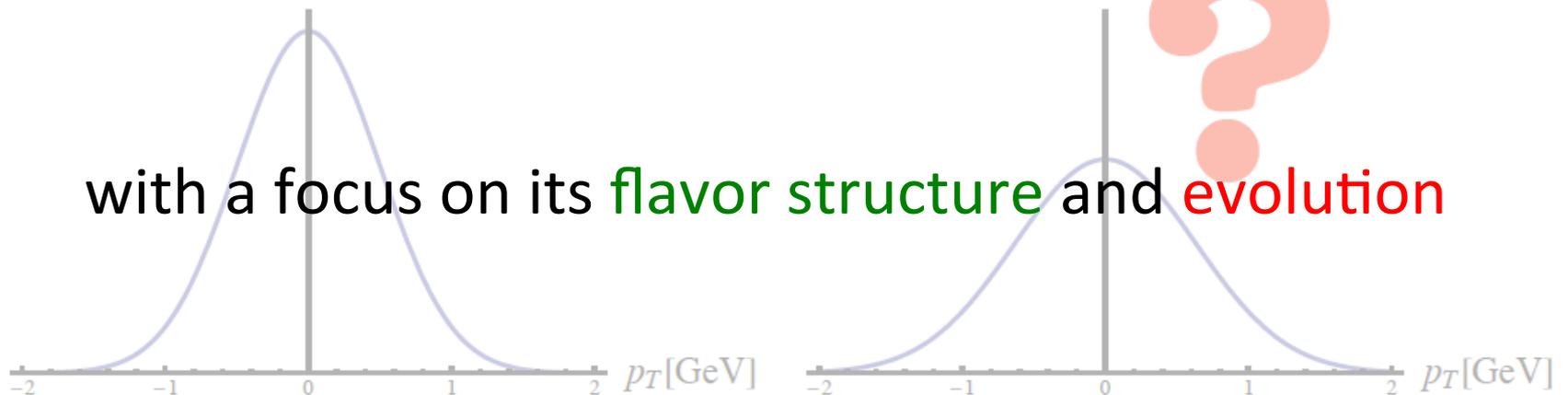
# Goals

Investigate the **3D structure** of hadrons



in momentum space (access **TMD** distributions)

with a focus on its **flavor structure** and **evolution**



# The to-do list

- **SIDIS** fit with no evolution at all (Hermes) [*done*]
- Get hints on the non-perturbative parameters involved in evolution from **e+e- annihilations**, compatible with the previous fit [*in progress*]
- **SIDIS** fit with QCD **evolution** (Hermes + Compass)
- **Global** SIDIS + DY + e+e- fits



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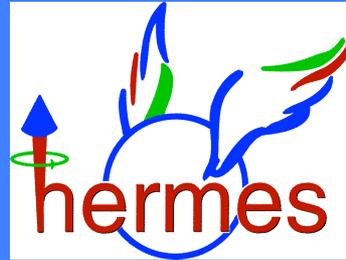
Investigations into the flavor dependence of partonic transverse momentum

---

Andrea Signori,<sup>a,b</sup> Alessandro Bacchetta,<sup>c,d</sup> Marco Radici<sup>c</sup> and Gunar Schnell<sup>e,f</sup>

TODAY:

- 1) SIDIS multiplicities
- 2) e+e- multiplicities



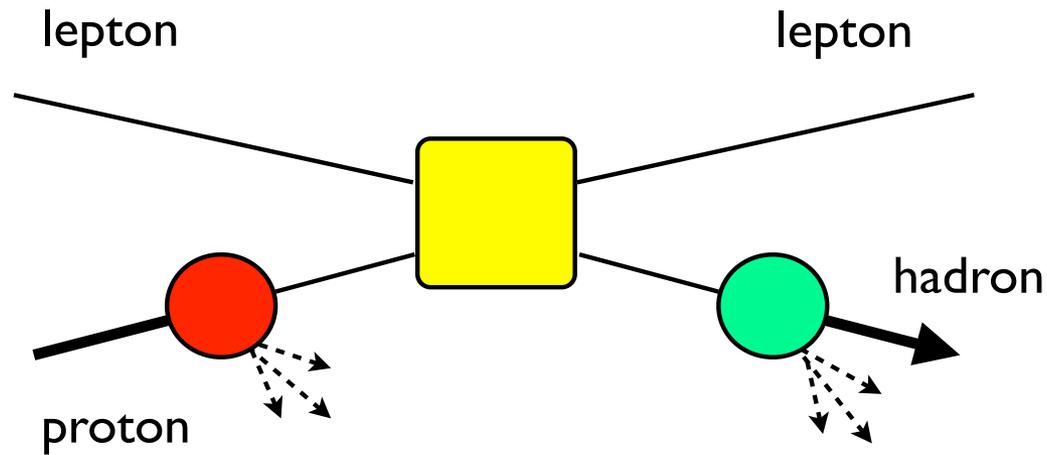
SIDIS

multiplicities

# TMDs in SIDIS

UNPOLARIZED

SIDIS



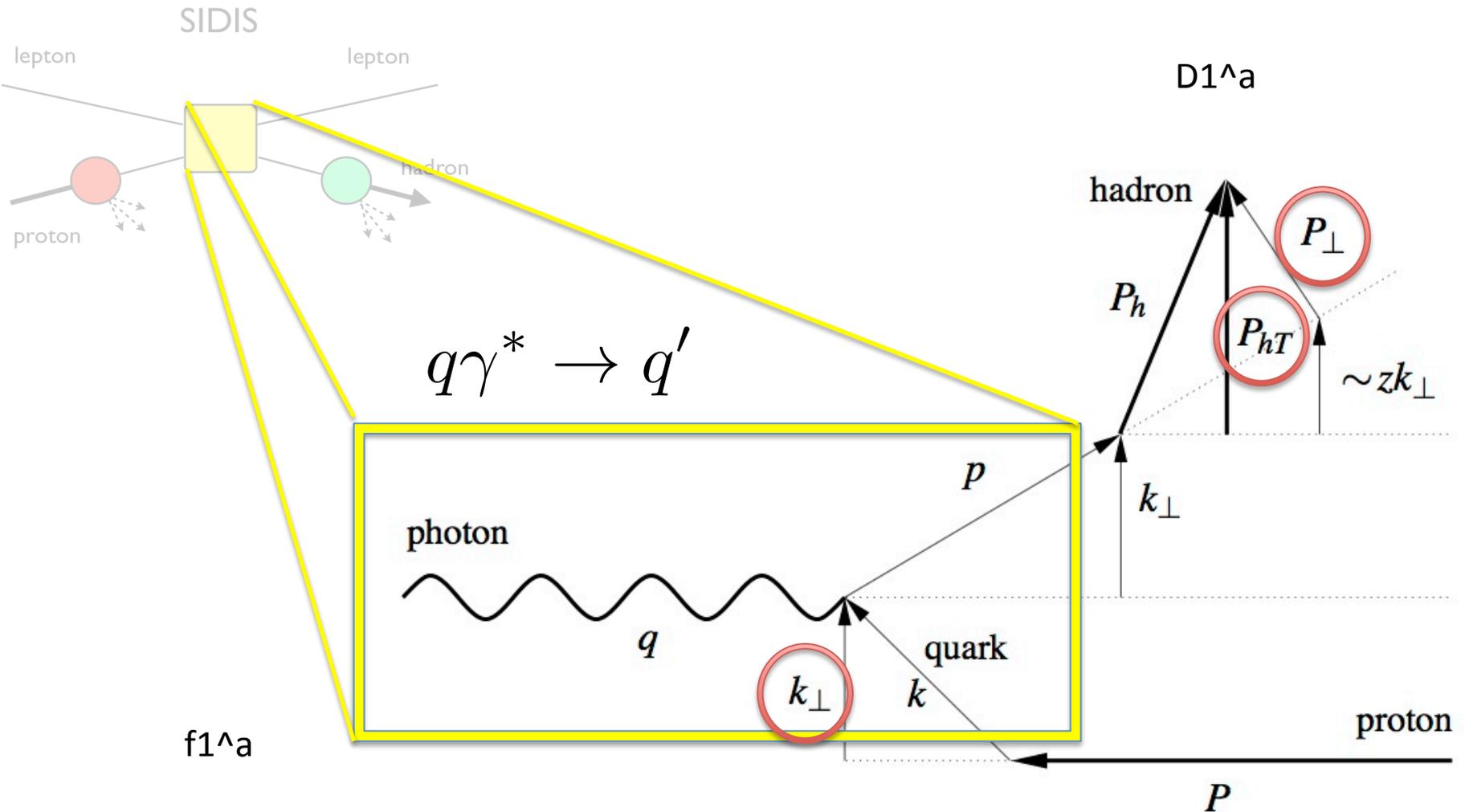
Parton model picture

LO QED

Zero-order QCD  
Leading twist

$$\sigma(P_{hT}^2) \sim \sum_a f_1^a(x, k_{\perp}^2) \otimes D_1^{a \rightarrow h}(z, P_{\perp}^2)$$

# Which transverse momenta ?

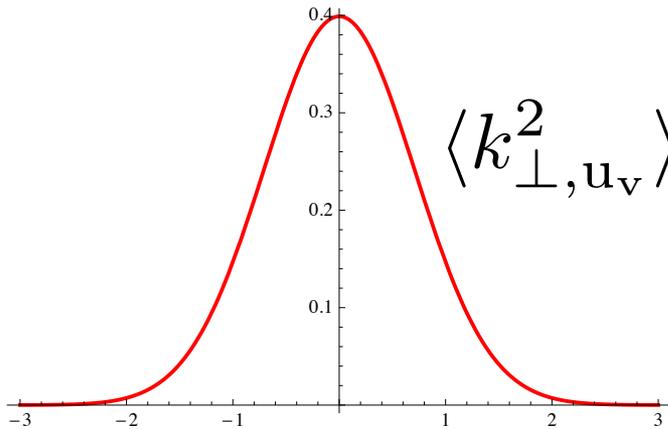


# Flavor dependent Gaussians

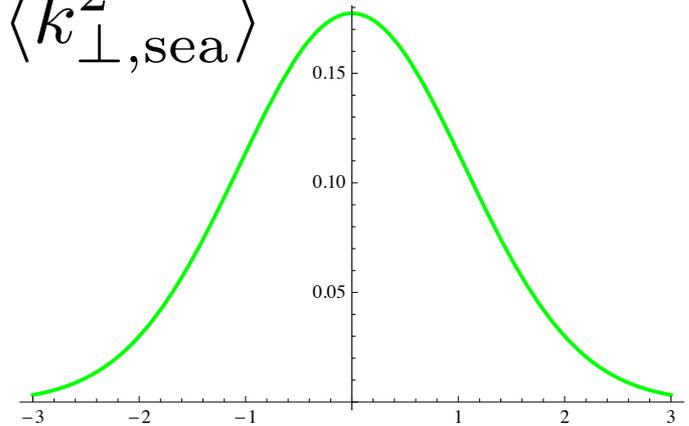
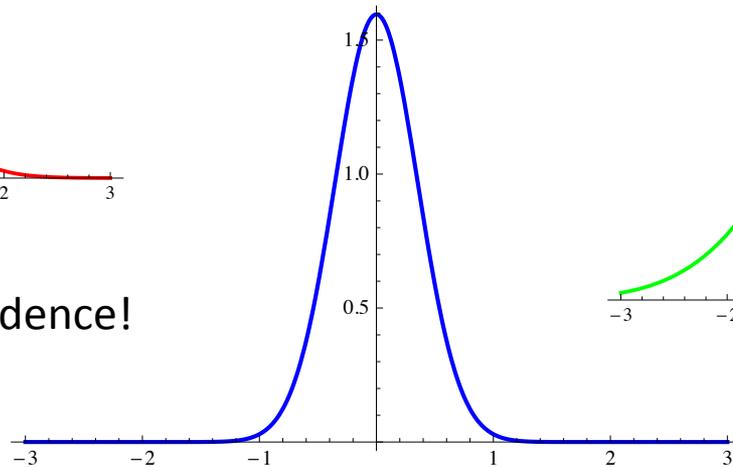
## Different Gaussian parametrizations of TMD parts

$$f_1^a(x, k_{\perp}^2) = f_1^a(x) \frac{1}{\pi \langle k_{\perp, a}^2 \rangle} \exp \left\{ -\frac{k_{\perp}^2}{\langle k_{\perp, a}^2 \rangle} \right\}$$

$$\langle k_{\perp, u_v}^2 \rangle \neq \langle k_{\perp, d_v}^2 \rangle \neq \langle k_{\perp, sea}^2 \rangle$$



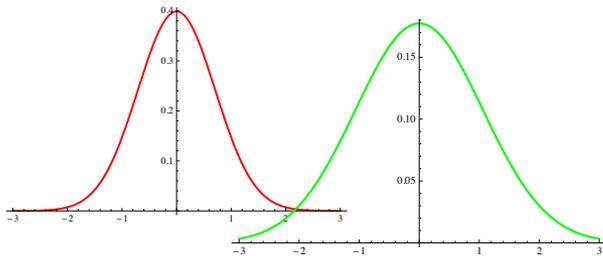
Simplified flavor dependence!



# Flavor dependent Gaussians

## Different Gaussian parametrization of TMD parts

$$D_1^{a \rightarrow h}(z, P_\perp) = D_1^{a \rightarrow h}(z) \frac{1}{\pi \langle P_{\perp, a \rightarrow h}^2 \rangle} \exp \left\{ - \frac{P_\perp^2}{\langle P_{\perp, a \rightarrow h}^2 \rangle} \right\}$$



4 different combinations out of

$$u, d, s \longleftrightarrow \pi^\pm, K^\pm$$

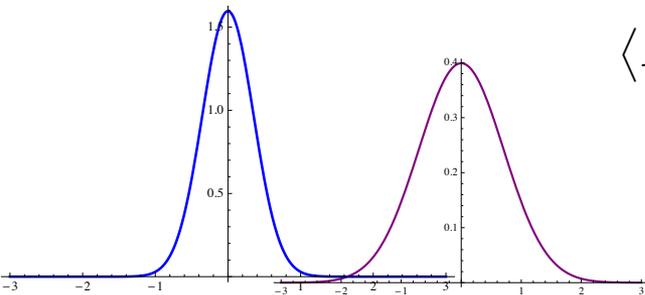
$$\langle P_{\perp, u \rightarrow \pi^+}^2 \rangle = \langle P_{\perp, \bar{d} \rightarrow \pi^+}^2 \rangle = \langle P_{\perp, \bar{u} \rightarrow \pi^-}^2 \rangle = \langle P_{\perp, d \rightarrow \pi^-}^2 \rangle \equiv \langle P_{\perp, \text{fav}}^2 \rangle,$$

$$\langle P_{\perp, u \rightarrow K^+}^2 \rangle = \langle P_{\perp, \bar{u} \rightarrow K^-}^2 \rangle \equiv \langle P_{\perp, uK}^2 \rangle,$$

$$\langle P_{\perp, \bar{s} \rightarrow K^+}^2 \rangle = \langle P_{\perp, s \rightarrow K^-}^2 \rangle \equiv \langle P_{\perp, sK}^2 \rangle,$$

---


$$\langle P_{\perp, \text{all others}}^2 \rangle \equiv \langle P_{\perp, \text{unf}}^2 \rangle.$$



# Kinematic dependence

$$\langle \mathbf{k}_{\perp, q}^2 \rangle(x) = \langle \widehat{\mathbf{k}_{\perp, q}^2} \rangle \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma} \quad \longrightarrow \quad \text{Flavor independent Kinematic dependence}$$

$$\langle \widehat{\mathbf{k}_{\perp, q}^2} \rangle = \langle \mathbf{k}_{\perp, q}^2 \rangle(\hat{x} = 0.1)$$

$$\langle \mathbf{P}_{\perp, q \rightarrow h}^2 \rangle(z) = \langle \widehat{\mathbf{P}_{\perp, q \rightarrow h}^2} \rangle \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

$$\langle \widehat{\mathbf{P}_{\perp, q \rightarrow h}^2} \rangle = \langle \mathbf{P}_{\perp, q \rightarrow h}^2 \rangle(\hat{z} = 0.5)$$

Flavor independent  
Kinematic dependence

# Flavor analysis

Parameters for TMD PDFs					
	$\langle \hat{k}_{\perp, d_v}^2 \rangle$ [GeV <sup>2</sup> ]	$\langle \hat{k}_{\perp, u_v}^2 \rangle$ [GeV <sup>2</sup> ]	$\langle \hat{k}_{\perp, sea}^2 \rangle$ [GeV <sup>2</sup> ]	$\alpha$ (random)	$\sigma$ (random)

5 parameters

interval [0,2]

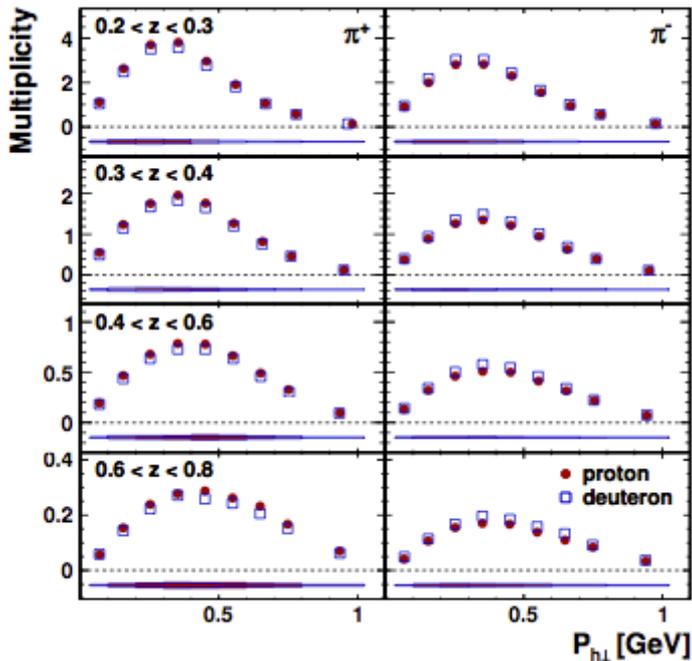
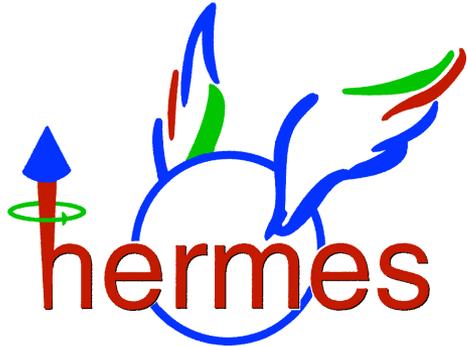
interval [-0.3,0.1]

Parameters for TMD FFs							
	$\langle \hat{P}_{\perp, fav}^2 \rangle$ [GeV <sup>2</sup> ]	$\langle \hat{P}_{\perp, unf}^2 \rangle$ [GeV <sup>2</sup> ]	$\langle \hat{P}_{\perp, sK}^2 \rangle$ [GeV <sup>2</sup> ]	(random)	$\beta$	$\delta$	$\gamma$

7 parameters

interval [0.125,0.250]

# Fitting procedure



× 200

Monte Carlo fit

***Distributions of best-values***

# Fitting procedure

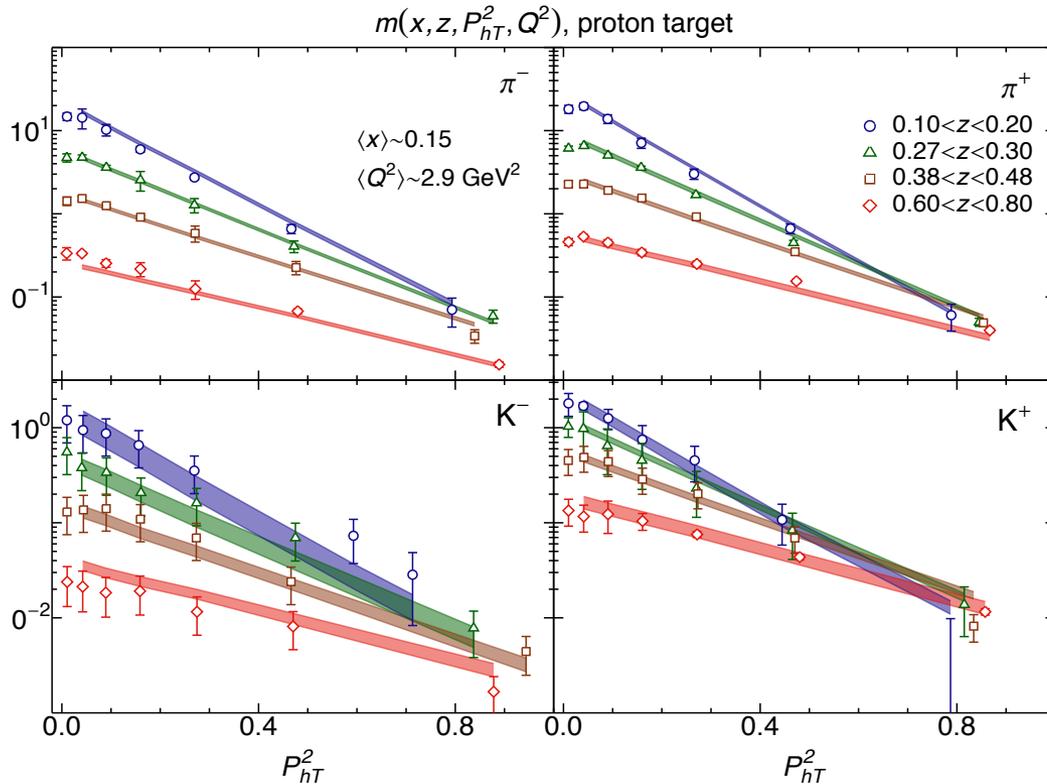
proton target    global  $\chi^2 / \text{d.o.f.} = 1.63 \pm 0.12$   
                   no flavor dep.                     $1.72 \pm 0.11$

$\pi^-$   
 $1.80 \pm 0.27$   
 $1.83 \pm 0.25$

$K^-$   
 $0.78 \pm 0.15$   
 $0.87 \pm 0.16$

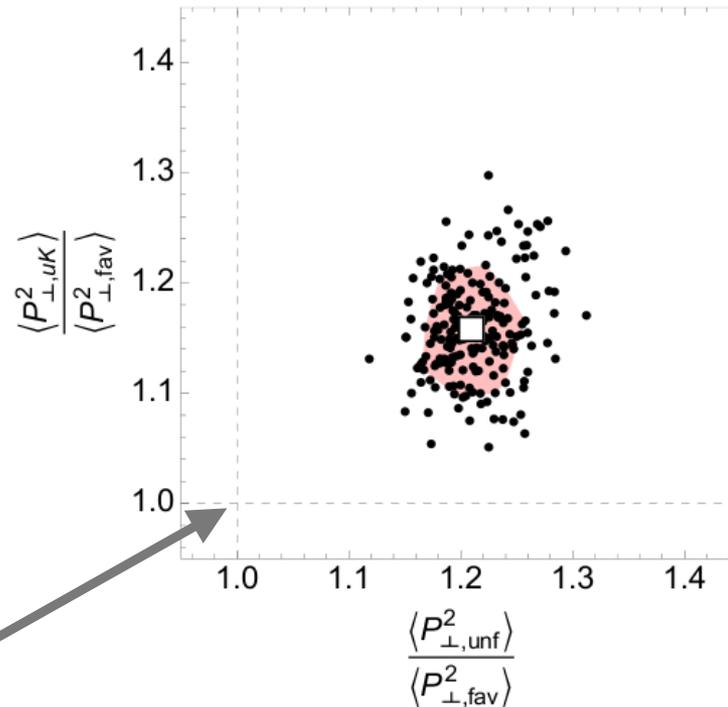
$\pi^+$   
 $2.64 \pm 0.21$   
 $2.89 \pm 0.23$

$K^+$   
 $0.46 \pm 0.07$   
 $0.43 \pm 0.07$



# TMD FFs – full analysis

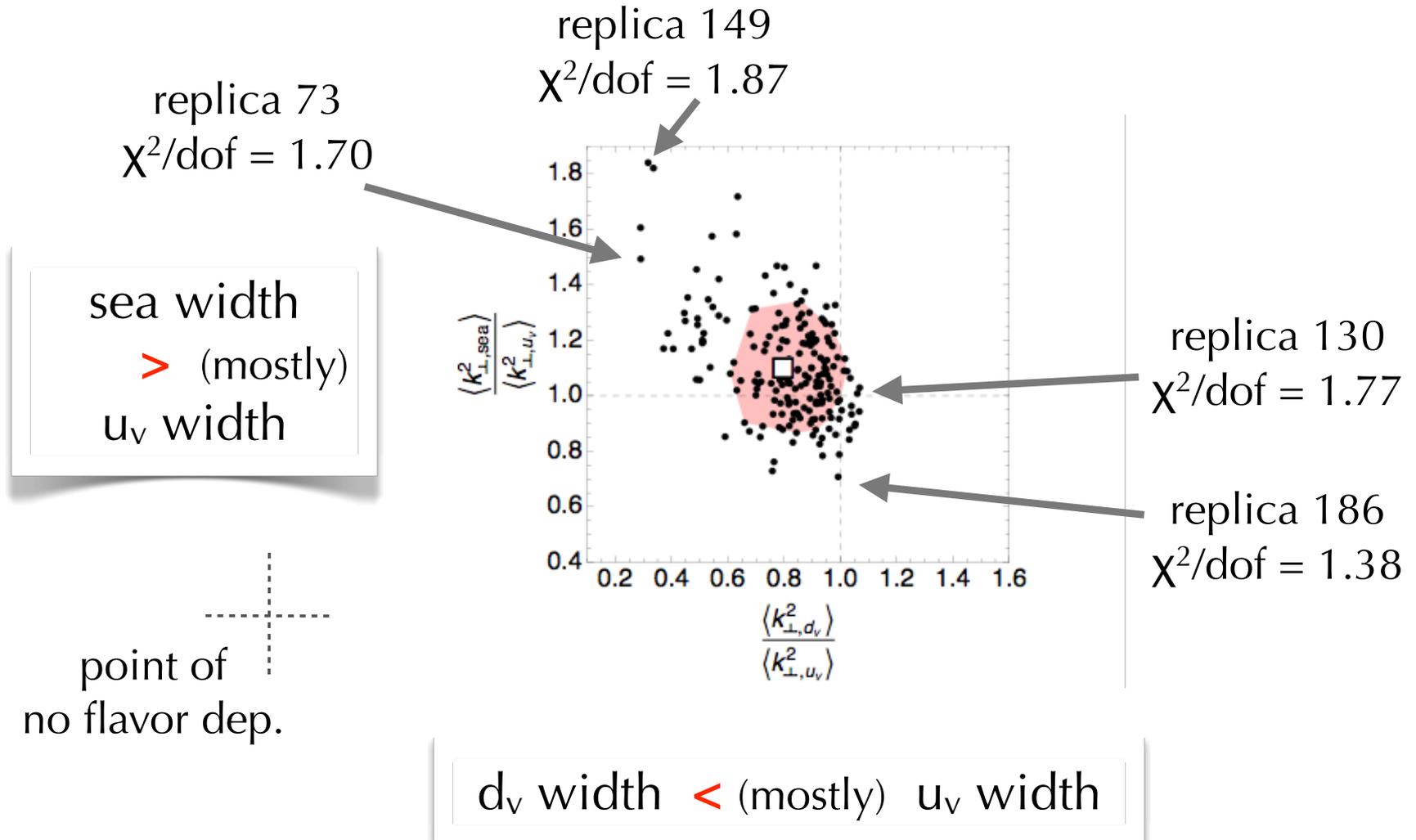
$q \rightarrow \pi$  favored width  
<  
 $q \rightarrow K$  favored width



point of  
no flavor dep.

$q \rightarrow \pi$  favored width < unfavored

# TMD PDFs – full analysis



# QCD evolution

Looking towards **SIDIS + DY +  $l^+l^-$  global fits** ...

Collins (2011)  
EIS (2012-2013)

$$\sigma \sim \int d^2\mathbf{b} e^{-i\mathbf{b}\cdot\mathbf{q}_T} \mathcal{H}\left[\frac{Q_f}{\mu}\right] \times$$

$$[C_{f/j} \otimes f_{j/N}](x, b_*, Q_0) \underline{\mathcal{F}_{PDF}^{NP}} \times$$

Intrinsic (large  $b$ )  
transverse momenta

Collinear OPE  
for small  $b_T$



$$[C_{D/i} \otimes D_{i/h}](z, b_*, Q_0) \underline{\mathcal{F}_{FF}^{NP}} \times (Q_0 \sim 1/b^*)$$

Perturbative  
transverse momenta  
and evolution  
(resummed logs)

$$\exp\left\{\int_{Q_0}^{Q_f} \frac{d\mu}{\mu} \gamma_{PDF}\left[\ln\frac{Q_f^2}{\mu^2}, \alpha_S(\mu)\right]\right\} \left[\frac{Q_f}{Q_0}\right]^{-D(b^*, Q_0) + \underline{NP}} \times$$

Soft evolution

$$\exp\left\{\int_{Q_0}^{Q_f} \frac{d\mu}{\mu} \gamma_{FF}\left[\ln\frac{Q_f^2}{\mu^2}, \alpha_S(\mu)\right]\right\} \left[\frac{Q_f}{Q_0}\right]^{-D(b^*, Q_0) + \underline{NP}}$$

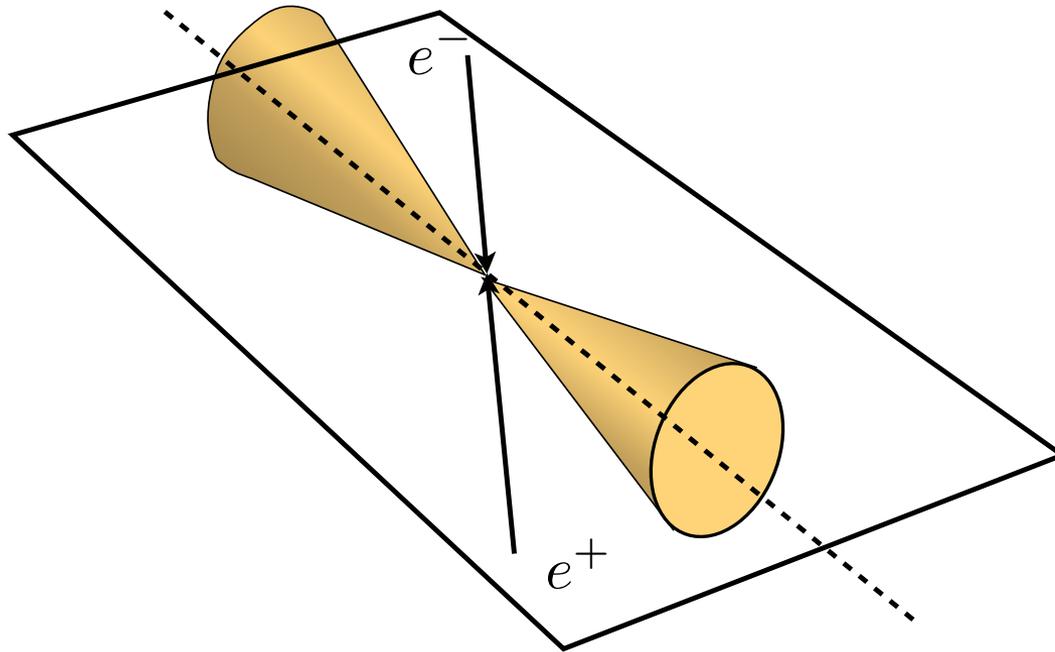


# $e^+e^-$ multiplicities

double goal:

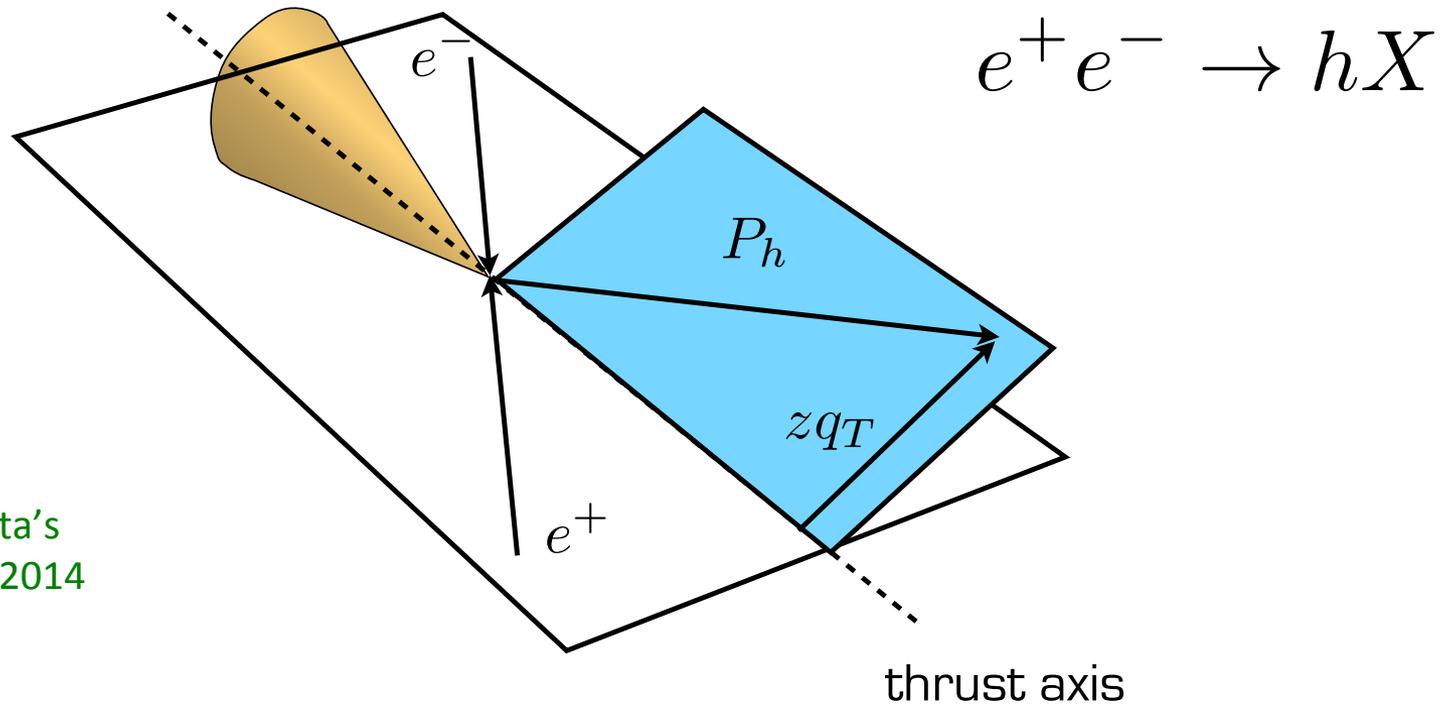
- pin down flavor dependence in TMD FFs
- get info on the non-perturbative evolution

# Hadron production



$$e^+e^- \rightarrow \text{jets}$$

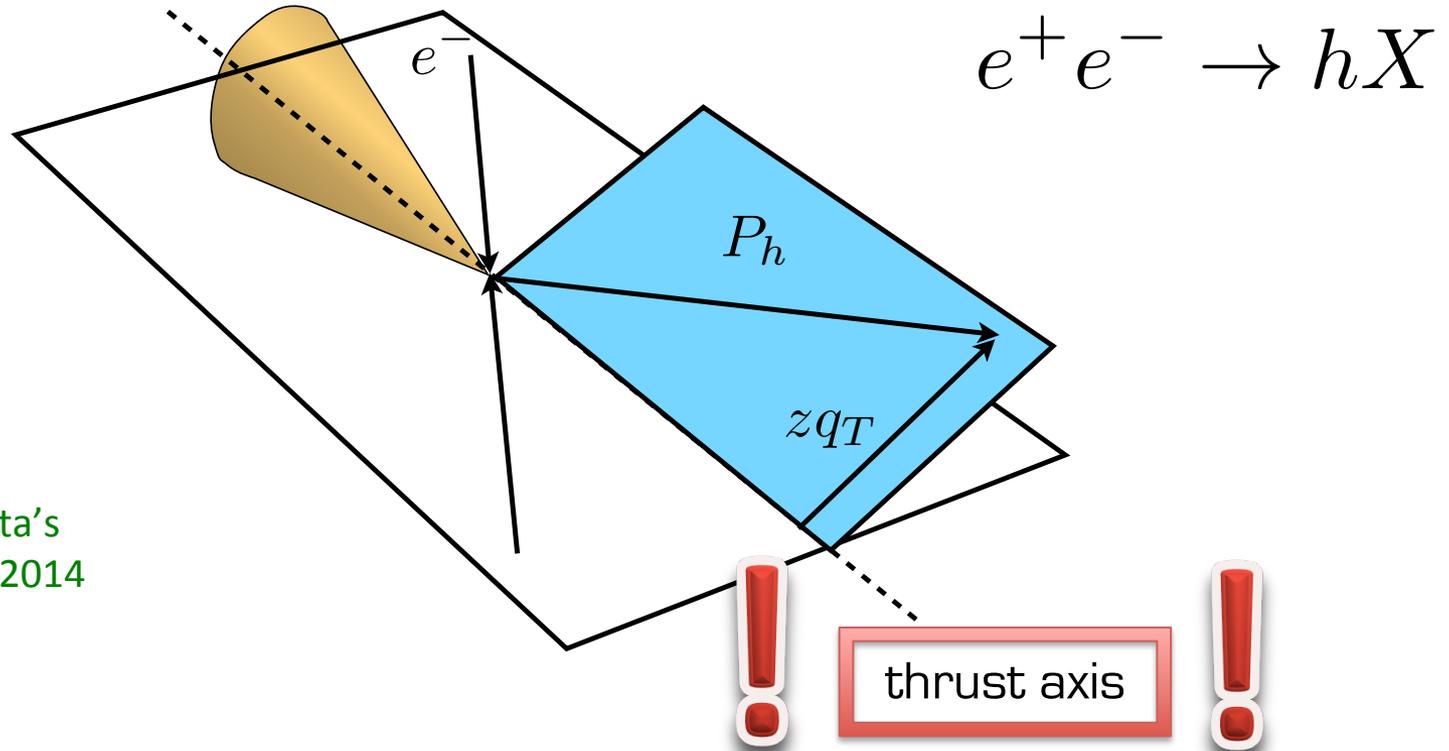
# One hadron



See A. Bacchetta's  
talk in QCD Evo 2014

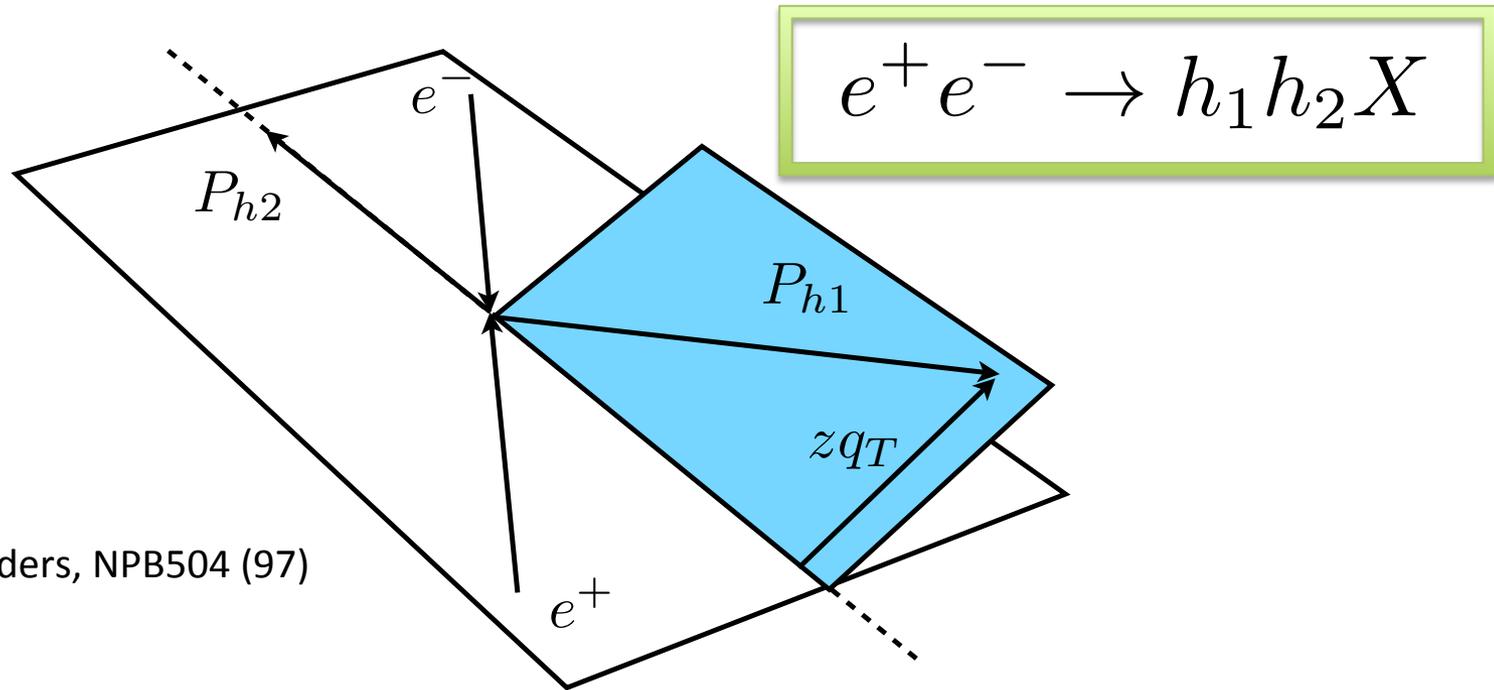
$$\frac{d\sigma}{dzdq_T^2 dy} = \frac{6\pi\alpha^2}{Q^2} A(y) z^2 \mathcal{H}(Q/\mu) \sum_q e_q^2 \int_0^\infty db_T b_T J_0(q_T b_T) D_1^{q \rightarrow h}(z, b_T, \mu) + Y(q_T^2, Q^2)$$

# One hadron



$$\frac{d\sigma}{dz dq_T^2 dy} = \frac{6\pi\alpha^2}{Q^2} A(y) z^2 \mathcal{H}(Q/\mu) \sum_q e_q^2 \int_0^\infty db_T b_T J_0(q_T b_T) D_1^{q \rightarrow h}(z, b_T, \mu) + Y(q_T^2, Q^2)$$

# Two hadrons



Boer, Jakob, Mulders, NPB504 (97)

$$\frac{d\sigma}{dz_1 dz_2 dq_T^2 dy} = \frac{6\pi\alpha^2}{Q^2} A(y) z_1^2 z_2^2 \mathcal{H}(Q/\mu) \sum_q e_q^2 \int_0^\infty db_T b_T J_0(q_T b_T) \times$$

$$\left[ \underline{D_1^{q \rightarrow h_1}(z_1, b_T, \mu) D_1^{\bar{q} \rightarrow h_2}(z_2, b_T, \mu)} + (q \rightarrow \bar{q}) \right] + Y(q_T^2, Q^2)$$

# $e^+e^-$ multiplicities

$$\{e^+e^- \rightarrow h_1 h_2 X\} / \{e^+e^- \rightarrow h_1 X\}$$

**TMD**

collinear

$$M(h_1, h_2) = \frac{\frac{d\sigma^{2h}}{dz_1 dz_2 dy dq_T^2}}{\frac{d\sigma^{1h}}{dz_1 dy}}$$

The same structure  
as in the SIDIS case

NB The experimental transverse momentum is

$$P_{1\perp} = -z_1 q_T$$

# Evolved TMD FFs

OPE of the perturbative part on the collinear distribution

Intrinsic  $b_T$  part

Gaussians from Hermes!

$$D_1^{a \rightarrow h}(z, b_T, Q^2, \mu^2) = \sum_i [C_{a/i} \otimes D_{i/h}](z, b^*, \mu_b^2, \mu_b) \mathcal{F}_{FF}^{NP}(z, b_T) \times$$

$$\exp \left[ \int_{\mu_b}^{\mu} \frac{ds}{s} \gamma_{FF} \left( \ln \frac{Q^2}{s^2}, \alpha_S(s) \right) \right] \times$$

$$\exp \left[ - \ln \frac{Q^2}{\mu_b^2} \left( D(b^*, \mu_b) + \frac{g_2}{4} b_T^2 \right) \right]$$

Perturbative evolution

Soft evolution

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

b-star prescription

Collins (2011)

EIS (2012-2013)

# Input from the Hermes fit

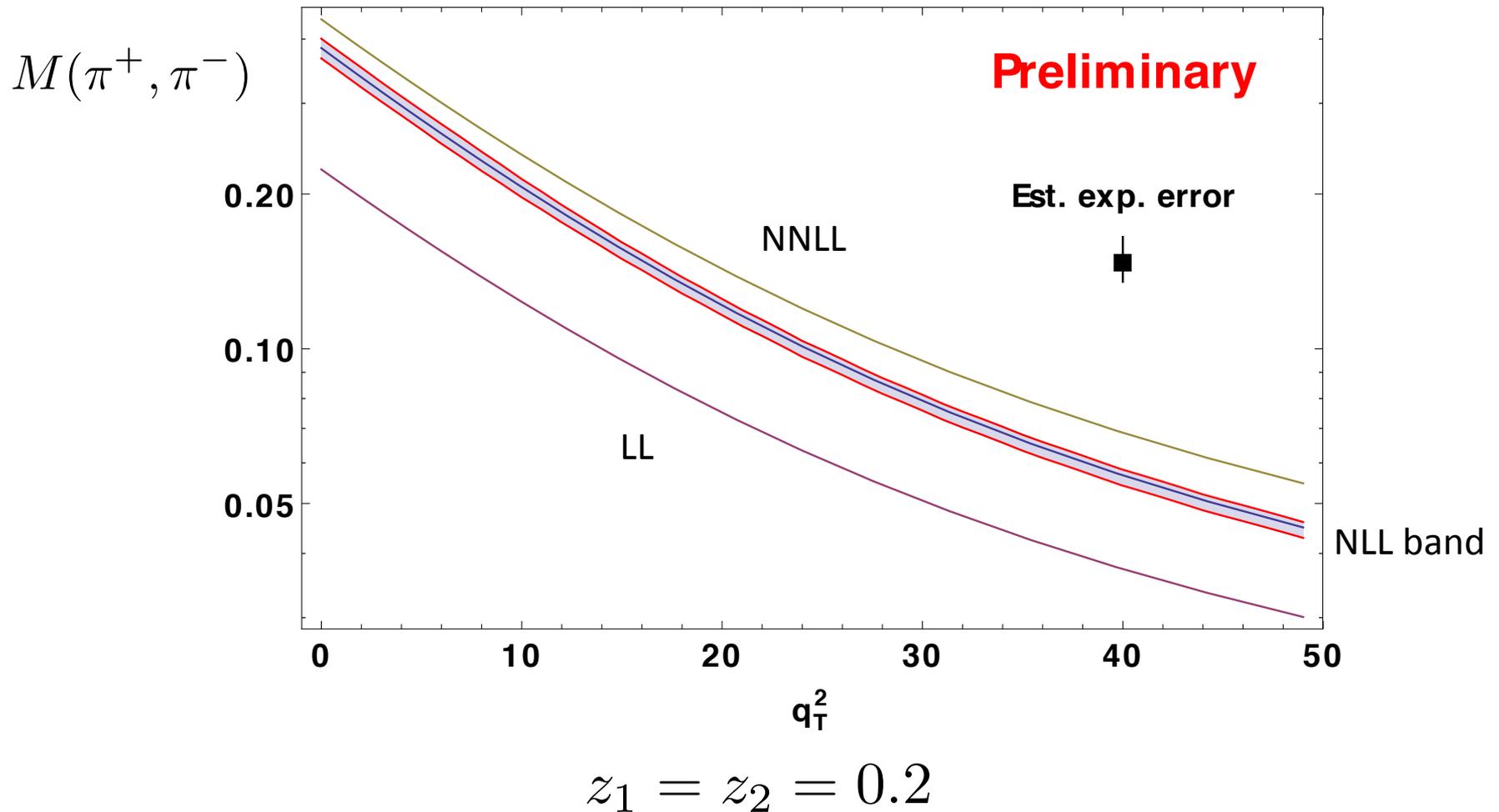
$$\begin{aligned}\langle P_{\perp, u \rightarrow \pi^+}^2 \rangle &= \langle P_{\perp, \bar{d} \rightarrow \pi^+}^2 \rangle = \langle P_{\perp, \bar{u} \rightarrow \pi^-}^2 \rangle = \langle P_{\perp, d \rightarrow \pi^-}^2 \rangle \equiv \langle P_{\perp, \text{fav}}^2 \rangle, \\ \langle P_{\perp, u \rightarrow K^+}^2 \rangle &= \langle P_{\perp, \bar{u} \rightarrow K^-}^2 \rangle \equiv \langle P_{\perp, uK}^2 \rangle, \\ \langle P_{\perp, \bar{s} \rightarrow K^+}^2 \rangle &= \langle P_{\perp, s \rightarrow K^-}^2 \rangle \equiv \langle P_{\perp, sK}^2 \rangle, \\ \langle P_{\perp, \text{all others}}^2 \rangle &\equiv \langle P_{\perp, \text{unf}}^2 \rangle.\end{aligned}$$

## 200 VALUES

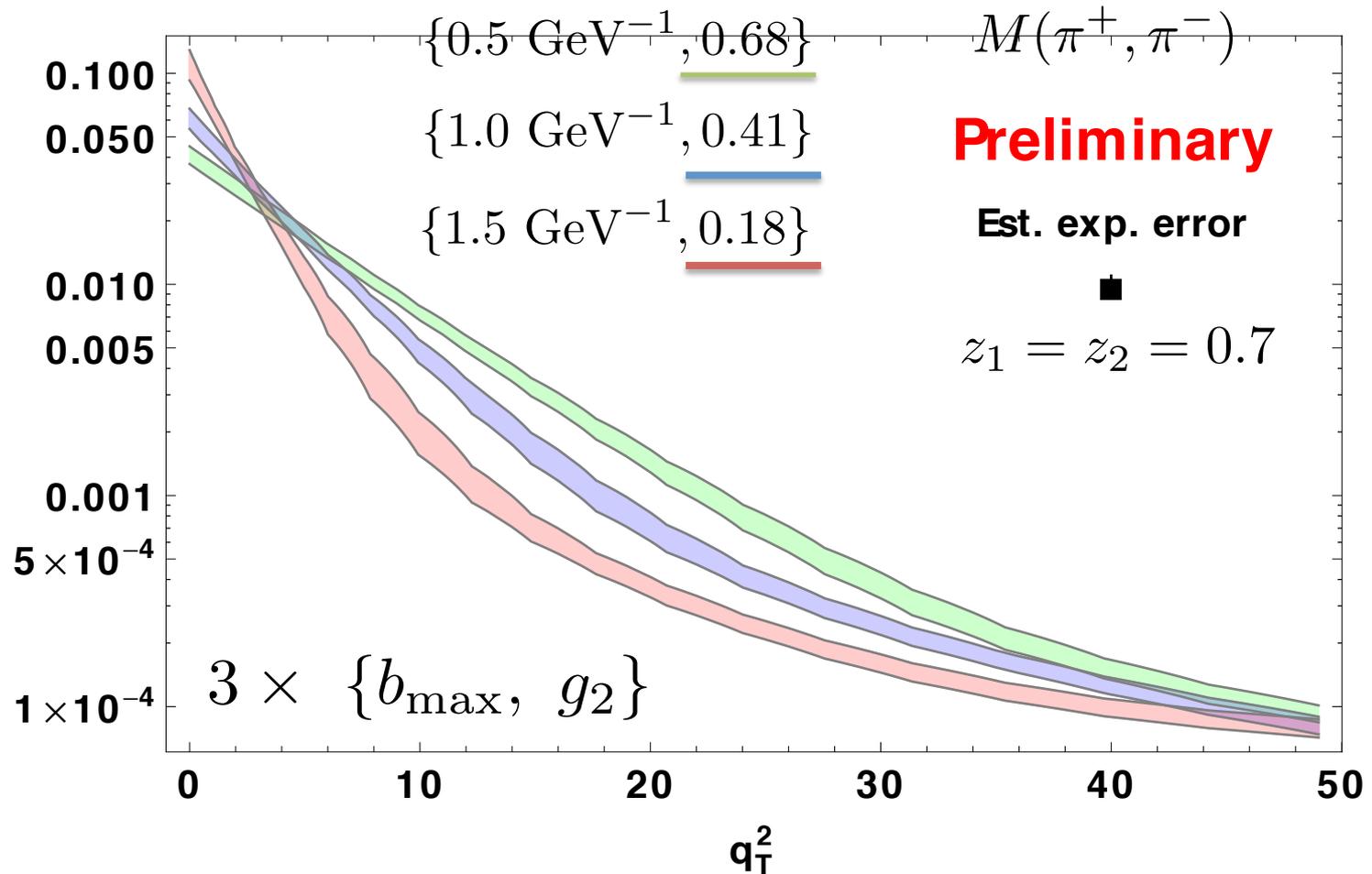
Kinematic dependence

$$\langle P_{\perp, q \rightarrow h}^2 \rangle(z) = \widehat{\langle P_{\perp, q \rightarrow h}^2 \rangle} \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

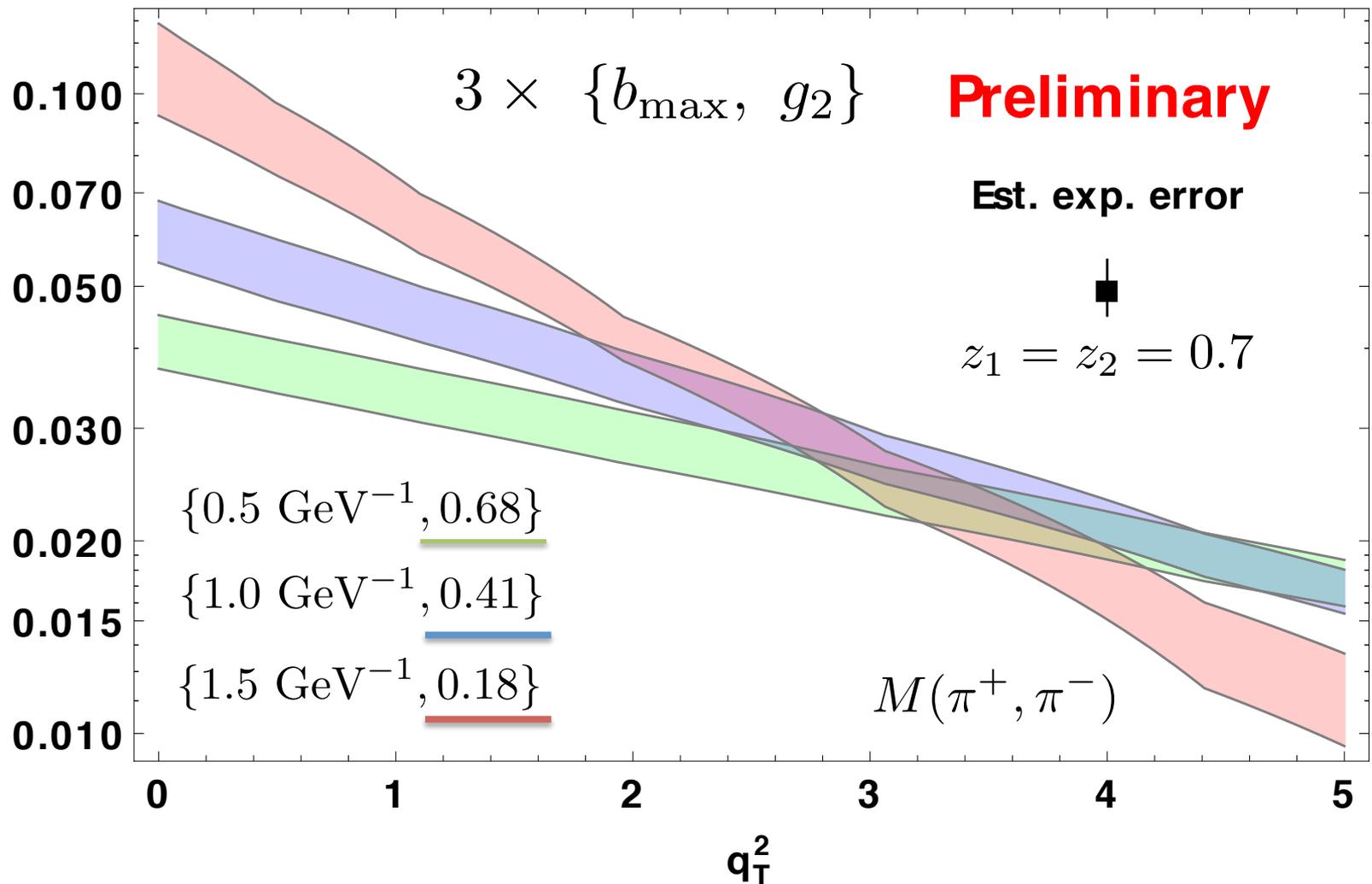
# Effects of theoretical accuracy



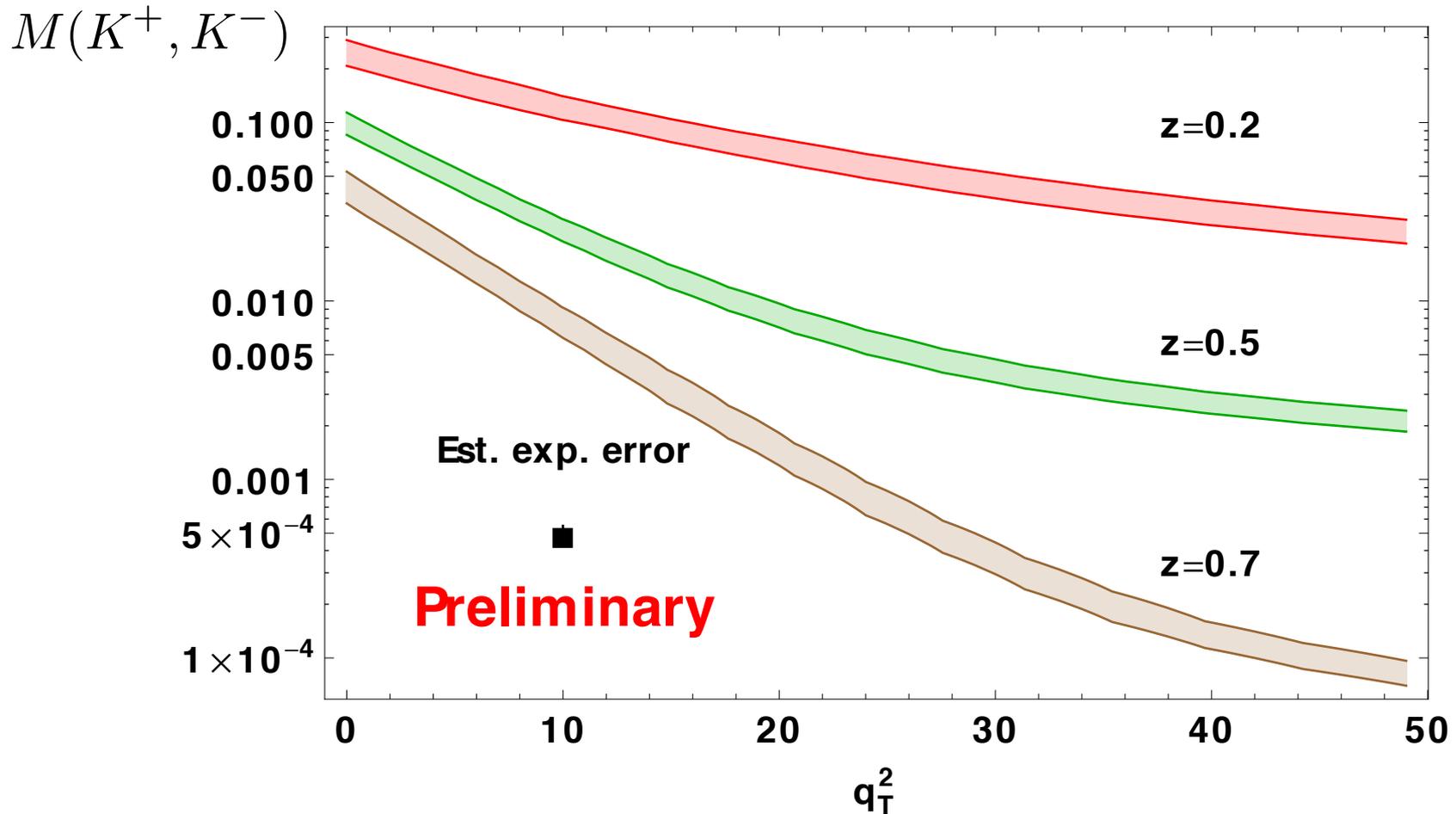
# Sensitivity to evolution parameters



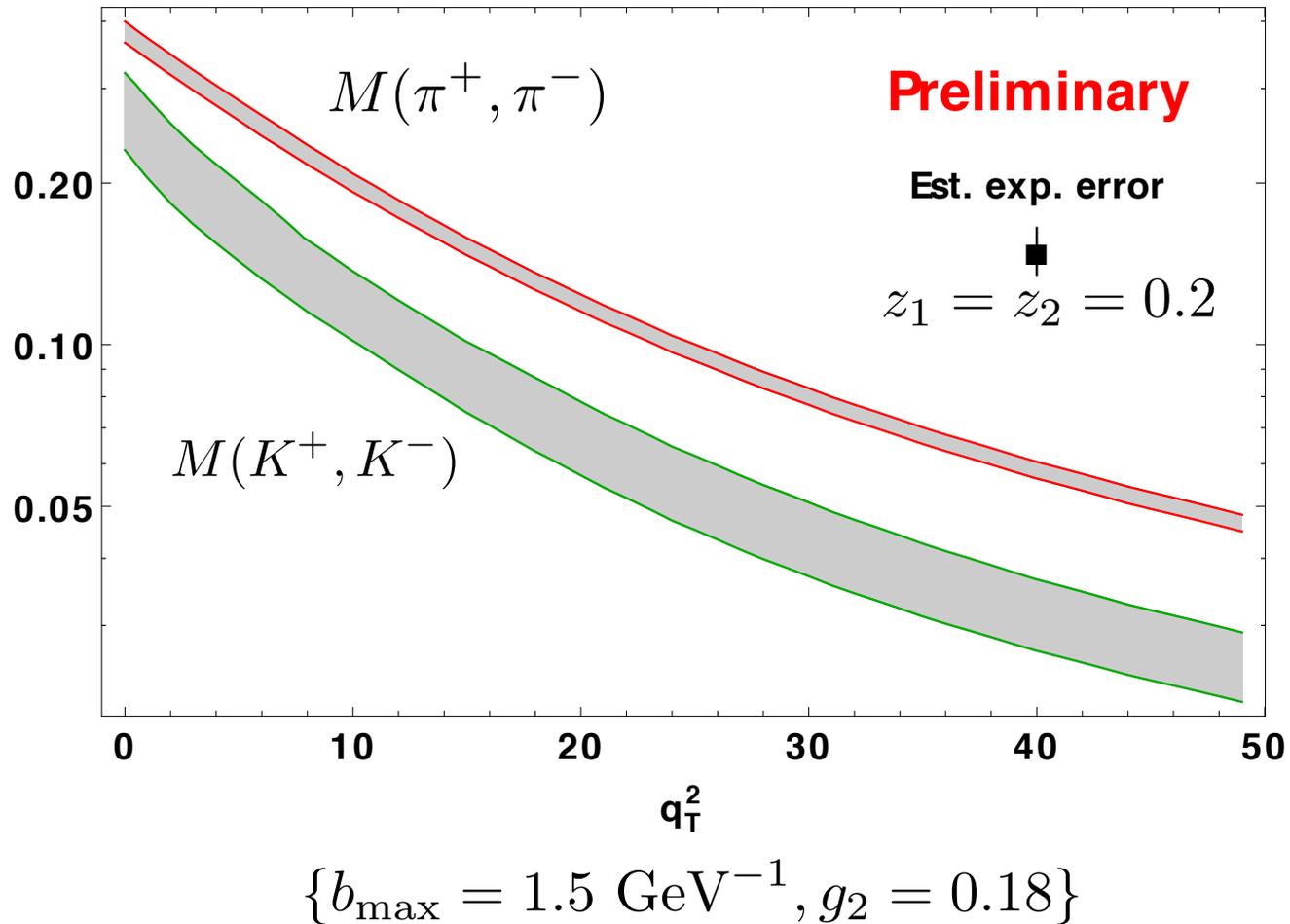
# Sensitivity to evolution parameters



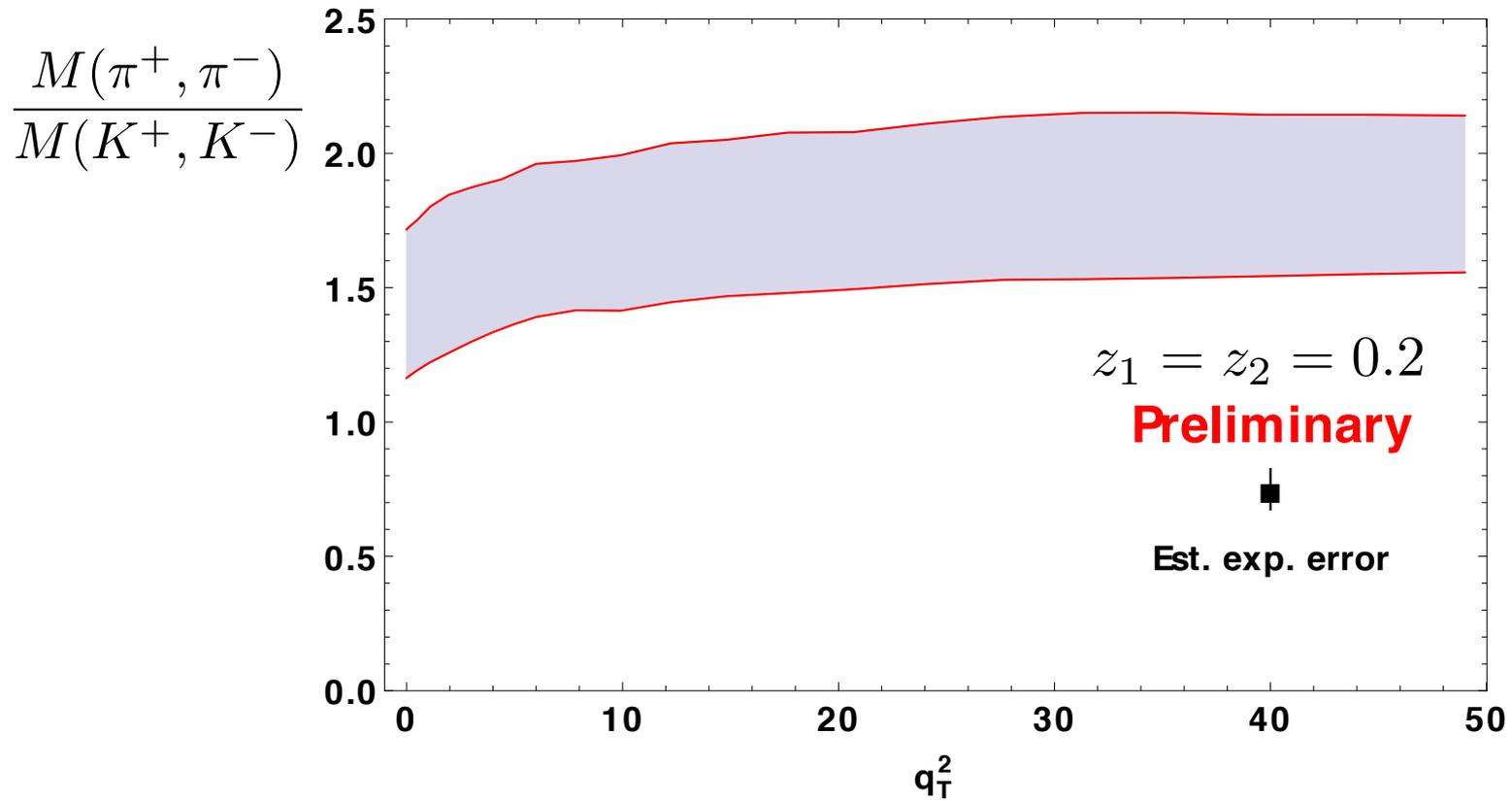
# Intrinsic parameters: z-dependence



# Intrinsic parameters: flavor

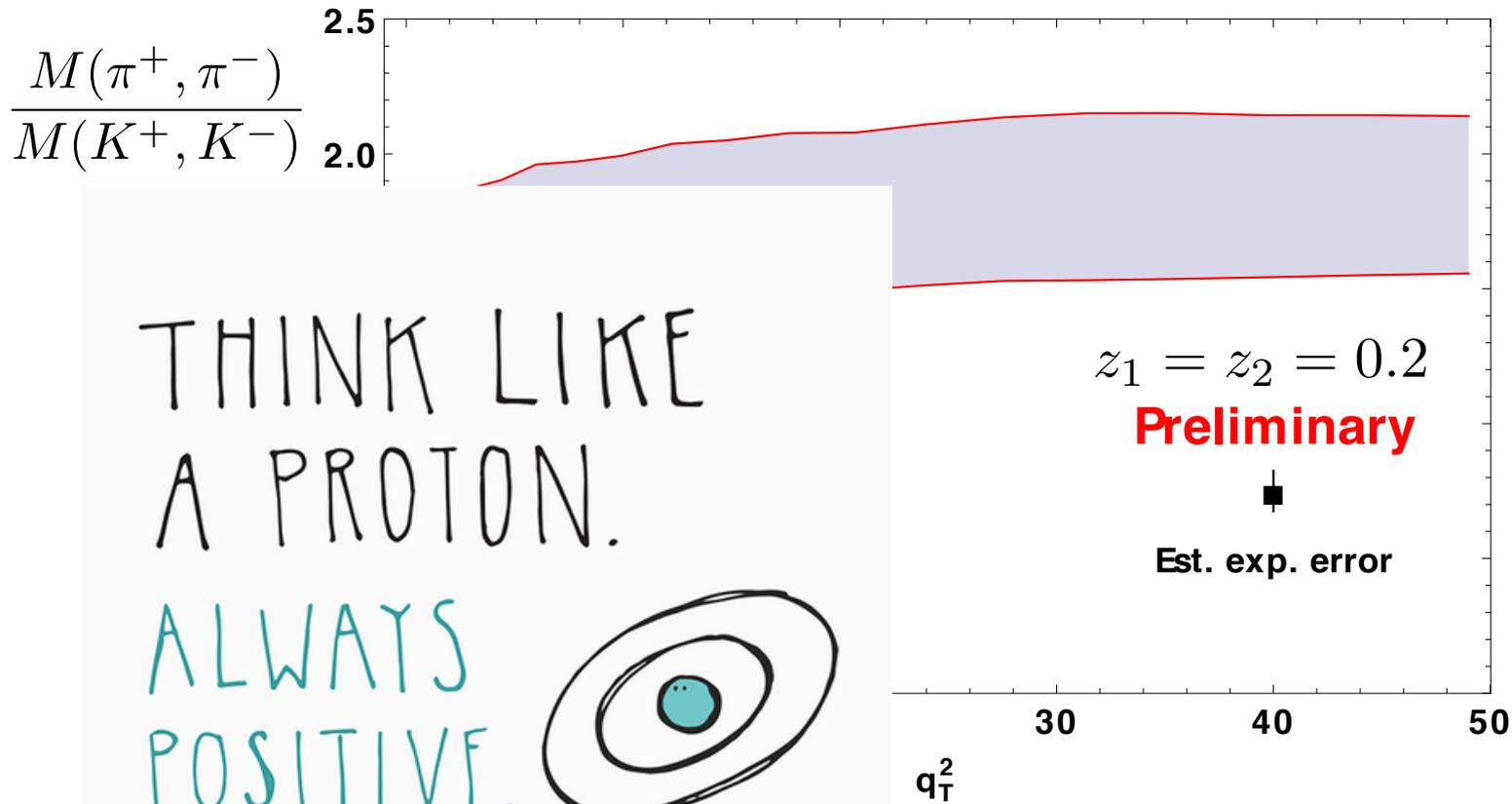


# Intrinsic parameters: flavor



$$\{b_{\max} = 1.5 \text{ GeV}^{-1}, g_2 = 0.18\}$$

# Intrinsic parameters: flavor



THINK LIKE  
A PROTON.

ALWAYS  
POSITIVE.

$\{g_1 = 0.18, g_2 = 0.18\}$

# Conclusions

## SIDIS multiplicities

*There is a lot of room for flavor dependence :*

- **clear** indication in TMD FFs that “ $q \rightarrow \pi$  favored” width < “unfavored” & “ $q \rightarrow K$  favored”
- TMD PDFs: hints that  $d_v$  width <  $u_v$  width < sea width
- flavor-independent fit is not ruled out
- **anticorrelation**: many intrinsic  $\{\mathbf{k}_\perp, \mathbf{P}_\perp\}$  give same  $\mathbf{P}_{hT}$

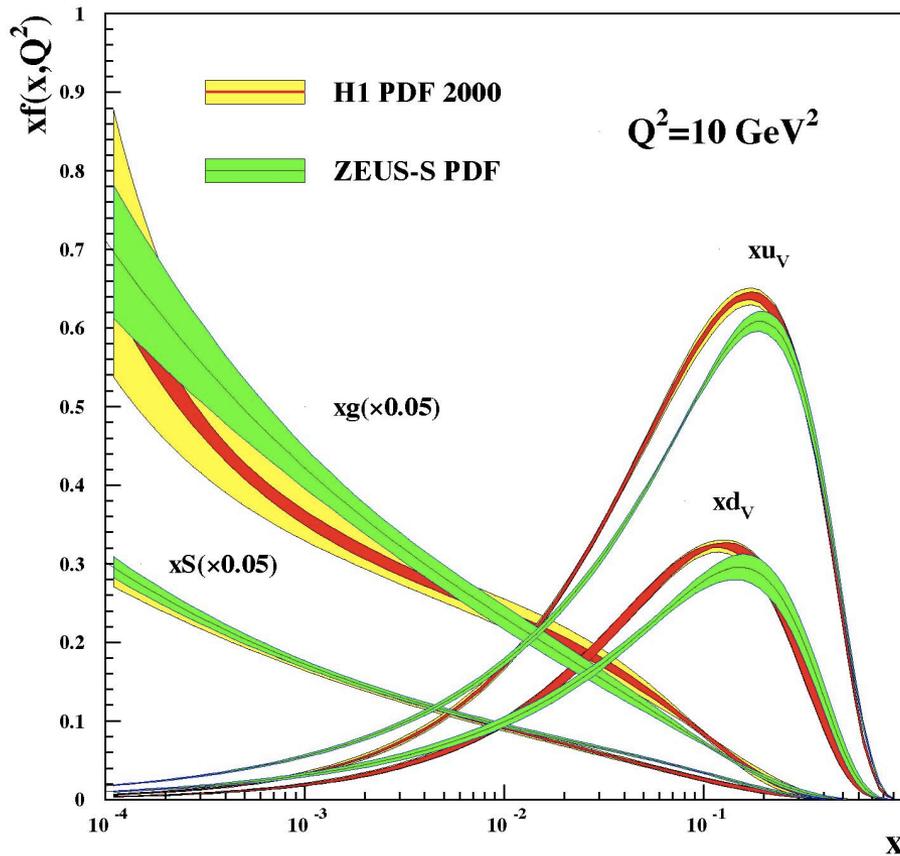
# Conclusions

## *e+e- multiplicities*

- Electron-positron data at  $100 \text{ GeV}^2$  can be extremely valuable to **pin down evolution parameters**
- They are useful to **constrain flavor dependence** of the TMD fragmentation functions
- They are needed to **determine the nonperturbative parameters** of TMD fragmentation functions
- Indirectly, the knowledge of TMD fragmentation functions will help constraining TMD **distribution** functions

# Backup slides

# Flavor in transverse space



Since the **flavor** dependence  
In the **collinear** case is **strong** ...

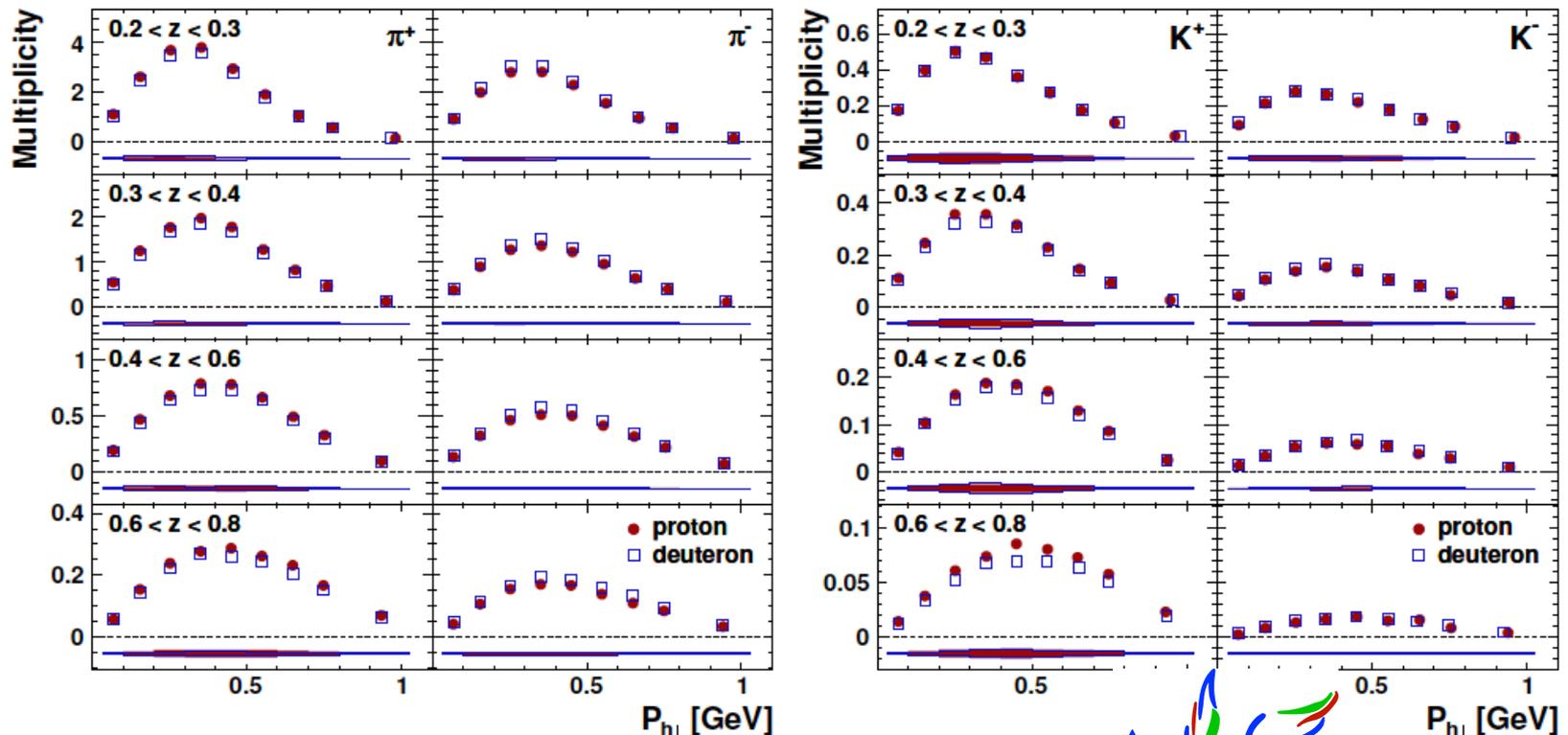
... WHY NOT  
LOOKING FOR IT IN  
 $K_{\perp}$  DEPENDENCE  
OF TMDs ?

# Flavor in transverse space

- ✓ **Lattice** QCD calculations  
Musch *et al.*, PRD **83** (11) 094507  
[ ... ]
- ✓ **Model** calculations  
Chiral quark soliton model [ Schweitzer *et al.*, JHEP 1301 (913) 163 ]  
Diquark spectator model [ Bacchetta *et al.*, PRD **78** (08) 074010 ]  
Statistical approach [ Bourrely *et al.*, PRD **83** (11) 074008 ]  
NJL-jet model [ Matevosyan *et al.*, PRD **85** (12) 014021 ]  
[ ... ]
- ✓ **Previous** fits  
JLab Hall C [ Asaturyan *et al.*, (E00-108), PRC **85** (12) 015202 ]

# Flavor in transverse space

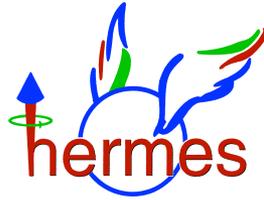
With **flavor dependence** we can account *theoretically* for **different cross sections** for different target/final state hadron combinations.



# Hermes

$$e^{\pm} + \boxed{P/D} \rightarrow e^{\pm} + \underline{\{\pi^{\pm} / K^{\pm}\}} + X$$

6 bins in x,  
8 bins in z,  
7 bins in  $P_{hT}$ ,



2 targets, 4 final-state hadrons

Our selection

- Remove the first bin x- $Q^2$  ( $Q^2 > 1.4 \text{ GeV}^2$ )
- $0.1 < z < 0.8$
- $P_{hT}^2 < Q^2/3$

2688 points

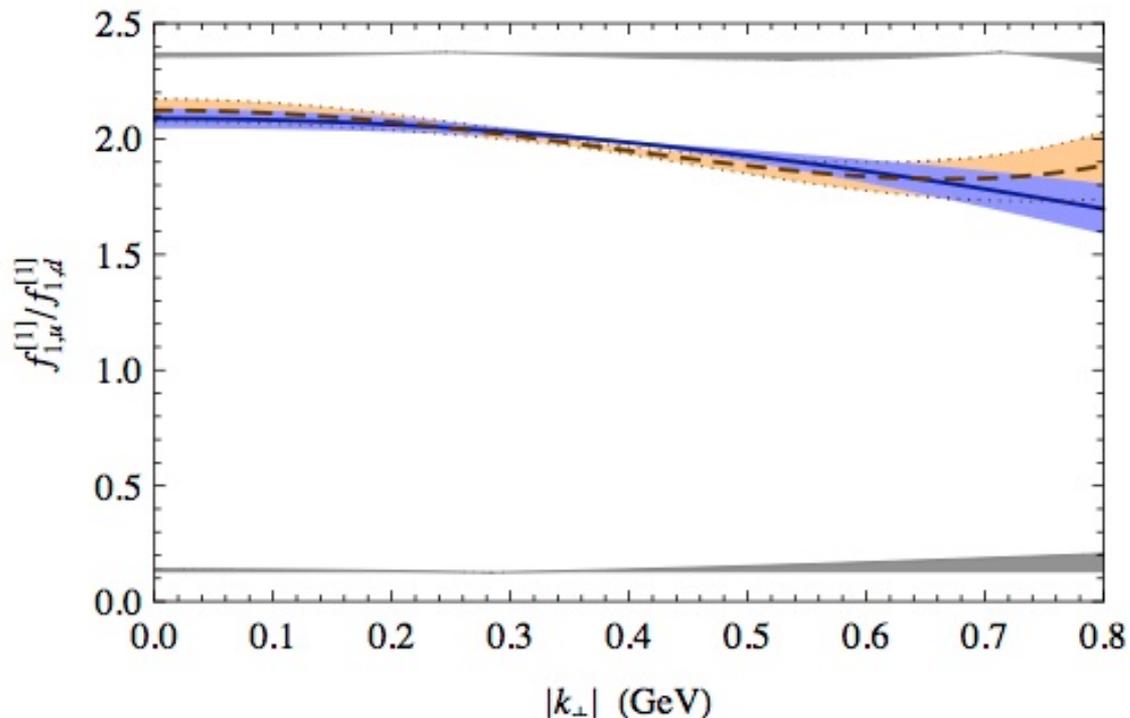
1538 analyzed points

limited  $Q^2$  range  $\Rightarrow$  **safely neglect evolution everywhere**

# Flavor in transverse space

valence picture of proton, #u / #d = 2

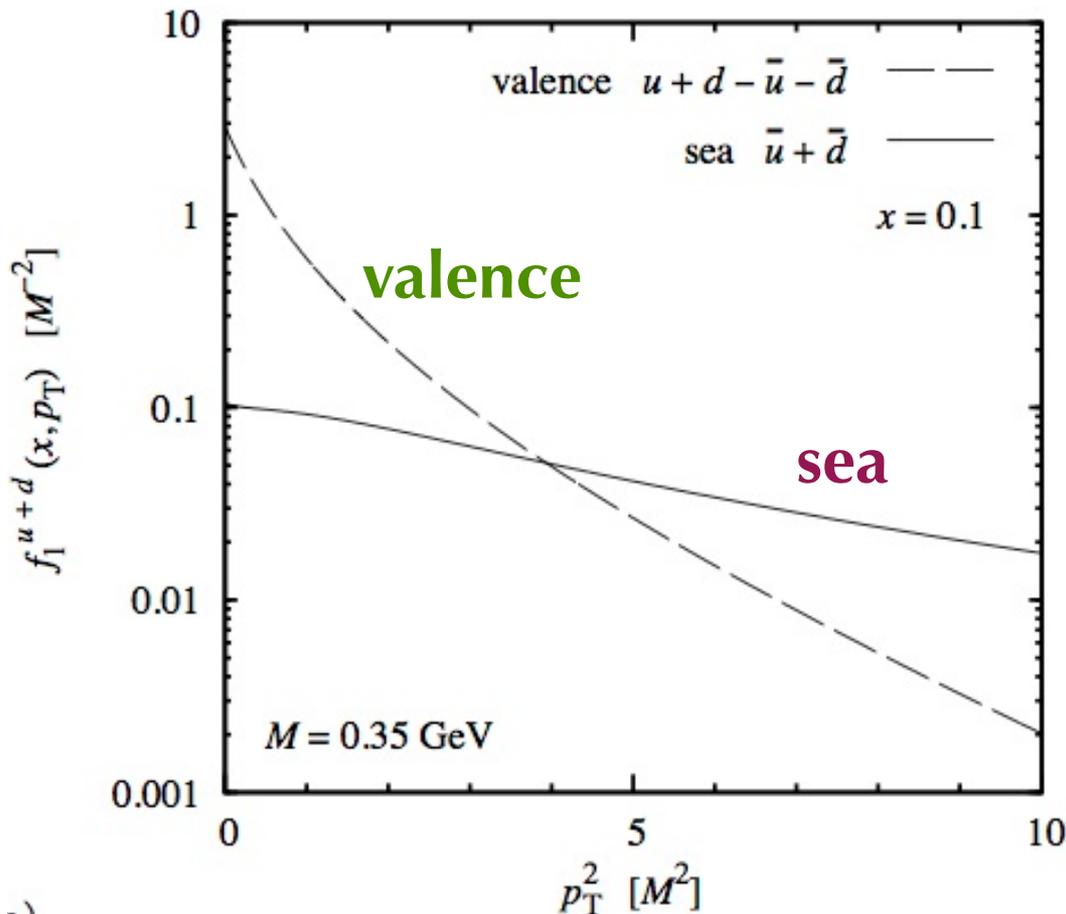
Less up at larger transverse momentum..!



Lattice QCD

Musch *et al.*, P.R. D83 (11) 094507

# Flavor in transverse space



**Chiral quark soliton model**  
 (Schweitzer, Strikman, Weiss)  
 JHEP 1301 (913) 163

And other models...

**Diquark spectator**  
 (Bacchetta, Courtoy, Radici –  
 PRD 78 (08) 074010)

**Statistical approach**  
 (Bourrely, Buccella, Soffer  
 - PRD 83 (11) 074008)

# Flavor in transverse space

TMD FFs - NJL-jet model

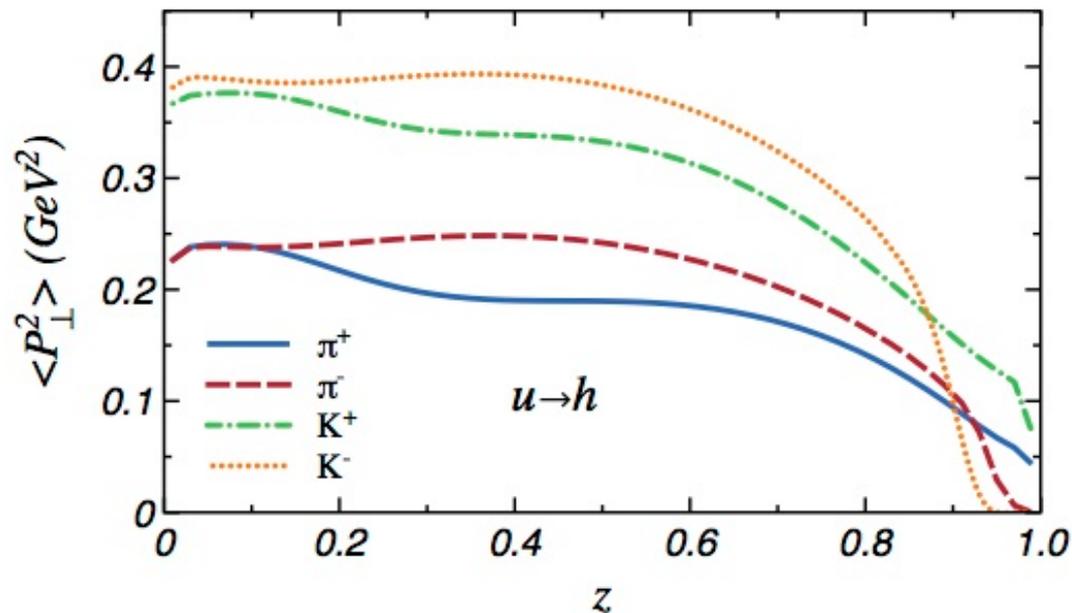
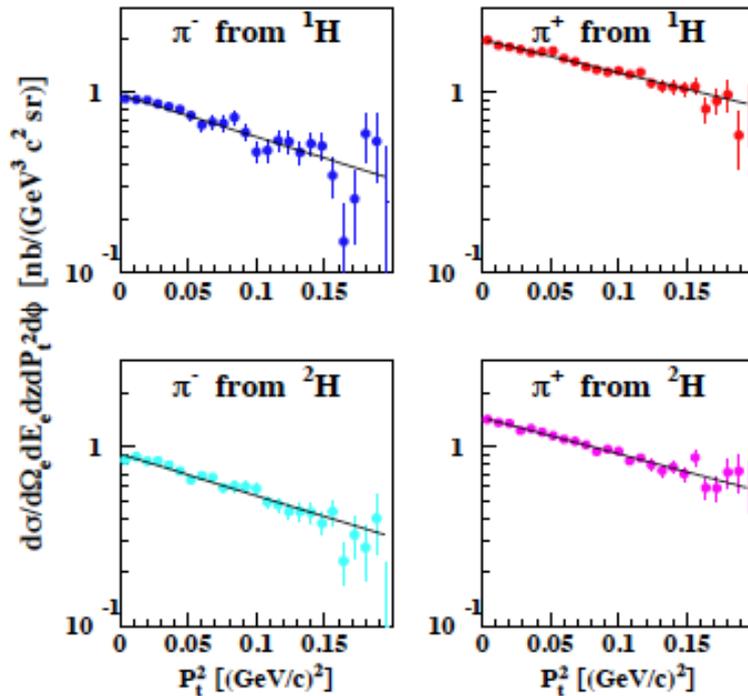


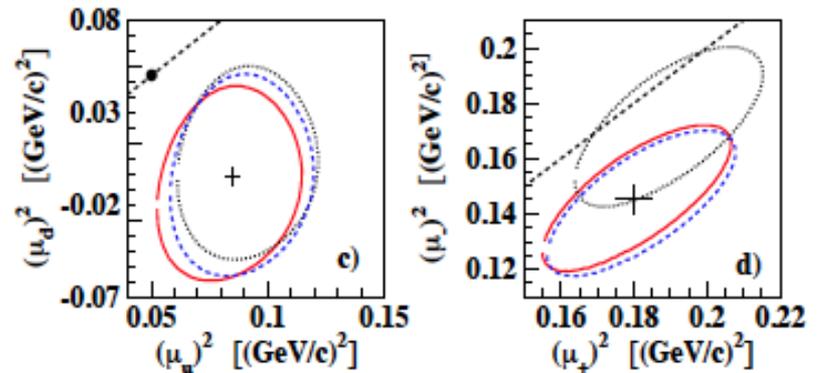
FIG. 14. The averaged transverse momentum of  $\pi$  and  $K$  mesons emitted by a  $u$  quark.

Matevosyan *et al.*, P. R. D85 (12) 014021

# Flavor in transverse space



no kaons, no sea,  
no X-Z dependence



up is wider than down

avored wider than unfavored

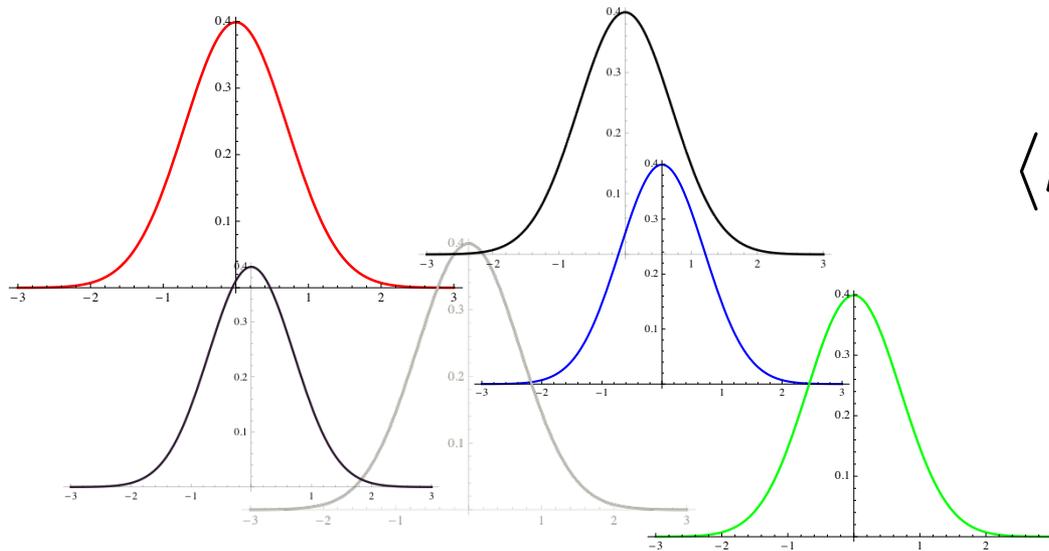
 **Jefferson Lab**

Asaturyan *et al.* (E00-108), P. R. C85 (12) 015202

# Flavor independent Gaussianity

## Gaussian parametrization of TMD parts

$$f_1^a(x, k_{\perp}^2) = f_1^a(x) \frac{1}{\pi \langle k_{\perp}^2 \rangle} \exp \left\{ -\frac{k_{\perp}^2}{\langle k_{\perp}^2 \rangle} \right\}$$



$$\langle k_{\perp,u}^2 \rangle = \langle k_{\perp,d}^2 \rangle = \dots$$

The same variance  
for all the flavors!

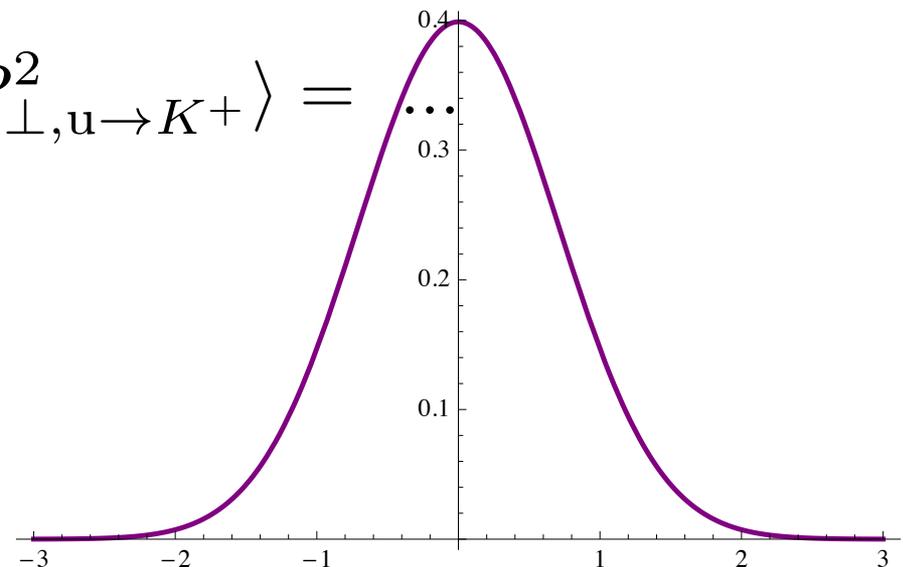
# Flavor independent Gaussianity

## *Gaussian parametrization of TMD parts*

$$D_1^{a \rightarrow h}(z, P_\perp^2) = D_1^{a \rightarrow h}(z) \frac{1}{\pi \langle P_\perp^2 \rangle} \exp \left\{ -\frac{P_\perp^2}{\langle P_\perp^2 \rangle} \right\}$$

$$\langle P_{\perp, u \rightarrow \pi^+}^2 \rangle = \langle P_{\perp, u \rightarrow \pi^-}^2 \rangle = \langle P_{\perp, u \rightarrow K^+}^2 \rangle = \dots$$

The same variance  
for all the fragmentation  
processes



# TMD PDFs – full analysis

68% confidence intervals of best-fit parameters for TMD PDFs in the different scenarios

Parameters for TMD PDFs				
	Default	$Q^2 > 1.6 \text{ GeV}^2$	Pions only	Flavor-indep.
$\langle \hat{\mathbf{k}}_{\perp, d_v}^2 \rangle [\text{GeV}^2]$	$0.30 \pm 0.17$	$0.33 \pm 0.19$	$0.34 \pm 0.12$	$0.30 \pm 0.10$
$\langle \hat{\mathbf{k}}_{\perp, u_v}^2 \rangle [\text{GeV}^2]$	$0.36 \pm 0.14$	$0.37 \pm 0.17$	$0.35 \pm 0.12$	$0.30 \pm 0.10$
$\langle \hat{\mathbf{k}}_{\perp, \text{sea}}^2 \rangle [\text{GeV}^2]$	$0.41 \pm 0.16$	$0.31 \pm 0.18$	$0.29 \pm 0.13$	$0.30 \pm 0.10$
$\alpha$ (random)	$0.95 \pm 0.72$	$0.93 \pm 0.70$	$0.95 \pm 0.68$	$1.03 \pm 0.64$
$\sigma$ (random)	$-0.10 \pm 0.13$	$-0.10 \pm 0.13$	$-0.09 \pm 0.14$	$-0.12 \pm 0.12$

# TMD FFs – full analysis

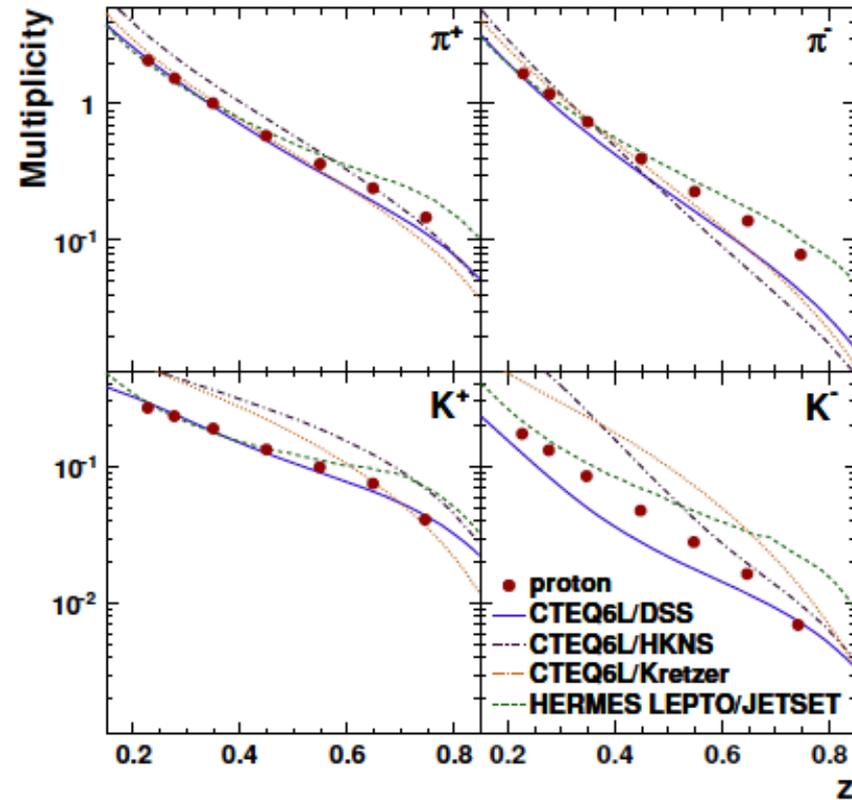
68% confidence intervals of best-fit parameters for TMD FFs in the different scenarios

Parameters for TMD FFs				
	Default	$Q^2 > 1.6 \text{ GeV}^2$	Pions only	Flavor-indep.
$\langle \hat{P}_{\perp, \text{fav}}^2 \rangle [\text{GeV}^2]$	$0.15 \pm 0.04$	$0.15 \pm 0.04$	$0.16 \pm 0.03$	$0.18 \pm 0.03$
$\langle \hat{P}_{\perp, \text{unf}}^2 \rangle [\text{GeV}^2]$	$0.19 \pm 0.04$	$0.19 \pm 0.05$	$0.19 \pm 0.04$	$0.18 \pm 0.03$
$\langle \hat{P}_{\perp, sK}^2 \rangle [\text{GeV}^2]$	$0.19 \pm 0.04$	$0.19 \pm 0.04$	-	$0.18 \pm 0.03$
$\langle \hat{P}_{\perp, uK}^2 \rangle [\text{GeV}^2]$	$0.18 \pm 0.05$	$0.18 \pm 0.05$	-	$0.18 \pm 0.03$
$\beta$	$1.43 \pm 0.43$	$1.59 \pm 0.45$	$1.55 \pm 0.27$	$1.30 \pm 0.30$
$\delta$	$1.29 \pm 0.95$	$1.41 \pm 1.06$	$1.20 \pm 0.63$	$0.76 \pm 0.40$
$\gamma$	$0.17 \pm 0.09$	$0.16 \pm 0.10$	$0.15 \pm 0.05$	$0.22 \pm 0.06$

# Results

'Tension' in the **collinear** case

	$Q^2 > 1.4 \text{ GeV}^2$
global	2.86
$p \rightarrow K^-$	2.25
$p \rightarrow \pi^-$	3.39
$p \rightarrow \pi^+$	1.87
$p \rightarrow K^+$	0.89
$D \rightarrow K^-$	4.26
$D \rightarrow \pi^-$	5.05
$D \rightarrow \pi^+$	3.33
$D \rightarrow K^+$	1.80

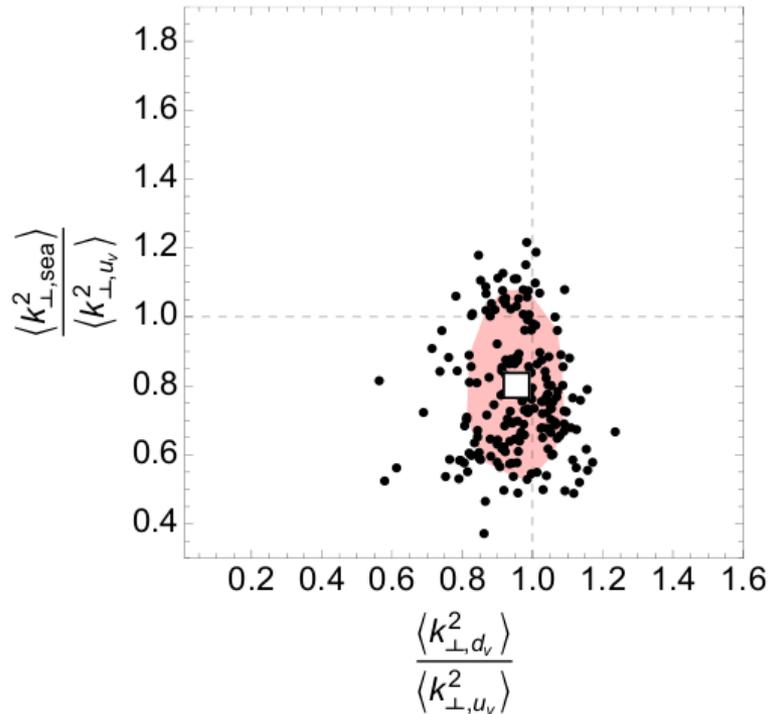


**Table 2.** Values of  $\chi^2/\text{d.o.f.}$  obtained from the comparison of the HERMES multiplicities  $m_N^h(x, z, Q^2)$  with the theoretical prediction using the MSTW08LO collinear PDFs [8] and the DSS LO collinear FFs [48]. In all cases, the range  $0.1 \leq z \leq 0.8$  was included.

# TMD PDFs – without Kaons

sea width  
< (mostly)  
 $u_v$  width

point of  
no flavor dep.

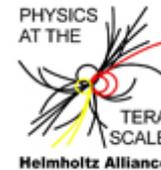


Strange quarks  
are relevant !

$d_v$  width  $\sim$  (mostly)  $u_v$  width

# TMD lib & plotter

**TMD lib :**  
a **library** of parametrizations of  
TMDs and uPDFs  
on the same footing of LHAPDF



F. Hautmann, H. Jung  
T. Rogers, P. Mulders, AS

## TMD Project

Webpage maintained by: **Ted Rogers, Andrea Signori**

This is the development page for the TMD project. The purpose of this project is to organize a repository of theoretical and phenomenological studies of transverse-momentum-dependent parton distribution functions (TMD PDFs) and fragmentation functions (TMD FFs). We provide access to parametrizations and fits of TMDs, with and without taking into account the perturbative QCD evolution.

**Coming soon!**

## High Energy Physics | TMD Plotter



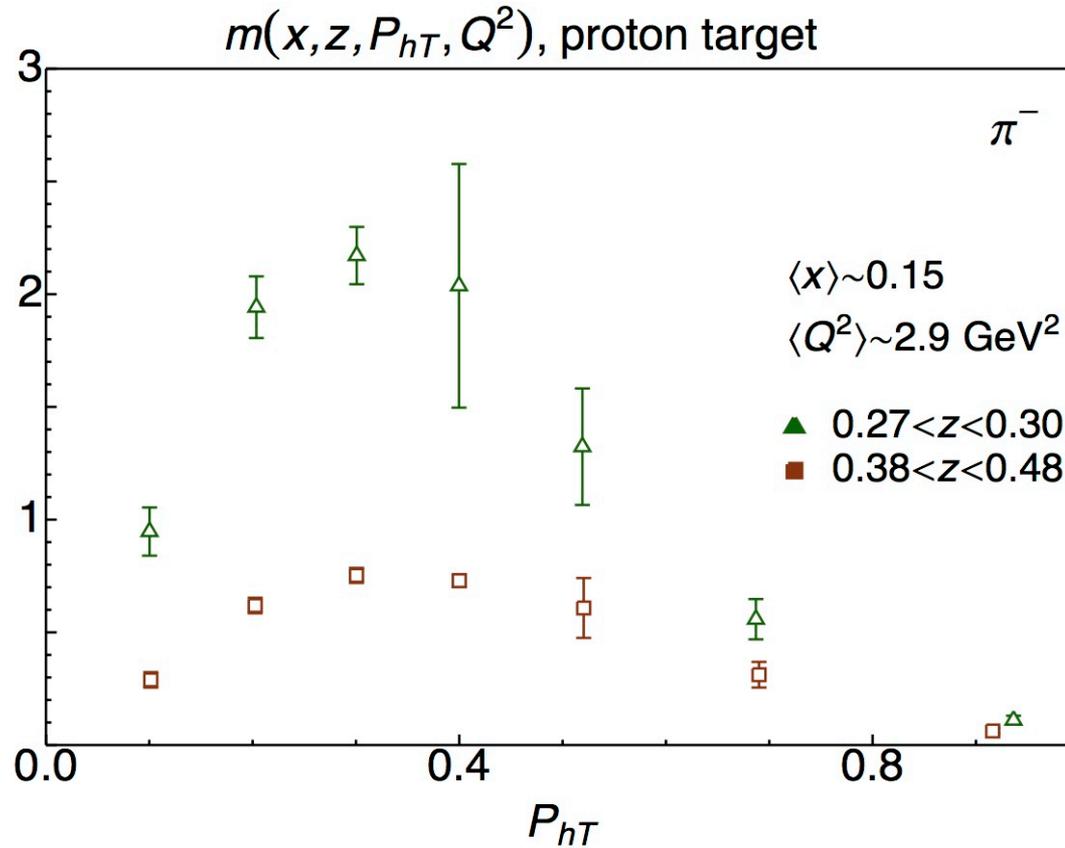
Home

TMD Plotter

Publications

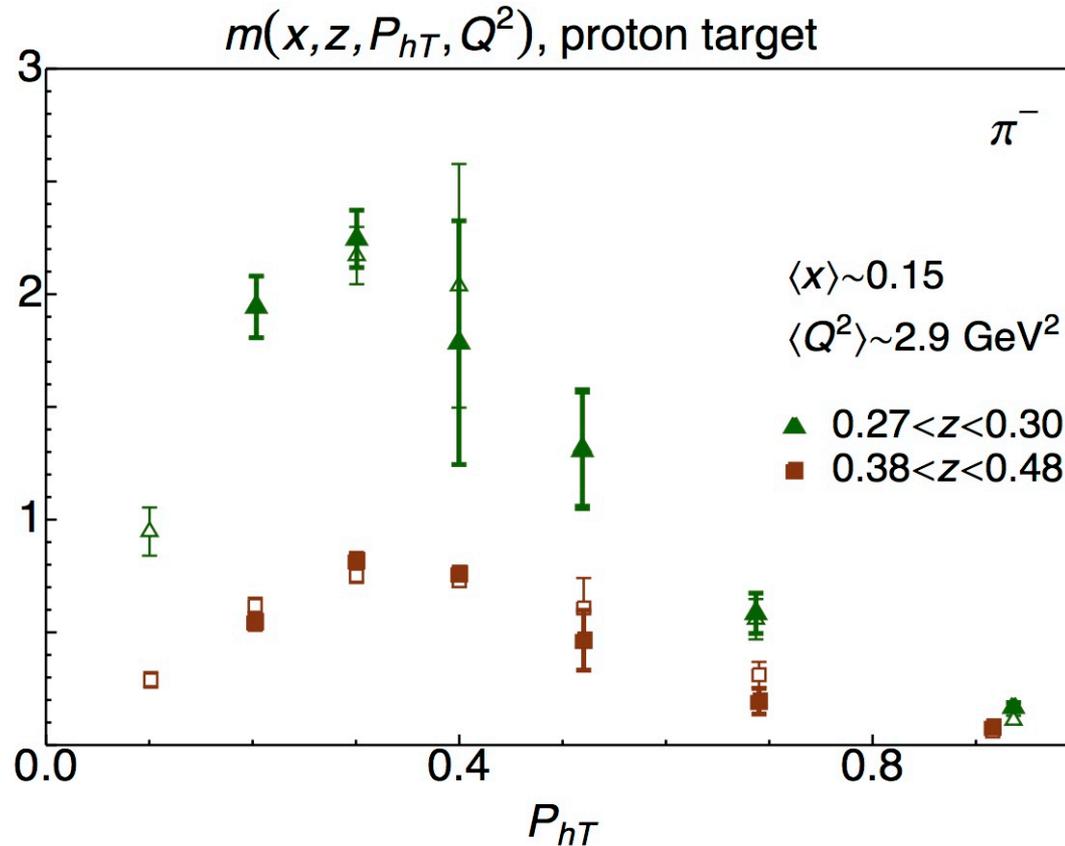
HEP Links

# Fitting procedure



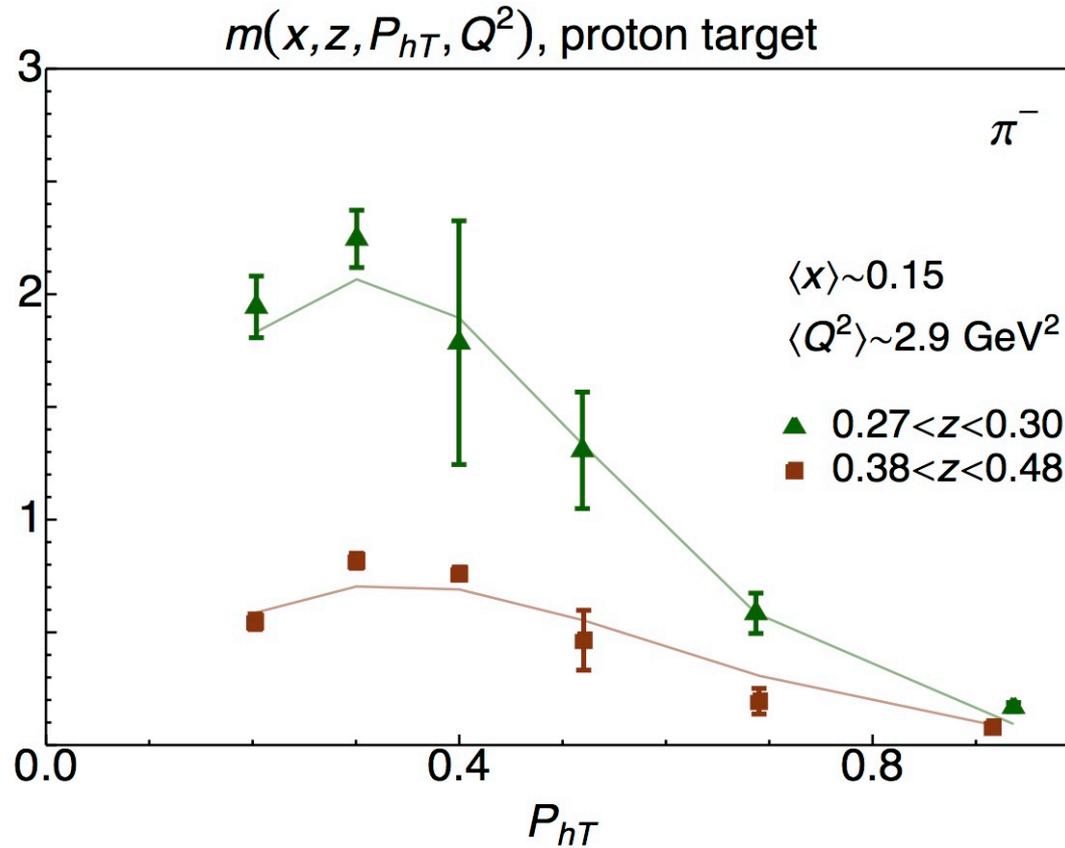
Sample of original data

# Fitting procedure



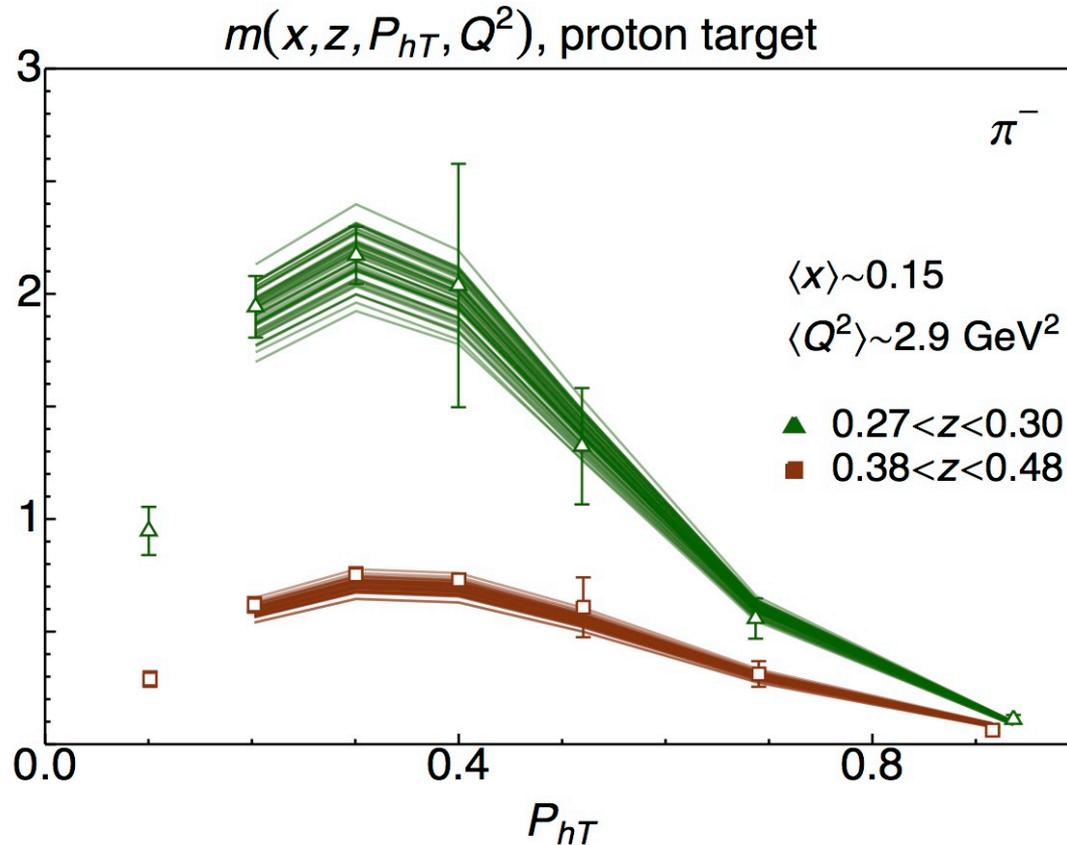
Replica of the original data with Gaussian noise

# Fitting procedure



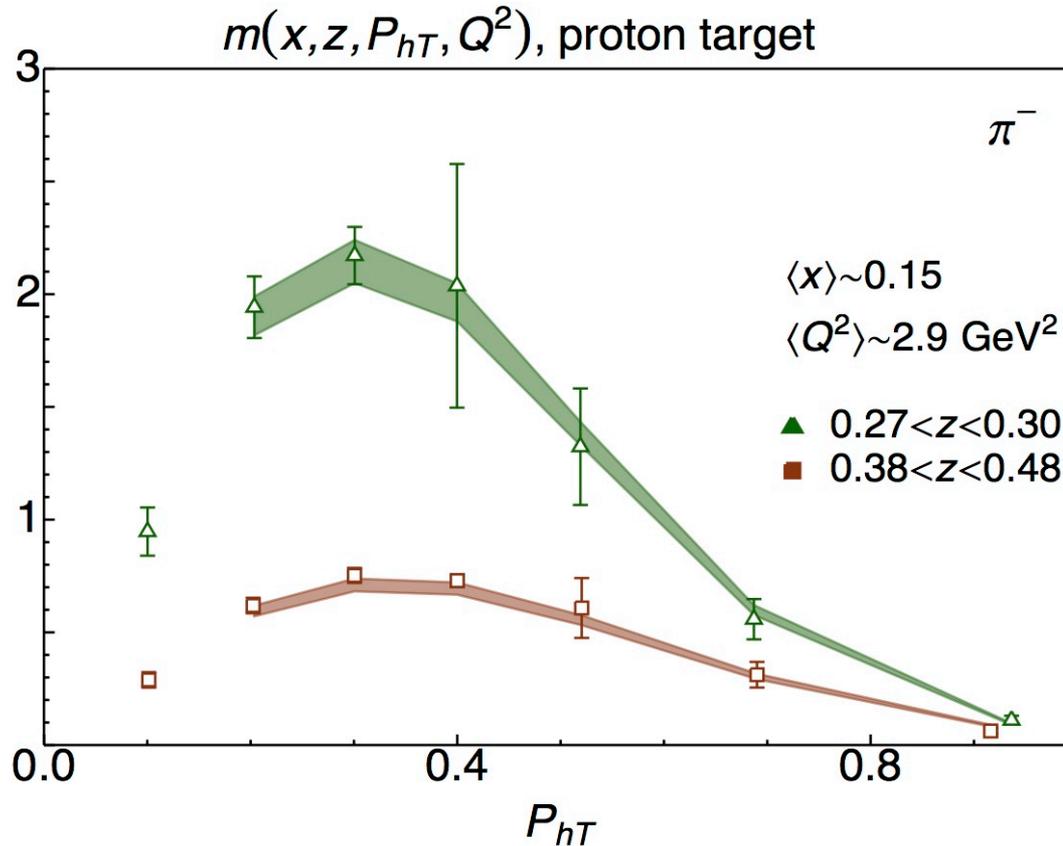
We fit the replicated data...

# Fitting procedure



... repeating the fit over the 200 replicas

# Fitting procedure



Plot of the 68% CL bands