ORBITAL ANGULAR MOMENTUM

HOW TO DEFINE IT AND HOW TO MEASURE IT

Elliot Leader

Imperial College London

Physics Report: E.L and Cédric Lorcê, 2014

Two tracks:

(1) Technical

(2) Emphasis on physical implications

OUTLINE

- Ambiguities or Variants in definition of $\boldsymbol{L}$
- THREE fundamental versions
- How to measure them
- Model calculations
- Puzzles

Throughout this talk

- Ambiguities, Variants Versions: all illustrated mainly in QED

- Avoid technical details

- Nucleon moving along OZ, longitudinally polarized: only discuss L_z.

Two kinds of variants

(1) Difference between CANONICAL and KINETIC angular momentum

(2) Difference between INSTANT FORM and LIGHT-FRONT dynamics

Two kinds of variants

(1) Difference between CANONICAL and KINETIC angular momentum

(2) Difference between INSTANT FORM and LIGHT-FRONT dynamics

Difference between canonical and kinetic has nothing to do with Field Theory
It is hidden in Undergraduate Physics!

REMINDER: Undergrad Dynamics

Kinetic momentum

.

Defined as mass times velocity

$$\boldsymbol{p}_{\mathsf{kin}} = m\boldsymbol{v} = m\dot{\boldsymbol{x}}$$

Follows motion of particle.

Non-relativistic expression for the particle kinetic energy

$$E_{\mathsf{kin}} = \boldsymbol{p}_{\mathsf{kin}}^2/2m$$

Quantum Mechanics

Canonical momentum

Heisenberg uncertainty relations between position and momentum

$$[x_i, p_j] = i\hbar\,\delta_{ij}$$

Quantum Mechanics

Canonical momentum

Heisenberg uncertainty relations between position and momentum

$$[x_i, p_j] = i\hbar\,\delta_{ij}$$

This $\boldsymbol{p}$ is NOT the kinetic momentum

It is canonical momentum, defined as

$$\boldsymbol{p}_{\mathsf{can}} = \partial L / \partial \dot{\boldsymbol{x}}$$

where L is the Lagrangian of the system

Comparison of $\boldsymbol{p}_{\mathsf{can}}$ with $\boldsymbol{p}_{\mathsf{kin}}$

For a particle moving in a potential $V(\boldsymbol{x})$

$$L = E_{\mathsf{kin}} - V = \frac{1}{2} m\dot{\boldsymbol{x}}^2 - V(\boldsymbol{x})$$

so that

$$\boldsymbol{p}_{\mathsf{can}} = m\dot{\boldsymbol{x}} = \boldsymbol{p}_{\mathsf{kin}},$$

and there is no distinction between kinetic and canonical momentum.

What happens if an electromagnetic field is present?

Classical problem: charged particle, say an electron with charge e, moving in a fixed homogeneous *external* magnetic field $\boldsymbol{B} = (0, 0, B)$.

Particle follows a helical trajectory, so that at each instant, the particle kinetic momentum $\boldsymbol{p}_{\text{kin}}$ points toward a different direction.

What happens if an electromagnetic field is present?

Classical problem: charged particle, say an electron with charge e, moving in a fixed homogeneous *external* magnetic field $\boldsymbol{B} = (0, 0, B)$.

Particle follows a helical trajectory, so that at each instant, the particle kinetic momentum $\boldsymbol{p}_{\text{kin}}$ points toward a different direction.

The Lagrangian is given by

$$L = \frac{1}{2} m \dot{\boldsymbol{x}}^2 - e \dot{\boldsymbol{x}} \cdot \boldsymbol{A}$$

where $\boldsymbol{A}$ is the **vector potential** responsible for the magnetic field $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$. It leads to

$$\boldsymbol{p}_{\text{can}} = \boldsymbol{p}_{\text{kin}}[\boldsymbol{x}(t)] - e\boldsymbol{A}[\boldsymbol{x}(t)]$$

Under a gauge transformation $\boldsymbol{A}$ changes, but that does not affect the physical motion of the particle.

But, it clearly changes $\boldsymbol{p}_{\text{can}}$.
$\boldsymbol{p}_{\text{can}}$ is a gauge **non-invariant** quantity.

key issue in the recent controversy: is such a quantity measurable?

How does this show up in QCD?

$$\frac{1}{2} = \langle\langle S_z^q \rangle\rangle + \langle\langle L_z^q \rangle\rangle + \langle\langle S_z^G \rangle\rangle + \langle\langle L_z^G \rangle\rangle$$

How does this show up in QCD ?

$$\frac{1}{2} = \langle\langle S_z^q \rangle\rangle + \langle\langle L_z^q \rangle\rangle + \langle\langle S_z^G \rangle\rangle + \langle\langle L_z^G \rangle\rangle$$

Totally intuitive; can't be incorrect.

How does this show up in QCD ?

$$\frac{1}{2} = \langle\langle S_z^q \rangle\rangle + \langle\langle L_z^q \rangle\rangle + \langle\langle S_z^G \rangle\rangle + \langle\langle L_z^G \rangle\rangle$$

Totally intuitive; can't be incorrect.

But: Operators $L^{q,G}$ and S^G are not gauge invariant.

Based on the CANONICAL version of $\boldsymbol{J}$. In QED
$q \to$ electron, $\quad G \to$ photon

$$\begin{aligned}\boldsymbol{J}_{\text{can}} \;&=\; \underbrace{\int d^3x\,\psi^\dagger \tfrac{1}{2}\boldsymbol{\Sigma}\psi}_{\boldsymbol{S}^e_{\text{can}}} + \underbrace{\int d^3x\,\psi^\dagger(\boldsymbol{x}\times \tfrac{1}{i}\boldsymbol{\nabla})\psi}_{\boldsymbol{L}^e_{\text{can}}} \\ &\quad + \underbrace{\int d^3x\,\boldsymbol{E}\times\boldsymbol{A}}_{\boldsymbol{S}^\gamma_{\text{can}}} + \underbrace{\int d^3x\,E^i(\boldsymbol{x}\times\boldsymbol{\nabla}A^i)}_{\boldsymbol{L}^\gamma_{\text{can}}}\end{aligned}$$

Based on the CANONICAL version of $\boldsymbol{J}$. In QED $q \to$ electron, $G \to$ photon

$$\boldsymbol{J}_{\text{can}} = \underbrace{\int d^3x\, \psi^\dagger \tfrac{1}{2}\boldsymbol{\Sigma}\psi}_{\boldsymbol{S}^e_{\text{can}}} + \underbrace{\int d^3x\, \psi^\dagger (\boldsymbol{x} \times \tfrac{1}{i}\boldsymbol{\nabla})\psi}_{\boldsymbol{L}^e_{\text{can}}}$$

$$+ \underbrace{\int d^3x\, \boldsymbol{E} \times \boldsymbol{A}}_{\boldsymbol{S}^\gamma_{\text{can}}} + \underbrace{\int d^3x\, E^i (\boldsymbol{x} \times \boldsymbol{\nabla}) A^i}_{\boldsymbol{L}^\gamma_{\text{can}}}$$

Nice, because it splits $\boldsymbol{J}^{\gamma,G}$ into $\boldsymbol{S}^{\gamma,G} + \boldsymbol{L}^{\gamma,G}$ and we claim to measure the gluon spin ΔG.

Usually write this in the Jaffe-Manohar form:

$$\frac{1}{2} = \frac{1}{2}a_0 + \Delta G + \langle\langle L_z^q \rangle\rangle + \langle\langle L_z^G \rangle\rangle$$

where

$$a_0 = \text{axial charge of nucleon}$$

Should write Jaffe-Manohar in form :

$$\frac{1}{2} = \frac{1}{2}a_0 + \Delta G + \langle\langle L^q_{can,z} \rangle\rangle + \langle\langle L^G_{can,z} \rangle\rangle$$

Should write Jaffe-Manohar in form :

$$\frac{1}{2} = \frac{1}{2}a_0 + \Delta G + \langle\langle L^q_{can,z}\rangle\rangle + \langle\langle L^G_{can,z}\rangle\rangle$$

But still not completely accurate:

Danger! ΔG is a gauge invariant quantity. $\langle\langle S^G_{\mathrm{can},z}\rangle\rangle$ is (supposedly) not.
But as the nucleon momentum $P \to \infty$

$$\Delta G = \langle\langle S^G_{\mathrm{can},z}\rangle\rangle|_{\mathrm{Gauge}A^+=0}$$

Hence, correct way to write Jaffe-Manohar sum rule, for a longitudinally polarized nucleon, is

$$\frac{1}{2} = \frac{1}{2} a_0 + \Delta G + \lim_{P\to\infty} \left[\sum_q \langle\langle L^q_{\mathrm{can},z} \rangle\rangle \Big|_{A^+=0} + \langle\langle L^G_{\mathrm{can},z} \rangle\rangle \Big|_{A^+=0} \right]$$

Hence, correct way to write Jaffe-Manohar sum rule, for a longitudinally polarized nucleon, is

$$\frac{1}{2} = \frac{1}{2}a_0 + \Delta G + \lim_{P\to\infty}\left[\sum_q \langle\langle L^q_{\mathrm{can},z}\rangle\rangle\Big|_{A^+=0} + \langle\langle L^G_{\mathrm{can},z}\rangle\rangle\Big|_{A^+=0}\right]$$

NB It is $\langle\langle L^q_{\mathrm{can},z}\rangle\rangle\Big|_{A^+=0}$ that appears in the JM sum rule.

Another subtlety

$\boldsymbol{J}_{\mathsf{can}}$ was defined in terms of the generalised angular momentum density tensor $M^{\mu\nu\rho}(t,\boldsymbol{x})$ as

$$J^i_{\mathsf{can}} = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}(t,\boldsymbol{x})$$

Another subtlety

$\boldsymbol{J}_{\text{can}}$ was defined in terms of the generalised angular momentum density tensor $M^{\mu\nu\rho}(t, \boldsymbol{x})$ as

$$J^i_{\text{can}} = \frac{1}{2}\epsilon^{ijk} \int d^3x M^{0jk}(t, \boldsymbol{x})$$

This is the INSTANT FORM: integral over SPACE at fixed TIME of $M^{0jk}(t, \boldsymbol{x})$

Another subtlety

$\boldsymbol{J}_{\text{can}}$ was defined in terms of the generalised angular momentum density tensor $M^{\mu\nu\rho}(t, \boldsymbol{x})$ as

$$J^i_{\text{can}} = \frac{1}{2}\epsilon^{ijk} \int d^3x M^{0jk}(t, \boldsymbol{x})$$

This is the INSTANT FORM: integral over SPACE at fixed TIME of $M^{0jk}(t, \boldsymbol{x})$

In LIGHT-FRONT dynamics, role of time is played by x^+ and integral is over $dx^- d^2\boldsymbol{x}_\perp$ of $M^{+jk}(x^+, x^-, \boldsymbol{x}_\perp)$

So there is $\boldsymbol{J}^{inst}_{\text{can}}$ and $\boldsymbol{J}^{lf}_{\text{can}}$ and, analogously, $\boldsymbol{L}^{q,inst}_{\text{can}}$ and $\boldsymbol{L}^{q,lf}_{\text{can}}$

with

$$\lim_{P\to\infty} \langle\langle \boldsymbol{L}^{q,inst}_{\text{can}} \rangle\rangle = \langle\langle \boldsymbol{L}^{q,lf}_{\text{can}} \rangle\rangle$$

The KINETIC version, called Belinfante in Field Theory

$$\boldsymbol{J}_{\mathsf{Bel}} = \underbrace{\int \mathrm{d}^3x\, \overline{\psi} \left[\boldsymbol{x} \times \tfrac{1}{2} (\gamma^0\, i\boldsymbol{D} + \boldsymbol{\gamma}\, iD^0) \right] \psi}_{\boldsymbol{J}^e_{\mathsf{Bel}}} + \underbrace{\int \mathrm{d}^3x\, \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})}_{\boldsymbol{J}^\gamma_{\mathsf{Bel}}}$$

where the covariant derivative is given by $\boldsymbol{D} = \boldsymbol{\partial} + ie\boldsymbol{A} \equiv -\boldsymbol{\nabla} + ie\boldsymbol{A}$ and $D^0 = \partial_t + ieA^0$

The KINETIC version, called Belinfante in Field Theory

$$\boldsymbol{J}_{\text{Bel}} = \underbrace{\int \mathrm{d}^3x\, \overline{\psi} \left[\boldsymbol{x} \times \tfrac{1}{2}(\gamma^0\, i\boldsymbol{D} + \boldsymbol{\gamma}\, iD^0)\right] \psi}_{\boldsymbol{J}^e_{\text{Bel}}} + \underbrace{\int \mathrm{d}^3x\, \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})}_{\boldsymbol{J}^\gamma_{\text{Bel}}}$$

where the covariant derivative is given by $\boldsymbol{D} = \boldsymbol{\partial} + ie\boldsymbol{A} \equiv -\boldsymbol{\nabla} + ie\boldsymbol{A}$ and $D^0 = \partial_t + ieA^0$

Notice: No electron spin, no photon spin. But each term gauge invariant.

Using Equations of Motion and discarding a surface term at infinity, coming from integrating $\boldsymbol{\nabla}\cdot$ **term** , yields the form used by Ji:

$$\boldsymbol{J}_{\mathsf{Ji}} = \underbrace{\int \mathrm{d}^3x\, \psi^\dagger \tfrac{1}{2}\boldsymbol{\Sigma}\psi}_{\boldsymbol{S}^e_{\mathsf{Ji}}} + \underbrace{\int \mathrm{d}^3x\, \psi^\dagger (\boldsymbol{x} \times i\boldsymbol{D})\psi}_{\boldsymbol{L}^e_{\mathsf{Ji}}} + \underbrace{\int \mathrm{d}^3x\, \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})}_{\boldsymbol{J}^\gamma_{\mathsf{Ji}}}$$

Using Equations of Motion and discarding a surface term at infinity, coming from integrating $\boldsymbol{\nabla}\cdot$ **term** , yields the form used by Ji:

$$\boldsymbol{J}_{\mathrm{Ji}} = \underbrace{\int \mathrm{d}^3x\,\psi^\dagger \tfrac{1}{2}\boldsymbol{\Sigma}\psi}_{\boldsymbol{S}^e_{\mathrm{Ji}}} + \underbrace{\int \mathrm{d}^3x\,\psi^\dagger(\boldsymbol{x}\times i\boldsymbol{D})\psi}_{\boldsymbol{L}^e_{\mathrm{Ji}}} + \underbrace{\int \mathrm{d}^3x\,\boldsymbol{x}\times(\boldsymbol{E}\times\boldsymbol{B})}_{\boldsymbol{J}^\gamma_{\mathrm{Ji}}}$$

All terms are gauge invariant, but $\boldsymbol{J}^{\gamma,G}_{\mathrm{Ji}}$ not split into spin and orbital parts.

These are INSTANT FORM expressions.

As with the canonical case can define LIGHT-FRONT forms

But in this case, in QCD,

$$\langle\langle \boldsymbol{L}_{\mathsf{Ji}}^{q,\mathit{inst}} \rangle\rangle = \langle\langle \boldsymbol{L}_{\mathsf{Ji}}^{q,\mathit{lf}} \rangle\rangle$$

Summary

There exist THREE different OAM expectation values of interest

$$\langle\langle L^{inst,q}_{can,z}\rangle\rangle\Big|_{A^+=0} \quad \langle\langle L^{lf,q}_{can,z}\rangle\rangle\Big|_{A^+=0} \quad \langle\langle L^{q}_{Ji,z}\rangle\rangle$$

and, don't forget,

they are renormalization scale dependent

MEASUREMENT OF THE OAM

(1) The kinetic version : $\langle\langle L^q_{\mathsf{Ji},z}\rangle\rangle$

a) Ji relation with Generalized Parton Distributions (GPDs H and E)

$$\langle\langle J^q_{\mathrm{Ji},z}\rangle\rangle = \tfrac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x\,[H_q(x,0,0)+E_q(x,0,0)]$$

Thus

$$\langle\langle L^q_{\mathrm{Ji},z}\rangle\rangle = \tfrac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x\,[H_q(x,0,0)+E_q(x,0,0)] - \tfrac{1}{2}\, a^q_0$$

where a^q_0 is the contribution to a_0 (or $g_A^{(0)}$), the flavor-singlet axial charge of the nucleon, from a quark plus antiquark of given flavor

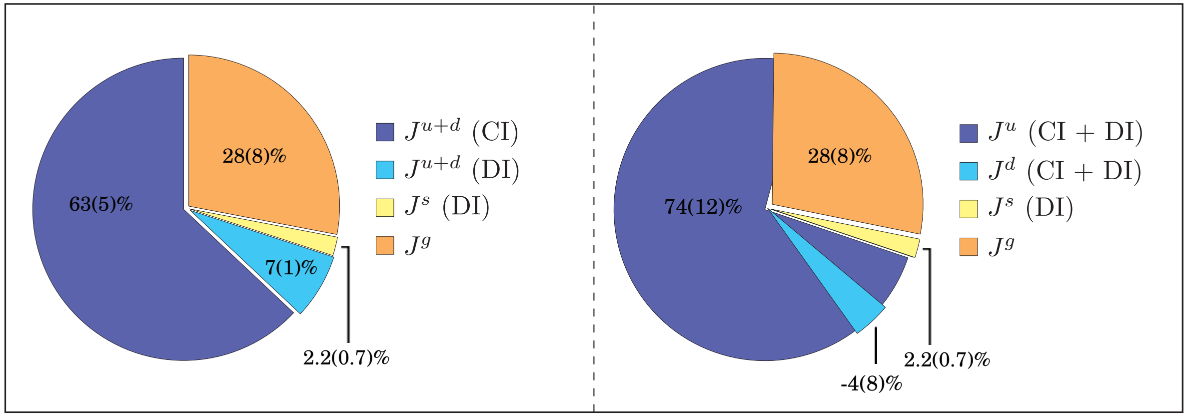
Source of data

(i) Lattice calculation (Deka et al arXiv:1312.4816)

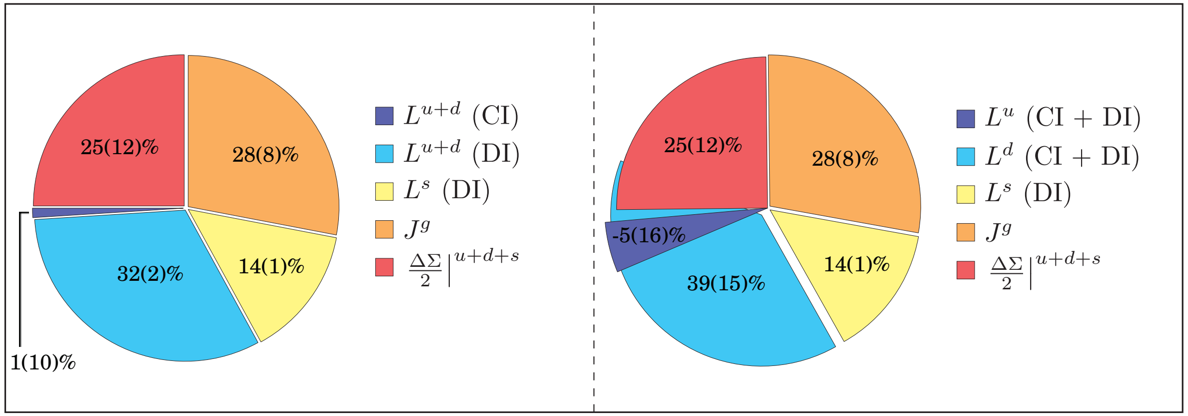
▶ Quenched approximation: no quark-antiquark loops
▶ Connected insertions (CI): current connects only to valence quark lines
▶ Disconnected insertions (DI): current also connects to quark loops (but still quenched)

Beautiful results courtesy of Keh-Fei Liu

$$L_q \equiv \langle\langle L^q_{\mathsf{Ji},z} \rangle\rangle \qquad J_q \equiv \langle\langle J^q_{\mathsf{Ji},z} \rangle\rangle$$



28(8)%
63(5)%
7(1)%
2.2(0.7)%
J^{u+d} (CI)
J^{u+d} (DI)
J^{s} (DI)
J^{g}
28(8)%
74(12)%
-4(8)%
2.2(0.7)%
J^{u} (CI + DI)
J^{d} (CI + DI)
J^{s} (DI)
J^{g}


25(12)%
28(8)%
32(2)%
14(1)%
1(10)%
L^{u+d} (CI)
L^{u+d} (DI)
L^{s} (DI)
J^{g}
$\frac{\Delta\Sigma}{2}\Big|^{u+d+s}$
25(12)%
28(8)%
-5(16)%
39(15)%
14(1)%
L^{u} (CI + DI)
L^{d} (CI + DI)
L^{s} (DI)
J^{g}
$\frac{\Delta\Sigma}{2}\Big|^{u+d+s}$

Key Lattice Results

	CI	DI	$Total$
L_u	-0.11 ± 0.08	0.08 ± 0.005	-0.025 ± 0.080
L_d	0.11 ± 0.08	0.08 ± 0.005	0.19 ± 0.07

NB $L_{u+d}|_{CI} \approx 0$

NB $L_u - L_d = -0.22 \pm 0.11$

Source of data

(ii) Extraction of E from DVCS, EM Form Factors etc, not easy.

Diehl and Kroll arXiv:1302.4604
Parametrization mainly determined by EM Form Factors: therefore Valence

Find $J^u_{val} = 0.230^{+0.009}_{-0.024}$ $J^d_{val} = -0.004^{+0.010}_{-0.016}$
Lattice $J^u_{val} = 0.317 \pm 0.008$ $J^d_{val} = -0.140 \pm 0.083$

b) Relation to twist-3 GPD G_2^q of Kiptily and Polyakov

$$\langle\langle L^q_{\mathrm{Ji},z}\rangle\rangle = -\int_{-1}^{1} \mathrm{d}x\, x\, G_2^q(x,0,0). \qquad (1)$$

This relation was first obtained by Penttinen, Polyakov, Shuvaev and Strikman in the parton model

b) Relation to twist-3 GPD G_2^q of Kiptily and Polyakov

$$\langle\langle L^q_{\mathsf{Ji},z}\rangle\rangle = -\int_{-1}^{1} \mathrm{d}x\, x\, G_2^q(x,0,0). \qquad (2)$$

This relation was first obtained by Penttinen, Polyakov, Shuvaev and Strikman in the parton model and later confirmed in QCD by Hatta and Yoshida

Perhaps hopelessly difficult to extract information on such a twist-3 GPD

c) Lorcé and Pasquini relation to Generalized Transverse Momentum Distributions (GTMDs)

$$\langle\langle L^q_{\mathrm{Ji},z}\rangle\rangle = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp \,\frac{\boldsymbol{k}_\perp^2}{M^2}\,F^q_{1,4}(x,\boldsymbol{k}_\perp,\Delta = 0;\mathcal{W}_{\mathrm{straight}}),$$

where the Wilson line $\mathcal{W}_{\mathrm{straight}}$ connects the points $-\frac{z}{2}$ and $\frac{z}{2}$ by a direct straight line

c) Lorcé and Pasquini relation to Generalized Transverse Momentum Distributions (GTMDs)

$$\langle\langle L^q_{\mathrm{Ji},z}\rangle\rangle = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{\boldsymbol{k}_\perp^2}{M^2}\,F^q_{1,4}(x,\boldsymbol{k}_\perp,\Delta=0;\mathcal{W}_{\text{straight}}),$$

where the Wilson line $\mathcal{W}_{\text{straight}}$ connects the points $-\frac{z}{2}$ and $\frac{z}{2}$ by a direct straight line

At present there is no clear way of extracting the twist-2 GTMDs from experimental data, but can be calculated in models

MEASUREMENT OF THE OAM

(2) The canonical version : $\langle\langle L^{\text{lf},q}_{\text{can},z}\rangle\rangle\Big|_{A^+=0}$

Lorcé, Pasquini relation to GTMDs

$$\langle\langle L^{\mathsf{lf},q}_{\mathsf{can},z}\rangle\rangle\Big|_{A^+=0} = -\int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \frac{\boldsymbol{k}_\perp^2}{M^2}\, F^q_{1,4}(x,\boldsymbol{k}_\perp,\Delta=0;\mathcal{W}_{\mathsf{LF}}),$$

where the staple-like Wilson line $\mathcal{W}_{\mathsf{LF}}$ connects the points $-\frac{z}{2}$ and $\frac{z}{2}$ *via* the intermediary points $-\frac{z}{2}\pm\infty^-$ and $\frac{z}{2}\pm\infty^-$ by straight lines

Lorcé, Pasquini relation to GTMDs

$$\langle\langle L^{lf,q}_{\text{can},z}\rangle\rangle\Big|_{A^+=0} = -\int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \frac{\boldsymbol{k}_\perp^2}{M^2}\, F^q_{1,4}(x, \boldsymbol{k}_\perp, \Delta = 0; \mathcal{W}_{\text{LF}}),$$

where the staple-like Wilson line $\mathcal{W}_{\text{LF}}$ connects the points $-\frac{z}{2}$ and $\frac{z}{2}$ *via* the intermediary points $-\frac{z}{2} \pm \infty^-$ and $\frac{z}{2} \pm \infty^-$ by straight lines

NB. changing the shape of the Wilson line, one obtains either the kinetic or the canonical quark orbital angular momentum

Actually, choosing the light-front gauge, can ignore this Wilson line ......this is done in models

Actually, choosing the light-front gauge, can ignore this Wilson line ......this is done in models

In lattice calculations, it is technically very difficult to fix a gauge. Forced to make calculations including explicitly the Wilson line. There is progress. [Musch et al PR D85 094510 (2012)]

MODEL CALCULATIOS

Four types of QCD models: none have genuine gluon degrees of freedom

- Light-Front Constituent Quark Model (LFCQM)
- Light-Front Chiral Quark-Soliton Model (LFχ QSM)
- MIT Bag Model
- Myher-Thomas Cloudy Bag Model with OGE

(a) The sign of $L_u - L_d$

All models, with exception of LFχ QSM lead to POSITIVE values of $L_u - L_d$

Key question: at what scale is model valid?

(a) The sign of $L_u - L_d$

All models, with exception of LFχ QSM lead to POSITIVE values of $L_u - L_d$

Key question: at what scale is model valid?

Presumably should be very low scale

(a) The sign of $L_u - L_d$

All models, with exception of LFχ QSM lead to POSITIVE values of $L_u - L_d$

Key question: at what scale is model valid?

Presumably should be very low scale

Usually fixed by forcing model to agree with ONE measured observable

Typically $0.16 - 0.36\, GeV^2$

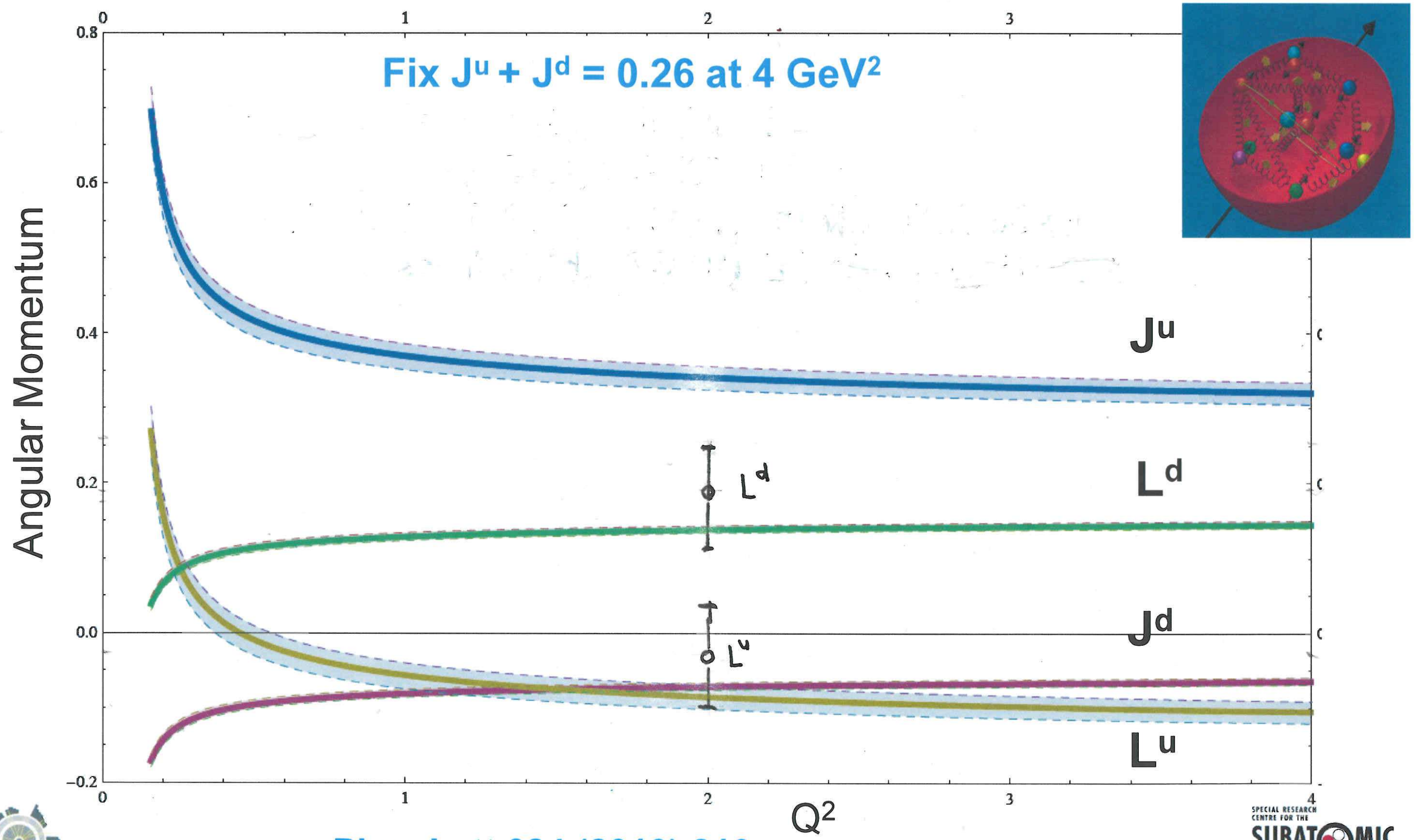
Usually model result $L_u - L_d > 0$ is considered failure of model

Thomas disagrees: suggests cross-over in $L_u - L_d$ due to evolution

Courtesy of Tony Thomas.

NLO Evolution

Remarkable agreement between model and LQCD



- Phys Lett 684 (2010) 216

Similar evolution starting with correct sign $L_u - L_d$ from LFχ QSM at low scale, as used by Wakamatsu, gives poor agreement with Lattice results

Similar evolution starting with correct sign $L_u - L_d$ from LFχ QSM at low scale, as used by Wakamatsu, gives poor agreement with Lattice results

Problem: Can evolution be trusted at such low scales where α_s is not small???

(b) Kinetic vs Canonical in models

Recall

$$\boldsymbol{L}^q_{Ji} = \int \mathrm{d}^3x\, \psi^\dagger (\boldsymbol{x} \times i\boldsymbol{D})\psi$$

$$\boldsymbol{L}^q_{can} = \int d^3x\, \psi^\dagger (\boldsymbol{x} \times \tfrac{1}{i}\boldsymbol{\nabla})\psi$$

(b) Kinetic vs Canonical in models

Recall

$$\boldsymbol{L}^q_{Ji} = \int \mathrm{d}^3x\,\psi^\dagger(\boldsymbol{x} \times i\boldsymbol{D})\psi$$

$$\boldsymbol{L}^q_{can} = \int d^3x\,\psi^\dagger(\boldsymbol{x} \times \tfrac{1}{i}\boldsymbol{\nabla})\psi$$

Since models usually have no gluon degrees of freedom, $\boldsymbol{D} = -\boldsymbol{\nabla}$, so expect

$$\boldsymbol{L}^q_{Ji} = \boldsymbol{L}^q_{can}$$

What do the models calculate?

(i) Expanding the nucleon state in terms of light-front wave functions in the definition of $\boldsymbol{L}^q_{can}$ restricted to the 3-quark sector,

$$\ell^q_{\mathrm{can},z} \equiv \langle\langle L^{lf,q}_{\mathrm{can},z}\rangle\rangle\big|^{\mathrm{model}}$$
$$= \sum_{\{\lambda\}} \int [\mathrm{d}x]_3\, [\mathrm{d}^2 k_\perp]_3\, \Psi^{*+}_3(\{x, \boldsymbol{k}_\perp, \lambda\})$$
$$\times \sum_{l,r(q)} (\delta_{rl} - x_l) \left(\boldsymbol{k}_{r\perp} \times \frac{1}{i}\boldsymbol{\nabla}_{\boldsymbol{k}_{l\perp}}\right)_z \Psi^+_3(\{x, \boldsymbol{k}_\perp, \lambda\})$$

Explanation of structure

$$\sum_{l,r(q)} (\delta_{rl} - x_l) \left(\boldsymbol{k}_{r\perp} \times \frac{1}{i} \boldsymbol{\nabla}_{\boldsymbol{k}_{l\perp}} \right)$$

This is INTRINSIC OAM defined wrt with respect to the transverse center of momentum

NRel: Centre of mass: $\boldsymbol{R}_{CM} = \sum_l \left(\frac{m_l}{M}\right) \boldsymbol{r}_l$

This is INTRINSIC OAM defined wrt with respect to the transverse center of momentum

NRel: Centre of mass: $\boldsymbol{R}_{CM} = \sum_l \left(\frac{m_l}{M}\right) \boldsymbol{r}_l$

Relativity: Centre of inertia or Centre of momentum:

$$\boldsymbol{R} = \sum_l \left(\frac{E_l}{E}\right) \boldsymbol{r}_l \quad \overbrace{=}^{\text{fast quark}} \quad \sum_l x_l \boldsymbol{r}_l$$

This is INTRINSIC OAM defined wrt with respect to the transverse center of momentum

NRel: Centre of mass: $\boldsymbol{R}_{CM} = \sum_l \left(\frac{m_l}{M}\right) \boldsymbol{r}_l$

Relativity: Centre of inertia or Centre of momentum:

$$\boldsymbol{R} = \sum_l \left(\frac{E_l}{E}\right) \boldsymbol{r}_l \quad \overbrace{=}^{\text{fast quark}} \quad \sum_l x_l \boldsymbol{r}_l$$

Transverse Centre of momentum (Burkardt)

$$\boldsymbol{R}_\perp = \sum_{l=1}^{3} x_l \, \boldsymbol{r}_{l\perp}$$

Should define Z-component of intrinsic OAM for quark q using $-(\boldsymbol{k}_{q\perp} \times \boldsymbol{b}_q)$

$\boldsymbol{b}_q$ impact parameter

$$\begin{aligned} \boldsymbol{b}_q &= \boldsymbol{r}_{q,\perp} - \boldsymbol{R}_\perp = \boldsymbol{r}_{q,\perp} - \sum_{l=1}^{3} x_l \, \boldsymbol{r}_{l\perp} \\ &= (1 - x_q)\boldsymbol{r}_{q,\perp} - \sum_{l \neq q} x_l \, \boldsymbol{r}_{l\perp} \end{aligned}$$

Should define Z-component of intrinsic OAM for quark q using $-(\boldsymbol{k}_{q\perp} \times \boldsymbol{b}_q)$

$\boldsymbol{b}_q$ impact parameter

$$
\begin{aligned}
\boldsymbol{b}_q &= \boldsymbol{r}_{q,\perp} - \boldsymbol{R}_\perp = \boldsymbol{r}_{q,\perp} - \sum_{l=1}^{3} x_l\, \boldsymbol{r}_{l\perp} \\
&= (1 - x_q)\boldsymbol{r}_{q,\perp} - \sum_{l \neq q} x_l\, \boldsymbol{r}_{l\perp}
\end{aligned}
$$

In momentum representation $\frac{1}{i}\boldsymbol{\nabla}_{\boldsymbol{k}_l} \to \boldsymbol{r}_l$

Thus

$$
\sum_l (\delta_{ql} - x_l)\left(\boldsymbol{k}_{q\perp} \times \tfrac{1}{i}\boldsymbol{\nabla}_{\boldsymbol{k}_{l\perp}}\right) \to (\boldsymbol{k}_{q\perp} \times \boldsymbol{b}_q)
$$

(ii) Get same result from

$$\ell^q_{\mathrm{can},z} = -\int \mathrm{d}x\, \mathrm{d}^2 k_\perp \,\frac{\boldsymbol{k}_\perp^2}{M^2}\, F^q_{1,4}(x, \boldsymbol{k}_\perp, \Delta = 0; \mathcal{W}_{\mathrm{LF}})\big|^{\mathrm{model}}$$

(iii) Obtain $\langle\langle L^q_{\mathsf{Ji},z}\rangle\rangle$ via

$$\ell^q_{\mathsf{kin},z} \equiv \langle\langle L^q_{\mathsf{Ji},z}\rangle\rangle|^{\mathsf{model}} = \int_{-1}^{1} \mathsf{d}x\, \ell^q_{\mathsf{kin},z}(x)$$

where

$$\ell^q_{\mathsf{kin},z}(x) = \tfrac{1}{2}\left\{x\left[H_q(x,0,0) + E_q(x,0,0)\right] - 2S^q_z(x)\right\}|^{\mathsf{model}}$$

(iv) "Naive" version of $\langle\langle L^{lf,q}_{\text{can},z}\rangle\rangle$ from light-front wave functions

$$
\begin{aligned}
\mathcal{L}^q_{\text{can},z} \;&\equiv\; \langle\langle L^{lf,q}_{\text{can},z}\rangle\rangle\big|^{\text{naive,model}} \\
&= \sum_{\{\lambda\}} \sum_{l,r(q)} \int [\mathrm{d}x]_3\, [\mathrm{d}^2 k_\perp]_3\, \Psi_3^{*+}(\{x, \boldsymbol{k}_\perp, \lambda\}) \\
&\quad \times \left(\boldsymbol{k}_{r\perp} \times \tfrac{1}{i}\boldsymbol{\nabla}_{\boldsymbol{k}_{l\perp}}\right)_z \Psi_3^{+}(\{x, \boldsymbol{k}_\perp, \lambda\})
\end{aligned}
$$

(iv) "Naive" version of $\langle\langle L^{lf,q}_{\text{can},z}\rangle\rangle$ from light-front wave functions

$$\mathcal{L}^{q}_{\text{can},z} \equiv \langle\langle L^{lf,q}_{\text{can},z}\rangle\rangle\big|^{\text{naive,model}}$$
$$= \sum_{\{\lambda\}} \int [\mathrm{d}x]_3\,[\mathrm{d}^2k_\perp]_3\,\Psi^{*+}_3(\{x,\boldsymbol{k}_\perp,\lambda\})$$
$$\times \sum_{r(q)} \left(\boldsymbol{k}_{r\perp} \times \tfrac{1}{i}\boldsymbol{\nabla}_{\boldsymbol{k}_{r\perp}}\right)_z \Psi^{+}_3(\{x,\boldsymbol{k}_\perp,\lambda\})$$

Factor $(\delta_{rl} - x_l)$ is replaced by δ_{rl}

Therefore "angular momentum about ORIGIN "

not INTRINSIC

(v) "Naive" $\mathcal{L}^q_{\text{can},z}$ from Pretzelosity

In SOME models

She et al: PR D79, 054008 (2009); Avakian et al: PR D81, 074035 (2010)

find

$$\mathcal{L}^q_{\text{can},z} = -\int \mathrm{d}x\, \mathrm{d}^2k_\perp \frac{\boldsymbol{k}_\perp^2}{2M^2}\, h_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$$

Only valid in a restricted class of models

Lorce and Pasquini: PL B710 (2012) 486

Requires the instant-form wave function $\psi(\{\boldsymbol{k},\sigma\})$ to be a pure s-wave and related to the light-front wave function $\Psi(\{x,\boldsymbol{k}_\perp,\lambda\})$ by just a Wigner rotation

Only valid in a restricted class of models

Lorce and Pasquini: PL B710 (2012) 486

Requires the instant-form wave function $\psi(\{\boldsymbol{k}, \sigma\})$ to be a pure s-wave and related to the light-front wave function $\Psi(\{x, \boldsymbol{k}_\perp, \lambda\})$ by just a Wigner rotation

This is the case for following model results

Model	LFCQM			LFχQSM		
q	u	d	Total	u	d	Total
$\ell^q_{\mathrm{kin},z}$	0.071	0.055	0.126	-0.008	0.077	0.069
$\ell^q_{\mathrm{can},z}$	0.131	-0.005	0.126	0.073	-0.004	0.069
$\mathcal{L}^q_{\mathrm{can},z}$	0.169	-0.042	0.126	0.093	-0.023	0.069

Model	LFCQM			LFχQSM		
q	u	d	Total	u	d	Total
$\ell^q_{\mathrm{kin},z}$	0.071	0.055	0.126	-0.008	0.077	0.069
$\ell^q_{\mathrm{can},z}$	0.131	-0.005	0.126	0.073	-0.004	0.069
$\mathcal{L}^q_{\mathrm{can},z}$	0.169	-0.042	0.126	0.093	-0.023	0.069

▶Two models give very different results

Model	LFCQM			LFχQSM		
q	u	d	Total	u	d	Total
$\ell^q_{\text{kin},z}$	0.071	0.055	0.126	−0.008	0.077	0.069
$\ell^q_{\text{can},z}$	0.131	−0.005	0.126	0.073	−0.004	0.069
$\mathcal{L}^q_{\text{can},z}$	0.169	−0.042	0.126	0.093	−0.023	0.069

▶Two models give very different results

▶In each model "Total" is same for $\ell_{\text{kin},z}$, $\ell_{\text{can},z}$,$\mathcal{L}_{\text{can},z}$

Model	LFCQM			LFχQSM		
q	u	d	Total	u	d	Total
$\ell^q_{\text{kin},z}$	0.071	0.055	0.126	−0.008	0.077	0.069
$\ell^q_{\text{can},z}$	0.131	−0.005	0.126	0.073	−0.004	0.069
$\mathcal{L}^q_{\text{can},z}$	0.169	−0.042	0.126	0.093	−0.023	0.069

- ▶Two models give very different results
- ▶In each model “Total” is same for $\ell_{\text{kin},z}$, $\ell_{\text{can},z}$,$\mathcal{L}_{\text{can},z}$
- ▶ Clear that $\ell^q_{\text{can},z} \neq \mathcal{L}^q_{\text{can},z}$, but $\sum_q \ell^q_{\text{can},z} = \sum_q \mathcal{L}^q_{\text{can},z}$

Model	LFCQM			LFχQSM		
q	u	d	Total	u	d	Total
$\ell^q_{\text{kin},z}$	0.071	0.055	0.126	-0.008	0.077	0.069
$\ell^q_{\text{can},z}$	0.131	-0.005	0.126	0.073	-0.004	0.069
$\mathcal{L}^q_{\text{can},z}$	0.169	-0.042	0.126	0.093	-0.023	0.069

- ▶ Two models give very different results
- ▶ In each model "Total" is same for $\ell_{\text{kin},z}$, $\ell_{\text{can},z}$, $\mathcal{L}_{\text{can},z}$
- ▶ Clear that $\ell^q_{\text{can},z} \neq \mathcal{L}^q_{\text{can},z}$, but $\sum_q \ell^q_{\text{can},z} = \sum_q \mathcal{L}^q_{\text{can},z}$
- ▶ Puzzle: No A^μ in models, so why is $\ell^q_{\text{kin},z} \neq \ell^q_{\text{can},z}$?

Model	LFCQM			LFχQSM		
q	u	d	Total	u	d	Total
$\ell^q_{\text{kin},z}$	0.071	0.055	0.126	−0.008	0.077	0.069
$\ell^q_{\text{can},z}$	0.131	−0.005	0.126	0.073	−0.004	0.069
$\mathcal{L}^q_{\text{can},z}$	0.169	−0.042	0.126	0.093	−0.023	0.069

- ▶Two models give very different results
- ▶In each model "Total" is same for $\ell_{\text{kin},z}$, $\ell_{\text{can},z}$,$\mathcal{L}_{\text{can},z}$
- ▶ Clear that $\ell^q_{\text{can},z} \neq \mathcal{L}^q_{\text{can},z}$, but $\sum_q \ell^q_{\text{can},z} = \sum_q \mathcal{L}^q_{\text{can},z}$
- ▶ Puzzle: No A^μ in models, so why is $\ell^q_{\text{kin},z} \neq \ell^q_{\text{can},z}$?
- ▶ Because Ji relation for $\ell^q_{\text{kin},z}$ uses QCD energy-momentum tensor, different from models ??
- ▶ Then why is $\sum_q \ell^q_{\text{kin},z} = \sum_q \ell^q_{\text{can},z}$??

Further puzzles

Burkardt and Hikmat [PR D79, 071501 (2009)] calculated $\ell^q_{\text{kin},z}$ via the Ji relation and $\ell^q_{\text{can},z}$ directly from the wave functions in the scalar diquark model (no A^μ)

Here they obtained $\ell^q_{\text{kin},z} = \ell^q_{\text{can},z}$

Further puzzles

Burkardt and Hikmat [PR D79, 071501 (2009)] calculated $\ell^q_{\text{kin},z}$ via the Ji relation and $\ell^q_{\text{can},z}$ directly from the wave functions in the scalar diquark model (no A^μ)

Here they obtained $\ell^q_{\text{kin},z} = \ell^q_{\text{can},z}$

BUT for the density in Bjorken-x found

$$\ell^q_{\text{kin},z}(x) \neq \ell^q_{\text{can},z}(x)$$

Suggests that the Ji relation does not hold for the densities in x-space

$$\langle\langle J^q_{\text{Ji},z}(x)\rangle\rangle \neq \tfrac{1}{2}x\,[H_q(x,0,0) + E_q(x,0,0)]$$

contrary to claim of Hoodbhoy, Ji and Lu [PR D59, 014013 (1998)]

CONCLUSIONS

- Exist many other, so called, Gauge Invariant Extensions. Theoretically interesting but no new physics.

CONCLUSIONS

- Exist many other, so called, Gauge Invariant Extensions. Theoretically interesting but no new physics.

- Physically relevant 3 versions of OAM:

$$\langle\langle L^{inst,q}_{\mathrm{can},z}\rangle\rangle\Big|_{A^+=0} \qquad \langle\langle L^{lf,q}_{\mathrm{can},z}\rangle\rangle\Big|_{A^+=0} \qquad \langle\langle L^{q}_{\mathrm{Ji},z}\rangle\rangle$$

CONCLUSIONS

- Exist. many other, so called, Gauge Invariant Extensions. Theoretically interesting but no new physics.

- Physically relevant 3 versions of OAM:

$$\langle\langle L^{\textit{inst},q}_{\text{can},z}\rangle\rangle\Big|_{A^+=0} \qquad \langle\langle L^{\textit{lf},q}_{\text{can},z}\rangle\rangle\Big|_{A^+=0} \qquad \langle\langle L^{q}_{\text{Ji},z}\rangle\rangle$$

- All can be related to, in principle, measurable quantities like GPDs and GTMDs, but difficult and is challenge for the future

- $\langle\langle L^q_{\mathrm{Ji},z}\rangle\rangle$ can be calculated on Lattice $\equiv L_q$. Beautiful results. Disconnected Insertions very important. At 2 $GeV \quad L_u - L_d < 0$.

- $\langle\langle L^q_{\text{Ji},z}\rangle\rangle$ can be calculated on Lattice $\equiv L_q$. Beautiful results. Disconnected Insertions very important. At $2\,GeV \quad L_u - L_d < 0$.

- $\langle\langle L^q_{\text{Ji},z}\rangle\rangle$ can be calculated in models $\equiv \ell^q_{\text{kin},z}$. Almost all give $\ell^u_{\text{kin},z} - \ell^d_{\text{kin},z} > 0$ AT LOW SCALE. Maybe compatible with Lattice via evolution

- $\langle\langle L^q_{\text{Ji},z}\rangle\rangle$ can be calculated on Lattice $\equiv L_q$. Beautiful results. Disconnected Insertions very important. At $2\,GeV \quad L_u - L_d < 0$.

- $\langle\langle L^q_{\text{Ji},z}\rangle\rangle$ can be calculated in models $\equiv \ell^q_{\text{kin},z}$. Almost all give $\ell^u_{\text{kin},z} - \ell^d_{\text{kin},z} > 0$ AT LOW SCALE. Maybe compatible with Lattice via evolution

- $\langle\langle L^q_{\text{can},z}\rangle\rangle$ can be calculated in models $\equiv \ell^q_{\text{can},z}$. Distinguish about what point the OAM is defined: "Naive" about Origin; or 'Intrinsic" about Transverse centre of momentum.

- Interesting Theoretical Puzzles:

- Seems that

$$\langle\langle J^q_{\mathrm{Ji},z}(x)\rangle\rangle \neq \frac{1}{2}x\left[H_q(x,0,0)+E_q(x,0,0)\right]$$

Why??

- Interesting Theoretical Puzzles:

- Seems that

$$\langle\langle J^q_{\text{Ji},z}(x)\rangle\rangle \neq \frac{1}{2}x\left[H_q(x,0,0)+E_q(x,0,0)\right]$$

  Why??

- When no vector potential A^μ, why is $\ell^q_{\text{kin},z} \neq \ell^q_{\text{can},z}$ yet

$$\sum_q \ell^q_{\text{kin},z} = \sum_q \ell^q_{\text{can},z} \quad ?$$