Phenomenology of TMDs

Transversity 2014
Transverse polarisation phenomena in hard processes
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Outline

- Unpolarized Drell-Yan
- Unpolarized SIDIS
- Sivers effect
- Conclusions

Unpolarized data phenomenology

Unpolarized data phenomenology

- Tmd factorization has been proved for two kinds of processes:

DRELL-YAN

- $\sqrt{s}$~20-69 GeV; 1-7 TeV
- 4<Q<9; 10.5<Q<25 GeV; M_{Z_0}
- 0.1<P_T<tens GeV; 1-hundreds GeV

- (Absolute) Cross sections
- $\langle P_T^2 \rangle$
- Azimuthal asymmetries

SIDIS

(JLAB,HERMES,COMPASS)

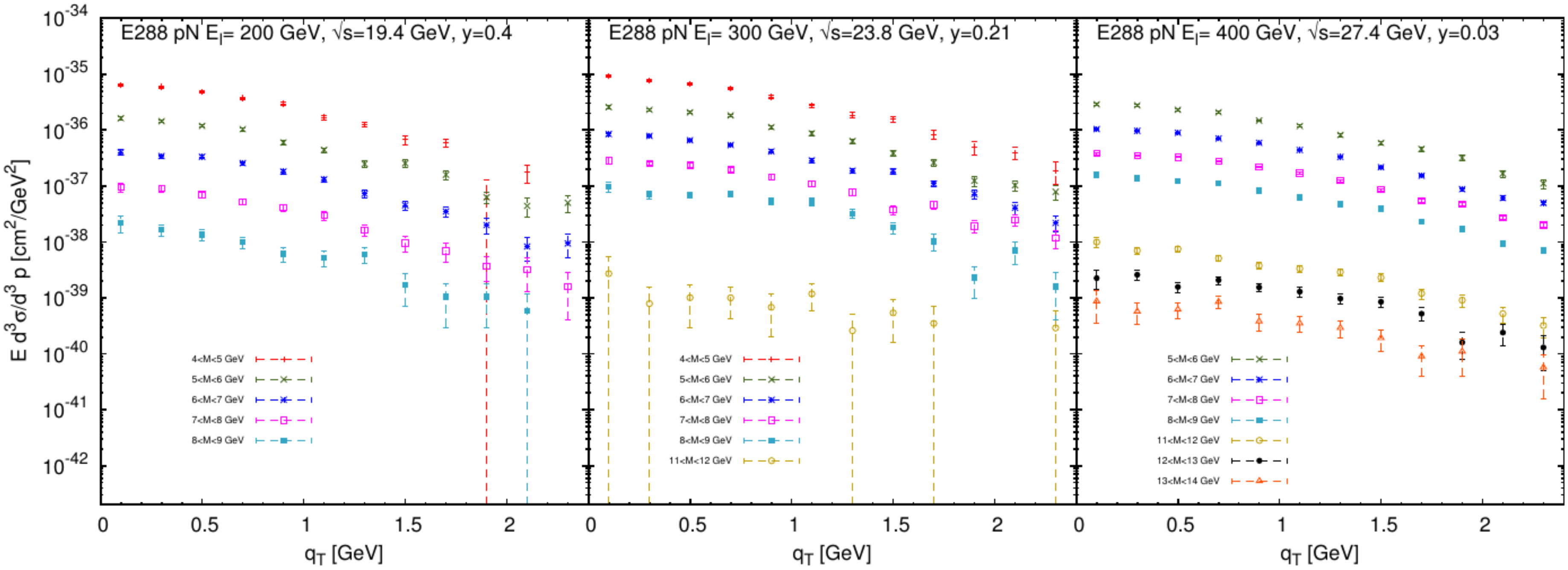
- $\sqrt{s}$~3.6-7-18 GeV
- 1<Q<3.2 GeV
- 0.1<P_T<few GeV

- Multiplicity
- $\langle P_T^2 \rangle$
- Azimuthal asymmetries

Unpolarized Drell-Yan phenomenology

Drell-Yan phenomenology

- Low energy data example: FERMILAB E288 at 3 different energies



- The P_T distributions seem to be Gaussian....

Drell-Yan phenomenology

- Simple phenomenological ansatz

$$f_{q/p}(x, k_\perp) = f(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

→ Factorization of longitudinal and transverse degrees of freedom;
Gaussian distribution of transverse momentum

In this way the distribution in P_T is just a guassian! →

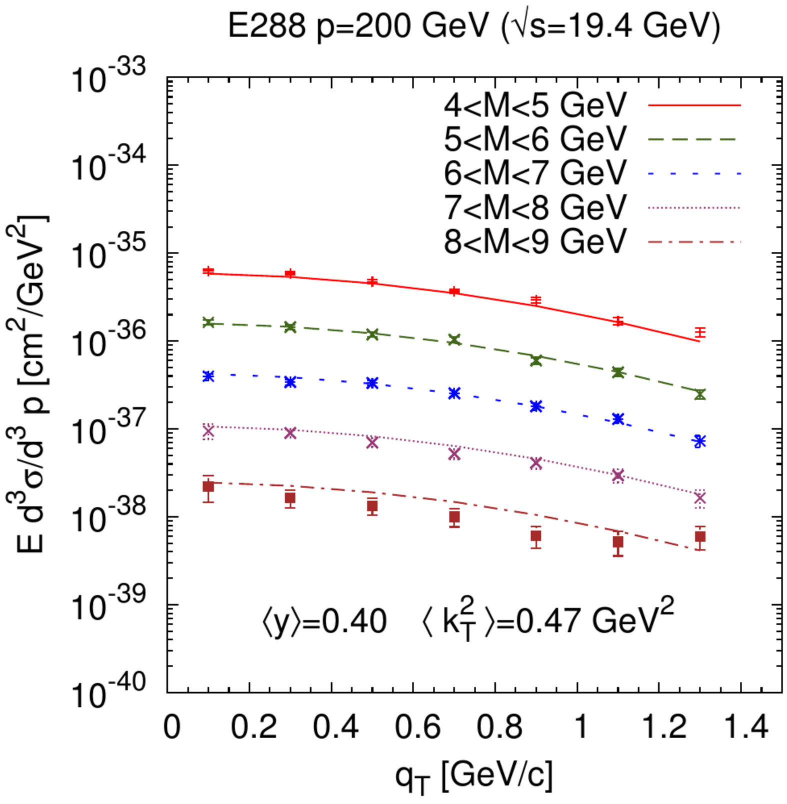
$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$

- Where for pp or pN scattering we just have: $\langle P_T^2 \rangle = 2 \langle k_\perp^2 \rangle$

Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1)\bar{f}_{q/h_2}(x_2)\frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$



Nice!

Further information: the M^2 dependence is described by model and it is given by the interplay between $1/M^2$ born cross section +DGLAP+Kinematics

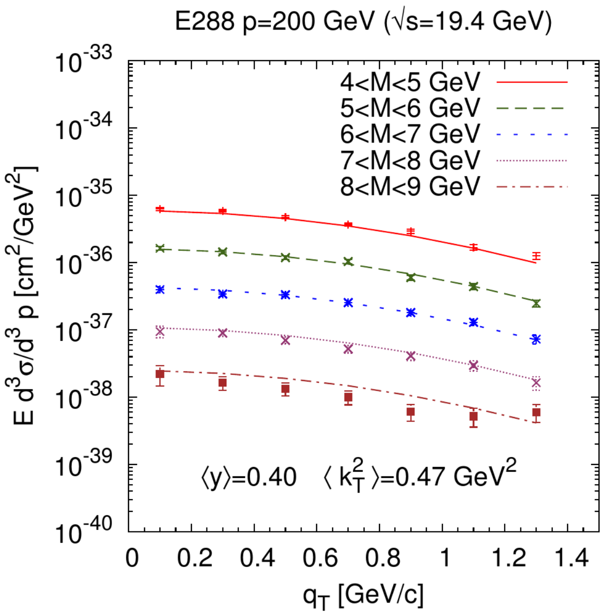
Is the width of the gaussian a measure of transverse momentum?

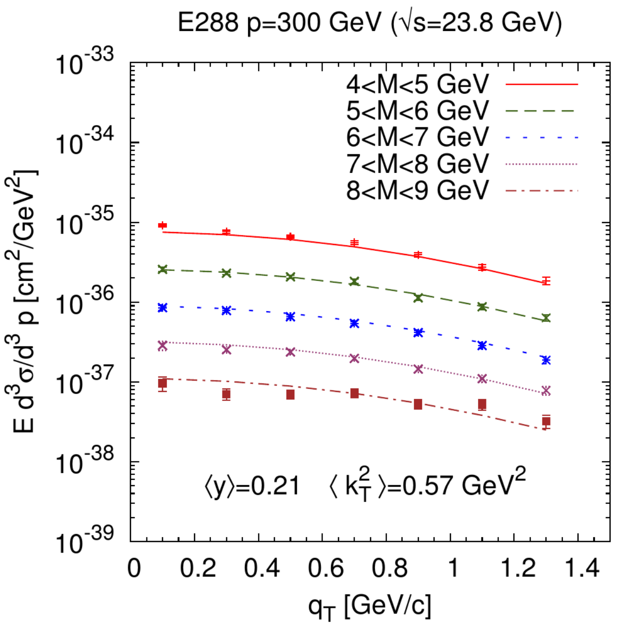
The model does not answer directly to this question.

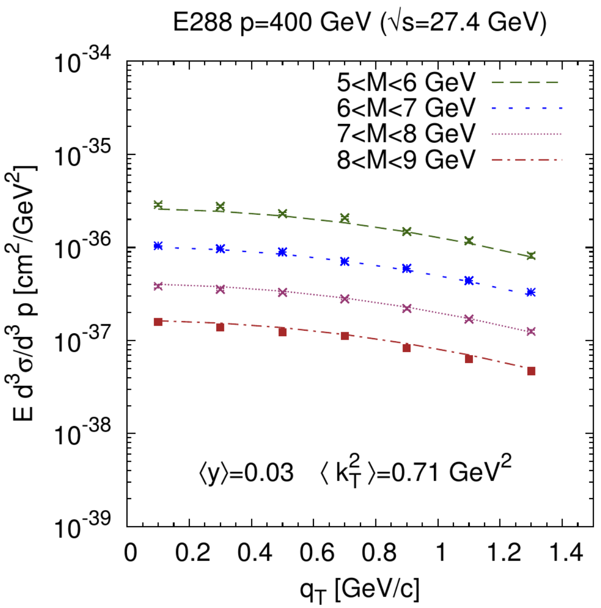
Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1)\bar{f}_{q/h_2}(x_2)\frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$


E288 p=200 GeV (√s=19.4 GeV)


E288 p=300 GeV (√s=23.8 GeV)

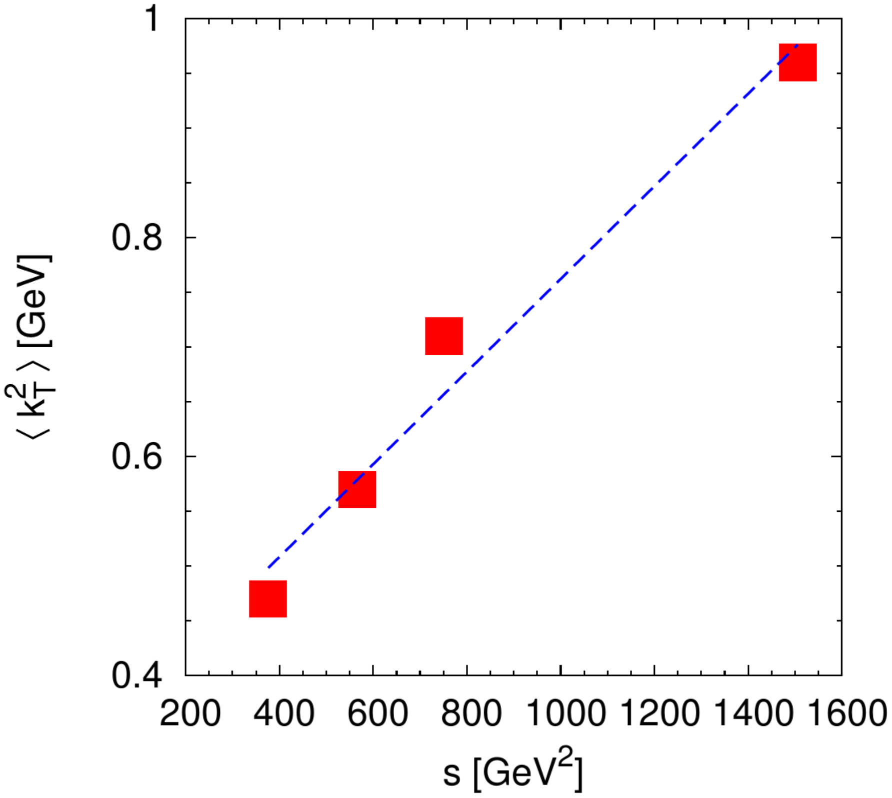

E288 p=400 GeV (√s=27.4 GeV)

- Each data set is gaussian but with a different width

Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$



- QCD prediction?

$$\langle K_\perp^2 \rangle^{\text{pert.}} = \alpha_s(Q^2)\, S\, f(\tau, \alpha_s(Q^2)) + \ldots$$

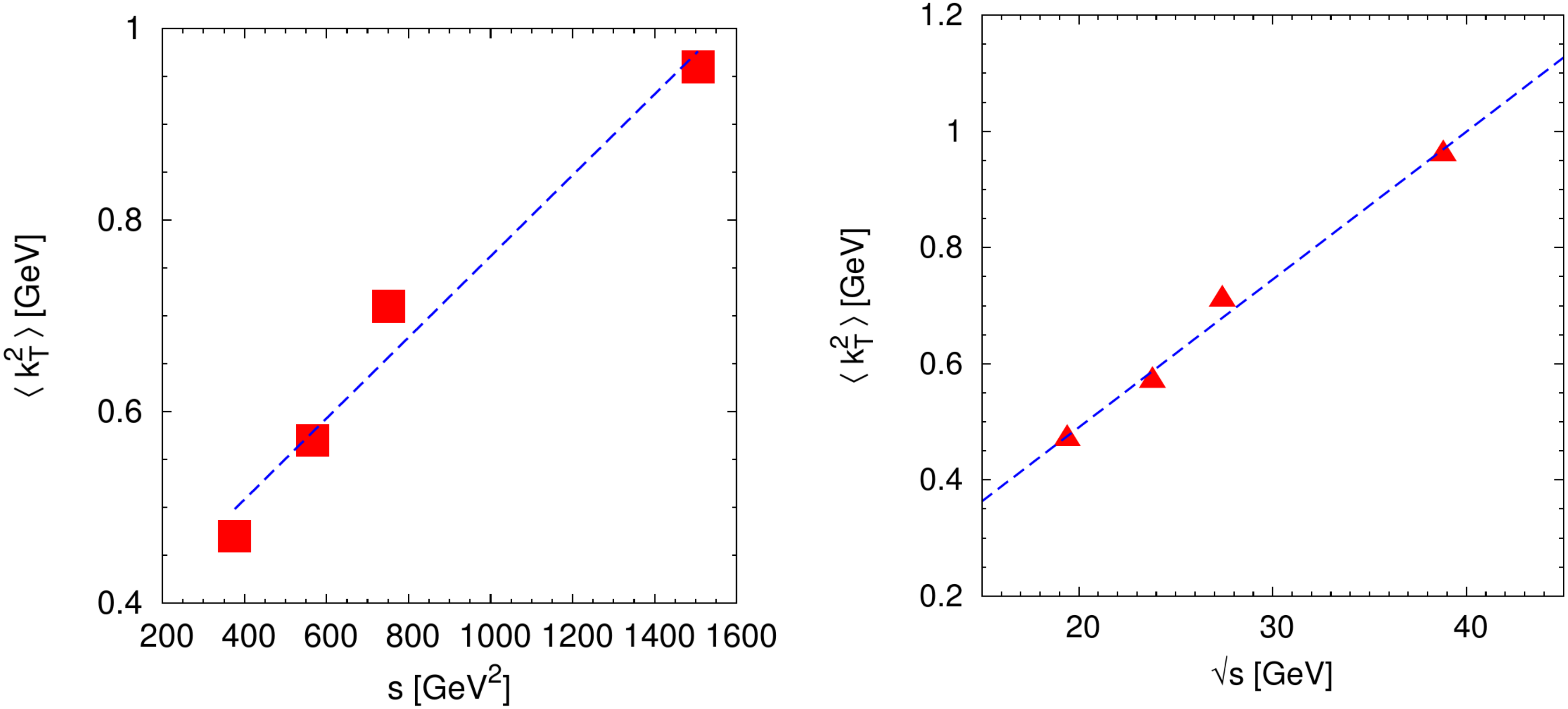
- Altarelli, Parisi and Petronzio
  Phys.Lett. B76 (1978) 351

See, for SIDIS, also
Schweitzer, Metz, Teckentrup
Phys.Rev. D81 (2010) 094019

Drell-Yan phenomenology

- Are data gaussian distributed?

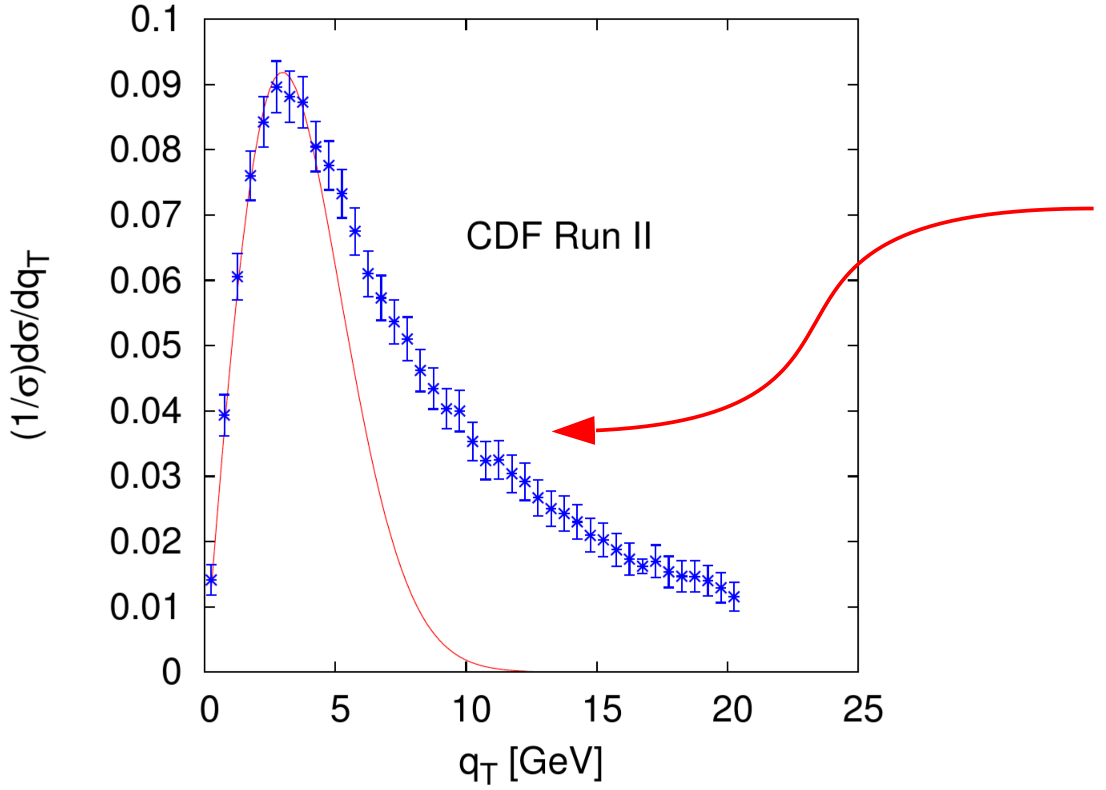
$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2}\sum_q f_{q/h_1}(x_1)\bar{f}_{q/h_2}(x_2)\frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$



Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1)\bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$

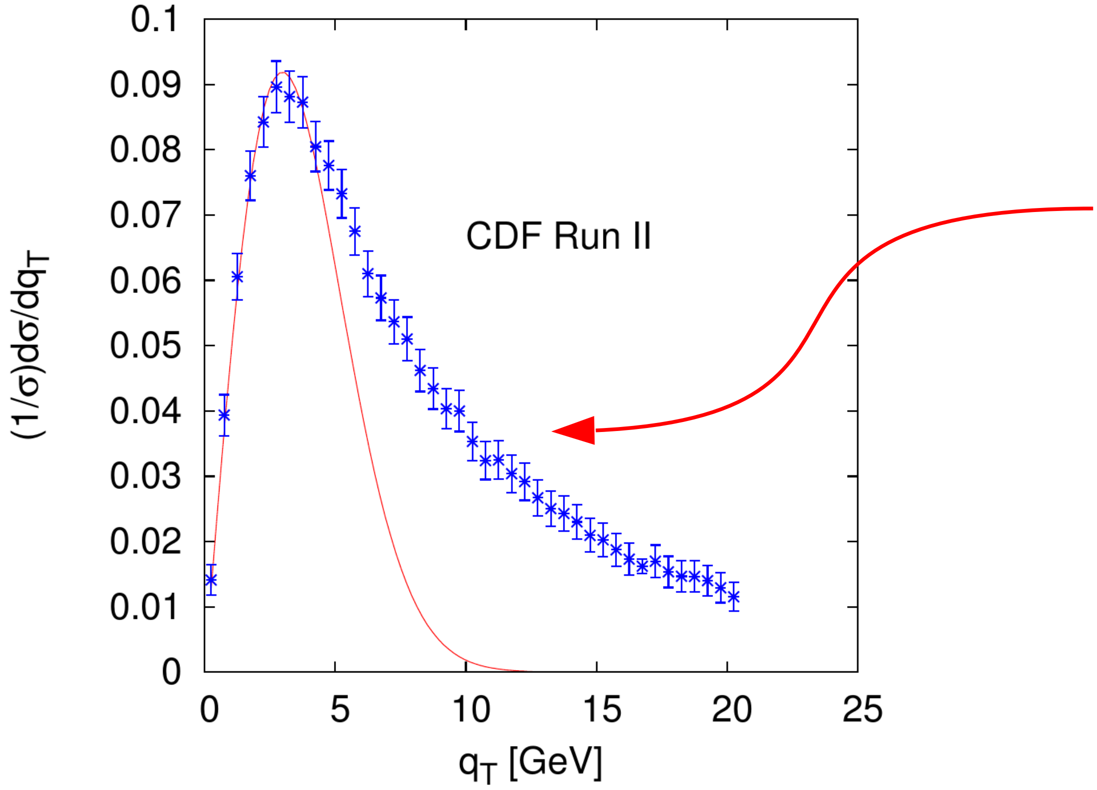


Clearly it is not a Gaussian tail.

Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$



Clearly it is not a Gaussian tail.

- The tail is generated by Soft Gluon emissions that can be treated using QCD

Drell-Yan phenomenology

- Resummation: CSS

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2dydq_T^2} = \int\frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j(x_1,x_2,b_T,Q) + Y(x_1,x_2,q_T,Q)$$

Soft gluon emissions resummed in b-space

Regular part

$$\boxed{W_j(x_1,x_2,b_T,Q) = \exp\left[S_j(b_T,Q)\right]\sum_{i,k} C_{ji}\otimes f_i(x_1,C_1^2/b_T^2)\; C_{\bar{j}k}\otimes f_k(x_2,C_1^2/b_T^2)}$$

Sudakov factor

$$S_j(b_T,Q) = \int_{C_1^2/b_T^2}^{Q^2}\frac{d\kappa^2}{\kappa^2}\left[A_j(\alpha_s(\kappa))\ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa))\right]$$

$$C_1 = 2\exp(-\gamma_E)$$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

CSS formalism

- We can resum pertubartively the soft gluon only up to some value of b_T!

$$W_j(x_1, x_2, b_T, Q) = \exp\left[S_j(b_T, Q)\right] \sum_{i,k} C_{ji} \otimes f_i(x_1, C_1^2/b_T^2)\, C_{\bar{j}k} \otimes f_k(x_2, C_1^2/b_T^2)$$

$$S_j(b_T, Q) = -\int_{C_1^2/b_T^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} \left[A_j(\alpha_s(\kappa)) \ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa))\right]$$

$$\mu = \frac{C_1}{b_T}$$

At large b_T the scale μ becomes too small!

This correspond to the case when: $Q^2 \gg q_T^2 \simeq \Lambda_{QCD}^2$

Very small transverse momenta are non perturbative
They cannot be treated by pQCD, we need a phenomenological prescription

$$C_1 = 2\exp(-\gamma_E)$$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

CSS formalism

- Let us freeze the scale when we reach a non perturbative region and define:

$$b_T \longrightarrow b_* = \frac{b_T}{\sqrt{1 + b_T^2 / b_{max}^2}} \qquad \mu = \frac{C_1}{b_T} \longrightarrow \mu_b = C_1 / b_*$$

And then we define a non perturbative function for large b_T:

$$\frac{W_j(x_1, x_2, b_T, Q)}{W_j(x_1, x_2, b_*, Q)} = F_{NP}(x_1, x_2, b_T, Q)$$

$$W_j(x_1, x_2, b_T, Q) = \sum_{i,k} \exp\left[S_j(b_*, Q)\right] \left[C_{ji} \otimes f_i\left(x_1, \mu_b\right)\right] \left[C_{\bar{j}k} \otimes f_k\left(x_2, \mu_b\right)\right] F_{NP}(x_1, x_2, b_T, Q)$$

$b_*,\ \mu_b$

b_T

$$C_1 = 2\exp(-\gamma_E)$$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

CSS Phenomenology

To perform phenomenological studies you need a non perturbative function.

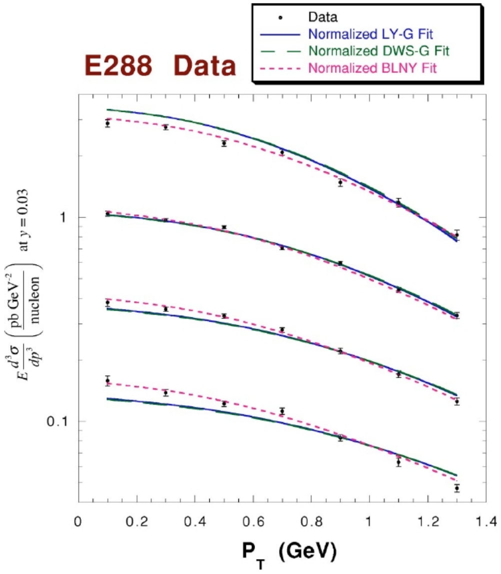
$$F_{NP}(x_1, x_2, b_T, Q)$$

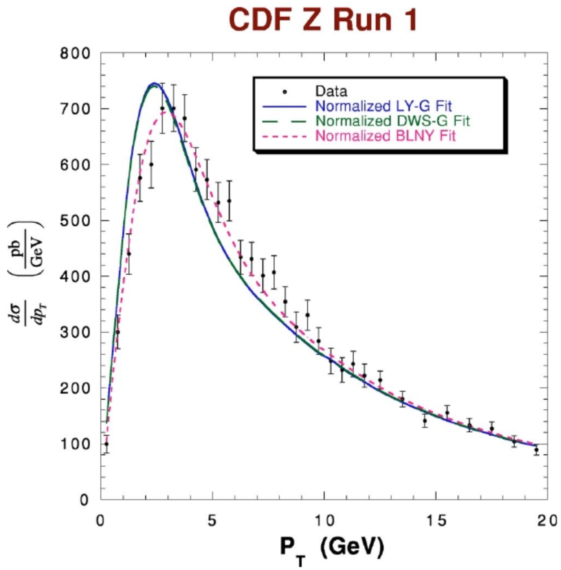
Davies-Webber-Stirling (DWS)	$\exp\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right)\right] b^2;$
Ladinsky-Yuan (LY)	$\exp\left\{\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right)\right] b^2 - [g_1 g_3 \ln(100 x_1 x_2)] b\right\};$
Brock-Landry- Nadolsky-Yuan (BLNY)	$\exp\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln(100 x_1 x_2)\right] b^2$

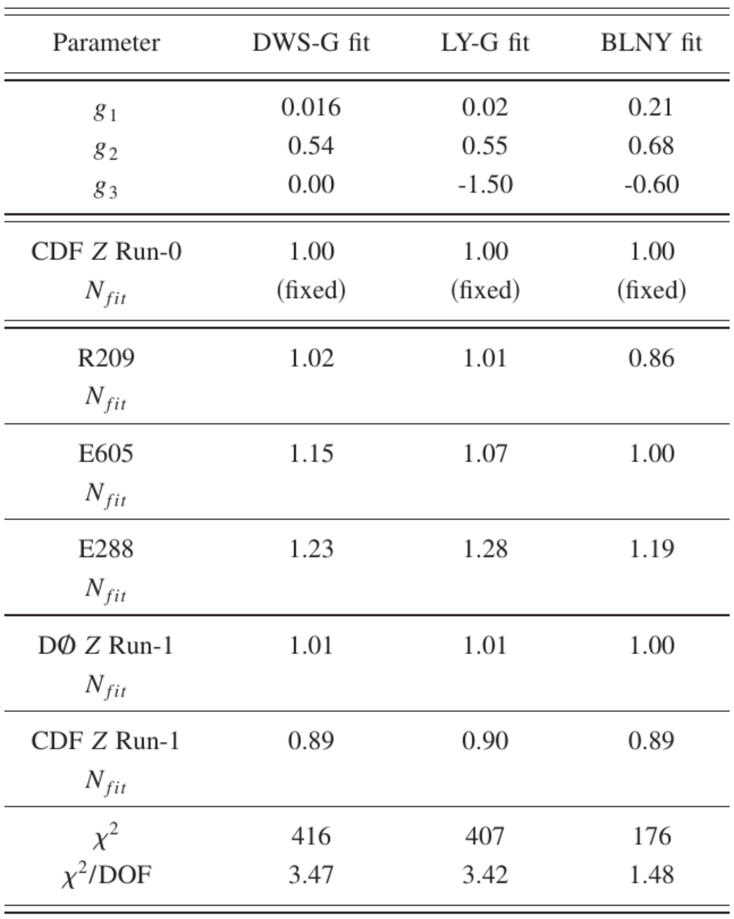
Nadolsky et al., Phys.Rev. D67,073016 (2003)

CSS Phenomenology

Nadolsky et al.* analyzed successfully low energy DY data and Z_0 production data using different parametrizations







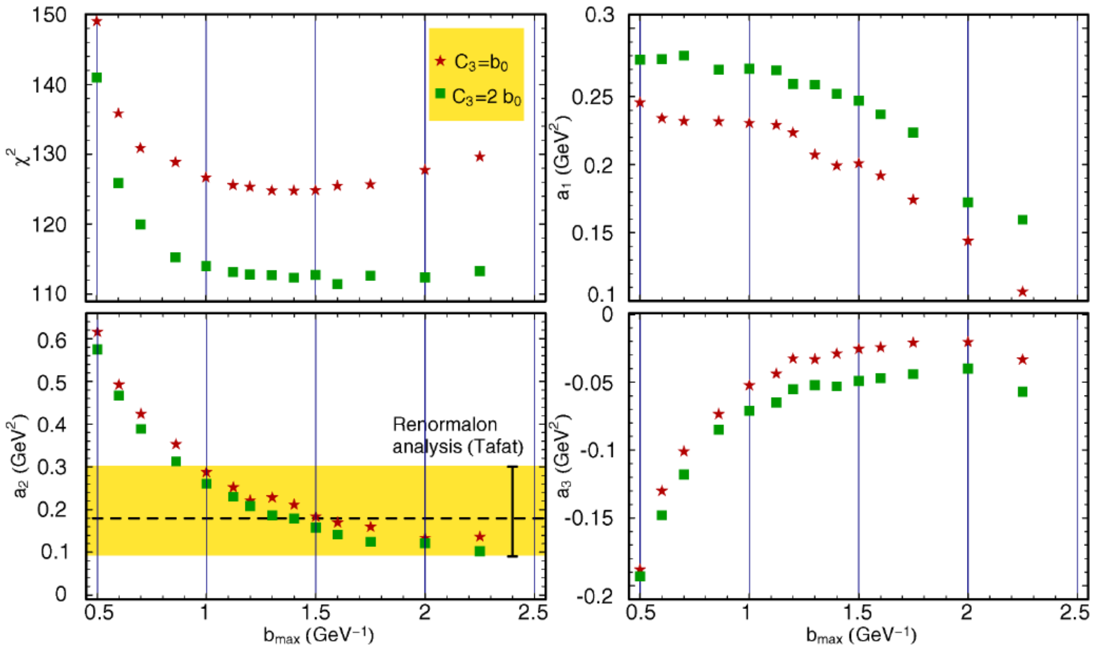
Parameter	DWS-G fit	LY-G fit	BLNY fit
g_1	0.016	0.02	0.21
g_2	0.54	0.55	0.68
g_3	0.00	-1.50	-0.60
CDF *Z* Run-0 N_{fit}	1.00 (fixed)	1.00 (fixed)	1.00 (fixed)
R209 N_{fit}	1.02	1.01	0.86
E605 N_{fit}	1.15	1.07	1.00
E288 N_{fit}	1.23	1.28	1.19
DØ *Z* Run-1 N_{fit}	1.01	1.01	1.00
CDF *Z* Run-1 N_{fit}	0.89	0.90	0.89
χ^2 χ^2/DOF	416 3.47	407 3.42	176 1.48

$b_{max} = 0.5 \text{ GeV}^{-1}$

*Nadolsky et al., Phys.Rev. D67,073016 (2003)

CSS cooking....

Konychev and Nadolsky Phys. Lett. B33,710 (2006)



$$C_3 = b_0 \qquad \mu_F \equiv \mu_b$$
$$f(x,\mu_F) \equiv f(x,\mu_b)$$

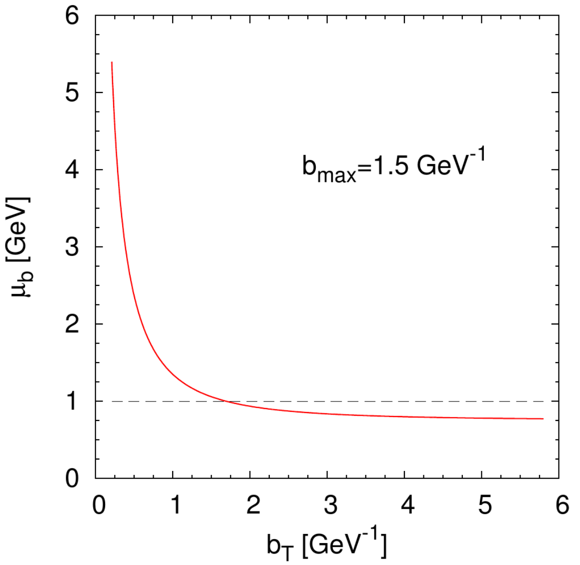
$$\mu_F = 2\mu_b$$
$$C_3 = 2b_0 \qquad f(x,\mu_F) \equiv f(x,2\mu_b)$$

- Larger b_{max} improves the convergence but...

$$b_* = \frac{b_T}{\sqrt{1+b_T^2/b_{max}^2}} \qquad \mu_b = C_1/b_* \qquad C_1 = 2\exp(-\gamma_E)$$

μ_b can become less then 1 GeV

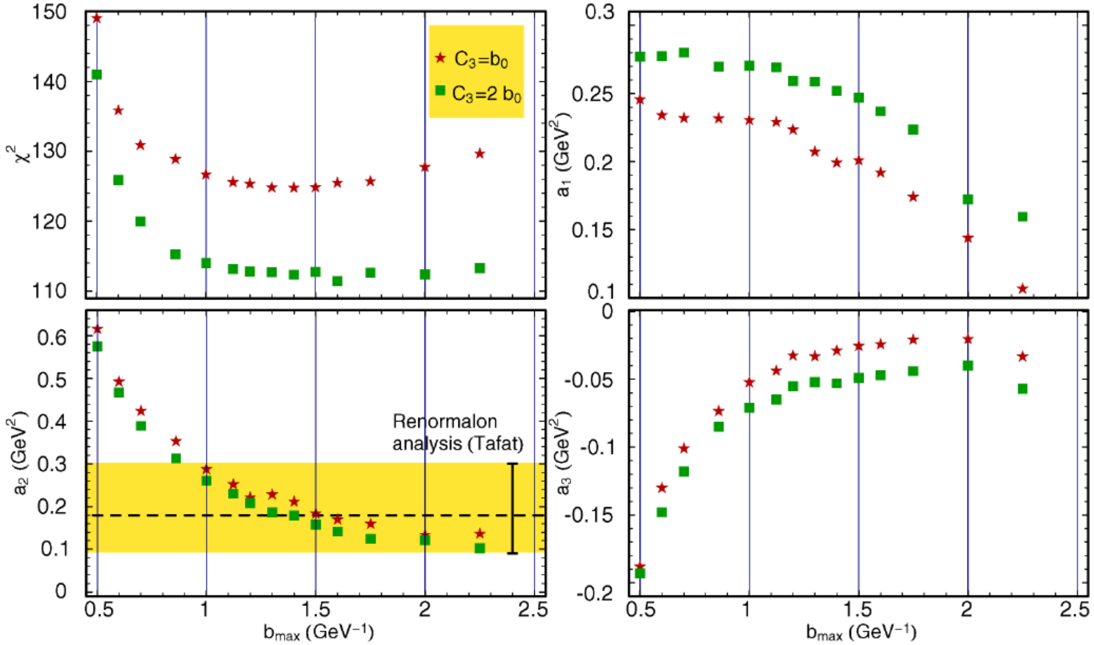
(it does for b_{max}=1.5 GeV^{-1} !)



- Freezing, Shifting, where?

CSS cooking....

Konychev and Nadolsky Phys. Lett. B33,710 (2006)



$$C_3 = b_0$$

$$\mu_F \equiv \mu_b$$

$$f(x,\mu_F) \equiv f(x,\mu_b)$$

$$C_3 = 2b_0$$

$$\mu_F = 2\mu_b$$

$$f(x,\mu_F) \equiv f(x,2\mu_b)$$

- From the other side... μ_b becomes infinity at b=0

$$b_* = \frac{b_T}{\sqrt{1+b_T^2/b_{max}^2}}$$

$$\mu_b = C_1/b_*$$

$$C_1 = 2\exp(-\gamma_E)$$

- Freezing? How? Where?

See Ellis et al. Nucl.Phys. B503 (1997) 309
Qiu et al. Phys.Rev. D63 (2001) 11401
Complex plane method(Werner Vogelsang)

- The b* prescrition is a model!

- and talk by Ignazio Scimemi

Unpolarized SIDIS phenomenology

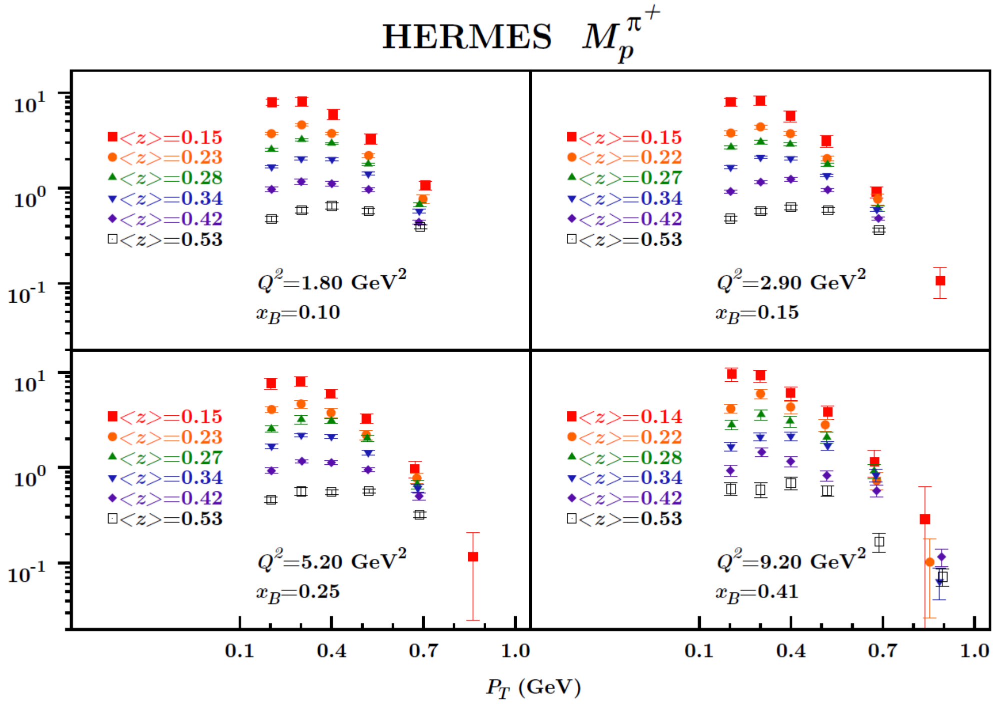
SIDIS phenomenology

SIDIS

(JLAB,HERMES,COMPASS)

- $\sqrt{s}$~3.6-7-18 GeV
- $1<Q<3.2$ GeV
- $0.1<P_T<$few GeV

- Multiplicity
- $\langle P_T^2\rangle$
- Azimuthal asymmetries



Selection of multidimensional HERMES data

SIDIS phenomenology

- Simple phenomenological ansatz

$$f_{q/p}(x,k_\perp) = f(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle} \qquad D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

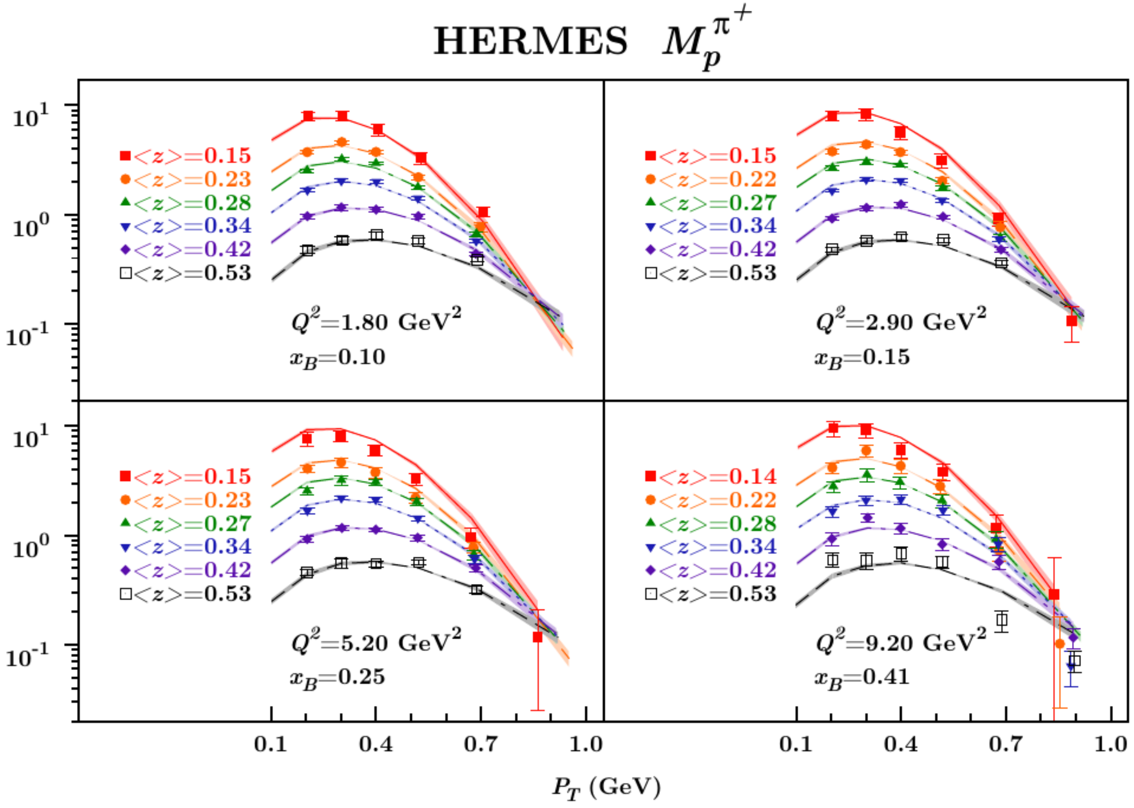
$$F_{UU} = \sum_q e_q^2\, f_{q/p}(x_B)\, D_{h/q}(z_h)\,\frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$$

SIDIS phenomenology

$$F_{UU} = \sum_q e_q^2 \, f_{q/p}(x_B) \, D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$



$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \ \mathrm{GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \ \mathrm{GeV}^2$$

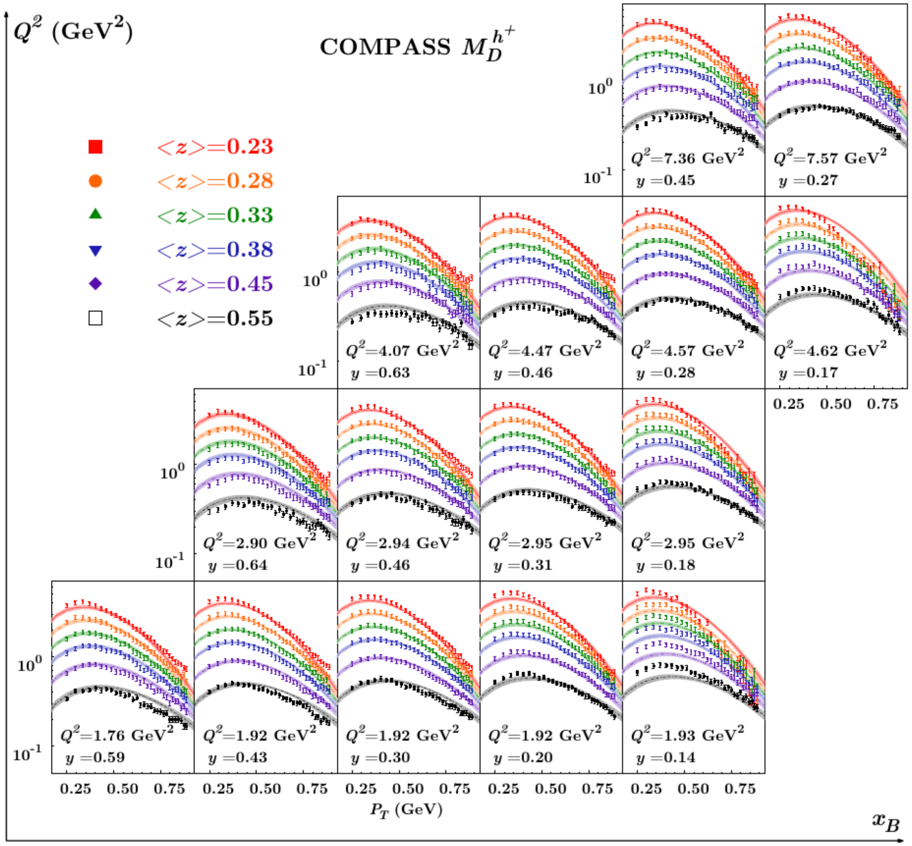
$$\chi^2_{\mathrm{dof}} = 1.69$$

See talk by Osvaldo Gonzales

SIDIS phenomenology

$$F_{UU} = \sum_q e_q^2 \, f_{q/p}(x_B) \, D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$



$$\langle k_\perp^2 \rangle = 0.60 \pm 0.14 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.20 \pm 0.02 \text{ GeV}^2$$

$$\chi^2_{\rm dof} = 3.42$$

$$N_y = A + B\, y$$

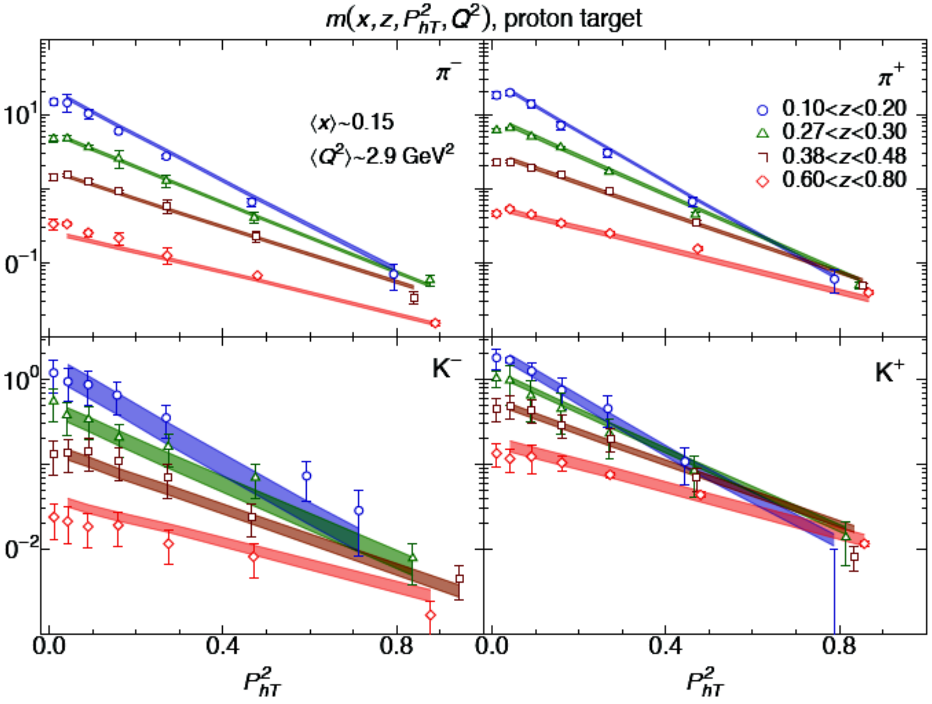
See talk by Osvaldo Gonzales

SIDIS phenomenology

- Gaussians but flavor dependent, x dependent, z dependent....

$$f_{q/p}(x,k_\perp) = f(x)\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle} \qquad D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$\langle \boldsymbol{k}_{\perp,\,q}^2\rangle(x) = \langle\widehat{\boldsymbol{k}_{\perp,\,q}^2}\rangle\,\frac{(1-x)^\alpha\,x^\sigma}{(1-\hat{x})^\alpha\,\hat{x}^\sigma} \qquad \langle \boldsymbol{P}_{\perp,\,q\to h}^2\rangle(z) = \langle\widehat{\boldsymbol{P}_{\perp,\,q\to h}^2}\rangle\,\frac{(z^\beta+\delta)\,(1-z)^\gamma}{(\hat{z}^\beta+\delta)\,(1-\hat{z})^\gamma}$$



proton target global χ^2 / d.o.f. = 1.63 ± 0.12

no flavor dep. 1.72 ± 0.11

<u>See talk by Andrea Signori</u>

CSS Resummation and TMD evolution (2011)

CSS Resummation and TMD evolution (2011)

- 2011 - Proper definition of a TMD (in b space):

$$
\begin{aligned}
\tilde{F}(x, b_T, Q, \zeta_F) \quad = \quad & \sum_j \int_x^1 \frac{dy}{y} \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) f_j(y, \mu_b) \\
& \exp\left\{ \ln\left(\frac{\sqrt{\zeta_F}}{\mu_b}\right) \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln\left(\frac{\sqrt{\zeta_F}}{\kappa}\right) \gamma_K(\kappa) \right\} \\
& \exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right) \right\}
\end{aligned}
$$

New scale ς_F related to rapidity divergences

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

CSS Resummation and TMD evolution (2011)

- In phenomenological applications:

$$\zeta_F = Q^2 \qquad b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu_b = C_1/b_*$$

- And previous expression simplify considerably:

$$\begin{aligned}
\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) \quad = \quad & \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \\
& \exp\left\{ \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; Q^2/\kappa^2) \right\} \\
& \exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) \right\}
\end{aligned}$$

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

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- And previous expression simplify considerably:

$$\tilde{F}(x,b_T,Q,\zeta_F \equiv Q^2) \;=\; \boxed{\sum_j \tilde{C}_{f/j}(x/y,b_*,\mu_b,\mu_b^2)\otimes f_j(y,\mu_b)}$$
$$\exp\left\{\int_{\mu_b}^{Q}\frac{d\kappa}{\kappa}\gamma_F(\kappa;Q^2/\kappa^2)\right\}$$
$$\exp\left\{-g_P(x,b_T)-g_K(b_T)\ln\left(\frac{Q}{Q_0}\right)\right\}$$

Convolution of the collinear PDFs
with the Wilson coefficient

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

CSS Resummation and TMD evolution (2011)

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$$\zeta_F = Q^2 \qquad b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu_b = C_1/b_*$$

- And previous expression simplify considerably:

$$\begin{aligned}\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) \quad = \quad & \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \\ & \boxed{\exp\left\{ \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; Q^2/\kappa^2) \right\}} \\ & \exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0} \right) \right\}\end{aligned}$$

Sudakov factor

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

CSS Resummation and TMD evolution (2011)

- In phenomenological applications:

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- And previous expression simplify considerably:

$$\begin{aligned}\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) \quad = \quad & \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \\ & \exp\left\{\int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; Q^2/\kappa^2)\right\} \\ & \boxed{\exp\left\{-g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right)\right\}}\end{aligned}$$

Non perturbative function

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

CSS Resummation and TMD evolution (2011)

- In phenomenological applications:

$$\zeta_F = Q^2 \qquad b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu_b = C_1/b_*$$

- And previous expression simplify considerably:

$$\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) = \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b)$$

$$\boxed{\exp\left\{ \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; Q^2/\kappa^2) \right\}}$$

Sudakov factor

$$\exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) \right\}$$

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

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- And previous expression simplify considerably:

$$\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) \;=\; \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b)$$
$$\boxed{\exp\left\{\frac{1}{2} S^{CSS}(b_*, \mu_b)\right\}}$$
$$\exp\left\{-g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right)\right\}$$

Sudakov factor is the same of CSS!
Roughly speaking the TMD evolution reduced to a CSS resummation.
CSS results can be used to study TMDs.

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

Alternative evolution equation

$$\begin{aligned}
\frac{\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2)}{\tilde{F}(x, b_T, Q_0, \zeta_{F0} \equiv Q_0^2)} &= \exp\left\{ \int_Q^{Q_0} \frac{d\kappa}{\kappa} \left[\gamma_F(\kappa; 1) - \gamma_K(\kappa) \ln(Q/\kappa) \right] \right\} \\
&\quad \exp\left[- \int_{\mu_b}^{Q_0} \frac{d\kappa}{\kappa} \gamma_K(\kappa) \ln(Q/Q_0) \right] \exp\left[-g_K(b_T) \ln(Q/Q_0) \right] \\
&= \tilde{R}(Q, Q_0, b_T) \exp\left[-g_K(b_T) \ln(Q/Q_0) \right]
\end{aligned}$$

$$\boxed{\tilde{F}(x, b_T, Q, Q^2) = \tilde{F}(x, b_T, Q_0, Q_0^2) \tilde{R}(Q, Q_0, b_T) \ \exp\left[-g_K(b_T) \ln(Q/Q_0) \right]}$$

Output function

Input function

Notice that:

$$\frac{\tilde{f}'^{\perp}_{1T}(x, b_T, Q, \zeta_F)}{\tilde{f}'^{\perp}_{1T}(x, b_T, Q_0, \zeta_{F0})} = \frac{\tilde{f}_1(x, b_T, Q, \zeta_F)}{\tilde{f}_1(x, b_T, Q_0, \zeta_{F0})} \equiv \frac{\tilde{F}(x, b_T, Q, \zeta_F)}{\tilde{F}(x, b_T, Q_0, \zeta_{F0})}$$

Aybat, Collins, Qiu, Rogers, Phys. Rev. D85, 034043 (2012)

EIKV phenomenology

- TMD evolution in the CSS-like version

$$\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) \quad = \quad \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b)$$
$$\exp\left\{\frac{1}{2} S^{CSS}(b_*, \mu_b)\right\}$$
$$\exp\left\{-g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right)\right\}$$

- Some approximations to make life simpler

$$\tilde{C}_{ji}(z, \alpha(\mu)) = \delta_{ij}\delta(1-z)$$ At LO; PDF at LO

- Simple parametrizations for the non-perturbative part:

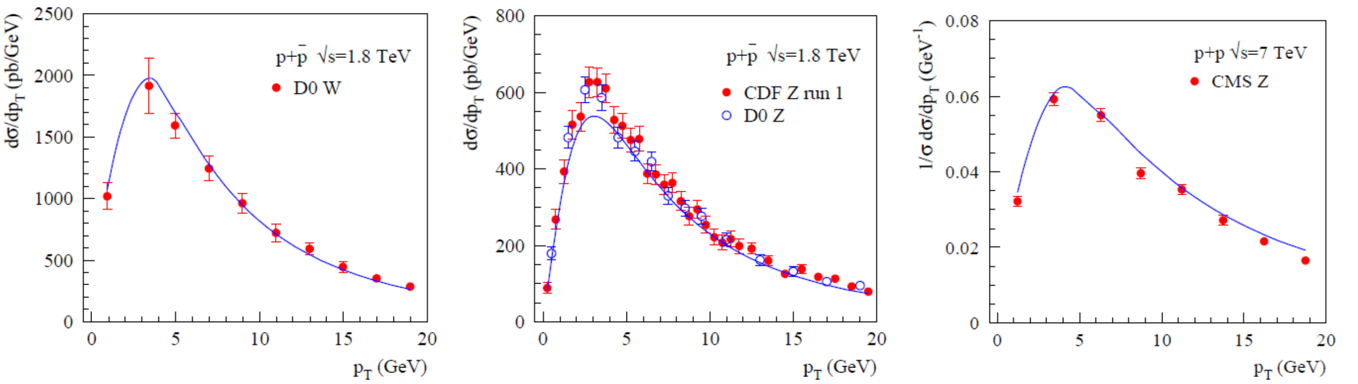
$$F_{NP}(b_T, Q)^{\mathrm{pdf}} = \exp\left[-b_T^2 \left(g_1^{\mathrm{pdf}} + \frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

$$F_{NP}(b_T, Q)^{\mathrm{ff}} = \exp\left[-b_T^2 \left(g_1^{\mathrm{ff}} + \frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

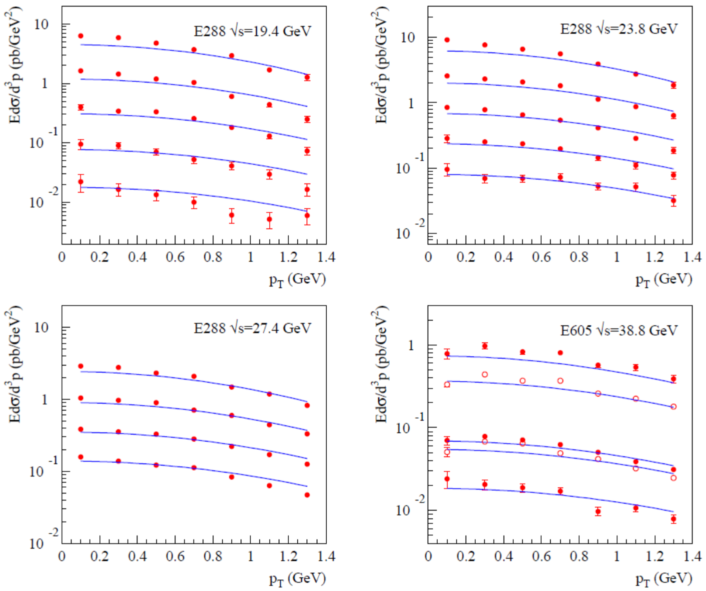
EIKV phenomenology

- Fit DY data and SIDIS data....



Z and W-Boson Production

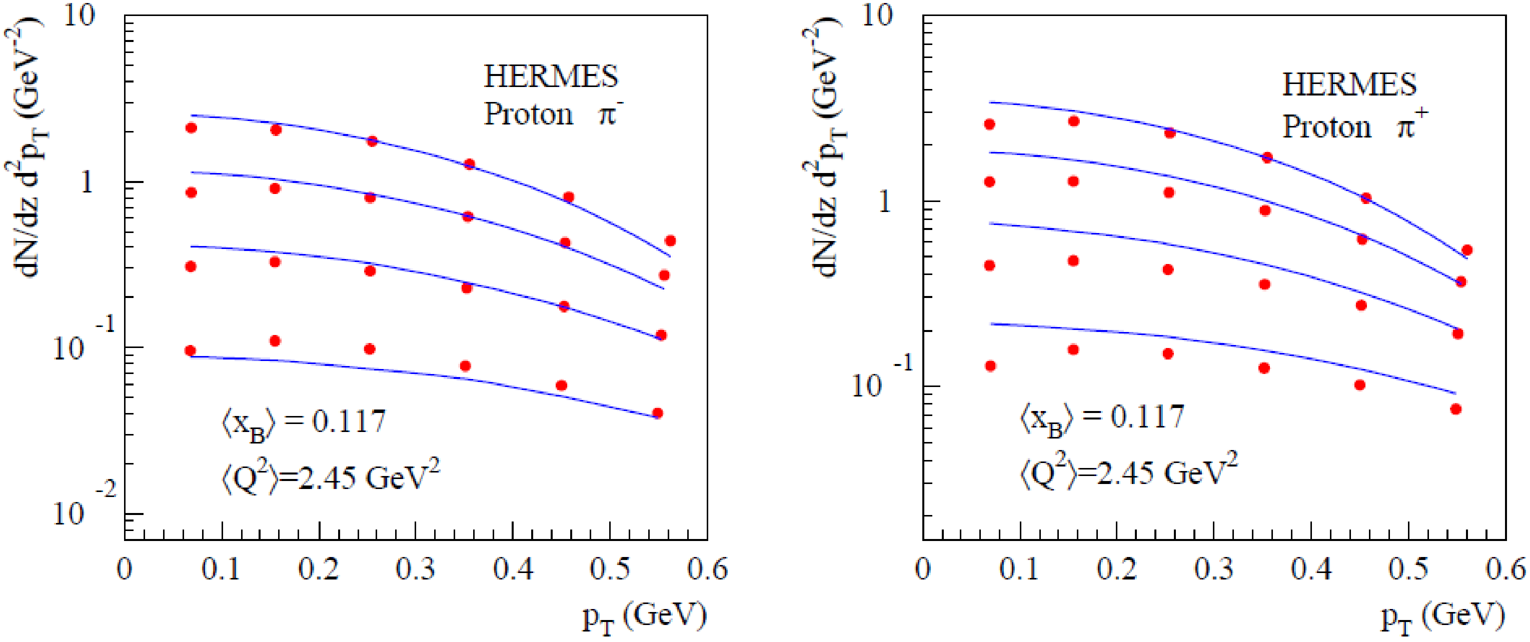
Low energy DY



Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

EIKV phenomenology

HERMES SIDIS data

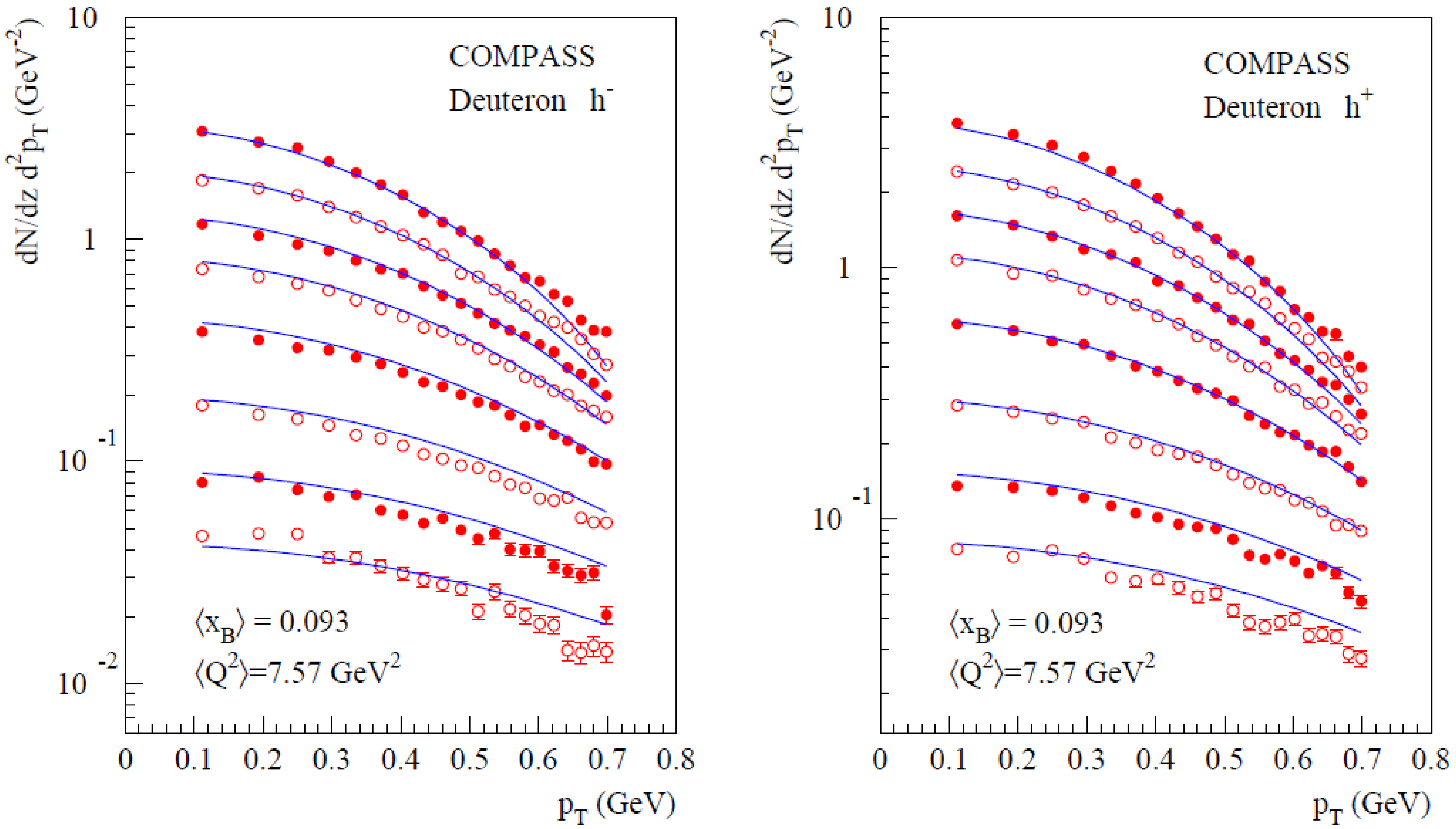


MSTW2008 PDF and DSS

Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

EIKV phenomenology

(some...only two bins?) COMPASS SIDIS data



MSTW2008 PDF and DSS

Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

TMD evolution modelling
Rogers & Aybat

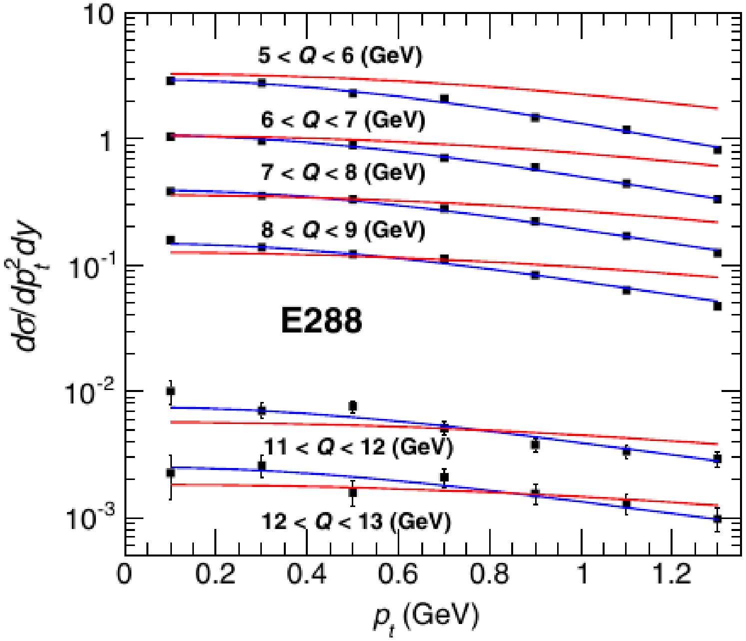
$$\widetilde{F}(x,\boldsymbol{b}_T;Q)=\widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$\tilde{F}(x,b_T,Q_0,Q_0^2)=f(x,Q_0)\exp\left[-\frac{\langle k_\perp^2\rangle}{4}b_T^2\right]$$

$$g_K(b_T)=\frac{1}{2}g_2b_T^2$$ g_2 from DY

Average transverse momentum from SIDIS (HERMES)

Red line, prediction based on the above formula with the parameter as in Rogers,Aybat 2011



Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Alternative TMD evolution
Yuan-Sun phenomenology

- Yuan-Sun explanation: the Sudakov form factor must be modified taking into account that low energy data are almost in a non perturbative region.

$$S_{\mathrm{Sud}} = 2C_F \int_{Q_0}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln \frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2} \right]$$

$$\tilde{F}_{UU}(Q; b) = e^{-S_{\mathrm{sud}}(Q, Q_0, b)} \tilde{F}_{UU}(Q_0; b),$$

$$\tilde{F}_{UU}(Q_0, b) = \sum_q e_q^2 f_q(x_B, \mu = Q_0) D_q(z_h, \mu = Q_0) e^{-g_0 b^2 - g_h b^2 / z_h^2}$$

- Notice that there is not any b* and therefore any b_{max}.

See for a interesting discussion Section VII of Aidala, Field, Gamberg, Rogers, Phys.Rev. D89 (2014) 094002

Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Alternative TMD evolution
Yuan-Sun phenomenology

- Gaussian parametrization for the PDF and the fragmentation function at the scale of HERMES.

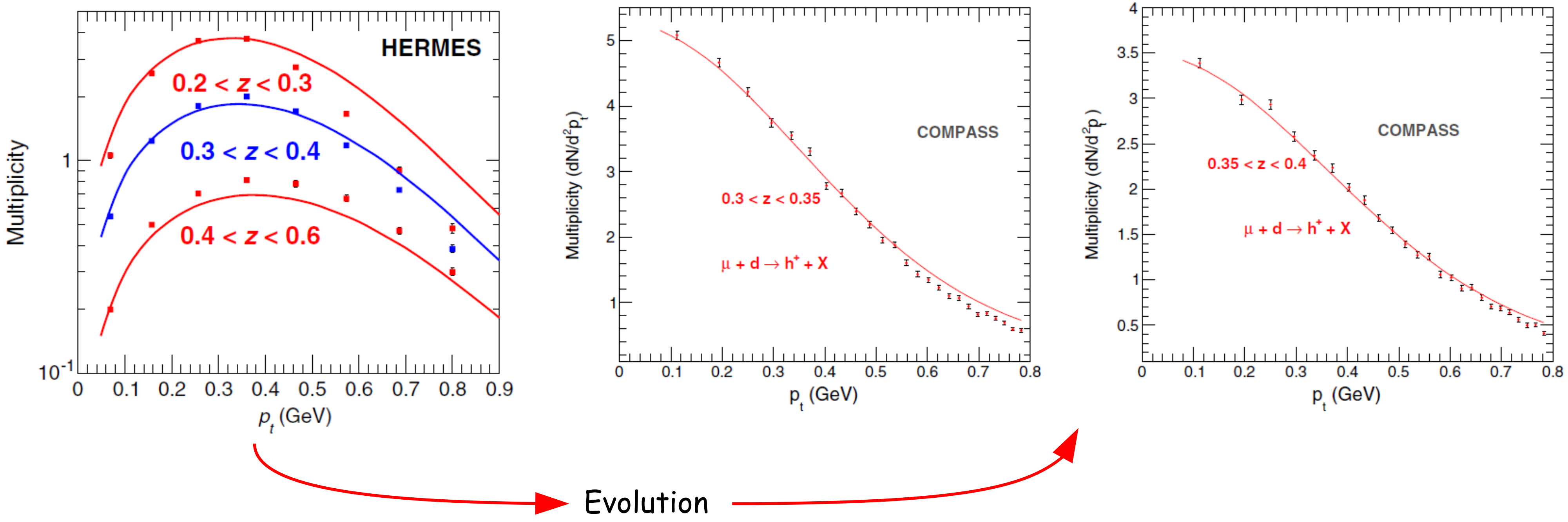
$$\tilde{F}_{UU}(Q_0, b) \quad = \sum_q e_q^2 f_q(x_B, \mu = Q_0) D_q(z_h, \mu = Q_0) e^{-g_0 b^2 - g_h b^2 / z_h^2}$$

$$\tilde{W}_{UU}(Q_0, b) = \sum_q e_q^2 f_q(x, \mu = Q_0) f_{\bar{q}}(x', \mu = Q_0) e^{-g_0 b^2 - g_0 b^2}$$

Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Alternative TMD evolution
Yuan-Sun phenomenology

- Gaussian parametrization for the PDF and the fragmentation function at the scale of HERMES.
- Parameters g_0 and g_h as in Schweitzer et al, Phys. ReV. D81,094019 (2010)



Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Alternative TMD evolution
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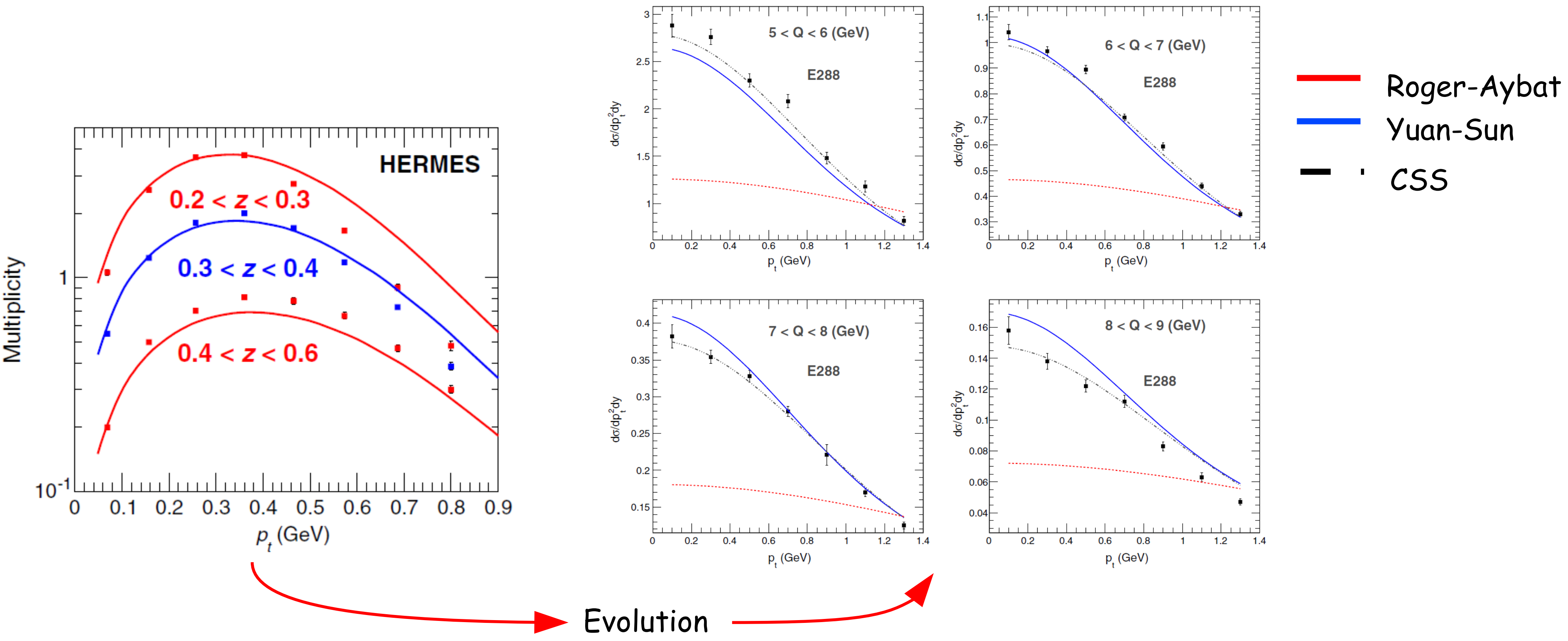


Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

The Sivers function from SIDIS data

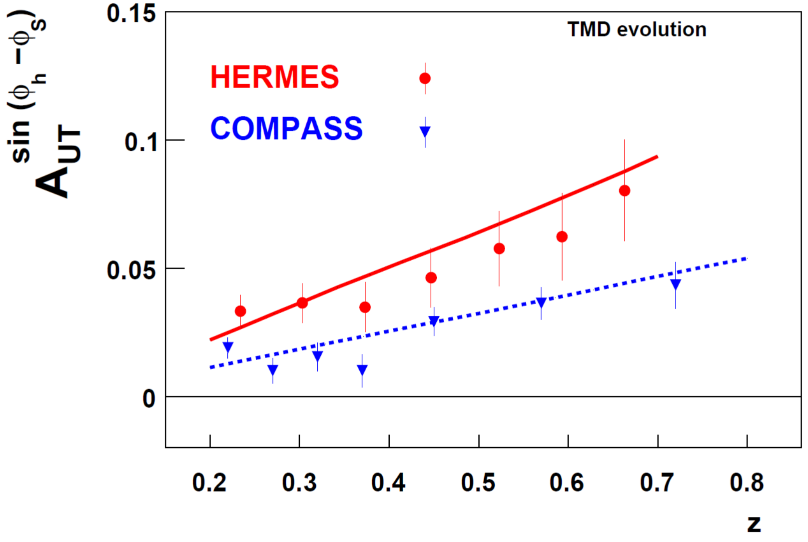
Sivers phenomenology

- Aybat-Roger-Prokudin: TMD EVO IO No FIT Qual. OK

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$\tilde{F}(x,b_T,Q_0,Q_0^2) = f(x,Q_0)\exp\left[-\frac{\langle k_\perp^2\rangle}{4}b_T^2\right]$$

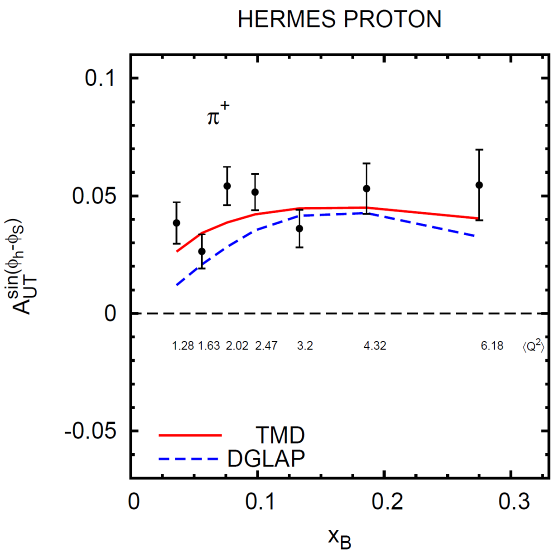
$$g_K(b_T) = \frac{1}{2}g_2 b_T^2$$ g_2 from DY

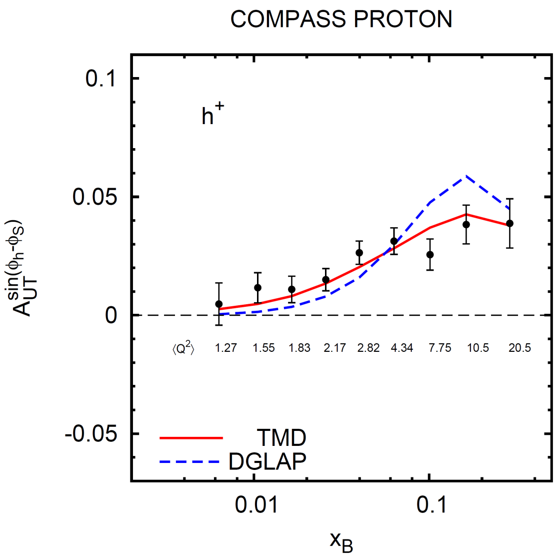


Aybat, Prokudin, Rogers, Phys.Rev.Lett. 108 (2012) 242003

Sivers phenomenology

- Aybat-Roger-Prokudin: TMD EVO IO   No FIT   Qual. OK
- Anselmino-Boglione-Melis: Gaussian   FIT χ^2=1.26

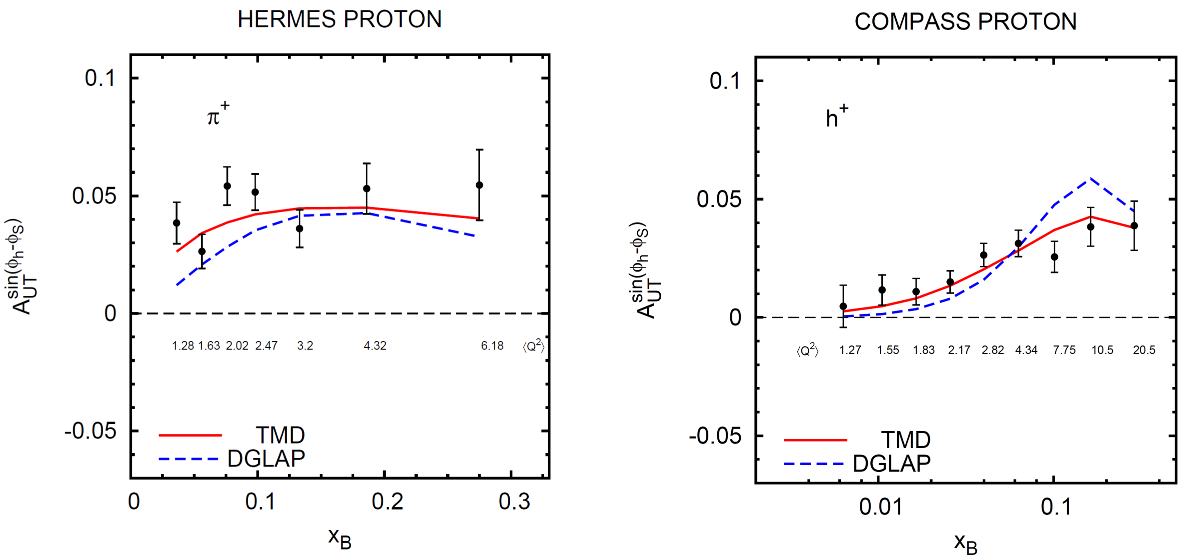




Anselmino, Boglione, Melis Phys.Rev. D86 (2012) 014028

Sivers phenomenology

Aybat-Roger-Prokudin: TMD EVO IO	No FIT	Qual. OK
Anselmino-Boglione-Melis: Gaussian	FIT	χ^2=1.26
Anselmino-Boglione-Melis: TMD EVO IO	FIT	χ^2=1.02

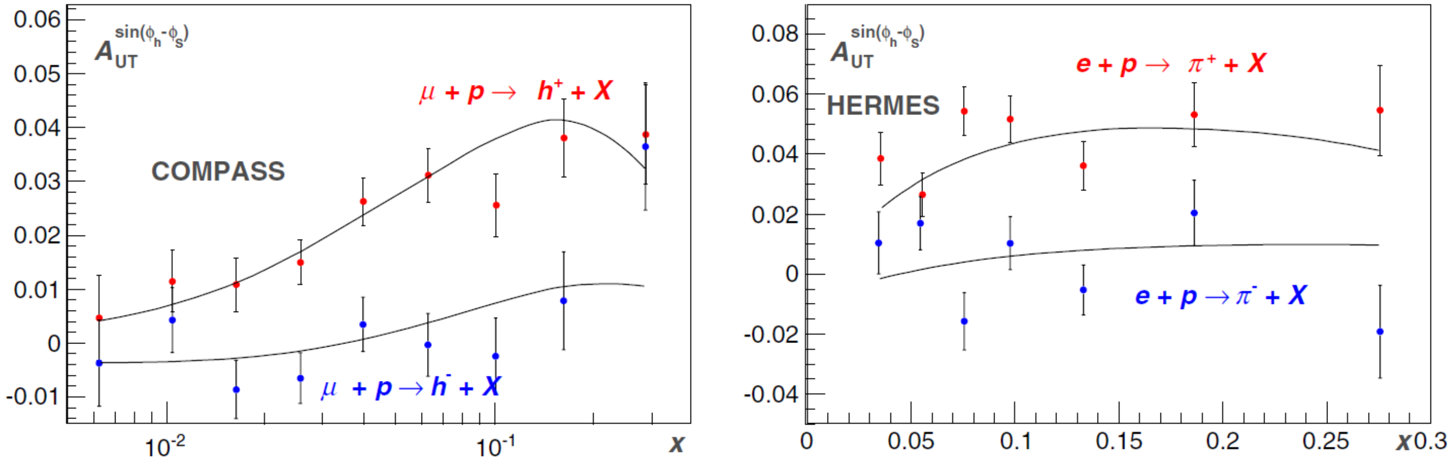


Anselmino, Boglione, Melis Phys.Rev. D86 (2012) 014028

Sivers phenomenology

➢Aybat-Roger-Prokudin: TMD EVO IO	No FIT	Qual. OK
➢Anselmino-Boglione-Melis: Gaussian	FIT	χ^2=1.26
➢Anselmino-Boglione-Melis: TMD EVO IO	FIT	χ^2=1.02
➢Sun-Yuan: TMD EVO IO+ Modified Sudakov	FIT	χ^2=1.08

$$\tilde{F}^{\alpha}_{\text{sivers}}(Q_0, b) = \frac{ib^{\alpha}_{\perp} M}{2} \sum_q e_q^2 \Delta f_q^{\text{sivers}}(x) D_q(z) e^{-(g_0-g_s)b^2 - g_h b^2/z_h^2} \qquad S_{\text{Sud}} = 2C_F \int_{Q_0}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2} \right]$$



Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Sivers phenomenology

Aybat-Roger-Prokudin: TMD EVO IO	No FIT	Qual. OK
Anselmino-Boglione-Melis: Gaussian	FIT	χ^2=1.26
Anselmino-Boglione-Melis: TMD EVO IO	FIT	χ^2=1.02
Sun-Yuan: TMD EVO IO+ Modified Sudakov	FIT	χ^2=1.08
EIKV: TMD Evo a la CSS+ C at LO	FIT	χ^2=1.3

$$F_{UT}^{\sin(\phi_h-\phi_s)} = \frac{1}{4\pi}\int_0^\infty db\, b^2 J_1(P_{h\perp}b/z_h)\sum_q e_q^2\, T_{q,F}(x_B,x_B,c/b_*)D_{h/q}(z_h,c/b_*)$$

$$\times \exp\left\{-\int_{c^2/b_*^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}\exp\left\{-b^2\left(g_1^{\rm ff}+g_1^{\rm sivers}+g_2\ln\frac{Q}{Q_0}\right)\right\}$$

Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

Sivers phenomenology

Aybat-Roger-Prokudin: TMD EVO IO	No FIT	Qual. OK
Anselmino-Boglione-Melis: Gaussian	FIT	χ^2=1.26
Anselmino-Boglione-Melis: TMD EVO IO	FIT	χ^2=1.02
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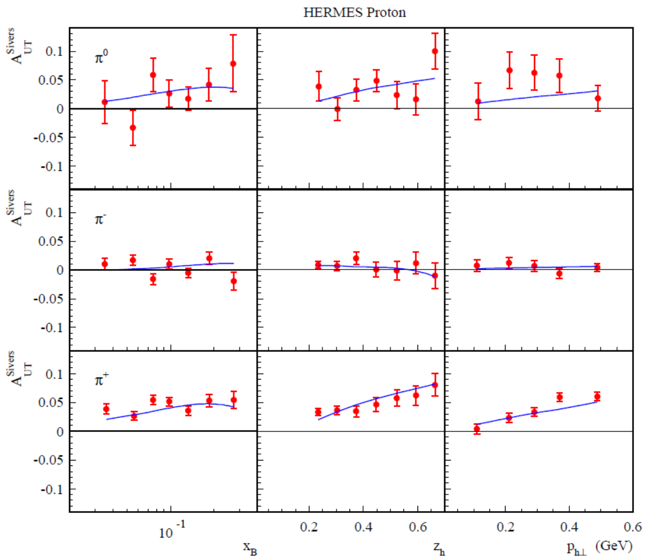

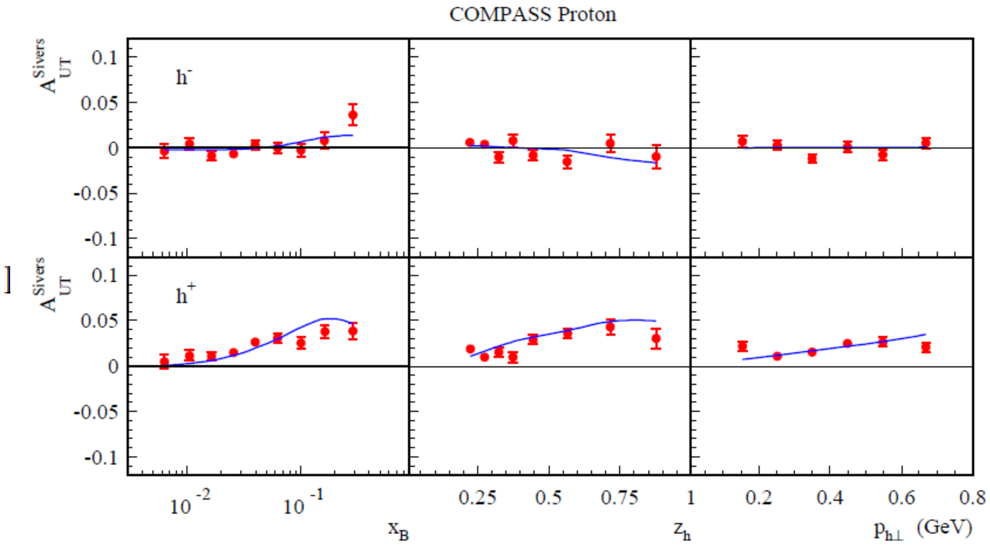

Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

Sivers phenomenology

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EIKV: TMD Evo a la CSS+ C at LO	FIT	χ^2=1.3

Unpolarized phenomenology

Can these methods
describe unpolarized processes?

	SIDIS	DY
Aybat-Roger-Prokudin: TMD EVO IO	No	No
Anselmino-Boglione-Melis: Gaussian	Maybe (separately)	Maybe low energy No High energy
Anselmino-Boglione-Melis: TMD EVO IO	No	No
Sun-Yuan: TMD EVO IO+ Modified Sudakov	No Hermes YES/Maybe COMPASS	Yes low energy No High energy
EIKV: TMD Evo a la CSS+ C at LO	No Hermes YES/Maybe COMPASS	YES

Unpolarized phenomenology

Can these methods describe unpolarized processes?

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Anselmino-Boglione-Melis: TMD EVO IO	No	No
Sun-Yuan: TMD EVO IO+ Modified Sudakov	No Hermes YES/Maybe COMPASS	Yes low energy No High energy
EIKV: TMD Evo a la CSS+ C at LO	No Hermes YES/Maybe COMPASS	YES

- This is a comparison list!
  There other works related
  to the unpolarized processes!

See talk by Osvaldo Gonzales, Andrea Signori
Leonard Gamberg, Ignazio Scimemi...

Conclusions

- Gaussian model can describe many features of low energy data but they are not able to describe all the data coherently

- CSS resummation describes DY data but the non perturbative part is important (and can be relevant in a fit SIDIS+DY).

- Different approaches can describe the Sivers asymmetry.

- Although we did a lot of progress, none of these approaches is able to describe all the data in a satisfactory way.

- Future machines like EIC could help to have data at energies closer to the low energy DY.

Yuan-Sun phenomenolgy

- Then Anselmino et al like parametrization for the Sivers function at the scale of HERMES

$$\tilde{F}^{\alpha}_{\text{sivers}}(Q_0, b) \quad = \frac{ib^{\alpha}_{\perp}M}{2}\sum_{q} e_q^2 \Delta f_q^{\text{sivers}}(x) D_q(z) e^{-(g_0 - g_s)b^2 - g_h b^2/z_h^2}$$

$$\Delta f_q(x) = N_q x^{\alpha_q}(1 - x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{\alpha_q + \beta_q}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} f_q(x)$$

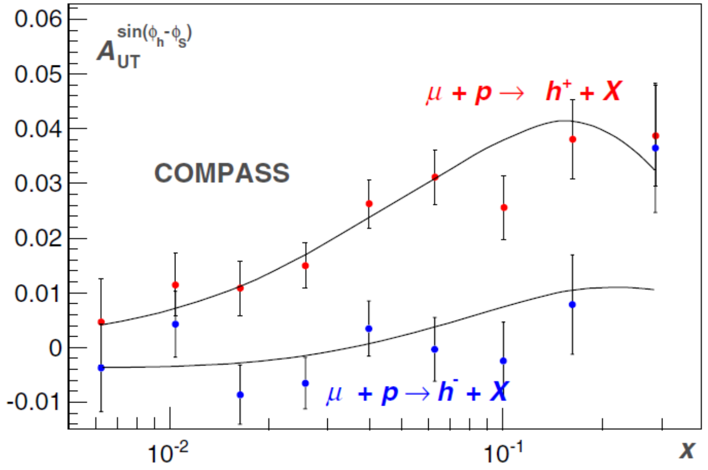
Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

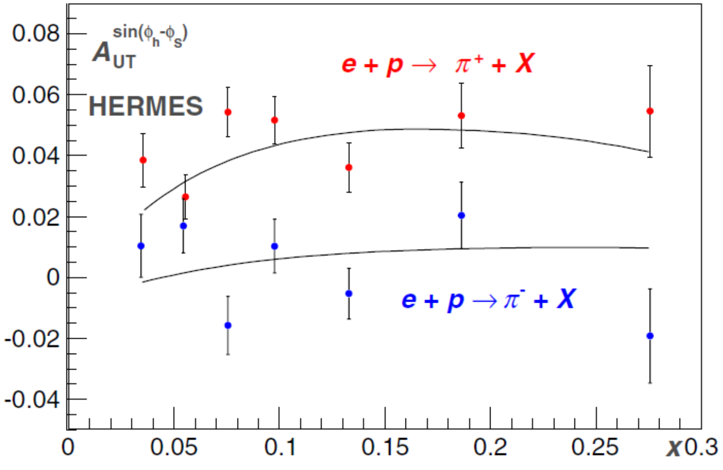
Yuan-Sun phenomenolgy

TABLE I. Parameters $\{a_i^0\}$ describing our optimum Δf_i in Eq. (5) at the input scale $Q^2 = 2.4$ GeV.

flavor i	N_i	α_i	β_i	g_s (GeV^2)
u	0.13 ± 0.023	0.81 ± 0.16	4.0 ± 1.2	0.062 ± 0.005
d	-0.27 ± 0.12	1.41 ± 0.28	4.0 ± 1.2	0.062 ± 0.005
s	0.07 ± 0.06	0.58 ± 0.39	4.0 ± 1.2	0.062 ± 0.005
$\bar{u}$	-0.07 ± 0.05	0.58 ± 0.39	4.0 ± 1.2	0.062 ± 0.005
$\bar{d}$	-0.19 ± 0.12	0.58 ± 0.39	4.0 ± 1.2	0.062 ± 0.005

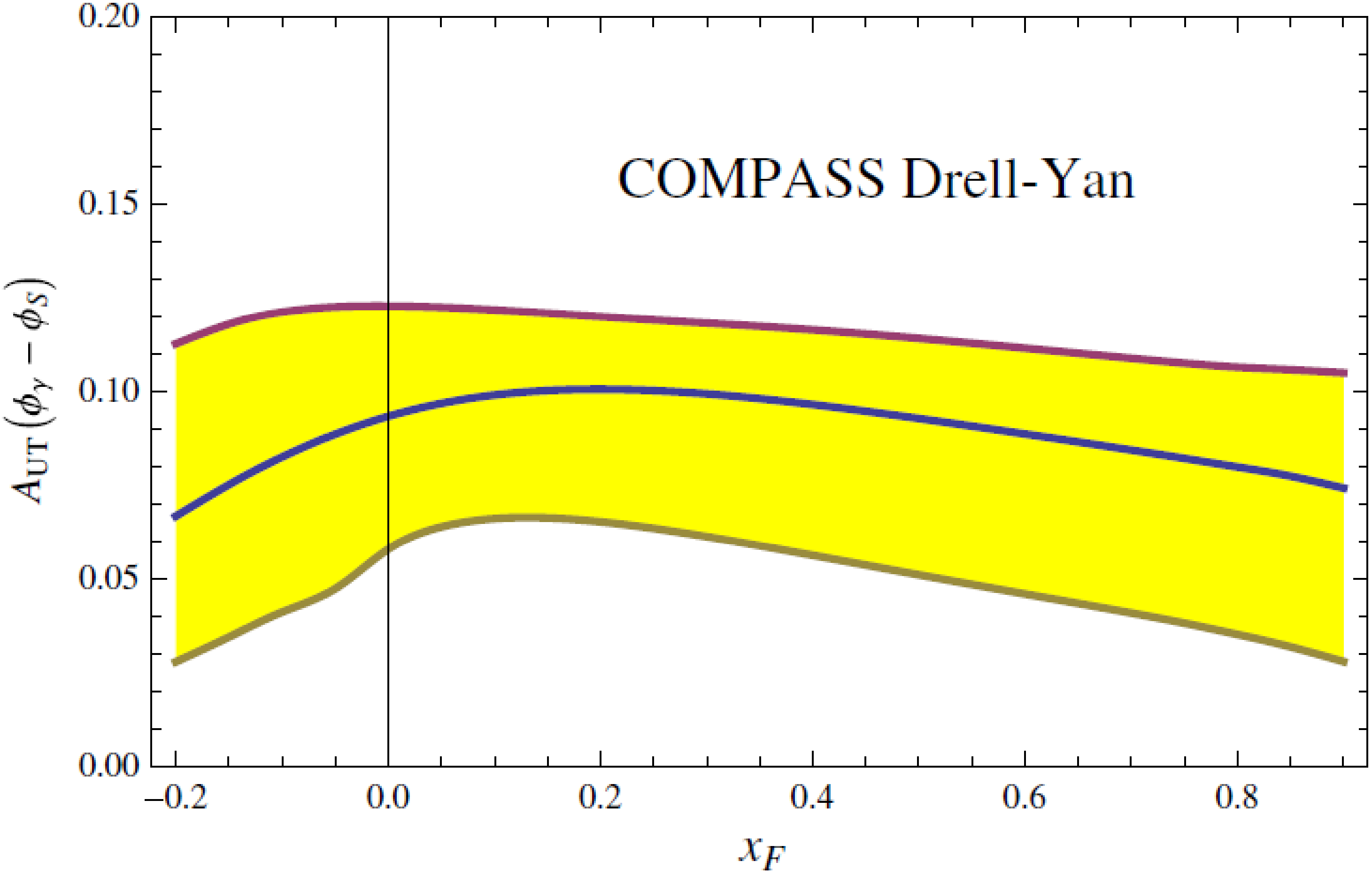
$$\chi^2/\bar{\mathrm{d}}.\mathrm{o.f} = 1.08$$





Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

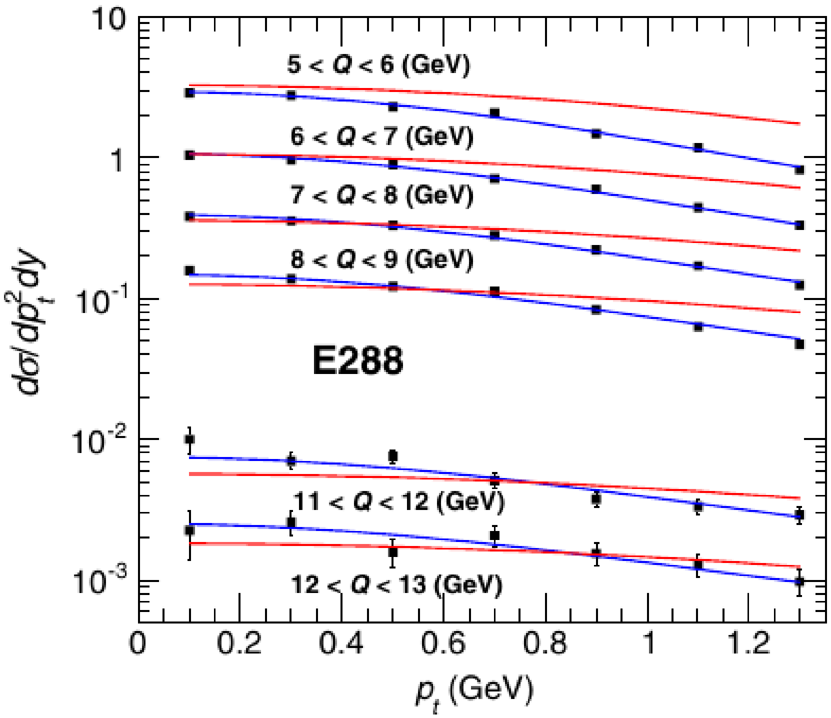
Yuan-Sun phenomenolgy



Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Yuan-Sun

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$



This formulation maximize the
Non perturbative input
Maybe not suitable for DY...

Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Echevarria-Idilbi-Kang-Vitev phenomenology

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

- Restart from the TMD evolution in the CSS-like version

$$\begin{aligned}
\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) &= \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \\
&\exp\left\{ \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln\left(\frac{Q}{\kappa}\right) \gamma_K(\kappa) \right\} \\
&\exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) \right\}
\end{aligned}$$

- Make some approximation to simply life

$\tilde{C}_{ji}(z, \alpha(\mu)) = \delta_{ij}\delta(1 - z)$ At LO; PDF at LO

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

- Simple parametrizations for the non-perturbative part:

$$F_{NP}(b_T,Q)^{\rm pdf}=\exp\left[-b_T^2\left(g_1^{\rm pdf}+\frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

$$F_{NP}(b_T,Q)^{\rm ff}=\exp\left[-b_T^2\left(g_1^{\rm ff}+\frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

- Choose Q_0^2=2.4 GeV^2 as reference scale. We know that simple gaussian models describe well SIDIS data...

$$g_1^{\rm pdf}=\frac{\langle k_\perp^2\rangle_{Q_0}}{4},\qquad g_1^{\rm ff}=\frac{\langle p_T^2\rangle_{Q_0}}{4z^2}$$

$$\langle k_\perp^2\rangle_{Q_0}=0.25-0.44\ {\rm GeV}^2,\qquad \langle p_T^2\rangle_{Q_0}=0.16-0.20\ {\rm GeV}^2$$

- We know that DY data can be described using:

$$b_{\rm max}=1.5\ {\rm GeV}^{-1}\qquad g_2=0.184\pm0.018\ {\rm GeV}^2$$

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

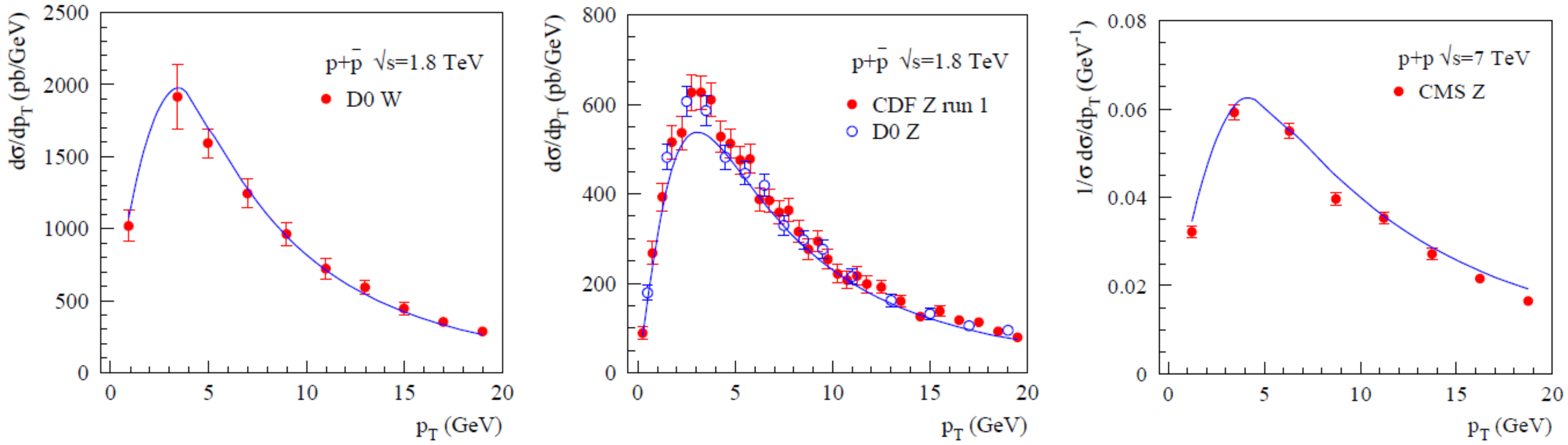
- Try to find reasonable parameters to describe data and see what happens...

$$\langle k_\perp^2 \rangle_{Q_0} = 0.38\ \mathrm{GeV}^2, \qquad \langle p_T^2 \rangle_{Q_0} = 0.19\ \mathrm{GeV}^2, \qquad g_2 = 0.16\ \mathrm{GeV}^2$$

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

Z and W-Boson Production

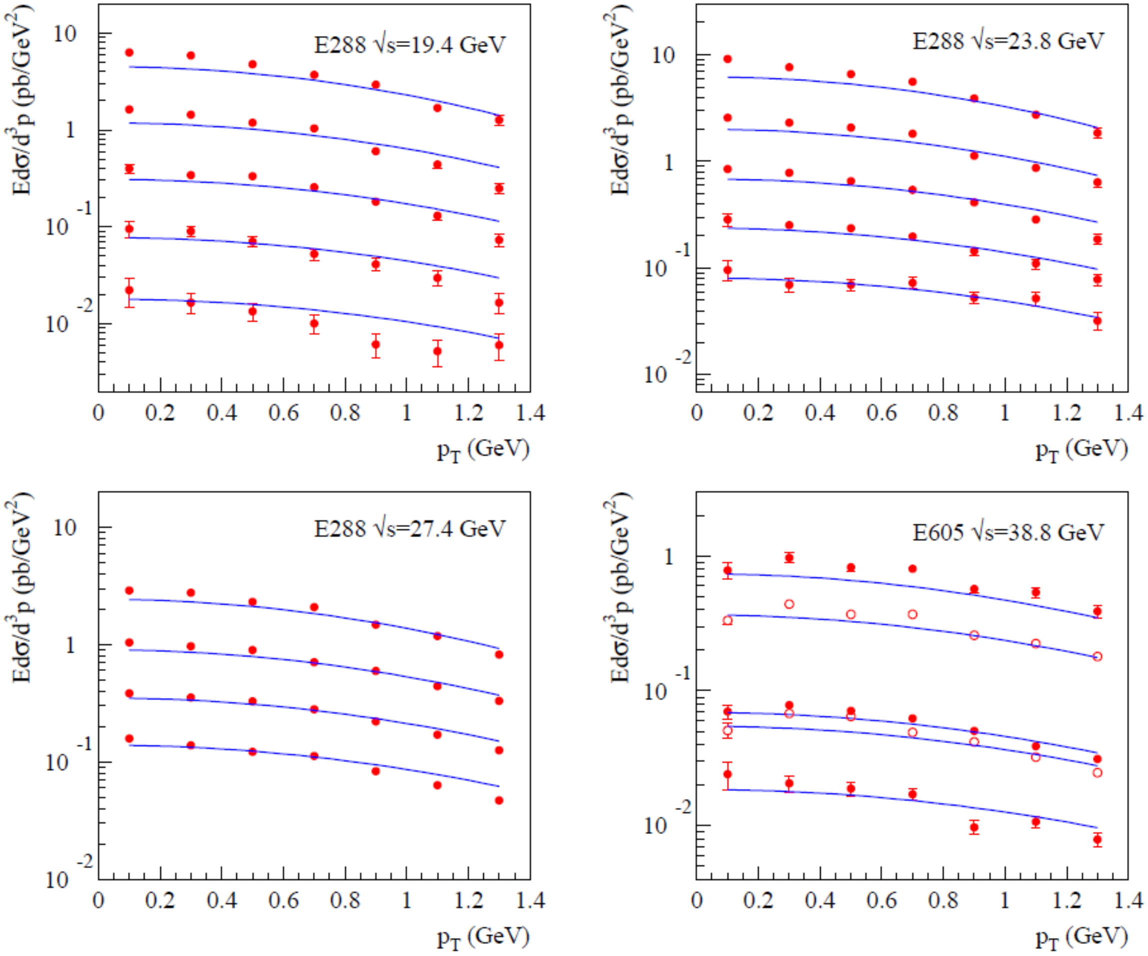


MSTW2008 PDF

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

Low energy Drell-Yan

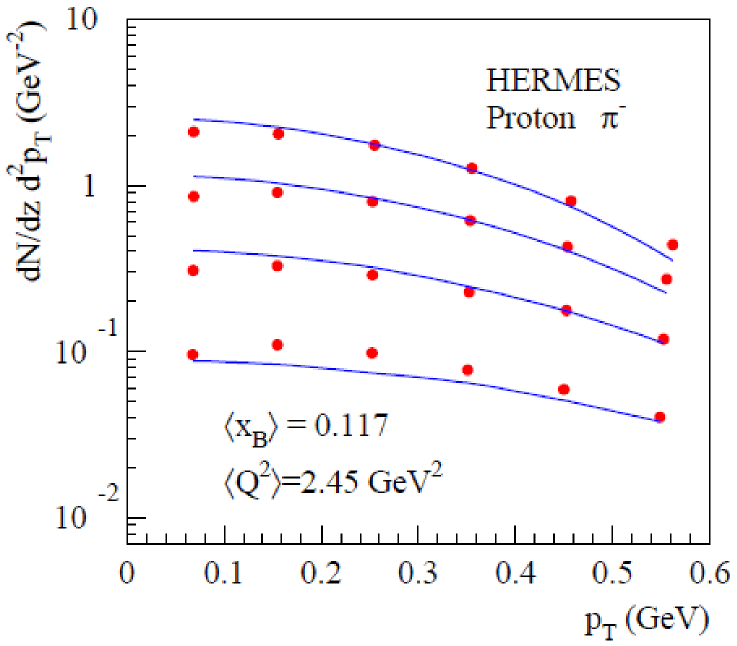


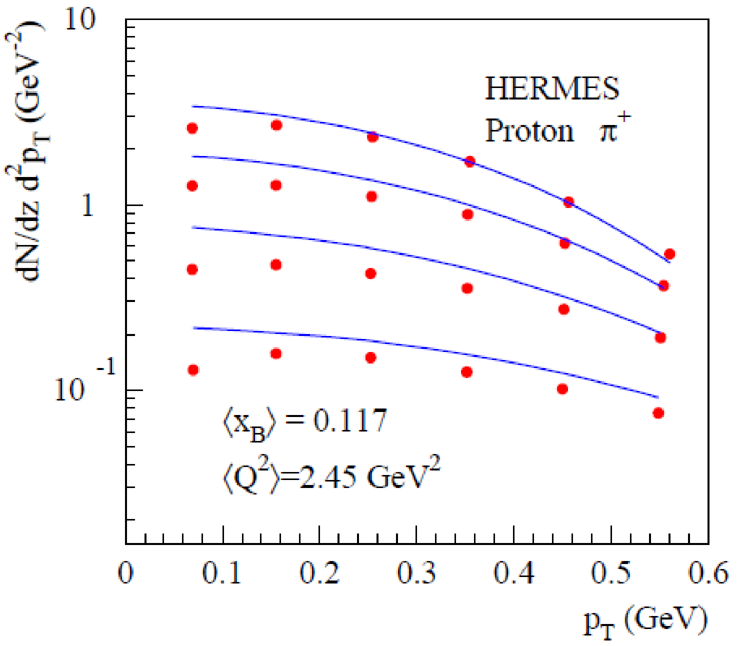
EKS98 Cu PDF

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

HERMES SIDIS data



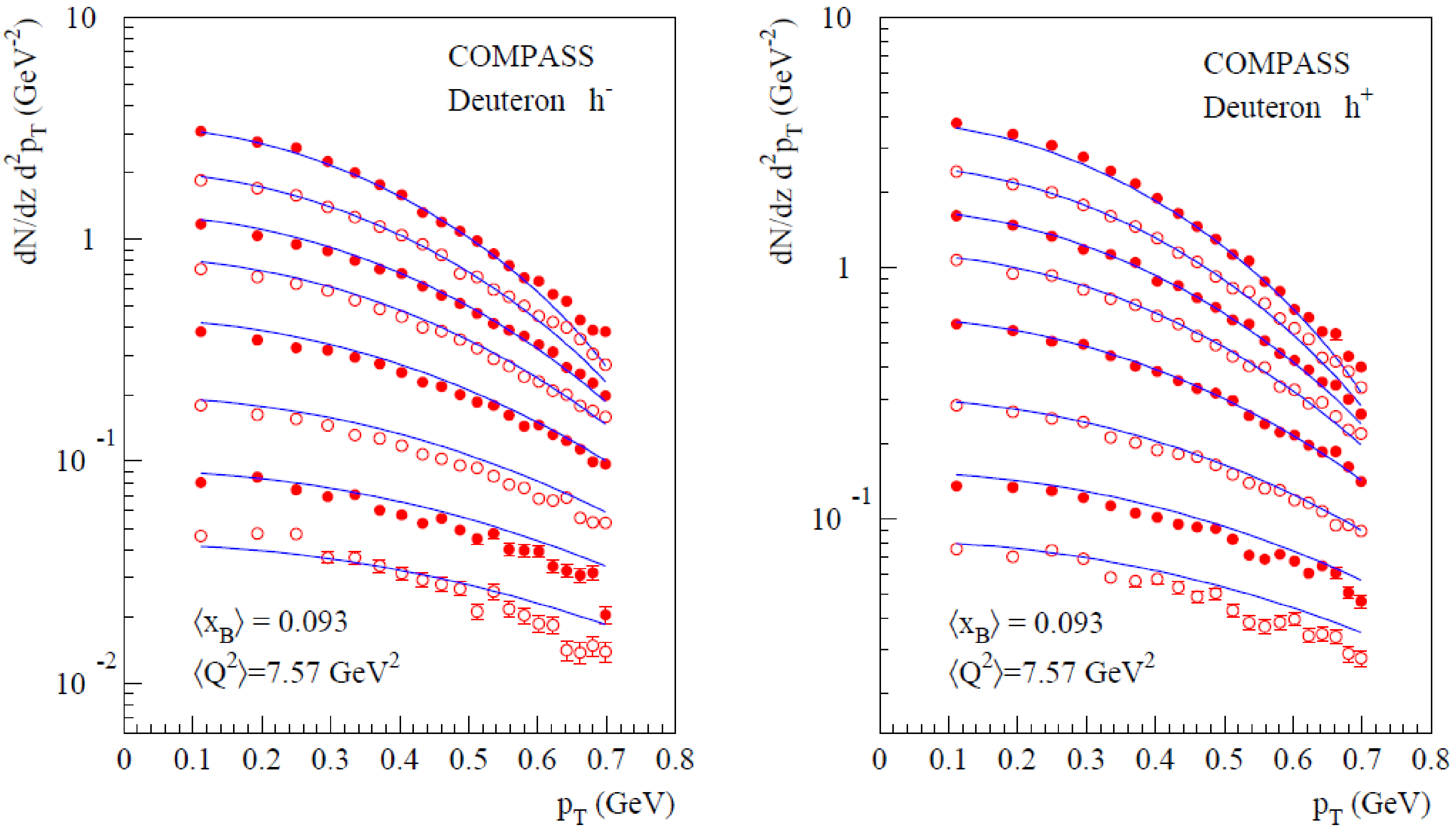


MSTW2008 PDF and DSS

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

(some…) COMPASS SIDIS data



MSTW2008 PDF and DSS

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

- Ready for Sivers! Again a CSS-like version approximated at LO

$$F_{UT}^{\sin(\phi_h-\phi_s)} = \frac{1}{4\pi}\int_0^\infty db\, b^2 J_1(P_{h\perp}b/z_h)\sum_q e_q^2 T_{q,F}(x_B,x_B,c/b_*)D_{h/q}(z_h,c/b_*)$$
$$\times \exp\left\{-\int_{c^2/b_*^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}\exp\left\{-b^2\left(g_1^{\rm ff}+g_1^{\rm sivers}+g_2\ln\frac{Q}{Q_0}\right)\right\}$$

- The Qiu-Sterman function treated at LO as a Sivers function.

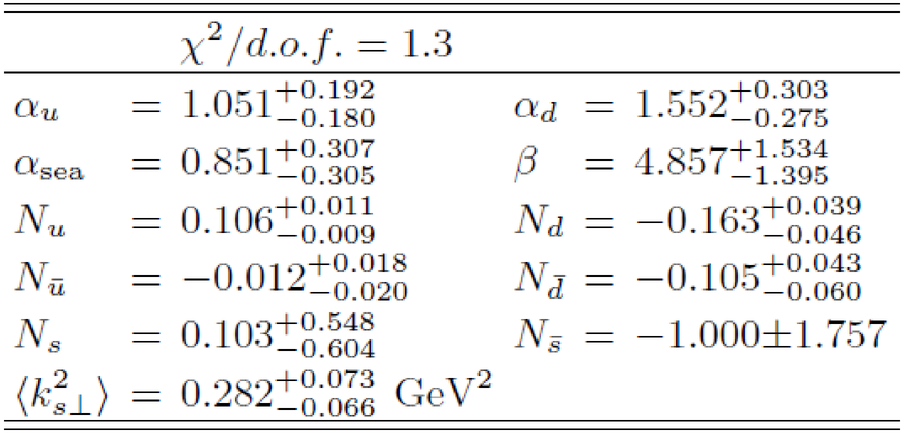
$$T_{q,F}(x,x,\mu) = N_q\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}x^{\alpha_q}(1-x)^{\beta_q}f_{q/A}(x,\mu)$$

Using an Anselmino-like parametrization.

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

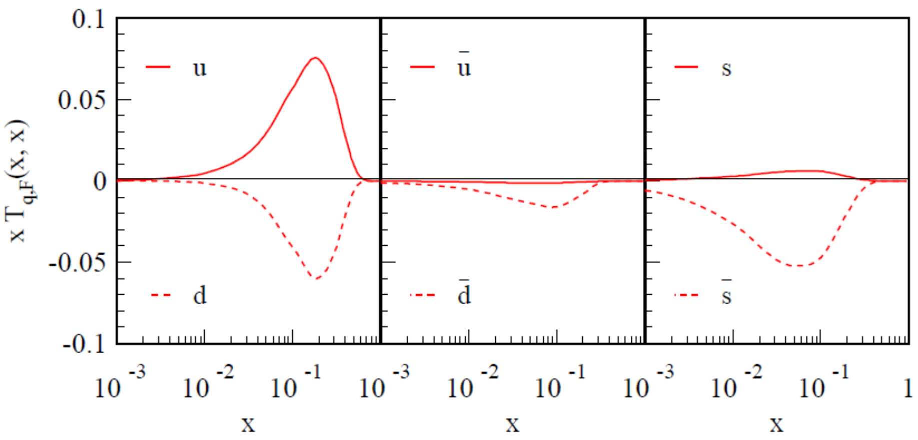
EIKV phenomenology

- Fit of HERMES, COMPASS and JLAB data



$\chi^2/d.o.f. = 1.3$

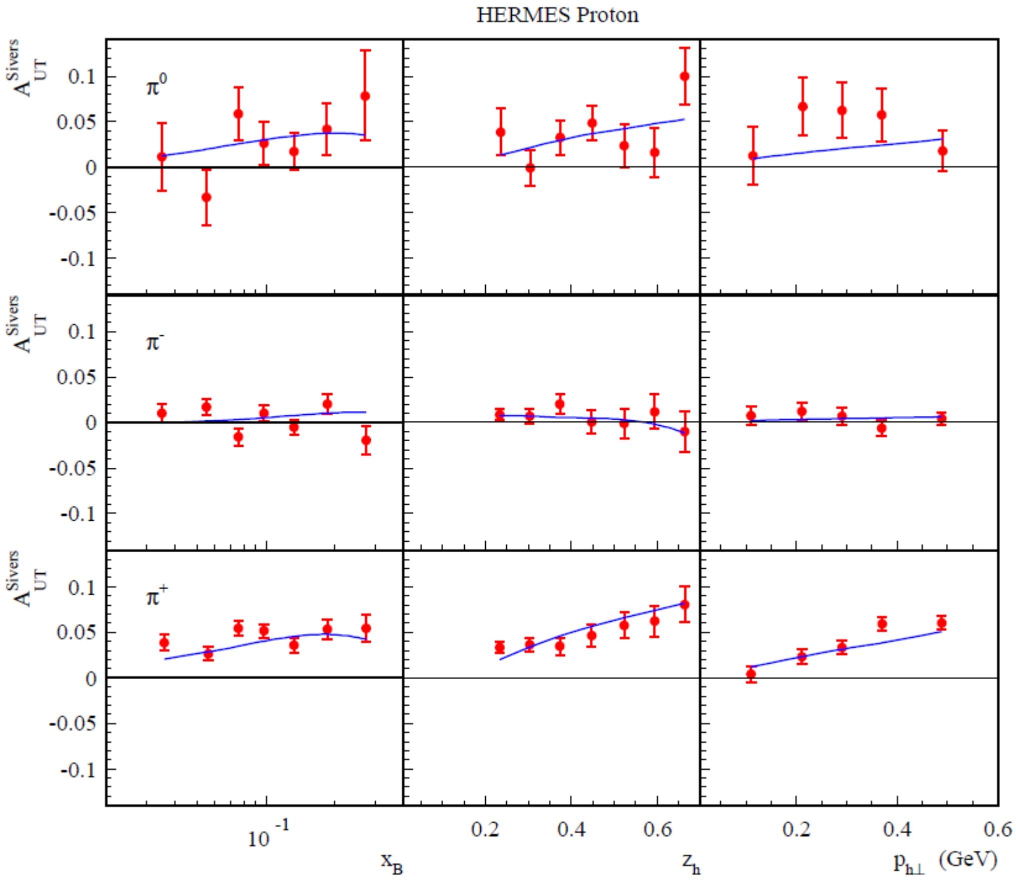
α_u	$= 1.051^{+0.192}_{-0.180}$	α_d	$= 1.552^{+0.303}_{-0.275}$
α_{sea}	$= 0.851^{+0.307}_{-0.305}$	β	$= 4.857^{+1.534}_{-1.395}$
N_u	$= 0.106^{+0.011}_{-0.009}$	N_d	$= -0.163^{+0.039}_{-0.046}$
$N_{\bar{u}}$	$= -0.012^{+0.018}_{-0.020}$	$N_{\bar{d}}$	$= -0.105^{+0.043}_{-0.060}$
N_s	$= 0.103^{+0.548}_{-0.604}$	$N_{\bar{s}}$	$= -1.000 \pm 1.757$
$\langle k_{s\perp}^2 \rangle$	$= 0.282^{+0.073}_{-0.066}$ GeV^2		



Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

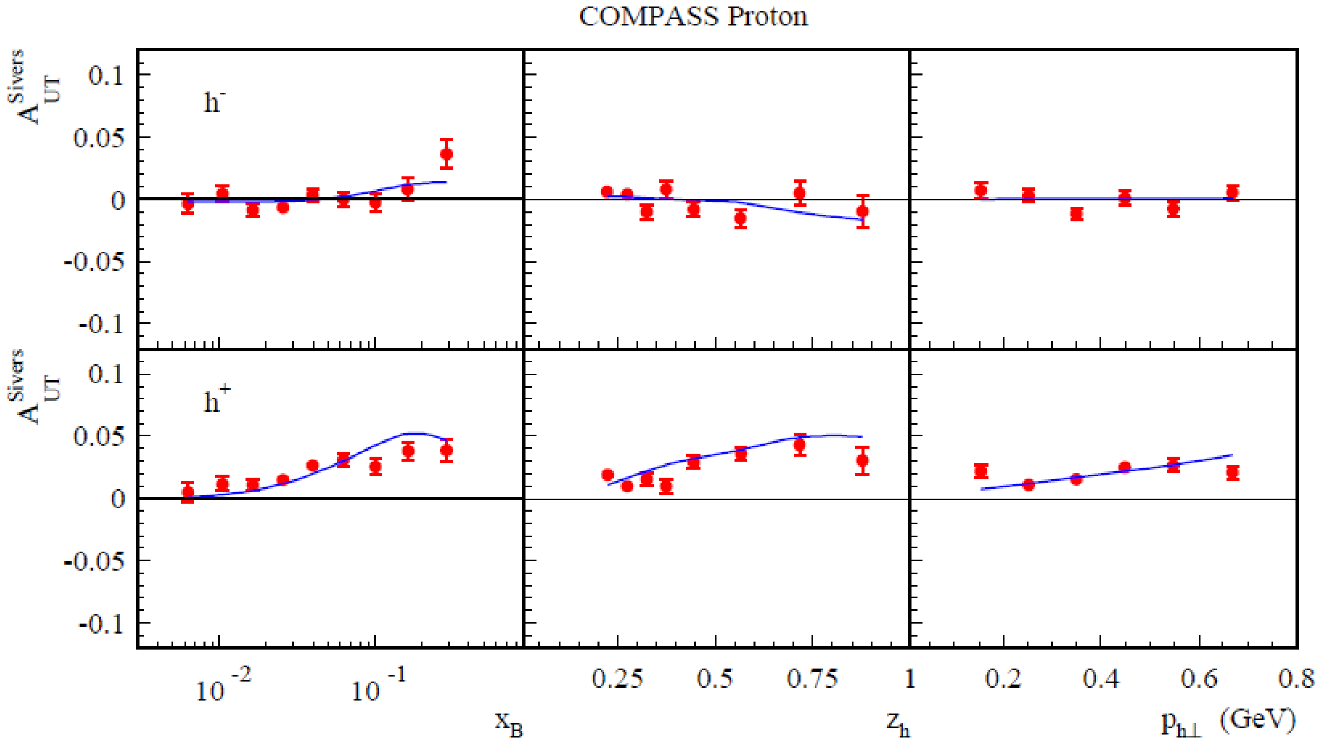
HERMES SIDIS data



Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

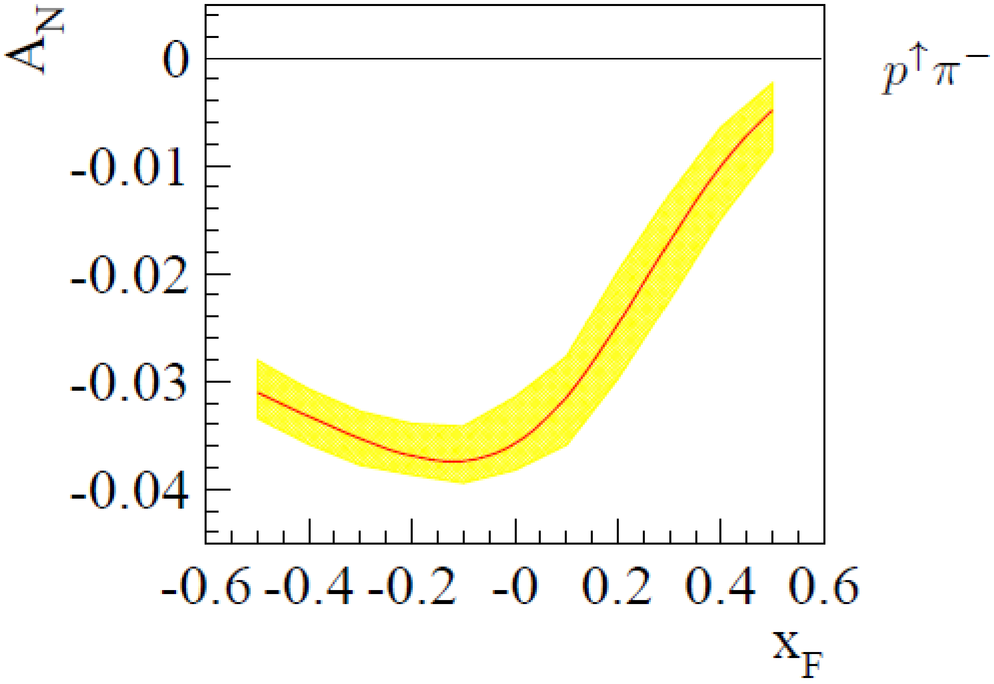
COMPASS SIDIS data



Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

Predicition for COMPASS Drell-Yan



Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

Extraction of transversity & Collins functions

- Azimuthal asymmetry in polarized SIDIS

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = \sum_q h_{1q}(x, k_\perp) \otimes \mathrm{d}\Delta\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z, \boldsymbol{p}_\perp)$$

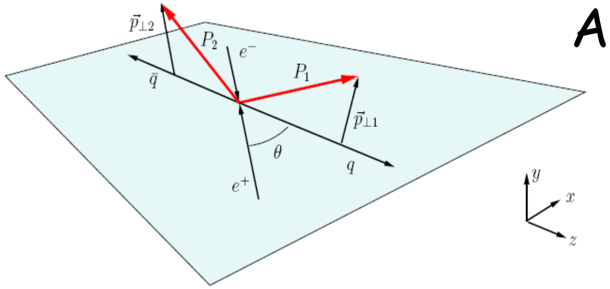
Transversity

Collins function

$$\boxed{A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\,\frac{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}]\,\sin(\phi+\phi_S)}{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}]}}$$

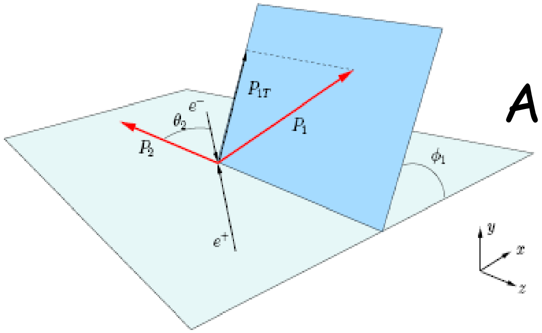
Extraction of transversity & Collins functions

- $e^+e^- \to h_1 h_2 X$ BELLE Data



A_{12} asymmetry → **Thrust axis method**

$$A(z_1, z_2, \theta, \varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1+\varphi_2)}$$
$$= 1 + \frac{1}{8} \frac{\sin^2\theta}{1+\cos^2\theta} \cos(\varphi_1+\varphi_2) \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$



A_0 asymmetry → **Hadronic plane method**

$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi} \frac{z_1\, z_2}{z_1^2+z_2^2} \frac{\sin^2\theta_2}{1+\cos^2\theta_2} \cos(2\,\phi_1) \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

Extraction of transversity & Collins functions

- To avoid acceptance effects the BELLE Collaboration considered ratio of different combinations of hadron pairs:

$$\frac{\text{Unlike-sign } (\pi^+\pi^- + \pi^-\pi^+)}{\text{Like-sign } (\pi^+\pi^+ + \pi^-\pi^-)} \longrightarrow A^{UL} \text{ asymmetry}$$

$$\frac{\text{Unlike-sign } (\pi^+\pi^- + \pi^-\pi^+)}{\text{Charged } (\pi^+\pi^+ + \pi^-\pi^- + \pi^+\pi^- + \pi^-\pi^+)} \longrightarrow A^{UC} \text{ asymmetry}$$

- A_{12}^{UL} A_{12}^{UC} A_{0}^{UL} A_{0}^{UC}

Parametrizations

- Gaussian parametrization of the unpolarized PDF & FF:

- $$f_{q/p}(x,k_\perp) \;=\; f_{q/p}(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

- $$D_{h/q}(z,p_\perp) \;=\; D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

[*] $\langle k_\perp^2\rangle = 0.25\ \mathrm{GeV}^2 \qquad \langle p_\perp^2\rangle = 0.20\ \mathrm{GeV}^2$

[*] Anselmino et al. Phys. Rev. D71 074006 (2005)

Parametrizations

- Parametrization of Transversity function:

✎ $$\Delta_T q(x, k_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) \left[\underbrace{f_{q/p}(x)}_{\text{Unpolarized PDF}} + \underbrace{\Delta q(x)}_{\text{Helicity PDF}} \right] \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T}$$

$$\boxed{\mathcal{N}_q^T(x) = N_q^T \, x^{\alpha_q} (1-x)^{\beta_q} \, \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q} \, \beta_q^{\beta_q}}}$$

N_q^T, α, β free parameters

Parametrizations

➢Parametrization of the Collins function:

✎ $$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\, h(p_\perp)\, D_{\pi/q}(z,p_\perp)$$

- $\mathcal{N}_q^C(z) = N_q^C\, z^\gamma (1-z)^\delta \, \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$

- $h(p_\perp) = \sqrt{2e}\,\frac{p_\perp}{M_h}\, e^{-p_\perp^2/M_h^2}$

Unpolarized FF

N_q^C, γ, δ, M_h free parameters

✔Bound:

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) \le 2\, D_{\pi/q}(z,k_\perp)$$

✔Torino vs Amsterdam notation

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z,p_\perp)$$

2013 Update of the extraction*

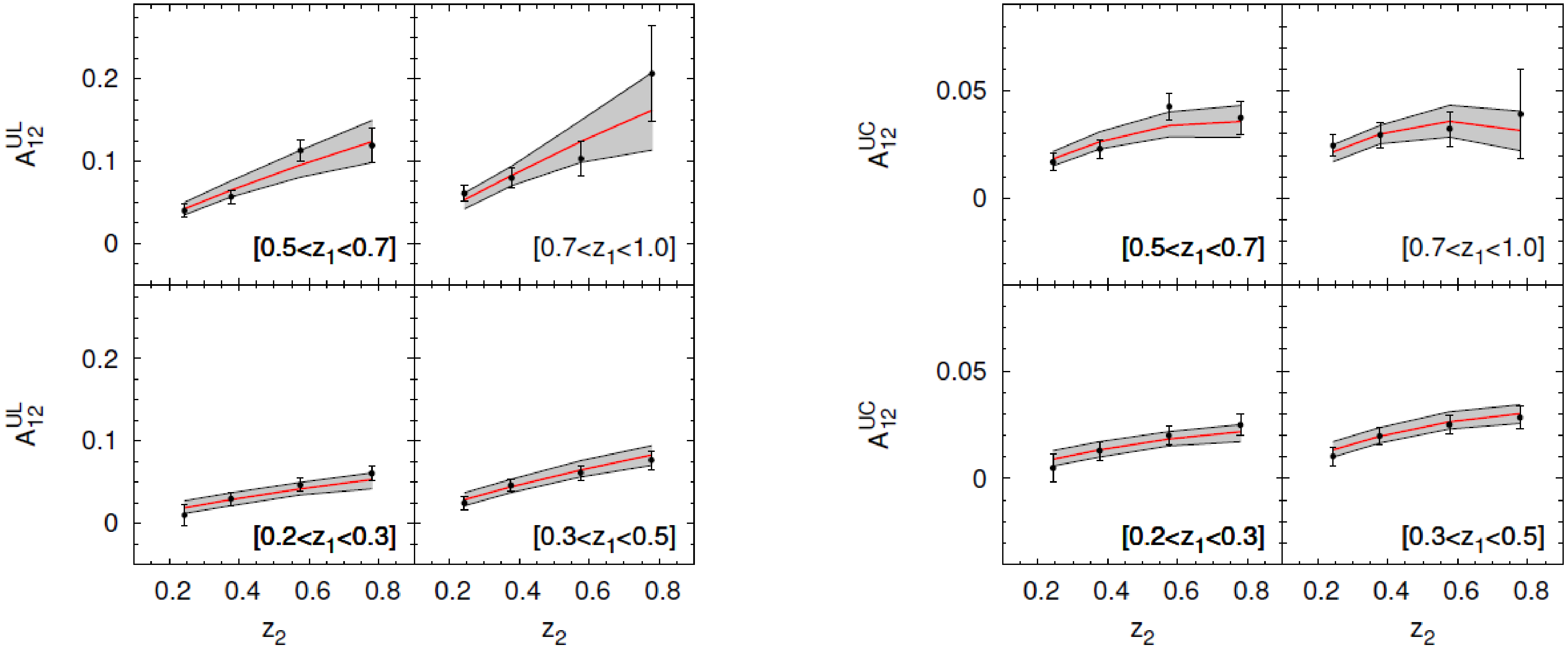
- New analysis (PRD87, 2013):
  - HERMES (2009) π+ π-
  - COMPASS Deuteron (2004) π+ π-
  - COMPASS Proton (2013) π+ π-
  - BELLE A_{12} or A_0 (BELLE ERRATUM 2012, PRD86)

- U and d quarks transversity, favored and disfavored Collins functions
- Two separate fits for A_{12} and A_0 sets

*Anselmino et al. Phys. Rev. D87,094019

Extraction of transversity & Collins functions

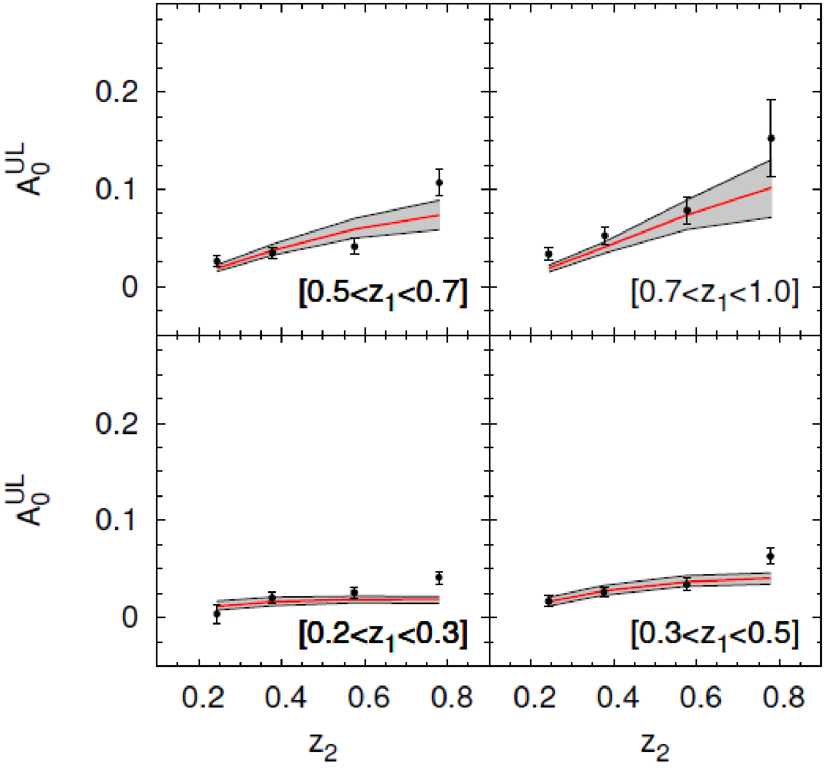
- FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES

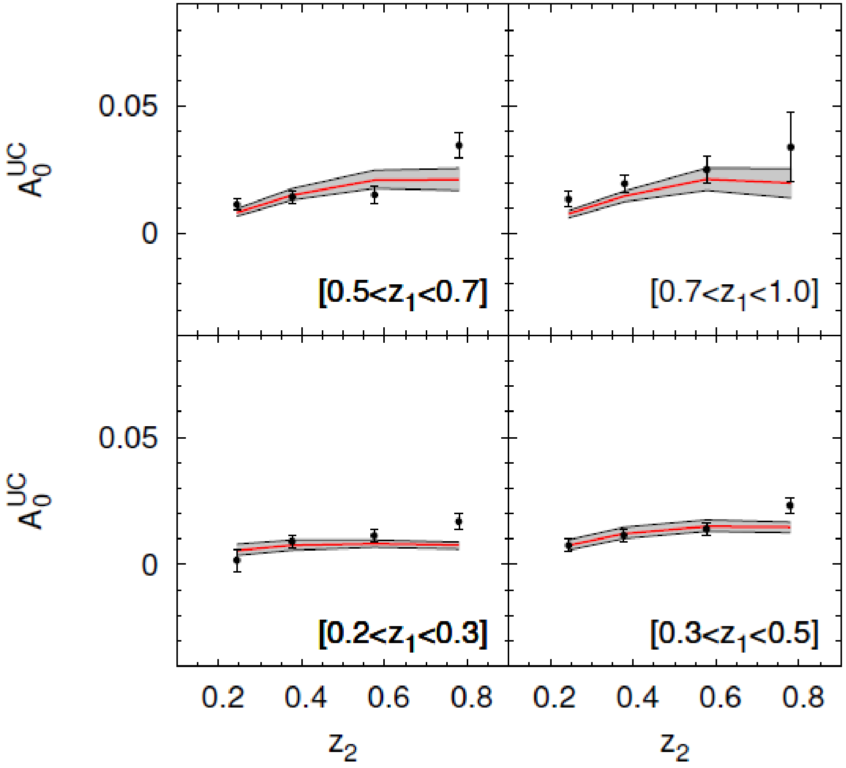


→Full compatibility between UL and UC, contrary to 2008 BELLE data

Extraction of transversity & Collins functions

- FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES

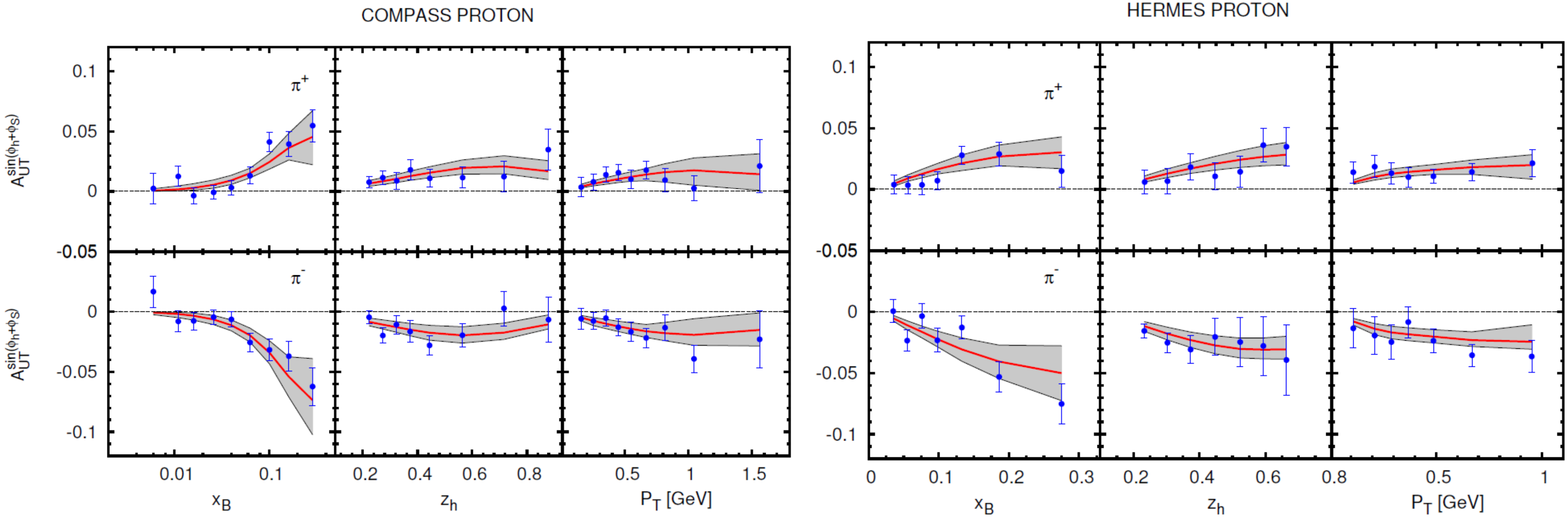




→Still tension between the two methods A_0 and A_{12}

Extraction of transversity & Collins functions

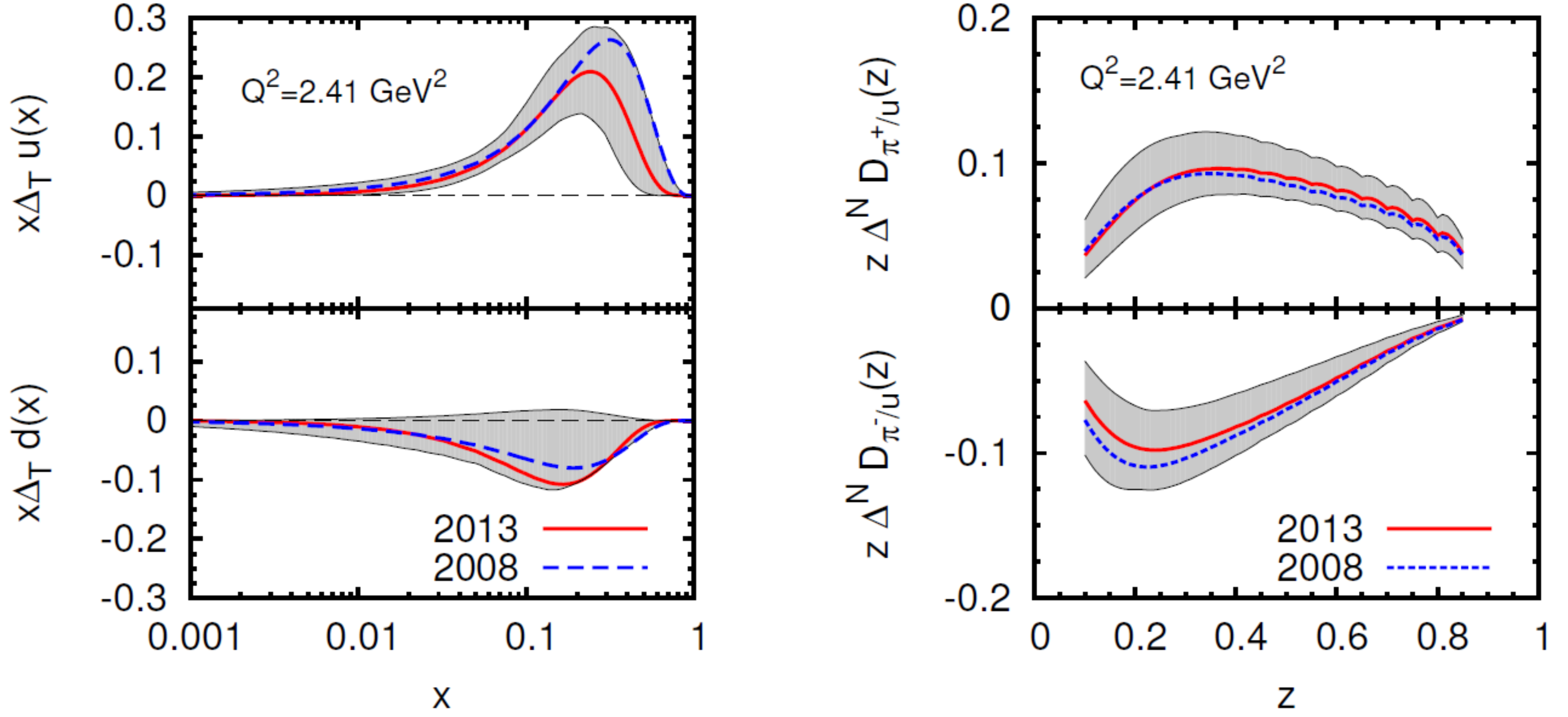
- FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES

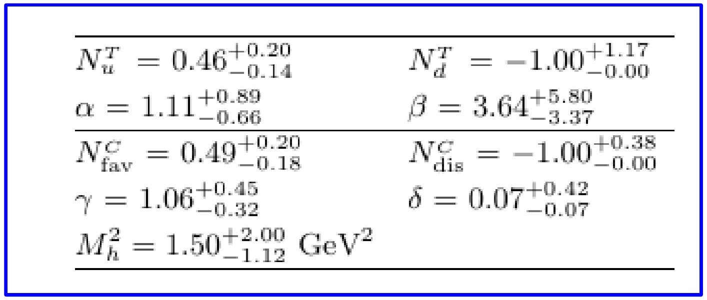


→Similar good description of HERMES and COMPASS

Extraction of transversity & Collins functions

- FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES





$N_u^T = 0.46^{+0.20}_{-0.14}$	$N_d^T = -1.00^{+1.17}_{-0.00}$
$\alpha = 1.11^{+0.89}_{-0.66}$	$\beta = 3.64^{+5.80}_{-3.37}$
$N_{\rm fav}^C = 0.49^{+0.20}_{-0.18}$	$N_{\rm dis}^C = -1.00^{+0.38}_{-0.00}$
$\gamma = 1.06^{+0.45}_{-0.32}$	$\delta = 0.07^{+0.42}_{-0.07}$
$M_h^2 = 1.50^{+2.00}_{-1.12}$ GeV²	

➔Results similar to 2008 extraction

Extraction of transversity & Collins functions

- FIT II: A_0 BELLE data UL & UC +COMPASS+ HERMES ???

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{\mathrm{d.o.f}} = 0.80$	$\chi^2_{\mathrm{tot}} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
Standard Parameterization $\chi^2_{\mathrm{d.o.f}} = 1.12$	$\chi^2_{\mathrm{tot}} = 190$	$\chi^2 = 125$	$\chi^2 = 20$ NO FIT	$\chi^2 = 12$ NO FIT	$\chi^2 = 35$	$\chi^2 = 30$

→ A_0 data cannot be nicely described even if fitted...

Standard parametrization of the Collins function

- Parametrization of the z-dependent part of the Collins function:

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z)\, h(p_\perp)\, D_{\pi/q}(z, p_\perp)$$

$$\boxed{\mathcal{N}_q^C(z) = N_q^C\; z^\gamma (1-z)^\delta\, \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}}$$

Our standard parametrization

- It is equal to 0 at z=0 and z=1

New parametrization of the Collins function

- Let us try to change the parametrization of the z-dependent part of the Collins function:

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\,h(p_\perp)\,D_{\pi/q}(z,p_\perp)$$

$$\boxed{\mathcal{N}_q^C(z) = N_q^C\; z\,[(1-a-b)+a\,z+b\,z^2]}$$

NEW Polynomial parametrization

- It is equal to 0 at z=0 and equal to N_q at z=1

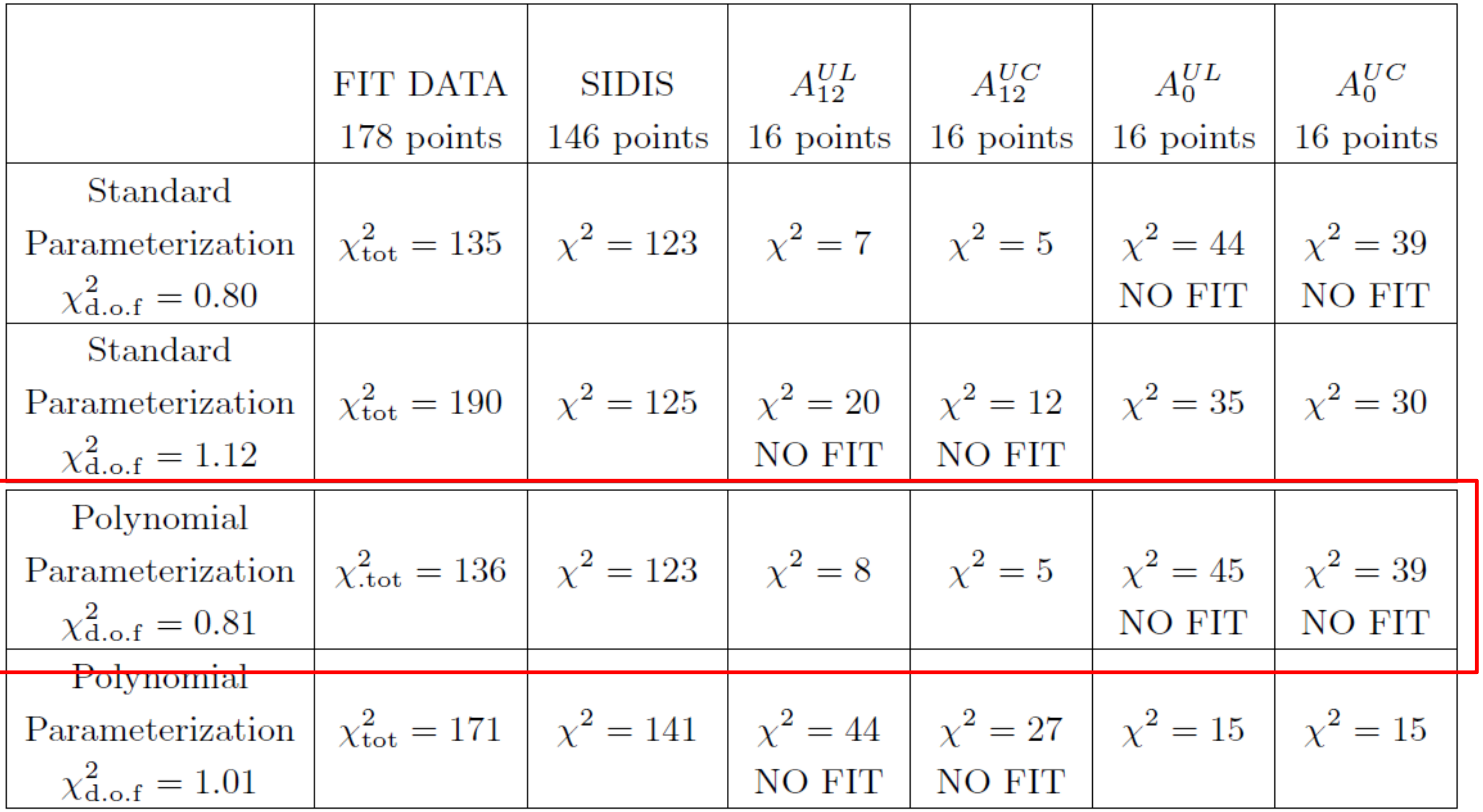
Extraction of transversity & Collins functions

- FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{\mathrm{d.o.f}} = 0.80$	$\chi^2_{\mathrm{tot}} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
Standard Parameterization $\chi^2_{\mathrm{d.o.f}} = 1.12$	$\chi^2_{\mathrm{tot}} = 190$	$\chi^2 = 125$	$\chi^2 = 20$ NO FIT	$\chi^2 = 12$ NO FIT	$\chi^2 = 35$	$\chi^2 = 30$
Polynomial Parameterization $\chi^2_{\mathrm{d.o.f}} = 0.81$	$\chi^2_{.\mathrm{tot}} = 136$	$\chi^2 = 123$	$\chi^2 = 8$	$\chi^2 = 5$	$\chi^2 = 45$ NO FIT	$\chi^2 = 39$ NO FIT
Polynomial Parameterization $\chi^2_{\mathrm{d.o.f}} = 1.01$	$\chi^2_{\mathrm{tot}} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

Extraction of transversity & Collins functions

- FIT III and IV: Polynomial Parametrization



	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{\mathrm{d.o.f}} = 0.80$	$\chi^2_{\mathrm{tot}} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
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Polynomial Parameterization $\chi^2_{\mathrm{d.o.f}} = 1.01$	$\chi^2_{\mathrm{tot}} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

→ If we fit A_{12} data we get the same description obtained with the std par.

→ Almost identical Collins function, again the description of A_0 is not so good

Extraction of transversity & Collins functions

- FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_{0}^{UL} 16 points	A_{0}^{UC} 16 points
Standard Parameterization $\chi^2_{\rm d.o.f} = 0.80$	$\chi^2_{\rm tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
Standard Parameterization $\chi^2_{\rm d.o.f} = 1.12$	$\chi^2_{\rm tot} = 190$	$\chi^2 = 125$	$\chi^2 = 20$ NO FIT	$\chi^2 = 12$ NO FIT	$\chi^2 = 35$	$\chi^2 = 30$
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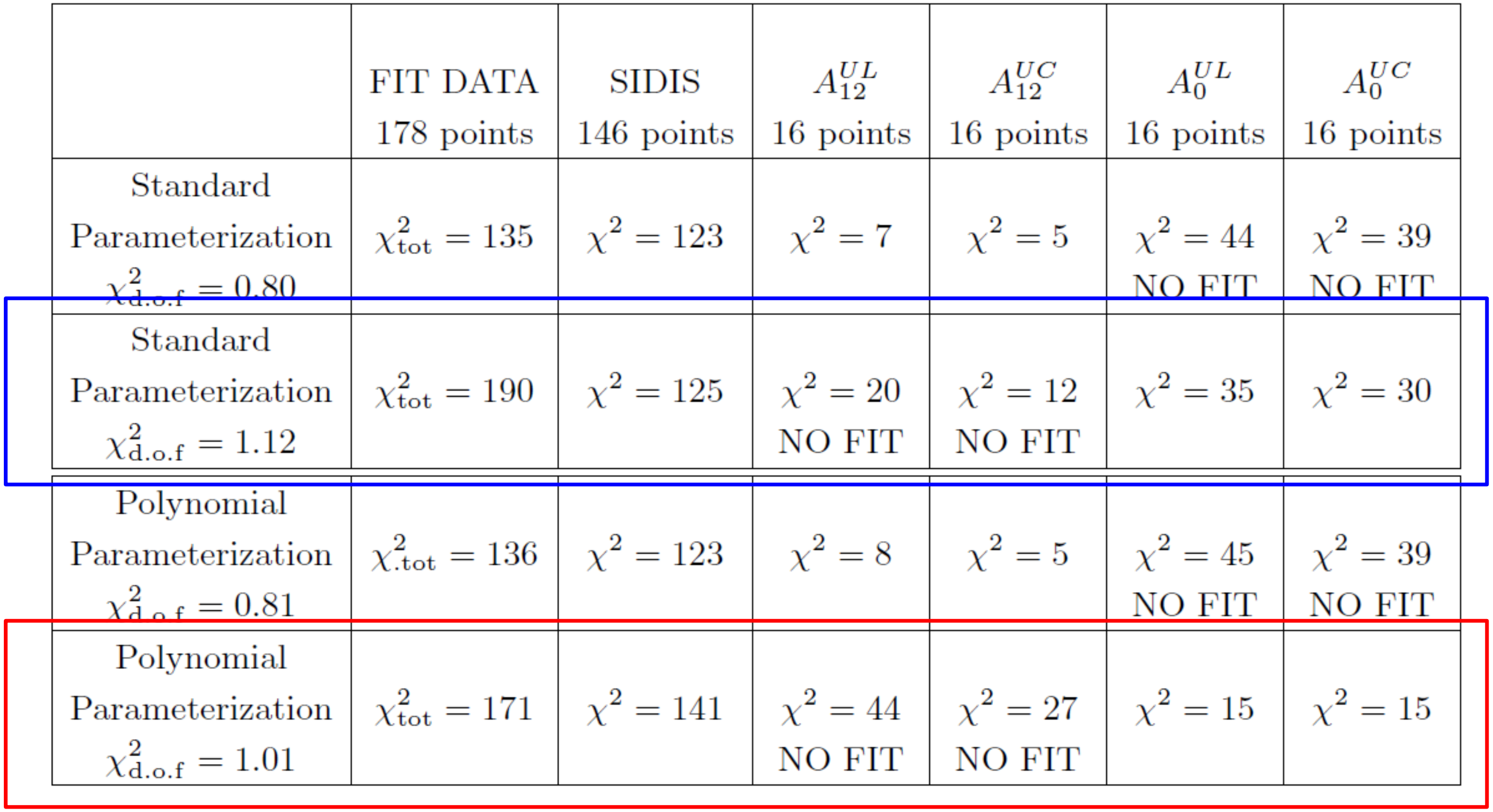
Polynomial Parameterization $\chi^2_{\rm d.o.f} = 1.01$	$\chi^2_{\rm tot} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

➔ If we fit A_o data we can improve their description

➔ Still tension with A_{12}

Extraction of transversity & Collins functions

- FIT III and IV: Polynomial Parametrization



	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{\rm d.o.f} = 0.80$	$\chi^2_{\rm tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
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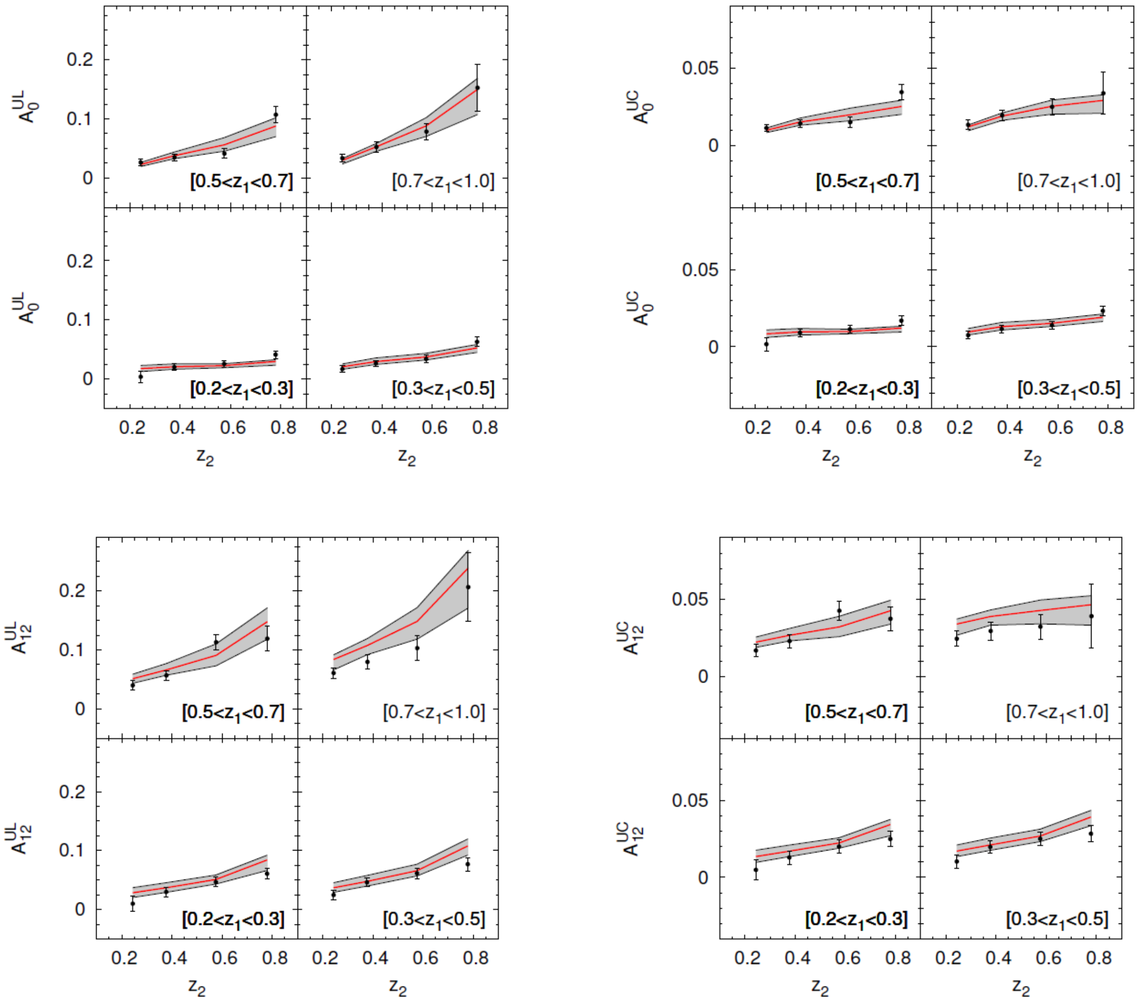
Polynomial Parameterization $\chi^2_{\rm d.o.f} = 1.01$	$\chi^2_{\rm tot} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

→ If we fit A_0 data we can improve their description

→ Still tension with A_{12}

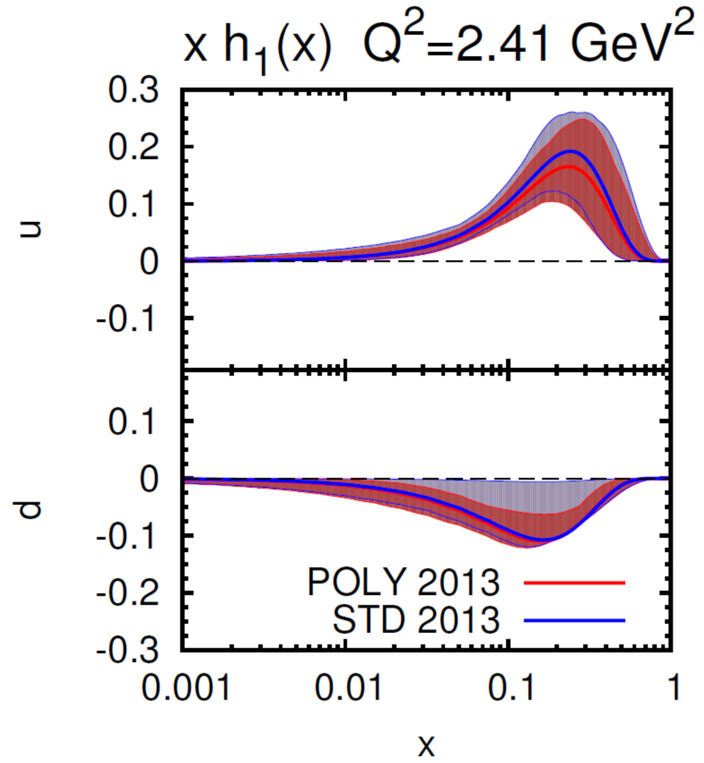
Extraction of transversity & Collins functions

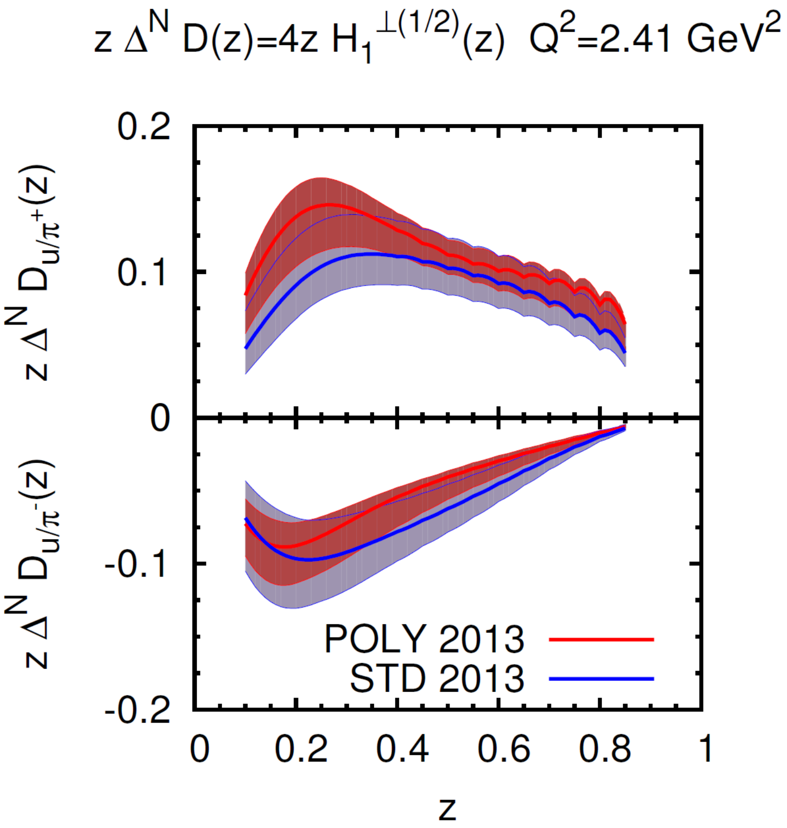
- FIT IV: A_0 BELLE data UL & UC +COMPASS+ HERMES-POLYNOMIAL



Extraction of transversity & Collins functions

- FIT II vs FIT IV (POLYNOMIAL vs STD; FITTED A_0)

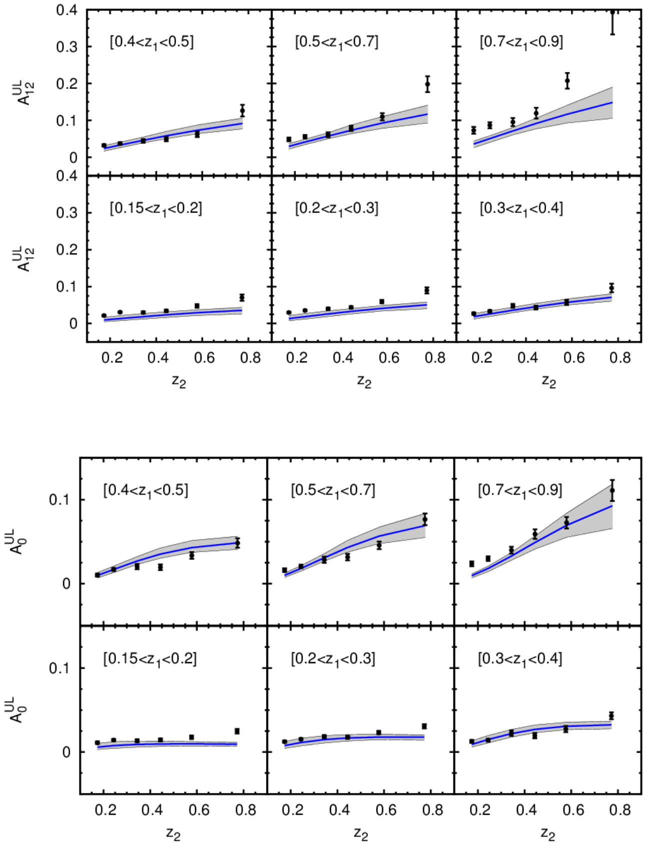




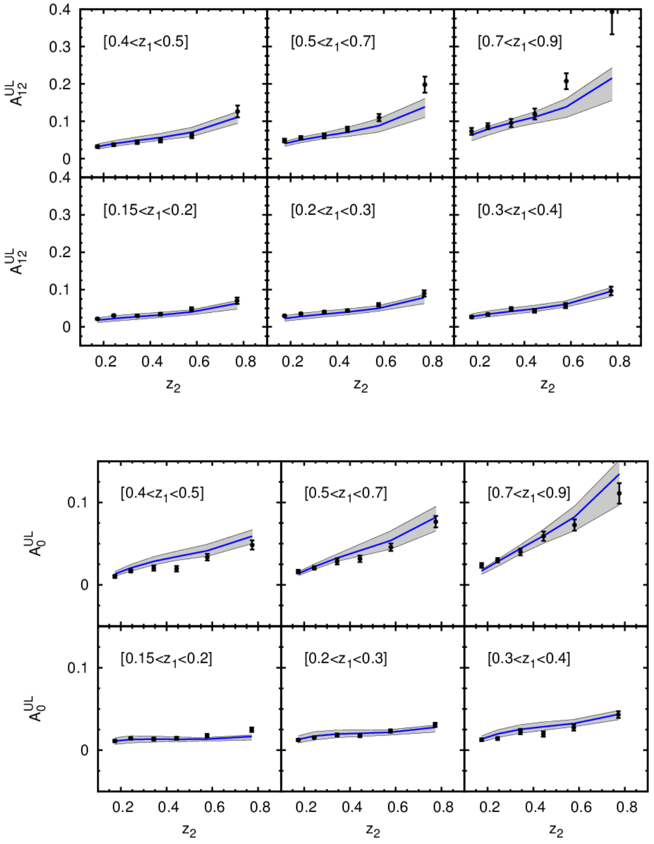
→ Same transversity

→ Different Collins functions (but not dramatically different)

BaBar Predictions



FIT I

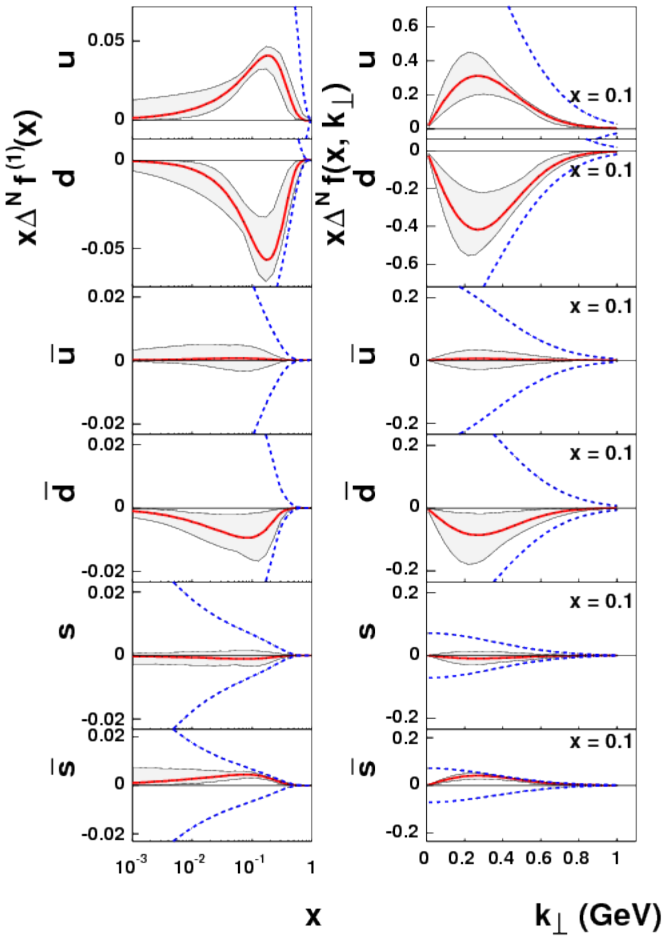


FIT IV

The Sivers function from SIDIS data

Sivers function in SIDIS

- In 2009 we performed a fit of HERMES (2002-5) and COMPASS (Deuteron 2003-4) data on π and K production



✔Valence quark

●$\Delta^N f_{u/p^\uparrow} > 0 \quad \Rightarrow f_{1T}^{\perp u} < 0$

●$\Delta^N f_{d/p^\uparrow} < 0 \quad \Rightarrow f_{1T}^{\perp d} > 0$

✔Sea quarks

●$\Delta^N f_{\bar{s}/p^\uparrow} > 0 \quad \Rightarrow f_{1T}^{\perp \bar{s}} < 0$

➔$\Delta^N f_q^{(1)}(x) \equiv \int d^2\boldsymbol{k}_\perp \, \frac{k_\perp}{4m_p} \, \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)$

Anselmino et al. , Eur. Phys. J. A39, 89-100 (2009)

Sivers function in SIDIS

- 2009 extraction: DGLAP evolution (No TMD evolution), No COMPASS proton data
- In 2012 we applied the Collins TMD evolution scheme to the analysis of the new data from HERMES (2009) and from COMPASS (proton target, 2010-11)

Anselmino et al. , Eur. Phys. J. A39, 89-100 (2009)

TMD evolution formalism

- The simplest version of the Collins TMD evolution equation can be summarized by the following expression:

✎ $$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

Corresponding to Eq. 44 of Ref [*] with $\tilde{K}$=0 and : $\boxed{\mu^2 = \zeta_F = \zeta_D = Q^2}$

- *[*]S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]*

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x,\boldsymbol{b}_T;Q)=\widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

Output function at the scale **Q**
in the impact parameter space

Input function at the scale **Q_0**
in the impact parameter space

Evolution kernel

TMD evolution formalism

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- **Perturbative** part of the evolution kernel

TMD evolution formalism

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✎ $$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0) \boxed{\widetilde{R}(Q, Q_0, b_T)} \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{\ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{\mathrm{d}\mu'}{\mu'} \gamma_K(\mu') + \int_{Q_0}^{Q} \frac{\mathrm{d}\mu}{\mu} \gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right)\right\}$$

TMD evolution formalism

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- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q,Q_0,b_T)\equiv\exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\gamma_K(\mu')+\int_{Q_0}^{Q}\frac{d\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

$$\gamma_K(\mu)=\alpha_s(\mu)\,\frac{2\,C_F}{\pi}$$

$$\gamma_F(\mu;\frac{Q^2}{\mu^2})=\alpha_s(\mu)\,\frac{C_F}{\pi}\left(\frac{3}{2}-\ln\frac{Q^2}{\mu^2}\right)$$

TMD evolution formalism

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Scale that separates the perturbative region
from the non perturbative one

TMD evolution formalism

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$$\mu_b=\frac{C_1}{b_*(b_T)}\qquad b_*(b_T)\equiv\frac{b_T}{\sqrt{1+b_T^2/b_{\max}^2}}\qquad C_1=2e^{-\gamma_E}$$

One of the possible prescription to separate the perturbative region from the non perturbative one

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

✎ $$\widetilde{F}(x,\boldsymbol{b}_T;Q)=\widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-\boxed{g_K(b_T)}\ln\frac{Q}{Q_0}\right\}$$

- **Non Perturbative** (scale independent) part of the evolution kernel that needs to be empirically modeled

$$\boxed{g_K(b_T)=\frac{1}{2}\,g_2\,b_T^2}$$

Common choice used in the unpolarized DY data analyses in the CSS formalism

Parametrization ot the input functions

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \boxed{\widetilde{F}(x,\boldsymbol{b}_T;Q_0)}\,\widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

- Model/parametrization: Different parametrizations here can give very different answers!

- Our approach: Let us apply our standard parametrizations i.e. gaussians factorized among collinear and transverse degree of freedom.
  It is not a unique choice or the best one!

Parametrization of the input functions

- TMD evolution equations using a gaussian model::

$$\widetilde{f}_{q/p}(x, b_T; Q) = f_{q/p}(x, Q_0)\ \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-b_T^2\left(\alpha^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

$$\widetilde{D}_{h/q}(z, b_T; Q) = \frac{1}{z^2} D_{h/q}(z, Q_0)\ \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-b_T^2\left(\beta^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

$$\widetilde{f}'^{\perp}_{1T}(x, b_T; Q) = -2\,\gamma^2\, f^{\perp}_{1T}(x; Q_0)\,\widetilde{R}(Q, Q_0, b_T)\, b_T\ \exp\left\{-b_T^2\left(\gamma^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

Fit of HERMES and COMPASS SIDIS data

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, \Delta^N f_{q/p^\uparrow}(x,k_\perp,Q) \sin(\varphi-\phi_S)\, \frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp,Q)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, f_{q/p}(x,k_\perp,Q)\, \frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp,Q)}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q_0) = 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q_0)$$

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\, \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\, e^{-k_\perp^2/M_1^2}$$

$$\widehat{f}_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\, e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\widehat{D}_{h/q}(z,p_\perp;Q_0) = D_{h/q}(z,Q_0)\,\frac{1}{\pi\langle p_\perp^2\rangle}\, e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

11 free parameters

N_{u_v}	N_{d_v}	N_s
$N_{\bar u}$	$N_{\bar d}$	$N_{\bar s}$
α_{u_v}	α_{d_v}	α_{sea}
β	M_1 (GeV/c) .	

Fixed parameters

$\langle k_\perp^2\rangle = 0.25$ GeV^2

$\langle p_\perp^2\rangle = 0.20$ GeV^2

$g_2 = 0.68$ GeV^2

$b_{max} = 0.5$ GeV^{-1}

Fit of HERMES and COMPASS SIDIS data

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$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q_0) = 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q_0)$$

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$$\widehat{D}_{h/q}(z,p_\perp;Q_0) = D_{h/q}(z,Q_0)\,\frac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

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N_{u_v}	N_{d_v}	N_s
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Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

TMD evolution (exact)
$\chi^2_{\rm tot} = 255.8$
$\chi^2_{\rm d.o.f} = 1.02$

DGLAP evolution
$\chi^2_{\rm tot} = 315.6$
$\chi^2_{\rm d.o.f} = 1.26$

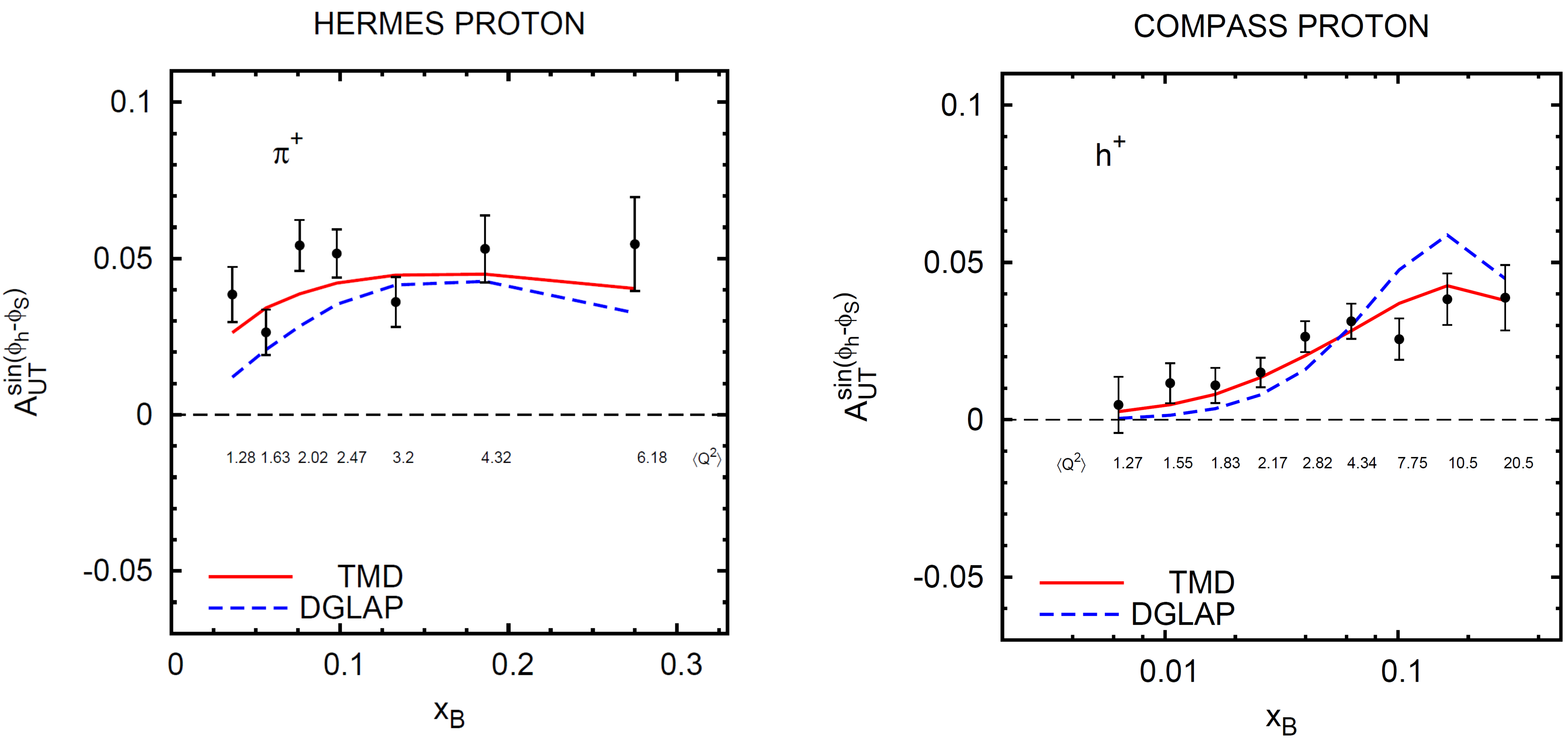
Fit of HERMES and COMPASS SIDIS data

χ^2 tables

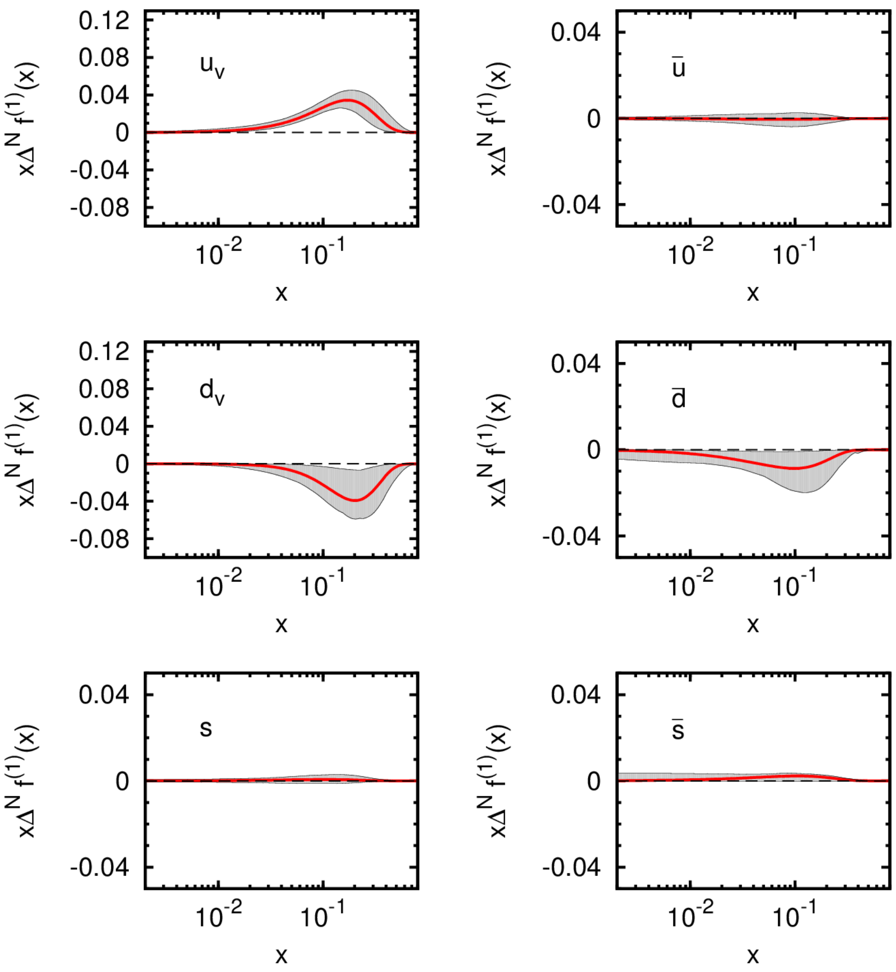
11 free parameters, 261 points

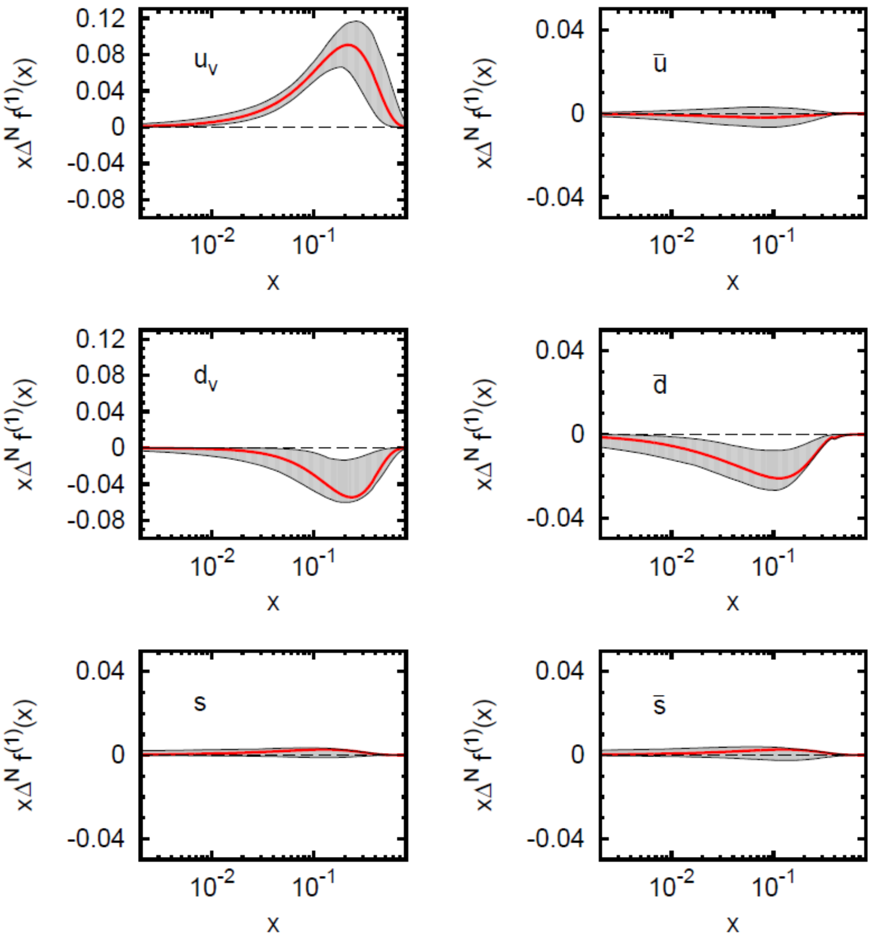
	TMD Evolution (Exact)		DGLAP Evolution
	$\chi^2_{tot} = 255.8$		$\chi^2_{tot} = 315.6$
	$\chi^2_{d.o.f} = 1.02$		$\chi^2_{d.o.f} = 1.26$
HERMES **π^+**	$\boxed{\chi^2_x = 10.7}$	7 points	$\boxed{\chi^2_x = 27.5}$
	$\chi^2_z = 4.3$		$\chi^2_z = 8.6$
	$\chi^2_{P_T} = 9.1$		$\chi^2_{P_T} = 22.5$
COMPASS **h^+**	$\boxed{\chi^2_x = 6.7}$	9 points	$\boxed{\chi^2_x = 29.2}$
	$\chi^2_z = 17.8$		$\chi^2_z = 16.6$
	$\chi^2_{P_T} = 12.4$		$\chi^2_{P_T} = 11.8$

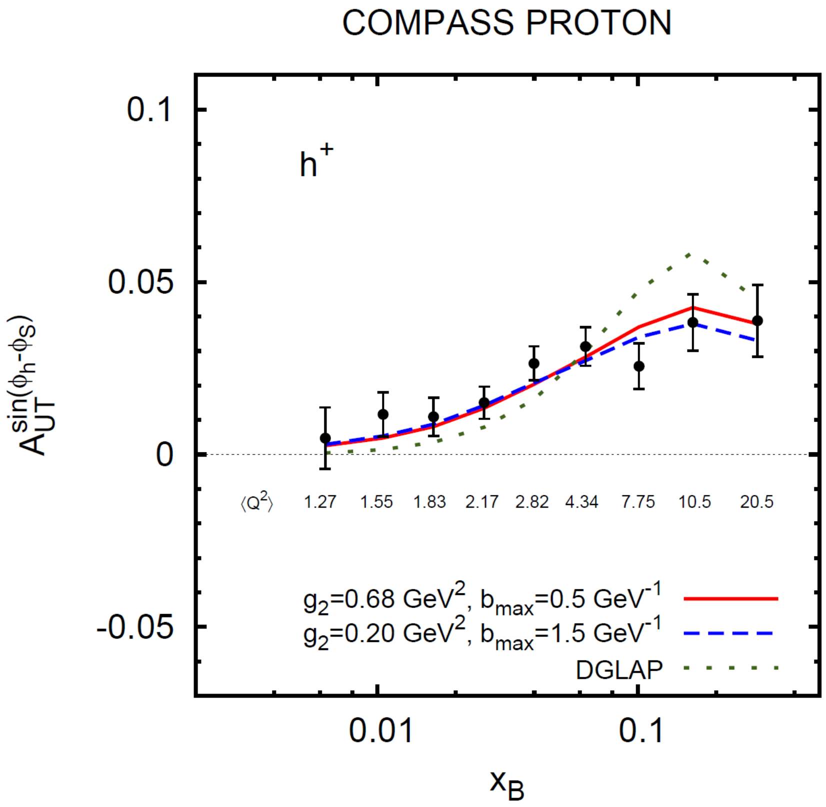
Fit of HERMES and COMPASS SIDIS data

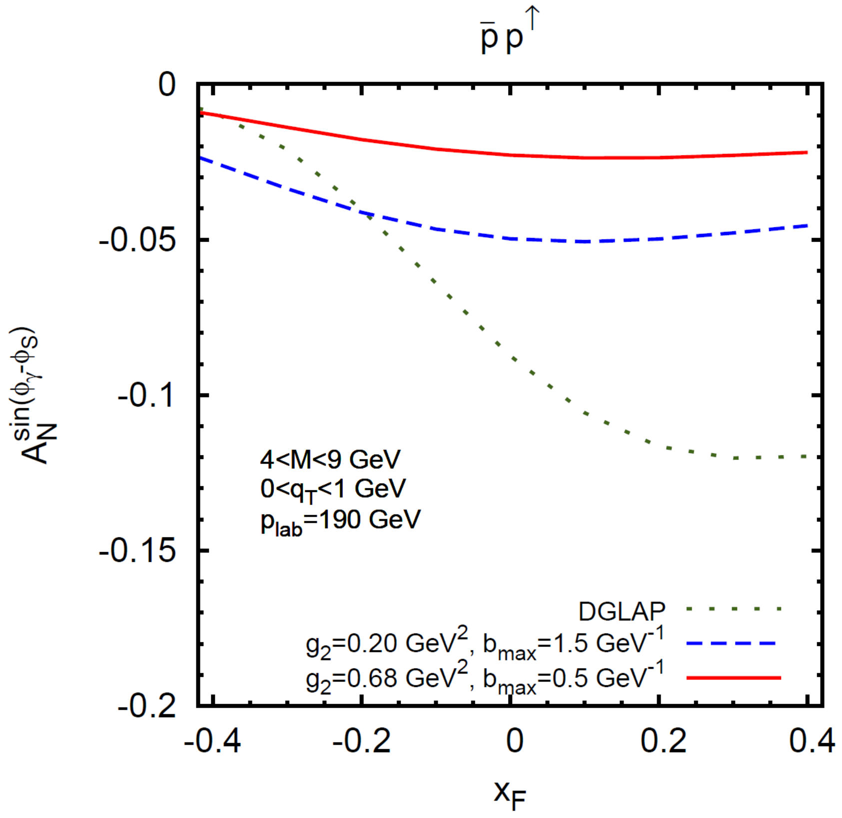


Sivers functions

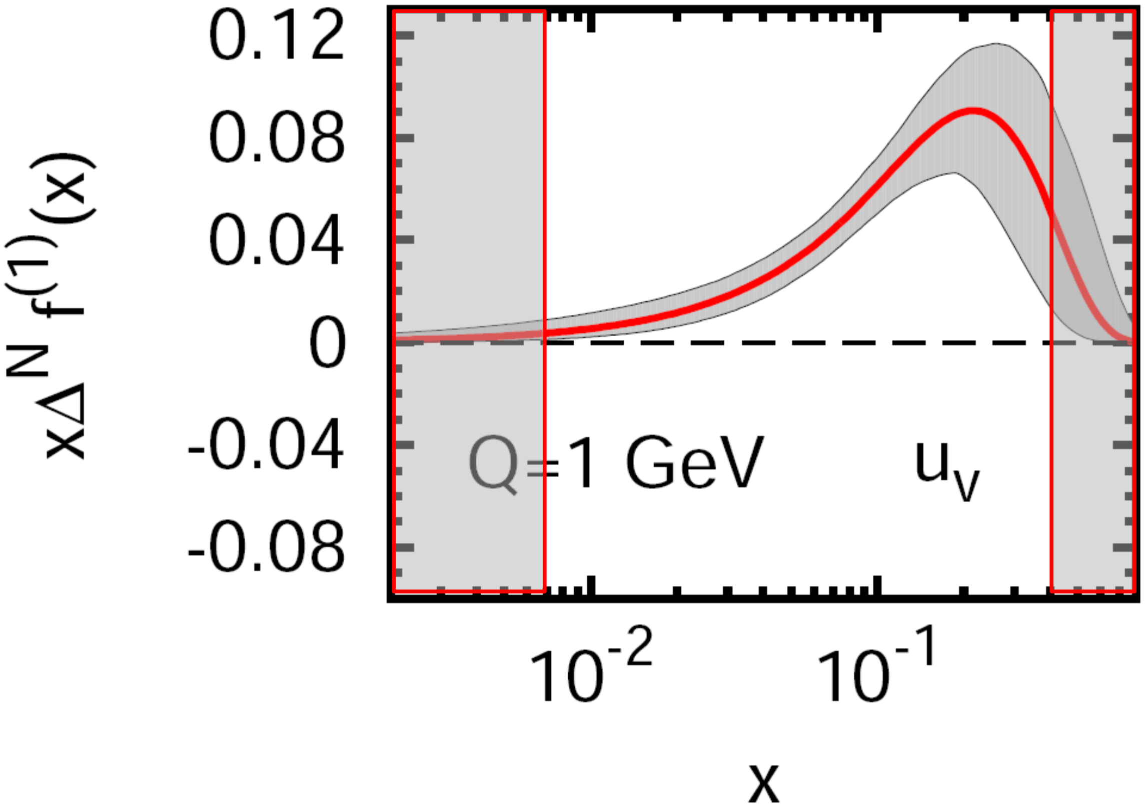

SIVERS FUNCTION - DGLAP


SIVERS FUNCTION - TMD


COMPASS PROTON
h$^+$
$A_{UT}^{\sin(\phi_h-\phi_S)}$
0.1
0.05
0
-0.05
⟨Q^2⟩ 1.27 1.55 1.83 2.17 2.82 4.34 7.75 10.5 20.5
g_2=0.68 GeV^2, b_{max}=0.5 GeV^{-1}
g_2=0.20 GeV^2, b_{max}=1.5 GeV^{-1}
DGLAP
0.01
0.1
x_B


$\bar{p}\,p^\uparrow$
$A_N^{\sin(\phi_\gamma-\phi_S)}$
0
-0.05
-0.1
-0.15
-0.2
4<M<9 GeV
0<q_T<1 GeV
p_{lab}=190 GeV
DGLAP
g_2=0.20 GeV^2, b_{max}=1.5 GeV^{-1}
g_2=0.68 GeV^2, b_{max}=0.5 GeV^{-1}
-0.4
-0.2
0
0.2
0.4
x_F

Sivers functions



Turin standard approach (DGLAP)

- Unpolarized TMDs are factorized in x and $k_\perp$. Only the collinear part evolves with DGLAP evolution equation. No evolution in the transverse momenta:

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

Collinear PDF (DGLAP evolution)

Normalized Gaussian: no evolution

Turin standard approach (DGLAP)

- The Sivers function is factorized in x and $k_\perp$ and proportional to the unpolarized PDF.

$$\begin{aligned}\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) &= 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q)\\ &= 2\mathcal{N}_q(x)f_{q/p}(x;Q)\sqrt{2e}\,\frac{k_\perp}{M_1}\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}\end{aligned}$$

Collinear PDF (DGLAP)

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2+\langle k_\perp^2\rangle}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{m_p}f_{1T}^\perp(x,k_\perp)$$

Collins TMD evolution of the Sivers function (PRD85,2012)

$$\tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F) = \tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu_0, Q_0^2) \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right.$$
$$\left. + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{\sqrt{\zeta_F}}{Q_0} \gamma_K(g(\mu')) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}. \quad (44)$$

$$\tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F) = \sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1\, d\hat{x}_2}{\hat{x}_1\, \hat{x}_2} \tilde{C}^{\mathrm{Sivers}}_{f/j}(\hat{x}_1, \hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b))\, T_{F\, j/P}(\hat{x}_1, \hat{x}_2, \mu_b)$$
$$\times \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp\left\{ -g^{\mathrm{Sivers}}_{f/P}(x, b_T) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}. \quad (47)$$

CSS formalism

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2dydq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j(x_1,x_2,b_T,Q) + Y(x_1,x_2,q_T,Q)$$

Resummed part

Regular part

$$W_j(x_1,x_2,b_T,Q) = \exp\left[S_j(b_T,Q)\right]\sum_{i,k} C_{ji}\otimes f_i(x_1,C_1^2/b_T^2)\; C_{\bar{j}k}\otimes f_k(x_2,C_1^2/b_T^2)$$

Pdfs convoluted with the Wilson Coefficients

$$[C_{ji}\otimes f_i](x,\mu^2) = \int_x^1 \frac{dz}{z} C_{ji}(z,\alpha_s(\mu)) f_i(x/z,\mu)$$

$$C_{ji}(z,\alpha(\mu)) \quad = \quad \delta_{ij}\delta(1-z) + \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n C_{ij}^{(n)}(z)$$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

CSS formalism

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2dydq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j(x_1,x_2,b_T,Q) + Y(x_1,x_2,q_T,Q)$$

Resummed part

Regular part

$$W_j(x_1,x_2,b_T,Q) = \exp\left[S_j(b_T,Q)\right]\sum_{i,k} C_{ji}\otimes f_i(x_1,C_1^2/b_T^2)\ C_{\bar{j}k}\otimes f_k(x_2,C_1^2/b_T^2)$$

Sudakov factor

$$S_j(b_T,Q) = \int_{C_1^2/b_T^2}^{Q^2}\frac{d\kappa^2}{\kappa^2}\left[A_j(\alpha_s(\kappa))\ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa))\right]$$

$$\begin{aligned} A_j(\alpha(\mu)) &= \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n A_j^{(n)} \\ B_j(\alpha(\mu)) &= \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n B_j^{(n)} \end{aligned}$$

$$\begin{array}{rcl} \text{Leading Log (LL)} & : & A^{(1)}; \\ \text{Next to LL (NLL)} & : & A^{(2)}, B^{(1)}, C^{(1)}; \\ \text{Next to NLL (NNLL)} & : & A^{(3)}, B^{(2)}, C^{(2)}; \\ \text{Fixed order } \alpha_s(FXO) & : & A^{(1)}, B^{(1)}, C^{(1)}; \end{array}$$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

CSS formalism

Evolution equations:

$$\frac{\partial W_j(x_1,x_2,b_T,Q)}{\partial \ln(Q^2)} = \Big[K(b_T\mu)+G(Q/\mu)\Big]W_j(x_1,x_2,b_T,Q)$$

$$\frac{dK(b_T\mu,\alpha_s(\mu)}{d\mu} = -\gamma_K(\alpha_s(\mu))$$

$$\frac{dG(Q/\mu,\alpha_s(\mu)}{d\mu} = +\gamma_K(\alpha_s(\mu))$$

$$\frac{\partial W_j(x_1,x_2,b_T,Q)}{\partial \ln(Q^2)} = \left\{-\int_{C_1^2/b_T^2}^{Q^2}\frac{d\kappa^2}{\kappa^2}\left[A_j(\alpha_s(\kappa))\ln\left(\frac{Q^2}{\kappa^2}\right)+B_j(\alpha_s(\kappa))\right]\right\}W_j(x_1,x_2,b_T,Q)$$

$$\longrightarrow S_j(b_T,Q)$$

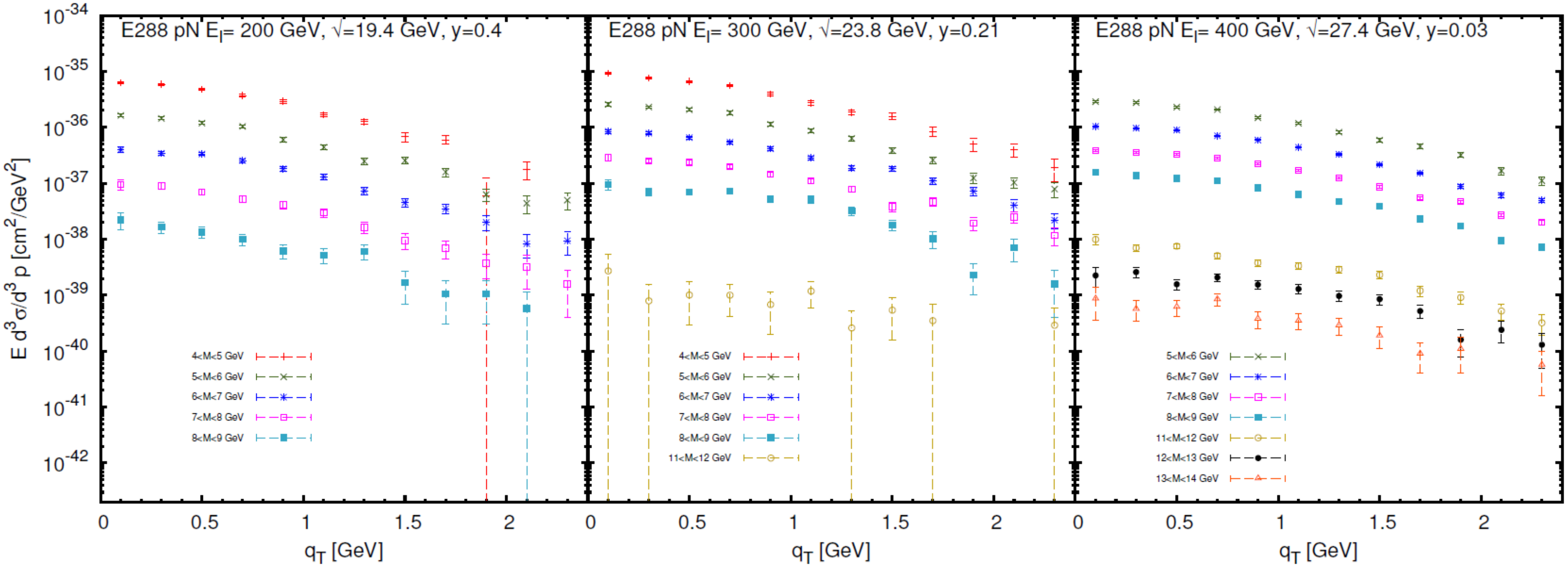
$$W_j(x_1,x_2,b_T,Q) = \exp\left[S_j(b_T,Q)\right]W_j(x_1,x_2,b_T,C_1/b_T)$$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

Drell-Yan phenomenology

- Low energy data

	E288 200	E288 300	E288 400	E605	R209
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV	38.8 GeV	62 GeV
E_{beam}	200 GeV	300 GeV	400 GeV	800 GeV	-
Beam/Target	p Cu	p Cu	p Cu	p Cu	p p
Q range	4-9 GeV	4-9; 11-12 GeV	5-9; 11-14 GeV	4-9; 10.5-18 GeV	5-8; 11-25 GeV
Other kin. var	y=0.4	y=0.21	y=0.03	$-0.1 < x_F < 0.2$	
Observable	$Ed^3\sigma/d^3\mathbf{p}$	$Ed^3\sigma/d^3\mathbf{p}$	$Ed^3\sigma/d^3\mathbf{p}$	$Ed^3\sigma/d^3\mathbf{p}$	$d\sigma/dq_T^2$



- The P_T distribution seems to be Gaussian....

Drell-Yan phenomenology

- Are data distributed as a Gaussian? Do data scale as 1/M2+DGLAP+KIN

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1)\bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$

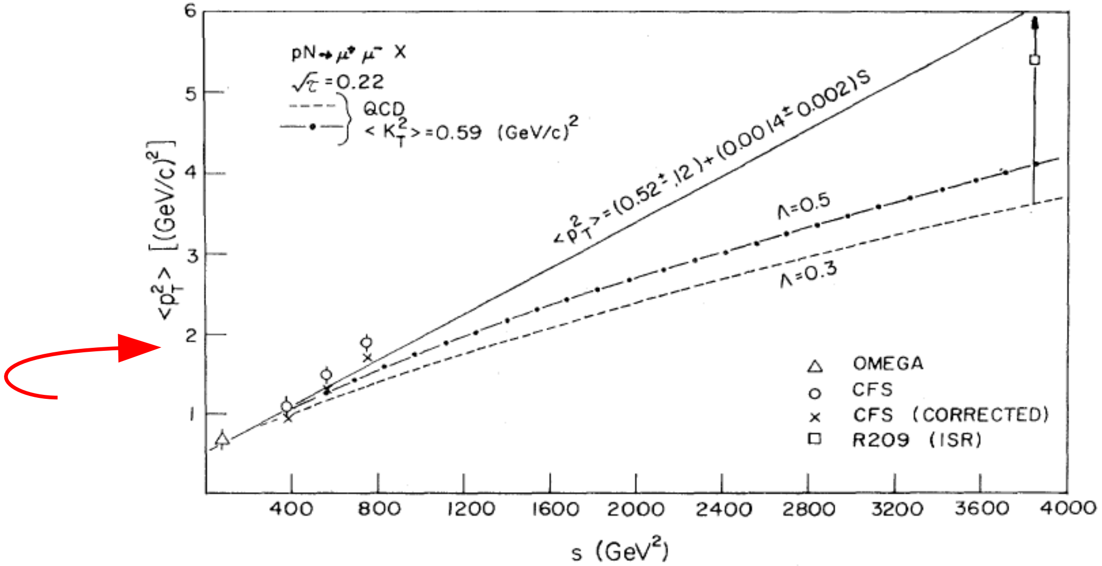


FIG. 3. $\langle {p_T}^2 \rangle$ vs s for dimuons produced in p-nucleon interactions. The solid curve is the linear fit to the data. The dashed and dot-dash curves are the predictions of first-order QCD using the Altarelli *et al.* prescription for different values of Λ.

$$< K_\perp^2 > = \alpha_s(Q^2)\sqrt{S}\, f(\tau, \alpha_s(Q^2)) + \ldots$$

Cox and Malhotra,Phys. Rev. D29(1984)

TMD evolution

TMD in the b space:

$$\begin{aligned}
\tilde{F}(x, b_T, Q, \zeta_F) \quad = \quad & \sum_j \int_x^1 \frac{dy}{y} \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) f_j(y, \mu_b) \\
& \boxed{\exp\left\{ \ln\left(\frac{\sqrt{\zeta_F}}{\mu_b}\right) \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln\left(\frac{\sqrt{\zeta_F}}{\kappa}\right) \gamma_K(\kappa) \right\}} \\
& \exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right) \right\}
\end{aligned}$$

Related to the evolution in the cut off parameter of the TMD:

$$\frac{\partial \ln \tilde{F}(x, \mathbf{b}_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(\mathbf{b}_T; \mu)$$

However.... at first order in the strong coupling constant:

$$\tilde{K}(\mu, b_T) = -\frac{\alpha_s(\mu)}{\pi} \ln(\mu^2 b_T^2 / C_1^2) \qquad \text{if } \mu_b = C_1/b_* \qquad \tilde{K}(b_*, \mu_b) = 0$$

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

TMD evolution

TMD in the b space:

$$\begin{aligned}
\tilde{F}(x, b_T, Q, \zeta_F) \quad = \quad & \sum_j \int_x^1 \frac{dy}{y} \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) f_j(y, \mu_b) \\
& \boxed{\exp\left\{ \ln\left(\frac{\sqrt{\zeta_F}}{\mu_b}\right) \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln\left(\frac{\sqrt{\zeta_F}}{\kappa}\right) \gamma_K(\kappa) \right\}} \\
& \exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right) \right\}
\end{aligned}$$

Second part of the part of the Sudakov form factor, notice that depends on ζ_F

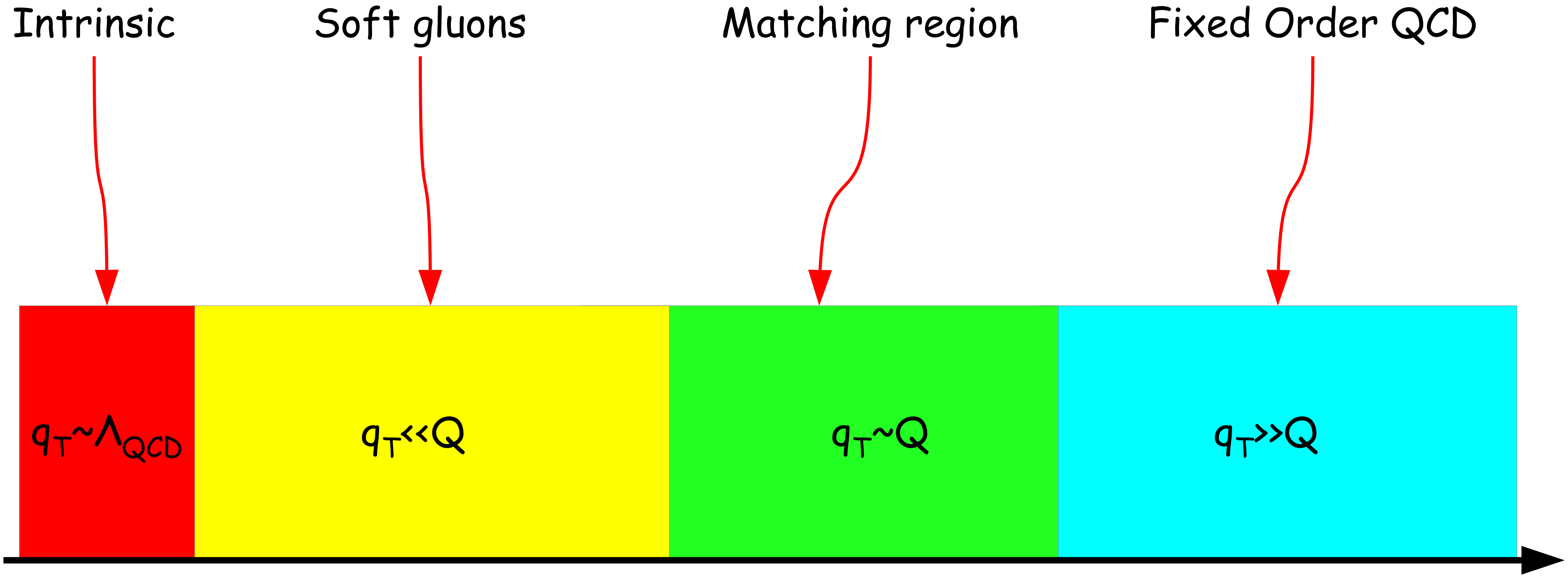
at order α_s:

$$\gamma_F(\mu; \zeta_F/\mu^2) \quad = \quad \alpha_s(\mu) \frac{C_F}{\pi} \left(\frac{3}{2} - \ln(\frac{\zeta_F}{\mu^2}) \right)$$

$$\gamma_K(\mu) = 2 C_F \frac{\alpha_s(\mu)}{\pi}$$

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

CSS formalism



Intrinsic

Soft gluons

Matching region

Fixed Order QCD

$q_T \sim \Lambda_{QCD}$

$q_T << Q$

$q_T \sim Q$

$q_T >> Q$

CSS Phenomenology

Nadolsky et al. Analyzed low energy DY data and Z boson production data Using different parametrizations

Parameter	DWS-G fit	LY-G fit	BLNY fit
g_1	0.016	0.02	0.21
g_2	0.54	0.55	0.68
g_3	0.00	-1.50	-0.60
CDF Z Run-0 N_{fit}	1.00 (fixed)	1.00 (fixed)	1.00 (fixed)
R209 N_{fit}	1.02	1.01	0.86
E605 N_{fit}	1.15	1.07	1.00
E288 N_{fit}	1.23	1.28	1.19
DØ Z Run-1 N_{fit}	1.01	1.01	1.00
CDF Z Run-1 N_{fit}	0.89	0.90	0.89
χ^2	416	407	176
χ^2/DOF	3.47	3.42	1.48

$$b_{max} = 0.5 \text{ GeV}^{-1}$$

See, Nadolsky et al., Phys.Rev. D67,073016 (2003)

TMD evolution

Collins suggest that: $\zeta_F = Q^2$ $\quad b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$ $\quad \mu_b = C_1/b_*$

$$\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) = \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \exp[S_{RAC}(b_*, Q^2)] F_{NP}(x, b_T, Q)$$

It can be show easily that at first order in the strong coupling constant:

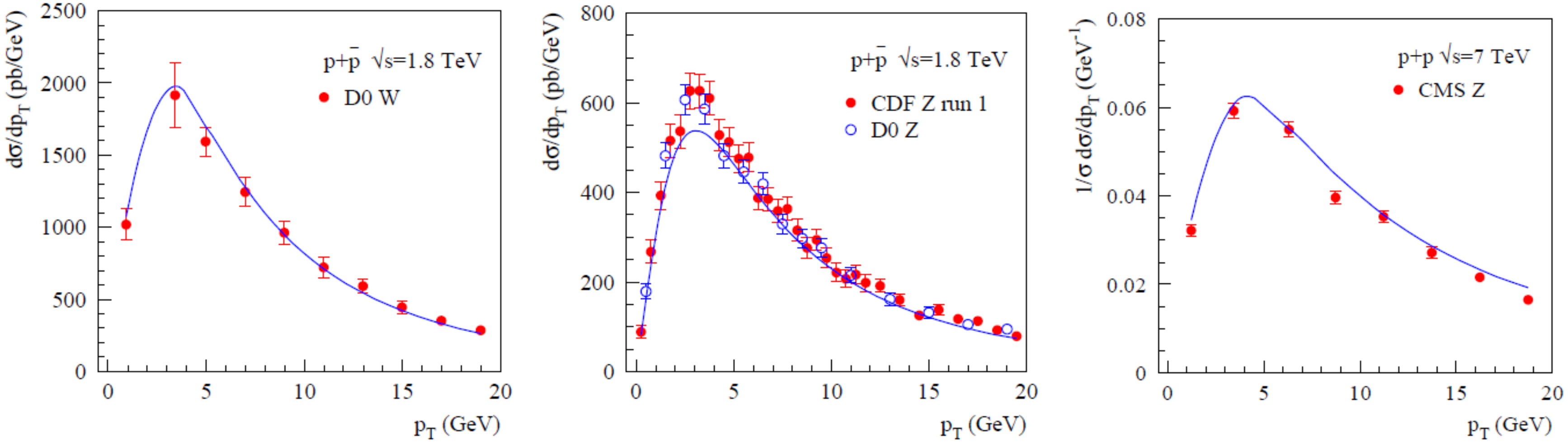
$$S_{RAC}(b_T, Q^2) = C_F \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \frac{\alpha_s(\kappa)}{\pi} \left[\frac{3}{2} - \ln\left(\frac{Q^2}{\kappa^2}\right)\right] \equiv \frac{1}{2} S_{CSS}(b_T, Q^2)$$

TMD evolution is more general than CSS which is a particular case of the TMD one
Previous studies performed with CSS are still valid!

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

EIKV phenomenology

Z and W-Boson Production



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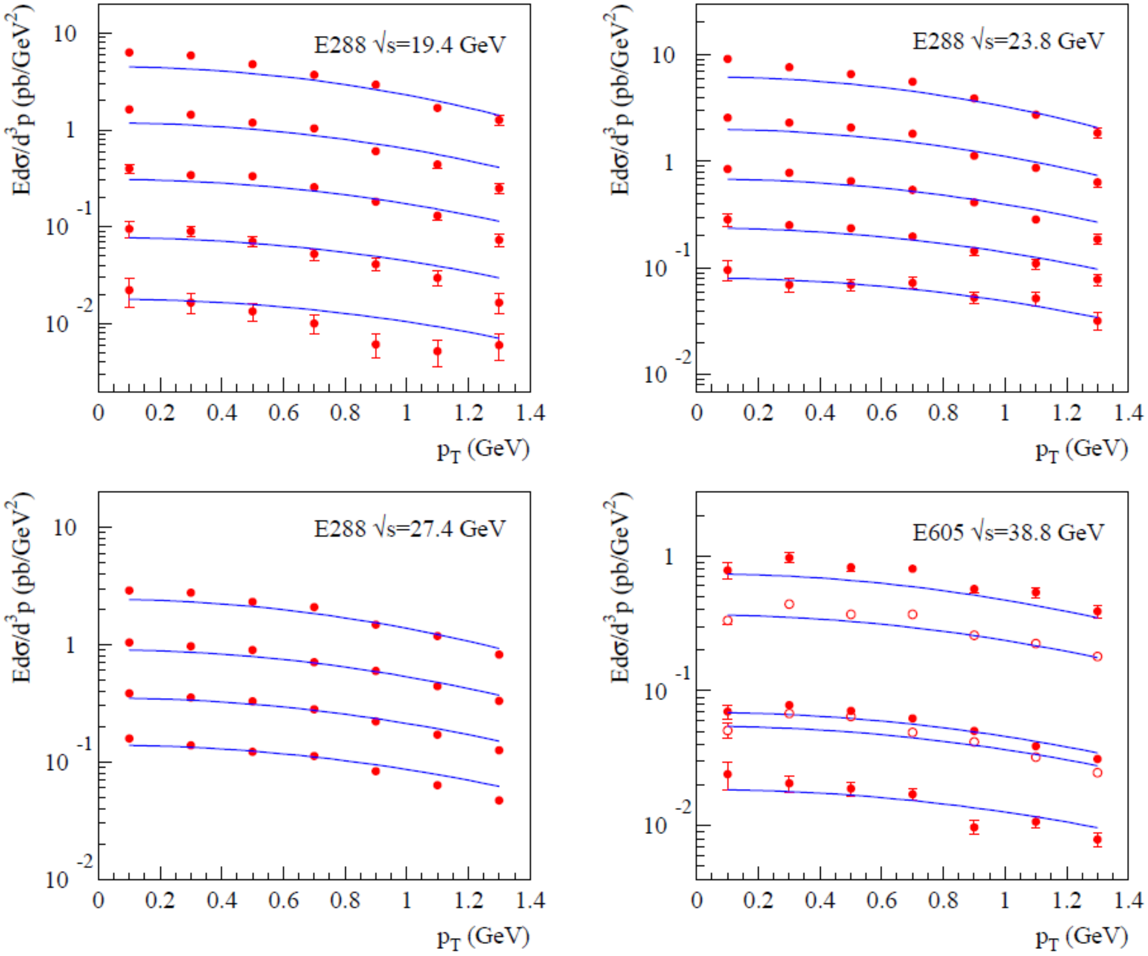
EIKV phenomenology

Low energy Drell-Yan



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