Study of TMD evolution in SIDIS at moderate Q

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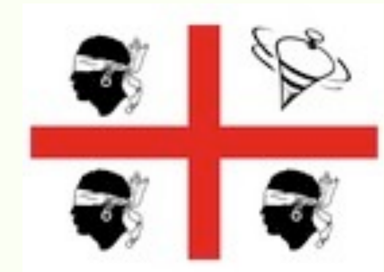
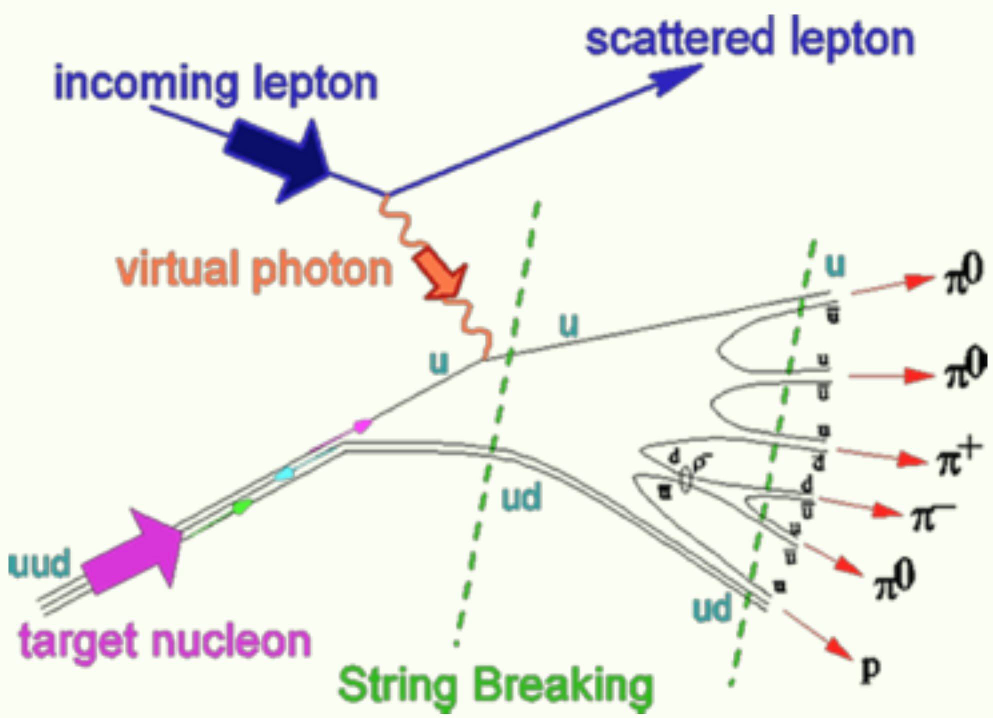
Chia Cagliari, Italy





Leonard Gamberg Penn State University-Berks

Based on Phys. Rev. D 89 (2014) C. Aidala, . B. Field, LG, T. Rogers

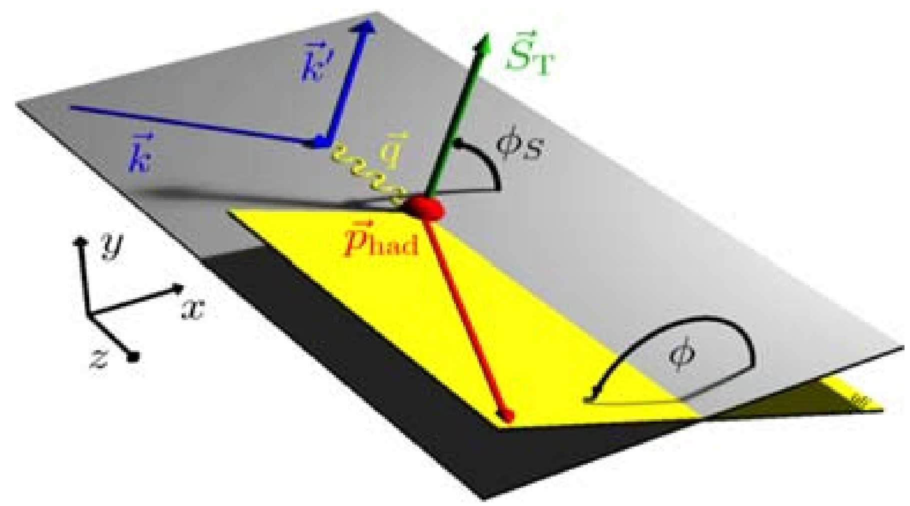
Outline

- Introductory comments on prediction of strong universality of TMD factorization
- Lightening Review of TMD factorization in parton model
- Lightening Review of elements TMD factorization in QCD...in particular the strong universal factor from CSS evolution kernal
- Study of evolution transverse momentum broadening SIDIS and role of universal role of NP content of evolution kernal
- Conclude

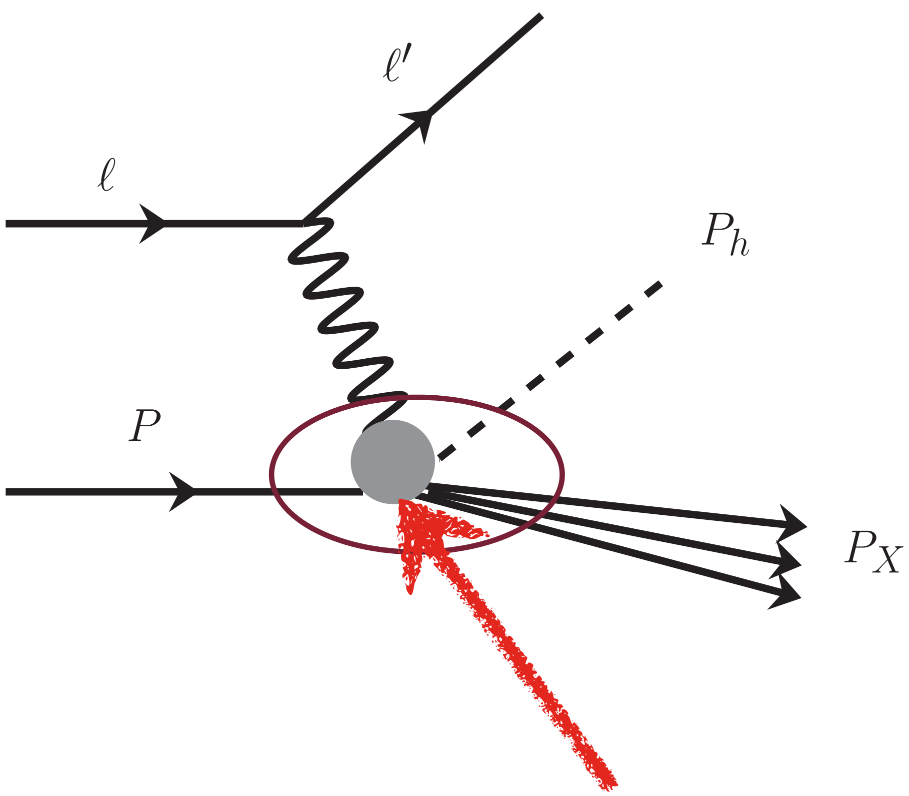
Comments

- Collins-Soper evol. kernel has perturbative-short distance & non-perturbative (**NP**) large-distance content
- Non-pertb. large-distance is ***strongly universal*** -many interesting predictions
- Universal character can exploited in observables "Bessel Weighting" another time and place
  (Boer Gamberg, Musch Prokudin JHEP 2011, Aghasyan, Avakian, Gamberg, Prokudin, Rossi et al 2014)
- Global fits, based on larger Q Drell-Yan–data/processes find substantial contributions from nonperturbative regions in the Collins-Soper evolution kernel-e.g. BNLY PRD 67(2003) & Konychev Nadolsky PLB 2005
- Many demonstrations that applying larger Q DY fits result in too rapid evolution for SIDIS data which are "HERMES/COMPASS/JLAB like"
- We investigate SIDIS measurements in the region of a few GeV, where sensitivity to *NP* transverse momentum dependence is more important or even dominate the evolution
- Performed a study that isolates/places bounds on it/we quantify it s.t. both high-energy DY fits as well respects the lower energy experiments

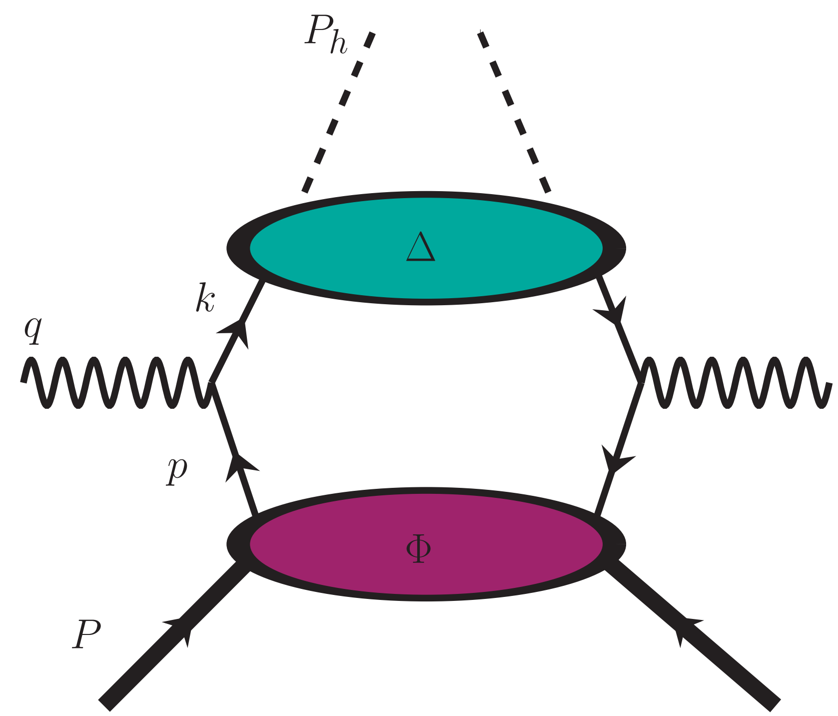
Factorization Parton Model



Kotzinian NPB 95,
Mulders Tangermann NPB 96,
Boer & Mulders PRD 97
Bacchetta et al JHEP 08



Factorize



$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}|\, d|\boldsymbol{P}_{h\perp}|} = \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right) L_{\mu\nu} W^{\mu\nu},$$

$$
\begin{aligned}
\frac{d\sigma}{dx_B\,dy\,d\psi\,dz_h\,d\phi_h\,dP_{h\perp}^2} &= \frac{\alpha^2}{x_B y Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&+ \quad \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&+ \quad S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+ \quad S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&+ \quad |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&+ \quad \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&+ \quad \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&+ \quad |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&+ \quad \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
\end{aligned}
$$

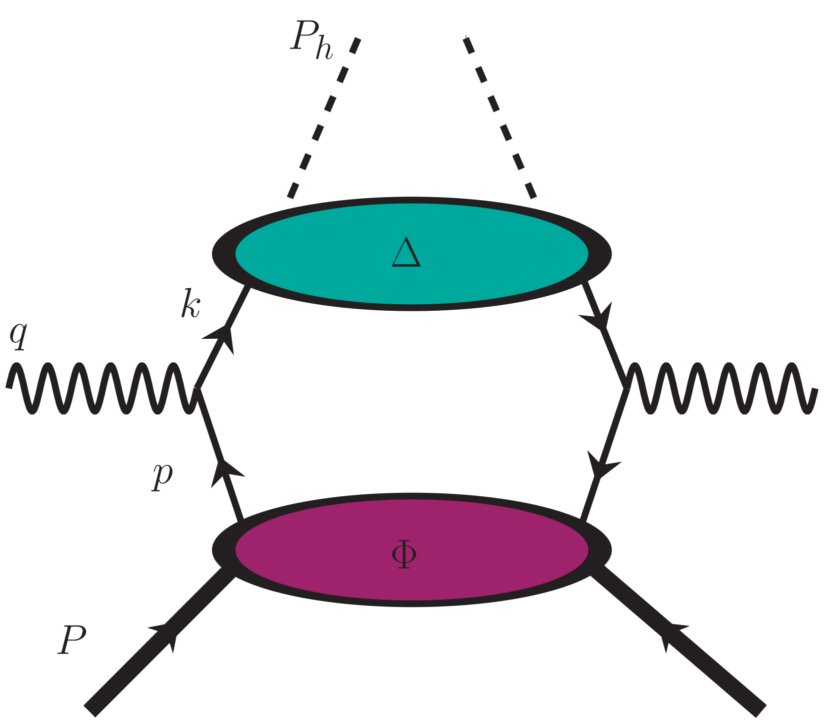
Bacchetta et al JHEP 08

Factorization P_T of hadron small sensitive to intrinsic transv. momentum of partons

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^2}\int \frac{d^2\mathbf{k}_T}{(2\pi)^2}\delta^2(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z_h} - \mathbf{k}_T)\mathrm{Tr}\left[\Phi(x,\mathbf{p}_T)\gamma^\mu\Delta(z,\mathbf{k}_T)\gamma^\nu\right]$$

$$\Phi(x,\mathbf{p}_T) = \int dp^- \Phi(p,P,S)|_{p^+=x_BP^+}\,, \qquad \Delta(z,\mathbf{k}_T) = \int dk^-\Delta(k,P_h)|_{k^-=\frac{P^-}{z_h}}$$

Small transverse momentum

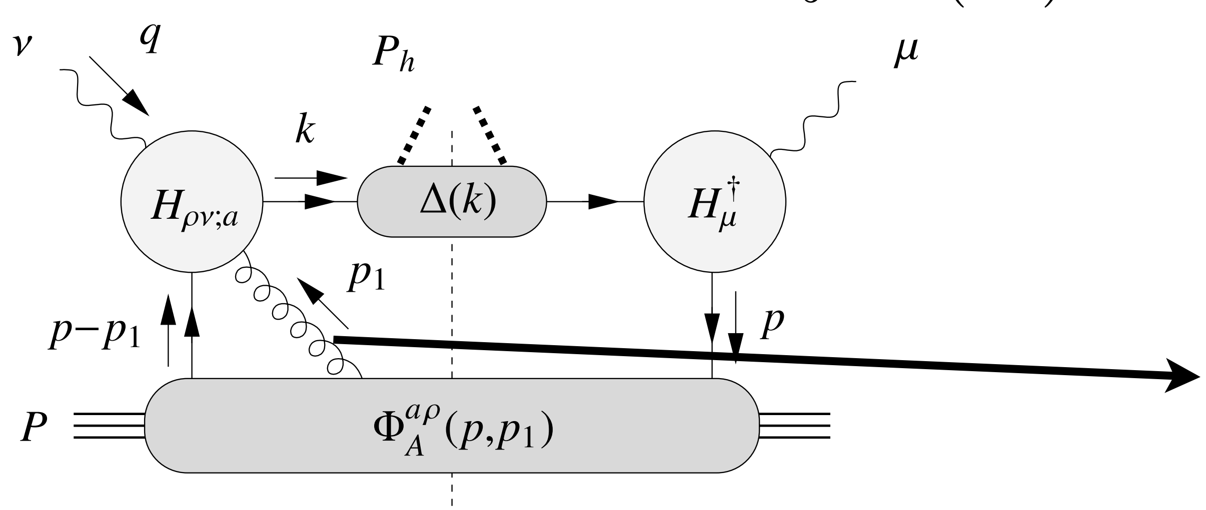


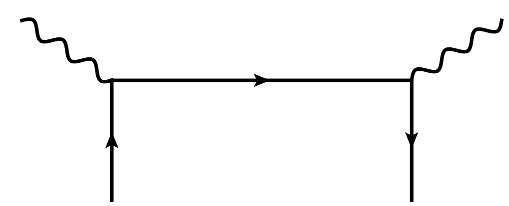
Purely Kinematic-integrate over small momentum component
Must also respect gauge invariance
Minimal requirement satisfy **color** gauge invariance

Minimal Requirement for PARTON MDL Factorization

Gauge link determined re-summing leading gluon interactions btwn soft and hard
Efremov,Radyushkin Theo. Math. Phys. 1981, Collins, Soper NPB 1981, 1982,Collins PLB 2002,
Belitsky, Ji, Yuan NPB 2003, Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD,

$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-,\xi_T)|P\rangle|_{\xi^+=0}$$





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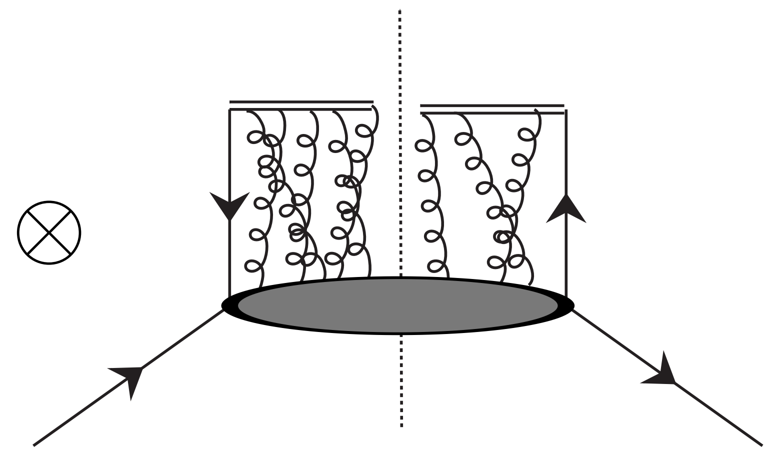
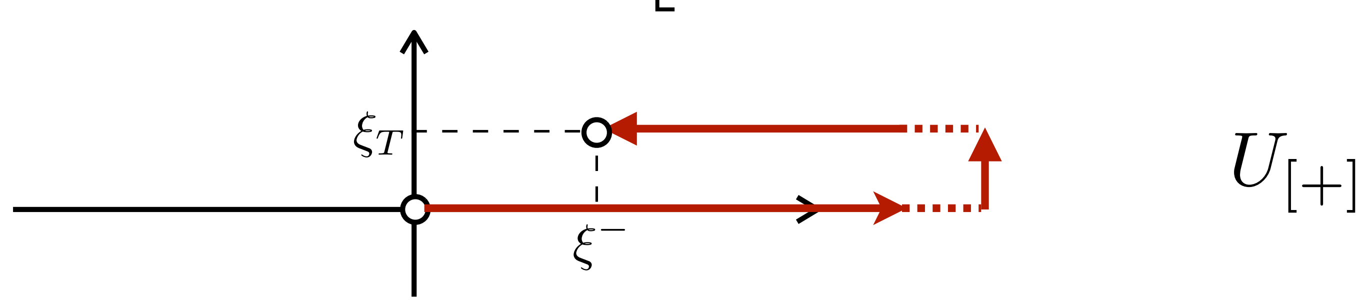


- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$W_{\mu\nu}(q,P,S,P_h) = \int d^4p\, d^4k\, \delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{\mathcal{U}^{[C]}_{[\infty;\xi]}}(p) H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$



Partonic picture Structure Functions momentum CONVOLUTION

$$\mathcal{C}\big[w\,f\,D\big] = x\sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

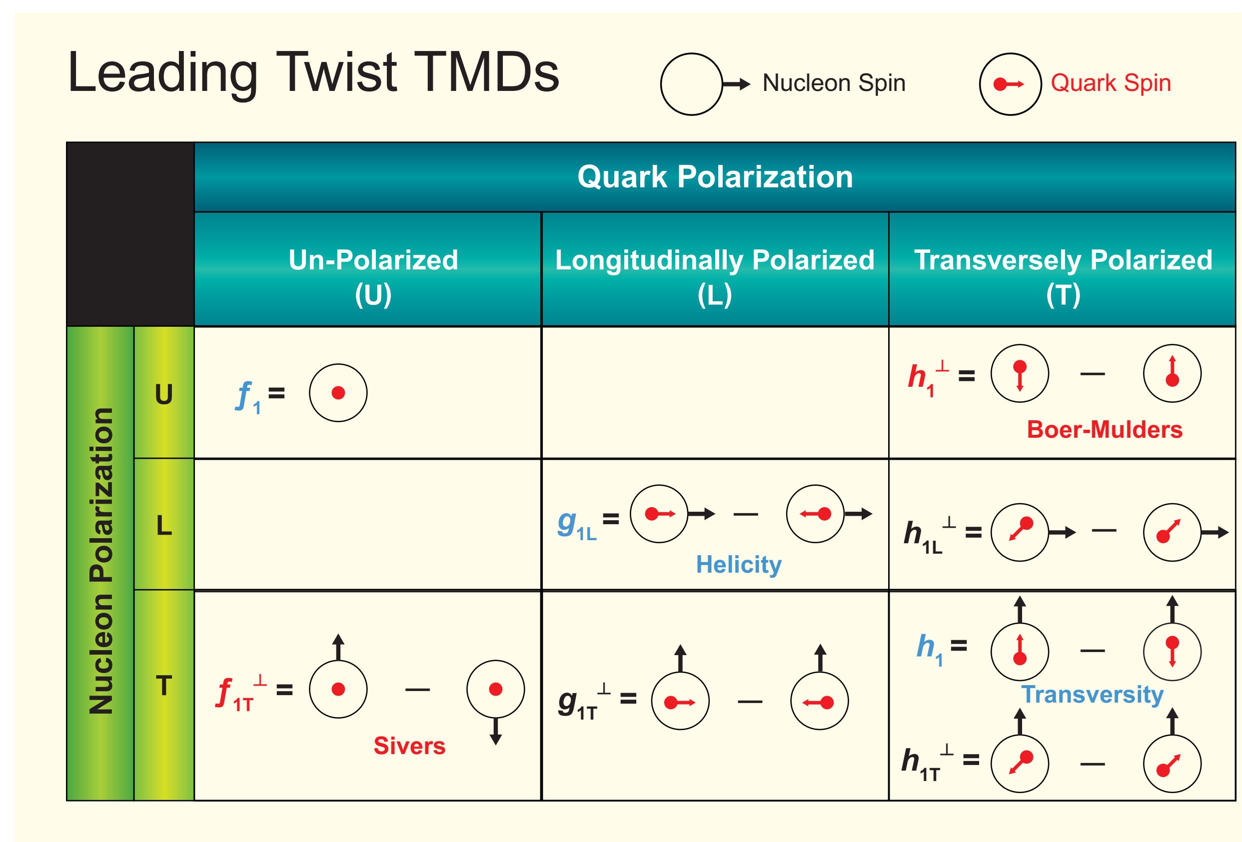
$$F_{LL} = \mathcal{C}\big[g_{1L} D_1\big],$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\, f_{1T}^{\perp} D_1\right],$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\, h_1 H_1^{\perp}\right],$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}\, h_{1L}^{\perp} H_1^{\perp}\right],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}\, h_1^{\perp} H_1^{\perp}\right],$$



★ CS has simple S/T interpretation--multipole expansion in terms of $b_T\,[\mathrm{GeV}^{-1}]$ conjugate to $\boldsymbol{P}_{h\perp}$

Boer, Gamberg, Musch, Prokudin JHEP 2011

$$
\begin{aligned}
\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} = \;&
\frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left(1+\frac{\gamma^2}{2x_B}\right) \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T| \Big\{ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU,T} + \varepsilon\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU,L} \\
&+ \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU}^{\cos(2\phi_h)} \\
&+ \lambda_e \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LU}^{\sin\phi_h} \\
&+ S_\parallel \Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UL}^{\sin 2\phi_h} \Big] \\
&+ S_\parallel \lambda_e \Big[\sqrt{1-\varepsilon^2}\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LL}^{\cos\phi_h} \Big] \\
&+ |\boldsymbol{S}_\perp| \Big[\sin(\phi_h-\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|) \left(\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, \mathcal{F}_{UT,L}^{\sin(\phi_h-\phi_S)} \right) \\
&\quad + \varepsilon\, \sin(\phi_h+\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} \\
&\quad + \varepsilon\, \sin(3\phi_h-\phi_S)\, J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin\phi_S} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h-\phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin(2\phi_h-\phi_S)} \Big] \\
&+ |\boldsymbol{S}_\perp| \lambda_e \Big[\sqrt{1-\varepsilon^2}\, \cos(\phi_h-\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LT}^{\cos(\phi_h-\phi_S)} \\
&\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LT}^{\cos\phi_S} \\
&\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h-\phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LT}^{\cos(2\phi_h-\phi_S)} \Big] \Big\} \qquad .)
\end{aligned}
$$

unpolarized

Sivers

“Parton Model”

Bessel weighting-projecting out Sivers

orthogonality of Bessel Fncts.

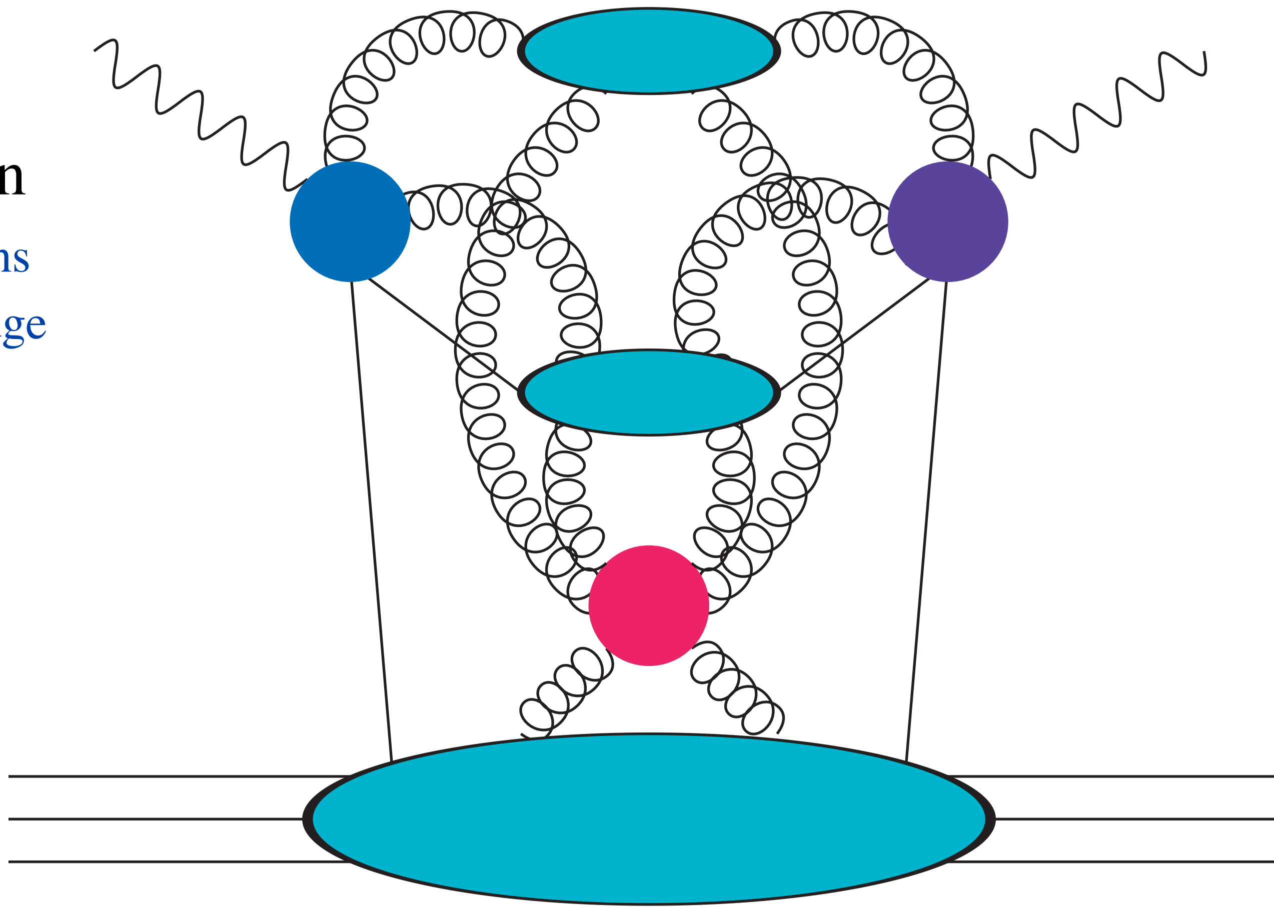
$$\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM} = \frac{2\,J_1(|\boldsymbol{P}_{hT}|\mathcal{B}_T)}{zM\mathcal{B}_T}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)}(\mathcal{B}_T) = 2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)\,\left(d\sigma^{\uparrow}-d\sigma^{\downarrow}\right)}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\;\mathcal{J}_0^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)\;\left(d\sigma^{\uparrow}+d\sigma^{\downarrow}\right)}$$

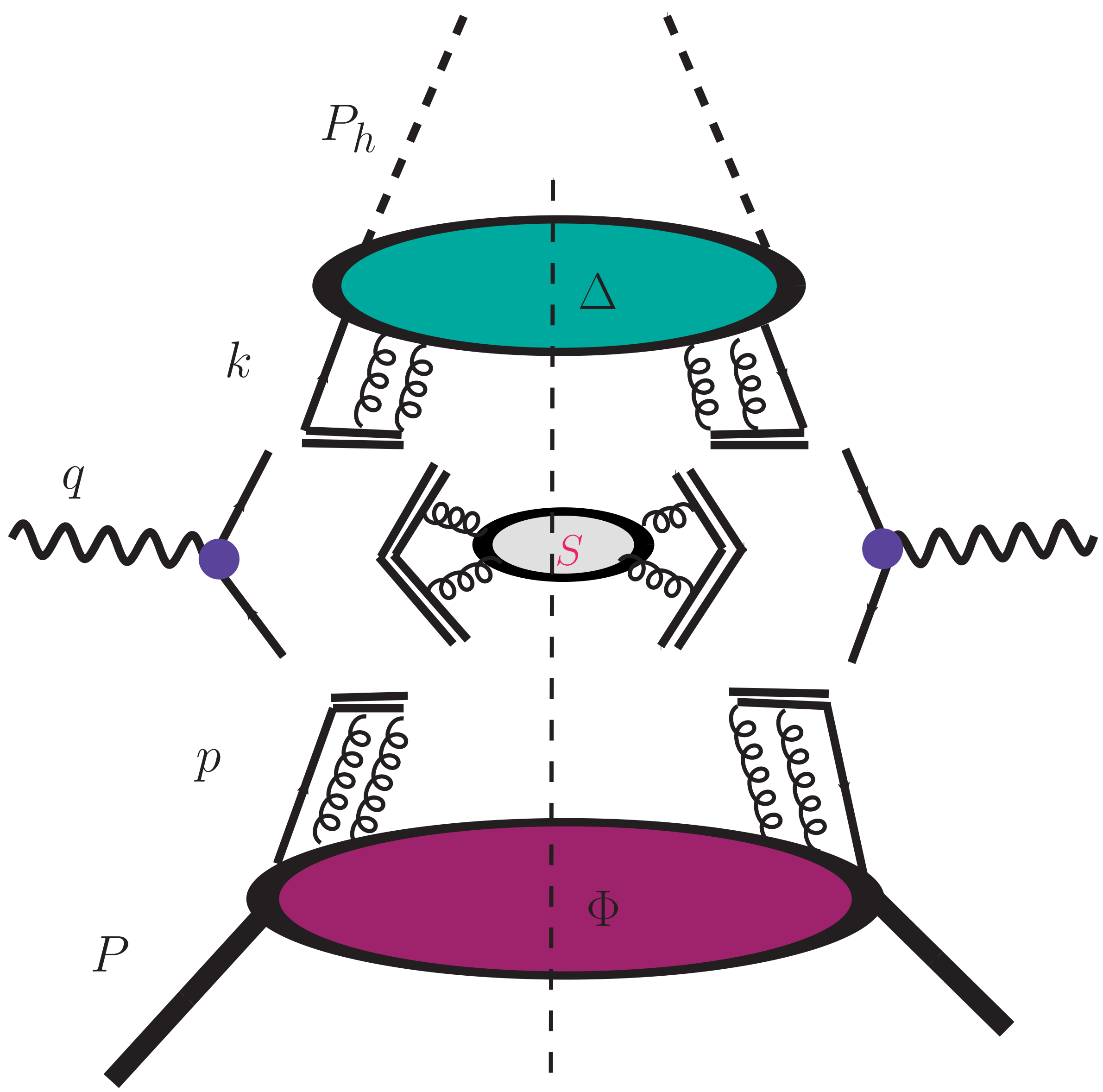
$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) = -2\,\frac{\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}{\sum_a e_a^2\;\tilde{f}_1^a(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}$$

QCD Factorization Procedure Beyond Parton Model include Glue

- Leading Regions-power counting Libby Sterman PRD 1978 (see Collins PRD 1980 nongauge theories, Collins Soperp NPB& CSS formalism 1982-85... Collins 2011 Cambridge Univ. Press)
- "Reduced Diagrams"
- Apply Ward Identities get factorized form
  - Soft Factor w/ gauge links
  - TMDs w/ gauge links



TMD factorization



Collins Soper NPB 1981,1982, CSS NPB 1985, Collins, Hautman PLB 00, Collins Metz PRL 2004, Collins Oxford Press 2011, Boer NPB 2001, 2009,2013, Ji, Ma, Yuan PLB 2004, PRD 2005, Ibildi, Ji, Yuan PRD 2004, Cherednikov, Karanikas, Stefanis NPB 2010, Abyat, Rogers PRD 2011, Abyat, Collins, Qiu, Rogers PRD 2012, Collins Rogers 2013, Echevarria, Idilbi, Scimemi JHEP 2012

ETC ....

- TMDs w/Gauge links: color invariant
- In addition Soft factor

Emergence of Soft Factor in Cross section

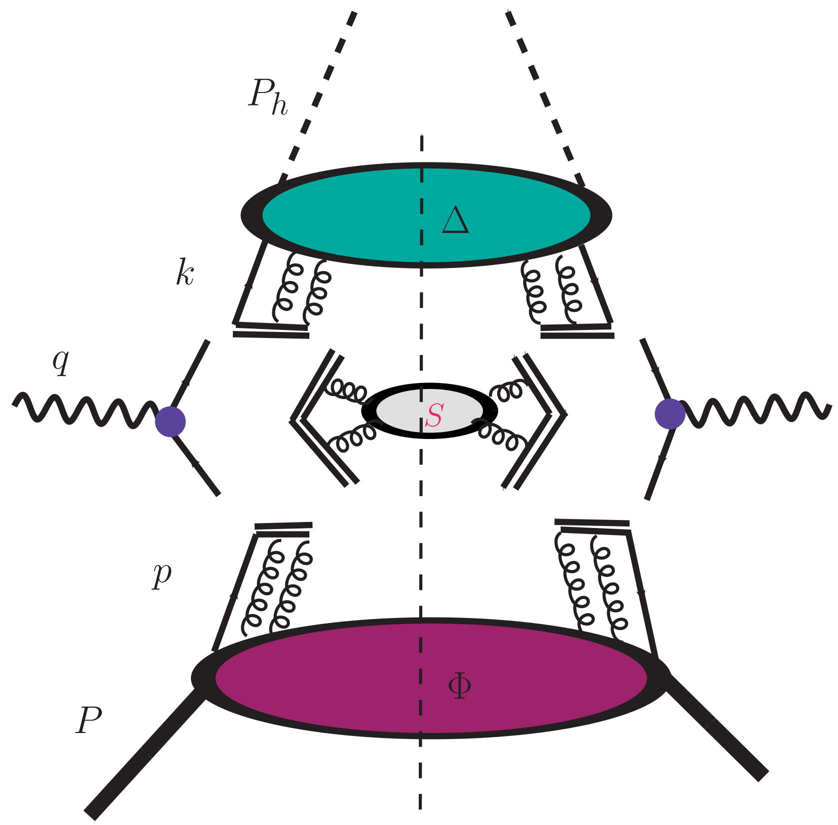
$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\rm unsub}(y_1 - (-\infty)) \times \tilde{F}_2^{\rm unsub}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}$$

Collins Act Pol. 2003
Ji Ma Yuan 2004, 2005

TMDs are still "entangled" not yet fully factorized

Use its properties to fully factorize and perform evolution

Collins 2011 Cam. Univ. Press see also Aybat Rogers PRD 2011

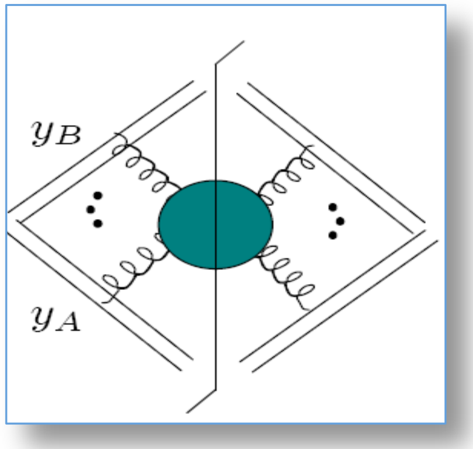


- Factorization introduces <u>Wilson lines</u> w/rapidity/LC divergences
- Extra variables needed to regulate these divergences
- Treatment of LC/Rapidity divergences Collins 2011, Aybat & Rogers PRD 2011

Collins Soper NPB 1981, Collins Metz PRL 2004, Ji, Ma, Yuan PRD 2004, Collins 2011, Collins Rogers 2012

Further treatment achieve full factorization using Soft Factor in CSS

- Lightlike Wilson lines in TMDs
  - Infinite rapidity QCD radiation in the wrong direction.
  - In soft factor/fragmentation function too.

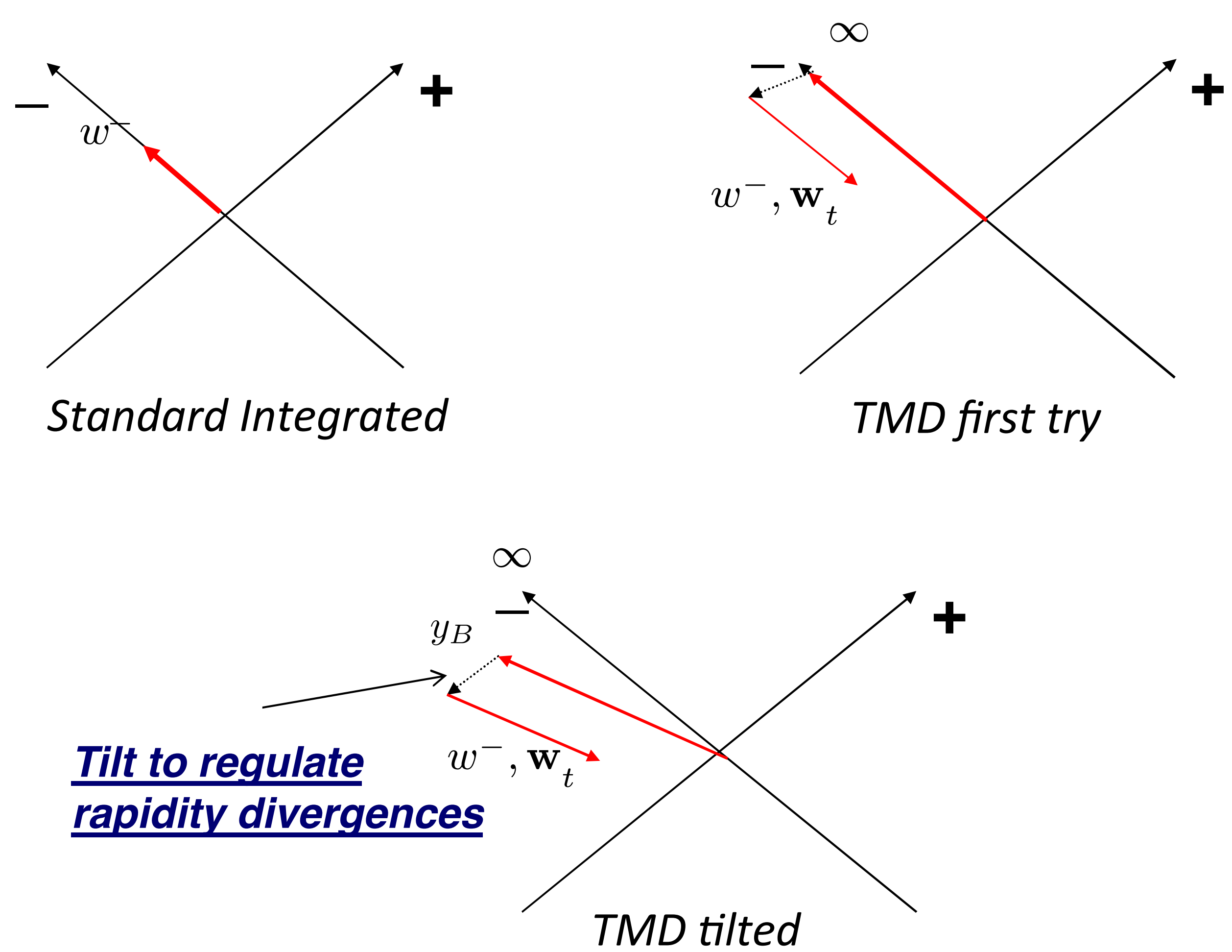


- Finite rapidity Wilson lines
  - Regulate rapidity of extra gluons.

Introduces rapidity scale parameter

$$n_B = (-e^{2y_B}, 1, \mathbf{0})$$

Paths of Wilson Lines in Coordinate Space



$$\zeta_F = M_P^2 x^2 e^{2(y_P - y_s)} \quad \Longleftrightarrow \quad y$$

Emergence of Soft Factor in TMDs

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\rm unsub}(y_1 - (-\infty)) \times \tilde{F}_2^{\rm unsub}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}$$

Soft factor further "repartitioned"
This is done to both

1) cancel LC divergences and
2) separate "right & left" movers i.e. full factorization

$$d\sigma = |\mathcal{H}|^2 \, \left\{ F_1^{\rm unsub}(y_1 - (-\infty)) \sqrt{\frac{\tilde{S}(+\infty, y_s)}{\tilde{S}(+\infty, -\infty)\tilde{S}(y_s, -\infty)}} \right\} \times \left\{ \tilde{F}_2^{\rm unsub}(+\infty - y_2) \sqrt{\frac{\tilde{S}(y_s, -\infty)}{\tilde{S}(+\infty, -\infty)\tilde{S}(+\infty, y_s)}} \right\}$$

Separately Well-defined

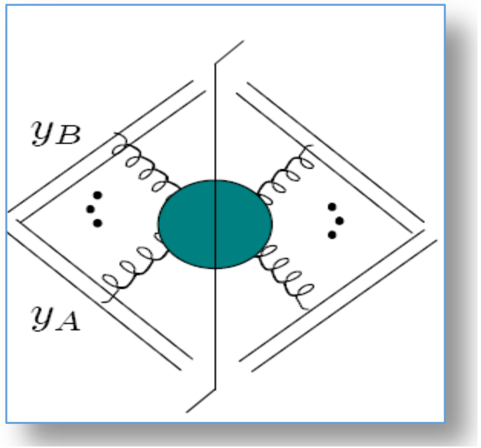
Factorization to TMD Evolution...CSS + JCC 2011

Evolution follows from their independence of rapidity scale

From operator definition get

Collins-Soper Equation:

$$-\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$



$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

Along with .... Renormalization group Equations

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln\tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

.... and RGE

Solve Collins Soper & RGE eqs. to obtain TMD Evolution kernal

Solve Collins Soper & RGE eqs. obtain TMD Evolution kernal
however....one more element ....

Solve Collins Soper & RGE eqs. obtain Evolution kernal

Collins Soper Sterman NPB 85

- Maximizes the perturbative content while providing a TMD formalism that is applicable over the entire range of P_T

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$

Nonperturbative part of evolution Kernal

$$\tilde{K}(b_T, \mu)$$

Collins Soper Sterman NPB 85

Totally universal related to derivative of soft factor independent of x & hadron

$$\tilde{K}(b_T;\mu) = \tilde{K}(b_*;\mu_b) - \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \gamma_K(g(\mu')) - g_K(b_T)$$

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$

$b_{\max}$ chosen so that b_* doesn't go too far beyond the pertb. region maximize perturbative content

Evolved Structure Function & *TMDs*

$$\mathcal{F}_{UU}(x,z,b,Q^2) = \sum_a \tilde{F}^a_{H1}(x,b_T,\mu,\zeta_F)\tilde{D}^a_{H2}(z_h,b_T,\mu,\zeta_D)H_{UU}(Q^2,\mu^2)$$

Non-perturbative large b_T behavior

Totally universal related to derivative of soft factor independent of x & hadron

$$\tilde{F}_{H_1}(x,b_T;Q,Q^2) = \tilde{F}_{H_1}(x,b_*;\mu_b,\mu_b^2)\exp\left\{-g_1(x,b_T;b_{\max}) - g_K(b_T;b_{\max})\ln\left(\frac{Q}{Q_0}\right) + \ln\left(\frac{Q}{\mu_b}\right)\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_{\text{PDF}}(\alpha_s(\mu');1) - \ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right\}$$

perform OPE on

$$\tilde{D}_{H_2}(z,b_T;Q,Q^2) = \tilde{D}_{H_2}(z,b_*;\mu_b,\mu_b^2)\exp\left\{-g_2(z,b_T;b_{\max}) - g_K(b_T;b_{\max})\ln\left(\frac{Q}{Q_0}\right) + \ln\left(\frac{Q}{\mu_b}\right)\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_{\text{FF}}(\alpha_s(\mu');1) - \ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right\}$$

Evolved TMD formalism for entire range of P_T

$$\frac{d\sigma}{dP_T^2} \propto \mathcal{H}(\alpha_s(Q)) \int d^2 b_T e^{i b_T \cdot P_T} \tilde{F}_{H_1}(x, b_T; Q, Q^2)\, \tilde{D}_{H_2}(z, b_T; Q, Q^2) \quad + \quad Y_{\mathrm{SIDIS}}$$

$$\frac{d\sigma}{dP_T^2} \propto \ \mathrm{F.T.}\exp\left\{ -g_{\mathrm{PDF}}(x, b_T; b_{\max}) - g_{\mathrm{FF}}(z, b_T; b_{\max}) - 2 g_K(b_T; b_{\max}) \ln\left(\frac{Q}{Q_0}\right) + \right.$$

$$\left. + \ 2\ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_{\mathrm{PDF}}(\alpha_s(\mu'); 1) + \gamma_{\mathrm{FF}}(\alpha_s(\mu'); 1) - 2\ln\left(\frac{Q}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$+ \ Y_{\mathrm{SIDIS}}$$

Comments Factorization

- This strong form of universality is, therefore, an important basic test of the TMD factorization theorem. It is related to the soft factors—the vacuum expectation values of Wilson loops—that are needed in the TMD definitions for consistent factorization with a minimal number of arbitrary cutoffs
- Constraining the nonperturbative component of the evolution probes fundamental aspects of soft QCD

Testing Factorization Theorem

see talks of Mauro Anselmino Stefano Melis & John Collins

Fixed scale phenomenology- Stage 1+

A.V. Efremov, K. Goeke, S. Menzel, A. Metz, and P. Schweitzer, Phys. Lett. B 612, 233 (2005).
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A. Signori, Bacchetta, Radici,Schnell, JHEP 1311 (2013)
Anselmino, Boglione, O. Gonzalez, S. Melis , Prokudin JHEP 1404 (2014) ....

Stage 2 w/ evolution of various forms

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Z.-B. Kang, B.-W. Xiao, and F. Yuan, Phys. Rev. Lett. 107, 152002 (2011).
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S. M. Aybat, J. C. Collins, J.-W. Qiu, and T. C. Rogers, Phys. Rev. D 85, 034043 (2012).
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M. G. Echevarria, A. Idilbi, A. Schafer, and I. Scimemi, Eur. Phys. J. C 73, 2636 (2013).
Bacchetta & Prokuding PLB 2013
P. Sun and F. Yuan, Phys. Rev. D 88, 034016 (2013).
Aidala, Field, Gamberg, Rogers, PRD 89 (2014)
M. Echevarria, A. Idilbi, Z-B.Kang, I. Vitev Phys.Rev. D89 (2014) 074013 & aXiv recently
....

Comments on Stage 2 Fitting

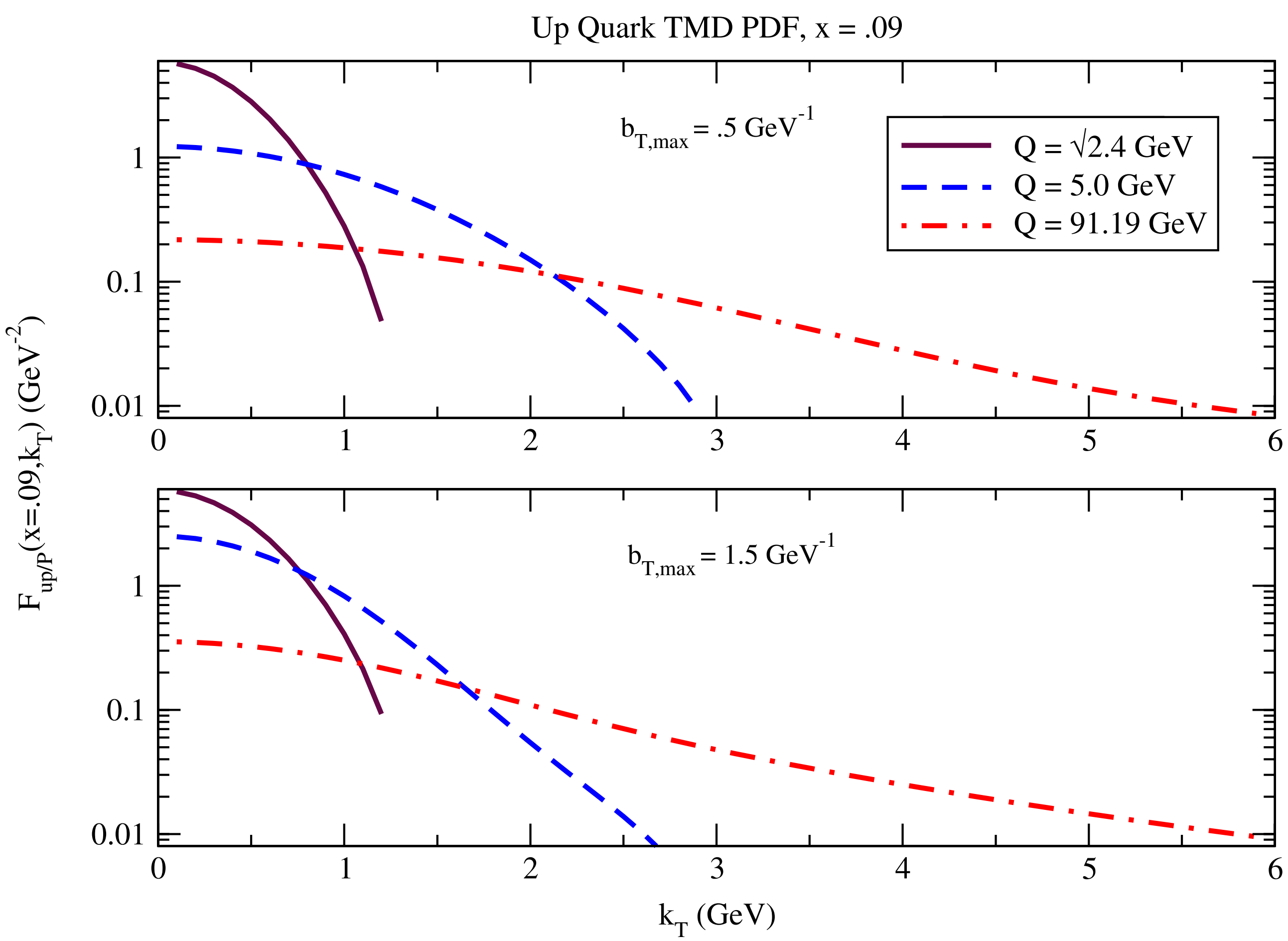
It was recently illustrated that the rapid evolution given by extrapolating the non-perturbative extractions from Drell-Yan cross sections at large Q is too fast to adequately account for data in the region of Q of order a few GeV.

The current phenomenological situation is further complicated by the observation that parametrizations obtained by extrapolating large Q fits to small Q implies suspiciously rapid evolution in the region of a few GeV, a result very clearly demonstrated in the recent work of Sun and Yuan and others ....

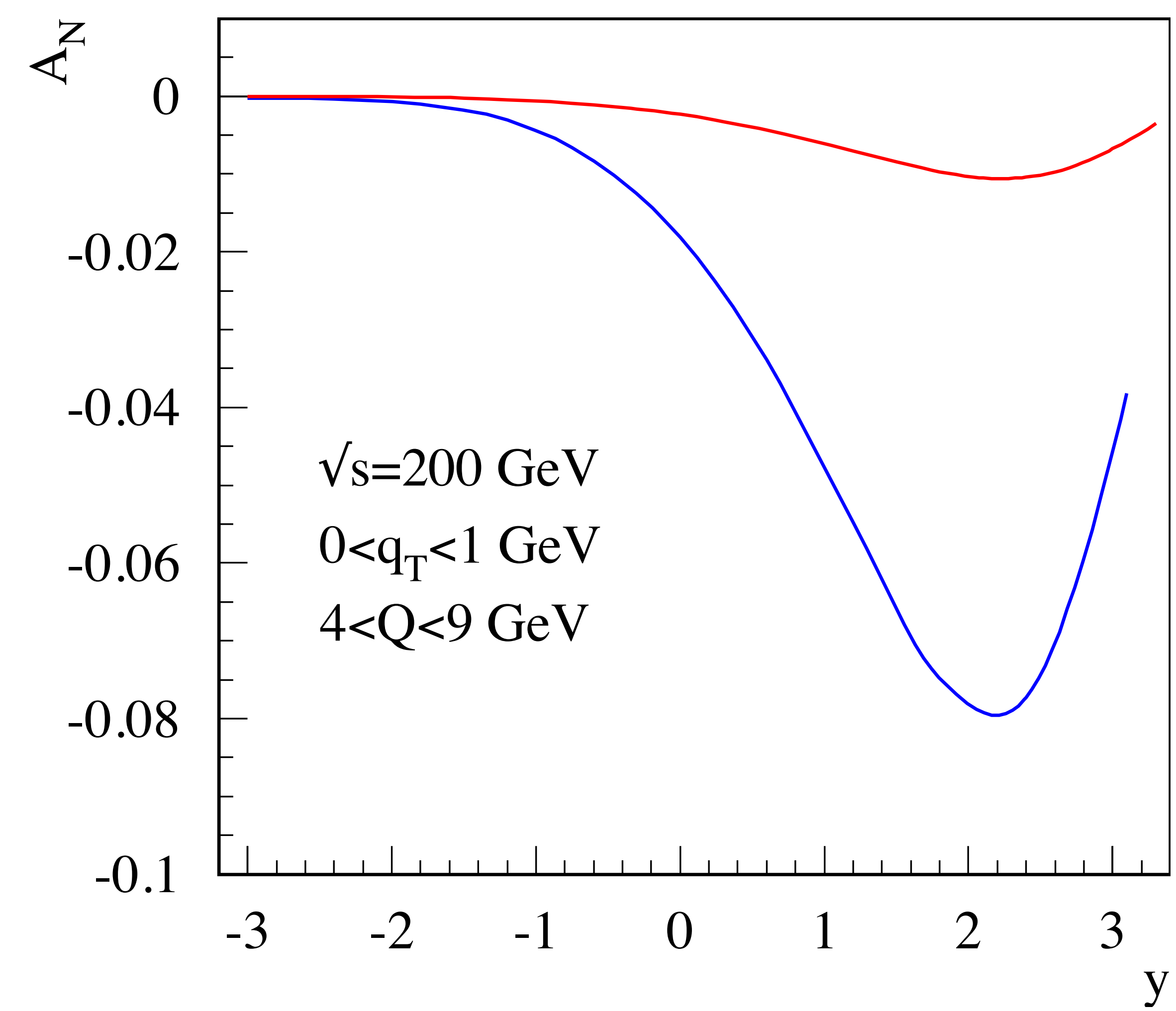
Rapid TMD Evolution ???

In the momentum-space TMD PDF, the evolution corresponds to rapid suppression at small k_T , of order $k_T \sim$ 1 GeV, with increasing Q.
The effect can be observed in the small k_T region of the curves

PHYSICAL REVIEW D **83,** 114042 (2011)

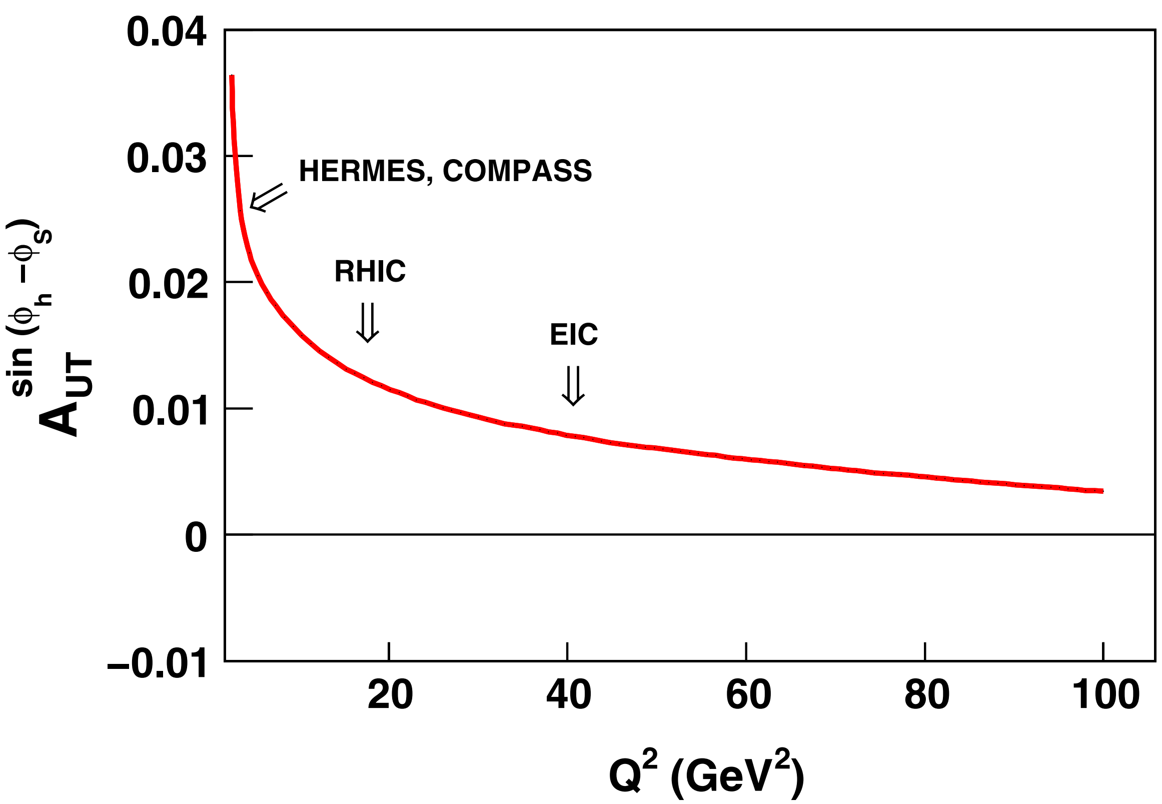


Kang QCD Evolution 2013



See also Sun & Yuan 2013 PRD, Boglione Prokudin Melis Anselmino et al ....

b_{max} = 0.5 and g_2 = 0.68 and start from Gaussian at HERMES



Sivers evolution integrated over x
Aybat Prokudin Rogers PRL 2012

Comments on Stage 2 Fitting

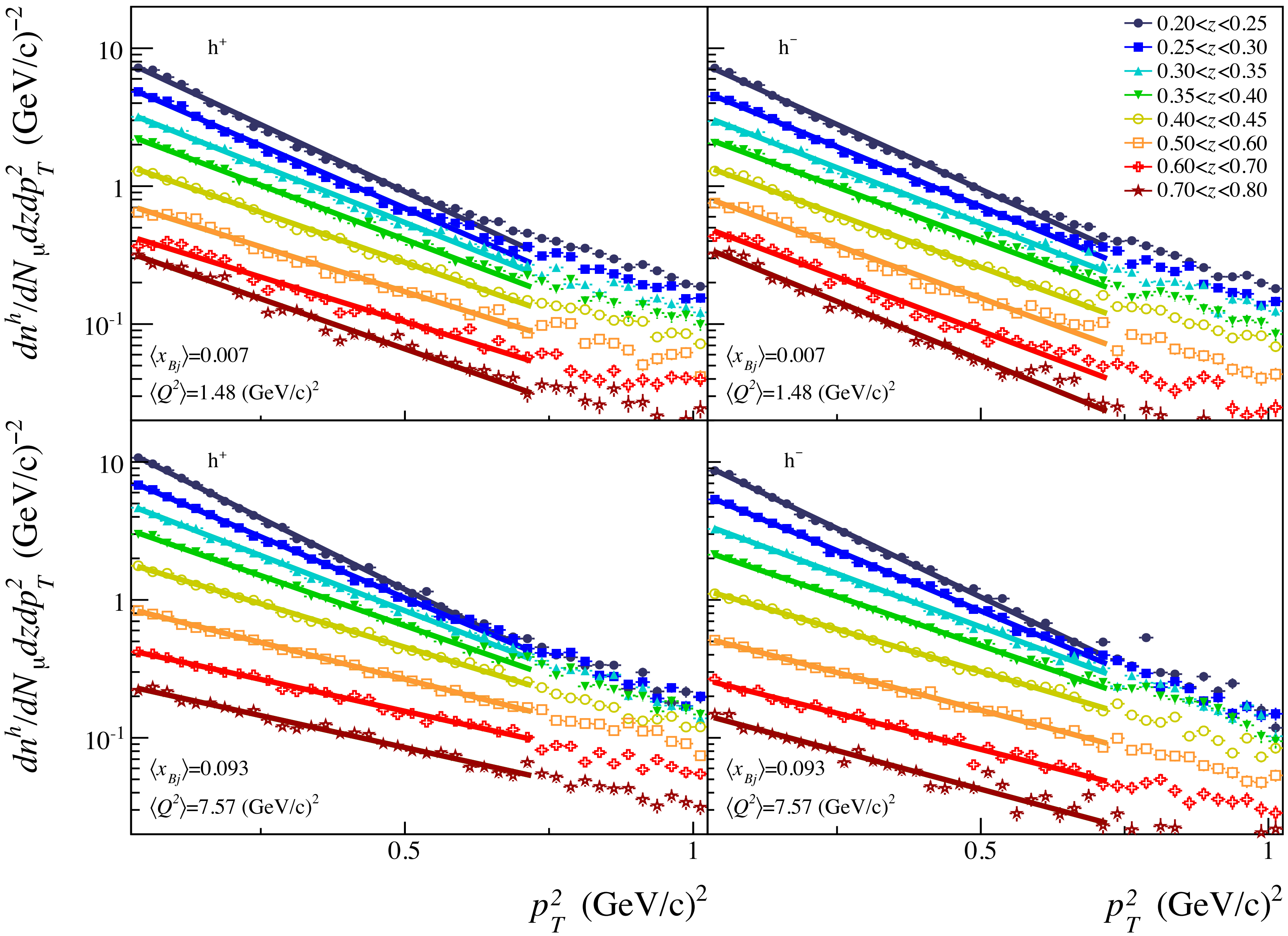
We explored this phenomena in PRD 89 2014

To maintain consistency with the general aim of extracting properties intrinsic to specific hadrons we would ideally vary Q while holding x, z, and hadron species fixed.

In experiments, however, these variables are correlated, and practical fitting becomes challenging.

We appeal to the multi-differential COMPASS data to study the variation in the multiplicity distribution with small variations in Q and roughly fixed x and z bins within the same experiment.

From COMPASS, C. Adolph et al., arXiv:1305.7317



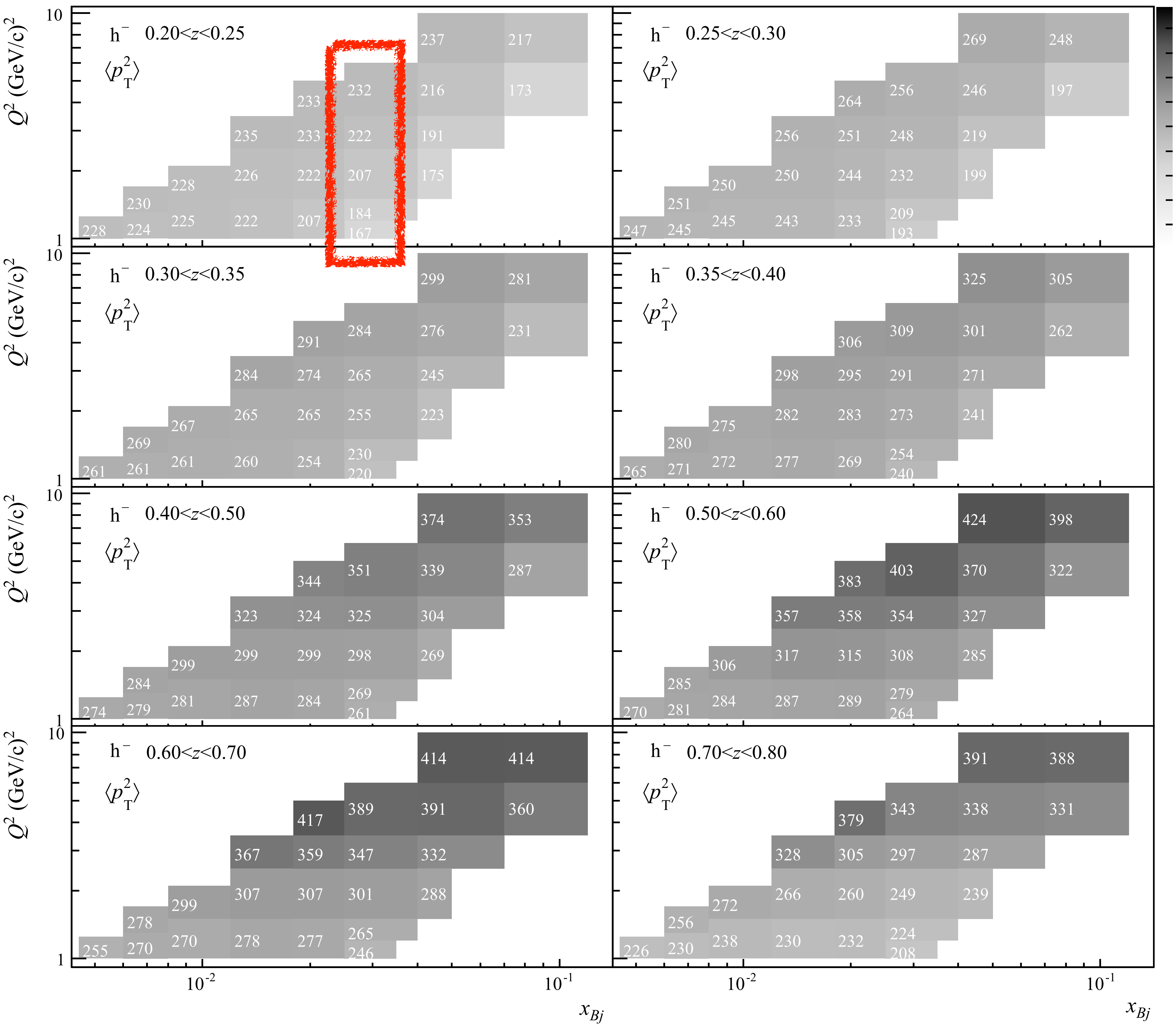
Approx. fixed x,z bins

Eur. Phys. J. C (2013) Adolph et al.

Bin	x_{bj}^{min}	x_{bj}^{max}	$\langle x_{bj} \rangle$	Q^2_{min}	Q^2_{max}	$\langle Q^2 \rangle$
1	0.0045	0.0060	0.0052	1.0	1.25	1.11
2	0.0060	0.0080	0.0070	1.0	1.30	1.14
3	0.0060	0.0080	0.0070	1.3	1.70	1.48
4	0.0080	0.0120	0.0099	1.0	1.50	1.22
5	0.0080	0.0120	0.0099	1.5	2.10	1.76
6	0.0120	0.0180	0.0148	1.0	1.50	1.22
7	0.0120	0.0180	0.0148	1.5	2.50	1.92
8	0.0120	0.0180	0.0150	2.5	3.50	2.90
9	0.0180	0.0250	0.0213	1.0	1.50	1.23
10	0.0180	0.0250	0.0213	1.5	2.50	1.92
11	0.0180	0.0250	0.0213	2.5	3.50	2.94
12	0.0180	0.0250	0.0216	3.5	5.00	4.07
13	0.0250	0.0350	0.0295	1.0	1.20	1.10
14	0.0250	0.0400	0.0316	1.2	1.50	1.34
15	0.0250	0.0400	0.0318	1.5	2.50	1.92
16	0.0250	0.0400	0.0319	2.5	3.50	2.95
17	0.0250	0.0400	0.0323	3.5	6.00	4.47
18	0.0400	0.0500	0.0447	1.5	2.50	1.93
19	0.0400	0.0700	0.0533	2.5	3.50	2.95
20	0.0400	0.0700	0.0536	3.5	6.00	4.57
21	0.0400	0.0700	0.0550	6.0	10.0	7.36
22	0.0700	0.1200	0.0921	3.5	6.00	4.62
23	0.0700	0.1200	0.0932	6.0	10.0	7.57

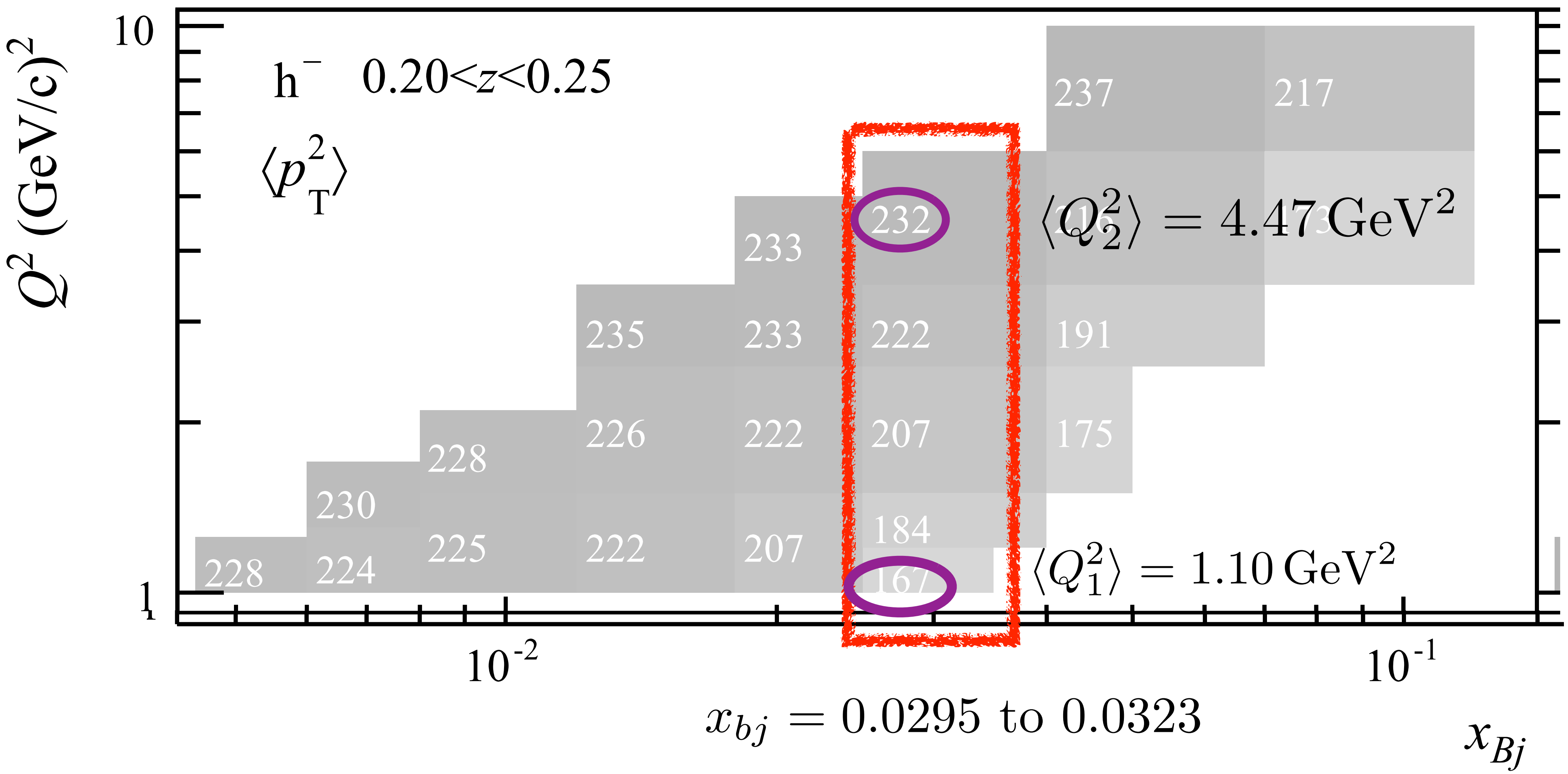
$\langle Q_1^2 \rangle$

$\langle Q_2^2 \rangle$



Window of fixed x and z

Panels are fixed z-bins & columns are fixed x bins for Q^2 vs. $\langle P_T^2 \rangle$



Quantifying the Evolution

COMPASS data for hadron multiplicities are fitted using a Gaussian form
We then quantified/bounded the P_{T} broadening

$$\tilde{\sigma}_{\text{TMD term}} \equiv \mathcal{H}(\alpha_s(Q))\tilde{F}_{H_1}(x, b_T; Q, Q^2)\tilde{D}_{H_2}(z, b_T; Q, Q^2)$$

$$\tilde{\sigma}_{\text{TMD term}} \approx \exp\left\{-\frac{b_T^2\langle P_T^2\rangle}{4}\right\}$$

$$g_{\text{PDF}}(x, b_T; b_{\max}) \propto g_{\text{FF}}(z, b_T; b_{\max}) \propto b_T^2$$

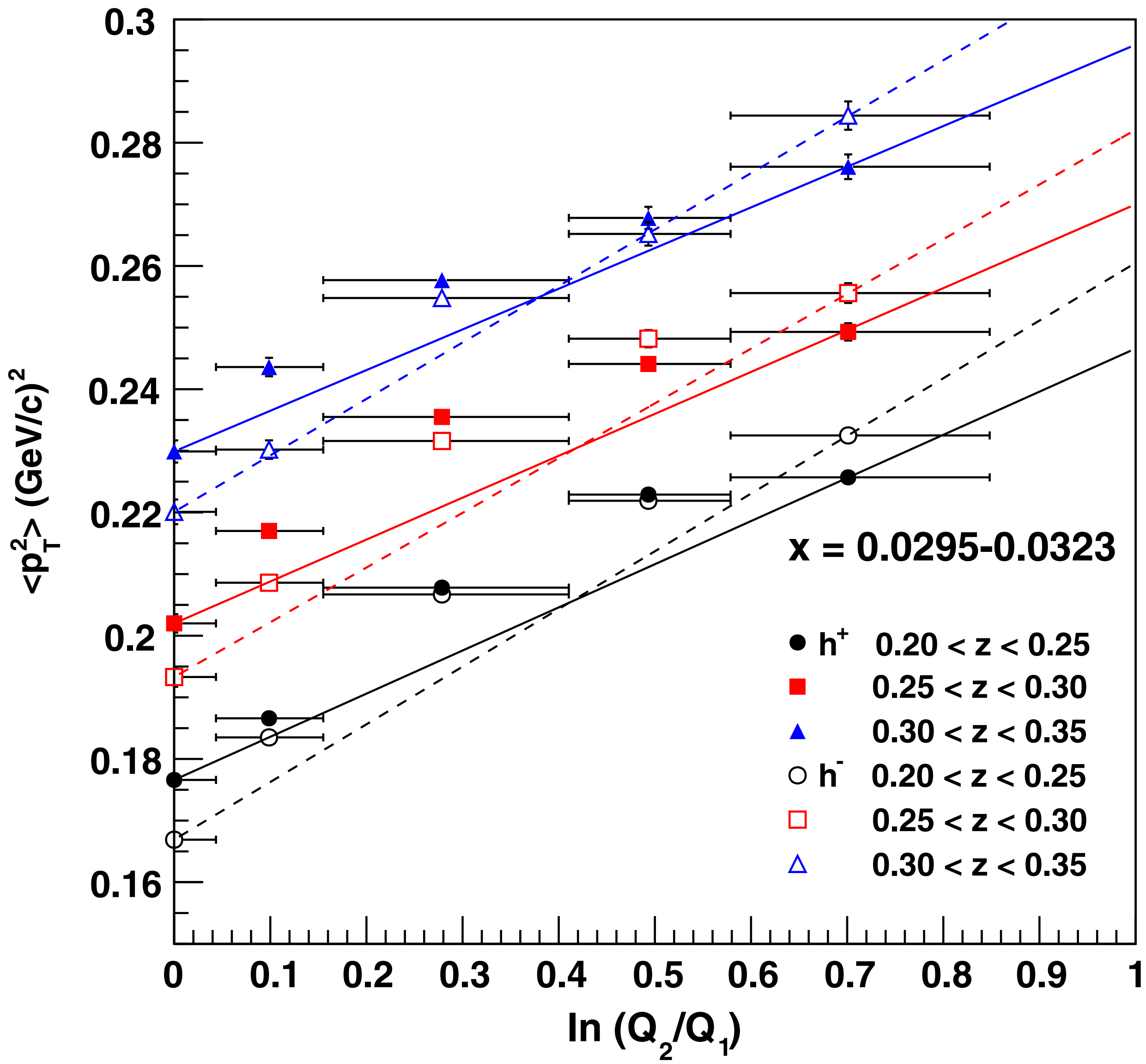
$$\left.\frac{d\ \ln\ \tilde{\sigma}_{\text{TMD term}}}{d\ \ln\ Q^2}\right|_{\mathrm{b_T dep}} = \tilde{K}(b_T; \mu_0)|_{\mathrm{b_T dep}}$$

$$\frac{d\sigma}{dP_T^2} \propto \text{F.T.} \exp\left\{-\frac{b_T^2}{4}\left(\langle P_T^2\rangle_0 + 4C_{\text{evol}}\ \ln\left(\frac{Q_2}{Q_1}\right)\right)\right\}$$

$$\Delta\langle P_T^2\rangle(Q_1, Q_2) \approx 4C_{\text{evol}}\ \ln\left(\frac{Q_2}{Q_1}\right)$$

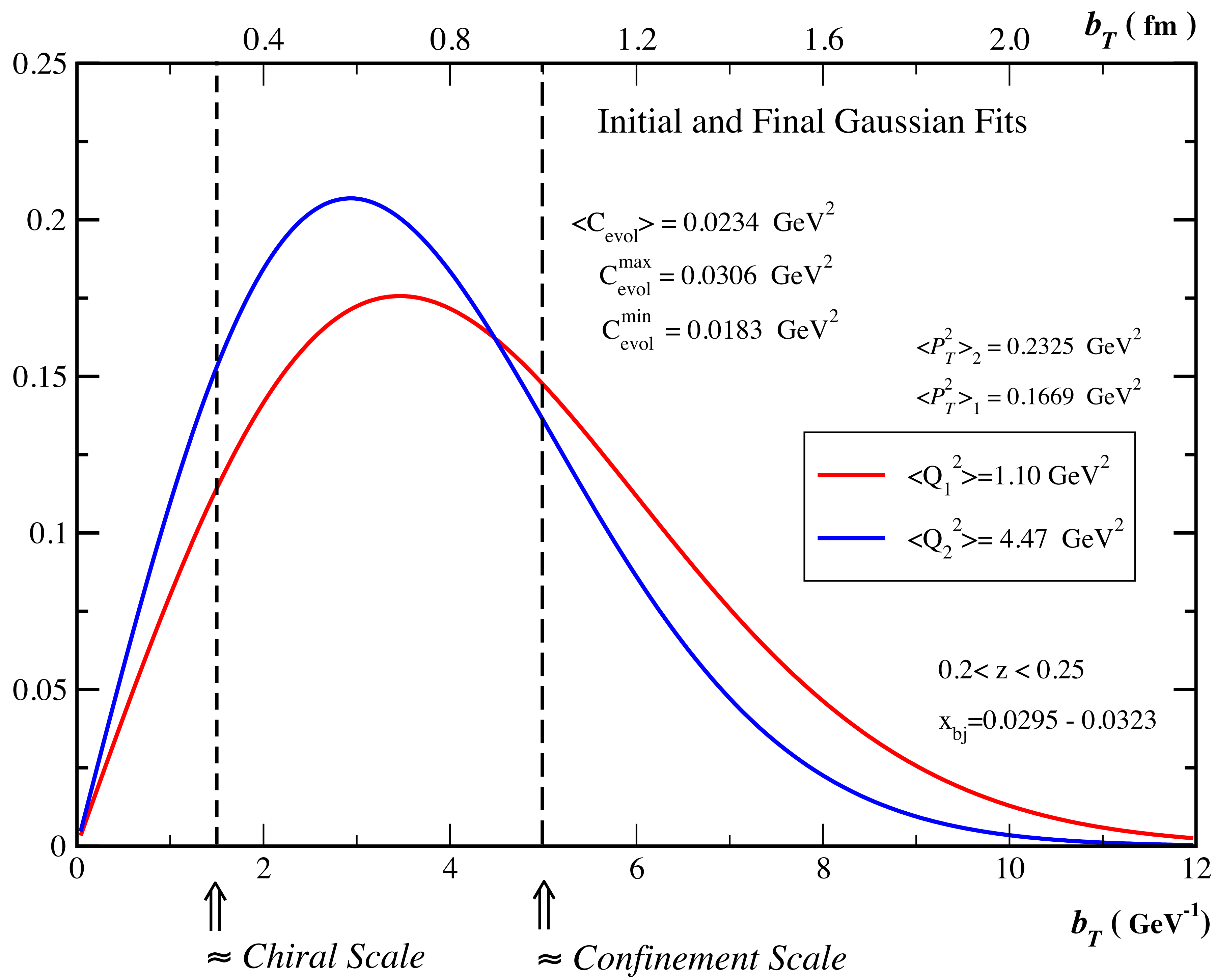
Looking for maximum range on Q to perform study

LIMITS ON TRANSVERSE MOMENTUM DEPENDENT …



$$\Delta\langle P_T^2\rangle(Q_1, Q_2) \approx 4C_{\text{evol}}\ \ln\left(\frac{Q_2}{Q_1}\right)$$

Quantify Broadening but in b-space



From the general features of Fig.we conclude that, for the differential cross section in the limit of $P_T \rightarrow 0$, the relevant range of b_T nearly dominated by the non-perturbative region of b_T for

$Q \sim 1.0$ GeV to ~ 2.0 GeV.

Comments

The only aspect of TMD factorization that we have used to parametrize broadening is CS equation & observation that one can fit COMPASS multiplicities w/ Gaussians parameterization

Specifically, we have applied it to the case of the COMPASS data for the small range of Q where the P_{T} distribution appears to remain approximately Gaussian even after evolution to obtain

$$\frac{d\sigma}{dP_T^2} \propto \text{F.T.} \exp\left\{-\frac{b_T^2}{4}\left(\langle P_T^2\rangle_0 + 4C_{\text{evol}} \ln\left(\frac{Q_2}{Q_1}\right)\right)\right\}$$

We will address the question of whether evolution is governed primarily by perturbative or nonperturbative b_{T} dependence.

N.B.While C_{evol} resembles g_2 in a quadratic approximation to g_{K}, here it should be emphasized that it is meant merely to approximate the collective effect of all the Q-dependent terms in the exponent of evolution kernal in a way consistent with CS equation, and it should not be identified at this stage with any specific perturbative or nonperturbative terms.

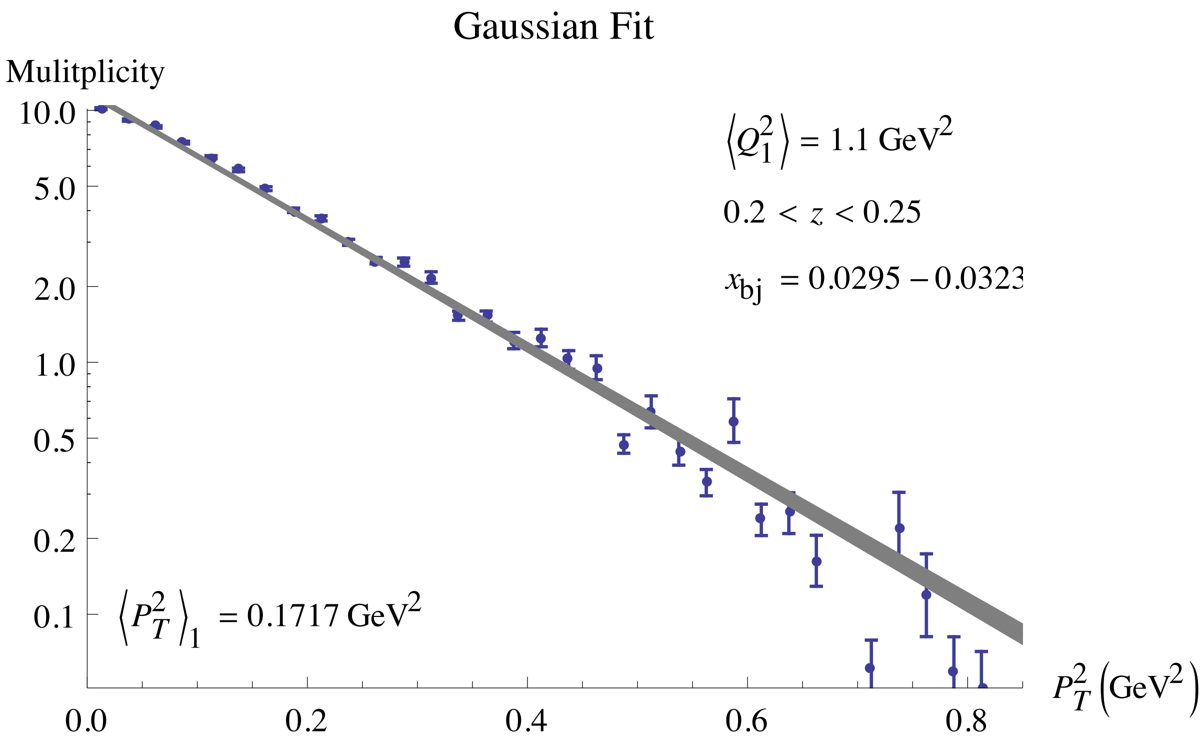
source of error

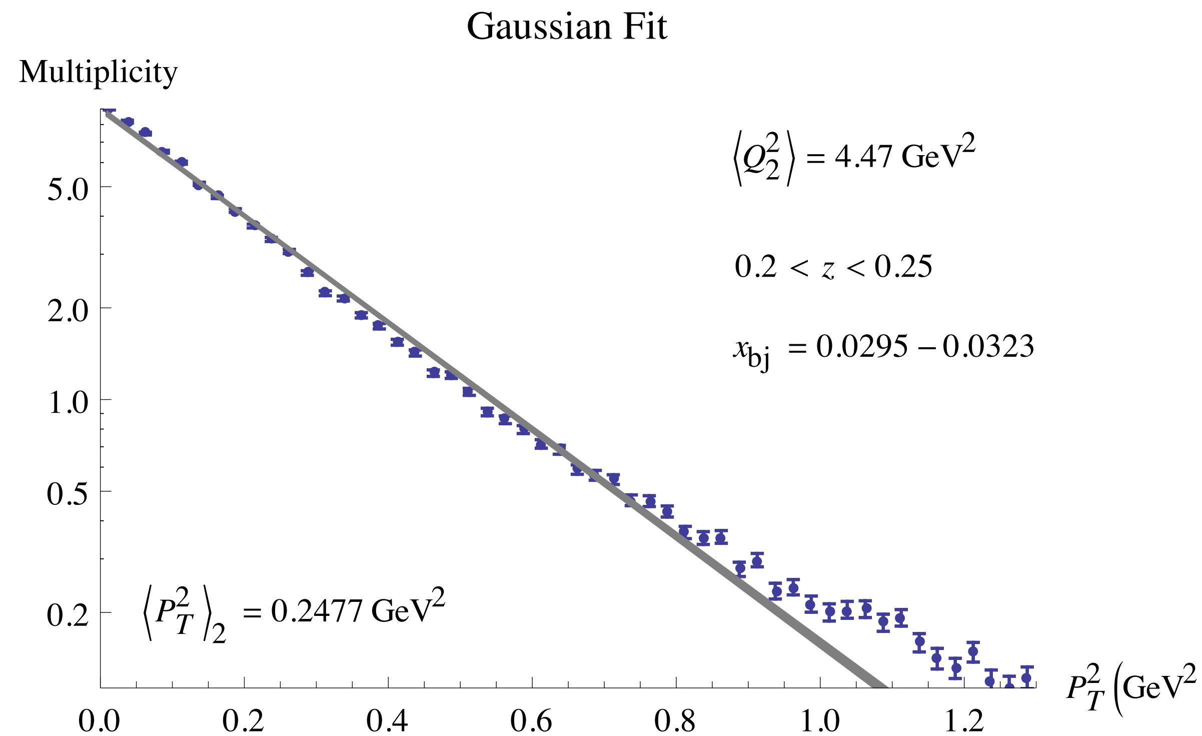
The cutoff at $P_T = 0.85$ GeV in the fits of COMPASS Data where the Gaussian description starts to break down.

One could speculate that including more of the large P_T tail might result in an enhanced relative contribution from small bT .

To address this, we have performed our own fit of the Gaussian form using the same data from COMPASS DATA that gave the two curves for $Q = 1.049$ GeV and $Q = 2.114$ GeV Fig. but now for the entire range of P_T (up to $P_T \sim 1.0$ GeV).

Refit-momentum space





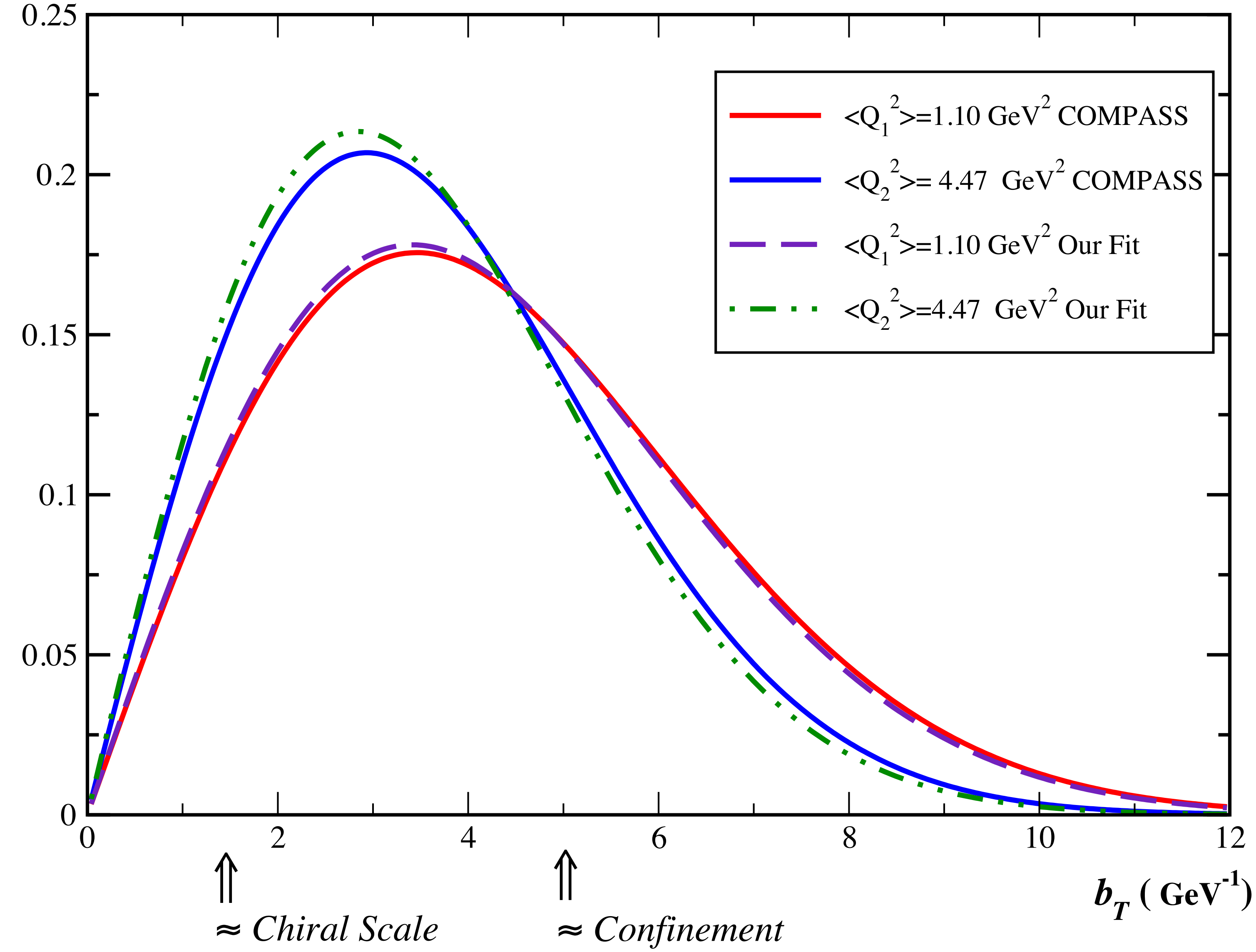
$$\langle P_T^2 \rangle^{\text{New Fits}}_{Q_1=1.049\,\text{GeV}} = 0.1717 \pm 0.0011\,\text{GeV}^2\,; \qquad \langle P_T^2 \rangle^{\text{New Fits}}_{Q_2=2.114\,\text{GeV}} = 0.2477 \pm 0.0008\,\text{GeV}^2$$

Was

$$\langle P_T^2 \rangle^{\text{Old Fits}}_{Q_1=1.049\,\text{GeV}} = 0.1669 \pm \ldots \qquad \langle P_T^2 \rangle^{\text{Old Fits}}_{Q_1=2.114\,\text{GeV}} = 0.2325 \pm \ldots\ \text{GeV}^2$$

Refit b space

Little change when we include "large" P_T data



The solid red and blue curves are the same as those in previous Fig. in where fit is restricted to region of $P_T \leq 0.85$ GeV.

Purple dashed and green dot-dashed curves are from the refit Gaussian curves above that use all P_T and correspond to Eq. (32) with the initial and final P_T from Eq. (34)

$$\langle P_T^2 \rangle^{\text{New Fits}}_{Q_1=1.049\,\text{GeV}} = 0.1717 \pm 0.0011\,\text{GeV}^2\,; \qquad \langle P_T^2 \rangle^{\text{New Fits}}_{Q_2=2.114\,\text{GeV}} = 0.2477 \pm 0.0008\,\text{GeV}^2$$

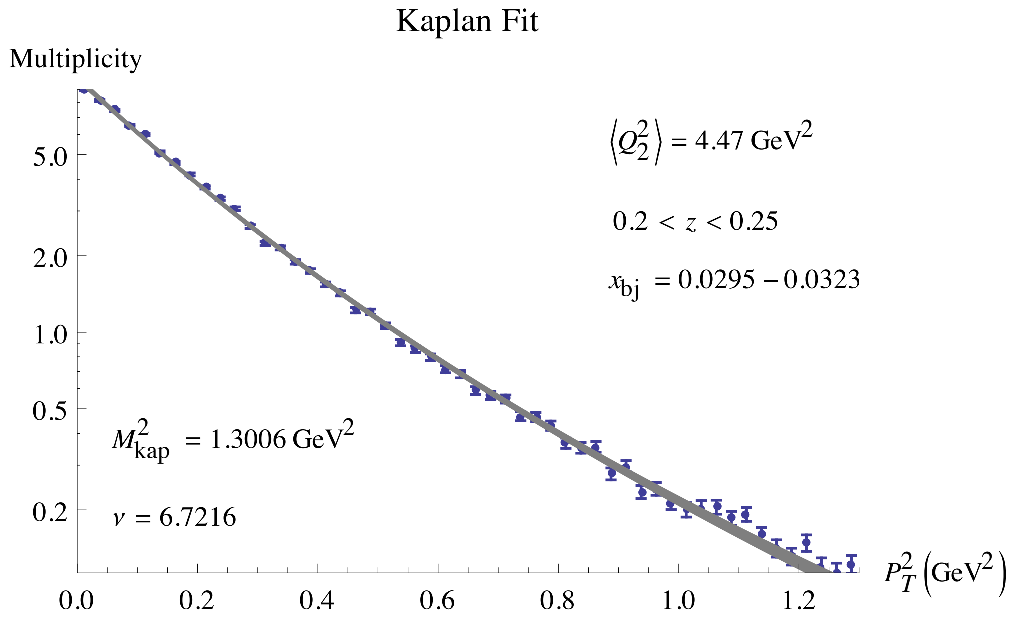
$$\langle P_T^2 \rangle^{\text{Old Fits}}_{Q_1=1.049\,\text{GeV}} = 0.1669 \pm \ldots \qquad \langle P_T^2 \rangle^{\text{Old Fits}}_{Q_1=2.114\,\text{GeV}} = 0.2325 \pm \ldots\ \text{GeV}^2$$

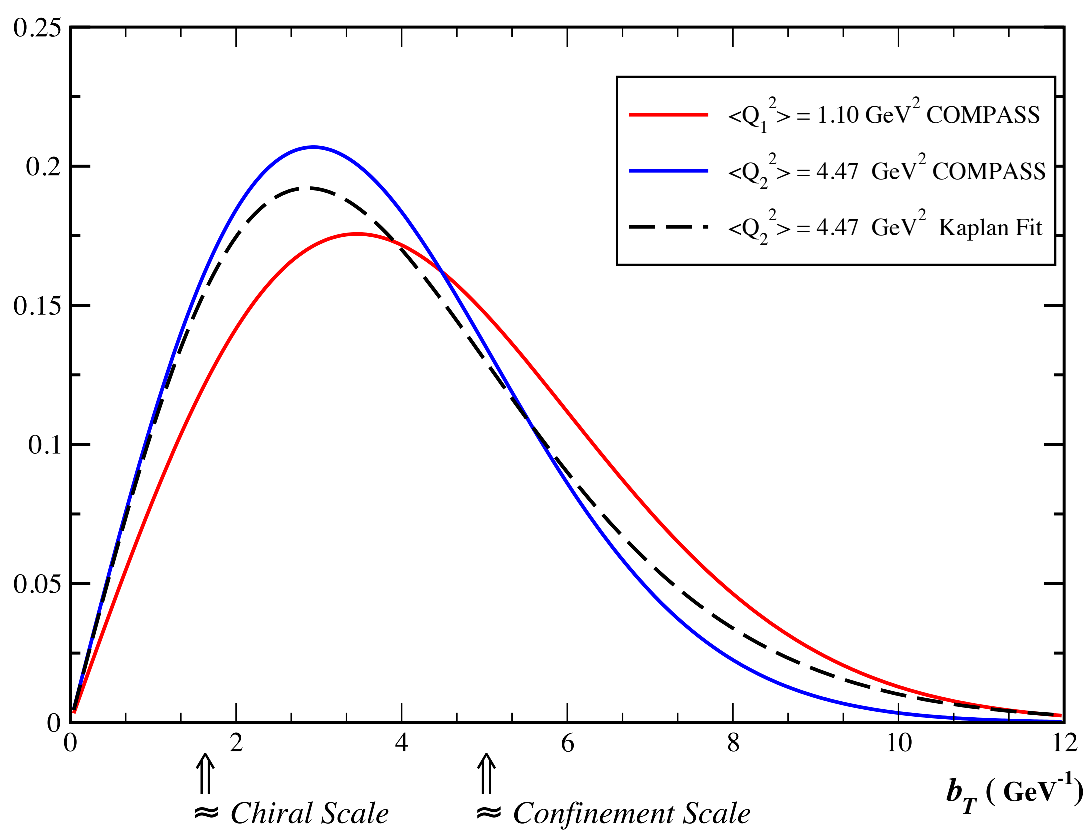
A critique could be made regarding the use of a Gaussian form on the grounds that analyticity considerations   imply a power law fall-off for the large PT behavior of TMD correlation functions.

Moreover, a power law behavior 1/PT2 (up to logarithmic corrections and the effects of evolution of collinear PDFs) is a prediction of pQCD .

This power law behavior is tied to singular behavior in the transverse position at small bT.

The true large PT behavior of the TMD functions is not directly meaningful at very large PT , since TMD factorization (without the Y term) is inapplicable once the PT is comparable with Q. Clearly, the Y -term will be need be incorporated in the future to deal with these issues.





$$\frac{d\sigma}{dP_T^2} \propto \frac{1}{\left(1 + \frac{P_T^2}{M_{\rm kap}^2}\right)^{\nu}}$$

FT

$$\frac{2 b_T^{\nu} M_{\rm kap}}{\Gamma(\nu)} \left(\frac{M_{\rm kap}}{2}\right)^{\nu} K_{1-\nu}\left(b_T M_{\rm kap}\right)$$

The black dashed curve shows the b_T space function for Q_2 = 2.114GeV. This corresponds to the fit obtained in transverse momentum space using the Kaplan function in momentum space

The fits themselves yield parameters M^2 = 1.3006 GeV^2 and ν = 6.7216.

For comparison, we have again included the solid red and blue curves corresponding to the original fits obtained by the COMPASS collaboration at $\langle Q_1 \rangle$ = 1.049 GeV and ⟨Q⟩ = 2.114 GeV, respectively

Comparison w TMD Evolution

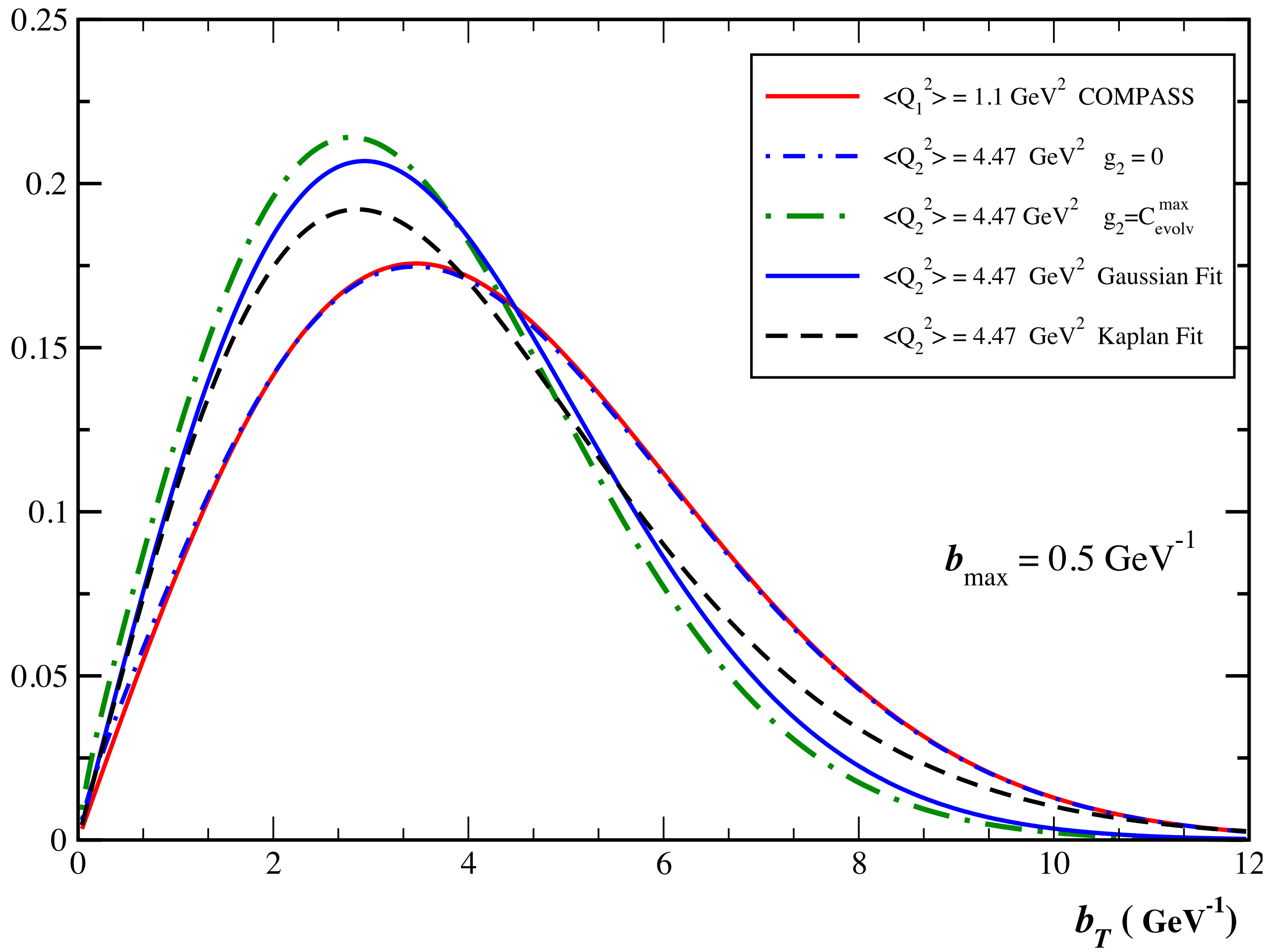
Next, we examine the evolved formula to estimate how well it matches the change in widths of the Gaussian fits observed in under different assumptions for g_K

$$b_T \tilde{\sigma}(b_T, \dots) =$$

$$\frac{b_T}{N(Q)} \exp\Bigg\{ -g_{\rm PDF}(x, b_T; b_{\rm max}) - g_{\rm FF}(z, b_T; b_{\rm max}) - 2 g_K(b_T; b_{\rm max}) \ln\left(\frac{Q}{Q_0}\right) + 2 \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_{\rm PDF}(\alpha_s(\mu'); 1) + \gamma_{\rm FF}(\alpha_s(\mu'); 1) - 2 \ln\left(\frac{Q}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right] \Bigg\}$$

We will require that for Q = Q_0=1.049 GeV, AND

$b_T \tilde{\sigma}(b_T, \dots)$ reduces to the Q = 1.049 GeV COMPASS Gaussian fit



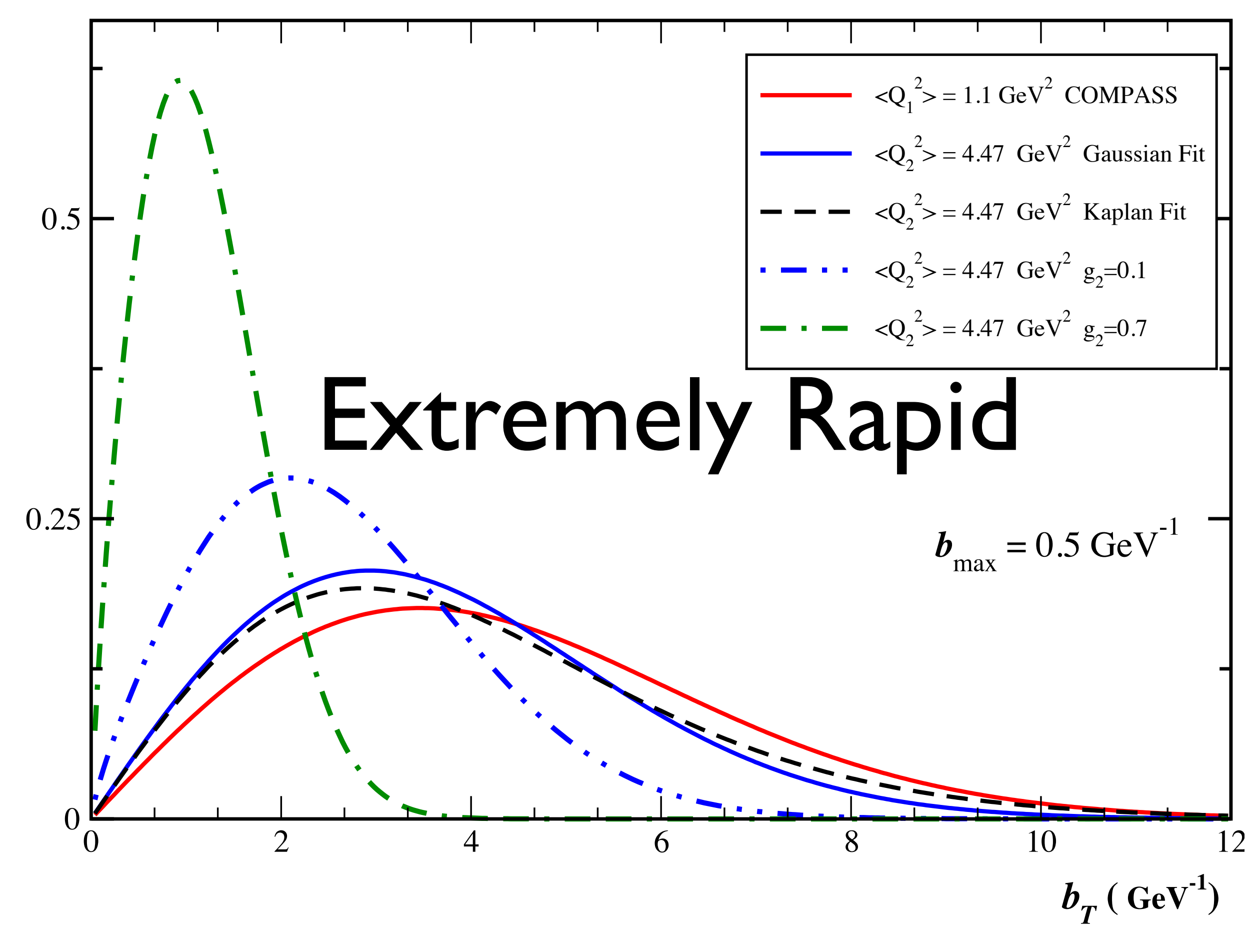
$$g_2(b_{\max}) = 0$$

very weak evolution

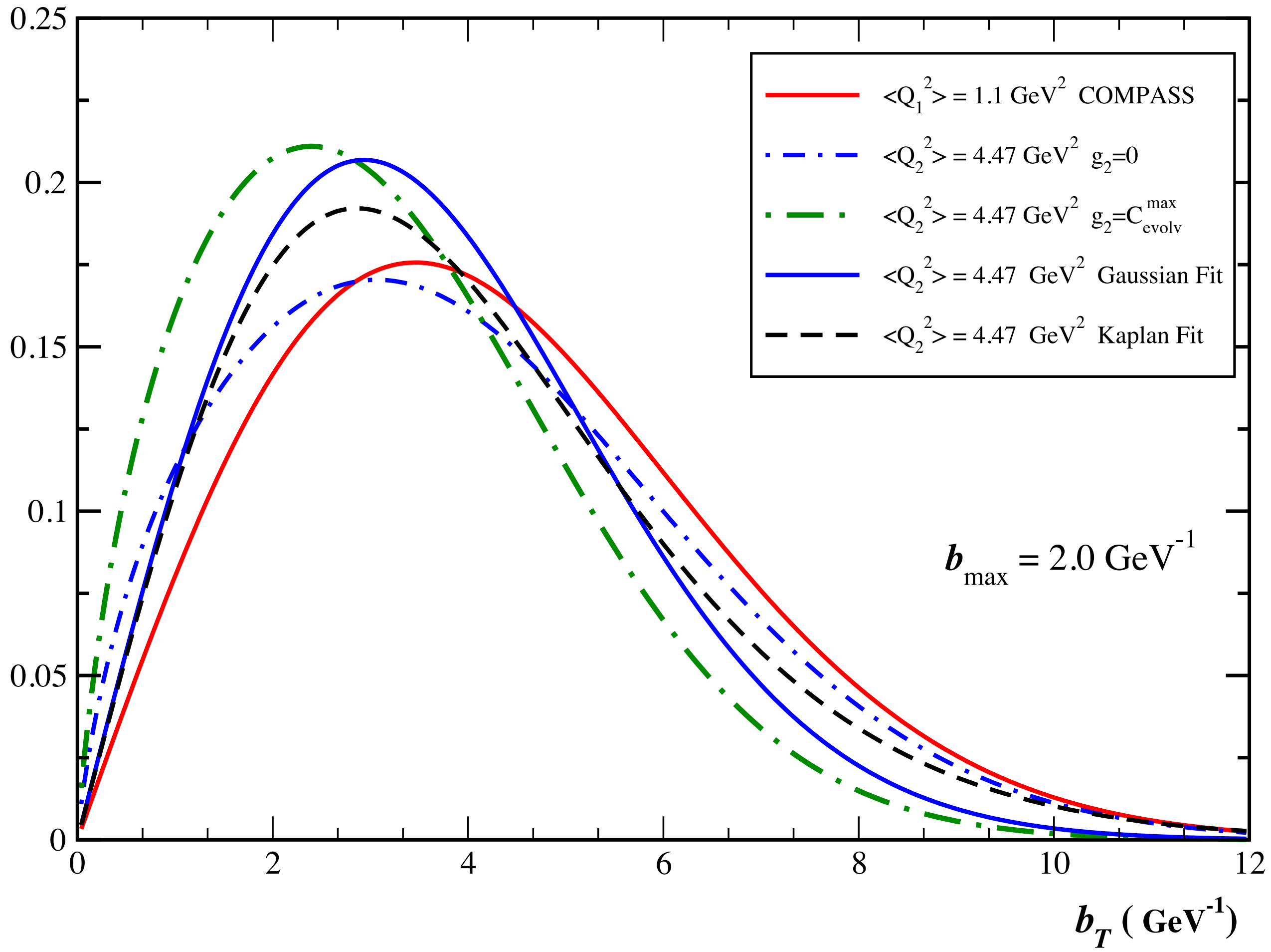
$$g_2(b_{\max}) \lesssim C^{\max}_{\text{evolv}}$$

vs.

$$g_2(b_{\max}) \geq 0.1\,\text{GeV}^2$$



Extremely Rapid



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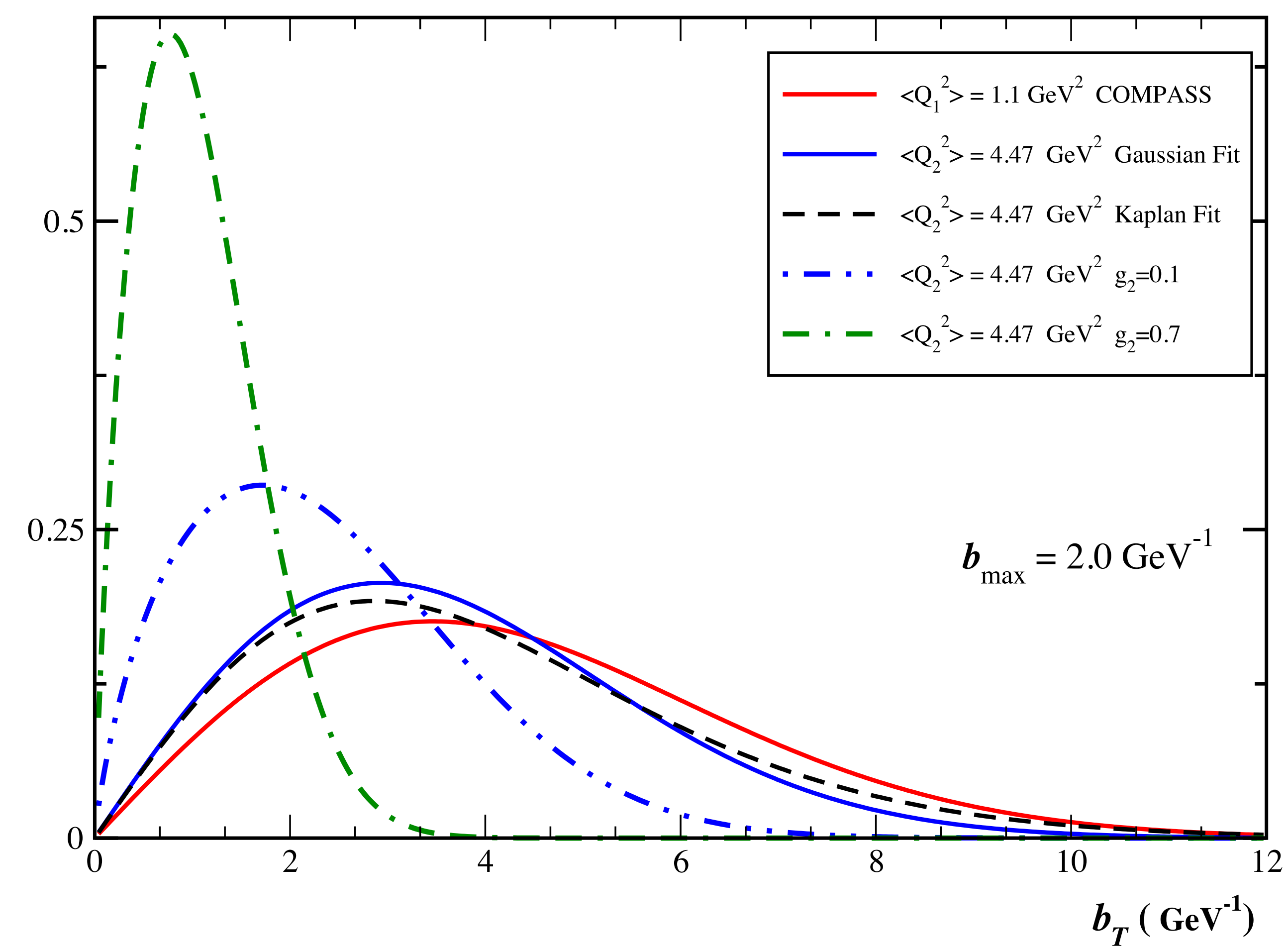
vs.

$$g_2(b_{\max}) \geq 0.1\,\mathrm{GeV}^2$$



COMMENTS

- Thus, if we demand the Gaussian ansatz in for the form of $g_K\,(b_T\,;\,b_{max})$ for all b_T, then we estimate that the true value of g_2, at least for the kinematics of our fit must lie roughly in the range of $0 < g_2 < 0.03\ \mathrm{GeV}^2$.
- Because of the strong universality of $g_K\,(b_T\,;\,b_{max})$, these results seem on the surface to indicate a discrepancy between the low Q data and detailed and successful fits of the past that focus on larger Q, which tend to find $g_2 > 0.1\ \mathrm{GeV}^2$

Setup g_K to respect *DY* fits

$$g_K(b_T; b_{\max}) = \frac{g_2(b_{\max}) b_{\mathrm{NP}}^2}{2} \ln\left(1 + \frac{b_T^2}{b_{\mathrm{NP}}^2}\right)$$

$$b_T \ll b_{\mathrm{NP}}$$

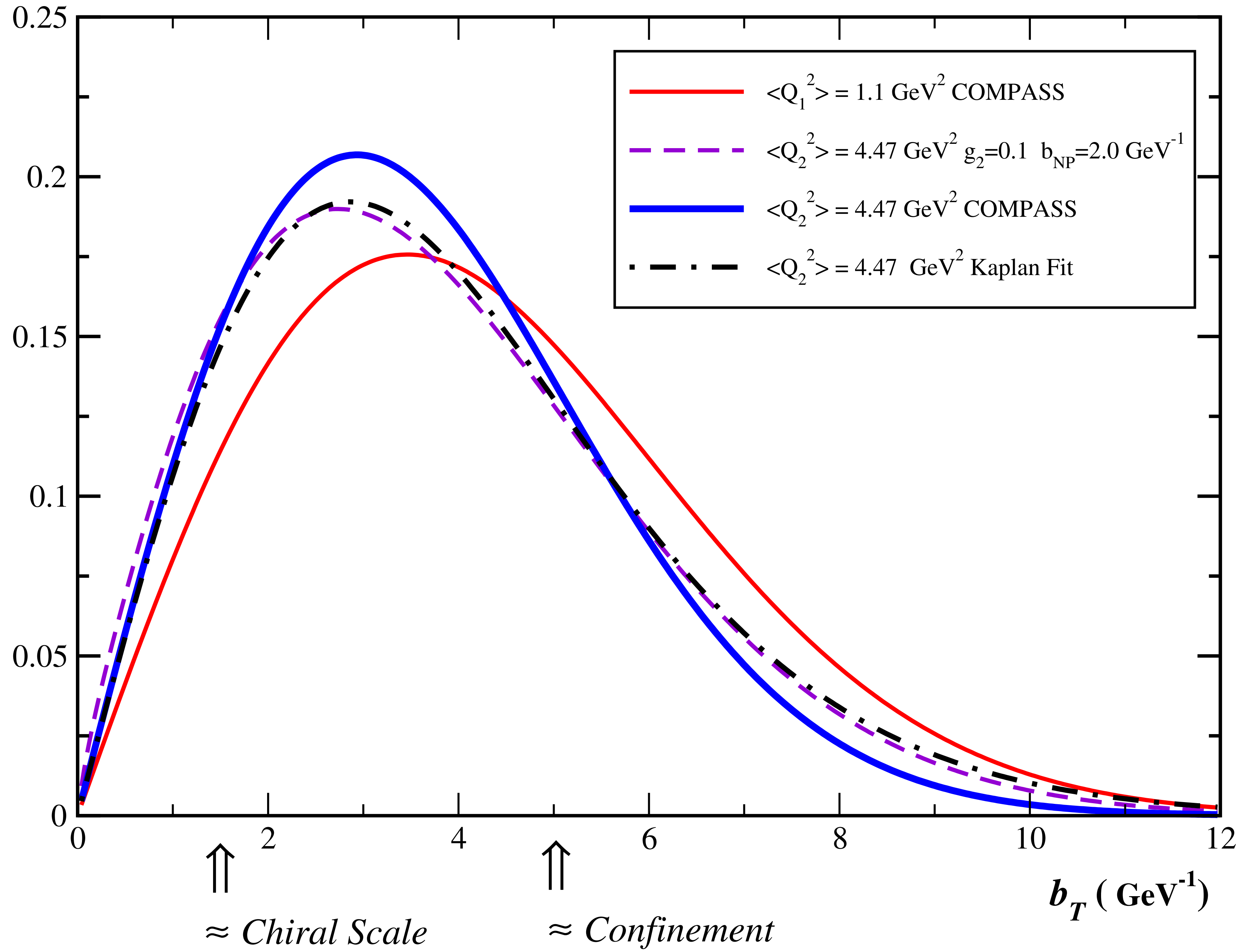
$$g_K(b_T; b_{\max}) \approx g_2(b_{\max}) \frac{1}{2} b_T^2 - g_2(b_{\max}) \frac{1}{4 b_{\mathrm{NP}}^2} b_T^4 + \ldots$$

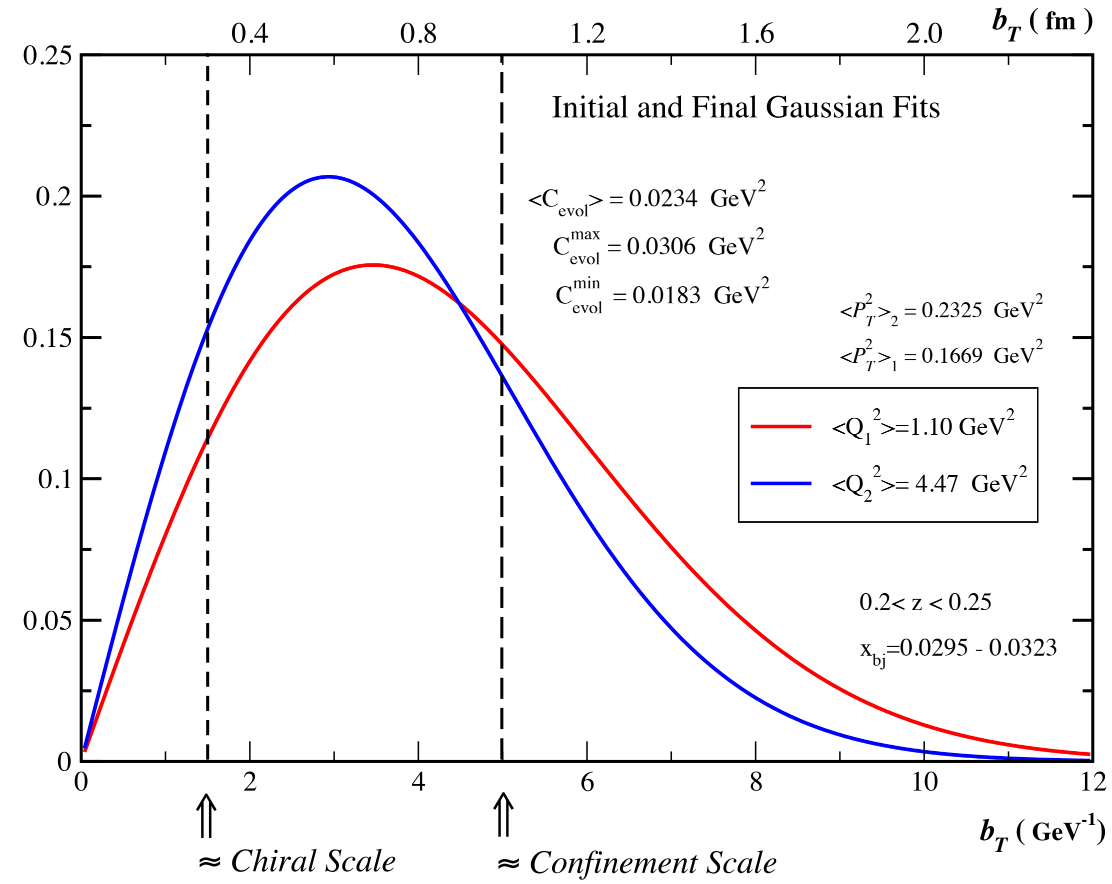
P. M. Nadolsky, D. Stump, and C. Yuan, Phys. Rev. D **61**, 014003 (1999).
P. M. Nadolsky, D. Stump, and C. Yuan, Phys. Rev. D **64**, 114011 (2001).

$b_{\max} = 0.5\,\mathrm{GeV}$, $g_2 = 0.1\,\mathrm{GeV}^2$ and $b_{\mathrm{NP}} = 2.0\,\mathrm{GeV}^{-1}$

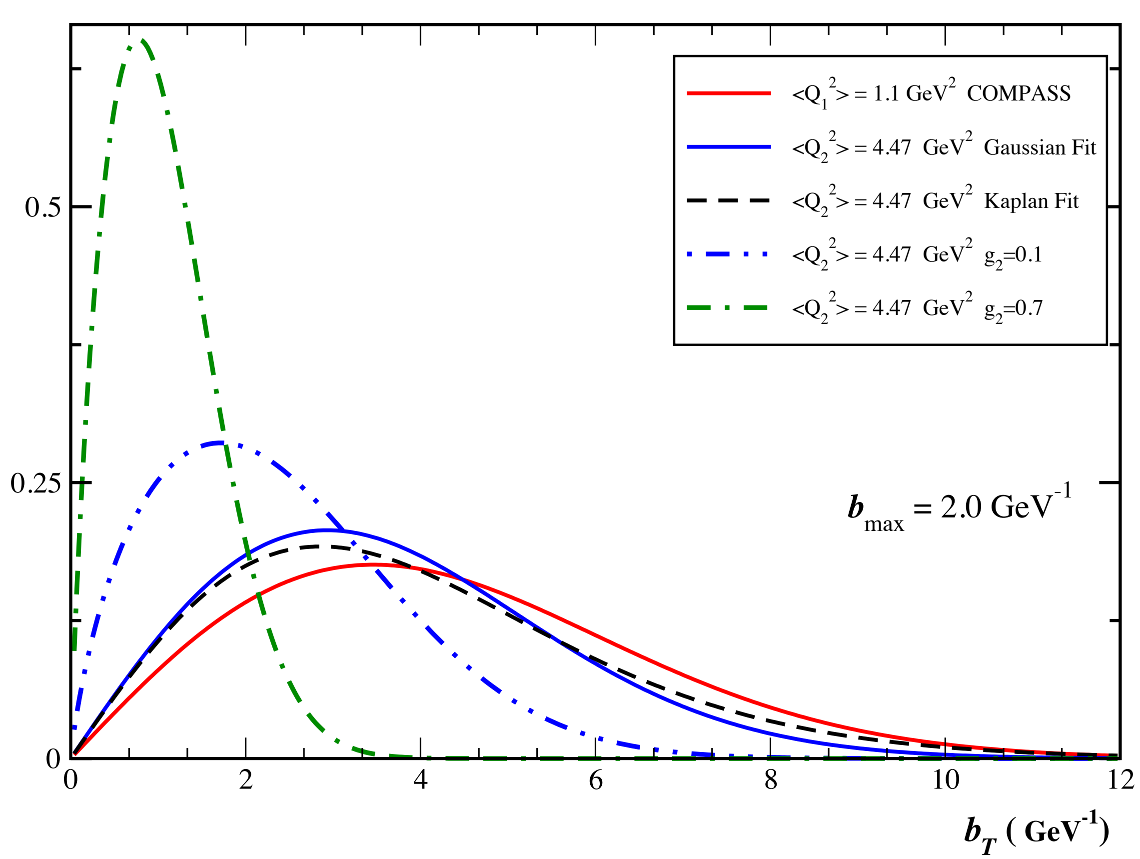
$b_{\max} = 0.5\,\text{GeV}$, $g_2 = 0.1\,\text{GeV}^2$ and $b_{NP} = 2.0\,\text{GeV}^{-1}$

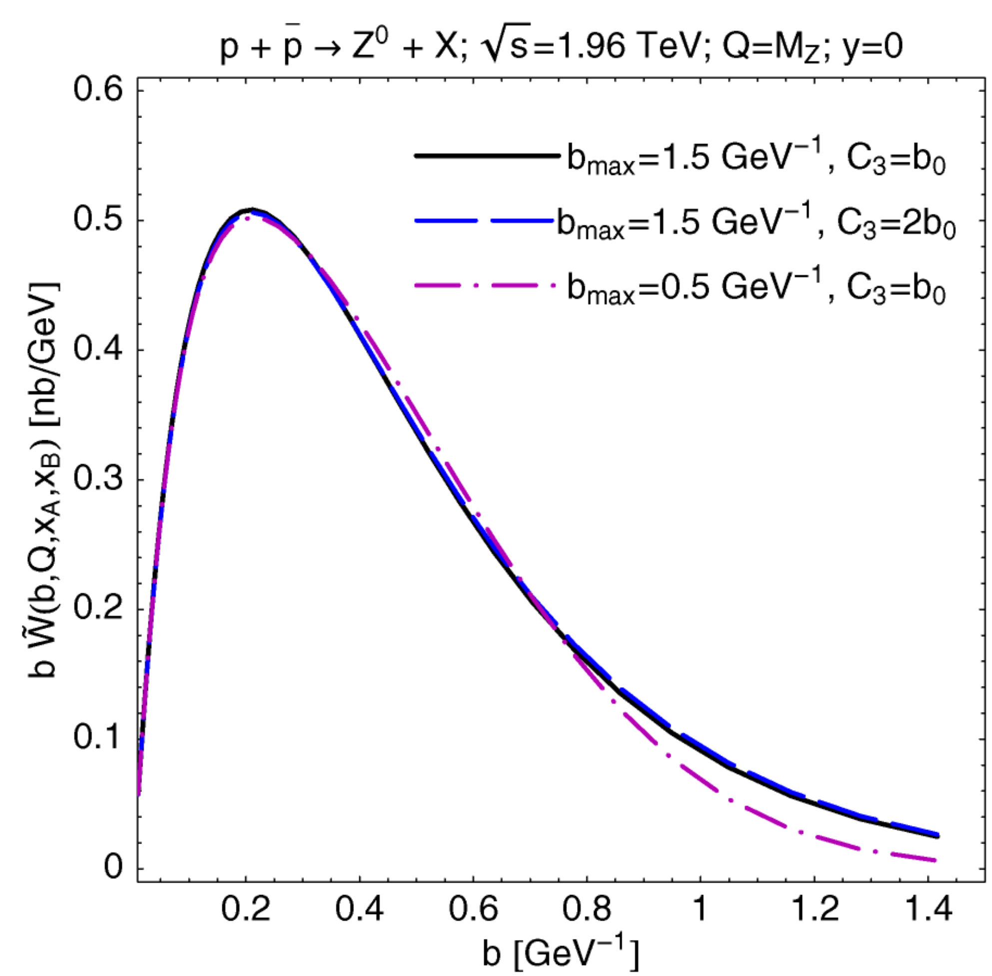
$$\frac{b_T}{N(Q)} \exp\Big\{ -g_{\text{PDF}}(x, b_T; b_{\max}) - g_{\text{FF}}(z, b_T; b_{\max}) - 2 g_K(b_T; b_{\max}) \ln\left(\frac{Q}{Q_0}\right) + 2 \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_{\text{PDF}}(\alpha_s(\mu'); 1) + \gamma_{\text{FF}}(\alpha_s(\mu'); 1) - 2 \ln\left(\frac{Q}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right] \Big\}$$





See, for example, Fig. of Konychev and Nadolsky and compare this with Fig. 3, where contributions from bT < 2.0 GeV^{-1} dominate.





Comments Factorization

- This strong form of universality is, an important basic test of the TMD factorization theorem. It is related to the soft factors—the vacuum expectation values of Wilson loops—that are needed in the TMD definitions for consistent factorization with a minimal number of arbitrary cutoffs.

- Constraining the nonperturbative component of the evolution probes fundamental aspects of soft QCD.

- CSS/JCC TMD-factorization formalism is tailored to the treatment of the individual, well-defined operator definitions for the TMDs, and it maps directly onto the partonic picture displayed in the TMD factorization

Conclusions

- Even with the small variations in Q discussed in this paper, however, one is able to constrain general properties of $g_K(b_T; b_{\max})$
- That the data are atrelatively low Q helps especially to constrain the form ofthe nonperturbative evolution function $g_K(b_T; b_{\max})$
- We find much greater sensitivity to the details of NP large b_{T} structure rather than evidence that nonperturbative contributions to evolution are unnecessary
- By accounting for nonperturbative behavior from at large b_{T} we find it is not difficult to reconcile past large Q fits e.g. from DY and SIDIS data