Transverse momentum distributions and nuclear effects

Emanuele Pace – Università di Roma "Tor Vergata" and INFN, Roma 2, Italy Alessio Del Dotto – Università di Roma Tre and INFN, Roma 3, Italy Leonid Kaptari – JINR, Dubna, Russia & Perugia Matteo Rinaldi – Università di Perugia and INFN, Sezione di Perugia, Italy Giovanni Salmè – INFN, Roma 1, Italy

Sergio Scopetta – Università di Perugia and INFN, Sezione di Perugia, Italy



Outline

- Importance of the ³He nucleus for fundamental studies in Hadronic Physics. In particular: the neutron information from ³He.
 - Recent theoretical developments in SiDIS studies (L. Kaptari, A. Del Dotto, E. P., G. Salmè, S.Scopetta, PRC 89 (2014) 035206) Crucial quantity: **the** (distorted) **spectral function** Importance of a relativistic treatment for the description of the JLab program @ 12 GeV
- The Light-Front spectral function of ³He (E. P., A. Del Dotto, M. Rinaldi, G. Salmè, S. Scopetta, Few Body Syst. 54 (2013) 1079) (work in progress): preliminary results
- The LF spectral function for the nucleon and the TMDs (A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, in preparation)

Conclusions



Importance of ³He for DIS structure studies

- ³He is theoretically well known. Even a relativistic treatment may be implemented.
- ³He has been used extensively as an effective neutron target, especially to unveil the spin content of the free neutron, due to its peculiar spin structure:



³He always promising when the neutron polarization properties have to be studied.

³He is a unique target:

```
*
```

in DIS, together with ³H, for the extraction of F_2^n (Marathon experiment, JLab);

```
*
```

in polarized DIS, for the extraction of the SSF g_1^n ;

*

in polarized SiDIS, for the extraction of neutron transversity and related observables;

*

in DVCS, for the extraction of neutron GPDs



Single Spin Asymmetries (SSAs)



The number of emitted hadrons at a given ϕ_h depends on the orientation of \vec{S}_{\perp} ! In SSAs 2 different mechanisms can be experimentally distinguished

$$A_{UT}^{Sivers(Collins)} = \frac{\int d\phi_S d^2 P_{h\perp} \sin(\phi_h - (+)\phi_S) d^6 \sigma_{UT}}{\int d\phi_S d^2 P_{h\perp} d^6 \sigma_{UU}}$$

with

$$d^{6}\sigma_{UT} = \frac{1}{2}(d^{6}\sigma_{U\uparrow} - d^{6}\sigma_{U\downarrow}) \qquad \qquad d^{6}\sigma_{UU} = \frac{1}{2}(d^{6}\sigma_{U\uparrow} + d^{6}\sigma_{U\downarrow})$$

$\textbf{SSAs} \rightarrow \textbf{the neutron} \rightarrow {}^{3}\textbf{He}$

SSAs in terms of parton distributions and fragmentation functions:

$$A_{UT}^{Sivers} = N^{Sivers}/D \qquad A_{UT}^{Collins} = N^{Collins}/D$$

$$N^{Sivers} \propto \sum_{q} e_{q}^{2} \int d^{2} \kappa_{T} d^{2} \mathbf{k}_{T} \delta^{2} (\mathbf{k}_{T} + \mathbf{q}_{T} - \kappa_{T}) \frac{\mathbf{P}_{\mathbf{h}\perp} \cdot \mathbf{k}_{T}}{\mathbf{M}} f_{1T}^{\perp q}(x, \mathbf{k}_{T}^{2}) D_{1}^{q,h}(z, (z\kappa_{T})^{2})$$

$$N^{Collins} \propto \sum_{q} e_{q}^{2} \int d^{2} \kappa_{T} d^{2} \mathbf{k}_{T} \delta^{2} (\mathbf{k}_{T} + \mathbf{q}_{T} - \kappa_{T}) \frac{\mathbf{P}_{\mathbf{h}\perp} \cdot \kappa_{T}}{\mathbf{M}_{\mathbf{h}}} h_{1}^{q}(x, \mathbf{k}_{T}^{2}) H_{1}^{\perp q,h}(z, (z\kappa_{T})^{2})$$

$$D \propto \sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1}^{q,h}(z)$$

LARGE A^{Sivers} measured in $\vec{p}(e, e'\pi)x$ HERMES PRL 94, 012002 (2005)
 SMALL A^{Sivers} measured in $\vec{D}(e, e'\pi)x$; COMPASS PRL 94, 202002 (2005)

A strong flavor dependence confirmed by recent data

Importance of the neutron for flavor decomposition!



The neutron information from ³He

As we have already seen 3 He is the ideal target to study the polarized neutron:



 $\ln S - \text{wave}$ ${}^3\vec{H}e = \vec{n} !$

... But the bound nucleons in ³He are moving! Dynamical nuclear effects in inclusive DIS (${}^{3}\vec{H}e(e,e')X$) were evaluated with a realistic spin-dependent spectral function for ${}^{3}\vec{H}e$

$$P_{\sigma,\sigma\prime}^{\mathcal{M}}(\vec{p},E) = \sum_{f_{(A-1)}} \langle \vec{p}, \sigma\tau; \psi_{f_{(A-1)}} | \psi_{\mathcal{J}\mathcal{M}} \rangle \langle \psi_{\mathcal{J}\mathcal{M}} | \psi_{f_{(A-1)}}; \vec{p}, \sigma'\tau \rangle \delta(E - E_{f_{(A-1)}} + E_A)$$

It was found that the formula $(f_p, f_n \quad dilution factors)$

 $A_n \simeq \frac{1}{p_n f_n} \left(A_3^{exp} - 2p_p f_p A_p^{exp} \right), \quad (Ciofi \ degli \ Atti \ et \ al., PRC48(1993)R968)$

can be safely used \longrightarrow widely used by experimental collaborations to extract g_1^n . The nuclear effects are hidden in the "effective polarizations"

 $p_p = -0.023$ (Av18) $p_n = 0.878$ (Av18)

June 12th, 2014

Transverse momentum distributions and nuclear effects - p.6/33

\vec{n} from ${}^{3}\vec{H}e$: SiDIS case

Can one use the same formula to extract the SSAs ? in SiDIS also the fragmentation functions can be modified by the nuclear environment !



The process ${}^3\vec{H}e(e,e'\pi)X$ has been evaluated :

in IA \rightarrow no FSI between the measured fast, ultrarelativistic π the remnant and the two nucleon recoiling system $E_{\pi} \simeq 2.4 \ GeV$ in JLAB exp at 6 GeV - Qian et al., PRL 107 (2011) 072003

SSAs involve convolutions of the spin-dependent nuclear spectral function, $P(\vec{p}, E)$, with parton distributions AND fragmentation functions [S.Scopetta, PRD 75 (2007) 054005] :

$$A \simeq \int d\vec{p} dE \dots P(\vec{p}, E) f_{1T}^{\perp q} \left(\frac{Q^2}{2p \cdot q}, \mathbf{k_T^2} \right) D_1^{q,h} \left(\frac{p \cdot h}{p \cdot q}, \left(\frac{p \cdot h}{p \cdot q} \kappa_{\mathbf{T}} \right)^2 \right)$$

The nuclear effects on fragmentation functions are new with respect to the DIS case and have been studied carefully, using models for $f_{1T}^{\perp q}$, $D_1^{q,h}$... and the Av18 (Pisa group w.f.) *spectral function*.



\vec{n} from ${}^{3}\vec{H}e$: SiDIS case

Ingredients of the calculations :

- A realistic spin-dependent spectral function of ³He (C. Ciofi degli Atti et al., PRC 46 (1992) R 1591; A. Kievsky et al., PRC 56 (1997) 64) obtained using the AV18 interaction and the wave functions evaluated by the Pisa group [A. Kievsky et al., NPA 577 (1994) 511] (small effects from 3-body interactions)
- Parametrizations of data for pdfs and fragmentation functions whenever available: $f_1^q(x, \mathbf{k_T^2})$, Glueck et al., EPJ C (1998) 461 , $f_{1T}^{\perp q}(x, \mathbf{k_T^2})$, Anselmino et al., PRD 72 (2005) 094007, $D_1^{q,h}(z, (z\kappa_T)^2)$, Kretzer, PRD 62 (2000) 054001



Models for the unknown pdfs and fragmentation functions: $h_1^q(x, \mathbf{k_T}^2)$, Glueck et al., PRD 63 (2001) 094005, $H_1^{\perp q, h}(z, (z\kappa_T)^2)$ Amrath et al., PRD 71 (2005) 114018

Results will be model dependent. Anyway, the aim for the moment is to study nuclear effects.



Results: \vec{n} from ${}^{3}\vec{H}e$: A_{UT}^{Sivers} , @ JLab



FULL: Neutron asymmetry (model)

DOTS: Neutron asymmetry extracted from ${}^{3}He$ (calculation) neglecting the contribution of the proton polarization $\bar{A}_{n} \simeq \frac{1}{f_{n}} A_{3}^{calc}$

DASHED : Neutron asymmetry extracted from ${}^{3}He$ (calculation) taking into account nuclear structure effects through the formula:

$$A_n \simeq \frac{1}{p_n f_n} \left(A_3^{calc} - 2p_p f_p A_p^{model} \right)$$

June 12th, 2014

Results: \vec{n} from ${}^{3}\vec{H}e$: $A_{UT}^{Collins}$, @ JLab



The extraction procedure successful in **DIS**

works also in SiDIS, for both the Collins and the Sivers SSAs !

- 1 What about FSI effects ? A pion is detected, now...
- **2** What about relativistic effects **(2)** 12 GeV JLab?

E12-09-018 experiment, approved with rate A, G. Cates et al.



FSI: *distorted* **spin-dependent spectral function of** ³**He**

L. Kaptari, A. Del Dotto, E. P., G. Salmè, S. Scopetta, PRC 89 (2014) 035206



Relative energy between A - 1 and the remnants: a few GeV \rightarrow eikonal approximation.

Relevant part of the (distorted) spin dependent spectral function:

$$\begin{aligned} \mathcal{P}_{||}^{IA(FSI)} &= \mathcal{O}_{\frac{1}{2}\frac{1}{2}}^{IA(FSI)} - \mathcal{O}_{-\frac{1}{2}-\frac{1}{2}}^{IA(FSI)}; \quad \text{with:} \\ \mathcal{O}_{\lambda\lambda'}^{IA(FSI)}(p_N, E) &= \sum_{\epsilon_{A-1}^*} \rho\left(\epsilon_{A-1}^*\right) \langle S_A, \mathbf{P}_{\mathbf{A}} | (\hat{S}_{Gl}) \{ \Phi_{\epsilon_{A-1}^*}, \lambda', \mathbf{p}_N \} \rangle \\ \langle (\hat{S}_{Gl}) \{ \Phi_{\epsilon_{A-1}^*}, \lambda, \mathbf{p}_N \} | S_A, \mathbf{P}_{\mathbf{A}} \rangle \delta\left(E - B_A - \epsilon_{A-1}^* \right). \end{aligned}$$

Glauber operator: $\hat{S}_{Gl}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \prod_{i=2,3} \left[1 - \theta(z_i - z_1) \Gamma(\mathbf{b}_1 - \mathbf{b}_i, z_1 - z_i) \right]$ (generalized) profile function: $\Gamma(\mathbf{b}_{1i}, z_{1i}) = \frac{(1 - i\alpha) \sigma_{eff}(z_{1i})}{4 \pi b_0^2} \exp \left[-\frac{\mathbf{b}_{1i}^2}{2 b_0^2} \right]$,

(hadronization model: Kopeliovich et al., NPA 2004; σ_{eff} model: Ciofi & Kopeliovich, EPJA 2003)

GEA = Generalized Eikonal Approximation

(succesfull application to unpolarized SiDIS: Ciofi & Kaptari PRC 2011)

June 12th, 2014

(preliminary) FSI effects on quantities relevant to SiDIS

FSI in the Parallel (along the target polarization) Spin Dependent Spectral Function $P_{II}(p_{mis}, \theta_{mis})$

$$\mathcal{F}^{A}(x,\{\alpha\}) \simeq \int_{x}^{A} \mathcal{F}^{N}(\xi/x,\{\alpha\}) f^{A}(\xi) d\xi; \quad f^{A}(\xi) = \int dE \int_{\mathbf{p_{min}}}^{p_{max}} P^{A}(\mathbf{p},E) \delta\left(\xi - \frac{pq}{m\nu}\right) d^{3}\mathbf{p}$$

 $\mathbf{p} = \mathbf{p_{min}} \rightarrow |\cos \theta_{\mathbf{p}}| = 1$ (FSI minimized, Spectral Function maximized !!!)



 $\mathcal{P}_{||}^{PWIA}$ and $\mathcal{P}_{||}^{FSI}$ can be very different, but observables involve integrals, dominated by the low momentum region: e.g., effective polarizations $p_{p(n)}$ differ by 10-15 %. Is this the effect in the extraction of the neutron information? June 12th, 2014 Actually, one should also consider the effect on dilution factors...

DILUTION FACTORS

$$A_{3}^{exp} \simeq \frac{\Delta \vec{\sigma}_{3}^{exp.}}{\sigma_{unpol.}^{exp.}} \Longrightarrow \frac{\langle \vec{\mathbf{s}}_{\mathbf{n}} \rangle \Delta \vec{\sigma}(\mathbf{n}) + 2 \langle \vec{\mathbf{s}}_{\mathbf{p}} \rangle \Delta \vec{\sigma}(\mathbf{p})}{\langle \mathbf{N}_{\mathbf{n}} \rangle \sigma_{\mathbf{unpol.}}(\mathbf{n}) + 2 \langle \mathbf{N}_{\mathbf{p}} \rangle \sigma_{\mathbf{unpol.}}(\mathbf{p})} = \langle \vec{\mathbf{s}}_{\mathbf{n}} \rangle \mathbf{f}_{\mathbf{n}} \mathbf{A}_{\mathbf{n}} + 2 \langle \vec{\mathbf{s}}_{\mathbf{p}} \rangle \mathbf{f}_{\mathbf{p}} \mathbf{A}_{\mathbf{p}}$$



Good news from preliminary GEA studies of FSI!

Results (Preliminary)

1) PWIA: $\langle p_n \rangle = 0.876, \ \langle p_p \rangle = -0.0237, \ \theta_e = 30^o, \ \theta_{\pi} = 14^o$							
$E_{beam},$	x_{Bj}	ν	p_{π}	$f_n(x,z)$	$\langle p_n \rangle f_n$	$f_p(x,z)$	$\langle p_p \rangle f_p$
${ m GeV}$		${\rm GeV}$	${ m GeV/c}$				
8.8	0.21	7.55	3.40	0.304	0.266	0.348	-8.410^{-3}
8.8	0.29	7.15	3.19	0.286	0.251	0.357	-8.510^{-3}
8.8	0.48	6.36	2.77	0.257	0.225	0.372	-8.910^{-3}
11	0.21	9.68	4.29	0.302	0.265	0.349	-8.310^{-3}
11	0.29	9.28	4.11	0.285	0.25	0.357	-8.510^{-3}

2) FSI: $\langle p_n \rangle = 0.756$, $\langle p_p \rangle = -0.0265$, $\langle N_n \rangle = 0.85$, $\langle N_p \rangle = 0.87$, $\langle \sigma_{eff} \rangle = 71 \ mb$

$E_{beam},$	x_{Bj}	ν	p_{π}	$f_n(x,z)$	$\langle p_n \rangle f_n$	$f_p(x,z)$	$\langle p_p \rangle f_p$
${ m GeV}$		${\rm GeV}$	${ m GeV/c}$				
8.8	0.21	7.55	3.40	0.353	0.267	0.405	-1.110^{-2}
8.8	0.29	7.15	3.19	0.332	0.251	0.415	-1.110^{-2}
8.8	0.48	6.36	2.77	0.298	0.225	0.432	-1.210^{-2}
11	0.21	9.68	4.29	0.351	0.266	0.405	-1.10^{-2}
11	0.29	9.28	4.11	0.331	0.250	0.415	-1.110^{-2}

$$A_n \simeq \frac{1}{p_n^{FSI} \ f_n^{FSI} \ f_n^{FSI}} \left(A_3^{exp} - 2p_p^{FSI} f_p^{FSI} A_p^{exp} \right) \simeq \frac{1}{p_n \ f_n} \left(A_3^{exp} - 2p_p f_p A_p^{exp} \right)$$



The effects of FSI in the dilution factors and in the effective polarizations are found to compensate each other to a large extent: the usual extraction seems to be safe!

What about Relativity?

3

A LF description of DIS processes off ³He - I

The role played by **Relativity** has to be investigated - it will become even more important with the upgrade of JLab @ 12 GeV.

A relativistic spectral function is very useful also for other studies (e.g., for nuclear GPDs, where final states have to be properly boosted)

The Relativistic Hamiltonian Dynamics (RHD) of an interacting system, introduced by Dirac (1949), *plus* the Bakamijan-Thomas construction of the Poincarè generators allow one to generate a description of DIS, SiDIS, DVCS off ³He which :



is fully Poincarè covariant



has a fixed number of on-mass-shell constituents

The LF form of RHD is adopted. It has 7 kinematical generators, a subgroup structure of the LF boosts (separation of the intrinsic motion from the global one: very important for us!) and a meaningful Fock expansion.



A LF description of DIS processes off ³He - II

Among the possible forms of RHD, the Light-Front one has several advantages:

- 7 Kinematical generators: i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) P^+ , \mathbf{P}_{\perp} , iii) Rotation around the z-axis.
- The LF boosts have a subgroup structure, then one gets a trivial separation of the intrinsic motion (as in the non relativistic case).
 - $P^+ \ge 0$ leads to a meaningful Fock expansion.
- No square roots in the dynamical operator P^- , propagating the state in the LF-time.
- The IMF description of DIS is easily included.

Drawback:

Although, using the BT construction, one can define a *kinematical*, intrinsic angular momentum (very important for us!), the transverse LF-rotations are dynamical.



Example: The SiDIS nuclear hadronic tensor in LF

In Impulse Approximation the LF hadronic tensor for the ³He nucleus is:

$$\mathcal{W}^{\mu\nu}(Q^{2}, x_{B}, z, \tau_{h}f, \hat{\mathbf{h}}, S_{He}) \propto \sum_{\sigma, \sigma'} \sum_{\tau_{hf}} \sum_{\epsilon_{S}^{min}} \sum_{\epsilon_{S}^{min}} \frac{\epsilon_{S}^{max}}{\epsilon_{S}^{min}} d\epsilon_{S} \int_{M_{N}^{2}}^{(M_{X} - M_{S})^{2}} dM_{f}^{2}$$

$$\times \int_{\xi_{lo}}^{\xi_{up}} \frac{d\xi}{\xi^{2}(1 - \xi)(2\pi)^{3}} \int_{P_{\perp}^{min}}^{P_{\perp}^{max}} \frac{dP_{\perp}}{\sin\theta} (P^{+} + q^{+} - h^{+})$$

$$\times w_{\sigma\sigma'}^{\mu\nu} \left(\tau_{hf}, \tilde{\mathbf{q}}, \tilde{\mathbf{h}}, \tilde{\mathbf{P}}\right) \mathcal{P}_{\sigma'\sigma}^{\tau_{hf}}(\tilde{\mathbf{k}}, \epsilon_{S}, S_{He})$$

where

$$(\tilde{\mathbf{v}} = \{v^+ = v^0 + v^3, \mathbf{v}_\perp\})$$

$$w^{\mu\nu}_{\sigma\sigma'}\left(au_{hf}, \mathbf{\tilde{q}}, \mathbf{\tilde{h}}, \mathbf{\tilde{P}}
ight)$$
 is the nucleon hadronic tensor

 $\mathcal{P}_{\sigma'\sigma}^{\tau_{hf}}(\tilde{\mathbf{k}}, \epsilon_S, S_{He}) \text{ is the LF nuclear spectral function} \\ \text{defined in terms of LF overlaps}$





June 12th, 2014

The ³He LF Spectral Function

$$\mathcal{P}_{\sigma'\sigma}^{\tau}(\tilde{\mathbf{k}},\epsilon_{S},S_{He}) \propto \sum_{\sigma_{1}\sigma_{1}'} D^{\frac{1}{2}} [\mathcal{R}_{M}^{\dagger}(\tilde{\mathbf{k}})]_{\sigma'\sigma_{1}'} \, \mathcal{S}_{\sigma_{1}'\sigma_{1}}^{\tau}(\mathbf{k},\epsilon_{S},S_{He}) \, D^{\frac{1}{2}} [\mathcal{R}_{M}(\tilde{\mathbf{k}})]_{\sigma_{1}\sigma_{1}'}$$

is obtained through the unitary Melosh Rotations : $D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})] = \frac{m+k^+ - i\boldsymbol{\sigma} \cdot (\hat{z} \times \mathbf{k}_\perp)}{\sqrt{(m+k^+)^2 + |\mathbf{k}_\perp|^2}}$ and the instant-form spectral function

$$\begin{aligned} \mathcal{S}_{\sigma_{1}'\sigma_{1}}^{\tau}(\mathbf{k},\epsilon_{S},S_{He}) &= \sum_{J_{S}J_{zS}\alpha}\sum_{T_{S}\tau_{S}} \left\langle T_{S},\tau_{S},\alpha,\epsilon_{S}J_{S}J_{zS};\sigma_{1}';\tau,\mathbf{k}|\Psi_{0}S_{He}\right\rangle \\ &\times \left\langle S_{He},\Psi_{0}|\mathbf{k}\sigma_{1}\tau;J_{S}J_{zS}\epsilon_{S},\alpha,T_{S},\tau_{S}\right\rangle \\ &= \left[B_{0,S_{He}}^{\tau}(|\mathbf{k}|,E) + \boldsymbol{\sigma}\cdot\mathbf{f}_{S_{He}}^{\tau}(\mathbf{k},E) \right]_{\sigma_{1}'\sigma_{1}} \end{aligned}$$

with $\mathbf{f}_{S_{He}}^{\tau}(\mathbf{k}, E) = \mathbf{S}_{A} B_{1,S_{He}}^{\tau}(|\mathbf{k}|, E) + \hat{\mathbf{k}} (\hat{\mathbf{k}} \cdot \mathbf{S}_{A}) B_{2,S_{He}}^{\tau}(|\mathbf{k}|, E)$

NOTICE: $S_{\sigma'_1\sigma_1}^{\tau}(\mathbf{k}, \epsilon_S, S_{He})$ is given in terms of THREE independent functions, $B_{0,1,2}$, once parity and t-reversal are imposed. Adding FSI, more terms could be included.



GOOD preliminary NEWS

We are now evaluating the SSAs using the LF hadronic tensor, to check whether the proposed extraction procedure still holds within the LF approach. We have preliminary encouraging indications:

LF longitudinal and transverse polarizations change little from the NR ones:

	proton NR	proton LF	neutronNR	neutron LF
$\int dE d\vec{p} \frac{1}{2} Tr(\mathcal{P}\sigma_z)_{\vec{S}_A = \hat{z}}$	-0.02263	-0.02231	0.87805	0.87248
$\int dE d\vec{p} \frac{1}{2} Tr(\mathcal{P}\sigma_y)_{\vec{S}_A = \hat{y}}$	-0.02263	-0.02268	0.87805	0.87494

The difference between the effective longitudinal and transverse polarizations is a measure of the relativistic content of the system (in a proton, it would correspond to the difference between axial and tensor charges).

The extraction procedure works well within the LF approach as it does in the non relativistic case.



Nucleon LF spectral function and TMDs - I

The LF spectral function for a J = 1/2 system of three spin 1/2 constituents can be used to find relations among the SIX T-even TMDs at the leading twist for the quarks inside a nucleon with momentum P and spin S.

Let us express the q-q correlator for a nucleon in terms of the functions A_i, \widetilde{A}_i (i = 1, 3)

$$\begin{split} \Phi(k,P,S) &= \int d^4 z \ e^{ik \cdot z} \langle PS | \ \bar{\psi}_q(0) \ \psi_q(z) | PS \rangle \\ &= \frac{1}{2} \left\{ A_1 \not\!\!\!P + A_2 \ S_L \ \gamma_5 \not\!\!\!P + A_3 \not\!\!\!P \gamma_5 \not\!\!S_\perp \right. \\ &+ \frac{1}{M} \ \widetilde{A}_1 \ \vec{k}_\perp \cdot \vec{S}_\perp \ \gamma_5 \not\!\!\!P + \widetilde{A}_2 \ \frac{S_L}{M} \not\!\!\!P \gamma_5 \not\!\!k_\perp + \frac{1}{M^2} \ \widetilde{A}_3 \ \vec{k}_\perp \cdot \vec{S}_\perp \not\!\!\!\!P \gamma_5 \not\!\!k_\perp \Big\} \end{split}$$

Particular combinations of the functions $A_i, \widetilde{A}_i \ (i = 1, 3)$ can be obtained by proper traces of $\Phi(k, P, S)$:

$$\frac{1}{2P^{+}} \operatorname{Tr}(\gamma^{+} \Phi) = A_{1},$$

$$\frac{1}{2P^{+}} \operatorname{Tr}(\gamma^{+} \gamma_{5} \Phi) = S_{L} A_{2} + \frac{1}{M} \vec{k}_{\perp} \cdot \vec{S}_{\perp} \tilde{A}_{1},$$

$$\frac{1}{2P^{+}} \operatorname{Tr}(i\sigma^{i+} \gamma_{5} \Phi) = S_{\perp}^{i} A_{3} + \frac{S_{L}}{M} k_{\perp}^{i} \tilde{A}_{2} + \frac{1}{M^{2}} \vec{k}_{\perp} \cdot \vec{S}_{\perp} k_{\perp}^{i} \tilde{A}_{3}.$$
The second distribution of the balance of the ba



Nucleon LF spectral function and TMDs - II

The SIX T-even twist-2 TMDs for the quarks inside a nucleon can be obtained by integration of the functions A_i , \tilde{A}_i on k^+ and k^- as follows

$$\begin{split} f(x, |\mathbf{k}_{\perp}|^2) &= \frac{1}{2} \int \frac{dk^+ dk^-}{(2\pi)^4} \,\delta[k^+ - xP^+] \,2P^+ A_1 \,, \\ \Delta f(x, |\mathbf{k}_{\perp}|^2) &= \frac{1}{2} \int \frac{dk^+ dk^-}{(2\pi)^4} \,\delta[k^+ - xP^+] \,2P^+ A_2 \,, \\ g_{1T}(x, |\mathbf{k}_{\perp}|^2) &= \frac{1}{2} \int \frac{dk^+ dk^-}{(2\pi)^4} \,\delta[k^+ - xP^+] \,2P^+ \widetilde{A}_1 \,, \\ \Delta'_T f(x, |\mathbf{k}_{\perp}|^2) &= \frac{1}{2} \int \frac{dk^+ dk^-}{(2\pi)^4} \,\delta[k^+ - xP^+] \,2P^+ \left(A_3 + \frac{|\mathbf{k}_{\perp}|^2}{2M^2} \widetilde{A}_3\right) \,, \\ h_{1L}^{\perp}(x, |\mathbf{k}_{\perp}|^2) &= \frac{1}{2} \int \frac{dk^+ dk^-}{(2\pi)^4} \,\delta[k^+ - xP^+] \,2P^+ \widetilde{A}_2 \,, \\ h_{1T}^{\perp}(x, |\mathbf{k}_{\perp}|^2) &= \frac{1}{2} \int \frac{dk^+ dk^-}{(2\pi)^4} \,\delta[k^+ - xP^+] \,2P^+ \widetilde{A}_3 \,, \end{split}$$



Nucleon LF spectral function and TMDs - III

Actually, in LF dynamics one has on-mass shell quarks. Let us consider therefore the contribution to the correlation function from on-mass-shell fermions

$$\begin{split} \Phi_p(k,P,S) &= \frac{(k_{on} + m)}{2m} \Phi(k,P,S) \frac{(k_{on} + m)}{2m} \\ &= \frac{1}{4m^2} \sum_{\sigma} \sum_{\sigma'} u_{LF}(\tilde{k},\sigma') \bar{u}_{LF}(\tilde{k},\sigma') \Phi(k,P,S) u_{LF}(\tilde{k},\sigma) \bar{u}_{LF}(\tilde{k},\sigma) \\ &= \frac{1}{4m^2} \sum_{\sigma} \sum_{\sigma'} u_{LF}(\tilde{k},\sigma') K \mathcal{P}_{\sigma'\sigma}(\tilde{k},\epsilon_S,S) \bar{u}_{LF}(\tilde{k},\sigma) \end{split}$$

Indeed the quantity $\bar{u}_{LF}(\tilde{k}, \sigma') \Phi(k, P, S) u_{LF}(\tilde{k}, \sigma)$ can be identified, up to a kinematical factor K with the LF nucleon spectral function

$$\bar{u}_{LF}(\tilde{k},\sigma') \Phi(k,P,S) u_{LF}(\tilde{k},\sigma) = K \mathcal{P}_{\sigma'\sigma}(\tilde{\mathbf{k}},\epsilon_S,S)$$

In a reference frame where $\mathbf{P}_{\perp} = 0$, the following relation holds between k^- and the spectator diquark energy ϵ_S :





Nucleon LF spectral function and TMDs - IV

If the full correlation function $\Phi(k, P, S)$ is approximated by its particle contribution $\Phi_p(k, P, S)$, then the valence approximations A_i^V , \widetilde{A}_i^V (i = 1, 3) for the functions A_i , \widetilde{A}_i can be obtained by proper traces

$$\frac{1}{2P^+} Tr\left[\gamma^+ \Phi_p(k, P, S)\right] = A_1^V$$

$$\frac{1}{2P^+} Tr \left[\gamma^+ \gamma_5 \Phi_p(k, P, S)\right] = A_2^V \lambda_N + \frac{1}{M} \widetilde{A}_1^V \mathbf{k}_\perp \cdot \mathbf{S}_\perp$$

$$\frac{1}{2P^+} Tr \left[\mathbf{k}_{\perp} \gamma^+ \gamma_5 \Phi_p(\mathbf{k}, P, S) \right] = A_3^V \mathbf{k}_{\perp} \cdot \mathbf{S}_{\perp} + \widetilde{A}_2^V \frac{\lambda_N}{M} |\mathbf{k}_{\perp}|^2 + \frac{1}{M^2} \widetilde{A}_3^V \mathbf{k}_{\perp} \cdot \mathbf{S}_{\perp} |\mathbf{k}_{\perp}|^2$$



Nucleon LF spectral function and TMDs - V

However these same traces can be also expressed through the LF spectral function

$$\frac{1}{2P^+} Tr\left[\gamma^+ \Phi_p(k, P, S)\right] = \frac{k^+}{4m^2P^+} K Tr\left[\mathcal{P}(\tilde{\mathbf{k}}, \epsilon_S, S)\right]$$

$$\frac{1}{2P^{+}} Tr \left[\gamma^{+} \gamma_{5} \Phi_{p}(k, P, S)\right] = \\ = \frac{k^{+}}{4m^{2}P^{+}} K \sum_{\sigma \sigma'} \chi^{\dagger}_{\sigma} \sigma_{z} \chi_{\sigma'} \mathcal{P}_{\sigma'\sigma}(\tilde{\mathbf{k}}, \epsilon_{S}, S) = \frac{k^{+}}{4m^{2}P^{+}} K Tr \left[\sigma_{z} \mathcal{P}(\tilde{\mathbf{k}}, \epsilon_{S}, S)\right]$$

$$\frac{1}{2P^{+}}Tr\left[\boldsymbol{k}_{\perp}\gamma^{+}\gamma_{5} \Phi_{p}(\boldsymbol{k}, P, S)\right] = \\ = \frac{k^{+}}{4m^{2}P^{+}} K \sum_{\sigma \sigma'} \chi^{\dagger}_{\sigma}\boldsymbol{k}_{\perp} \cdot \boldsymbol{\sigma} \chi_{\sigma'} \mathcal{P}_{\sigma'\sigma}(\tilde{\boldsymbol{k}}, \epsilon_{S}, S) = \frac{k^{+}}{4m^{2}P^{+}} K Tr\left[\boldsymbol{k}_{\perp} \cdot \boldsymbol{\sigma} \mathcal{P}(\tilde{\boldsymbol{k}}, \epsilon_{S}, S)\right]$$



Nucleon LF spectral function and TMDs - VI

$$\frac{1}{2} \operatorname{Tr}(\mathcal{P}I) = c B_{0}$$

$$\frac{1}{2} \operatorname{Tr}(\mathcal{P}\sigma_{z}) = S_{z} \left[a \left(B_{1} + B_{2} \cos^{2} \theta \right) + b \cos \theta \frac{|\mathbf{k}_{\perp}|^{2}}{k} B_{2} \right]$$

$$+ \mathbf{S}_{\perp} \cdot \mathbf{k}_{\perp} \left[a B_{2} \frac{\cos \theta}{k} + b \left(B_{1} + B_{2} \sin^{2} \theta \right) \right]$$

$$\frac{1}{2} \operatorname{Tr}(\mathcal{P}\sigma_{y}) = S_{y} \left[\left(a + d |\mathbf{k}_{\perp}|^{2} \right) B_{1} \right]$$

$$+ S_{z} k_{y} \left[a \frac{\cos \theta}{k} B_{2} - b \left(B_{1} + B_{2} \cos^{2} \theta \right) \right]$$

$$+ k_{y} \mathbf{S}_{\perp} \cdot \mathbf{k}_{\perp} \left[\left(\frac{a}{k^{2}} - b \frac{\cos \theta}{k} \right) B_{2} - d B_{1} \right]$$

Then in the valence approximation the SIX A_i^V, \tilde{A}_i^V (i = 1, 3) distributions can be expressed in terms of the 3 independent functions B_0, B_1, B_2 ! a, b, c, d are kinematical factors, predicted by the LF procedure!

In the LF approach only THREE of the SIX T-even TMDs are independent!

June 12th, 2014

Conclusions

We are studying **DIS processes off** 3 **He beyond** the **NR**, **IA** approach. We have encouraging results concerning:

- FSI effects evaluated through the GEA: a distorted spin dependent spectral function is studied
- Relativistic effects through an analysis of a LF spectral function (in IA)

Within LF dynamics and in the valence approximation only 3 of the 6 T-even TMDs are independent. The relations among them are precisely predicted within LF Dynamics, and could be experimentally checked to test the LF description of SiDIS.

Next steps:

June 12th, 2014

- complete this program!
- apply the LF spectral function to other processes (e.g., DVCS);

relativistic FSI?



³He: an effective neutron AND a LAB for Light-Front studies

Help from ³He for proton studies?

³He is a spin 1/2 system with three spin 1/2 constituents. The same as the proton, in the valence region \Rightarrow they have the same symmetries.

One could investigate the analogy:

proton	\Leftrightarrow	$^{3}\mathrm{He}$
Valence quark contribution	\Leftrightarrow	nucleon contribution
twist-2Approximation	\Leftrightarrow	Impulse Approximation
SiDIS	\Leftrightarrow	$(e,e^\prime p)$ at high Q^2
$6 \ asymmetries$	\Leftrightarrow	$6 \ Response functions$
$6 \ TMDS @ twist - 2$	\Leftrightarrow	$6\ momentum\ distributions$

A one-to-one correspondence can be obtained \Rightarrow check of the relations among the TMDS found in LF dynamics looking at ³He data (may be, in part, already available) \Rightarrow test of LF dynamics !

Hint for TMDs data analysis?



backup: the hadronization model

hadronization model: Kopeliovich et al., Predazzi, Hayashigaki, NPA 2004; σ_{eff} model: Ciofi & Kopeliovich, EPJA 2003)



At the interaction point, a color string, denoted X1, and a nucleon N1, arising from target fragmentation, are formed; the color string propagates and gluon radiation begins. The first π is created at $z_0 = 0.6$ by the breaking of the color string, and pion production continues until it stops at a maximum value of $z = z_{max}$, when energy conservation does not allow further pions to be created, and the number of pions remains constant. Once the total effective cross section has been obtained, the elastic slope b_0 and the ratio α of the real to the imaginary parts of the elastic amplitude remain to be determined.



GPDs of ³He

For a $J = \frac{1}{2}$ target, in a hard-exclusive process, $(Q^2, \nu \to \infty)$ such as (coherent) DVCS (Definition of GPDs from X. Ji PRL 78 (97) 610):



$$\Delta = P' - P, q^{\mu} = (q_0, \vec{q}), \text{ and } \bar{P} = (P + P')^{\mu}/2$$

$$x = k^+/P^+; \quad \xi = \text{``skewness''} = -\Delta^+/(2\bar{P}^+)$$

$$x \leq -\xi \longrightarrow \text{GPDs describe } antiquarks; \\ -\xi \leq x \leq \xi \longrightarrow \text{GPDs describe } q\bar{q} \text{ pairs}; x \geq \xi \longrightarrow \text{GPDs describe } quarks$$

the GPDs $H_q(x,\xi,\Delta^2)$ and $E_q(x,\xi,\Delta^2)$ are introduced:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \quad \gamma^{\mu} \quad \psi_q(\lambda n/2) | P \rangle = H_q(x,\xi,\Delta^2) \bar{U}(P') \gamma^{\mu} U(P)$$
$$+ \quad E_q(x,\xi,\Delta^2) \bar{U}(P') \frac{i\sigma^{\mu\nu} \Delta_{\nu}}{2M} U(P) + \dots$$



GPDs of ³He

For a $J = \frac{1}{2}$ target, in a hard-exclusive process, $(Q^2, \nu \to \infty)$ such as (coherent) DVCS (Definition of GPDs from X. Ji PRL 78 (97) 610):



$$\begin{aligned} & \Delta = P' - P, q^{\mu} = (q_0, \vec{q}), \text{ and } \bar{P} = (P + P')^{\mu}/2 \\ & \mathbf{x} = k^+/P^+; \quad \xi = \text{``skewness''} = -\Delta^+/(2\bar{P}^+) \\ & \mathbf{x} \le -\xi \longrightarrow \text{GPDs describe } antiquarks; \\ & -\xi \le x \le \xi \longrightarrow \text{GPDs describe } q\bar{q} \text{ pairs}; x \ge \xi \longrightarrow \text{GPDs describe } quarks \end{aligned}$$

and the helicity dependent ones, $\tilde{H}_q(x,\xi,\Delta^2)$ and $\tilde{E}_q(x,\xi,\Delta^2)$, obtained as follows:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \quad \gamma^{\mu} \gamma_5 \quad \psi_q(\lambda n/2) | P \rangle = \tilde{H}_q(x,\xi,\Delta^2) \bar{U}(P') \gamma^{\mu} U(P)$$
$$+ \quad \tilde{E}_q(x,\xi,\Delta^2) \bar{U}(P') \frac{\gamma_5 \Delta^{\mu}}{2M} U(P) + \dots$$



GPDs of ³He: the Impulse Approximation

coherent DVCS in I.A. (³He does not break-up, $\Delta^2 \ll M^2, \xi^2 \ll 1$,):

In a symmetric frame ($\bar{p}=(p+p')/2$) :



$$k^{+} = (x+\xi)\bar{P}^{+} = (x'+\xi')\bar{p}^{+} ,$$

$$k+\Delta)^{+} = (x-\xi)\bar{P}^{+} = (x'-\xi')\bar{p}^{+} ,$$

one has, for a given GPD, $H_q,\, \tilde{G}^q_M = H_q + E_q,\, {\rm or}\; \tilde{H}_q$

$$GPD_{q}(x,\xi,\Delta^{2}) \simeq \sum_{N} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' | \hat{O}_{q}^{\mu,N} | PS \rangle_{A} |_{z^{+}=0,z_{\perp}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix\bar{P}^{+}z^{-}}{}_{A} \langle P'S' |_{z^{+}=0} + \frac{1}{2} \int \frac{dz^{-}}{4\pi} e^{ix$$



GPDs of ³He: the Impulse Approximation

coherent DVCS in I.A. (³He does not break-up, $\Delta^2 \ll M^2, \xi^2 \ll 1$,):

In a symmetric frame ($\bar{p}=(p+p')/2$) :



$$k^{+} = (x+\xi)\bar{P}^{+} = (x'+\xi')\bar{p}^{+} ,$$

$$k+\Delta)^{+} = (x-\xi)\bar{P}^{+} = (x'-\xi')\bar{p}^{+} ,$$

one has, for a given GPD, H_q , $\tilde{G}^q_M=H_q+E_q$, or \tilde{H}_q

$$GPD_q(x,\xi,\Delta^2) \simeq \sum_N \int \frac{dz^-}{4\pi} e^{ix\bar{P}^+z^-} {}_A \langle P'S' | \hat{O}_q^{\mu,N} | PS \rangle_A |_{z^+=0,z_\perp=0}$$

By properly inserting a tensor product complete basis for the nucleon (PW) and the fully interacting recoiling system :



GPDs of ³He: the Impulse Approximation

coherent DVCS in I.A. (³He does not break-up, $\Delta^2 \ll M^2, \xi^2 \ll 1$,):

In a symmetric frame ($\bar{p}=(p+p')/2$) :



$$k^{+} = (x+\xi)\bar{P}^{+} = (x'+\xi')\bar{p}^{+} ,$$

$$k+\Delta)^{+} = (x-\xi)\bar{P}^{+} = (x'-\xi')\bar{p}^{+} ,$$

one has, for a given GPD, $H_q,\, \tilde{G}^q_M = H_q + E_q,\, {\rm or}\,\, \tilde{H}_q$

$$\begin{aligned} GPD_{q}(x,\xi,\Delta^{2}) \simeq \sum_{N} \int \frac{dz^{-}}{4\pi} e^{ix'\bar{p}^{+}z^{-}} \langle P'S'| \sum_{\vec{P}_{R}',f_{A-1}',\vec{p}\,',s'} \{ |P_{R}',\Phi_{A-1}^{f'}\rangle \otimes |p's'\rangle \} \\ \langle P_{R}',\Phi_{A-1}^{f'}| \otimes \langle p's'| \ \hat{O}_{q}^{\mu,N} \sum_{\vec{P}_{R},f_{A-1},\vec{p},s} \{ |P_{R},\Phi_{A-1}^{f}\rangle \otimes |ps\rangle \} \{ \langle P_{R},\Phi_{A-1}^{f}| \otimes \langle ps| \} \ |PS\rangle \ , \end{aligned}$$

and, since $\{\langle P_R, \Phi_{A-1}^f | \otimes \langle ps |\} | PS \rangle = (2\pi)^3 \delta^3 (\vec{P} - \vec{P}_R - \vec{p}) \langle \Phi_{A-1}^f, ps | PS \rangle$,



June 12th, 2014

(NR! Separation of the global motion from the intrinsic one!)

Transverse momentum distributions and nuclear effects - p.30/33

GPDs of ³He in IA

 H_q^A can be obtained in terms of H_q^N (S.S. PRC 70, 015205 (2004), PRC 79, 025207 (2009)):

$$H_q^A(x,\xi,\Delta^2) = \sum_N \int dE \int d\vec{p} \,\overline{\sum_{\mathcal{M}}} \sum_s P_{\mathcal{M}\mathcal{M},ss}^N(\vec{p},\vec{p'},E) \frac{\xi'}{\xi} H_q^N(x',\Delta^2,\xi') ,$$

 $\tilde{G}_{M}^{3,q}$ in terms of $\tilde{G}_{M}^{N,q}$ (M. Rinaldi, S.S. PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)):

$$\tilde{G}_{M}^{3,q}(x,\Delta^{2},\xi) = \sum_{N} \int dE \int d\vec{p} \left[P_{+-,+-}^{N} - P_{+-,-+}^{N} \right] (\vec{p},\vec{p}',E) \frac{\xi'}{\xi} \tilde{G}_{M}^{N,q}(x',\Delta^{2},\xi') ,$$

and \tilde{H}_q^A can be obtained in terms of \tilde{H}_q^N :

$$\tilde{H}_{q}^{A}(x,\xi,\Delta^{2}) = \sum_{N} \int dE \int d\vec{p} \left[P_{++,++}^{N} - P_{++,--}^{N} \right] (\vec{p},\vec{p}',E) \frac{\xi'}{\xi} \tilde{H}_{q}^{N}(x',\Delta^{2},\xi') ,$$



GPDs of ³He in IA

 H_q^A can be obtained in terms of H_q^N (S.S. PRC 70, 015205 (2004), PRC 79, 025207 (2009)):

$$H_q^A(x,\xi,\Delta^2) = \sum_N \int dE \int d\vec{p} \,\overline{\sum_{\mathcal{M}}} \sum_s P_{\mathcal{M}\mathcal{M},ss}^N(\vec{p},\vec{p'},E) \frac{\xi'}{\xi} H_q^N(x',\Delta^2,\xi') ,$$

 $\tilde{G}_{M}^{3,q}$ in terms of $\tilde{G}_{M}^{N,q}$ (M. Rinaldi, S.S. PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)):

$$\tilde{G}_{M}^{3,q}(x,\Delta^{2},\xi) = \sum_{N} \int dE \int d\vec{p} \left[P_{+-,+-}^{N} - P_{+-,-+}^{N} \right] (\vec{p},\vec{p}',E) \frac{\xi'}{\xi} \tilde{G}_{M}^{N,q}(x',\Delta^{2},\xi') ,$$

where $P_{\mathcal{M}'\mathcal{M},s's}^{N}(\vec{p},\vec{p}',E)$ is the one-body, spin-dependent, off-diagonal spectral function for the nucleon N in the nucleus,

$$\mathbf{P}_{\mathcal{M}'\mathcal{M}\sigma'\sigma}^{N}(\vec{p},\vec{p}',E) = \sum_{f_{A-1}} \delta(E - E_{A-1} + E_{A})$$

$$S_{A}\langle \Psi_{A}; J_{A}\mathcal{M}\pi_{A} | \vec{p},\sigma;\phi_{f_{A-1}} \rangle \underbrace{\langle \phi_{f_{A-1}};\sigma'\vec{p}' | \pi_{A}J_{A}\mathcal{M}';\Psi_{A} \rangle_{S_{A}}}_{\langle \Phi_{f_{A-1}};\sigma'\vec{p}' | \pi_{A}J_{A}\mathcal{M}';\Psi_{A} \rangle_{S_{A}}}$$

 $^{\nwarrow}$ intrinsic overlaps $^{\nearrow}$



The spectral function: a few words more

$$\mathbf{P}_{\mathcal{M}'\mathcal{M}\sigma'\sigma}^{N}(\vec{p},\vec{p}',E) = \sum_{f} \delta(E - E_{min} - E_{f}^{*})$$
$$_{S_{A}}\langle\Psi_{A};J_{A}\mathcal{M}\pi_{A}|\vec{p},\sigma;\phi_{f}(E_{f}^{*})\rangle \ \langle\phi_{f}(E_{f}^{*});\sigma'\vec{p}'|\pi_{A}J_{A}\mathcal{M}';\Psi_{A}\rangle_{S_{A}}$$



- the two-body recoiling system can be either the deuteron or a scattering state;
- when a deeply bound nucleon, with high removal energy $E = E_{min} + E_f^*$, leaves the nucleus, the recoling system is left with high excitation energy E_f^* ;
- the three-body bound state and the two-body bound or scattering state are evaluated within the same interaction (in our case, Av18, from the Pisa group): the extension of the treatment to heavier nuclei would be very difficult



GPDs of ³He: importance of relativity

What we have:

- * An instant form, I.A. calculation of $H^3, \tilde{G}^3_M, \tilde{H}^3$, within AV18;
- * the neutron contribution dominates \tilde{G}_M^3 and \tilde{H}^3 at low Δ^2 ;

an extraction procedure of the neutron information, able to take into account all the nuclear effects encoded in an IA analysis;

What we can do now: to estimate X-sections (DVCS, BH, Interference) \rightarrow a proposal of coherent DVCS off ³He at JLab@12 GeV?

BUT



- In case experiments are performed at higher Δ^2 :
 - * a RELATIVISTIC TREATMENT is mandatory:

a sizable difference in momentum between the initial and final states requires proper boosting

 * The fulfillment of polinomiality requires covariance;
 In NR calculations, number of particle sum rule, momentum sum rule, (slightly) violated.



A relativistic extension of the ³He spectral function definition is necessary