Different approaches to TMD Evolution with Scale (corrected)

John Collins (Penn State)

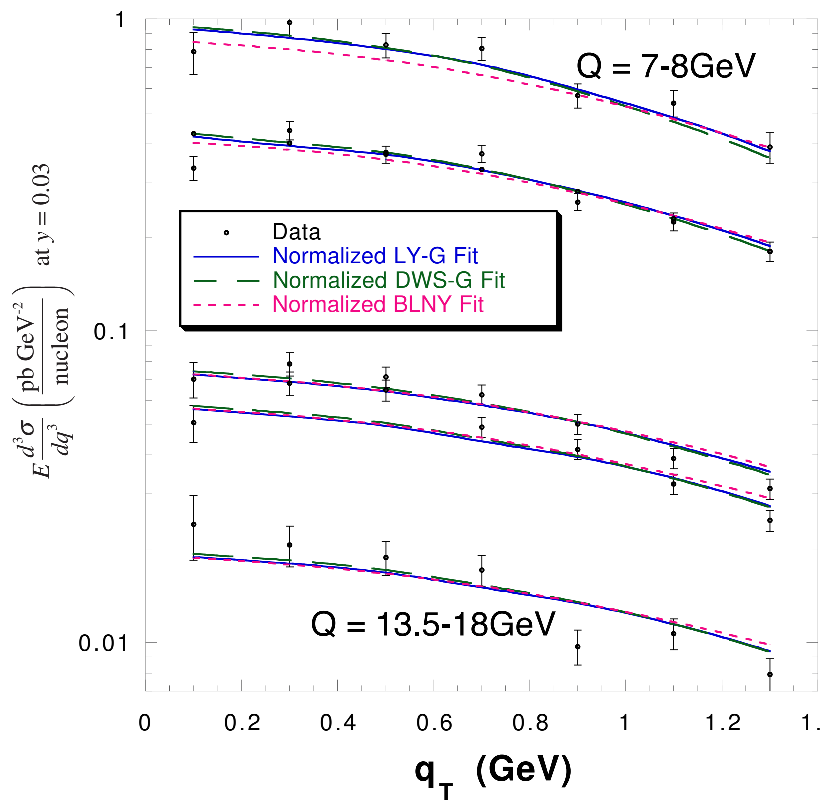
- Examine multiple views of TMD evolution, and non-perturbative contributions
- How to get a correct view (theoretically and phenomenologically)?

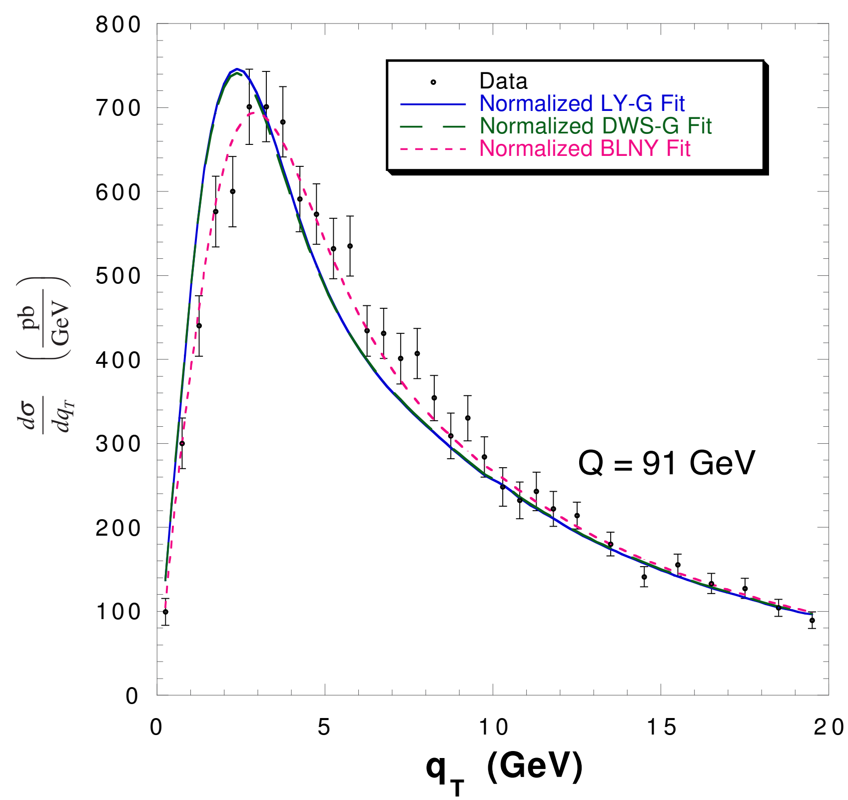
Formalisms used: They don't all appear compatible

Parton model:	QCD complications ignored
Non-TMD formalisms	E.g., Altarelli et al. NPB 246, 12 (1984)
Original CSS:	non-light-like axial gauge; soft factor
Ji–Ma–Yuan:	non-light-like Wilson lines; soft factor; parameter ρ
New CSS:	clean up, Wilson lines mostly light-like;
	absorb (square roots of) soft factor in TMD pdfs
Becher–Neubert:	SCET, but without actual finite TMD pdfs
Echevarría–Idilbi–Scimemi:	SCET
Mantry–Petriello:	SCET
Boer, Sun-Yuan:	Approximations on CSS

Disagreement on size of non-perturbative contribution to evolution ($\tilde{K}(b_{\mathsf{T}})$ at large b_{T}), or even whether it exists.

Symptom of QCD effects: Drell-Yan q_T distribution broadens





Q: 7–18 GeV, $\sqrt{s} = 38.8\,\mathrm{GeV}$ | $Q = m_Z$, $\sqrt{s} = 1800\,\mathrm{GeV}$

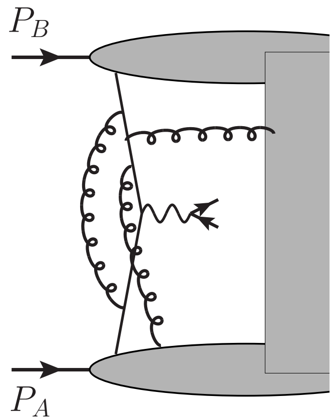
(Plot of $E\,\mathrm{d}\sigma/\mathrm{d}^3\mathbf{q}$) | (Plot of $\mathrm{d}\sigma/\mathrm{d}q_\mathsf{T}$: has q_T factor.)

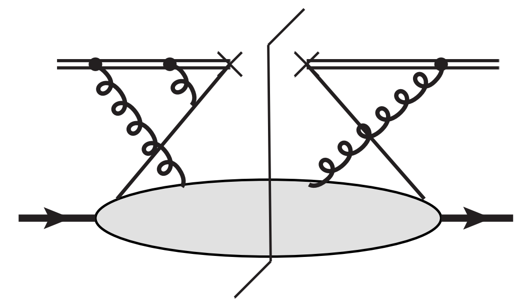
(Adapted from Landry et al., PRD 67,073016 (2003))

Width around $1\,\mathrm{GeV}$ | Width around $3\,\mathrm{GeV}$

(But values of x are different — perennial issue!)

Need for evolution from QCD





Fourier trans. of $\langle p|\bar{\psi}$ WL $\psi|p\rangle$

$\Longrightarrow$ Broadening from emitting pert. and non-pert. glue into increasing rapidity range.

- Non-trivial extraction of “misattached” glue onto Wilson lines in definitions of TMD pdfs etc.
- Can codify in separate soft factor or suitable redefinition of TMD functions.

TMD factorization (modernized Collins-Soper form) (Cf. Melis)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\Omega} = \frac{2}{s}\sum_j \frac{\mathrm{d}\hat{\sigma}_{j\bar{j}}(Q,\mu)}{\mathrm{d}\Omega} \int e^{i\boldsymbol{q}_\mathsf{T}\cdot\boldsymbol{b}_\mathsf{T}}\ \tilde{f}_{j/A}(x_A,\boldsymbol{b}_\mathsf{T};\zeta_A,\mu)\ \tilde{f}_{\bar{j}/B}(x_B,\boldsymbol{b}_\mathsf{T};\zeta_B,\mu)\,\mathrm{d}^2\boldsymbol{b}_\mathsf{T}$$

$$+ \text{ poln. terms } + \text{ high-}q_\mathsf{T}\text{ term } + \text{ power-suppressed}$$

where can set $\zeta_A = \zeta_B = Q^2$, $\mu = Q$.

Evolution:

$$\frac{\partial \ln \tilde{f}_{f/H}(x,b_\mathsf{T};\zeta;\mu)}{\partial \ln\sqrt{\zeta}} = \tilde{K}(b_\mathsf{T};\mu)$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}\ln\mu} = -\gamma_K(\alpha_s(\mu))$$

$$\frac{\mathrm{d}\ln \tilde{f}_{f/H}(x,b_\mathsf{T};\zeta;\mu)}{\mathrm{d}\ln\mu} = \gamma_f(\alpha_s(\mu);1) - \frac{1}{2}\gamma_K(\alpha_s(\mu))\ln\frac{\zeta}{\mu^2}$$

Small-b_T:

$$\tilde{f}_{f/H}(x,b_\mathsf{T};\zeta;\mu) = \sum_j \int_{x-}^{1+} \tilde{C}_{f/j}(x/\hat{x},b_\mathsf{T};\zeta,\mu,\alpha_s(\mu))\ f_{j/H}(\hat{x};\mu)\frac{\mathrm{d}\hat{x}}{\hat{x}}\ +\ O[(mb_\mathsf{T})^p]$$

Location of non-perturbative information

TMD-specific:

- Parton densities at large b_T (at one scale): $\tilde{f}_{j/A}(x_A, \boldsymbol{b}_\mathrm{T}; \zeta_A, \mu)$. "Intrinsic transverse momentum".
- Evolution kernel $\tilde{K}(b_\mathrm{T}; \mu)$ at large b_T. Universal "soft glue per unit rapidity".

Non-TMD:

- Ordinary parton densities, to give small b_T behavior of TMD pdfs by OPE.

Formalisms used: They don't all appear compatible

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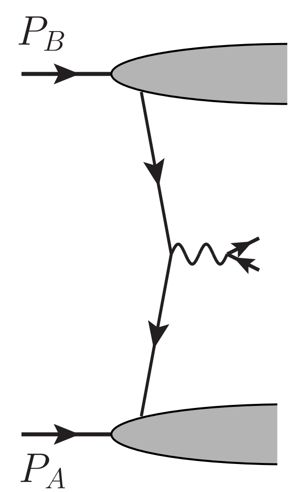
Parton Model

- Can apply at one energy or Q.
- But it ignores evolution, small-b_T/large-q_T behavior.
- q_T distribution is independent of s (or Q) — N.B. at fixed x_A, x_B.



Methods without TMD functions

- Based on collinear factorization + resummation of massless hard scattering. E.g.,
  - Altarelli, Ellis, Greco & Martinelli, NPB 246, 12 (1984)
  - . . .
  - Bozzi, Catani, de Florian & Grazzini, NPB 737, 73 (2006)
- Collinear factorization uses approximations valid for large Q when $q_\mathrm{T} \sim Q$ or q_T integrated over
- Logical foundation fails when $q_\mathrm{T} \ll Q$.
- Symptom: Effects of Boer-Mulders, Sivers functions missed.
- Integrals over scale include non-perturbative regions with, e.g., $\alpha_s(k^2)$ at small scale.
- TMD factorization shows what to do.

Original CSS

- Theoretical dimension:
  - Define TMD pdfs (etc) with use of non-light-like axial gauge
  - Separate soft function
  - Evolution equations have power-suppressed corrections

- Separation of non-perturbation large-b_T effects:
  - Proposed b_* prescription
  - TMD factorization & evolution determine what kinds of functions to use for non-perturbative part

- Classic fits to Drell-Yan ($5\,\mathrm{GeV} \lesssim Q \leq m_Z$)
  - BLNY: Landry et al., PRD 67,073016 (2003)
  - KN: Konychev & Nadolsky, PLB 633, 710 (2006)

Ji-Ma-Yuan

Theoretical dimension:

- Convert CSS to covariant gauge with non-light-like Wilson lines.
- Still have a separate soft factor
- Have extra ρ parameter in hard scattering etc, ρ large
- Should have evolution equation for ρ, but don't.

No fits known with this scheme.

New CSS (JCC, "Foundations of Perturbative QCD")

Theoretical developments

- Use covariant gauge, with suitable Wilson lines
- Full proofs (at least to all orders of perturbation theory)
- Absorb square root of soft factor into each TMD function (in strange way).
- Take as many Wilson lines light-like as possible. (Non-trivial!)
- Evolution equations are strictly homogeneous

Clean up, and at most scheme change from old CSS.

Becher-Neubert

- SCET-based, à la Beneke-Smirnov
- Expansion for large Q with $q_{\mathrm{T}} \ll Q$
- But restrict to $q_{\mathrm{T}} \gg \Lambda$
- Hence evade issues of full TMD formalism and non-perturbative information at large b_{T}.
- Hence also don't have Sivers, Boer-Mulders, etc
- Could not define separate TMD pdfs
- Important tool for certain NNLO calculations.

Echevarría–Idilbi–Scimemi

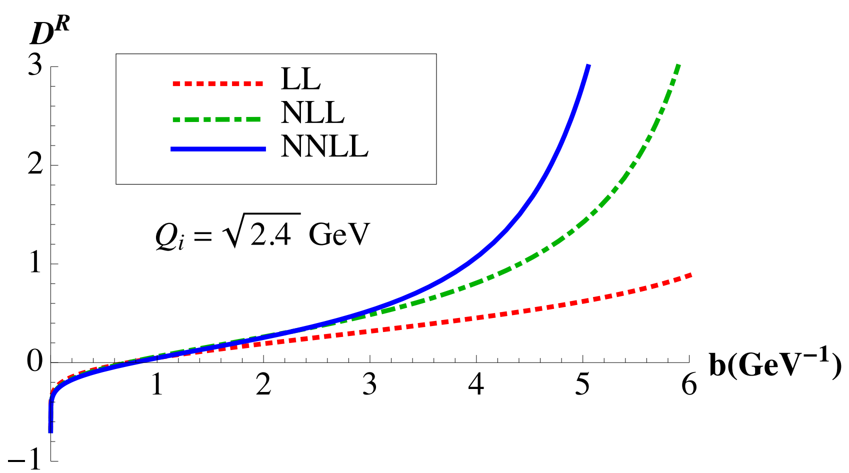
- SCET
- Scheme for regulating rapidity divergences without non-light-like Wilson lines. (But I don't think it obeys gauge-invariance)
- Absorb $\sqrt{\text{soft factor}}$ into each TMD pdf:

$$\text{TMD pdf} \times \text{TMD pdf} \times \text{soft} \;=\; \text{TMD pdf}' \times \text{TMD pdf}'$$

- Non-perturbative information at large b_{T}, or lack thereof:
  - In TMD pdfs use usual Gaussian parameterizations
  - But in $\tilde{K}$ use resummation of perturbation theory, e.g., up to $b_{\mathsf{T}} = 4\,\mathrm{GeV}^{-1} = 0.8\,\mathrm{fm}$ or beyond

Plot of $D^R(b_{\mathsf{T}}; Q_i) = -\tilde{K}(b_{\mathsf{T}}; Q_i)$:

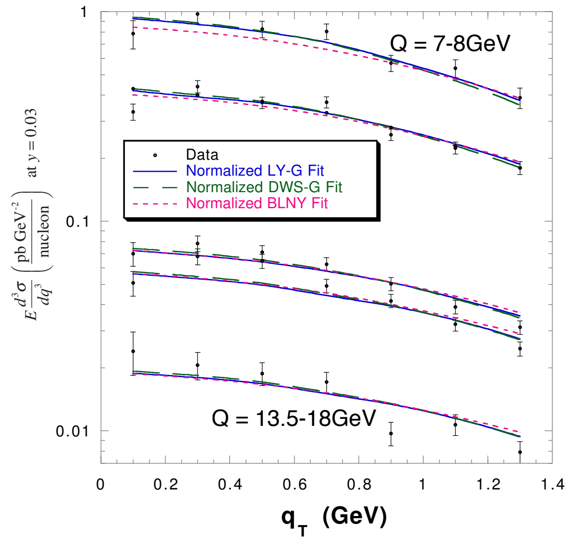
(Melis, QCD Evolution 2014 workshop)



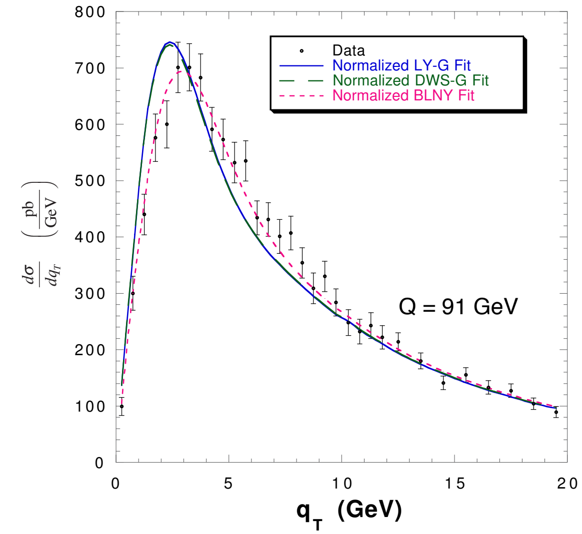
Geography of evolution of cross section

q_T

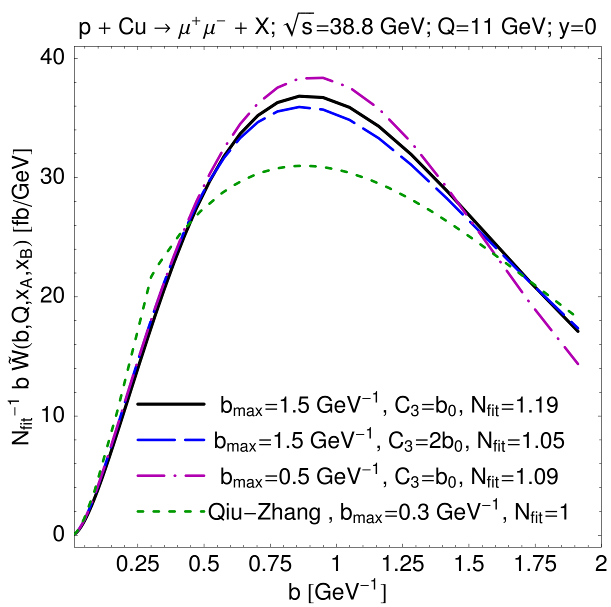
E605 Data

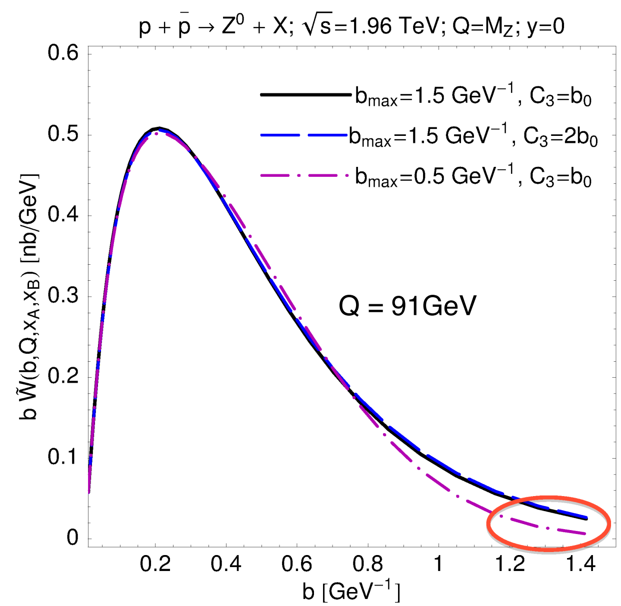


CDF Z Run 1



b_T





Q: 7–18 GeV, $\sqrt{s} = 38.8\,\text{GeV}$ $\qquad$ $Q = m_Z$, $\sqrt{s} = 1800\,\text{GeV}$

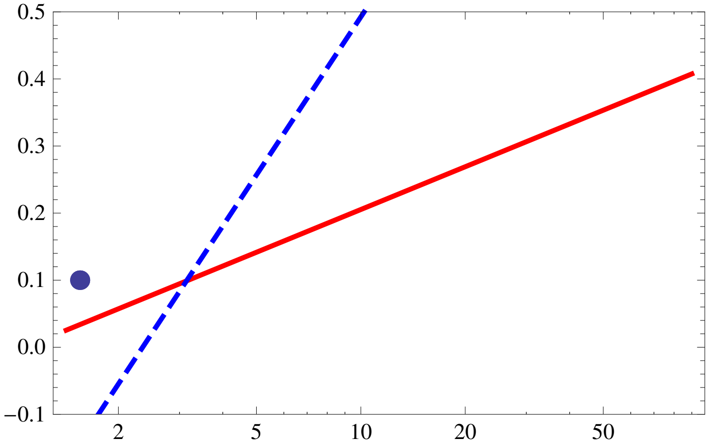
(Adapted from Landry et al., PRD 67,073016 (2003), Konychev & Nadolsky, PLB 633, 710 (2006))

Standard fits of TMD evolution give bad low-Q predictions

- Standard fits (to data at Q from $5\,\mathrm{GeV}$ to m_Z) have $\tilde{K}(b_\mathsf{T}, \mu) \propto -b_\mathsf{T}^2$ at large b_T.
- Then cross section is

$$\int \mathrm{d}^2\boldsymbol{b}_\mathsf{T}\, e^{i\boldsymbol{q}_\mathsf{T}\cdot\boldsymbol{b}_\mathsf{T}} e^{-b^2[\mathsf{coeff}(x)+\mathsf{const}\,\ln(Q^2/Q_0^2)]} \ldots$$

and exponent is too small or wrong sign (unphysical) at low Q:



Blue: BLNY, Red: KN

(Sun & Yuan, PRD 88, 114012 (2013))

Systematic analysis of non-perturbative part of evolution

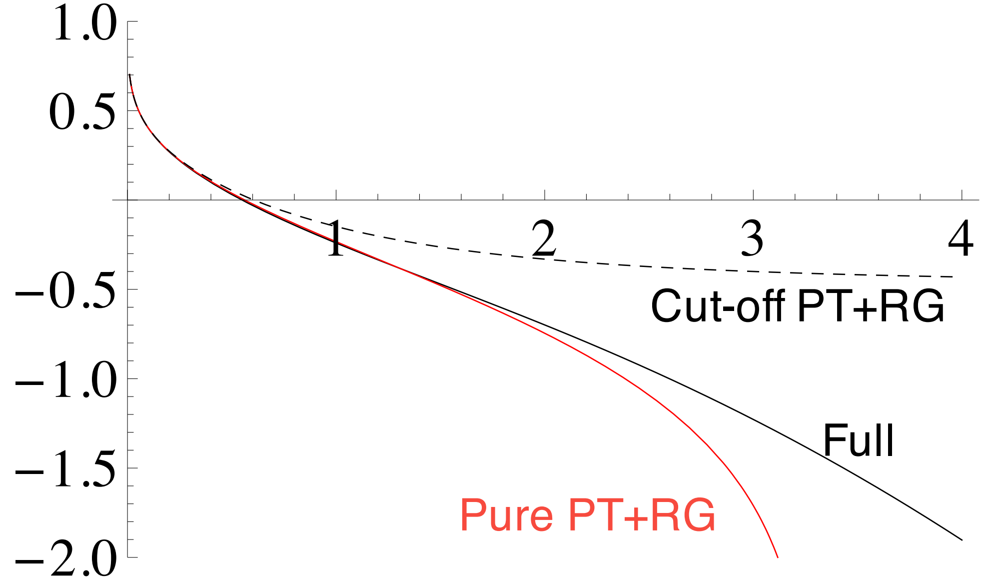
Issues for $\tilde{K}(b_{\mathsf{T}})$ at large b_{T}:

- Surely b_{T} above about $3\,\mathrm{GeV}^{-1} = 0.6\,\mathrm{fm}$ is in domain of non-perturbative physics
- It's difficult to avoid confounding x-dependence of transversity-momentum distribution with Q-dependence.
- Evolution appears to slow down at low Q
- Low Q involves larger (more non-perturbative) b_{T} than high Q

Hence:

- Assume the KN form (with its b_{T}^2 form) is OK for moderate b_{T}, to get the higher energy DY data correct.
- But it should flatten at higher b_{T}, which is relevant for lower Q experiments.

$\tilde{K}$ at large b_T



N.B. $b_{\mathrm{max}} = 1.5\,\mathrm{GeV}^{-1} = 0.3\,\mathrm{fm}$

My proposal:

- KN-fitted data constrain $\tilde{K}$ mostly at $b_\mathrm{T} \lesssim 2\,\mathrm{GeV}^{-1}$
- To get less evolution for low Q, flatten $\tilde{K}$ at b_T above region dominating KN fit.
- One idea: Instead of b_T^2, use $C\left[\sqrt{b_\mathrm{T}^2 + b_1^2} - b_\mathrm{T} - b_1\right]$

Simple ideas for physics constraints on large b_T behavior

- $\tilde{K}(b_\mathsf{T})$ codes emission of glue per unit extra rapidity
- So, for extra rapidity range Δy, let
  - $1 - c\Delta y =$ prob. of no relevant emission
  - $c\Delta y =$ prob. of emitting particle(s)
  - So

$$\tilde{K}(b_\mathsf{T})_{\mathrm{NP}} = \text{FT of } c\left[-\delta^{(2)}(\boldsymbol{k}_\mathsf{T}) + e^{-k_\mathsf{T}^2/k_{0\,\mathsf{T}}^2}/(\pi k_{0\,\mathsf{T}}^2)\right] = c\left[-1 + e^{-b_\mathsf{T}^2 k_{0\,\mathsf{T}}^2/4}\right]$$

- ?Change to exponential at large b_T instead of Gaussian?

Summary

- Surely we need non-perturbative contribution to TMD evolution
- It's governed by a single universal function
- Extrapolation of earlier DY fits to use at $b_{\rm T}$ relevant for lower energy SIDIS is incorrect.
- Rolling off of $\tilde{K}$ at large $b_{\rm T}$ is essential
- Physics and phenomenological arguments
- Redo global fits.
- Make sure measurements of TMD evolution are at fixed x!

Tool to compare different methods: The L function

(JCC & Rogers, in preparation)

- Shape change of transverse momentum distribution comes only from b_{T}-dependence of $\tilde{K}$

- Write cross section as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} = \text{norm.} \times \int e^{i\boldsymbol{q}_{\mathsf{T}}\cdot\boldsymbol{b}_{\mathsf{T}}} \widetilde{W}(b_{\mathsf{T}}, s, x_A, x_B) \, \mathrm{d}^2\boldsymbol{b}_{\mathsf{T}}$$

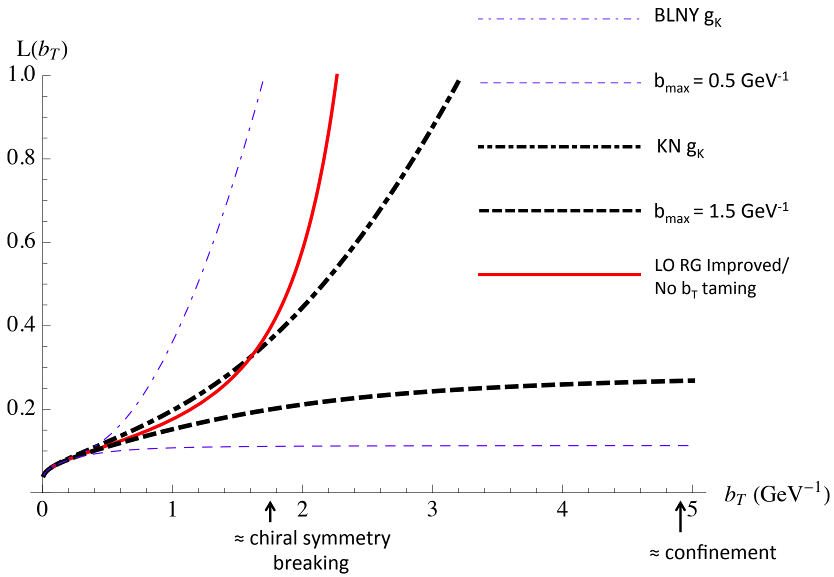
- So define scheme independent

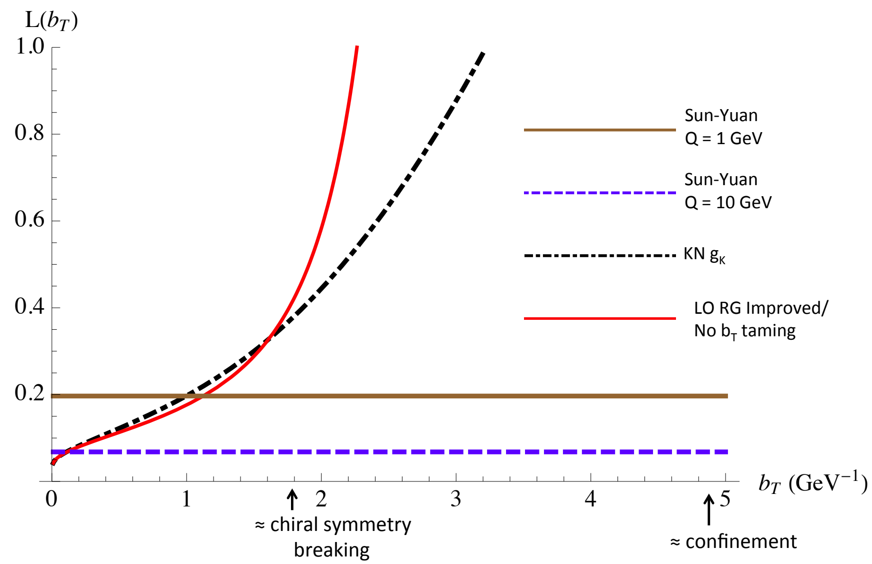
$$L(b_{\mathsf{T}}) = -\frac{\partial}{\partial \ln b_{\mathsf{T}}^2} \frac{\partial}{\partial \ln Q^2} \ln \tilde{W}(b_{\mathsf{T}}, Q, x_A, x_B) \stackrel{\text{CSS}}{=} -\frac{\partial}{\partial \ln b_{\mathsf{T}}^2} \tilde{K}(b_{\mathsf{T}}, \mu)$$

- QCD predicts it is
  - independent of Q, x_A, x_B
  - independent of light-quark flavor
  - RG invariant
  - perturbatively calculable at small b_{T}
  - non-perturbative at large b_{T}

Comparing different results using the L function

(Preliminary)





Q	Typical b_T
$2\,\mathrm{GeV}$	$3\,\mathrm{GeV}^{-1}$
$10\,\mathrm{GeV}$	$1.2\,\mathrm{GeV}^{-1}$
m_Z	$0.5\,\mathrm{GeV}^{-1}$

SY = Sun & Yuan (PRD 88, 114012 (2013)):

$$L_{\mathrm{SY}} = C_F \frac{\alpha_s(Q)}{\pi}$$

Depends on Q: contrary to QCD