Sivers effect in two hadron electroproduction (CFR)

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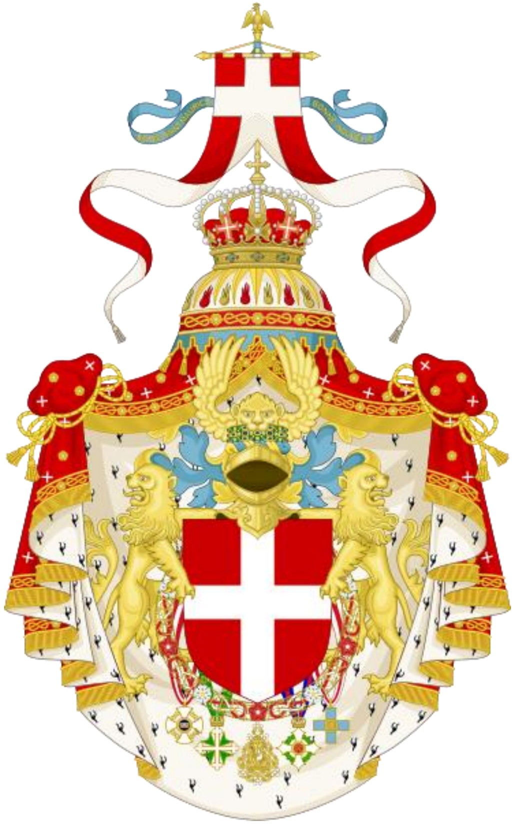
arXiv: 1403.5562, 1405.5059

House of Savoy



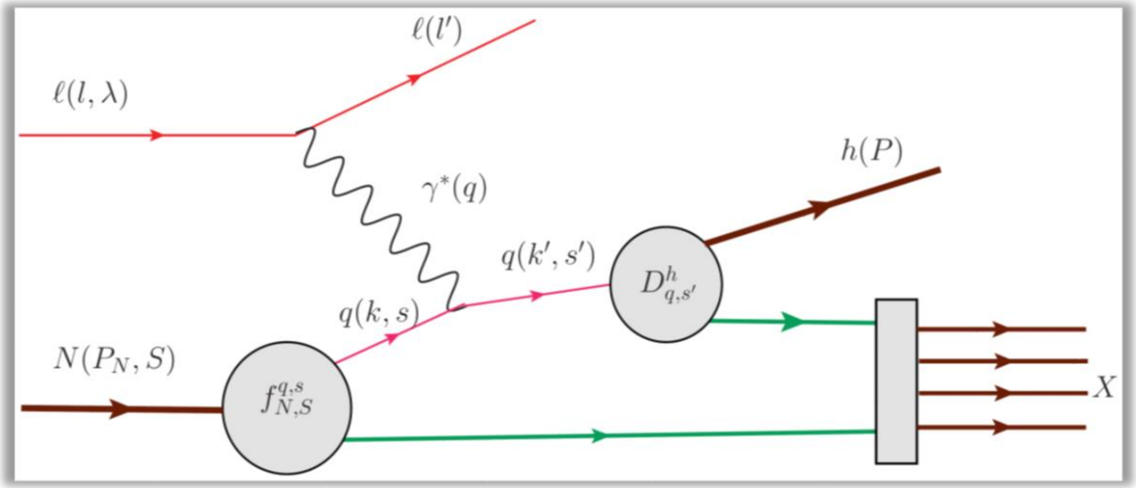
Bar Le Sarde à Douvaine

Country	Italy, Somalia, Ethiopia, Albania, Croatia, France, Spain
Titles	▪ Count of Savoy ▪ Duke of Savoy ▪ King of Sardinia ▪ King of Italy ▪ Emperor of Ethiopia ▪ King of the Albanians ▪ King of Croatia ▪ King of Spain ▪ King of Cyprus ▪ King of Armenia ▪ King of Jerusalem
Founded	1003
Founder	Umberto I
Final ruler	Umberto II
Current head	**Disputed:** Victor Emmanuel, Prince Amedeo

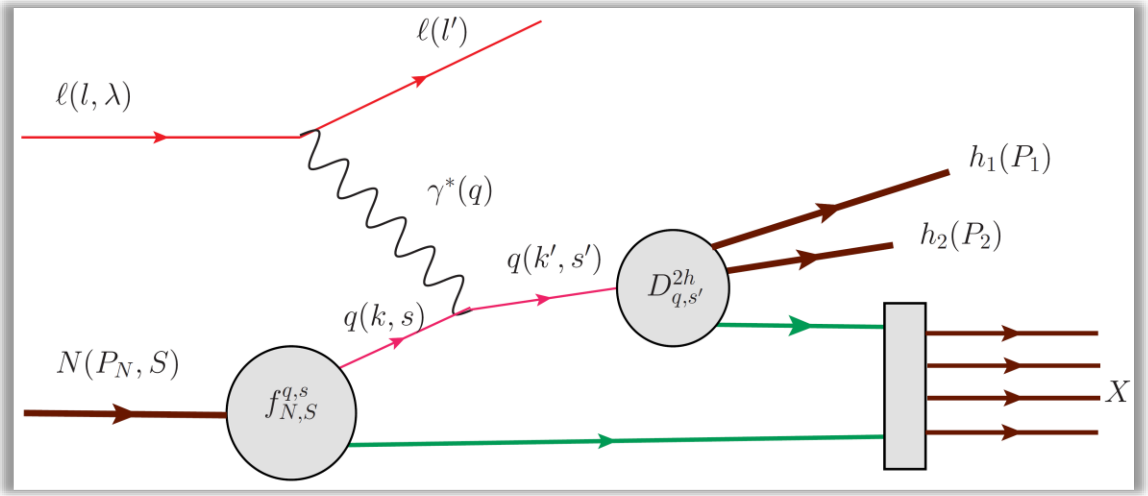
Outline

- Introduction
  - Relative transverse momentum
- Simple model parameterization
- mLEPTO
  - Numerical results
- Conclusions

TMDs @ SIDIS$_{1h}$ and SIDIS$_{2h}$



$$f_1(x,\mathbf{k}_T) \leftrightarrow D_{1h}(z,\mathbf{p}_\perp)$$
$$f_{1T}^\perp(x,\mathbf{k}_T) \leftrightarrow D_{1h}(z,\mathbf{p}_\perp)$$
$$h_1(x,\mathbf{k}_T) \leftrightarrow H_{1h}(z,\mathbf{p}_\perp)$$

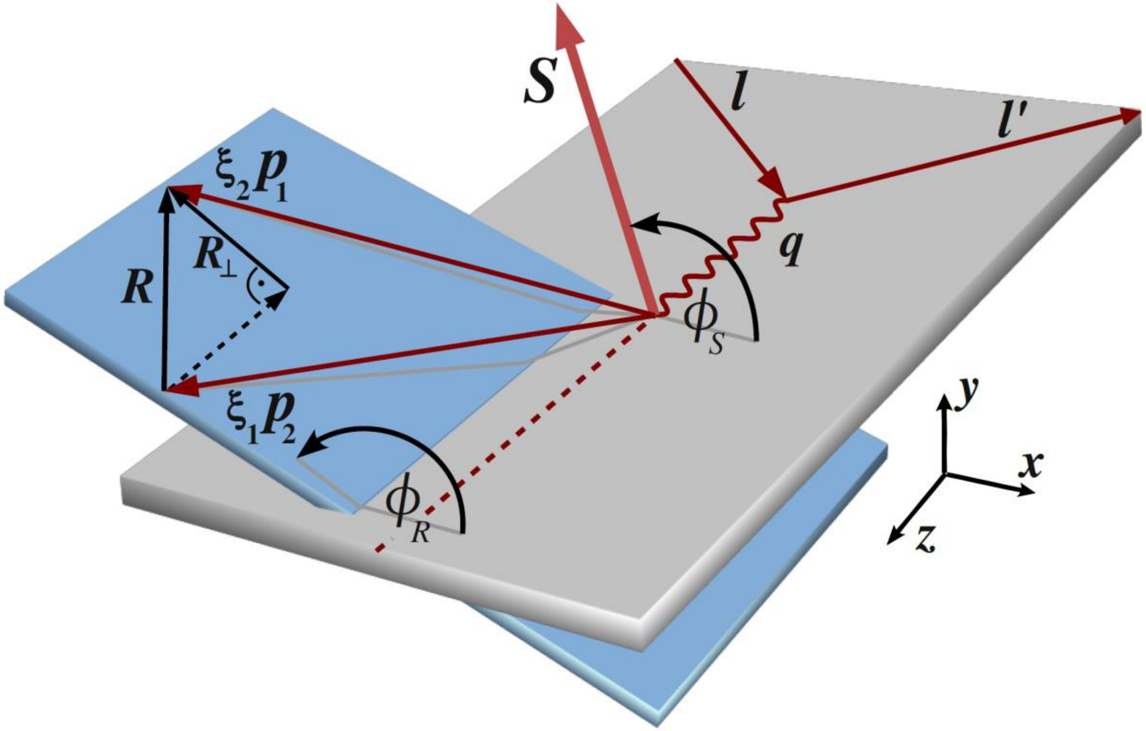


$$f_1 \leftrightarrow D_{2h}(z_1,z_2,\mathbf{p}_{1\perp},\mathbf{p}_{2\perp})$$
$$f_{1T}^\perp \leftrightarrow D_{2h}(z_1,z_2,\mathbf{p}_{1\perp},\mathbf{p}_{2\perp})$$
$$h_1 \leftrightarrow H_{2h}^1(z_1,z_2,\mathbf{p}_{1\perp},\mathbf{p}_{2\perp})$$
$$h_1 \leftrightarrow H_{2h}^2(z_1,z_2,\mathbf{p}_{1\perp},\mathbf{p}_{2\perp})$$
$$H_{2h}^\perp \leftrightarrow \mathbf{P}_T = \mathbf{P}_{1T} + \mathbf{P}_{2T},\ \phi_T$$
$$H_{2h}^\sphericalangle \leftrightarrow \mathbf{R}_T = \mathbf{P}_{1T} - \mathbf{P}_{2T},\ \phi_R$$

So far mainly D_{2h} and $H_{2h}^\sphericalangle$ integrated over total transverse momentum and also $H_{2h}^{1,2}$ (COMPASS: mirror symetry, interplay between $H_{2h}^{1,2}$ and $H_{2h}^\sphericalangle$) were studied

Definitions of relative transverse momentum.
I. Experiments

COMPASS Collaboration, *Physics Letters B 713 (2012) 10*



$$\mathbf{R} = \frac{z_2 \mathbf{p}_1 - z_1 \mathbf{p}_2}{z_1 + z_2} = \xi_2 \mathbf{p}_1 - \xi_1 \mathbf{p}_2$$

HERMES Collaboration, J. High Energy Phys. 06 (2008) 017. (SIDIS)

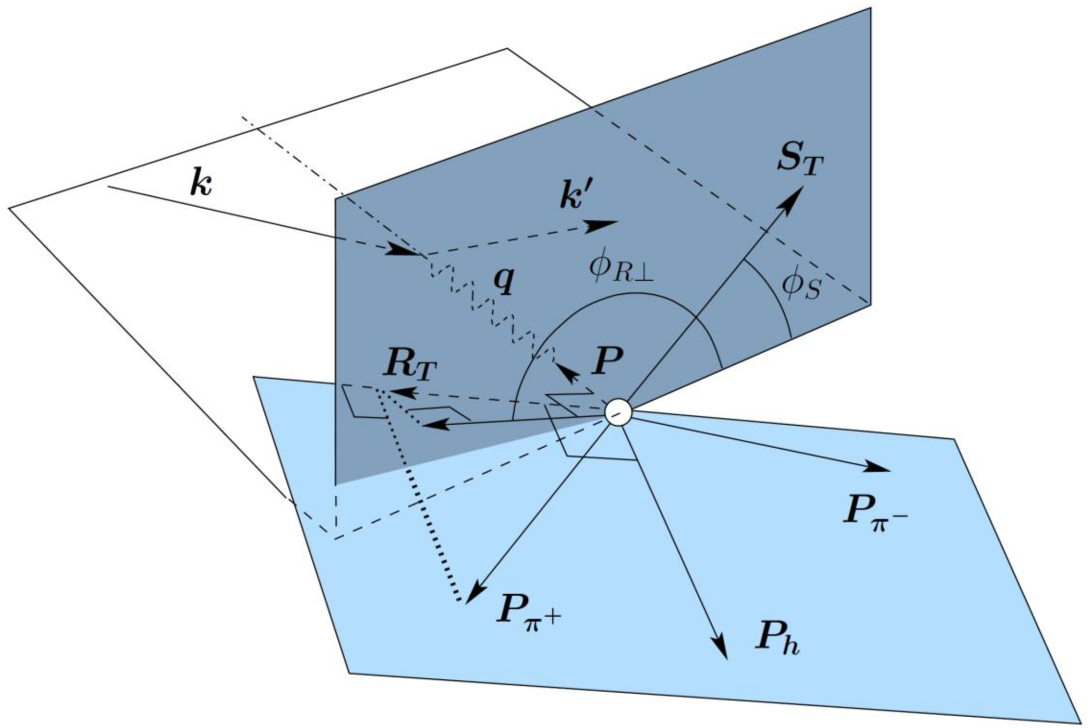


Figure 1: Depiction of the azimuthal angles $\phi_{R\perp}$ of the dihadron and ϕ_S of the component $\boldsymbol{S_T}$ of the target-polarization transverse to both the virtual-photon and target-nucleon momenta $\boldsymbol{q}$ and $\boldsymbol{P}$, respectively. Both angles are evaluated in the virtual-photon-nucleon center-of-momentum frame. Explicitly, $\phi_{R\perp} \equiv \frac{(\boldsymbol{q}\times\boldsymbol{k})\cdot\boldsymbol{R_T}}{|(\boldsymbol{q}\times\boldsymbol{k})\cdot\boldsymbol{R_T}|} \arccos \frac{(\boldsymbol{q}\times\boldsymbol{k})\cdot(\boldsymbol{q}\times\boldsymbol{R_T})}{|\boldsymbol{q}\times\boldsymbol{k}||\boldsymbol{q}\times\boldsymbol{R_T}|}$ and $\phi_S \equiv \frac{(\boldsymbol{q}\times\boldsymbol{k})\cdot\boldsymbol{S_T}}{|(\boldsymbol{q}\times\boldsymbol{k})\cdot\boldsymbol{S_T}|} \arccos \frac{(\boldsymbol{q}\times\boldsymbol{k})\cdot(\boldsymbol{q}\times\boldsymbol{S_T})}{|\boldsymbol{q}\times\boldsymbol{k}||\boldsymbol{q}\times\boldsymbol{S_T}|}$. Here, $\boldsymbol{R_T} = \boldsymbol{R} - (\boldsymbol{R}\cdot\hat{\boldsymbol{P}}_{\boldsymbol{h}})\hat{\boldsymbol{P}}_{\boldsymbol{h}}$, with $\boldsymbol{R} \equiv (\boldsymbol{P_{\pi^+}} - \boldsymbol{P_{\pi^-}})/2$, $\boldsymbol{P_h} \equiv \boldsymbol{P_{\pi^+}} + \boldsymbol{P_{\pi^-}}$, and $\hat{\boldsymbol{P}}_{\boldsymbol{h}} \equiv \boldsymbol{P_h}/ \mid \boldsymbol{P_h} \mid$, thus R_T is the component of P_{π^+} orthogonal to P_h, and $\phi_{R\perp}$ is the azimuthal angle of R_T about the virtual-photon direction. The dotted lines indicate how vectors are projected onto planes. The short dotted line is parallel to the direction of the virtual photon. Also included is a description of the polar angle θ, which is evaluated in the center-of-momentum frame of the pion pair.

Belle Collaboration. PRL 107, 072004 (2011) (e^+e^- annihilation)
Boer method (a_{12R})

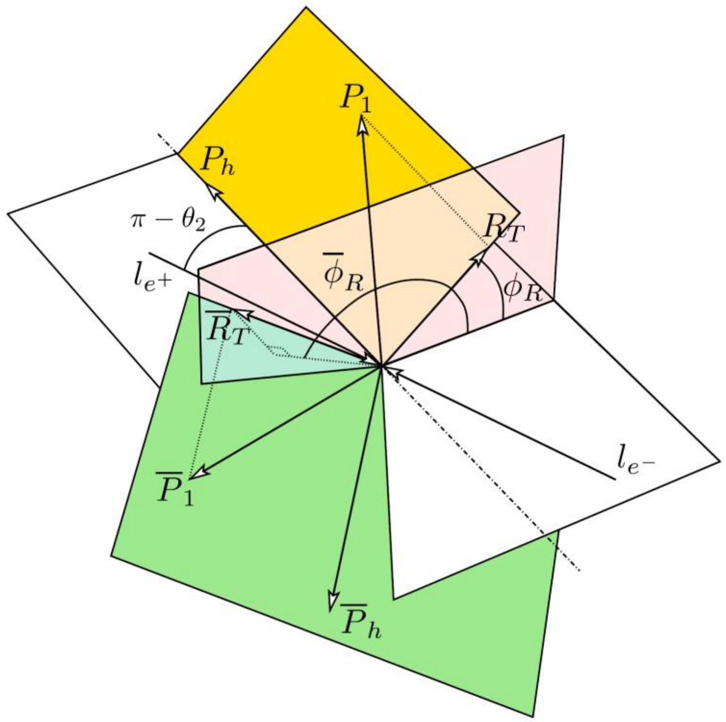


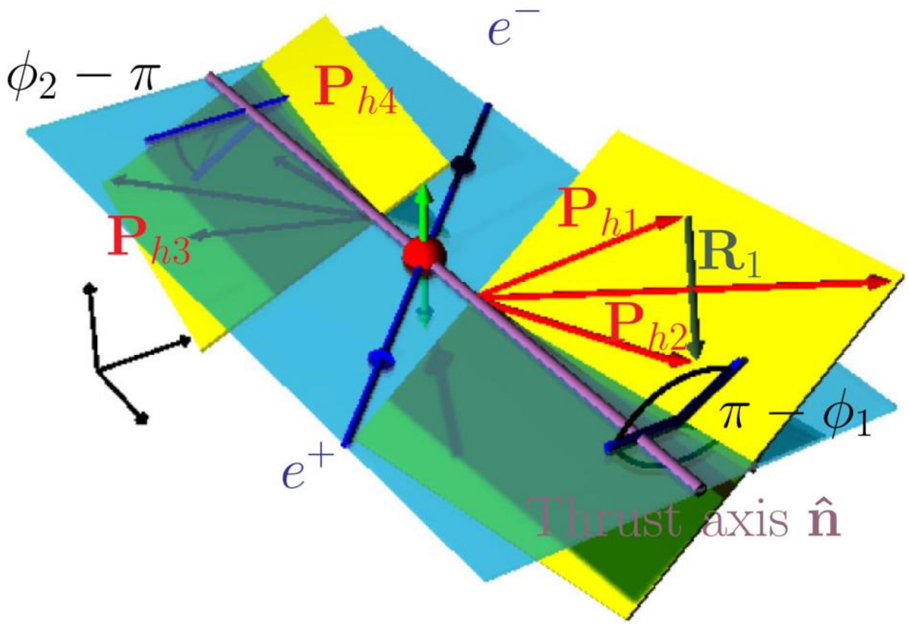
FIG. 1 (color online). Definition of the kinematics for the process $e^+e^- \to (\pi^+\pi^-)_{\text{jet1}}(\pi^+\pi^-)_{\text{jet2}}X$.

Similar to HERMES definition

$$\phi_R = \frac{(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h) \cdot \boldsymbol{R}_T}{|(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h) \cdot \boldsymbol{R}_T|} \arccos\left(\frac{\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h}{|\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h|} \cdot \frac{\boldsymbol{R}_T \times \boldsymbol{P}_h}{|\boldsymbol{R}_T \times \boldsymbol{P}_h|}\right)$$

$$\bar{\phi}_R = \frac{(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h) \cdot \bar{\boldsymbol{R}}_T}{|(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h) \cdot \bar{\boldsymbol{R}}_T|} \arccos\left(\frac{(\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h)}{|\boldsymbol{l}_{e^+} \times \boldsymbol{P}_h|} \cdot \frac{(\bar{\boldsymbol{R}}_T \times \boldsymbol{P}_h)}{|\bar{\boldsymbol{R}}_T \times \boldsymbol{P}_h|}\right)$$

Belle Collaboration. PRL 107, 072004 (2011) (e^+e^- annihilation)
Thrust axis method (a_{12})



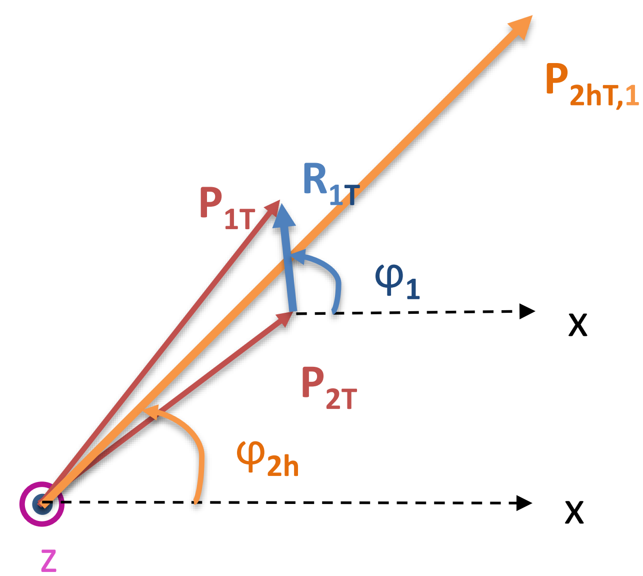


FIG. 1 (color online). Azimuthal angle definitions for ϕ_1 and ϕ_2 as defined relative to the thrust axis in the CMS.

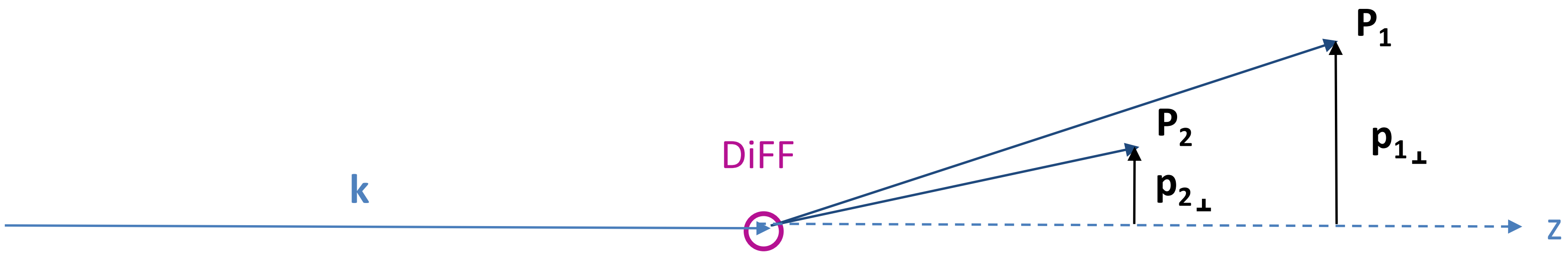
$$\mathbf{R}_1 = \mathbf{P}_{h1} - \mathbf{P}_{h2}$$
$$\mathbf{P}_{2h,1} = \mathbf{P}_{h1} + \mathbf{P}_{h2}$$

$$\phi_{\{1,2\}} = \mathrm{sgn}[\hat{\mathbf{n}} \cdot (\hat{\mathbf{z}} \times \hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \mathbf{R}_{1,2})\}] \times \arccos\left(\frac{\hat{\mathbf{z}} \times \hat{\mathbf{n}}}{|\hat{\mathbf{z}} \times \hat{\mathbf{n}}|} \cdot \frac{\hat{\mathbf{n}} \times \mathbf{R}_{1,2}}{|\hat{\mathbf{n}} \times \mathbf{R}_{1,2}|}\right)$$

For each pair the azimuthal angle of vector **R** around the trust axis is used

Definitions of relative transverse momentum.
II. Phenomenology

Xavier Artru, John Collins. Z. Phys. C 69, 277 (1996) (e^+e^- annihilation)



$$\mathbf{R}_A = \frac{z_2\mathbf{P}_1 - z_1\mathbf{P}_2}{z_1+z_2}, \quad \mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$$

$$\mathbf{r}_{\perp A} = \frac{z_2\mathbf{p}_{1\perp} - z_1\mathbf{p}_{2\perp}}{z_1+z_2}, \quad \mathbf{p}_\perp = \mathbf{p}_{1\perp} + \mathbf{p}_{2\perp}$$

$$D(z_1, z_2, \mathbf{p}_{1\perp}, \mathbf{p}_{2\perp}) = D'(z_1, z_2, \mathbf{p}_\perp, \mathbf{r}_{\perp A})$$

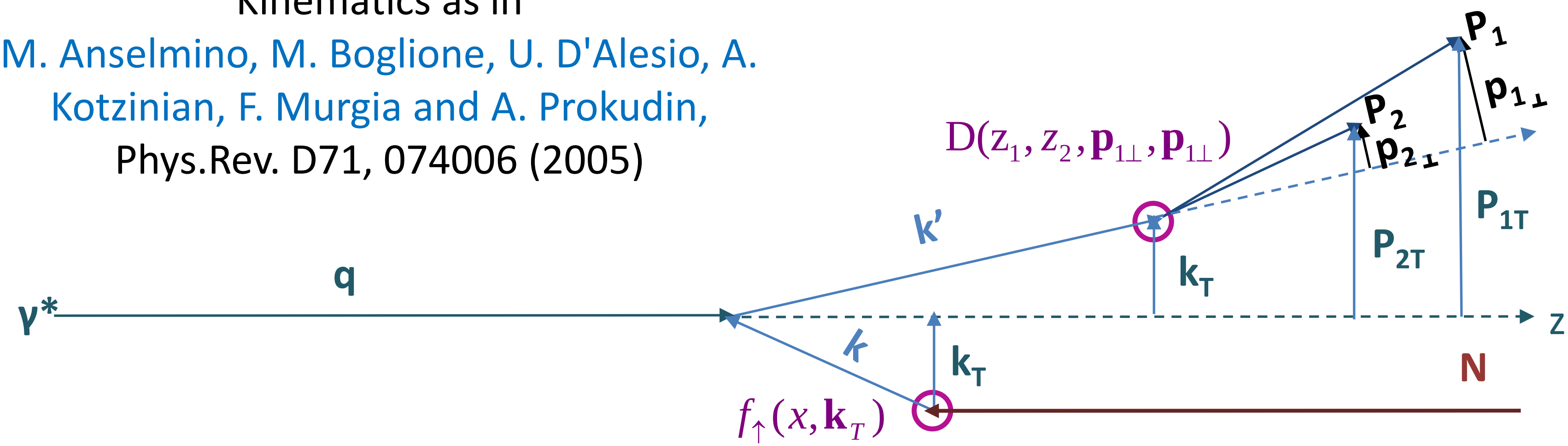
Note, that in the DIS limit and for small transverse momenta $\mathbf{R}_{A,3}\approx\mathbf{0}$

SIDIS in γ^*--N frame.

Relative transverse momentum according to Artru (COMPASS)

Kinematics as in

M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia and A. Prokudin, Phys.Rev. D71, 074006 (2005)



$$\mathbf{P}_{1T} \approx \mathbf{p}_{1\perp} + z_1\mathbf{k}_T, \quad \mathbf{P}_{2T} \approx \mathbf{p}_{2\perp} + z_2\mathbf{k}_T$$

$$\mathbf{P}_T = \mathbf{P}_{1T} + \mathbf{P}_{2T} \approx \mathbf{p}_\perp + z\mathbf{k}_T$$

$$\mathbf{R}_{T,A} = \frac{z_2\mathbf{P}_{1T} - z_1\mathbf{P}_{2T}}{z_1 + z_2} \approx \xi_2\mathbf{p}_{1\perp} - \xi_1\mathbf{p}_{2\perp} = \mathbf{r}_{\perp,A}$$

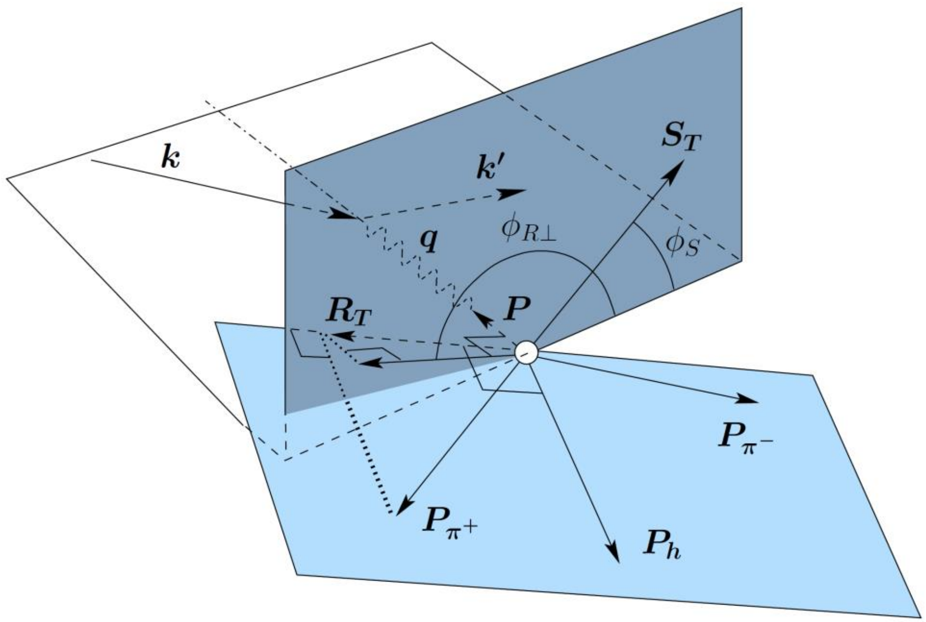
$$z \equiv z_1 + z_2, \quad \xi_1 = \frac{z_1}{z}, \quad \xi_2 = \frac{z_2}{z}$$

Note that $\mathbf{R}_{T,A}$ is independent on $\mathbf{k}_T$.

Its azimuthal angle, $\phi_{R,A}$, is well suited to study transverse spin effects in fragmentation.

But is not a good choice to study the k_T-dependent spin effects (Sivers effect)

Pavia definition of relative transverse momentum.
(Bianconi, Radici, Bacchetta, Jacob, Boer, Courtoy…, HERMES)



$$\mathbf{P}_h = \mathbf{P}_1 + \mathbf{P}_2, \quad \mathbf{R} = \frac{1}{2}(\mathbf{P}_1 - \mathbf{P}_2)$$

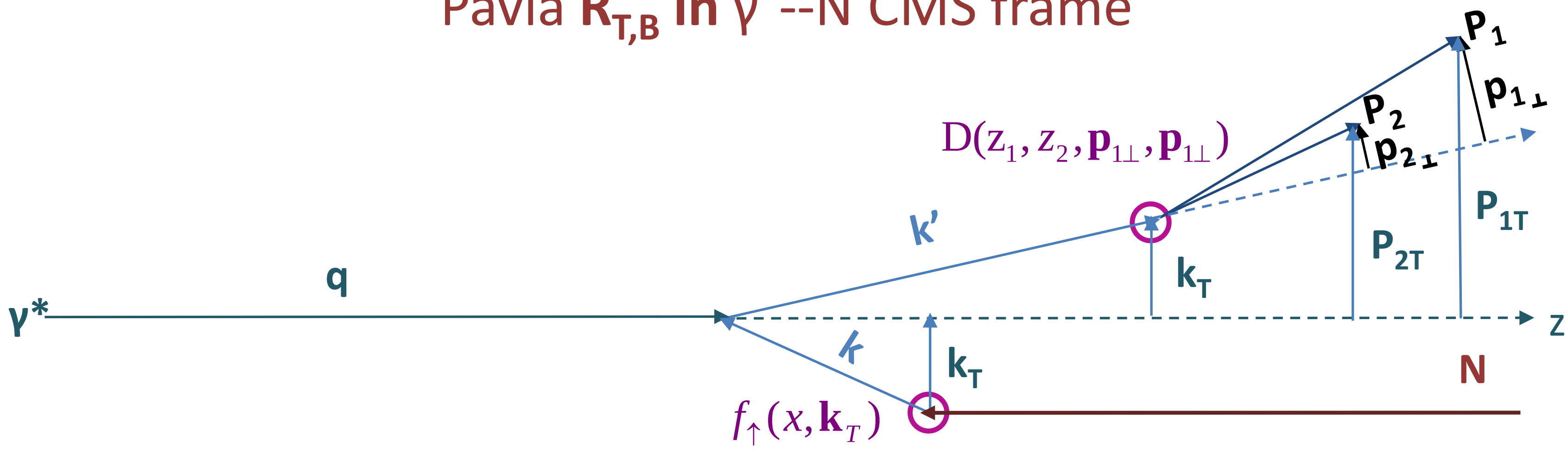
$$\mathbf{R}_{T,B} = \mathbf{R} - (\mathbf{R}\cdot\hat{\mathbf{P}}_h)\hat{\mathbf{P}}_h, \ \hat{\mathbf{P}}_h = \frac{\mathbf{P}_h}{|\mathbf{P}_h|}$$

$\phi_{R\perp}$ is defined as azimuthal angle of transverse to $\mathbf{q}$ component of $\mathbf{R}_T$:

$$\mathbf{R}_\perp = \mathbf{R}_{T,B} - (\mathbf{R}_{T,B}\cdot\hat{\mathbf{q}})\hat{\mathbf{q}}, \ \hat{\mathbf{q}} = \frac{\mathbf{q}}{|\mathbf{q}|}$$

Are the relative vectors $\mathbf{R}_{T,B}$ and $\mathbf{R}_{T,A}$ different?

Pavia $\mathbf{R_{T,B}}$ in γ^*--N CMS frame



SIDIS limit at low transverse momentum: $Q^2 \to \infty$, $W^2 \to \infty$ and all masses and $P_T \ll Q^2$

$$k'^0 = \frac{W}{2}\left(1 + \mathcal{O}\left(\frac{k_T^2}{Q^2}\right)\right) \approx \frac{W}{2}.$$ Neglect $\frac{1}{Q^2}$ corrections

$$\mathbf{P}_h = \mathbf{P}_1 + \mathbf{P}_2, \quad \mathbf{R} = \frac{1}{2}(\mathbf{P}_1 - \mathbf{P}_2), \quad \mathbf{R}_{T,B} = \mathbf{R} - \left(\mathbf{R}\cdot\hat{\mathbf{P}}_h\right)\hat{\mathbf{P}}_h = \frac{1}{2}\left(\mathbf{P}_{1T,B} - \mathbf{P}_{2T,B}\right), \quad \mathbf{P}_{iT,B} = \mathbf{P}_i - \frac{\left(\mathbf{P}_i\cdot\mathbf{P}_h\right)}{\left|\mathbf{P}_h\right|^2}\mathbf{P}_h$$

$$\left|\mathbf{P}_h\right|^2 = P_{h3}^2 + P_{hT}^2 \simeq z^2\left(k'^0\right)^2 + P_{hT}^2 \simeq z^2\left(k'^0\right)^2, \quad \mathbf{P}_i\cdot\mathbf{P}_h \simeq z_i z\left(k'^0\right)^2 + \mathbf{P}_{iT}\cdot\mathbf{P}_{hT} \simeq z_i z\left(k'^0\right)^2$$

$$\frac{\left(\mathbf{P}_i\cdot\mathbf{P}_h\right)}{\left|\mathbf{P}_h\right|^2} = \frac{z_i}{z} \quad \Rightarrow \quad \mathbf{P}_{1T,B} = -\mathbf{P}_{2T,B} = \mathbf{R}_{T,B} \approx \xi_2\mathbf{P}_1 - \xi_1\mathbf{P}_2$$

In the DIS limit for small transverse momenta $\mathbf{R_{T,B}} \approx \mathbf{R_{T,A}}$

SIDIS$_{2h}$ cross section in γ^*--N frame (Sivers part)

$$\frac{d\sigma}{d^2\mathbf{P}_{1T}d^2\mathbf{P}_{2T}} \propto \int d^2\mathbf{k}_T d^2\mathbf{p}_{1\perp} d^2\mathbf{p}_{2\perp} \delta^2\left(\mathbf{p}_{1\perp} + z_1\mathbf{k}_T - \mathbf{P}_{1T}\right)\delta^2\left(\mathbf{p}_{2\perp} + z_2\mathbf{k}_T - \mathbf{P}_{2T}\right) f_\uparrow(\mathbf{k}_T)\,\mathrm{D}(z_1, z_2, \mathbf{p}_{1\perp}, \mathbf{p}_{1\perp})$$

$$f_\uparrow(x,\mathbf{k}_T) = f_1(x,\mathbf{k}_T^2) + \varepsilon_{i,j} S_T^i \frac{\mathbf{k}_T^j}{M} f_{1T}^\perp(x,\mathbf{k}_T^2)$$

Change variables: Artru definition

$$\frac{d\sigma}{d^2\mathbf{P}_T d^2\mathbf{R}_{T,A}} \propto \int d^2\mathbf{k}_T d^2\mathbf{p}_\perp d^2\mathbf{r}_{\perp,A} \delta^2\left(\xi_1(\mathbf{p}_\perp - \mathbf{P}_T) + \mathbf{r}_{\perp,A} - \mathbf{R}_{T,A} + z_1\mathbf{k}_T\right)\delta^2\left(\xi_2(\mathbf{p}_\perp - \mathbf{P}_T) - \mathbf{r}_{\perp,A} + \mathbf{R}_{T,A} + z_2\mathbf{k}_T\right) f_\uparrow(\mathbf{k}_T)\,\mathrm{D}'(\mathbf{p}_\perp, \mathbf{r}_{\perp,A})$$

$$\mathrm{D}'(z_1, z_2, \mathbf{p}_\perp, \mathbf{r}_{\perp,A}) \doteq \mathrm{D}(z_1, z_2, \xi_1\mathbf{p}_\perp + \mathbf{r}_{\perp,A}, \xi_2\mathbf{p}_\perp - \mathbf{r}_{\perp,A})$$

$$\frac{d\sigma}{d^2\mathbf{P}_T d^2\mathbf{R}_{T,A}} \propto \int d^2\mathbf{k}_T d^2\mathbf{p}_\perp \delta^2\left(\mathbf{p}_\perp + z\mathbf{k}_T - \mathbf{P}_T\right) f_\uparrow(\mathbf{k}_T)\,\mathrm{D}'(\mathbf{p}_\perp, \mathbf{R}_{T,A})$$

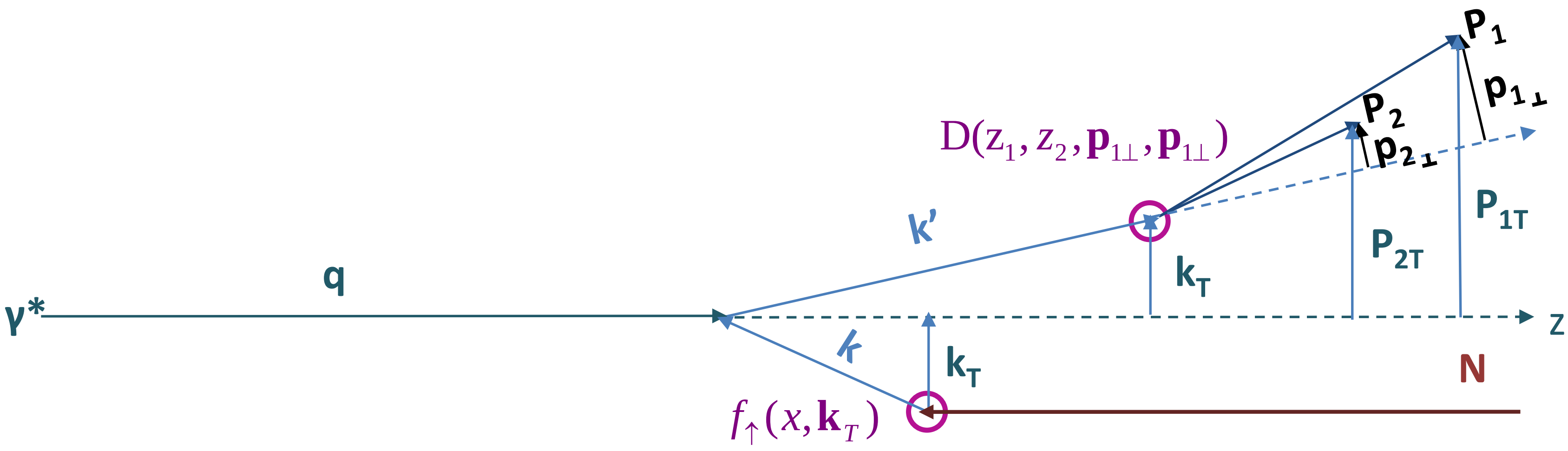
$$\frac{d\sigma}{d^2\mathbf{R}_{T,A}} = \int d^2\mathbf{P}_T \frac{d\sigma}{d^2\mathbf{P}_T d^2\mathbf{R}_{T,A}} \propto \int d^2\mathbf{k}_T \left(f_1(x,\mathbf{k}_T^2) + \varepsilon_{i,j} S_T^i \frac{\mathbf{k}_T^j}{M} f_{1T}^\perp(x,\mathbf{k}_T^2) \right) \int d^2\mathbf{p}_\perp\, \mathrm{D}'(z_1, z_2, \mathbf{p}_\perp, \mathbf{R}_{T,A})$$

$$= f_1(x)\,\mathrm{D}''(z_1, z_2, \mathbf{R}_{T,A}^2), \quad \mathrm{D}''(z_1, z_2, \mathbf{R}_{T,A}^2) \doteq \int d^2\mathbf{p}_\perp\, \mathrm{D}'(z_1, z_2, \mathbf{p}_\perp, \mathbf{R}_{T,A})$$

Well known statement (Bianconi, Boffi, Jakob, Radici, PRD D62, 34008, 2000)

No Sivers-like effect in terms of $\phi_{R,B}$ or $\phi_{R,A}$

Simple definition of relative transverse momentum



$$\mathbf{p}_\perp \doteq \mathbf{p}_{1\perp} + \mathbf{p}_{2\perp}, \quad \mathbf{r}_\perp \doteq \frac{\mathbf{p}_{1\perp} - \mathbf{p}_{2\perp}}{2}$$

$$\mathbf{P}_T = \mathbf{P}_{1T} + \mathbf{P}_{2T} \approx \mathbf{p}_\perp + z\mathbf{k}_T$$

$$\mathbf{R}_\mathrm{T} = \frac{\mathbf{P}_{1T} - \mathbf{P}_{2T}}{2} \approx \mathbf{r}_\perp - \frac{1}{2}(z_1 - z_2)\mathbf{k}_T$$

$$\bar{D}'(z_1, z_2, \mathbf{p}_\perp, \mathbf{r}_\perp) \doteq D(z_1, z_2, \frac{1}{2}\mathbf{p}_\perp + \mathbf{r}_\perp, \frac{1}{2}\mathbf{p}_\perp - \mathbf{r}_\perp)$$

$$\bar{D}''(z_1, z_2, \mathbf{r}_\perp^2) \doteq \int d^2\mathbf{p}_\perp\, D'(z_1, z_2, \mathbf{p}_\perp, \mathbf{r}_\perp)$$

$$\frac{d\sigma}{d^2\mathbf{R}_T} \propto \int d^2\mathbf{k}_T \left(f_1(x, \mathbf{k}_T^2) + \varepsilon_{i,j} S_T^i \frac{\mathbf{k}_T^j}{M} f_{1T}^\perp(x, \mathbf{k}_T^2) \right) \int d^2\mathbf{p}_\perp\, \bar{D}'\left(z_1, z_2, \mathbf{p}_\perp, \mathbf{R}_\mathrm{T} - 0.5(z_1 - z_2)\mathbf{k}_T\right)$$

$$= \int d^2\mathbf{k}_T \left(f_1(x, \mathbf{k}_T^2) + \varepsilon_{i,j} S_T^i \frac{\mathbf{k}_T^j}{M} f_{1T}^\perp(x, \mathbf{k}_T^2) \right) \bar{D}''\left(z_1, z_2, (\mathbf{R}_\mathrm{T} - 0.5(z_1 - z_2)\mathbf{k}_T)^2\right)$$

Quite similar to 1h SIDIS. Non zero Sivers-like effect for $z_1 \neq z_2$

2h SIDIS cross section and TMDs parameterization

Calculations are similar to Anselmino *et al,* PRD71, 074006 (2005)

$$\frac{d\sigma^{h_1h_2}}{dx\,dQ^2\,d\phi_S\,dz_1\,dz_2\,d^2P_{1T}\,d^2P_{2T}} = C(x,Q^2)\sum_q e_q^2 \int d^2k_T\;\; f_{\uparrow}^{q}(x,\mathbf{k}_T)\, D_q^{h_1h_2}(z_1,z_2,\mathbf{P}_{1T}-z_1\mathbf{k}_T,\mathbf{P}_{2T}-z_2\mathbf{k}_T)$$

$$f_{\uparrow}^{q}(x,\mathbf{k}_T) = f_1^{q}(x,k_T) + \frac{[\mathbf{S}_T\times\mathbf{k}_T]_3}{M}\, f_{1T}^{\perp q}(x,k_T)$$

$$f_1^{q}(x,k_T) = f_1^{q}(x)\frac{1}{\pi\mu_0^2}e^{-\mathbf{k}_T^2/\mu_0^2}, \quad f_{1T}^{\perp q}(x,k_T) = f_{1T}^{\perp q}(x)\frac{1}{\pi\mu_S^2}e^{-\mathbf{k}_T^2/\mu_S^2}$$

$$D_{1q}^{h_1h_2}\left(z_1,z_2,\mathrm{p}_{1\perp},\mathrm{p}_{2\perp},\mathbf{p}_{1\perp}\cdot\mathbf{p}_{2\perp}\right) = D_{1q}^{h_1h_2}(z_1,z_2)\frac{1}{\pi^2\nu_1^2\nu_2^2}e^{-P_{1\perp}^2/\nu_1^2-P_{2\perp}^2/\nu_2^2}\left(1+\frac{\mathbf{p}_{1\perp}\cdot\mathbf{p}_{2\perp}}{a^2}\right)$$

ϕ_1 and ϕ_2 correlation in 2h sample

EMC Z. Phys. C31, 333 (1986)

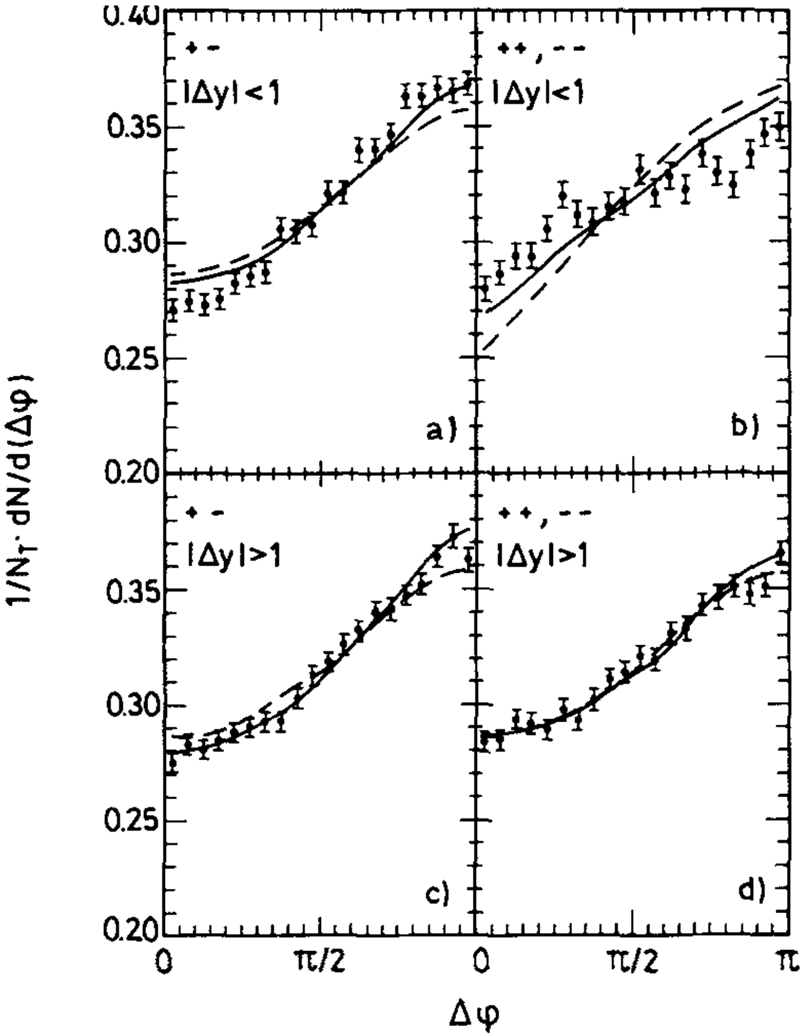
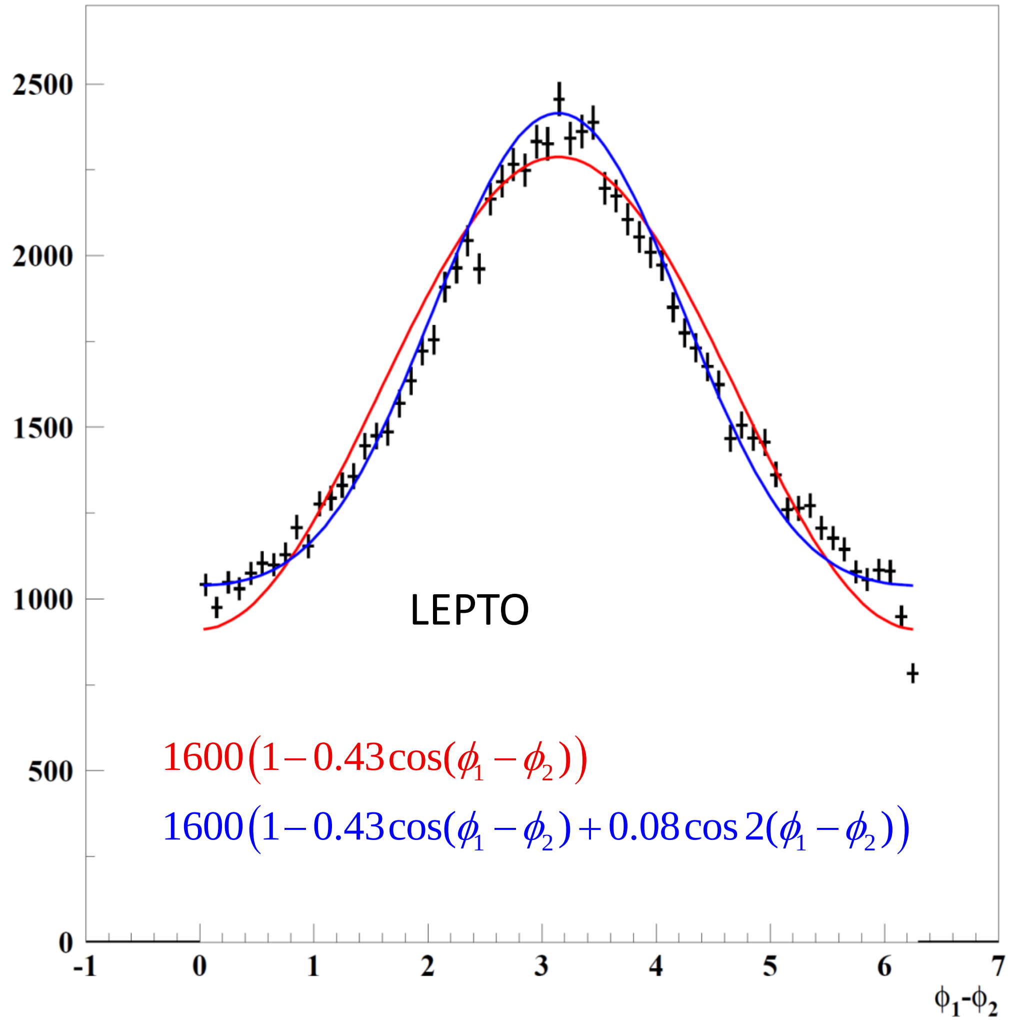


Fig. 8a–d. Normalized distribution of $\Delta\varphi$ for **a, c** oppositely and **b, d** equally charged pairs of hadrons with $|\Delta y| < 1$ **a, b** and with $|\Delta y| > 1$ **c, d**. The predictions of the Lund model (solid lines) and of the randomized $p_\perp$ model (dashed lines) are also shown



General expression for 2h Sivers effect in terms of $\mathbf{P}_{1T}$ and $\mathbf{P}_{2T}$

$$\frac{d\sigma^{h_1h_2}}{dx\,dQ^2\,d\phi_S\,dz_1\,dz_2\,d^2\mathbf{P}_{1T}\,d^2\mathbf{P}_{2T}} = C(x,Q^2)(\sigma_U + \sigma_S)$$

$$\sigma_U = \sum_q e_q^2 \int d^2k_T \; f_1^q \; D_q^{h_1h_2}, \qquad \sigma_S = \sum_q e_q^2 \int d^2k_T \frac{[\mathbf{S}_T \times \mathbf{k}_T]_3}{M} f_{1T}^{\perp q} \; D_{1q}^{h_1h_2}$$

It is easy to see using rotational and parity invariance, that the most general dependence of σ_S on the azimuthal angles ϕ_1, ϕ_2 and ϕ_S is given by two "Sivers-like" terms:

$$\frac{d\sigma^{h_1h_2}}{d\mathbf{P}_{1T}\,d^2\mathbf{P}_{2T}} = C\left(x,Q^2\right)\left[\sigma_U + S_T\left(\sigma_1 \frac{P_{1T}}{M}\sin\left(\phi_2 - \phi_S\right) + \sigma_2 \frac{P_{2T}}{M}\sin\left(\phi_2 - \phi_S\right)\right)\right]$$

where σ_U, σ_1 and σ_2 depend on $x, Q^2, z_1, z_2, P_{1T}, P_{2T}$ and $\mathbf{P}_{1T}\cdot\mathbf{P}_{1T}$ $\left(\text{or}\cos\left(\phi_1 - \phi_S\right)\right)$

Explicit expressions for all σ-s within model with Gaussian parameterization of PDFs and DiFFs are given in AK, Matevosyan and Thomas, arXiv: 1405.5059

1h Sivers asymmetries in 2h sample

$$\frac{d\sigma^{h_1h_2}}{P_{1T}dP_{1T}\,d^2\mathbf{P}_{2T}} = C\left(x,Q^2\right)\left[\sigma_{U,0} + S_T\left(\frac{P_{1T}}{2M}\sigma_{1,1} + \sigma_{2,0}\frac{P_{2T}}{M}\right)\sin\left(\phi_2 - \phi_S\right)\right]$$

$$\frac{d\sigma^{h_1h_2}}{d^2\mathbf{P}_{1T}P_{2T}dP_{2T}} = C\left(x,Q^2\right)\left[\sigma_{U,0} + S_T\left(\frac{P_{1T}}{M}\sigma_{1,0} + \sigma_{2,1}\frac{P_{2T}}{M}\right)\sin\left(\phi_1 - \phi_S\right)\right]$$

where $\sigma_{U,0}$ and $\sigma_{1(2),0(1)}$ denote the moments of $\cos\left(\phi_1 - \phi_2\right)$

Fourier expansion of the corresponding cross section terms:

$$\sigma_i = \frac{1}{2\pi}\sum_{n=0}^{\infty}\sigma_{i,n}\cos\left(n\phi\right), \qquad \phi = \phi_1 - \phi_2, \quad i \in \left\{U,1,2\right\}$$

$$\sigma_{i,m} = \frac{2}{1+\delta_m^0}\int_{-\pi}^{\pi} d\phi\cos\left(m\phi\right)\sigma_i$$

All σ's depend on x, Q^2, z_1, z_2, P_{1T} and P_{2T}.

General expression for 2h Sivers effect in terms of $\mathbf{P}_T$ and $\mathbf{R}_T$

$$\mathbf{P}_T = \mathbf{P}_{1T} + \mathbf{P}_{2T}, \quad \mathbf{R}_T = \frac{1}{2}\left(\mathbf{P}_{1T} - \mathbf{P}_{2T}\right)$$

$$\frac{d\sigma^{h_1h_2}}{d\mathbf{P}_T\, d^2\mathbf{R}_T} = C\left(x,Q^2\right)\left[\sigma_U + S_T\left(\sigma_T \frac{P_T}{M}\sin\left(\phi_T - \phi_S\right) + \sigma_R \frac{R_T}{M}\sin\left(\phi_R - \phi_S\right)\right)\right]$$

$$\sigma_T = \frac{1}{2}\left(\sigma_1 + \sigma_2\right), \quad \sigma_R = \sigma_1 - \sigma_2$$

where σ_U, σ_T and σ_R depend on $x, Q^2, z_1, z_2, P_T, R_T$ and $\mathbf{P}_T \cdot \mathbf{R}_T$ $\left(\text{or} \cos\left(\phi_T - \phi_R\right)\right)$

$$\frac{d\sigma^{h_1h_2}}{R_T dR_T\, d^2\mathbf{P}_T} = C\left(x,Q^2\right)\left[\sigma_{U,0} + S_T\left(\frac{P_T}{M}\sigma_{T,0} + \sigma_{R,1}\frac{R_T}{2M}\right)\sin\left(\phi_P - \phi_S\right)\right]$$

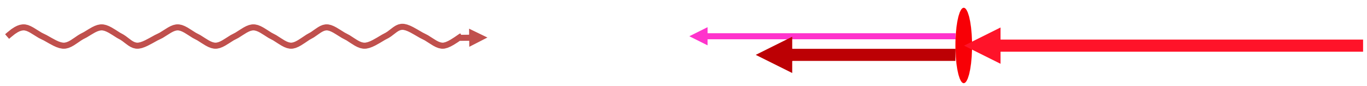
$$\frac{d\sigma^{h_1h_2}}{d^2\mathbf{R}_T P_T dP_T} = C\left(x,Q^2\right)\left[\sigma_{U,0} + S_T\left(\frac{P_T}{2M}\sigma_{T,1} + \sigma_{R,0}\frac{R_T}{M}\right)\sin\left(\phi_R - \phi_S\right)\right]$$

where $\sigma_{U,0}$ and $\sigma_{1(2),0(1)}$ denote the moments of $\cos\left(\phi_T - \phi_R\right)$ Fourier expansion of the corresponding cross section terms. And all σ's depend on x, Q^2, z_1, z_2, P_T and R_T.

Initial quark $\mathbf{k_T}$ in MC generators PYTHIA and LEPTO

- Generate virtual photon – quark scattering in collinear configuration:

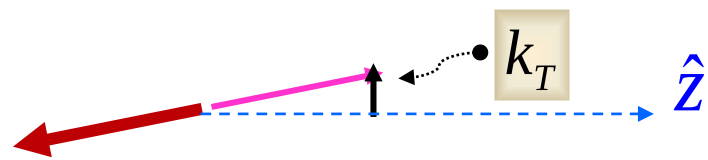
- Before



- After hard scattering

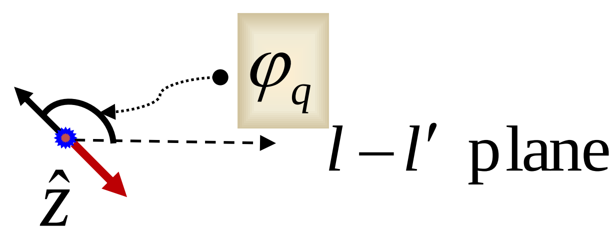
- Generate intrinsic transverse momentum of quark (Gaussian $\boldsymbol{k_T}$)

- Rotate in *l*-*l'* plane



- Generate uniform azimuthal distribution of quark

- Rotate around virtual photon



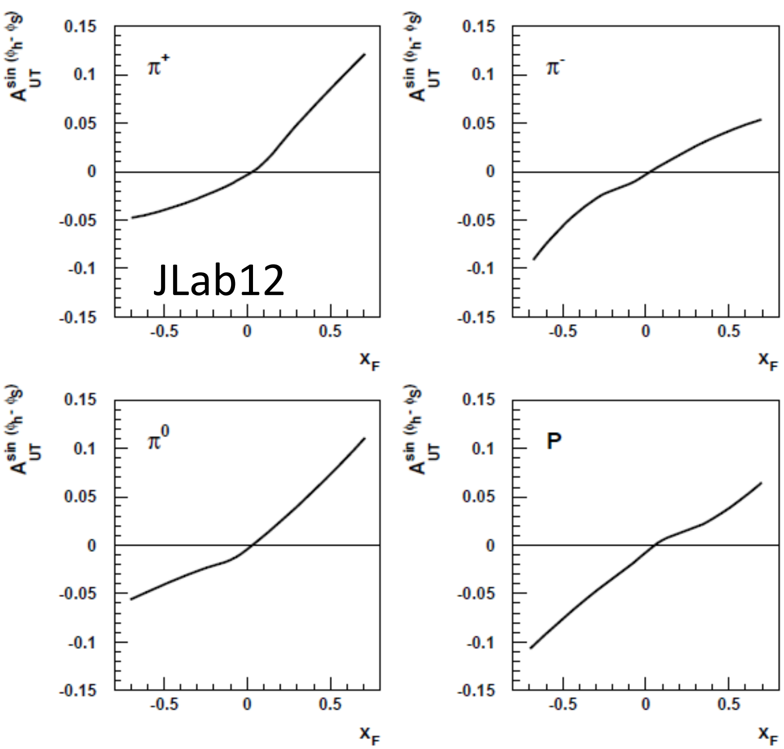
mLEPTO – modified LEPTO, includes Sivers modulation of the quark intrinsic transverse momentum in the transversely polarized nucleon

A.K. hep-ph/0504081, 0510359

Generate initial quark azimuth according

$$1+\left|S_T\right|\frac{f_{1T}^{\perp}(x,k_T)}{f_1(x,k_T)}\frac{k_T}{M}\sin(\phi_q-\phi_S)$$

No 1/Q suppression for Sivers effect



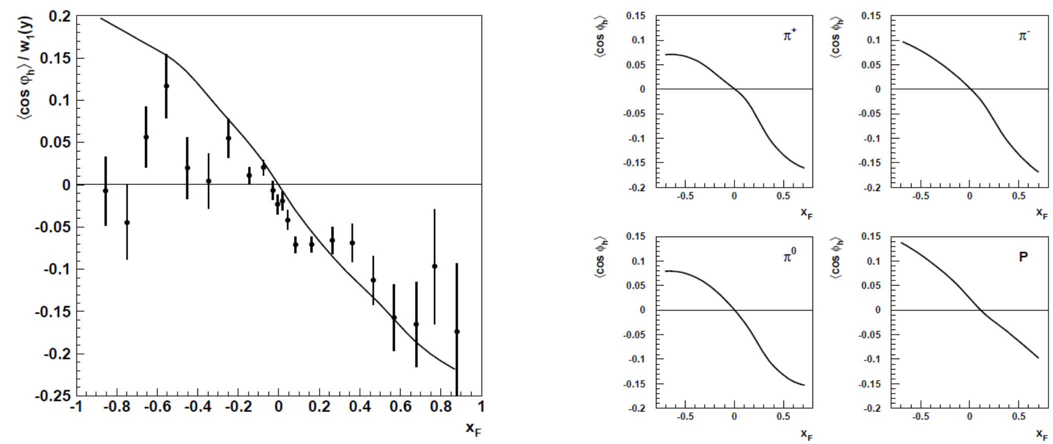


Figure 1. Left: The x_F dependence of $\langle\cos\phi_h\rangle/w_1(y)$ for charged hadrons compared with EMC data. Right: Predictions of modified LEPTO for x_F dependence of $\langle\cos\phi_h\rangle$ for different hadrons produced in 12 GeV unpolarized SIDIS process.

Results from mLEPTO for charged hadron production (proton target)

COMPASS kinematics: E_μ=160 GeV, Q^2 >1 GeV^2, 0.1<y<0.9, 0.03<x<0.7, W>5 GeV

$SIDIS_{1h}$: z>0.2, P_T>0.1 GeV

$SIDIS_{2h}$ symmetric pairs: $z_{1(2)}$>0.1, $P_{1(2)T}$>0.1 GeV

$SIDIS_{2h}$ asymmetric pairs: z_1>0.3, P_{1T}>0.3 GeV

Hadrons ordering in pairs:

Opposite charge hadrons pairs -- first hadron is the positive one

Same charge hadrons pairs -- first hadron is highest z one ($z_1>z_2$)

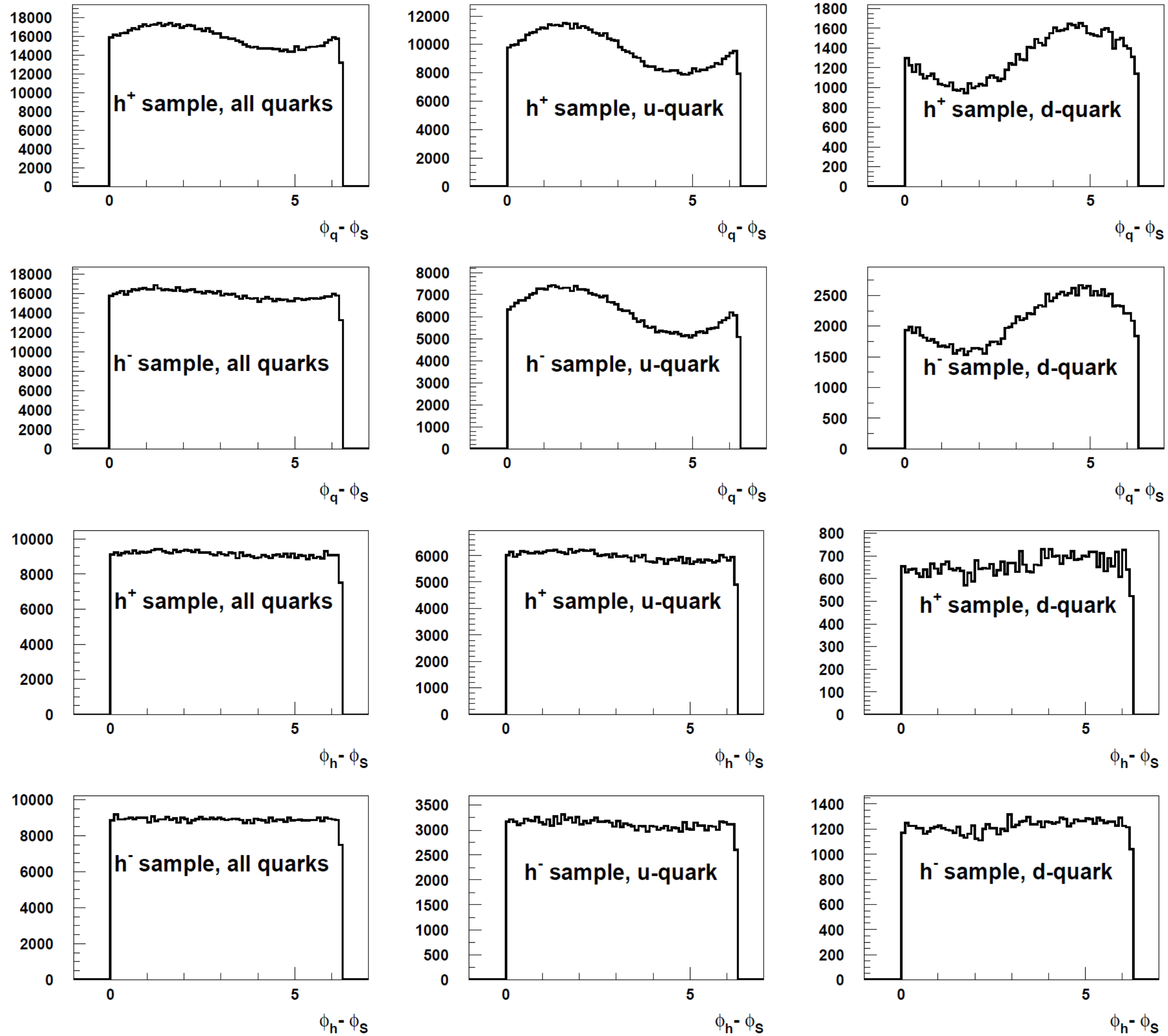
For numerical results the parameterization of Sivers function from Torino-Cagliari fits (last version from Stefano Melis) with slightly adjusted (within their uncertainties) parameters

$$\sigma_{TV} \propto C\left[\sigma_U + S_T \sigma_S \sin\left(\phi_{TV} - \phi_S\right)\right]$$

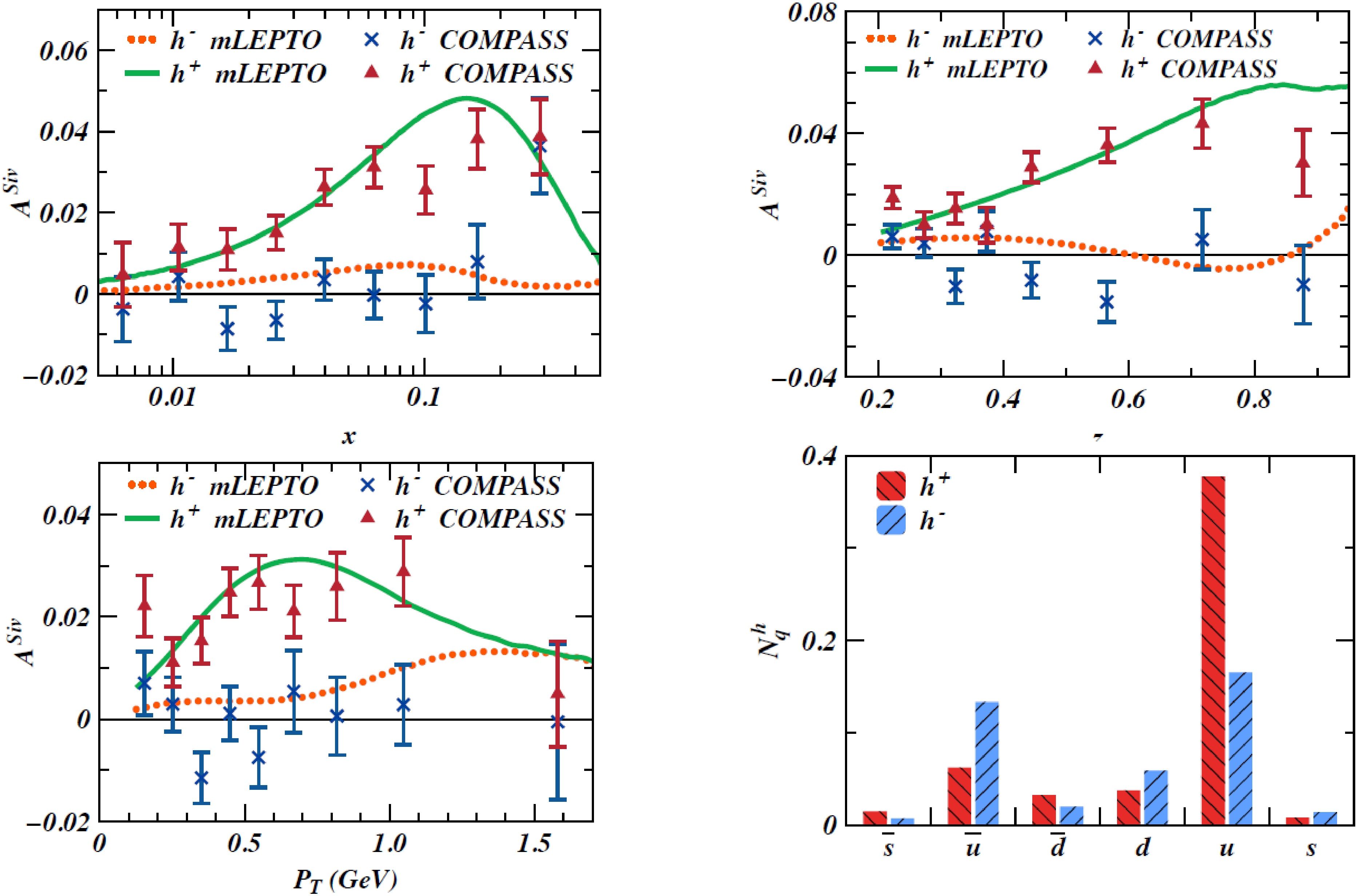
$$A^{Siv} \doteq \frac{\sigma_S}{\sigma_U}$$

10^{11} DIS events generated

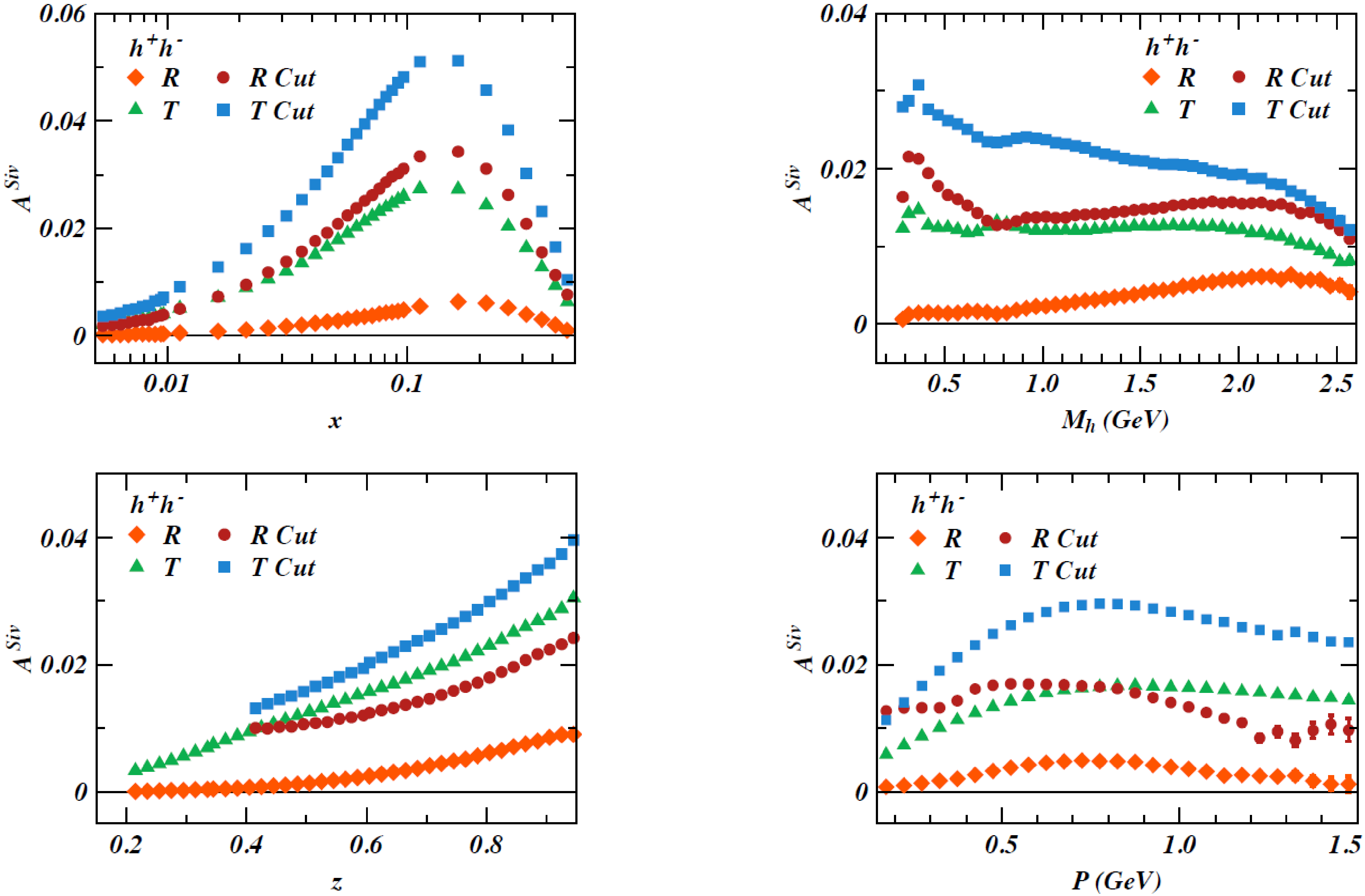
Quark Sivers angle distribution



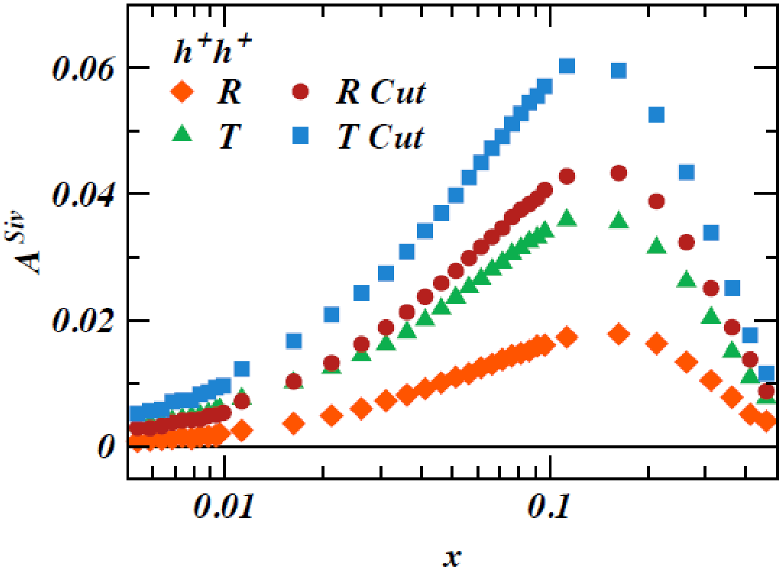
$SIDIS_{1h}$


h- mLEPTO
h- COMPASS
h+ mLEPTO
h+ COMPASS
A Siv
x
0.01
0.1
0.06
0.04
0.02
0
−0.02
0.08
−0.04
0.2
0.4
0.6
0.8
P_T (GeV)
0.5
1.0
1.5
h+
h-
N_q^h
s̄
ū
d̄
d
u
s

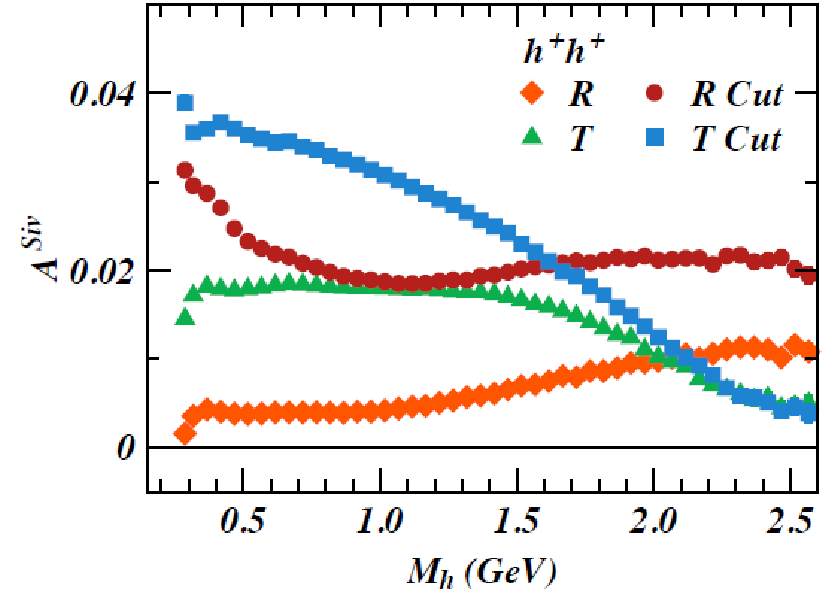
$SIDIS_{2h}$ h^+h^- pairs ($T \leftrightarrow P_{1T}+P_{2T}$)



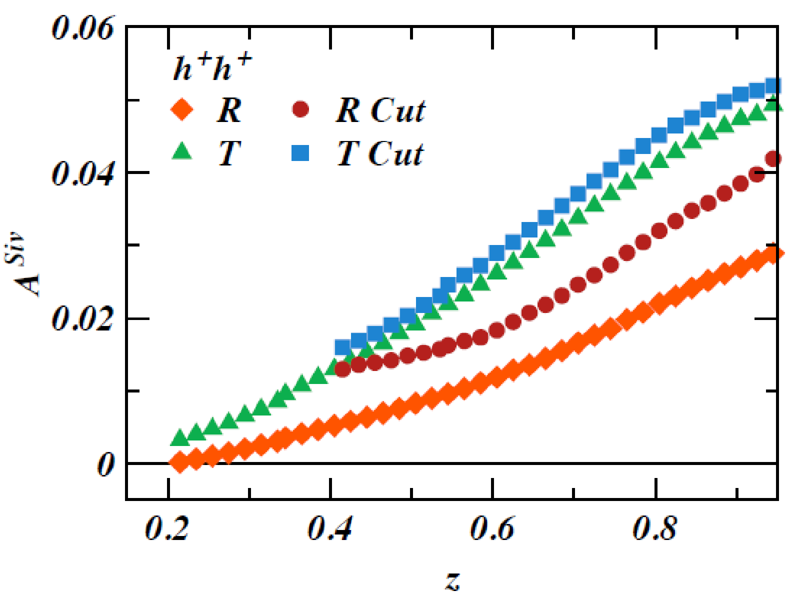
SIDIS$_{2h}$ h^+h^+ pairs ($T \leftrightarrow P_{1T}+P_{2T}$)

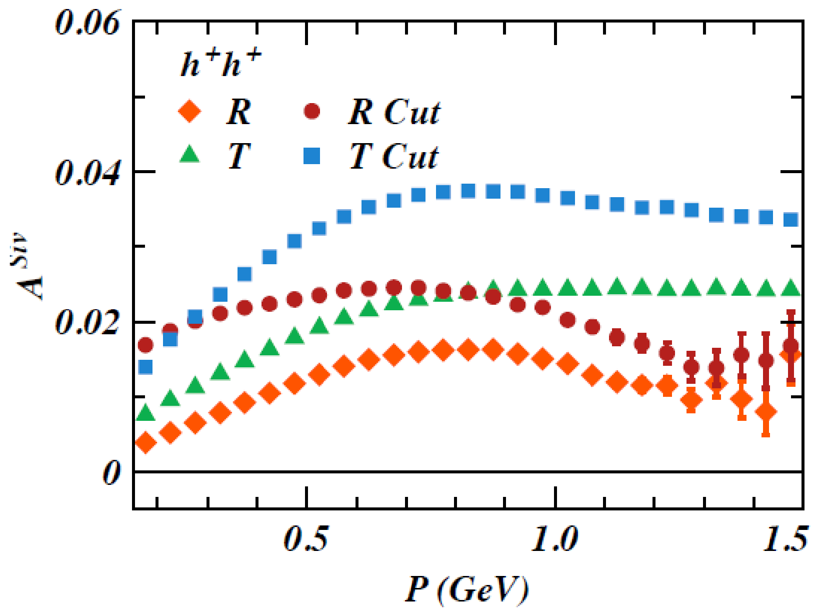


(a)

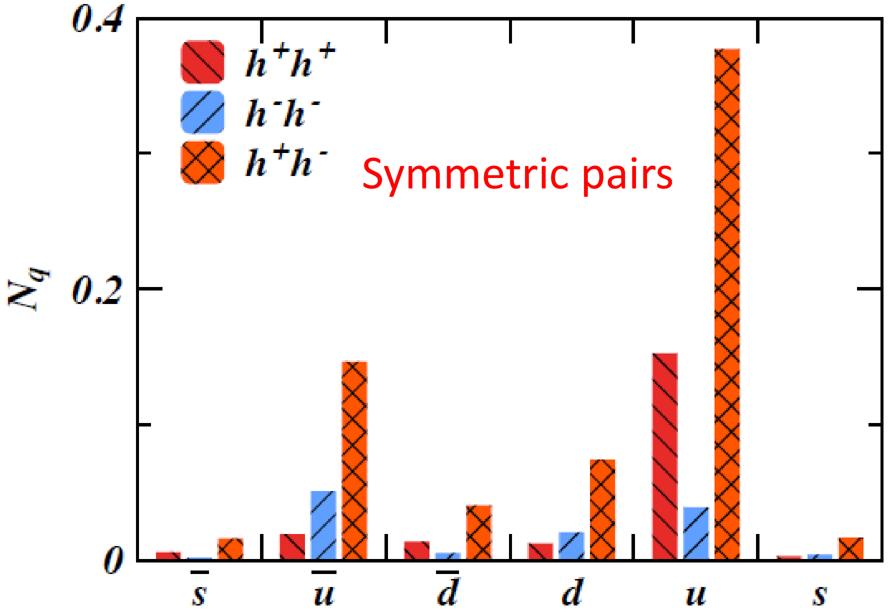


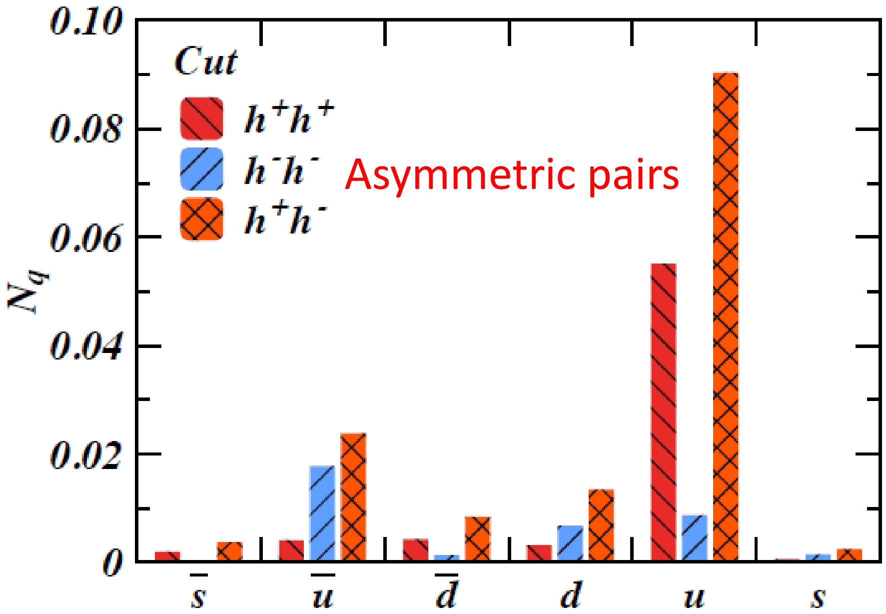
(a)





SIDIS$_{2h}$ quark flavor distributions





CONCLUSIONS

- In $SIDIS_{2h}$ we can study asymmetries in terms of $\mathbf{P}_{1T}$ and $\mathbf{P}_{2T}$ or using their different linear combinations
- $\mathbf{R}_{B,T} = \mathbf{R}_{A,T}$ is well suited for transverse spin effects in fragmentation since they are disconnected from $\mathbf{k}_{T.}$
- By the same reason the integrated over $\mathbf{P}_{2h,T}$ cross-section doesn't contain Sivers-like asymmetry
- We are using another definition $\mathbf{R}_T=(\mathbf{P}_{1T}- \mathbf{P}_{2T})/2$ which is linked to $\mathbf{k}_T$ for asymmetric pairs ($z_1 \neq z_2$)
- The explicit expressions for the Sivers effect description in $SIDIS_{2h}$ *a la* Torino parameterization are derived. They can be used in the data fittings similarly to $SIDIS_{1h}$ case.
- $SIDIS_{2h}$ will provide significant new information for extracting Sivers PDF
  - Extraction of asymmetries with different choices of analyzing variables
    - Check of self consistency
  - Existing COMPASS data can be used
  - JLab12
  - EIC: high multiplicity