

# Status of DVCS, DVMP and GPDs

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## Outline:

- The handbag, power corrections and GPDs
- Analysis of meson electroproduction
- DVCS
- The GPD  $\tilde{H}$
- The GPD  $E$
- Ji's sum rule
- Summary

# Can we apply the asymp. fact. formula ?

rigorous proofs of collinear factorization in generalized Bjorken regime:

for  $\gamma_L^* \rightarrow V_L(P)$  and  $\gamma_T^* \rightarrow \gamma_T$  amplitudes

( $Q^2, W \rightarrow \infty, x_{Bj}$  fixed)

$$\mathcal{K} = \int dx K(x, \xi, t) \mathcal{H}(x, \xi, Q^2)$$

Radyushkin, Collins et al, Ji et al

possible power corrections not under control  $\implies$

unknown at which  $Q^2$  asymptotic result can be applied

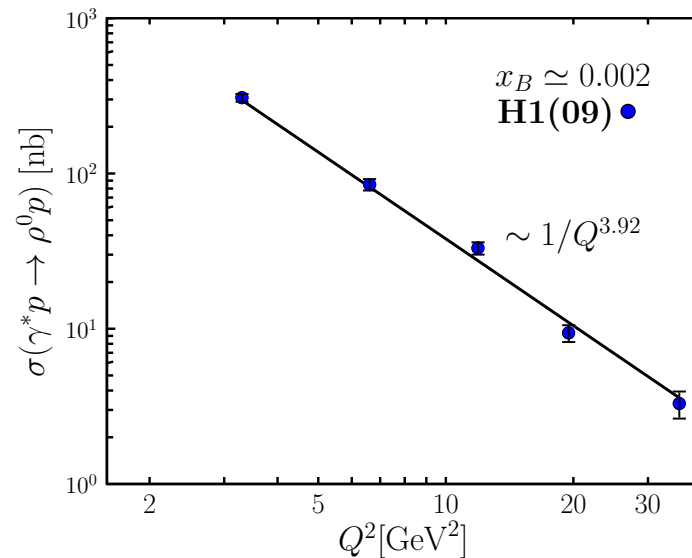
e.g.  $\rho^0$  production:  $\sigma_L/\sigma_T \propto Q^2$

experiment:  $\simeq 2$  for  $Q^2 \leq 10 \text{ GeV}^2$

$\gamma_T^* \rightarrow V_T$  transitions substantial

$\sigma_L \propto 1/Q^6$  at fixed  $x_{Bj}$

modified by  $\ln^n(Q^2)$  experiment:



# Two concepts to solve problem with $\gamma_L^* \rightarrow V_L$ ampl.:

at small  $x_B$                       only GPD  $H$  relevant

Mueller et al (11,13): absorb effects into GPDs

⇒ strong  $\ln^n(Q^2)$  from evolution of GPDs

only shown for HERA data (i.e. at  $W \simeq 90$  GeV) with  $H_{g,sea}$

- can this be extended to lower  $W$ ?

fits to only DVCS or to DVCS+DVMP data from HERA lead to different GPDs

Goloskokov-K (06): take into account transverse size of meson,

i.e. power corrections  $1/Q^n$  to subprocess  $\gamma_L^* q(g) \rightarrow V_L q(g)$

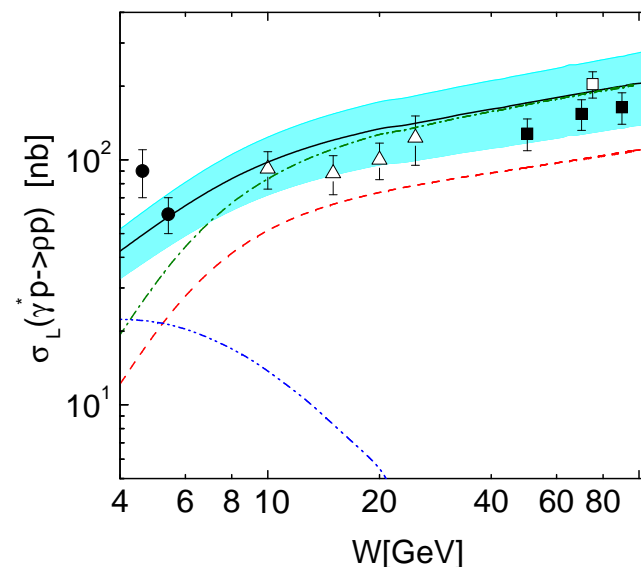
H for gluon, sea and valence

$W \gtrsim 4$  GeV             $Q^2 = 4$  GeV<sup>2</sup>

gluon + sea, gluon

valence + (gluon + sea)-valence

interference

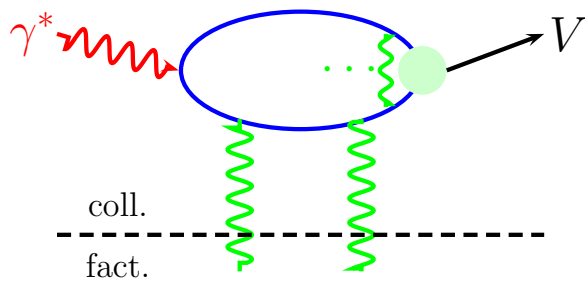


# The subprocess amplitude for DVMP

mod. pert. approach - quark trans. momenta in subprocess

(emission and absorption of partons from proton collinear to proton momenta)

transverse separation of color sources  $\implies$  gluon radiation



LO pQCD

+ quark trans. mom.

+ Sudakov supp.

$\implies$  asymp. fact. formula

(lead. twist) for  $Q^2 \rightarrow \infty$

Sudakov factor [Sterman et al\(93\)](#)

$$S(\tau, \mathbf{b}, Q^2) \propto \ln \frac{\ln(\tau Q / \sqrt{2} \Lambda_{\text{QCD}})}{-\ln(b \Lambda_{\text{QCD}})} + \text{NLL}$$

resummed gluon radiation to NLL  $\implies \exp[-S]$

provides rather sharp cut-off at  $b = 1/\Lambda_{\text{QCD}}$

$$\mathcal{H}_{0\lambda,0\lambda}^M = \int d\tau d^2b \hat{\Psi}_M(\tau, -\vec{b}) e^{-S} \hat{\mathcal{F}}_{0\lambda,0\lambda}(\bar{x}, \xi, \tau, Q^2, \vec{b})$$

$\hat{\Psi}_M \sim \exp[\tau \bar{\tau} b^2 / 4a_M^2]$  LC wave fct of meson

$\hat{\mathcal{F}}$  FT of hard scattering kernel

e.g.  $\propto 1/[k_\perp^2 + \tau(\bar{x} + \xi)Q^2/(2\xi)] \implies$  Bessel fct

Sudakov factor generates series of power corr.  $\sim (\Lambda_{\text{QCD}}^2/Q^2)^n$

(from region of soft quark momenta  $\tau, \bar{\tau} \rightarrow 0$ )

from intrinsic  $k_\perp$  in wave fct: series  $\sim (\langle k_\perp^2 \rangle / Q^2)^n$  (from all  $\tau$ )

# Parametrizing the GPDs

double distribution representation

Mueller *et al* (94), Radyushkin (99)

$$K^i(x, \xi, t) = \int_{-1}^1 d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta \delta(\rho + \xi\eta - x) K^i(\rho, \xi = 0, t) w_i(\rho, \eta) + D_i \Theta(\xi^2 - \bar{x}^2)$$

weight fct  $w_i(\rho, \eta) \sim [(1 - |\rho|)^2 - \eta^2]^{n_i}$  ( $n_g = n_{\text{sea}} = 2, n_{\text{val}} = 1$ , generates  $\xi$  dep.)

zero-skewness GPD  $K^i(\rho, \xi = 0, t) = k^i(\rho) \exp[(b_{ki} + \alpha'_{ki} \ln(1/\rho))t]$

$$k = q, \Delta q, \delta^q \text{ for } H, \tilde{H}, H_T \text{ or } N_{ki} \rho^{-\alpha_{ki}(0)} (1 - \rho)^{\beta_{ki}} \text{ for } E, \tilde{E}, \bar{E}_T$$

Regge-like  $t$  dep. (for small  $-t$  reasonable appr.),  $D$ -term neglected

**advantage:** polynomiality and reduction formulas automatically satisfied

positivity bounds respected (checked numerically)

$H_{\text{val}}, E_{\text{val}}$  and  $\tilde{H}_{\text{val}}$  at  $\xi = 0$  from GPD analysis of nucleon form factors

(sum rules)

Diehl *et al*(04), Diehl-K (13)

# Ansätze for zero-skewness GPDs

simplest ansatz:

VGG(98), Freund et al (01)

e.g.  $H^q(x, \xi = 0, t) = c_q q(x) F_1^q(t)$

PDF times form factor

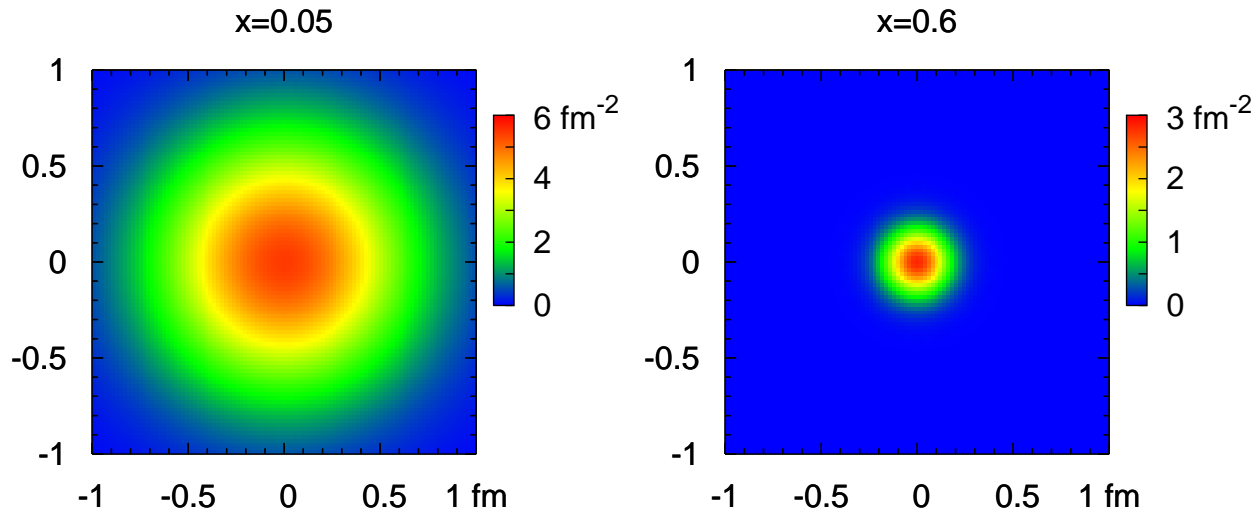
respects reduction formula and sum rules  $\int dx H^q(x, \xi = 0, t) = F_1^q(t)$

Burkhardt(00,03):  $q(x, \mathbf{b}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{-i\mathbf{b}\Delta_{\perp}} H^q(x, \xi = 0, t = -\Delta_{\perp}^2)$

density interpretation  $\mathbf{b}$  transverse distance between struck parton and

hadron's center of momentum  $\sum x_i \mathbf{b}_i = 0$ ;

partons with large (small)  $x_i$  must (can) have small (large)  $\mathbf{b}_i$



$x - b$  ( $x - t$ )  
correlation required

improvement: Regge-like ansatz (frequently used now)

$$H^q(x, \xi = 0, t) = q(x) e^{t f_q(x)} \quad f_q(x) = B_q + \alpha'_q \ln(x)$$

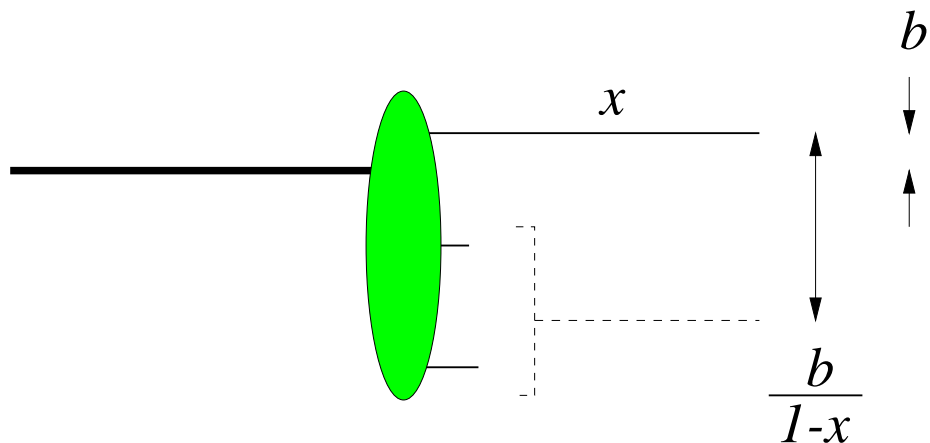
at small  $x$ :  $q \sim x^{-\alpha(0)} \implies H^q \sim x^{-\alpha(t)}$

standard Regge trajectory -  $H^q \sim 1/\sqrt{x}$  at  $t \simeq 0$  (fact. ansatz for all  $t$ )  
 $\sim \sqrt{x}$  at  $t \simeq -1 \text{ GeV}^2$

$$\text{FT: } q(x, \mathbf{b}) = \frac{1}{4\pi} \frac{q(x)}{f_q(x)} \exp[-b^2/4f_q(x)] \quad \text{and} \quad \langle b^2 \rangle_x^q = 4f_q(x)$$

distance between active parton and cluster of spectators

(rough estimate of proton radius)  $d_q(x) = \frac{\sqrt{\langle b^2 \rangle_x^q}}{1-x} \sim 1/(1-x)$  for  $x \rightarrow 1!$



Regge-like ansatz only suitable  
 at small  $x$ , i.e. at small  $-t$

further improvement:

DFJK04, Diehl-K(13)

used in analysis of form factors for proton and neutron at  $\xi = 0$

$$F_1^{p(n)} = e_{u(d)} \int_{-1}^1 dx H_i^u(x, \xi = 0, t) + e_{d(u)} \int_{-1}^1 dx H_i^d(x, \xi = 0, t)$$

Pauli form factor  $H \rightarrow E$

normalization fixed from  $\kappa_q = \int_0^1 dx E_v^q(x, \xi = 0, t = 0)$

profile fct:  $f_q = (B_q + \alpha'_q \ln 1/x)(1-x)^3 + A_q x(1-x)^2$

fixes valence quark GPDs  $H, E, \tilde{H}$  at  $\xi = 0$

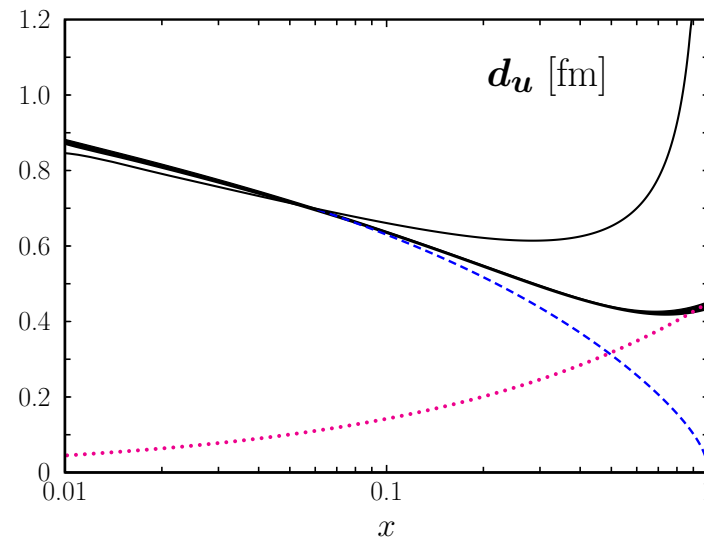
$$d_q(x) = \frac{2\sqrt{f_q(x)}}{1-x} \rightarrow 2\sqrt{A_q}$$

for  $x \rightarrow 1$

Regge-like profile fct can (only) be used

at small  $x$  (small  $-t$ )

its FT not meaningful at large  $x$





# Analysis of hard exclusive meson leptonproduction

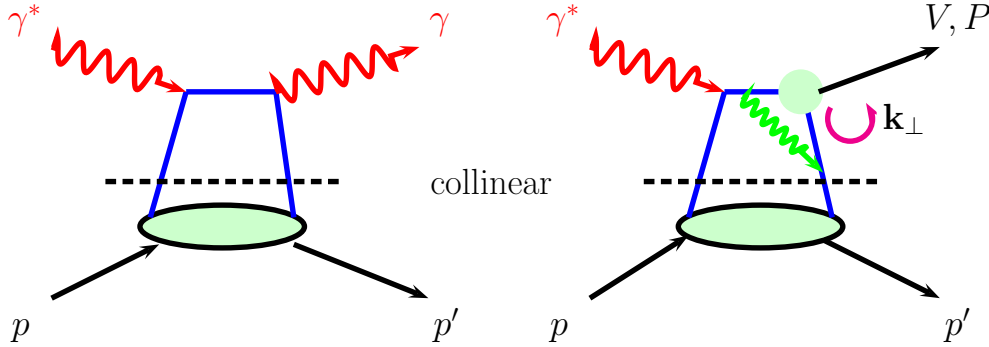
- analysis of nucleon form factor data DFJK(04), Diehl-K(13)  
fixes GPDs  $H, E$  and  $\tilde{H}$  for valence quarks at zero skewness
- fit to all available long. cross section data for  $\rho^0$  and  $\phi$  production  
from HERMES, COMPASS, E665, H1, ZEUS  
cover large range of kinematics  $Q^2 \simeq 3 - 100 \text{ GeV}^2$   $W \simeq 4 - 180 \text{ GeV}$   
but small  $-t' (\simeq -t)$ , skewness  $\xi (\simeq x_B/2) \lesssim 0.1$   
fixes the GPD  $H$  for gluons and sea quarks GK (05,07,08,13)
- analysis of transverse cross sections, SDMEs, asymmetries for  $\rho^0, \phi$  from  
HERMES, COMPASS and HERA  
analysis of cross section and asymmetries for  $\pi^+$  from HERMES  
typically  $2 \text{ GeV}^2 \lesssim Q^2 \lesssim 5 \text{ GeV}^2$   
provides information on  $\tilde{H}, E, H_T, \bar{E}_T$  (mainly for val. quarks)  
and on pion pole  
typically of lesser quality than that of  $H$  GK (10,11)  
(requires extension of handbag appr. to  $\gamma_T^* \rightarrow M$  transitions)

# Exploiting universality

our set of GPDs allows for parameter free calculations of other hard exclusive reactions

- $\nu_l p \rightarrow l P p$  Kopeliovich et al (13)  
V-A structure leads to different combinations of GPDs no data
- timelike DVCS Pire et al (13) no data
- $\gamma^* p \rightarrow \omega p$  GK(14)  
compared with SDMEs from HERMES(14)  
prominent role of pion pole
- DVCS K-Moutarde-Sabatie(13)

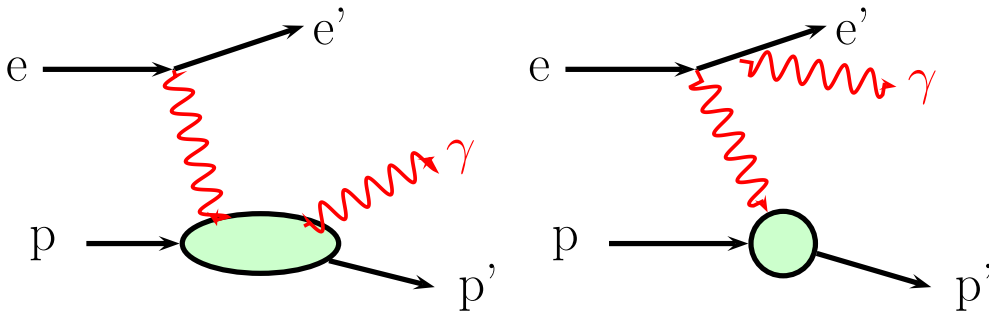
# DVCS



leading-twist, LO accuracy

collinear for consistency

NLO: gluon GPDs contribute as well



$$d\sigma(lp \rightarrow lp\gamma) = d\sigma_{BH} + d\sigma_I + d\sigma_{DVCS}$$

$$d\sigma_i \propto \sum_{n=0}^3 [c_n^i \cos(n\phi) + s_n^i \sin(n\phi)]$$

DVCS convolutions

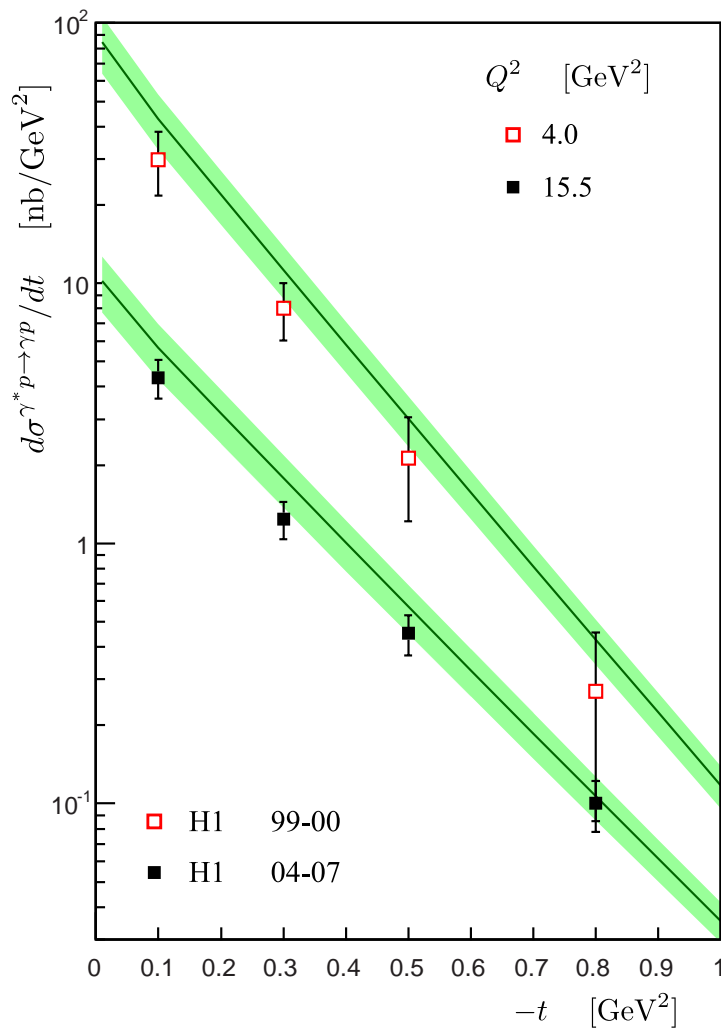
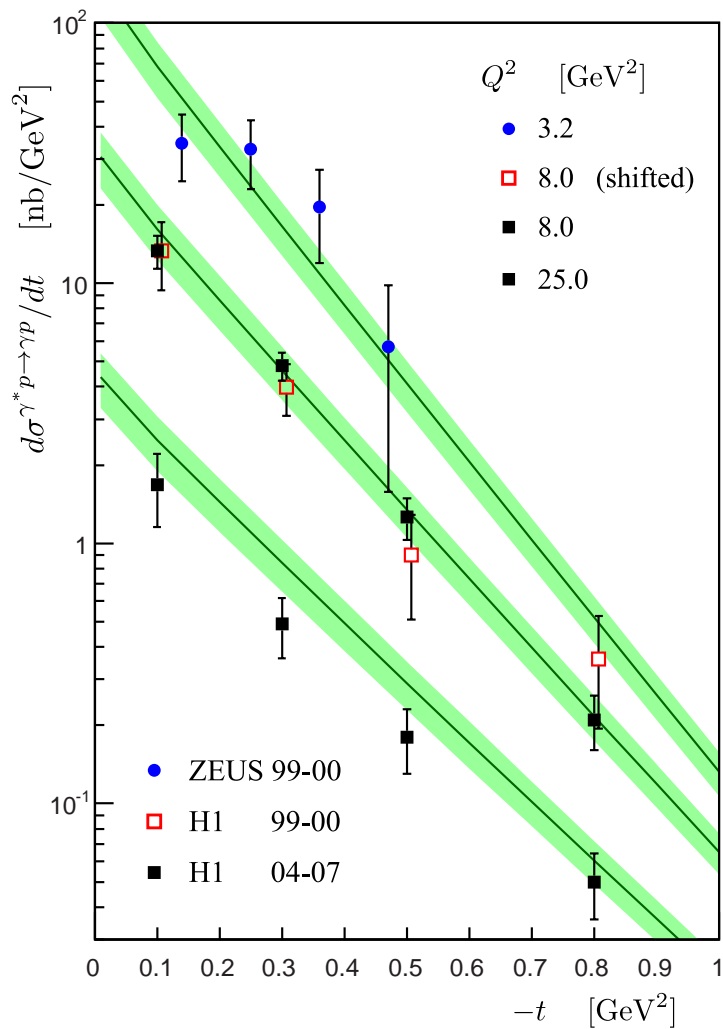
$$\mathcal{K} = e_u^2 \mathcal{K}^u + e_d^2 \mathcal{K}^d + e_s^2 \mathcal{K}^s$$

# GPD contributions to DVCS observables

Experiment	Observable	Normalized convolutions
HERMES	$A_C^{\cos 0\phi}$	$\text{Re}\mathcal{H} + 0.06\text{Re}\mathcal{E} + 0.24\text{Re}\tilde{\mathcal{H}}$
	$A_C^{\cos \phi}$	$\text{Re}\mathcal{H} + 0.05\text{Re}\mathcal{E} + 0.15\text{Re}\tilde{\mathcal{H}}$
	$A_{\text{LU,I}}^{\sin \phi}$	$\text{Im}\mathcal{H} + 0.05\text{Im}\mathcal{E} + 0.12\text{Im}\tilde{\mathcal{H}}$
	$A_{\text{UL}}^{+,\sin \phi}$	$\text{Im}\tilde{\mathcal{H}} + 0.10\text{Im}\mathcal{H} + 0.01\text{Im}\mathcal{E}$
	$A_{\text{UL}}^{+,\sin 2\phi}$	$\text{Im}\tilde{\mathcal{H}} - 0.97\text{Im}\mathcal{H} + 0.49\text{Im}\mathcal{E} - 0.03\text{Im}\tilde{\mathcal{E}}$
	$A_{\text{LL}}^{+,\cos 0\phi}$	$1 + 0.05\text{Re}\tilde{\mathcal{H}} + 0.01\text{Re}\mathcal{H}$
	$A_{\text{LL}}^{+,\cos \phi}$	$1 + 0.79\text{Re}\tilde{\mathcal{H}} + 0.11\text{Im}\mathcal{H}$
	$A_{\text{UT,DVCS}}^{\sin(\phi-\phi_S)}$	$\text{Im}\mathcal{H}\text{Re}\mathcal{E} - \text{Im}\mathcal{E}\text{Re}\mathcal{H}$
	$A_{\text{UT,I}}^{\sin(\phi-\phi_S)\cos \phi}$	$\text{Im}\mathcal{H} - 0.56\text{Im}\mathcal{E} - 0.12\text{Im}\tilde{\mathcal{H}}$
CLAS	$A_{\text{LU}}^{-,\sin \phi}$	$\text{Im}\mathcal{H} + 0.06\text{Im}\mathcal{E} + 0.21\text{Im}\tilde{\mathcal{H}}$
	$A_{\text{UL}}^{-,\sin \phi}$	$\text{Im}\tilde{\mathcal{H}} + 0.12\text{Im}\mathcal{H} + 0.04\text{Im}\mathcal{E}$
	$A_{\text{UL}}^{-,\sin 2\phi}$	$\text{Im}\tilde{\mathcal{H}} - 0.79\text{Im}\mathcal{H} + 0.30\text{Im}\mathcal{E} - 0.05\text{Im}\tilde{\mathcal{E}}$
HALL A	$\Delta\sigma^{\sin \phi}$	$\text{Im}\mathcal{H} + 0.07\text{Im}\mathcal{E} + 0.47\text{Im}\tilde{\mathcal{H}}$
	$\sigma^{\cos 0\phi}$	$1 + 0.05\text{Re}\mathcal{H} + 0.007\mathcal{H}\mathcal{H}^*$
	$\sigma^{\cos \phi}$	$1 + 0.12\text{Re}\mathcal{H} + 0.05\text{Re}\tilde{\mathcal{H}}$
HERA	$\sigma_{\text{DVCS}}$	$\mathcal{H}\mathcal{H}^* + 0.09\mathcal{E}\mathcal{E}^* + \tilde{\mathcal{H}}\tilde{\mathcal{H}}^*$

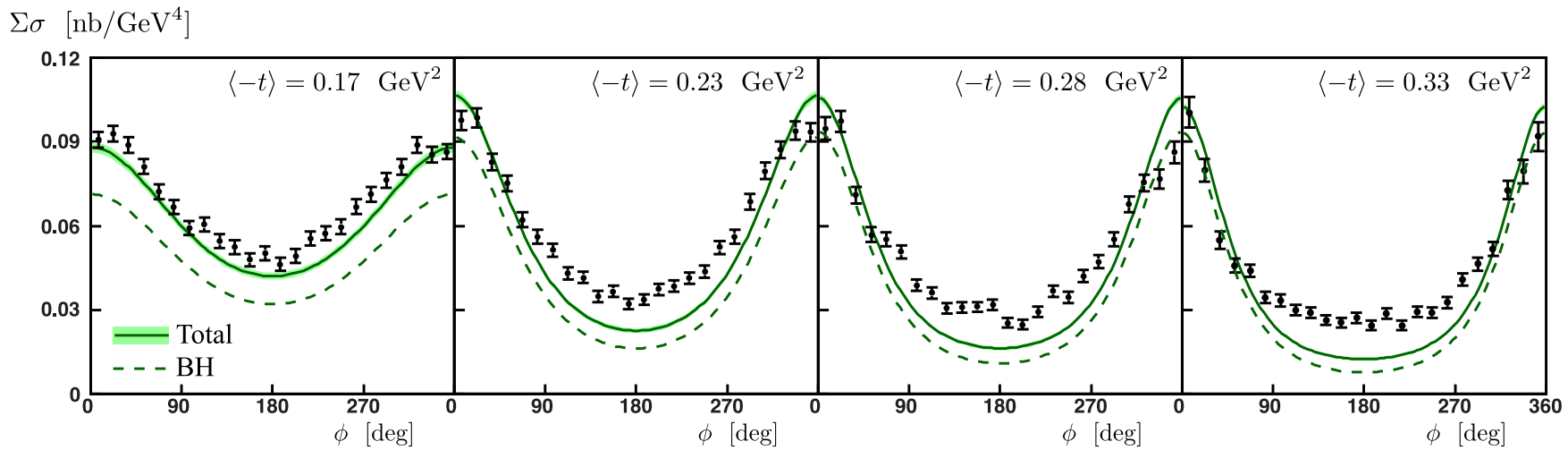
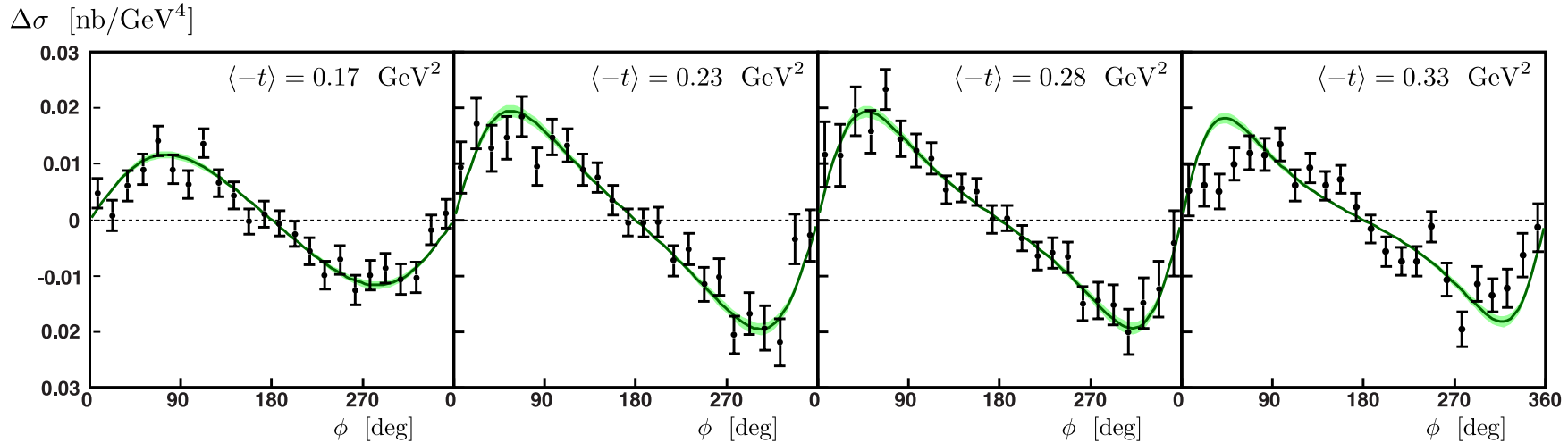
coeff. are normalized to the largest one, only relative coeff. larger than 1% are kept  
with  $\mathcal{H}$  most of the DVCS observables can be computed

# DVCS at HERA



$W \simeq 90$  GeV      data from **ZEUS, H1**

# DVCS at JLAB



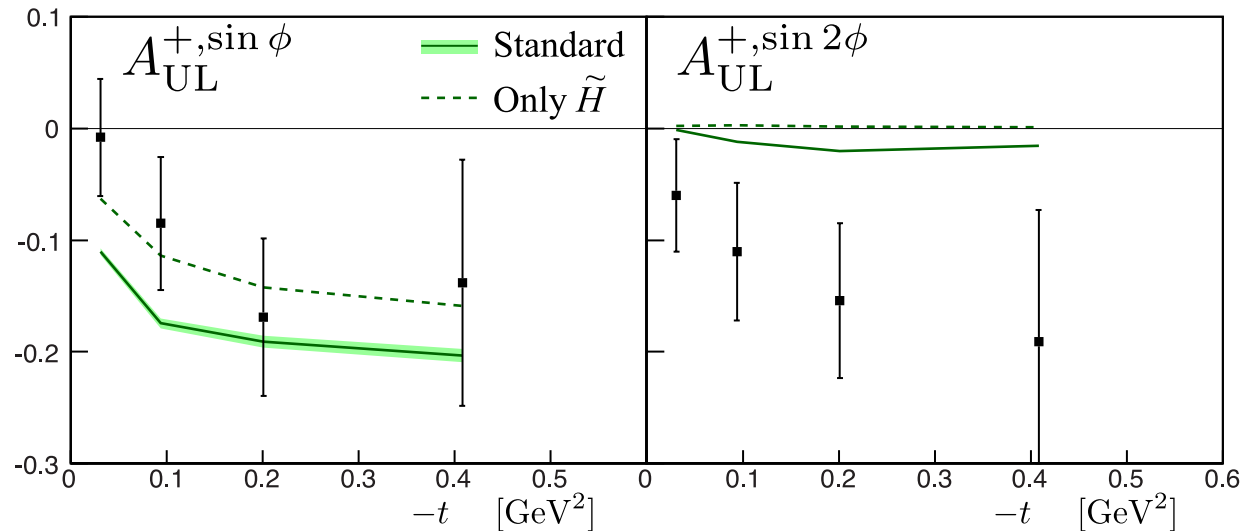
Hall A data  $x_{Bj} = 0.36$   $Q^2 = 2.3 \text{ GeV}^2$

dashed lines: BH

difference and sum of electron-helicity cross section

less satisfactory description of JLAB data (large skewness, small  $W$ )

# Long. polarized target asymmetry



Data from [HERMES\(10\)](#)  $x_{Bj} = 0.1$ ,  $Q^2 = 2.46 \text{ GeV}^2$  with positron beam dominated by DVCS-BH interference

sensitive to  $\tilde{H}$

[KMS\(13\)](#)

surprisingly strong  $\sin 2\phi$  modulation; theor. strongly suppressed  
the only small- $\xi$  observable which we don't fit

# $E$ for gluons and sea quarks

$E$  for valence quarks from FF analysis

Diehl-K(13)

Teryaev(99): sum rule (Ji's s.r. and momentum s.r. of DIS) at  $t = \xi = 0$

$$\int_0^1 dx x e_g(x) = e_{20}^g = - \sum e_{20}^{a_v} - 2 \sum e_{20}^{\bar{a}}$$

valence term very small

$\Rightarrow$  2nd moments of gluon and sea quarks cancel each other almost completely (holds approximately for other moments too provided GPDs don't have nodes)

positivity bound for FTs forbids large sea  $\Rightarrow$  gluon small too

$$\frac{b^2}{m^2} \left( \frac{\partial e_s(x,b)}{\partial b^2} \right)^2 \leq s^2(x,b) - \Delta s^2(x,b)$$

parameterization as described:  $\beta_e^s = 7$ ,  $\beta_e^g = 6$  Regge-like parameters as for  $H$

$$e_i = N_i x^{-\alpha_g(0)} (1-x)^{\beta_i}$$

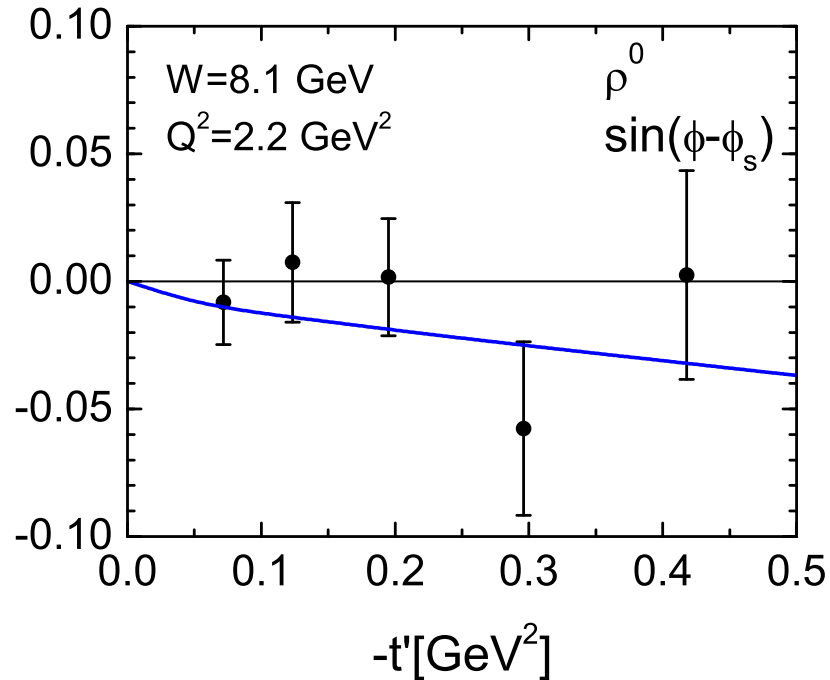
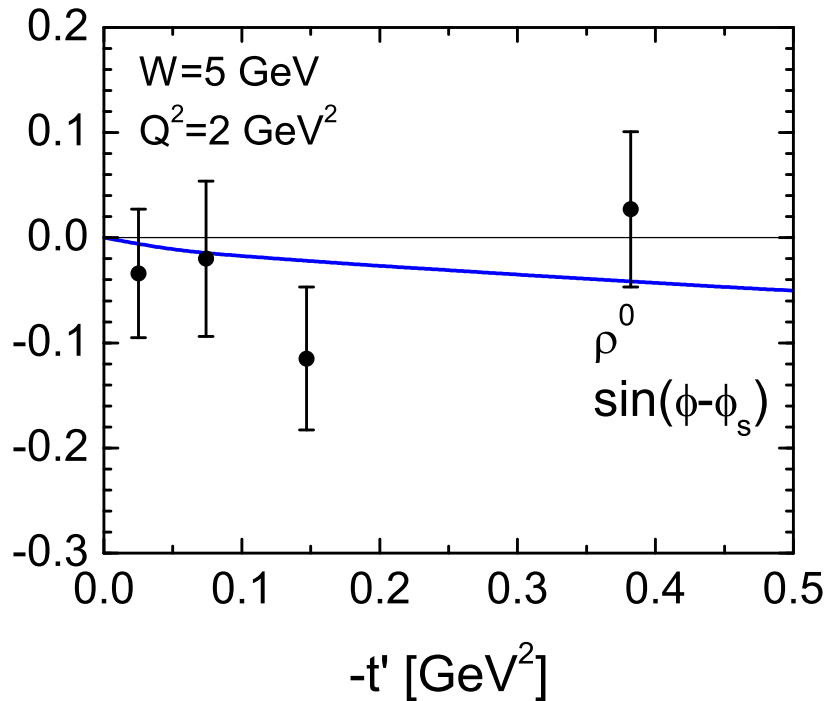
flavor symm. sea for  $E$  assumed

$N_s$  fixed by saturating bound ( $N_s = \pm 0.155$ ),  $N_g$  from sum rules

for  $\xi \neq 0$  input to double distribution ansatz



# $A_{UT}^{\sin(\phi-\phi_s)}$ for $\rho^0$ production



data: HERMES(08)

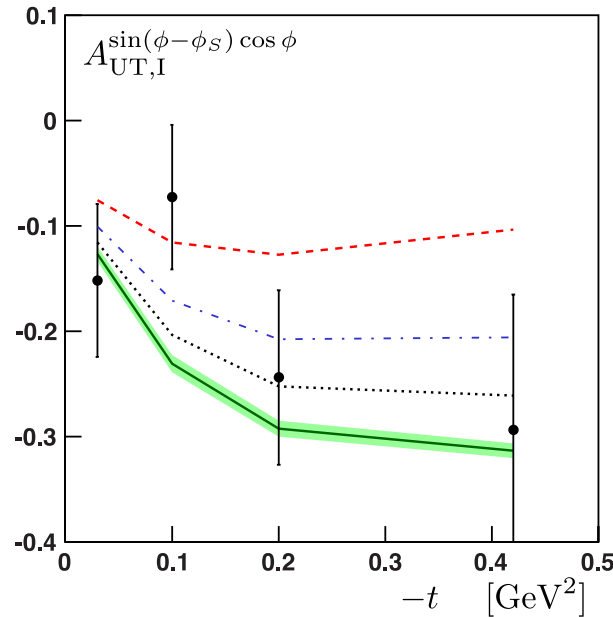
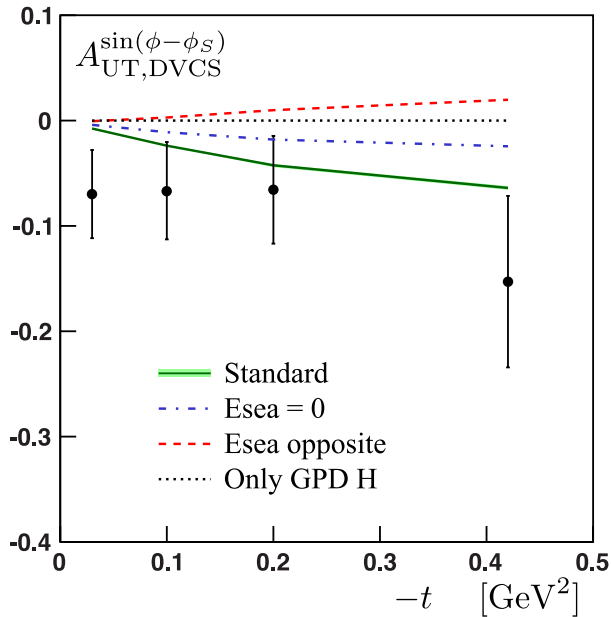
COMPASS(12)

theor. result: Goloskokov-K(09)

$$A_{UT}^{\sin(\phi-\phi_s)} \sim \text{Im}[\mathcal{E}^* \mathcal{H}]$$

gluon and sea contr. from  $E$  cancel to a large extent  
dominated by valence quark contr. from  $E$

# Target asymmetry in DVCS



data: HERMES(08)

$$\langle Q^2 \rangle \simeq 2.5 \text{ GeV}^2$$

$$\langle x_{Bj} \rangle \simeq 0.09$$

theory: KMS(12)

$A_{UT,DVCS}^{\sin(\phi-\phi_S)} \sim \text{Im}[\mathcal{E}^* \mathcal{H}]$   
 no cancellation between  
 sea and gluon  
 $\Rightarrow \mathcal{E}^{\text{sea}}$  seen

from BH-DVCS interference  
 separate contr. from  
 $\text{Im} \mathcal{H}$  and  $\text{Im} \mathcal{E}$

negative  $\mathcal{E}^{\text{sea}}$  favored in both cases

$\mathcal{E}^g \geq 0$  Koempel et al(11) transverse target polarisation in  $J/\Psi$  photo- and electroproduction, dominated by gluonic GPDs

# Application: Angular momenta of partons

$$J^a = \frac{1}{2} [q_{20}^a + e_{20}^a] \quad J^g = \frac{1}{2} [g_{20} + e_{20}^g] \quad (\xi = t = 0)$$

$q_{20}^a, g_{20}$  from ABM11 (NLO) PDFs

$e_{20}^{a_v}$  from form factor analysis Diehl-K. (13):

$$J_v^u = 0.230_{-0.024}^{+0.009} \quad J_v^d = -0.004_{-0.016}^{+0.010}$$

with  $e_{20}^s, e_{20}^g$  from analysis of  $A_{UT}$  in DVMP and DVCS

$$\begin{aligned} J^{u+\bar{u}} &= 0.261; J^{d+\bar{d}} = 0.035; J^{s+\bar{s}} = 0.018; J^g = 0.186 \quad (E^s = 0) \\ &= 0.235; \quad = 0.009; \quad = -0.008; \quad = 0.263 \quad (E^s < 0, E^g > 0) \\ & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (N_s = -0.155) \end{aligned}$$

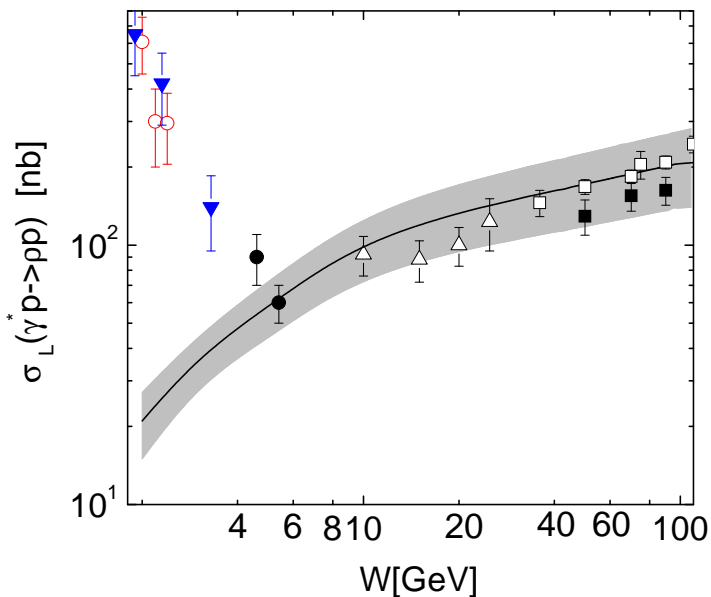
need better determ. of  $E^s$  (smaller errors of  $A_{UT}$  in DVCS)

$J^i$  quoted at scale 2 GeV

$$\sum J^i = 1/2 \quad \text{spin of the proton} \quad (\text{Ji's sum rule})$$

there is no spin crisis

# Why restriction to small skewness data?



at  $Q^2 = 4 \text{ GeV}^2$

data: E665, HERMES,  
CORNELL, H1, ZEUS, CLAS

breakdown of handbag physics?

at large  $x_{Bj}$  (small  $W$ )

- power corrections are strong at least in some cases
- kinematic corrections strong, e.g.

$$\xi \simeq \frac{x_{Bj}}{2-x_{Bj}} \left[ 1 + \frac{1}{(1-x_{Bj}/2)Q^2} (m_M^2 - x_{Bj}^2 m^2 - x_{Bj}(1-x_{Bj})t') \right]$$

- GPD parameterization can be applied to large skewness region but success is not guaranteed

$$t_0 = -4m^2\xi^2/(1-\xi^2) \text{ large}$$

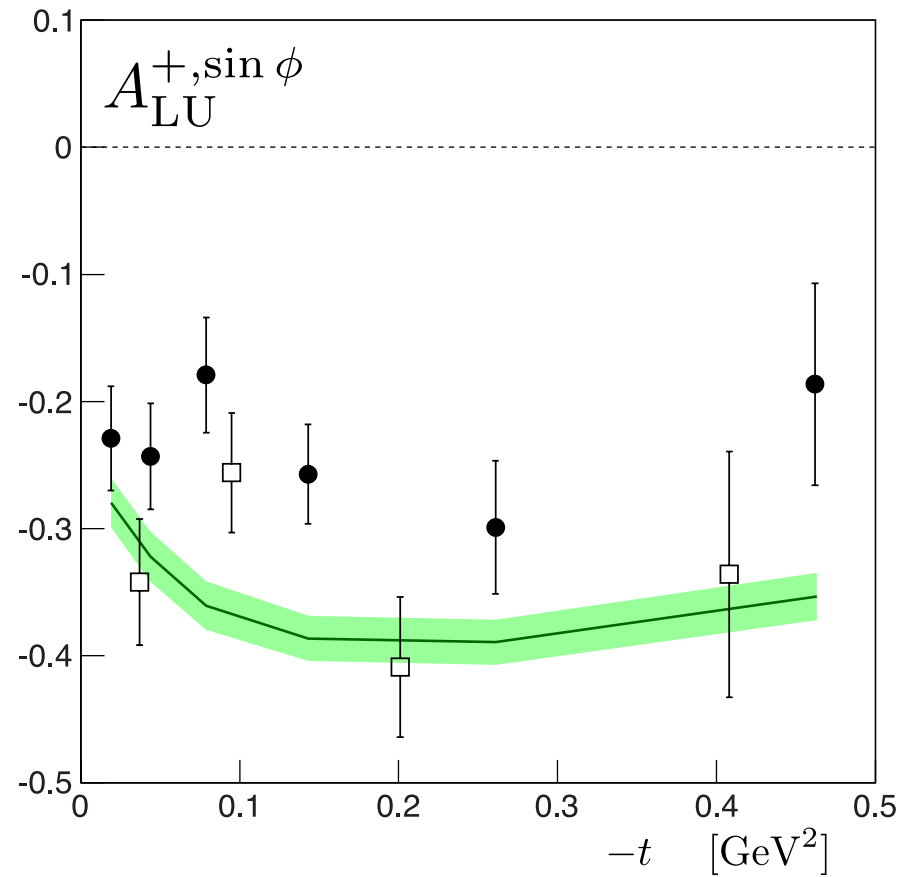
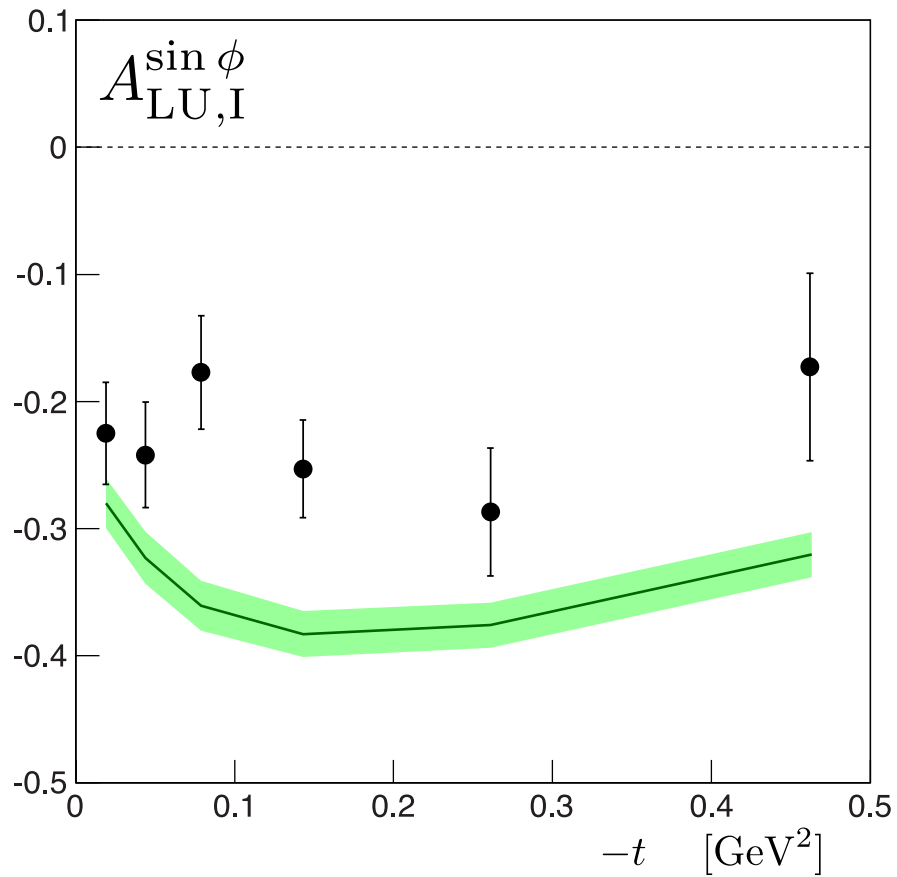
probes GPD in different region of  $t$

$$(W = 2 \text{ GeV}, Q^2 = 4 \text{ GeV}^2:$$

$$t_0 = -0.86 \text{ GeV}^2)$$

# Summary

- exclusive electroproduction of mesons allows to **extract** the GPDs  $H, E, \tilde{H}, H_T, \bar{E}_T$  at small  $\xi$  and  $W \gtrsim 4 \text{ GeV}$
- with exception of  $H$  little is known about the gluon and sea-quark sector, experimental information insufficient as yet
- double distr. ansatz is 'flexible' enough to account for all small  $\xi$  data; constraints from positivity, PDFs, form factors used
- these GPDs allow to **calculate** DVCS free of parameters, to study transverse localization of partons (at least for valence quarks) and to evaluate Ji's sum rule
- **future improvements:** use of new PDFs, more complicated profile fcts. for all GPDs,  $D$ -term, kinematical corrections at low  $Q^2$ , low  $W$ , large  $\xi$  and **new data from COMPASS, JLAB12 and EIC?**



HERMES 1203.6287 (solid circles)

and recoil data HERMES 1206.5683 (open squares)

$$x_B \simeq 0.097 \quad Q^2 = 2.51 \text{ GeV}^2$$