Monte Carlo methods for TMD analysis

round-table discussion at "Transversity 2014"
Chia, Sardinia, June 9th -13th, 2014

Gunar.Schnell @ desy.de

ikerbasque
Basque Foundation for Science

eman ta zabal zazu
Universidad del País Vasco
Euskal Herriko Unibertsitatea

Disclaimers

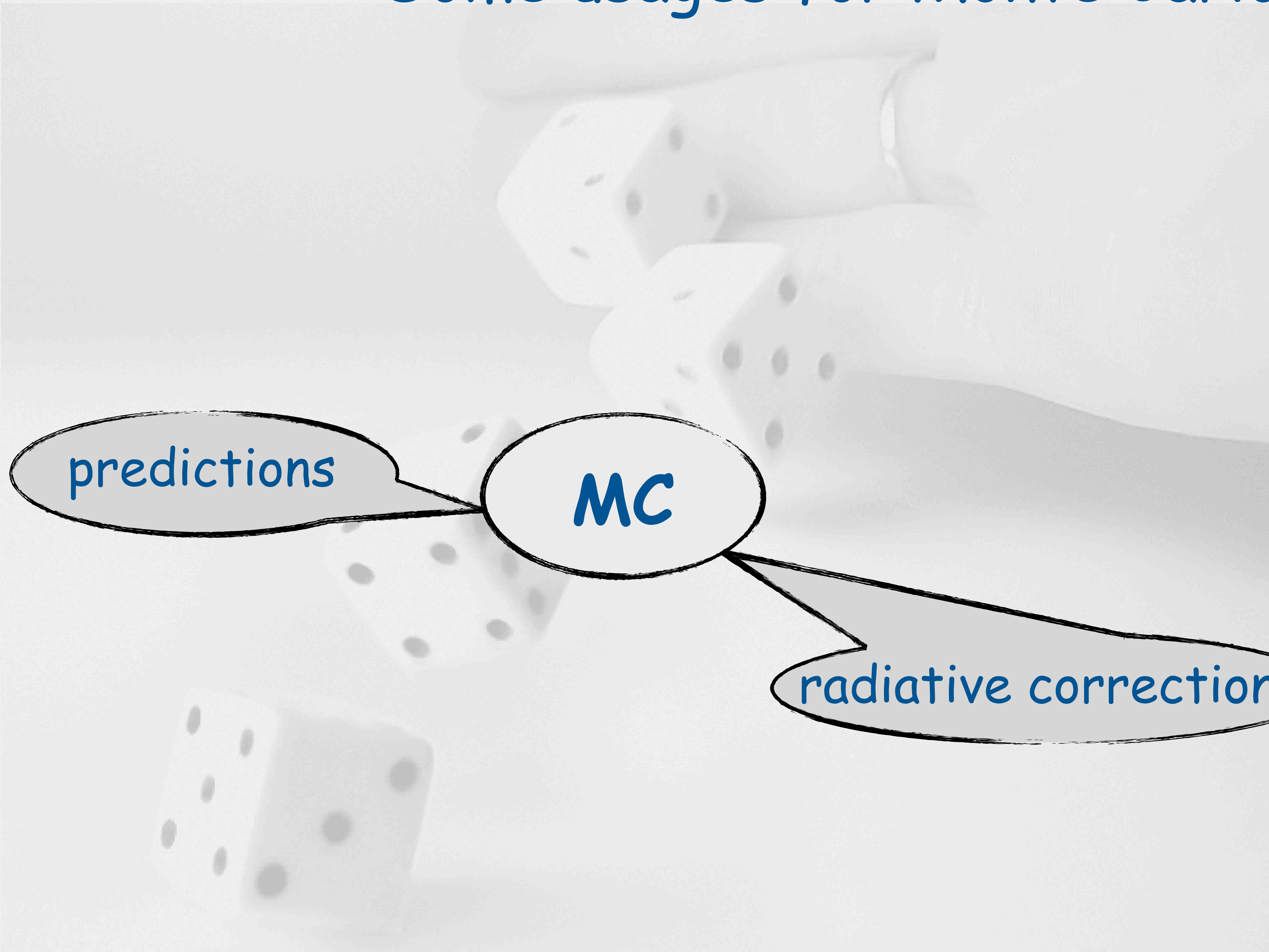
- focus rather on "MC in TMD analyses at HERMES"
- contains a number of actual trivial, but hopefully still useful, statements
- can not offer a general recipe, though hopefully some guidance for particular cases

Some usages for Monte Carlo

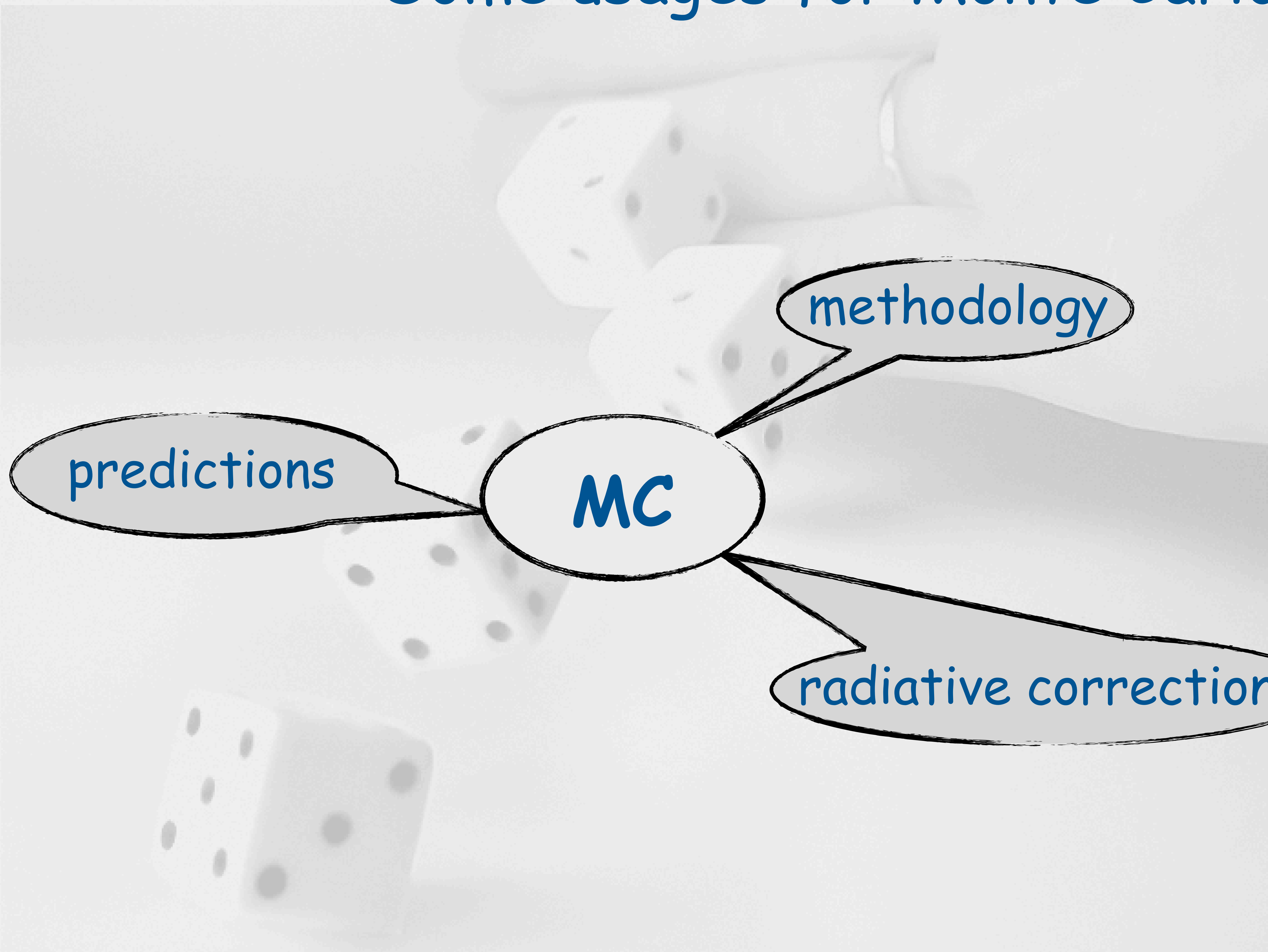
MC

Some usages for Monte Carlo

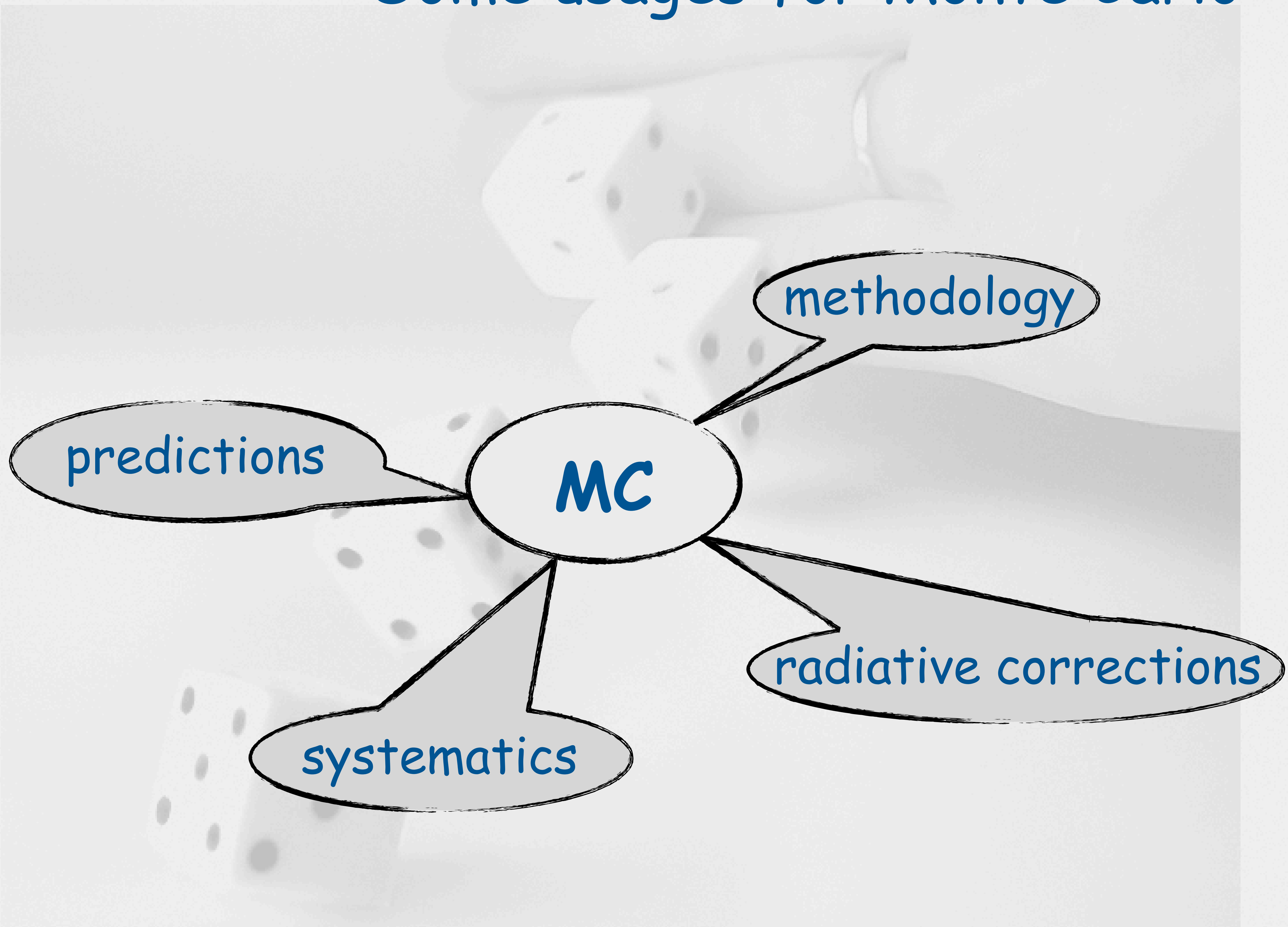
Some usages for Monte Carlo



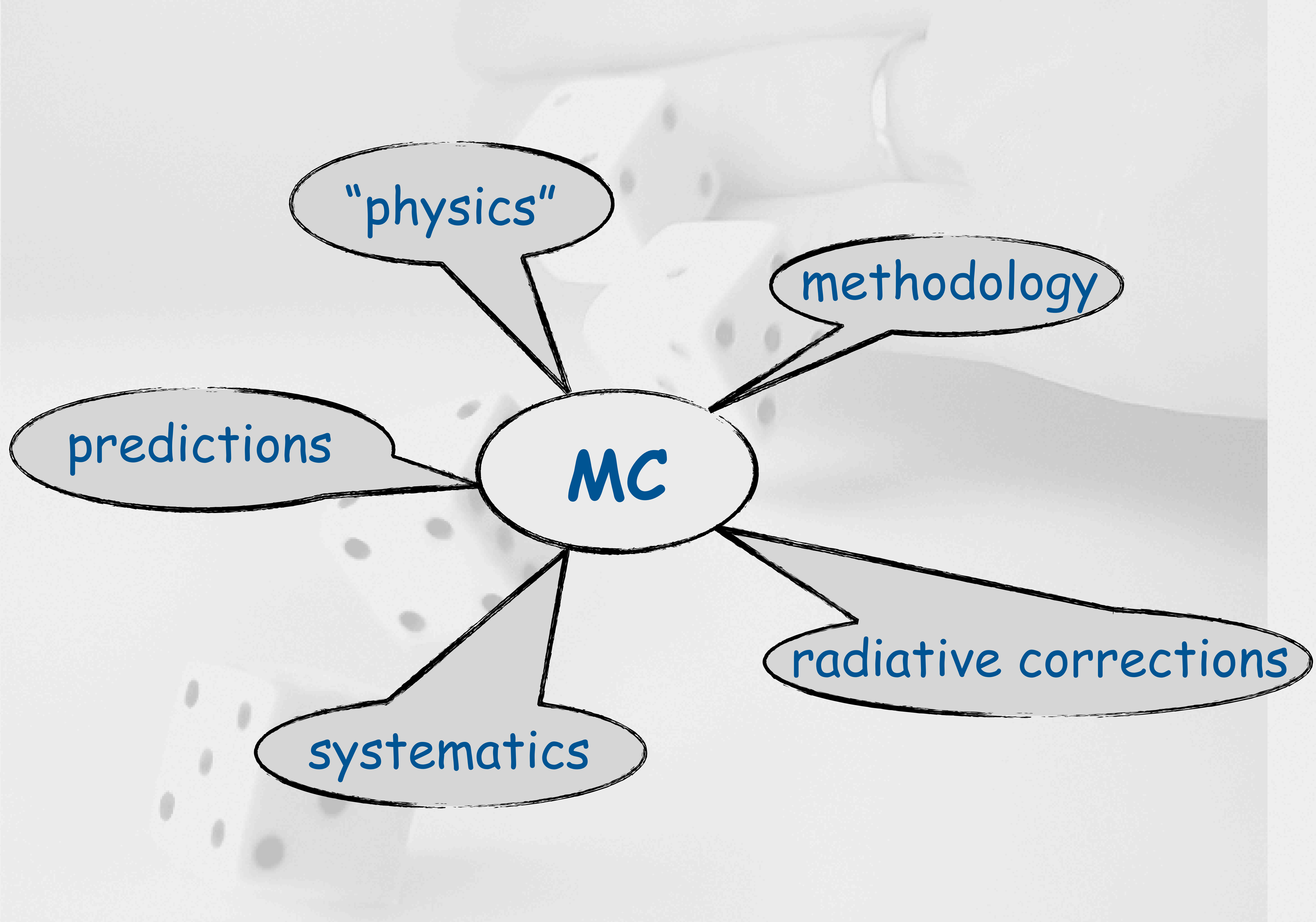
Some usages for Monte Carlo



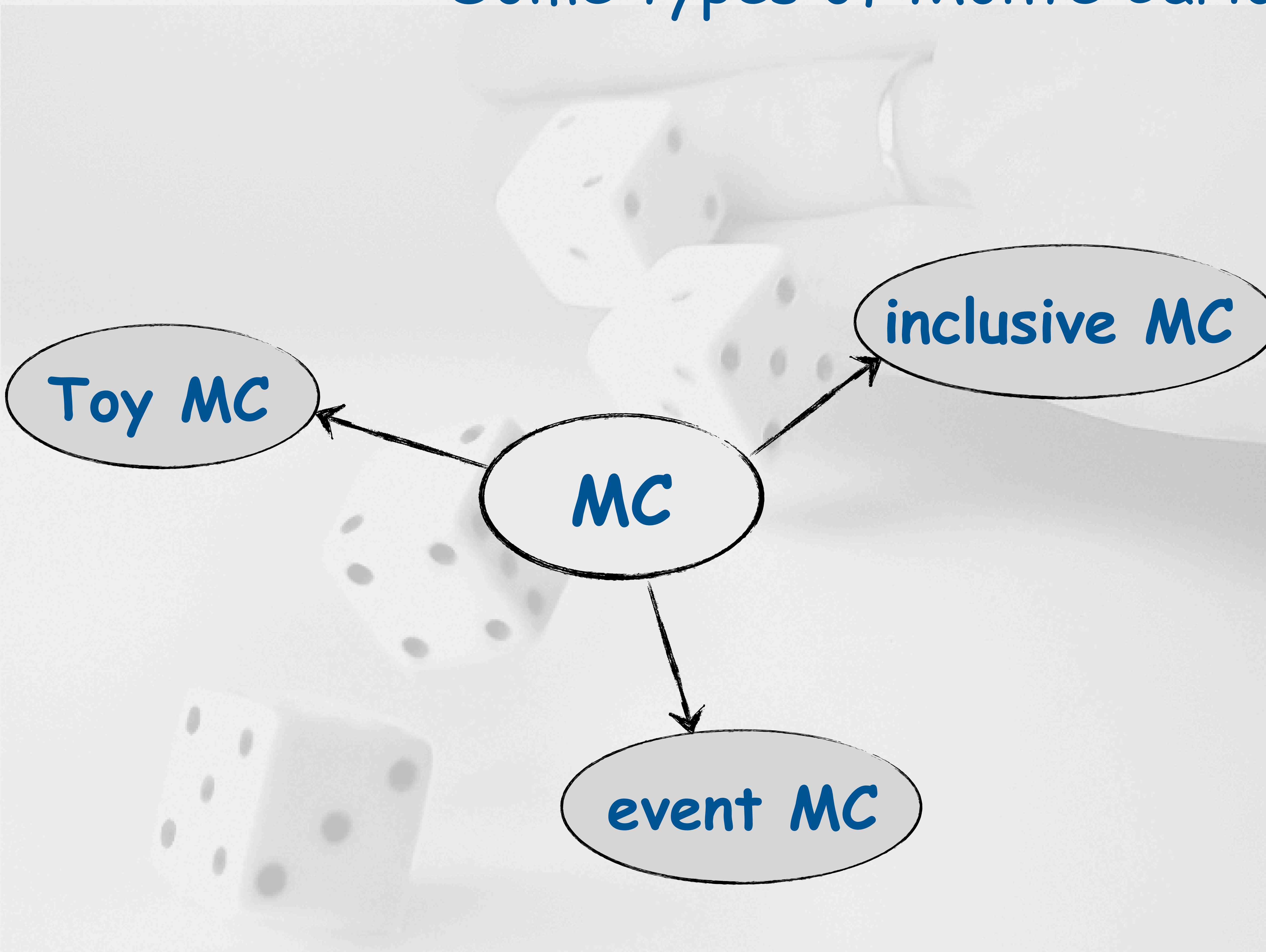
Some usages for Monte Carlo



Some usages for Monte Carlo



Some types of Monte Carlo



Prelude: role of acceptance in experiments

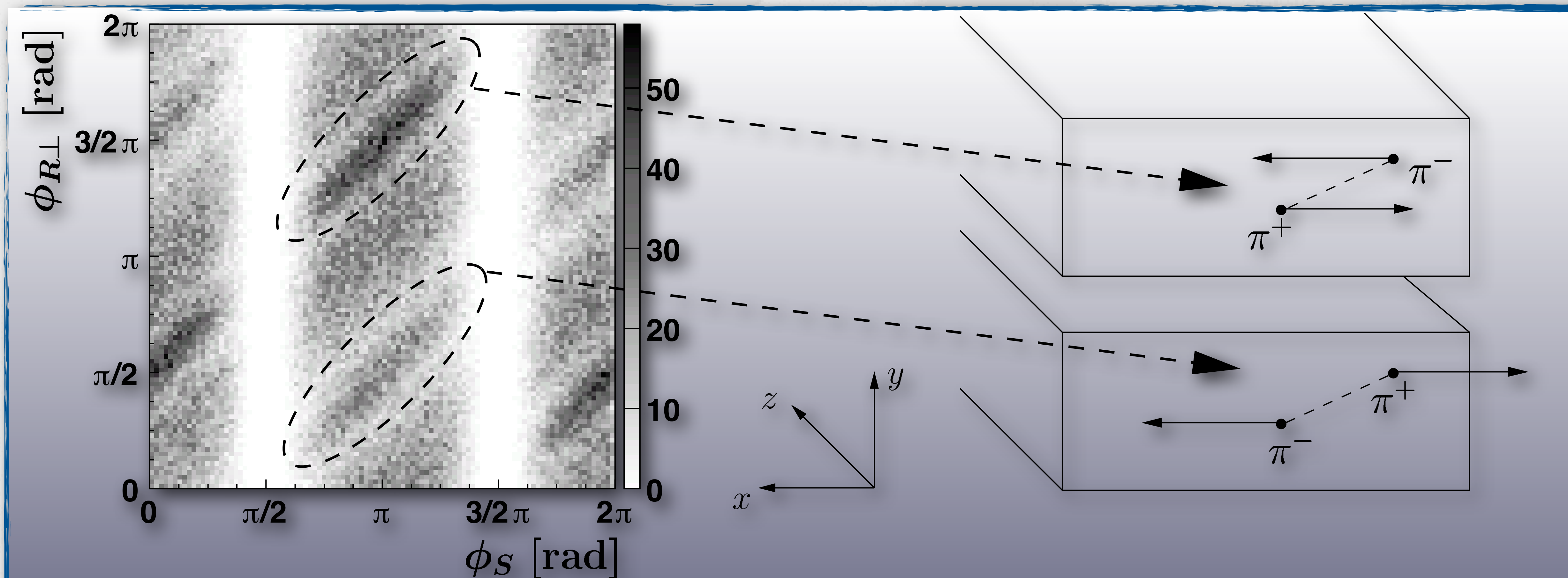
An unfortunate Lemma

- "No particle-physics experiment has a perfect acceptance!"

  - obvious for detectors with gaps/holes
  - but also for "4π", especially when looking at complicated final states

An unfortunate Lemma

- "No particle-physics experiment has a perfect acceptance!"

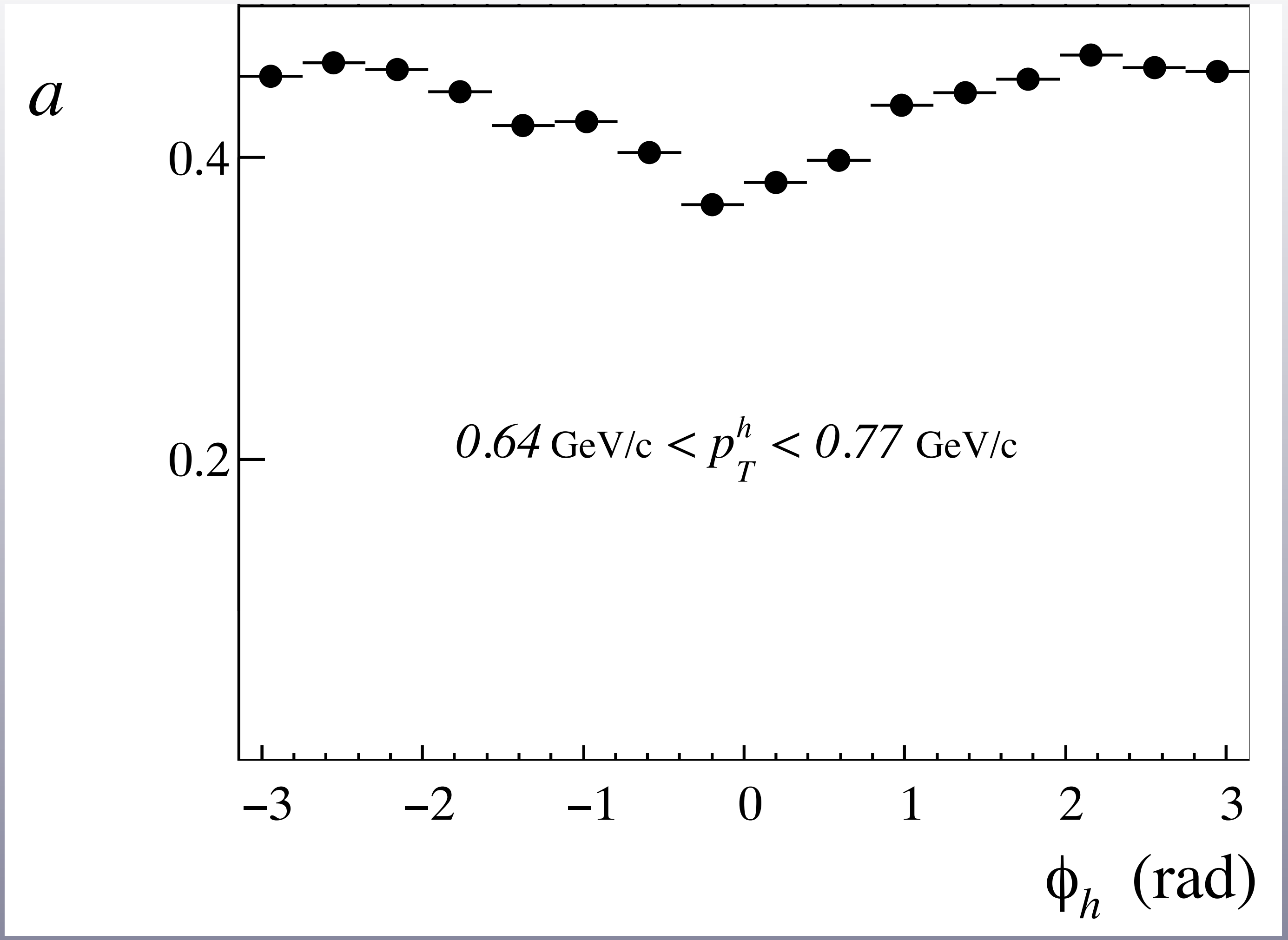


HERMES azimuthal acceptance for 2-hadron production

[P. van der Nat, Ph.D. thesis, Vrije Universiteit (2007)]

An unfortunate Lemma

- "No particle-physics experiment has a perfect acceptance!"



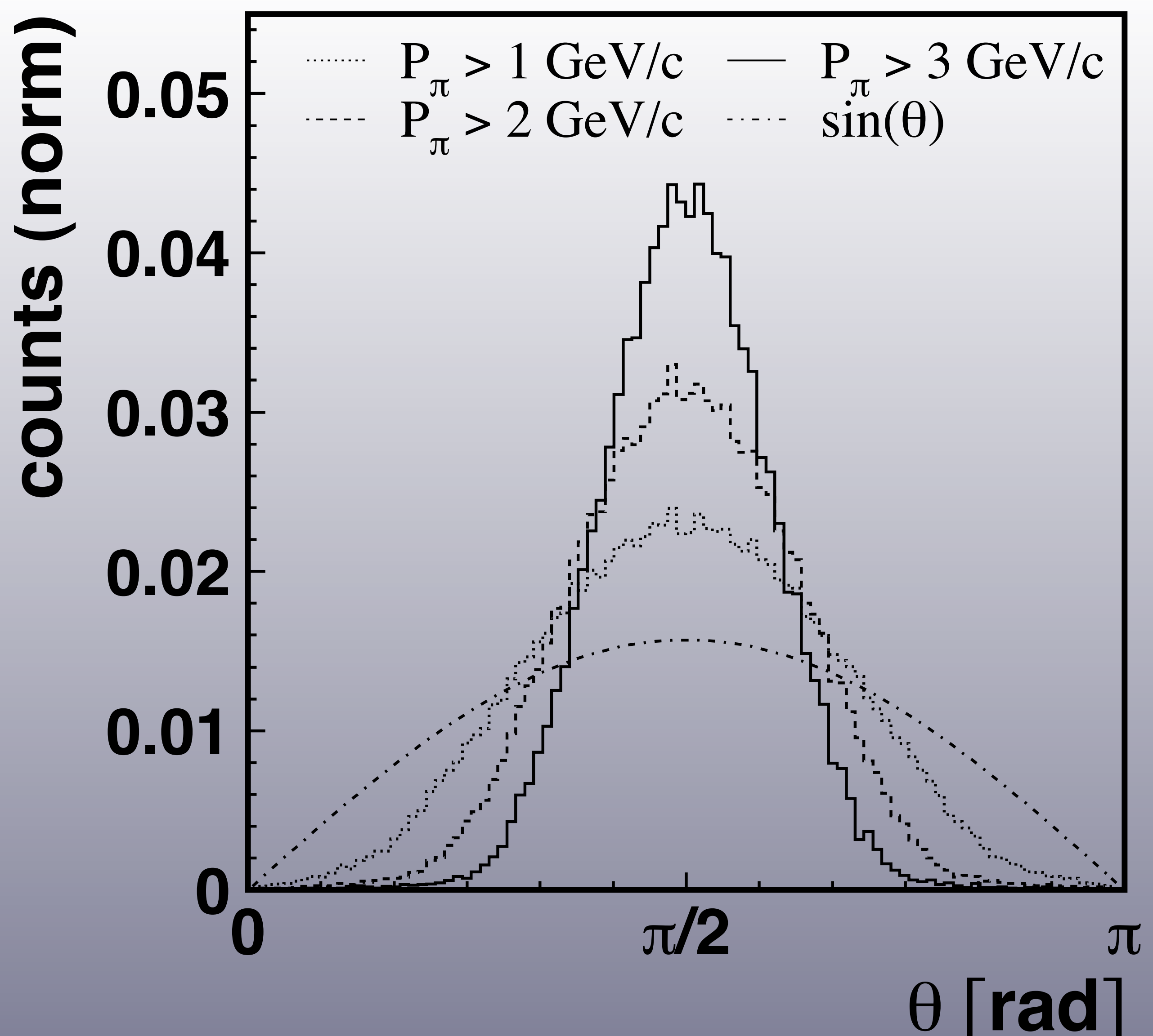
[C. Adolph, arXiv:1401.6284]

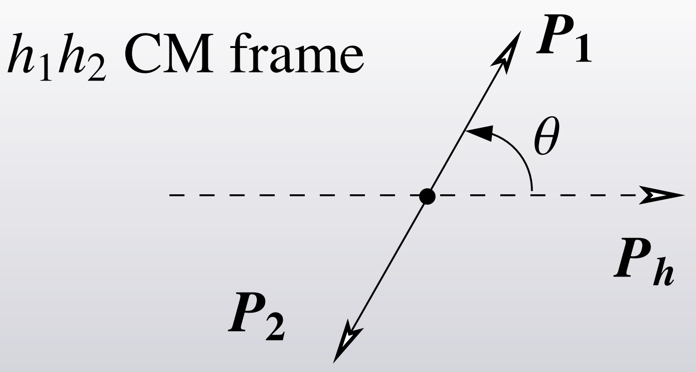
maybe "2π" around beam axis, but not around virtual-photon axis, e.g., because of lower limit on θ

[see also A. Bianconi et al., Eur.Phys.J. A49 (2013) 42]

An unfortunate Lemma

- "No particle-physics experiment has a perfect acceptance!"





momentum cuts strongly distort kinematic distributions even for "4π" geom. acceptance

[P. van der Nat, Ph.D. thesis, Vrije Universiteit (2007)]

An unfortunate Lemma

- "No particle-physics experiment has a perfect acceptance!"
  - obvious for detectors with gaps/holes
  - but also for "4π", especially when looking at complicated final states
- How acceptance effects are handled is one of the essential questions in experiments!

some typical acceptance effects

- acceptance in kinematic variable studied, e.g., azimuthal coverage in extraction of azimuthal moments
  - formally orthogonal modulations become correlated through incomplete acceptance
  - simple example: acceptance ~$\delta(\phi_S)$ cannot distinguish between Collins, Sivers and most other SSA moments
- acceptance in kinematic variables integrated over, e.g., due to limited statistics not being able to do fully differential analysis

a common misconception

- "acceptance cancels in asymmetries"

a common misconception

- "acceptance cancels in asymmetries"

$$A_{UT}(\phi, \Omega) \quad = \quad \frac{\sigma_{UT}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \qquad\qquad \Omega = x, y, z, \ldots$$

a common misconception

- "acceptance cancels in asymmetries"

$$A_{UT}(\phi,\Omega) = \frac{\sigma_{UT}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)} = \frac{\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)}$$

$\Omega = x, y, z, \ldots$

ϵ : **detection efficiency**

a common misconception

- "acceptance cancels in asymmetries"

$$
\begin{aligned}
A_{UT}(\phi,\Omega) &= \frac{\sigma_{UT}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)} \qquad \Omega = x, y, z, \ldots \\
&= \frac{\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)} \qquad \epsilon: \textbf{detection efficiency} \\
&\neq \frac{\int \mathrm{d}\Omega\,\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\int \mathrm{d}\Omega\,\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)} \neq \frac{\int \mathrm{d}\Omega\,\sigma_{UT}(\phi,\Omega)}{\int \mathrm{d}\Omega\,\sigma_{UU}(\phi,\Omega)} \equiv A_{UT}(\phi)
\end{aligned}
$$

a common misconception

- "acceptance cancels in asymmetries"

$$
\begin{aligned}
A_{UT}(\phi,\Omega) &= \frac{\sigma_{UT}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)} \qquad\qquad \Omega = x, y, z, \ldots \\
&= \frac{\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)} \qquad \epsilon : \textbf{detection efficiency} \\
&\neq \frac{\int \mathrm{d}\Omega\,\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\int \mathrm{d}\Omega\,\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)} \neq \frac{\int \mathrm{d}\Omega\,\sigma_{UT}(\phi,\Omega)}{\int \mathrm{d}\Omega\,\sigma_{UU}(\phi,\Omega)} \equiv A_{UT}(\phi)
\end{aligned}
$$

Acceptance does **not cancel** in general when **integrating** numerator and denominator over (large) ranges in kinematic variables!

… geometric acceptance …

extract acceptance from Monte Carlo simulation?

$$\epsilon(\phi, \Omega) \quad = \quad \frac{\epsilon(\phi, \Omega)\sigma_{UU}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \qquad\qquad \Omega = x, y, z, \ldots$$

simulated acceptance
e.g., GEANT

simulated cross section
e.g., PYTHIA

extract acceptance from Monte Carlo simulation?

$$\epsilon(\phi,\Omega) = \frac{\epsilon(\phi,\Omega)\sigma_{UU}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)} \neq \frac{\int \mathrm{d}\Omega\, \sigma_{UU}(\phi,\Omega)\, \epsilon(\phi,\Omega)}{\int \mathrm{d}\Omega\, \sigma_{UU}(\phi,\Omega)}$$

$$\Omega = x, y, z, \ldots$$

"Aus Differenzen und Summen
kürzen nur die Dummen."

extract acceptance from Monte Carlo simulation?

$$\epsilon(\phi,\Omega) = \frac{\epsilon(\phi,\Omega)\sigma_{UU}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)}$$

$$\neq \frac{\int d\Omega\, \sigma_{UU}(\phi,\Omega)\, \epsilon(\phi,\Omega)}{\int d\Omega\, \sigma_{UU}(\phi,\Omega)}$$

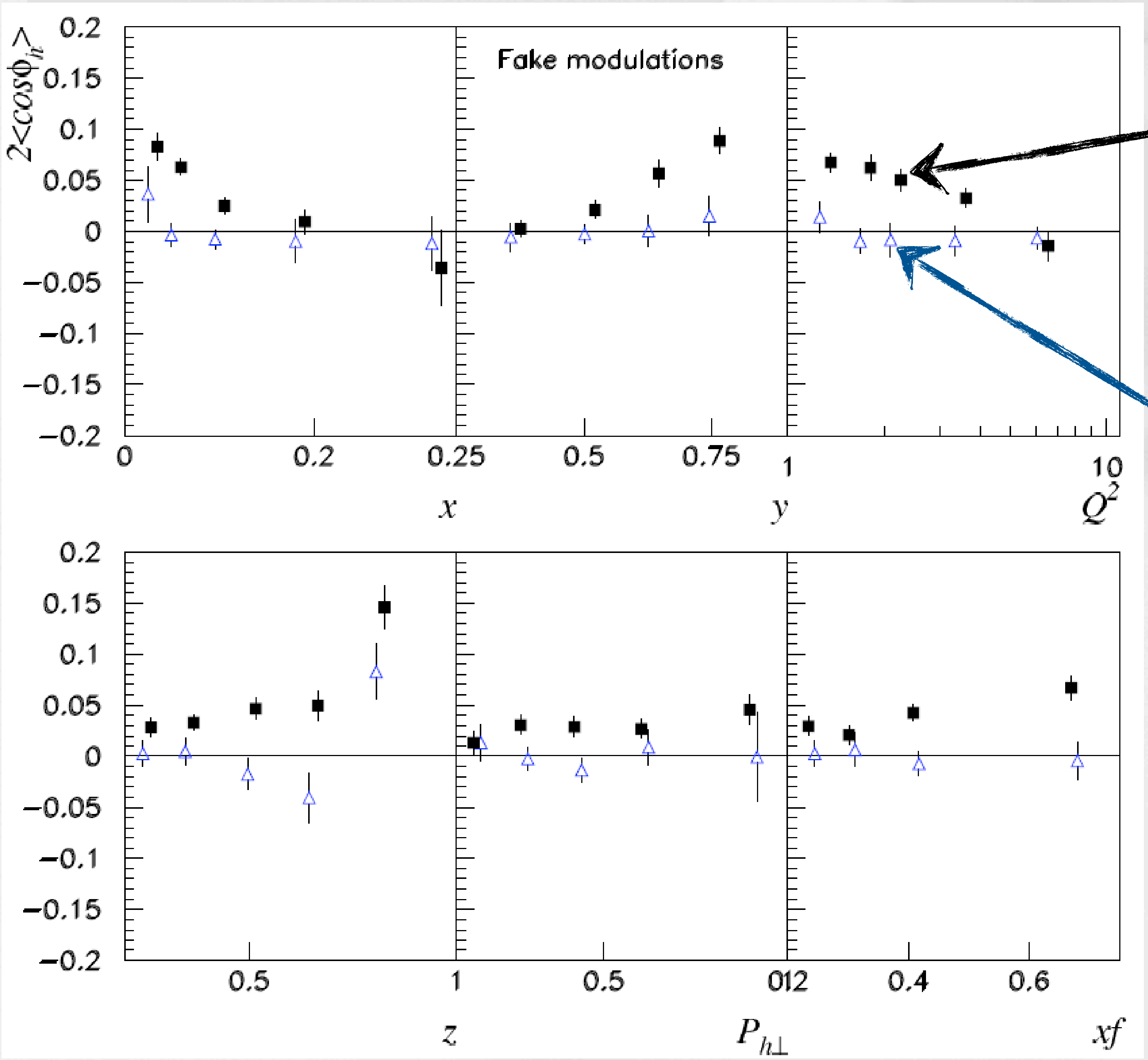
$$\neq \int d\Omega\, \epsilon(\phi,\Omega) \equiv \epsilon(\phi)$$

$\Omega = x, y, z, \ldots$

"Aus Differenzen und Summen kürzen nur die Dummen."

Cross-section model does NOT CANCEL in general when integrating numerator and denominator over (large) ranges in kinematic variables!

“Classique” example: $\langle\cos\phi\rangle_{UU}$



1D correction

(input: MC without azimuthal modulation)

5D correction

[F. Giordano, **Transversity 2008**]

... averaging ...

often enough one has to average observables over available phase space:

properly normalized for simplicity

$$\langle A(\Omega)\rangle_{\epsilon} \equiv \int \mathrm{d}\Omega\, A(\Omega)\epsilon(\Omega)$$

… averaging …

often enough one has to average observables over available phase space:

$$\langle A(\Omega)\rangle_\epsilon \equiv \int \mathrm{d}\Omega\, A(\Omega)\epsilon(\Omega)$$

$$\neq \int \mathrm{d}\Omega\, A(\Omega) \equiv \langle A(\Omega)\rangle_{\text{“}4\pi\text{”}}$$

... averaging ...

often enough one has to average observables over available phase space:

$$\langle A(\Omega)\rangle_\epsilon \equiv \int \mathrm{d}\Omega\, A(\Omega)\epsilon(\Omega)$$

$$\neq \int \mathrm{d}\Omega\, A(\Omega) \equiv \langle A(\Omega)\rangle_{\text{“}4\pi\text{”}}$$

life (of the experimentalist) simplifies if asymmetries are weakly (i.e. not more than linearly) dependent on kinematics:

$$\langle A(\Omega)\rangle_\epsilon = A(\langle\Omega\rangle_\epsilon) \quad \text{for} \quad A(\Omega) = A_0 + A_1\Omega$$

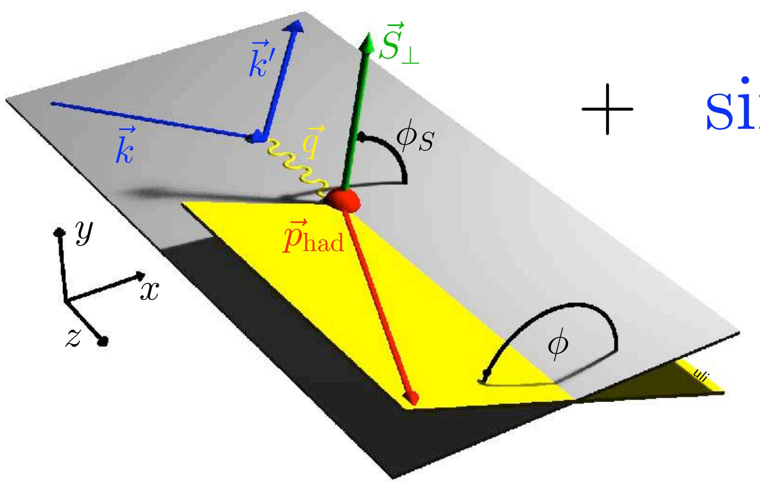
Measuring azimuthal SSA

$$A_{UT}(\phi,\phi_S) = \frac{1}{\langle|S_\perp|\rangle}\frac{N_h^\uparrow(\phi,\phi_S) - N_h^\downarrow(\phi,\phi_S)}{N_h^\uparrow(\phi,\phi_S) + N_h^\downarrow(\phi,\phi_S)}$$

$$\sim \sin(\phi+\phi_S)\sum_q e_q^2\,\mathcal{I}\left[\frac{\boldsymbol{k_T}\hat{\boldsymbol{P}}_{\boldsymbol{h}\perp}}{M_h} h_1^q(x,p_T^2) H_1^{\perp,q}(z,k_T^2)\right]$$

$$+ \sin(\phi-\phi_S)\sum_q e_q^2\,\mathcal{I}\left[\frac{\boldsymbol{p_T}\hat{\boldsymbol{P}}_{\boldsymbol{h}\perp}}{M} f_{1T}^{\perp,q}(x,p_T^2) D_1^q(z,k_T^2)\right]$$

$$+ \quad \cdots$$



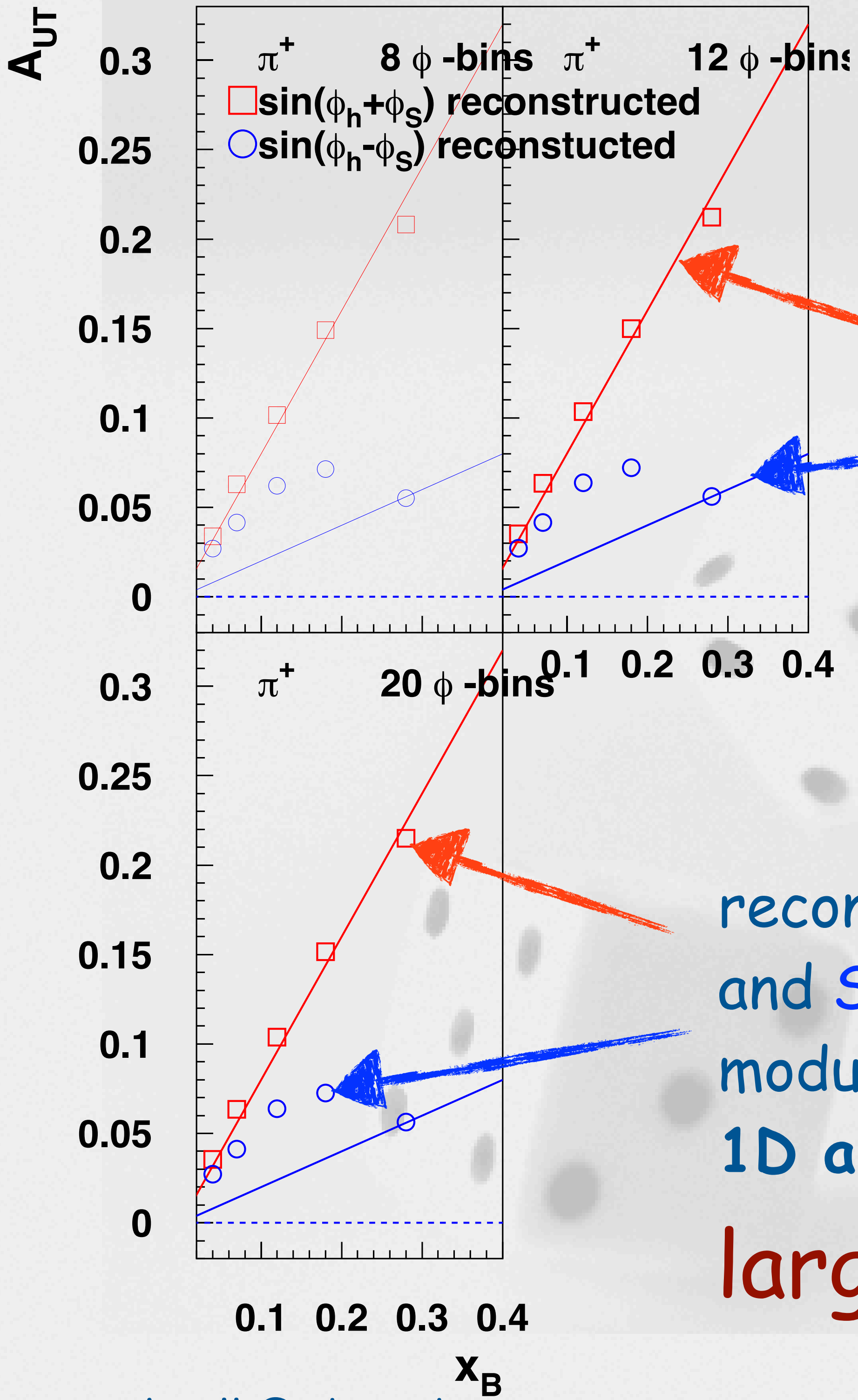
$\mathcal{I}[\ldots]$: convolution integral over initial (p_T) and final (k_T) quark transverse momenta

$\Rightarrow$ 2D Max.Likelihd. fit of to get Collins and Sivers amplitudes:

$$PDF(2\langle\sin(\phi\pm\phi_S)\rangle_{UT},\ldots,\phi,\phi_S) = \tfrac{1}{2}\{1 + P_T(2\langle\sin(\phi\pm\phi_S)\rangle_{UT}\,\sin(\phi\pm\phi_s) + \ldots)\}$$

1D vs. 2D fitting

- limited acceptance introduces correlations to originally orthogonal azimuthal Fourier amplitudes



input models for Collins- and Sivers-type modulations

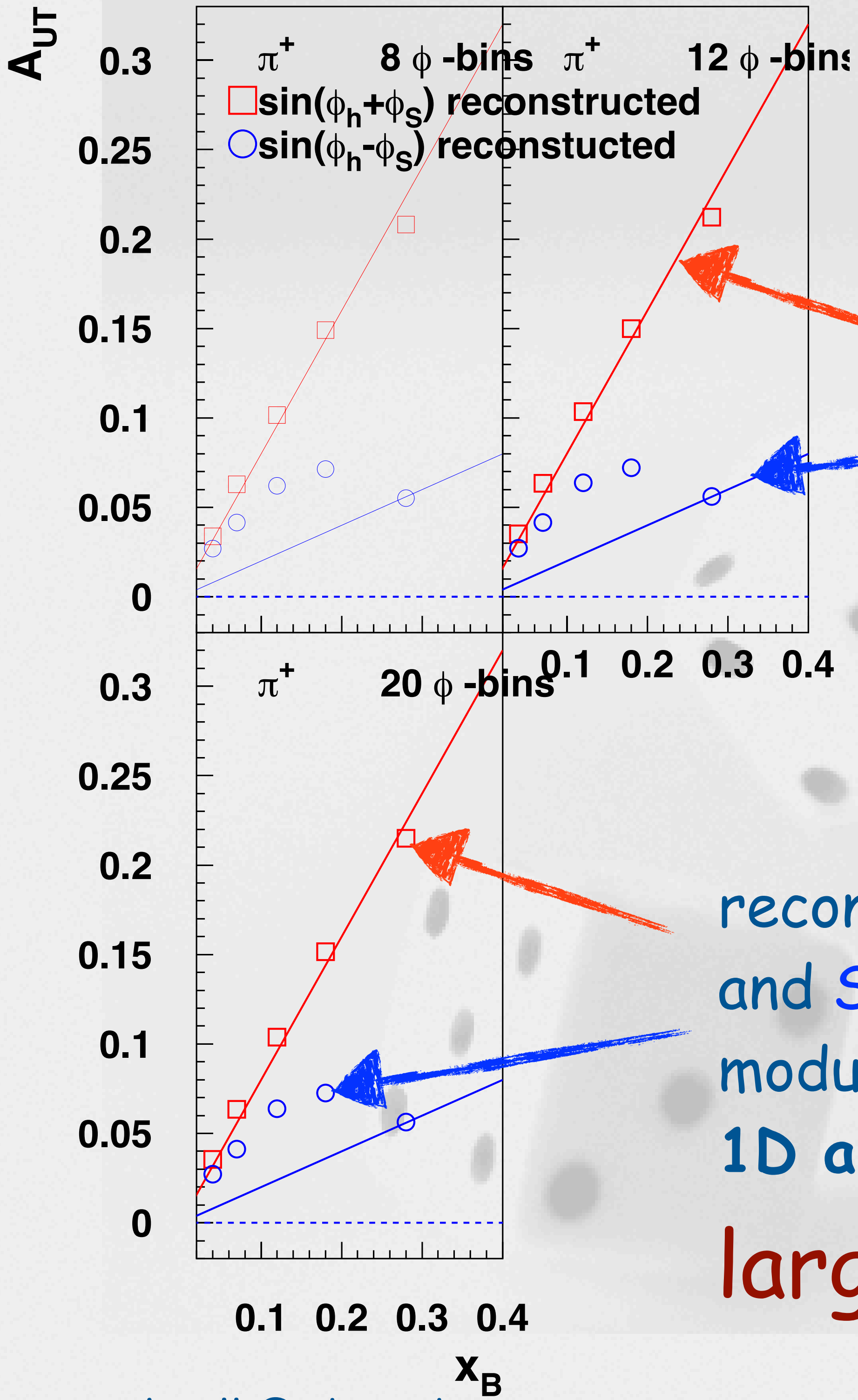
reconstructed Collins- and Sivers-type modulations

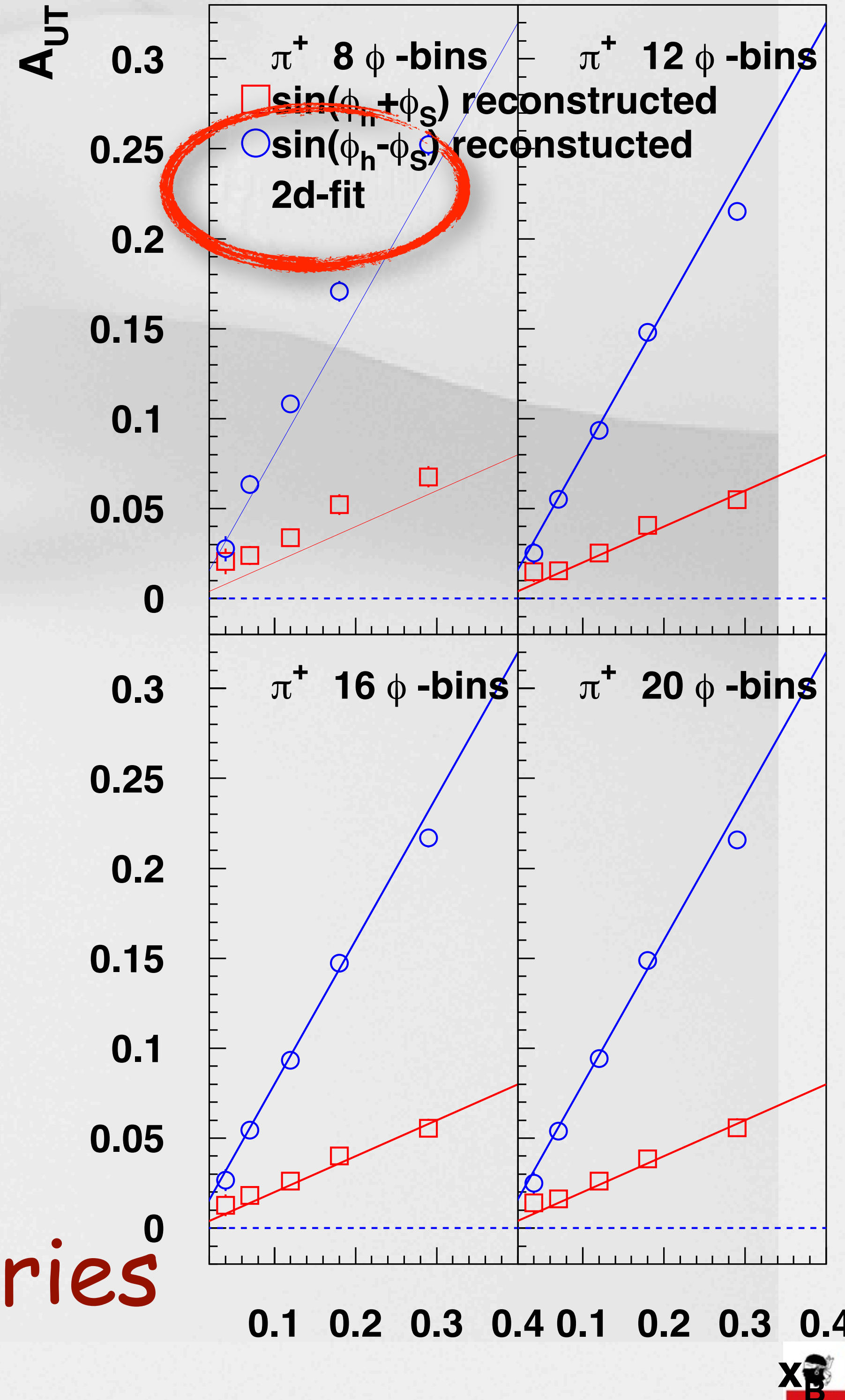
1D analysis

large false asymmetries

1D vs. 2D fitting

- limited acceptance introduces correlations to originally orthogonal azimuthal Fourier amplitudes

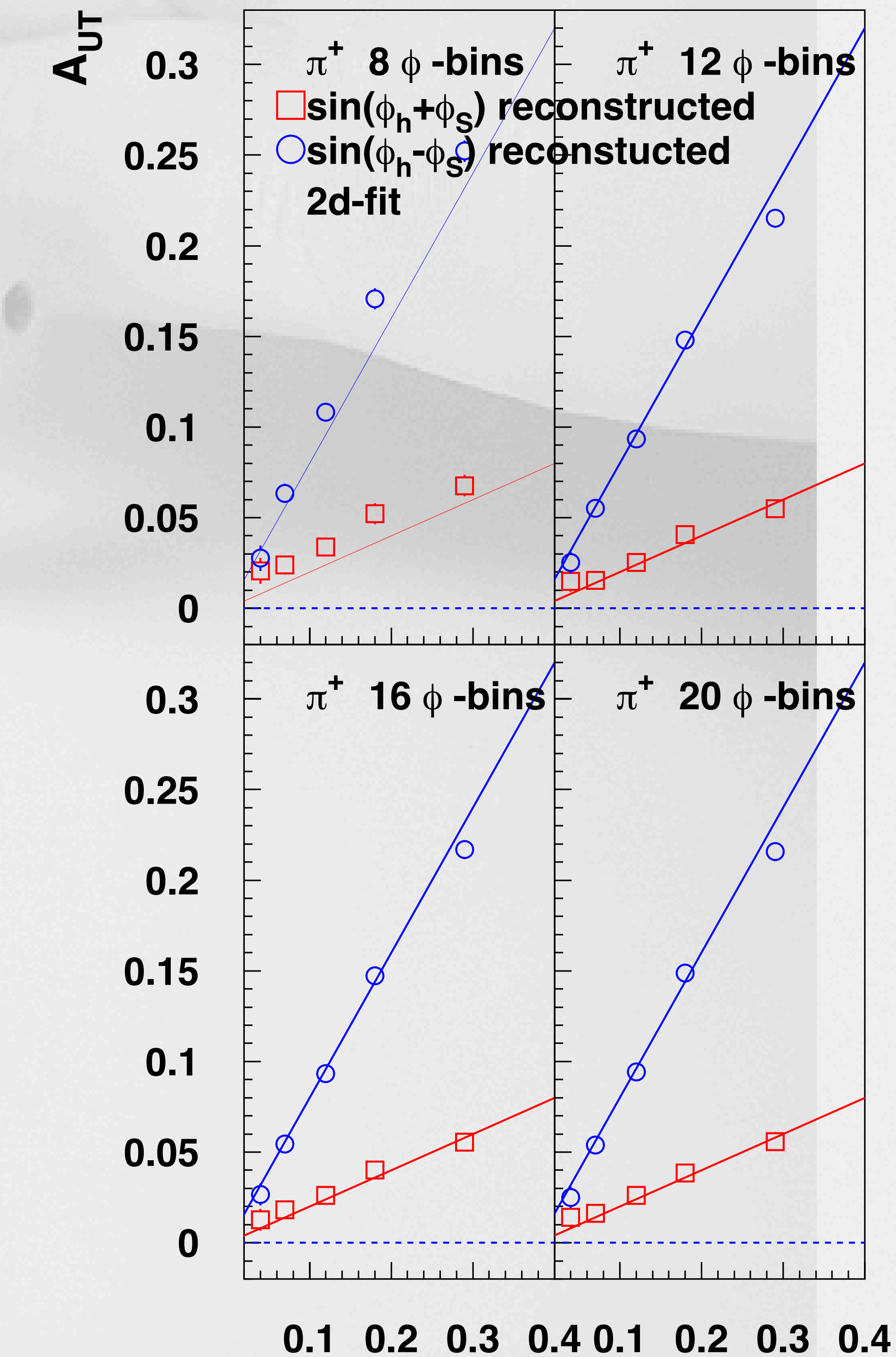




large false asymmetries

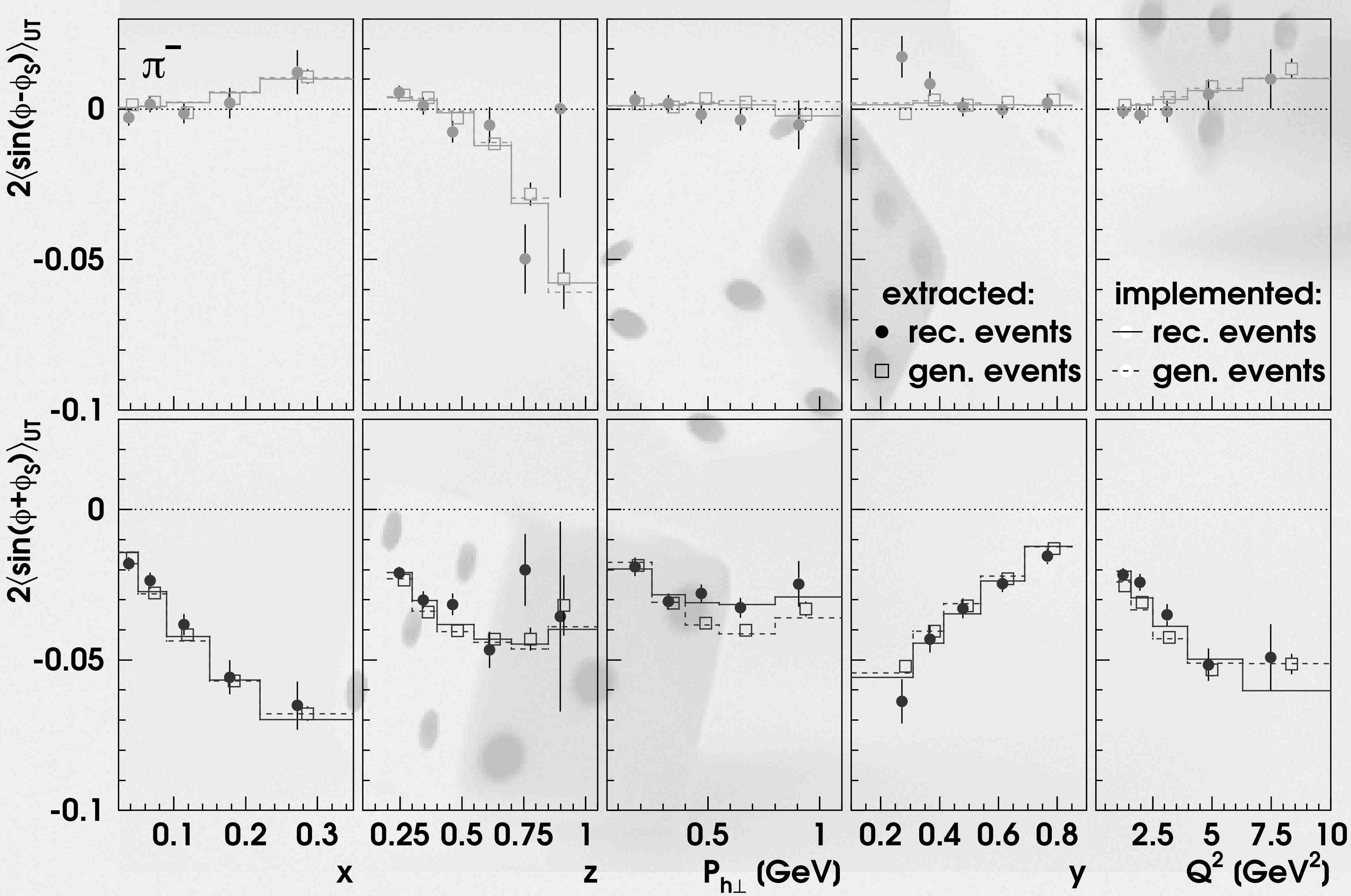
choice of models

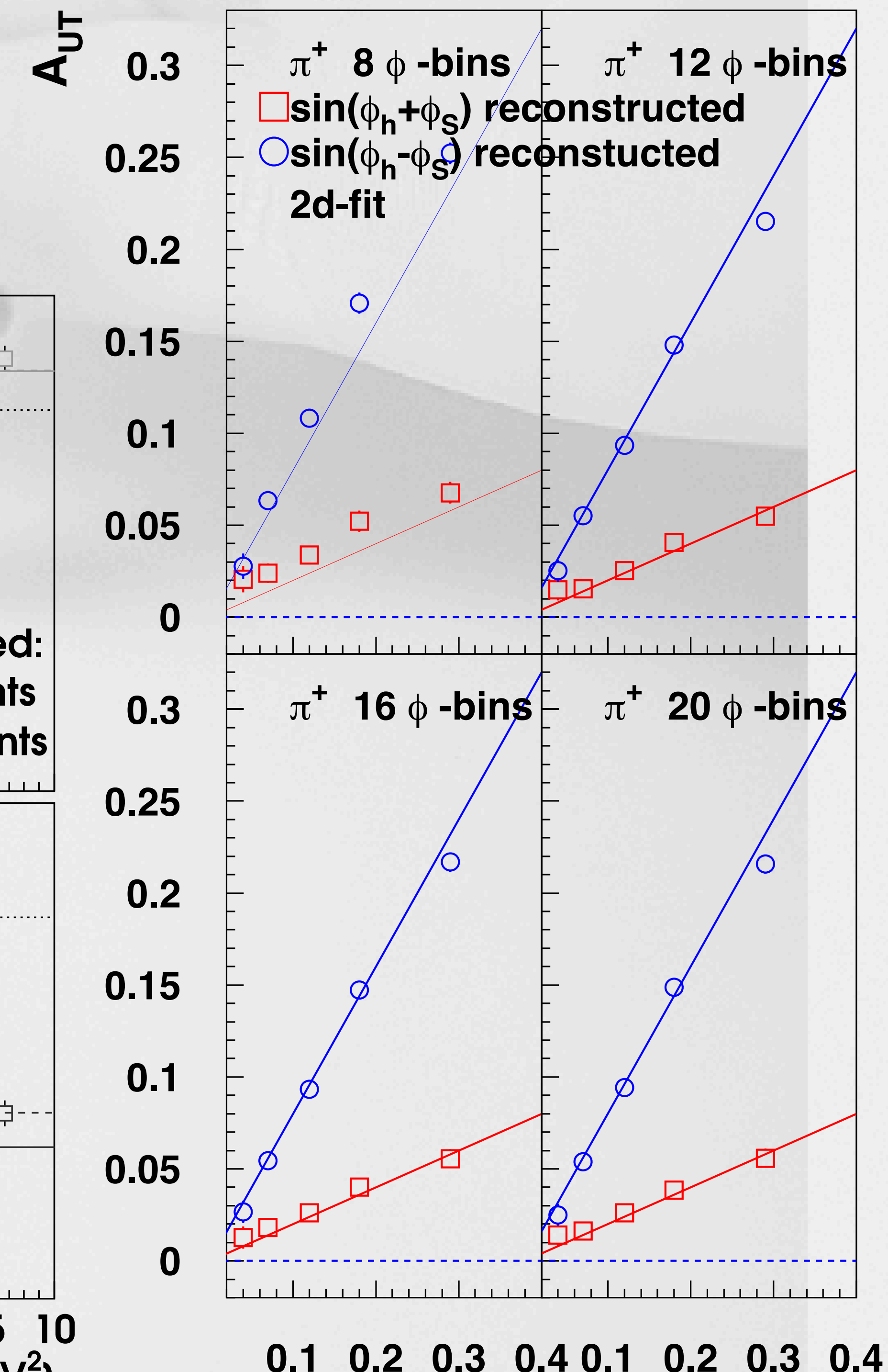
- linear dependence kind of trivial to reproduce (see earlier slide)



choice of models

- linear dependence kind of trivial to reproduce (see earlier slide)
- need more realistic model, e.g., GMCTRANS





GMCTRANS - a TMD MC generator

Initial goals

- physics generator for SIDIS pion production
- include transverse-momentum dependence, in particular simulate Collins and Sivers effects
- be fast
- allow comparison of input model and reconstructed amplitudes
- to be used with standard HERMES Monte Carlo
- be extendable (e.g., open for new models)

Basic workings

- use cross section that can be calculated analytically
- do not simulate full event
- start from 1-hadron SIDIS expressions of Mulders & Tangerman (Nucl.Phys.B461:197-237,1996) and others
- use Gaussian Ansatz for all transverse-momentum dependences of DFs and FFs
- unpolarized DFs (as well as helicity distribution) and FFs from fits/parametrizations (e.g., Kretzer FFs etc.)
- "polarized" DFs and FFs either related to unpolarized ones (e.g., saturation of Soffer bound for transversity)or some parametrizations used

SIDIS Cross Section incl. TMDs

$$d\sigma_{UT} \equiv d\sigma_{UT}^{\text{Collins}} \cdot \sin(\phi + \phi_S) + d\sigma_{UT}^{\text{Sivers}} \cdot \sin(\phi - \phi_S)$$

$$d\sigma_{UT}^{\text{Collins}}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} B(y) \sum_q e_q^2 \, \mathcal{I}\left[\left(\frac{k_T \cdot \hat{P}_{h\perp}}{M_h}\right) \cdot h_1^q H_1^{\perp q}\right]$$

$$d\sigma_{UT}^{\text{Sivers}}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2 \, \mathcal{I}\left[\left(\frac{p_T \cdot \hat{P}_{h\perp}}{M_N}\right) \cdot f_{1T}^{\perp q} D_1^q\right]$$

$$d\sigma_{UU}(x, y, z, \phi_S, P_{h\perp}) \equiv \frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2 \, \mathcal{I}\left[f_1^q D_1^q\right]$$

where

$$\mathcal{I}\left[\mathcal{W}\, f\, D\right] \equiv \int d^2 p_T\, d^2 k_T\, \delta^{(2)}\left(p_T - \frac{P_{h\perp}}{z} - k_T\right) \left[\mathcal{W}\, f(x, p_T)\, D(z, k_T)\right]$$

Gaussian Ansatz

- want to deconvolve convolution integral over transverse momenta
- easy Ansatz: Gaussian dependences of DFs and FFs on intrinsic (quark) transverse momentum:

$$\mathcal{I}[f_1(x, \boldsymbol{p_T^2}) D_1(z, z^2 \boldsymbol{k_T^2})] = f_1(x) \cdot D_1(z) \cdot \frac{R^2}{\pi z^2} \cdot e^{-R^2 \frac{P_{h\perp}^2}{z^2}}$$

with $\boldsymbol{f_1(x, p_T^2) = f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}}$ $\qquad \frac{1}{R^2} \equiv \langle k_T^2 \rangle + \langle p_T^2 \rangle = \frac{\langle P_{h\perp}^2 \rangle}{z^2}$

(similar: $D_1(z, z^2 \boldsymbol{k_T^2})$)

Caution: different notations for intrinsic transverse momenta exist! (Here: "Amsterdam notation")

Positivity Constraints

- DFs (FFs) have to fulfill various positivity constraints (resulting cross section has to be positive!)
- based on probability considerations one can derive positivity limits for leading-twist functions: Bacchetta et al., Phys. Rev. Lett. 85 (2000) 712-715

➡ transversity: e.g., Soffer bound

➡ Sivers and Collins functions: e.g., loose bounds:

$$\frac{|p_T|}{2M_N} f_{1T}^{\perp}(x, p_T^2) \equiv f_{1T}^{\perp(1/2)}(x, p_T^2) \leq \frac{1}{2} f_1(x, p_T^2)$$

$$\frac{|k_T|}{2M_h} H_1^{\perp}(z, z^2 k_T^2) \equiv H_1^{\perp(1/2)}(z, z^2 k_T^2) \leq \frac{1}{2} D_1(z, z^2 k_T^2)$$

Positivity and the Gaussian Ansatz

$$\frac{|\boldsymbol{p_T}|}{2M_N} f_{1T}^{\perp}(x, \boldsymbol{p_T^2}) \;\leq\; \frac{1}{2} f_1(x, \boldsymbol{p_T^2})$$

with
$$f_1(x, p_T^2) \;=\; f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}$$

$$f_{1T}^{\perp}(x, p_T^2) \;=\; f_{1T}^{\perp}(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}$$

$$\Rightarrow \quad |p_T| f_{1T}^{\perp}(x) \leq M_N f_1(x)$$

Positivity and the Gaussian Ansatz

$$\frac{|\boldsymbol{p_T}|}{2M_N} f_{1T}^{\perp}(x, \boldsymbol{p_T^2}) \;\leq\; \frac{1}{2} f_1(x, \boldsymbol{p_T^2})$$

with
$$\begin{aligned} f_1(x, p_T^2) &= f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}} \\ f_{1T}^{\perp}(x, p_T^2) &= f_{1T}^{\perp}(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}} \end{aligned}$$

$$\Rightarrow \quad |p_T| f_{1T}^{\perp}(x) \leq M_N f_1(x)$$

No (useful) solution for non-zero Sivers function!

Modify Gaussian width

$$f_{1T}^{\perp}(x, p_T^2) = f_{1T}^{\perp}(x) \, \frac{1}{(1-C)\pi\langle p_T^2\rangle} \, e^{-\frac{p_T^2}{(1-C)\langle p_T^2\rangle}}$$

➡ **positivity limit:**

$$f_{1T}^{\perp}(x) \, \frac{|p_T|}{2M_N} \, \frac{1}{\pi(1-C)\langle p_T^2\rangle} \, e^{-\frac{p_T^2}{(1-C)\langle p_T^2\rangle}} \leq 1/2 \, f_1(x) \, \frac{1}{\pi\langle p_T^2\rangle} \, e^{-\frac{p_T^2}{\langle p_T^2\rangle}}$$

➡ $$\frac{|p_T|}{1-C} \, e^{-\frac{C}{1-C}\frac{p_T^2}{\langle p_T^2\rangle}} \leq M_N \frac{f_1(x)}{f_{1T}^{\perp}(x)}$$

SIDIS Cross Section incl. TMDs

$$\sum_q \frac{e_q^2}{4\pi} \frac{\alpha^2}{(MExyz)^2} \left[X_{UU} + |\mathbf{S}_T| X_{SIV} \sin(\phi_h - \phi_s) + |\mathbf{S}_T| X_{COL} \sin(\phi_h + \phi_s)\right]$$

using Gaussian Ansatz for transverse-momentum dependence of DFs and FFs:

$$X_{UU} = R^2 e^{-R^2 P_{h\perp}^2/z^2} \left(1 - y + \frac{y^2}{2}\right) f_1(x) \cdot D_1(z)$$

$$X_{COL} = + \frac{|P_{h\perp}|}{M_\pi z} \frac{(1-C)\langle k_T^2\rangle}{\left[\langle p_T^2\rangle + (1-C)\langle k_T^2\rangle\right]^2} \exp\left[-\frac{P_{h\perp}^2/z^2}{\langle p_T^2\rangle + (1-C)\langle k_T^2\rangle}\right]$$
$$\times \quad (1-y) \cdot h_1(x) \cdot H_1^\perp(z)$$

$$X_{SIV} = - \frac{|P_{h\perp}|}{M_p z} \frac{(1-C')\langle p_T^2\rangle}{\left[\langle k_T^2\rangle + (1-C')\langle p_T^2\rangle\right]^2} \exp\left[-\frac{P_{h\perp}^2/z^2}{\langle k_T^2\rangle + (1-C')\langle p_T^2\rangle}\right]$$
$$\times \quad \left(1 - y + \frac{y^2}{2}\right) f_{1T}^\perp(x) \cdot D_1(z)$$

Example: Sivers (azimuthal) moments

use cross section expressions to evaluate azimuthal moments:

$$-\langle \sin(\phi-\phi_s)\rangle_{UT} = \frac{\sqrt{(1-C)\langle p_T^2\rangle}}{\sqrt{(1-C)\langle p_T^2\rangle + \langle k_T^2\rangle}} \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1/2)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

$$-\langle \sin(\phi-\phi_s)\rangle_{UT} = \frac{M_N\sqrt{\pi}}{2\sqrt{(1-C)\langle p_T^2\rangle + \langle k_T^2\rangle}} \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

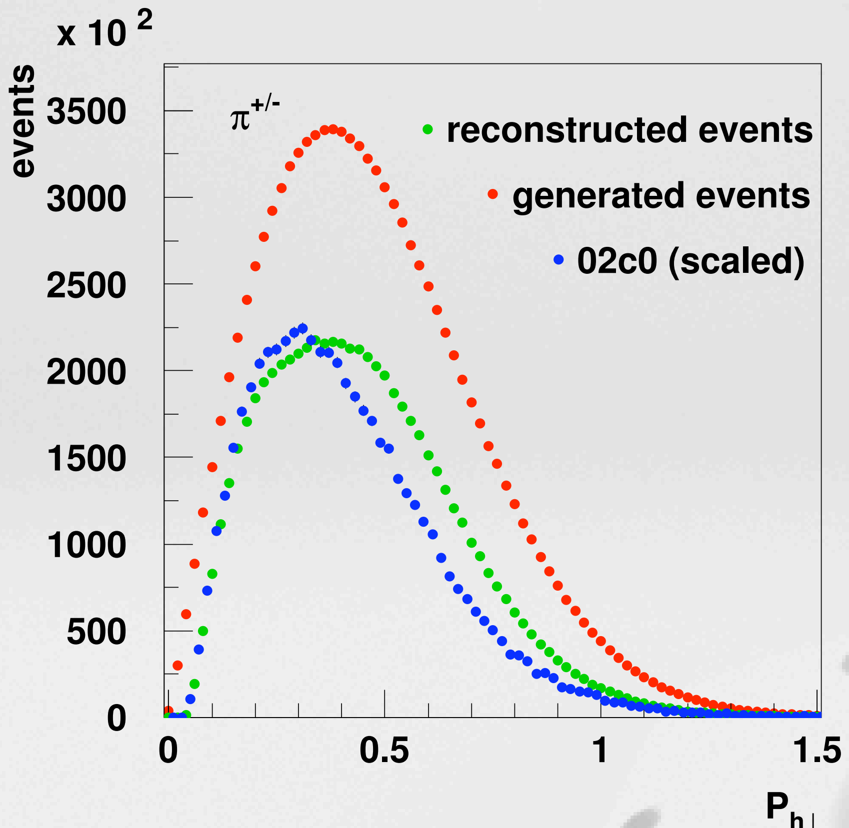
$$-\langle \frac{|P_{h\perp}|}{zM_N}\sin(\phi-\phi_s)\rangle_{UT} = \frac{2\sqrt{(1-C)\langle p_T^2\rangle}}{M_N\sqrt{\pi}} \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1/2)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

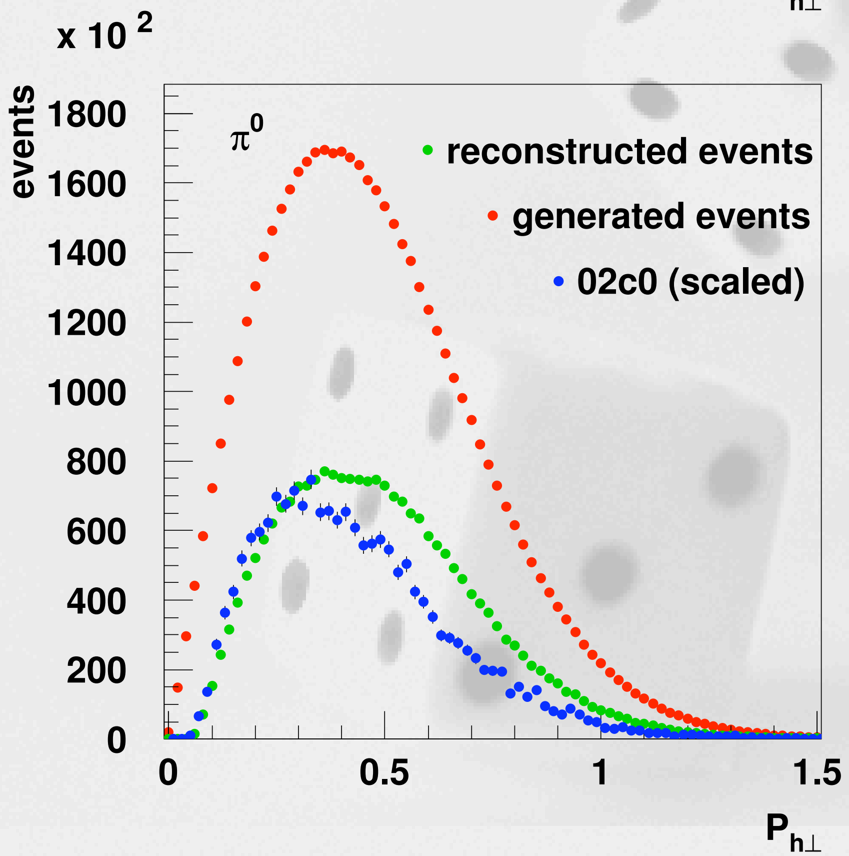
$$-\langle \frac{|P_{h\perp}|}{zM_N}\sin(\phi-\phi_s)\rangle_{UT} = \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

model-dependence on transverse momenta "swallowed" by p_T^2 - moment of Sivers fct.: $f_{1T}^{\perp(1)}$

Selected results

Tuning the Gaussians in gmc_trans





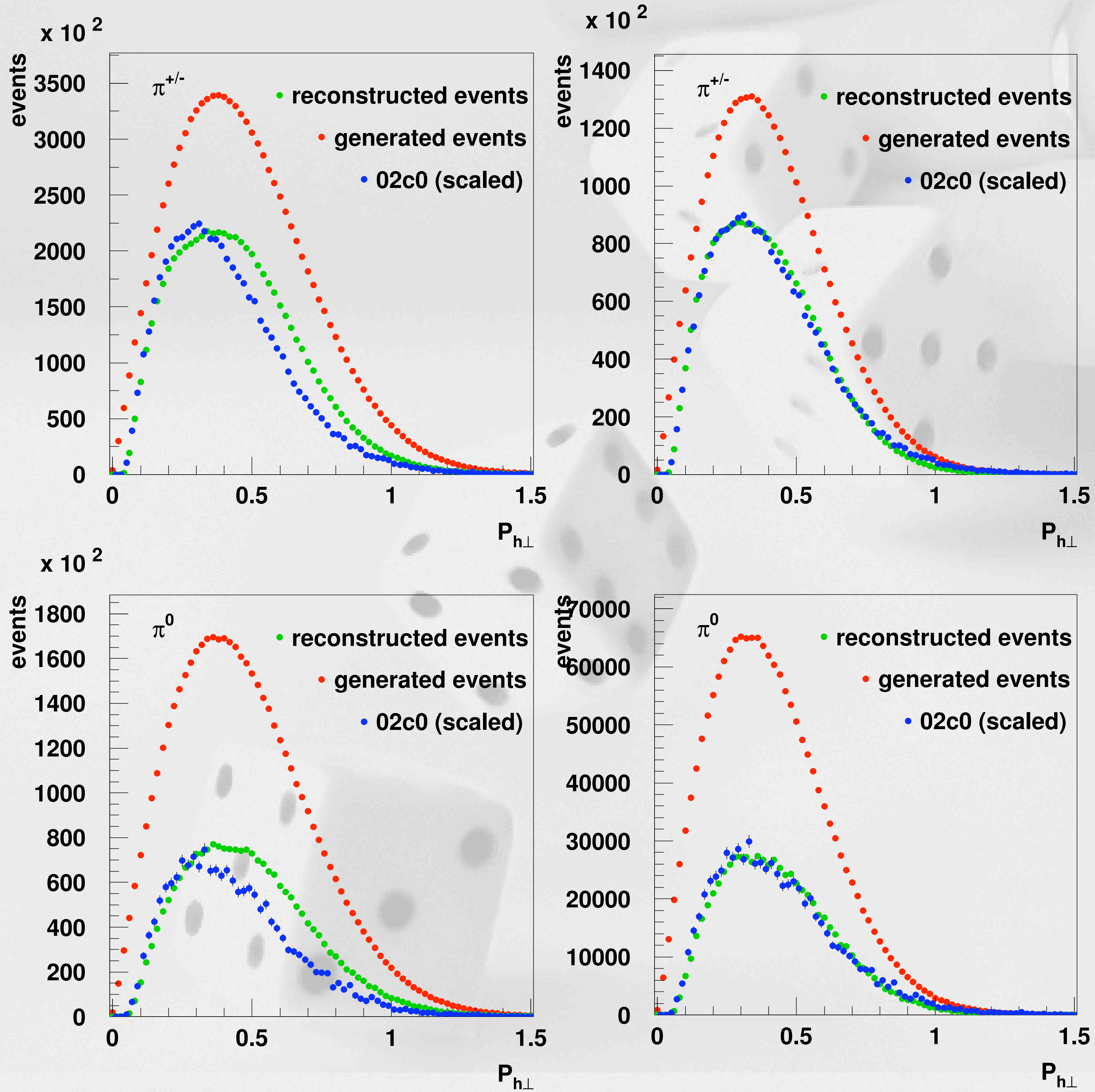
- constant Gaussian widths, i.e., no dependence on x or z:

$$\langle p_T \rangle = 0.44$$

$$\langle K_T \rangle = 0.44$$

- tune to data integrated over whole kinematic range

Tuning the Gaussians in gmc_trans



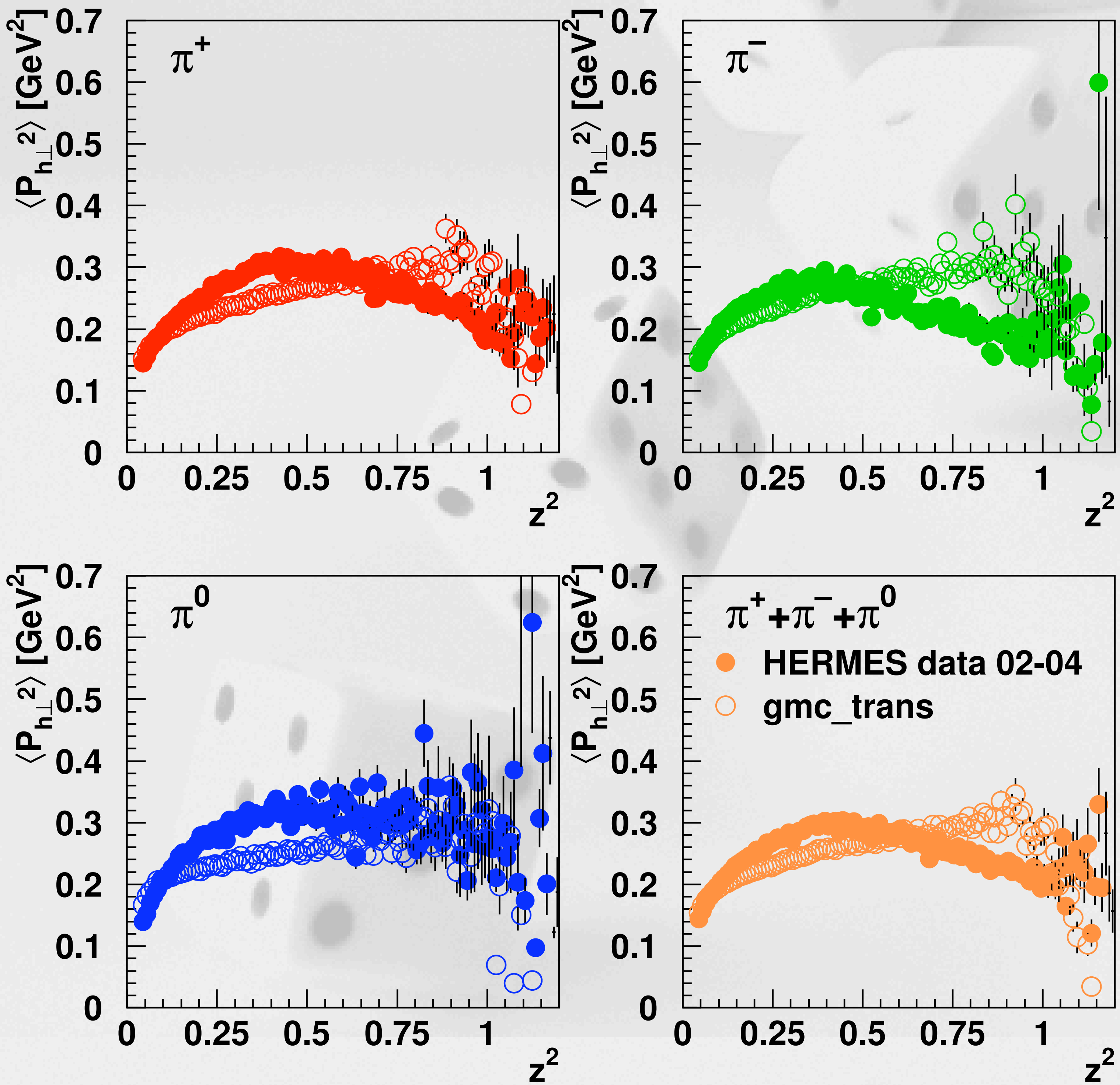
Better:

$\langle p_T \rangle = 0.38$

$\langle K_T \rangle = 0.38$

Tuning the Gaussians in gmc_trans

so far: $\langle P_{h\perp}^2(z) \rangle = z^2 \langle p_T^2 \rangle + \langle K_T^2 \rangle$



$\langle p_T \rangle = 0.38$

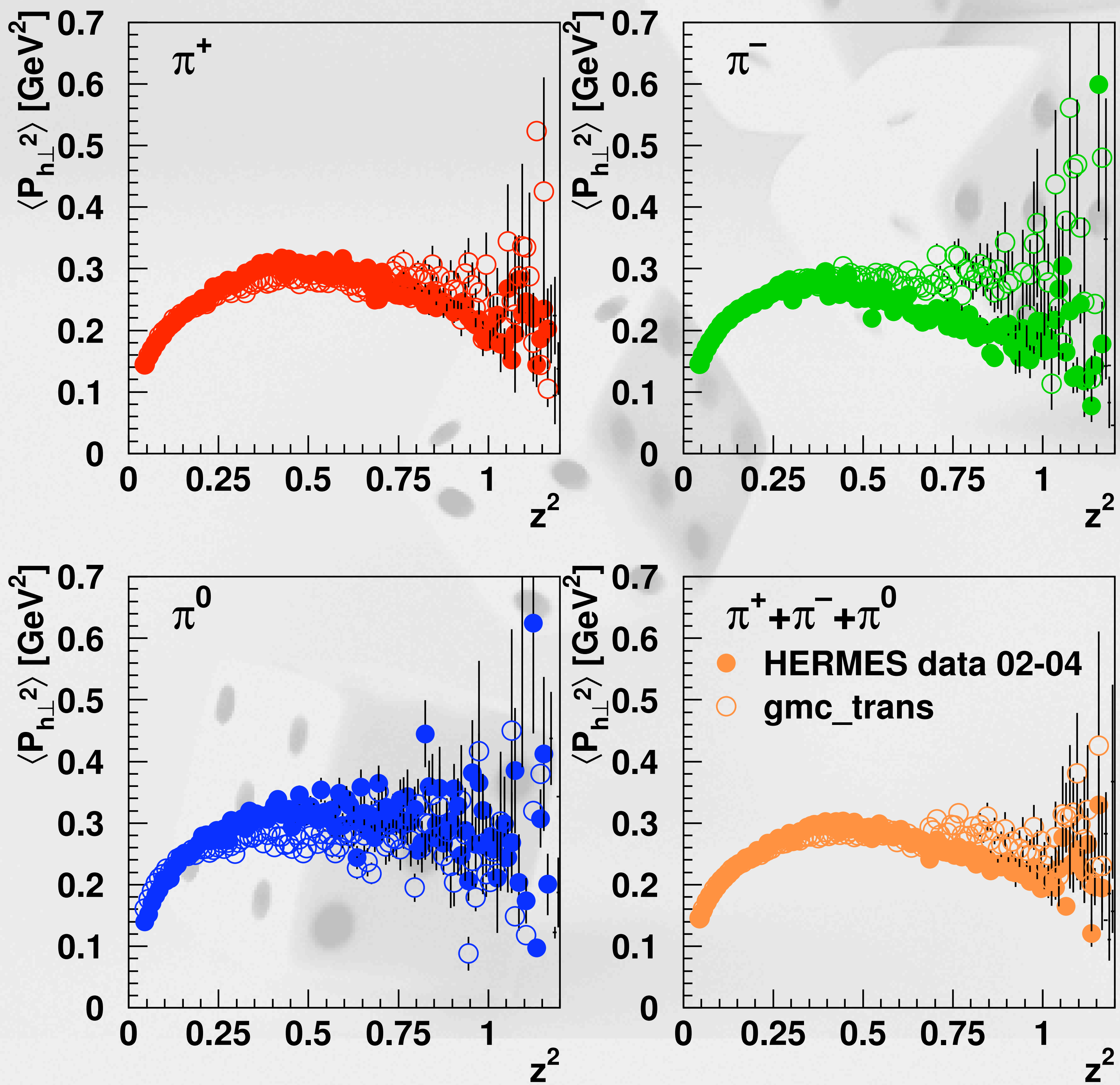
$\langle K_T \rangle = 0.38$

$\langle p_T^2 \rangle \simeq 0.185$

$\langle K_T^2 \rangle \simeq 0.185$

Tuning the Gaussians in gmc_trans

$$\left\langle P_{h\perp}^2(z) \right\rangle = z^2 \left\langle p_T^2 \right\rangle + \left\langle K_T^2(z) \right\rangle$$

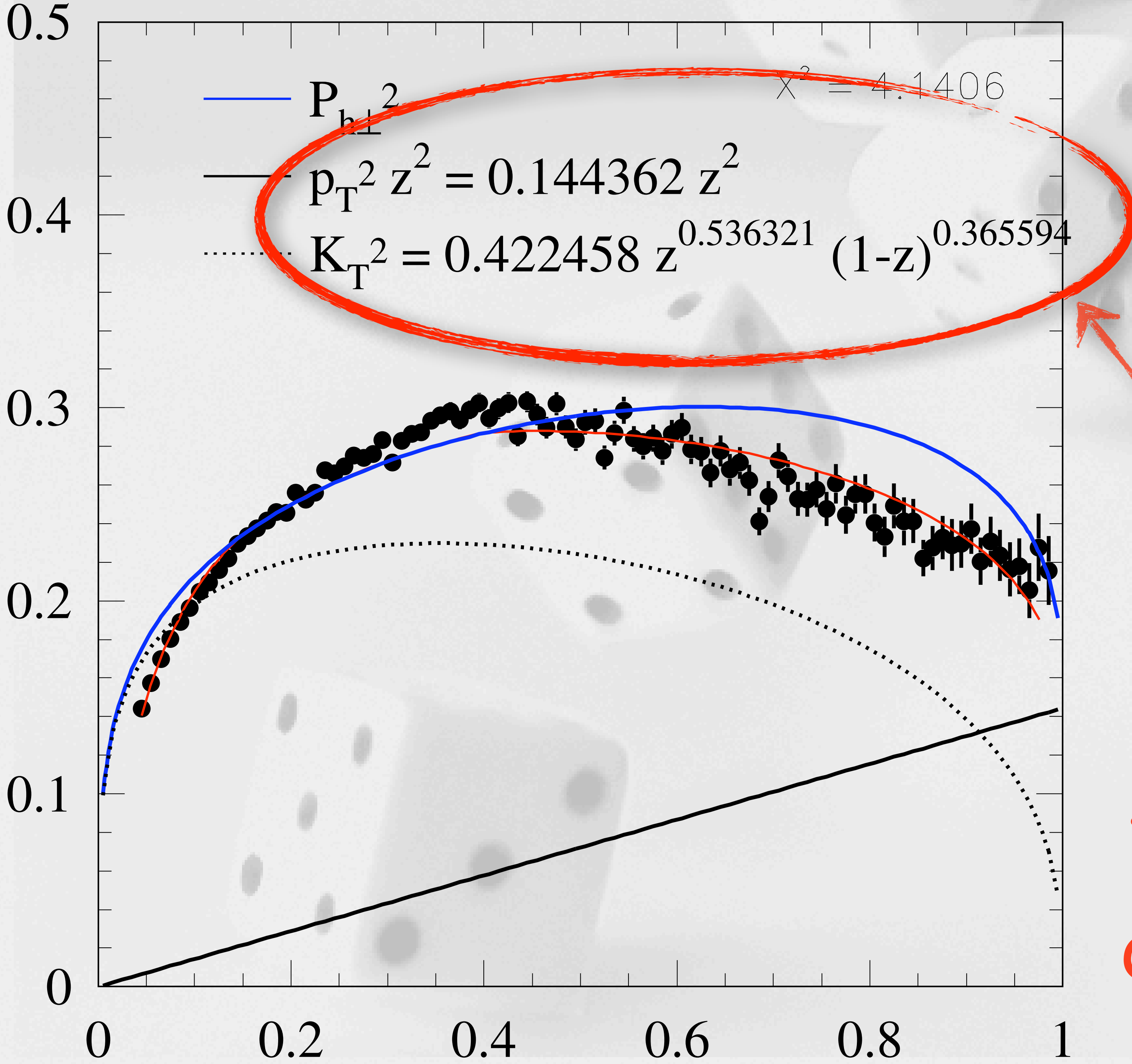


z-dependent!

“Hashi set”

Tuning the Gaussians in gmc_trans

now: $\langle P_{h\perp}^2(z) \rangle = z^2 \langle p_T^2 \rangle + \langle K_T^2(z) \rangle$



z-dependent!

"Hashi set"

tuned to HERMES data in acceptance

Some rather simple models for transversity & friends

$$\delta u(x) = 0.7 \cdot \Delta u(x) \qquad f_{1T}^{\perp u}(x) = -0.3 \cdot u(x)$$

$$\delta d(x) = 0.7 \cdot \Delta d(x) \qquad f_{1T}^{\perp d}(x) = 0.9 \cdot d(x)$$

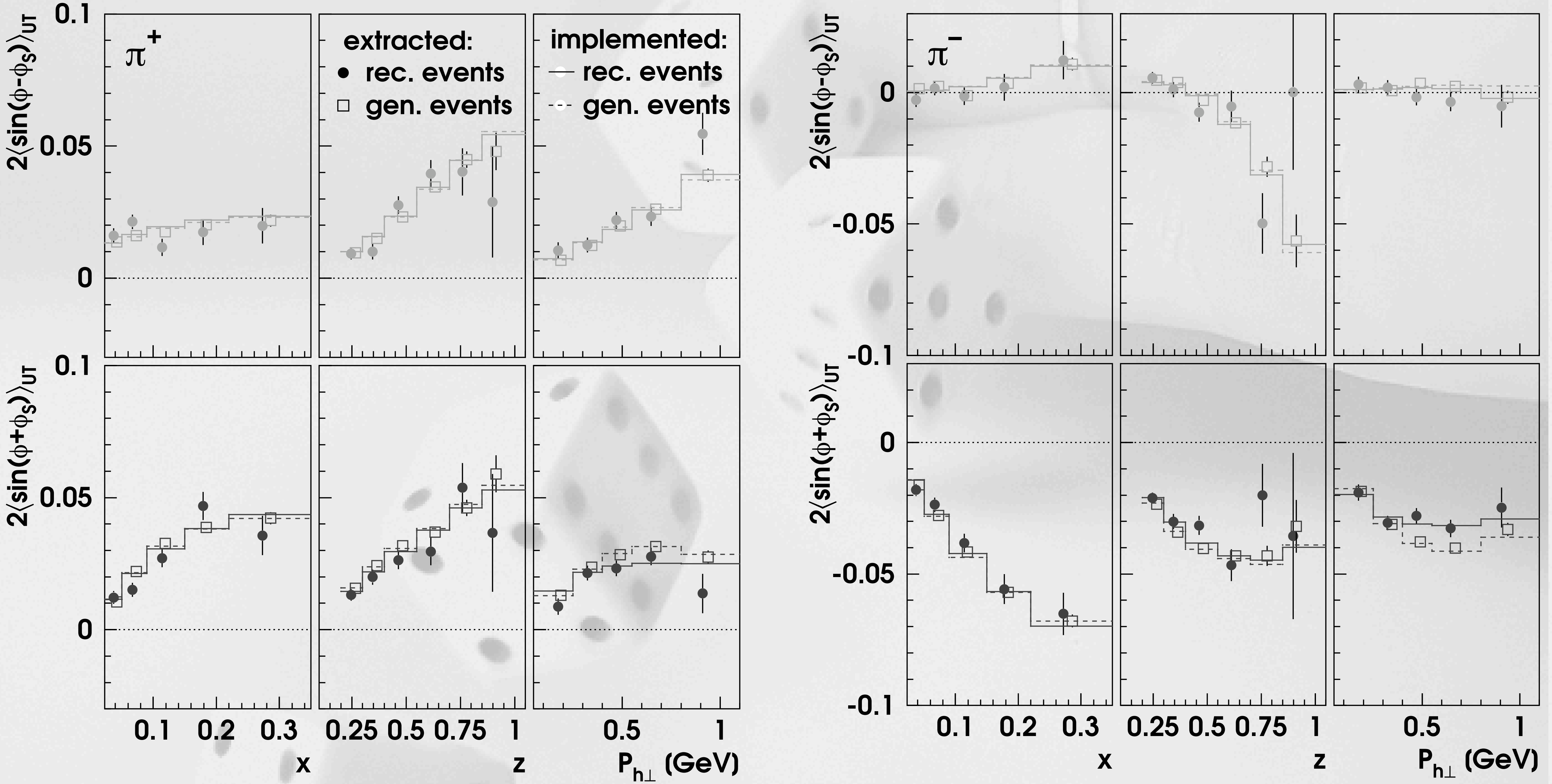
$$\delta q(x) = 0.3 \cdot \Delta q(x) \qquad f_{1T}^{\perp q}(x) = 0.0 \qquad q = \bar{u}, d, s, \bar{s}$$

$$H_{1,\mathrm{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\mathrm{fav}}(z)$$

$$H_{1,\mathrm{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\mathrm{dis}}(z)$$

GRSV for PDFs and Kretzer FF for D_1

Generated vs. extracted amplitudes



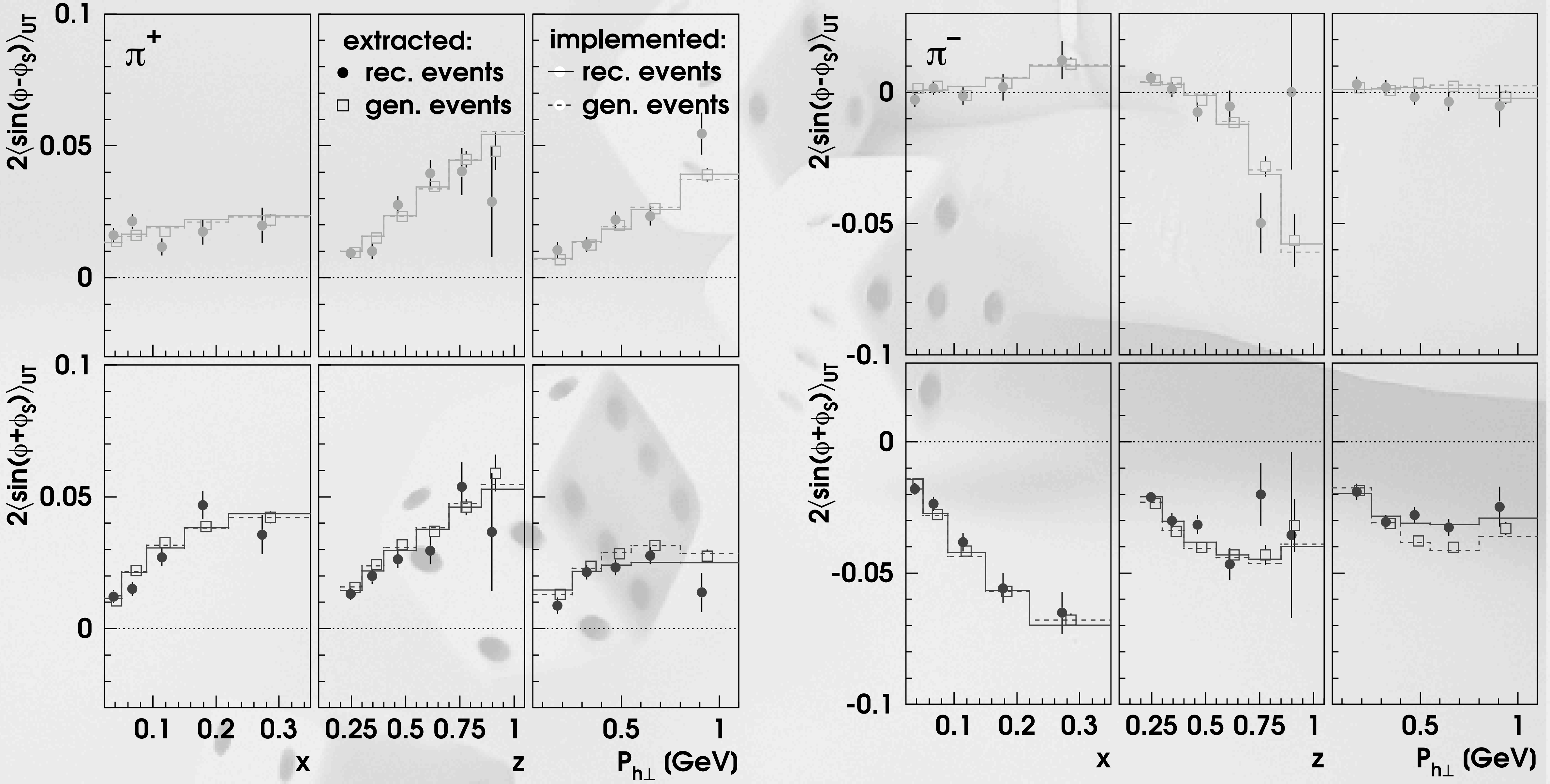
$$\delta u(x) = 0.7 \cdot \Delta u(x) \quad f_{1T}^{\perp u}(x) = -0.3 \cdot u(x) \quad H_{1,\text{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\text{fav}}(z)$$

$$\delta d(x) = 0.7 \cdot \Delta d(x) \quad f_{1T}^{\perp d}(x) = 0.9 \cdot d(x) \quad H_{1,\text{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\text{dis}}(z)$$

$$\delta q(x) = 0.3 \cdot \Delta q(x) \quad f_{1T}^{\perp q}(x) = 0.0 \quad q = \bar{u}, \bar{d}, s, \bar{s}$$

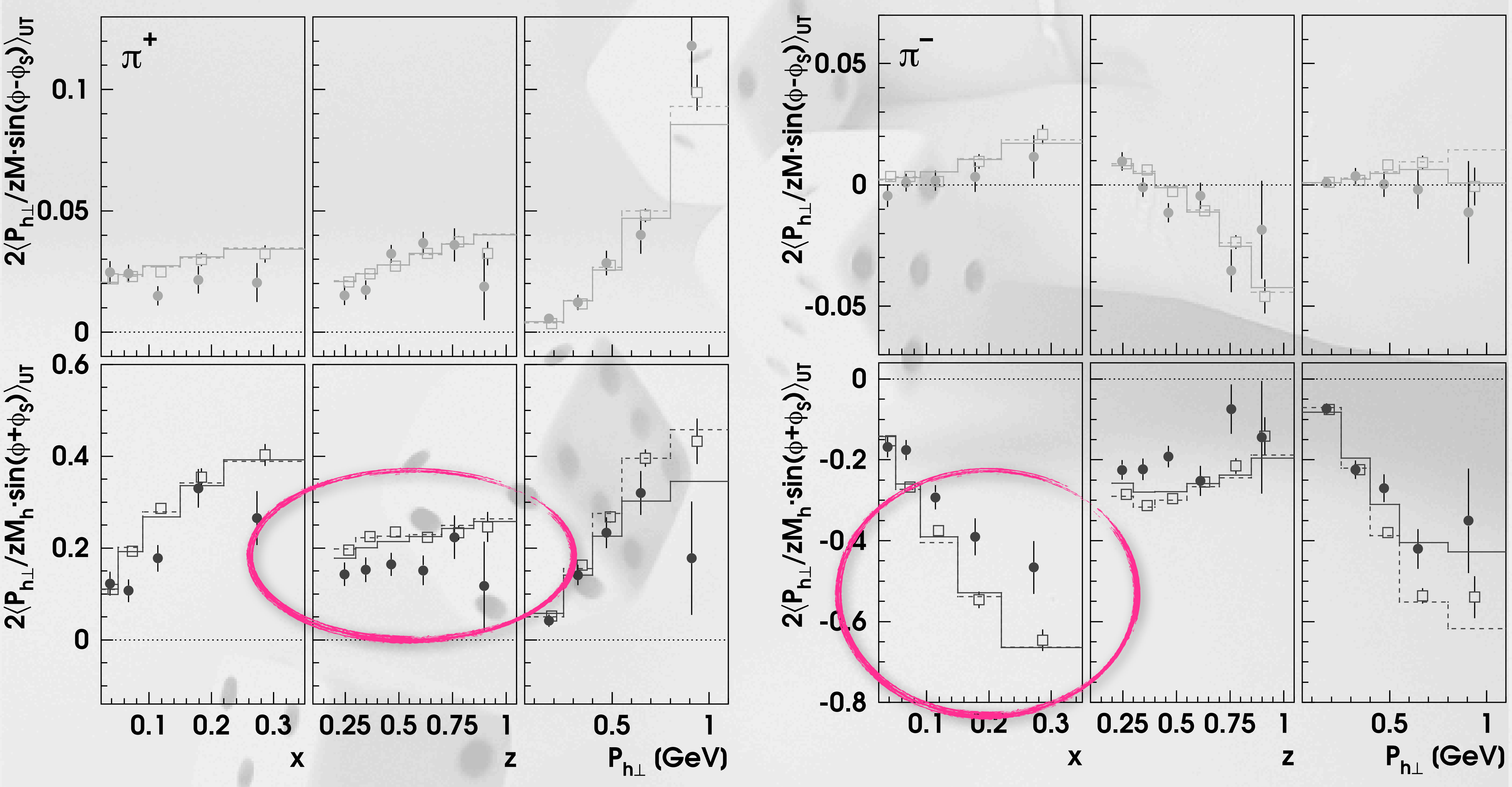
$$C_S = C_C = 0.25$$

Generated vs. extracted amplitudes



Extraction method works well!

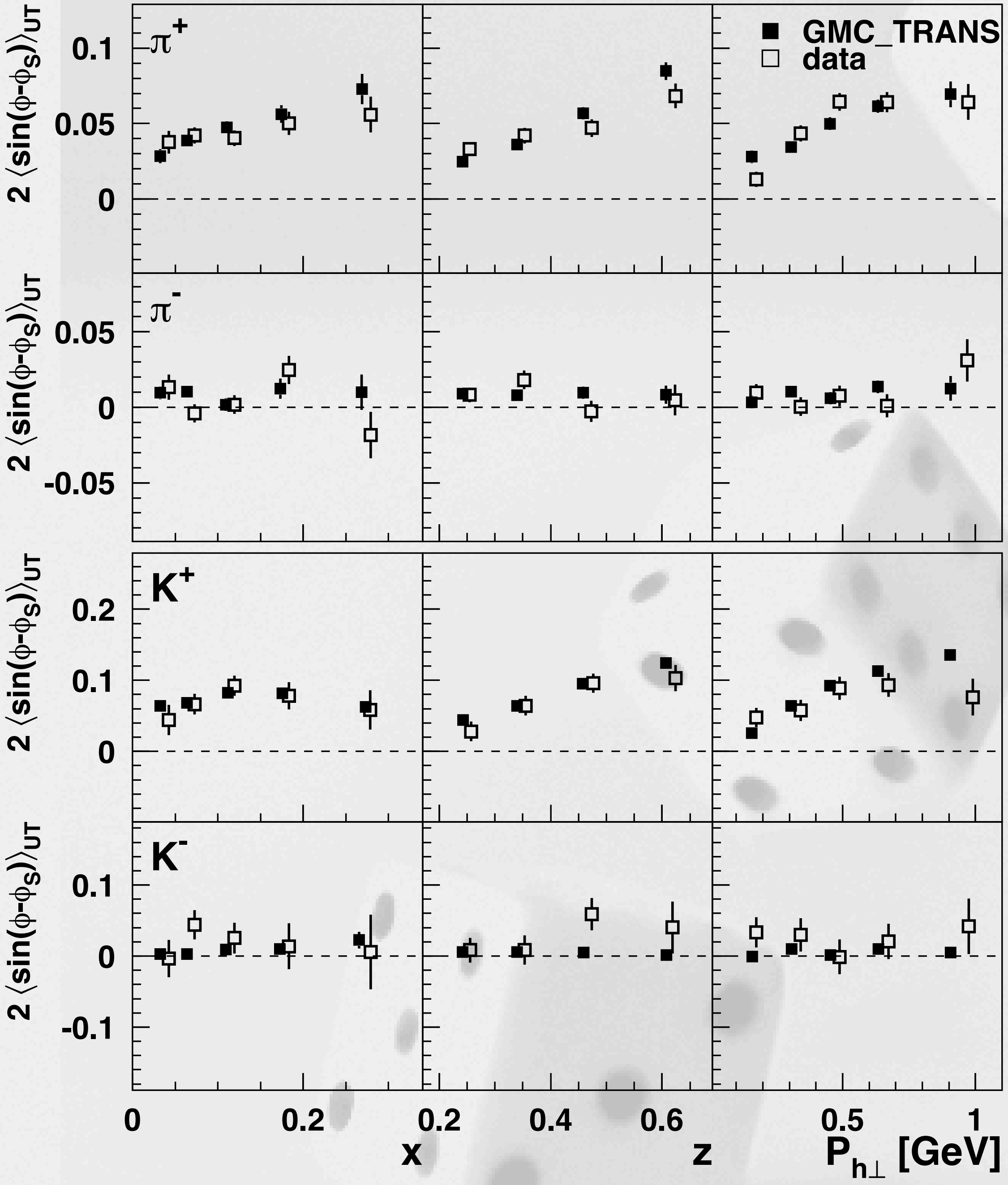
Extraction of weighted moments



Not so good news for weighted moments!

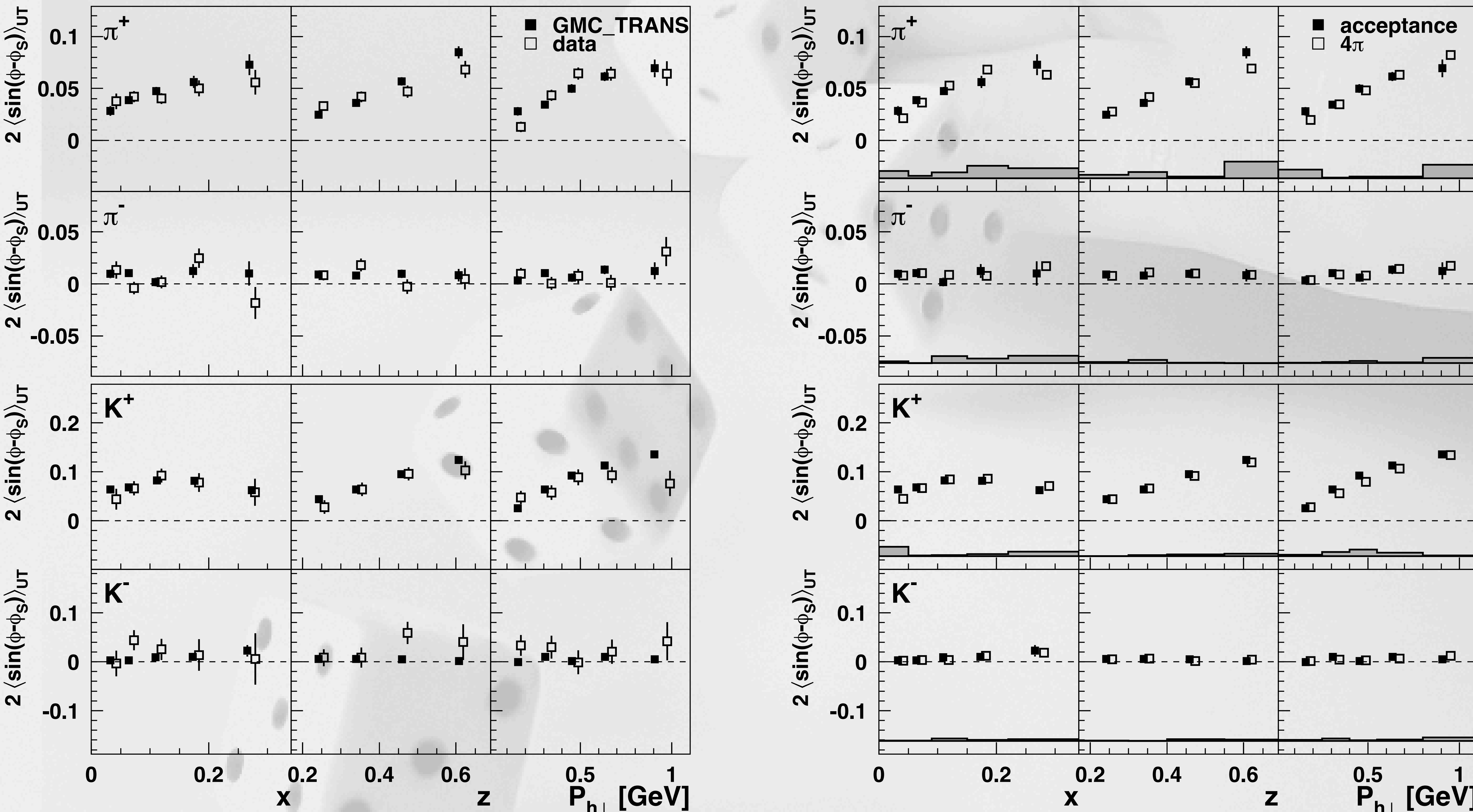
further improvement of the models

- DSS FFs and modified Anselmino et al. Sivers fit:



further improvement of the models

- DSS FFs and modified Anselmino et al. Sivers fit:



- could in principle be used for systematics, but ...

missing items in GMCTRANS

- not so good model for transversity & Collins FF
- missing models for other single- and double-spin asymmetries
- no azimuthal modulations of unpolarized cross section
- no radiative corrections
- no full event generation (missing track multiplicities and correlations etc.)

“polarize” PYTHIA

alternative: “reshuffling” PYTHIA events

- use model for azimuthal distribution to introduce spin dependence in PYTHIA
- throw random number ρ and assign spin state up if, e.g.,

$$\rho < \frac{1}{2}\left(1 + \sin(\phi - \phi_S)\,\Xi_{11}^{\sin(\phi-\phi_S),h} + \sin(\phi + \phi_S)\,\Xi_{11}^{\sin(\phi+\phi_S),h} + \sin(\phi_S)\,\Xi_{11}^{\sin(\phi_S),h}\right)$$

parametrization of azimuthal dependences (extracted, e.g., from real data)

Parametrization of azimuthal dependence

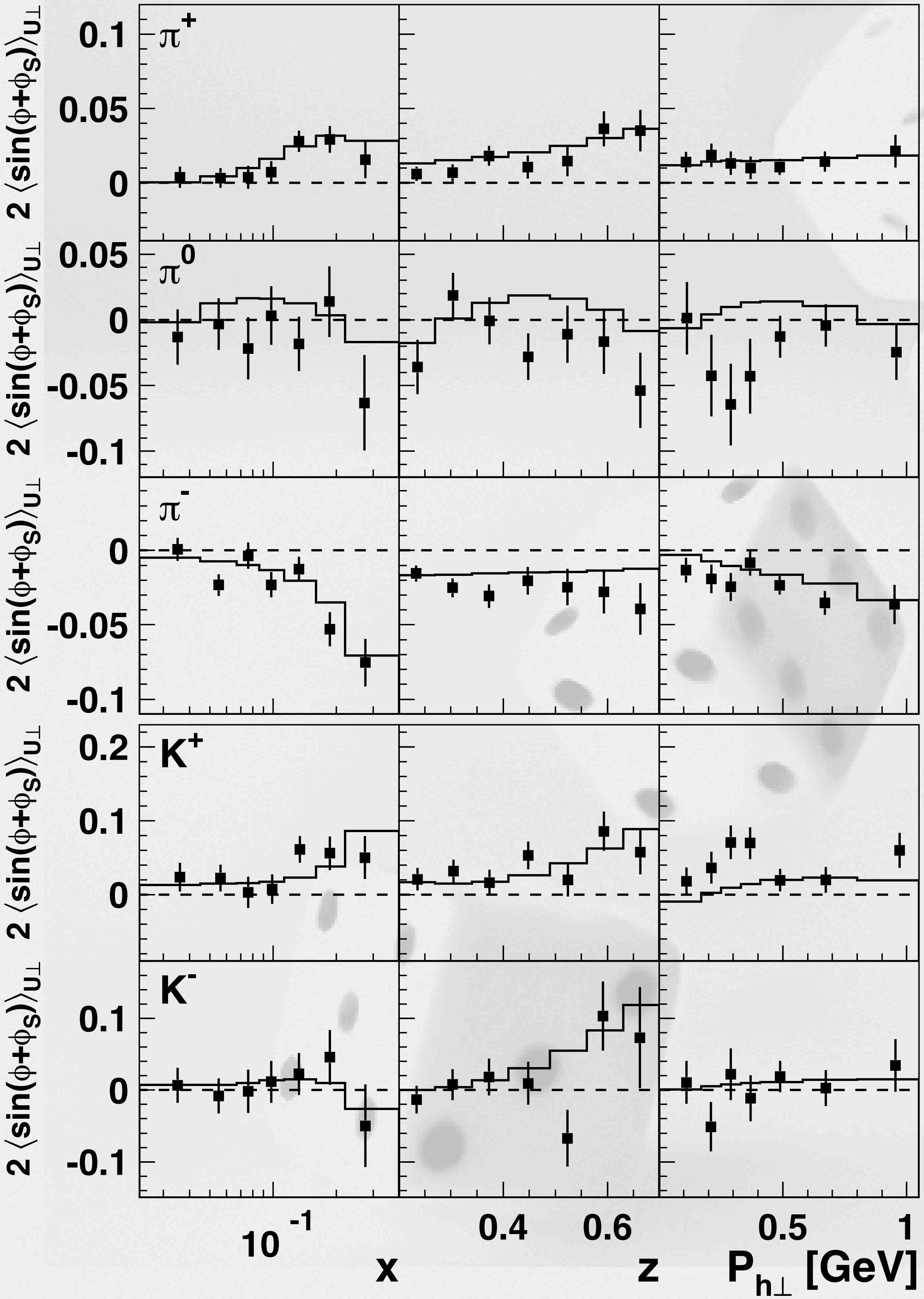
- fully differential model extracted in M.L. fit to data with PDF

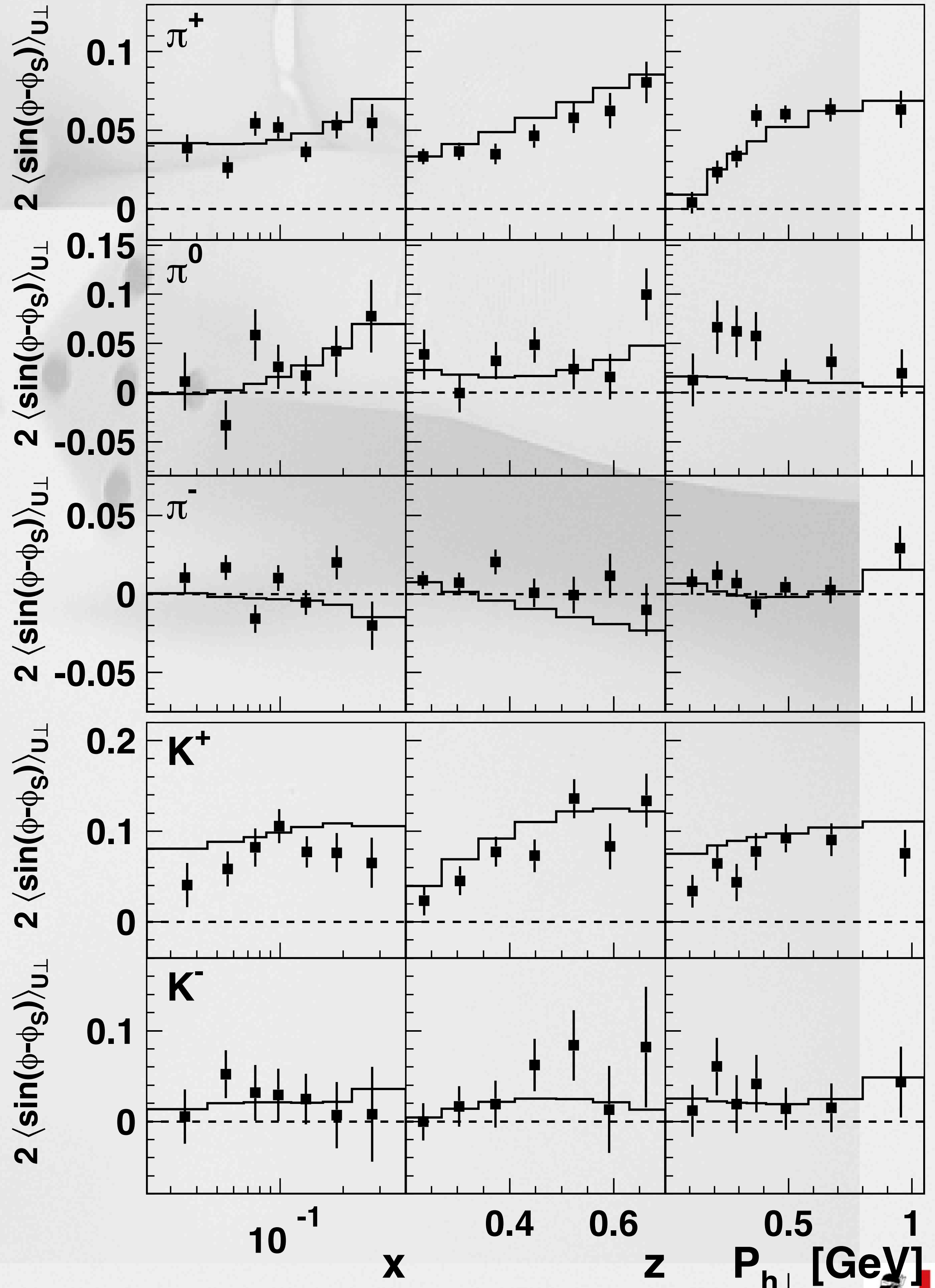
$$P\left(x, Q^2, z, |\mathbf{P}_{h\perp}|, \phi, \phi_S; \Xi_{22}^{\sin(\phi-\phi_S),h}, \Xi_{22}^{\sin(\phi+\phi_S),h}\right)$$
$$= 1 + S_\perp \left(\sin(\phi-\phi_S)\,\Xi_{22}^{\sin(\phi-\phi_S),h} + \sin(\phi+\phi_S)\,\Xi_{22}^{\sin(\phi+\phi_S),h}\right)$$

$$\begin{aligned}
\Xi_{22}^{\sin(\phi\pm\phi_S),h} = \; & \Xi_{22,1}^{\sin(\phi\pm\phi_S),h} + \Xi_{22,2}^{\sin(\phi\pm\phi_S),h} x' + \\
& \Xi_{22,3}^{\sin(\phi\pm\phi_S),h} Q^{2\prime} + \Xi_{22,4}^{\sin(\phi\pm\phi_S),h} z' + \\
& \Xi_{22,5}^{\sin(\phi\pm\phi_S),h} |\mathbf{P}_{h\perp}|' + \Xi_{22,6}^{\sin(\phi\pm\phi_S),h} x'^2 + \\
& \Xi_{22,7}^{\sin(\phi\pm\phi_S),h} z'^2 + \Xi_{22,8}^{\sin(\phi\pm\phi_S),h} |\mathbf{P}_{h\perp}|'^2 + \\
& \Xi_{22,9}^{\sin(\phi\pm\phi_S),h} x'z' + \Xi_{22,10}^{\sin(\phi\pm\phi_S),h} x' |\mathbf{P}_{h\perp}|' + \\
& \Xi_{22,11}^{\sin(\phi\pm\phi_S),h} z' |\mathbf{P}_{h\perp}|' + \Xi_{22,12}^{\sin(\phi\pm\phi_S),h} x'^3 + \\
& \Xi_{22,13}^{\sin(\phi\pm\phi_S),h} x'z'^2 + \Xi_{22,14}^{\sin(\phi\pm\phi_S),h} x'^2 z' + \\
& \Xi_{22,15}^{\sin(\phi\pm\phi_S),h} x'^2 |\mathbf{P}_{h\perp}|' + \Xi_{22,16}^{\sin(\phi\pm\phi_S),h} x' |\mathbf{P}_{h\perp}|'^2 + \\
& \Xi_{22,17}^{\sin(\phi\pm\phi_S),h} z'^2 |\mathbf{P}_{h\perp}|' + \Xi_{22,18}^{\sin(\phi\pm\phi_S),h} z' |\mathbf{P}_{h\perp}|'^2 + \\
& \Xi_{22,19}^{\sin(\phi\pm\phi_S),h} x'^2 |\mathbf{P}_{h\perp}|'^2 + \Xi_{22,20}^{\sin(\phi\pm\phi_S),h} z'^2 |\mathbf{P}_{h\perp}|'^2 + \\
& \Xi_{22,21}^{\sin(\phi\pm\phi_S),h} x'z' |\mathbf{P}_{h\perp}|' + \Xi_{22,22}^{\sin(\phi\pm\phi_S),h} x'^2 z' |\mathbf{P}_{h\perp}|' \;.
\end{aligned}$$

Description of data

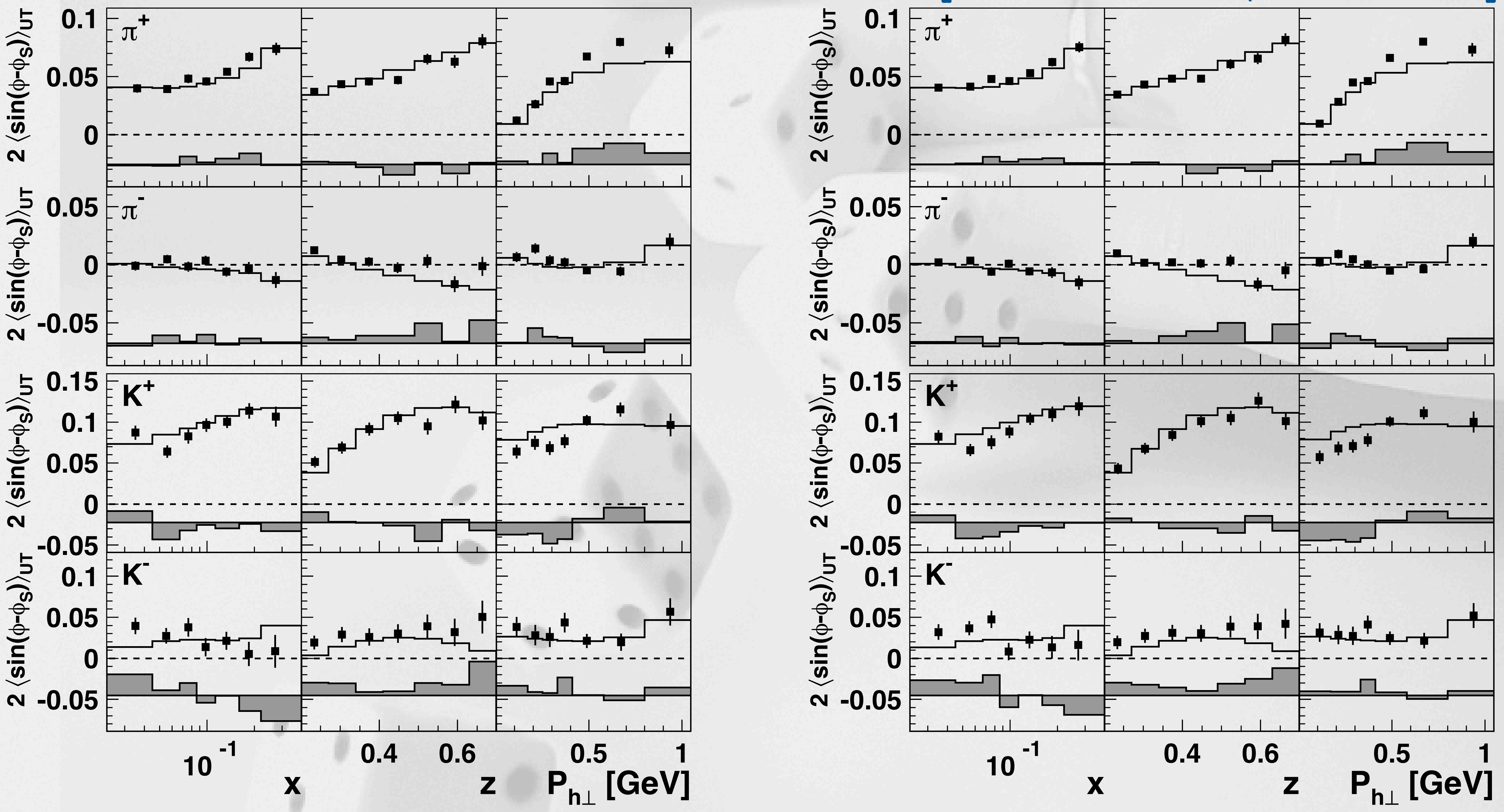
[M. Diefenthaler, Ph.D. thesis]





Evaluation of detector effects

[M. Diefenthaler, Ph.D. thesis]



- differences include effects from internal and external radiative effects, acceptance, PID, (mis)alignment etc.
- in further step "smoothed" to reduce statistical fluctuations

some Pro&Cons of “reshuffling”

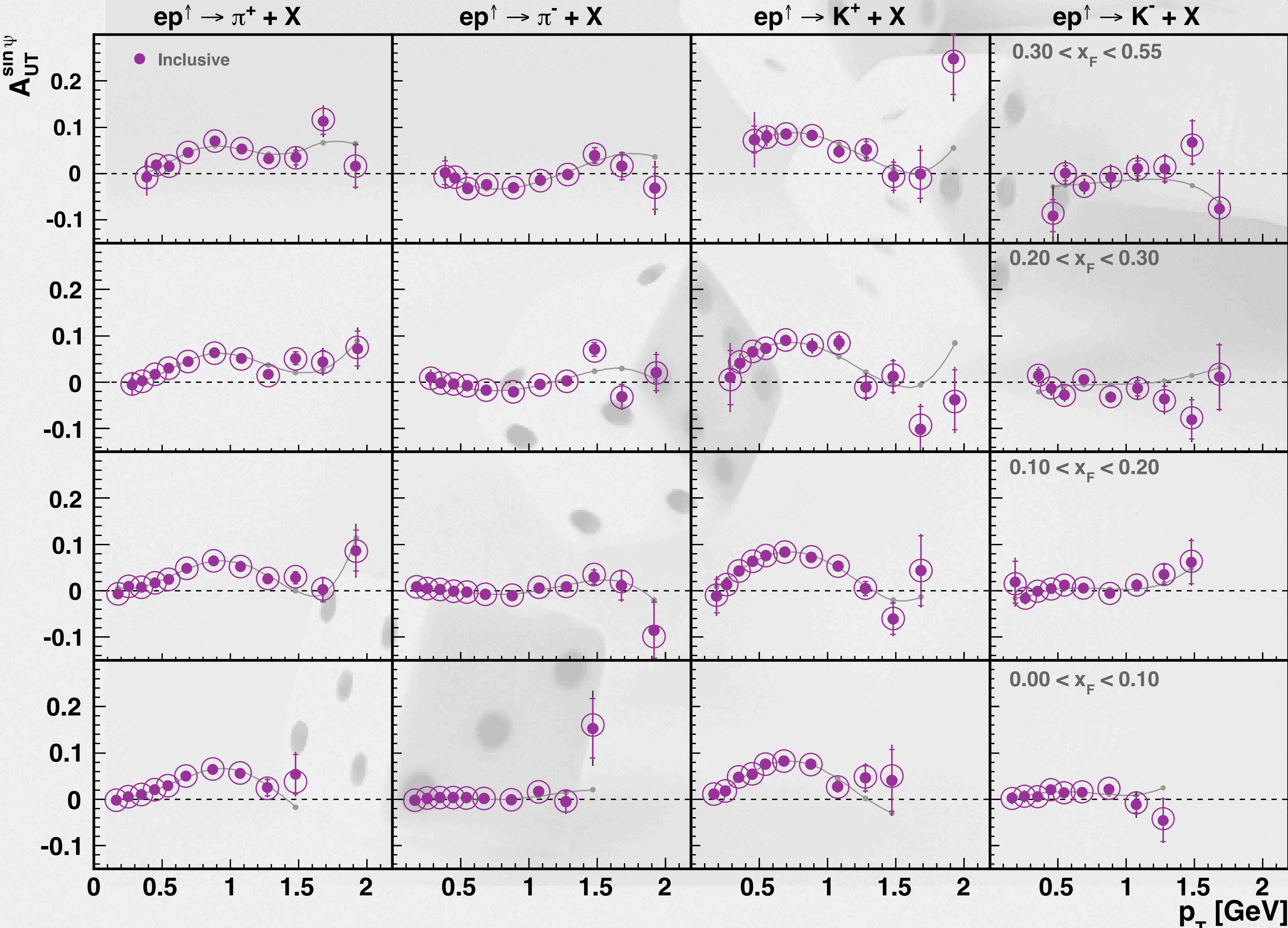
- whole event topology and correlations available
- flexible
- applicable also to cases where no guidance from theory available on shape/magnitude of modulations

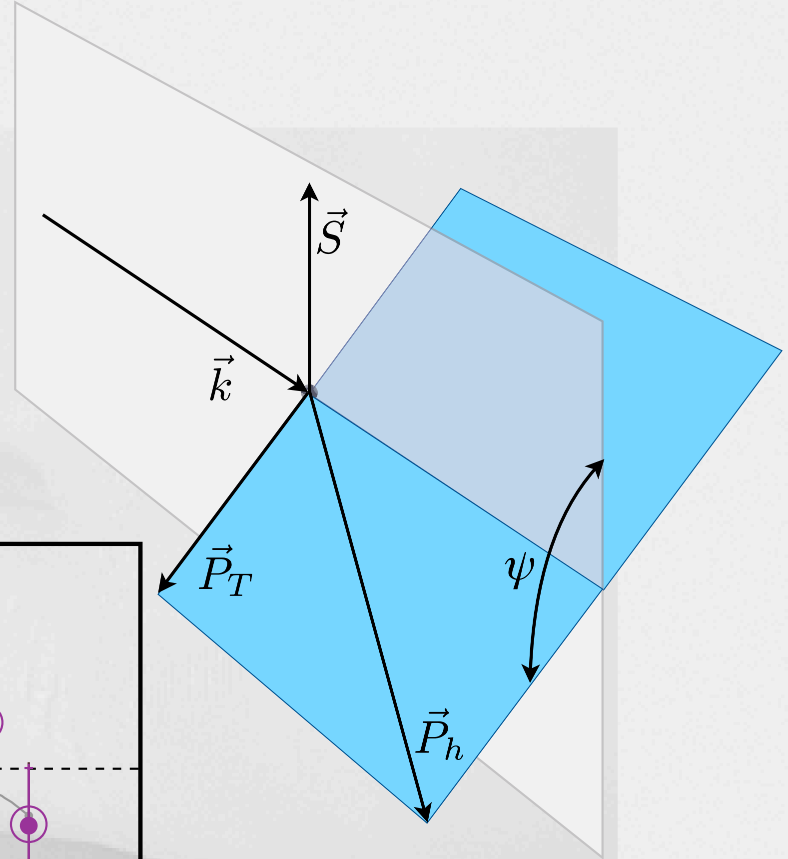
some Pro&Cons of "reshuffling"

- whole event topology and correlations available
- flexible
- applicable also to cases where no guidance from theory available on shape/magnitude of modulations

- need parametrization
  if from real data, where to stop Taylor (or other) expansion?
- large uncertainties on (some) parameters can introduce large spurious effects in systematics calculation
- relies on good description of unpolarized cross section in Monte Carlo

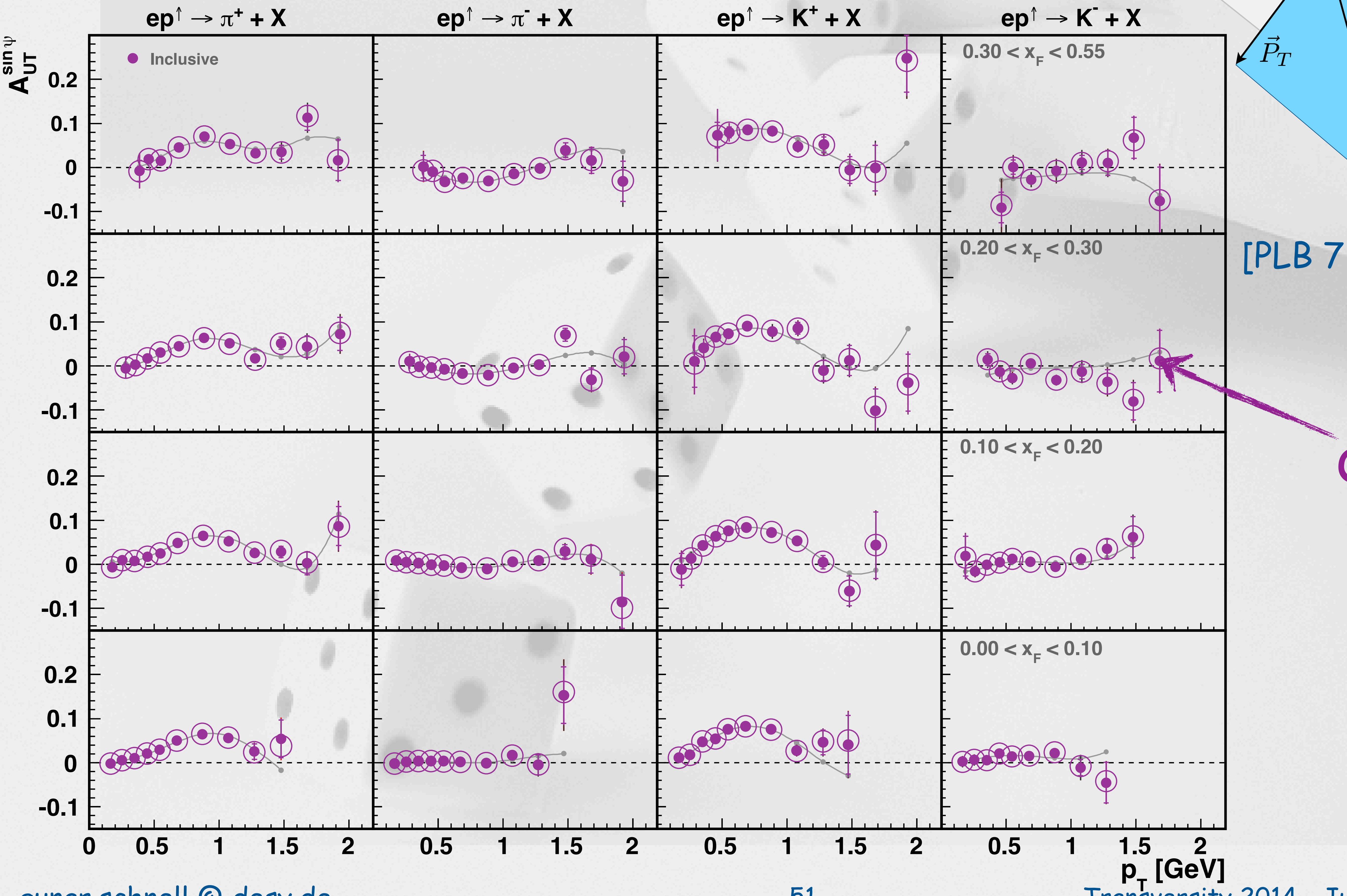
Another example: A_{UT} in inclusive hadron production

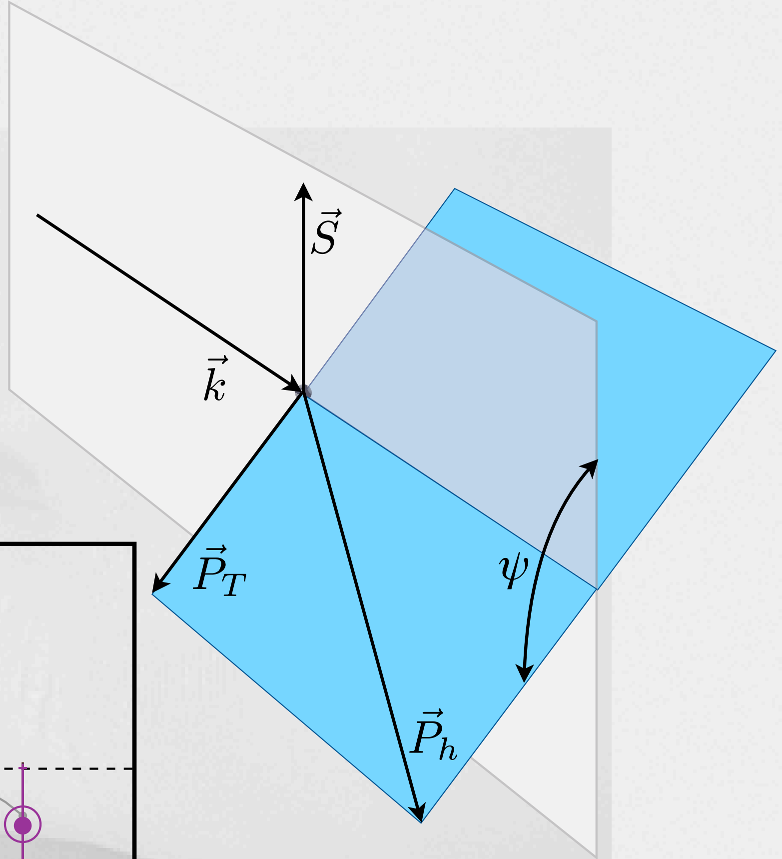




[PLB 728 (2014) 183]

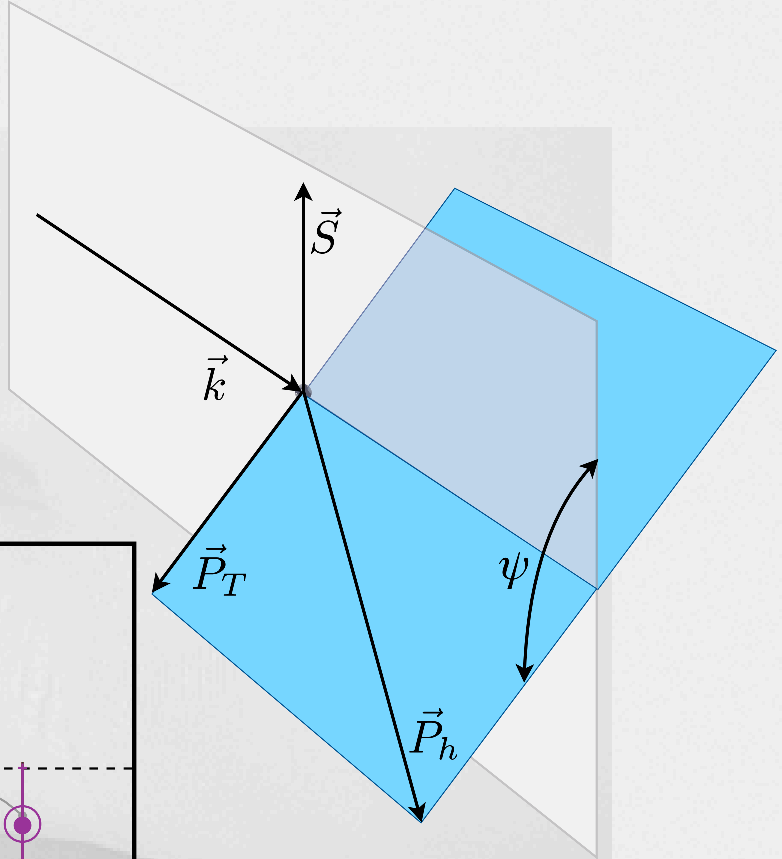
Another example: A_{UT} in inclusive hadron production

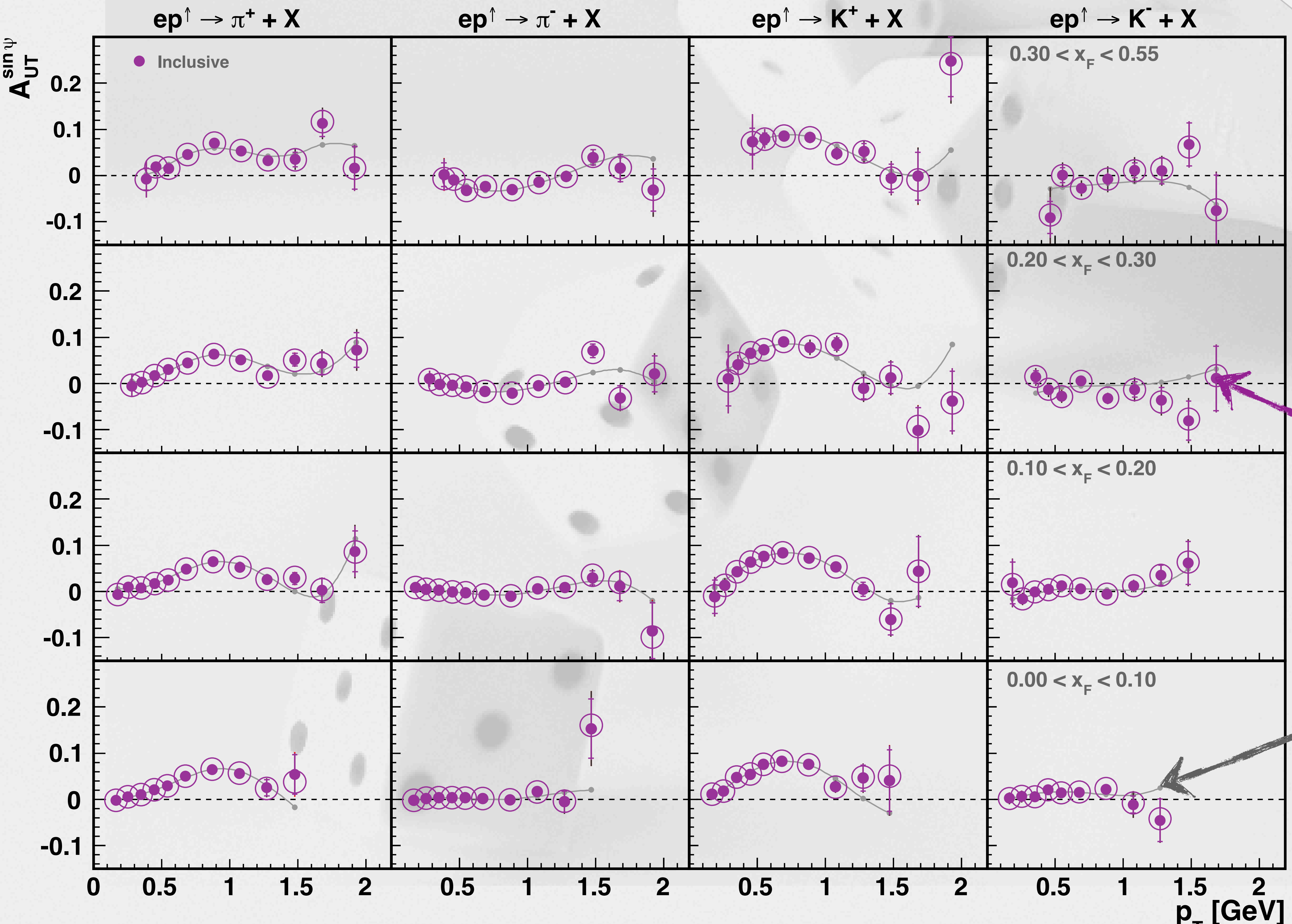




[PLB 728 (2014) 183]

Another example: A_{UT} in inclusive hadron production



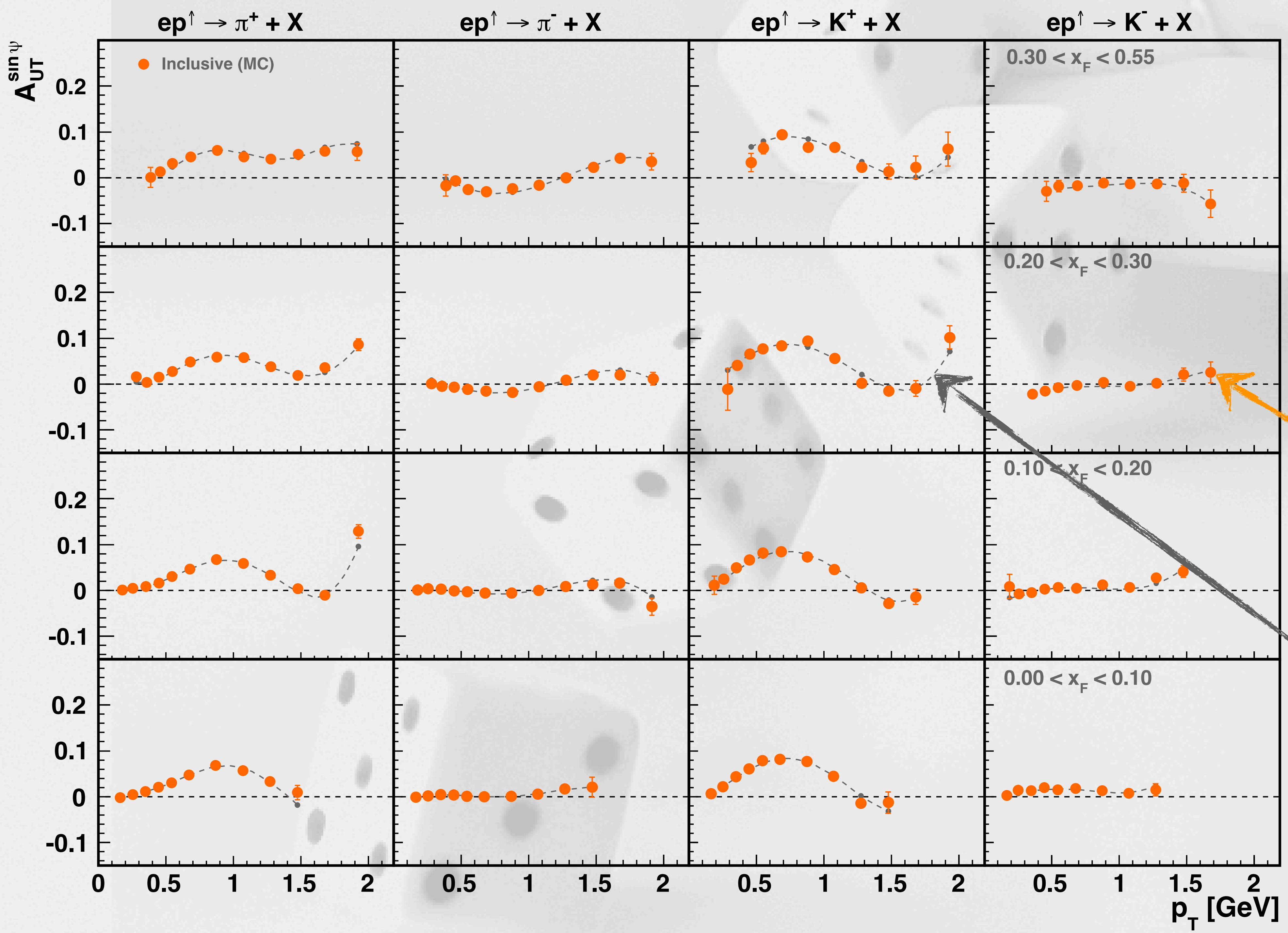


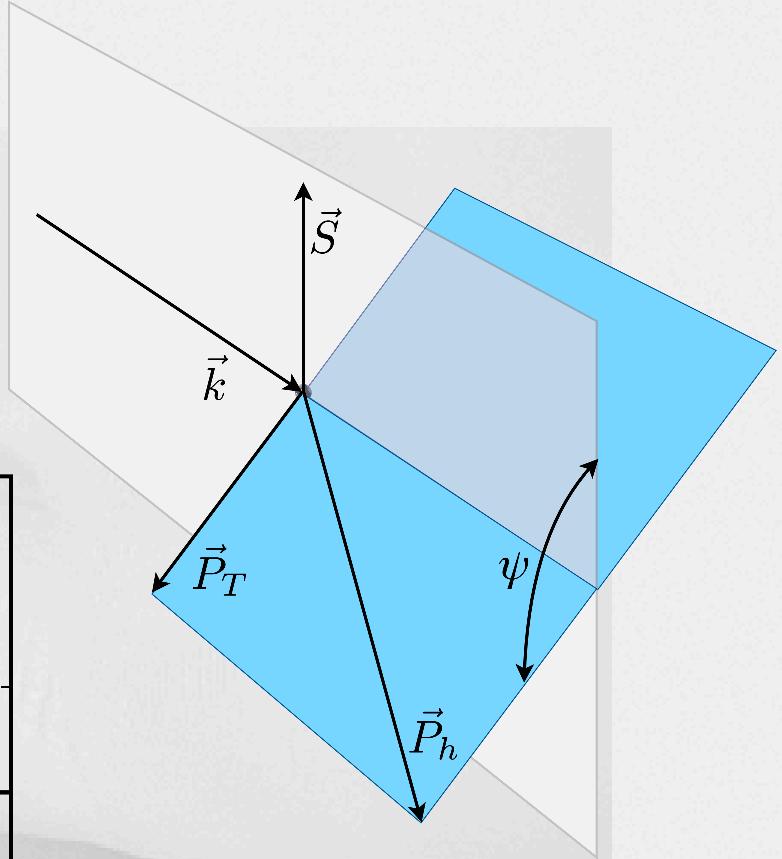
[PLB 728 (2014) 183]

data

fit to data

Another example: A_{UT} in inclusive hadron production

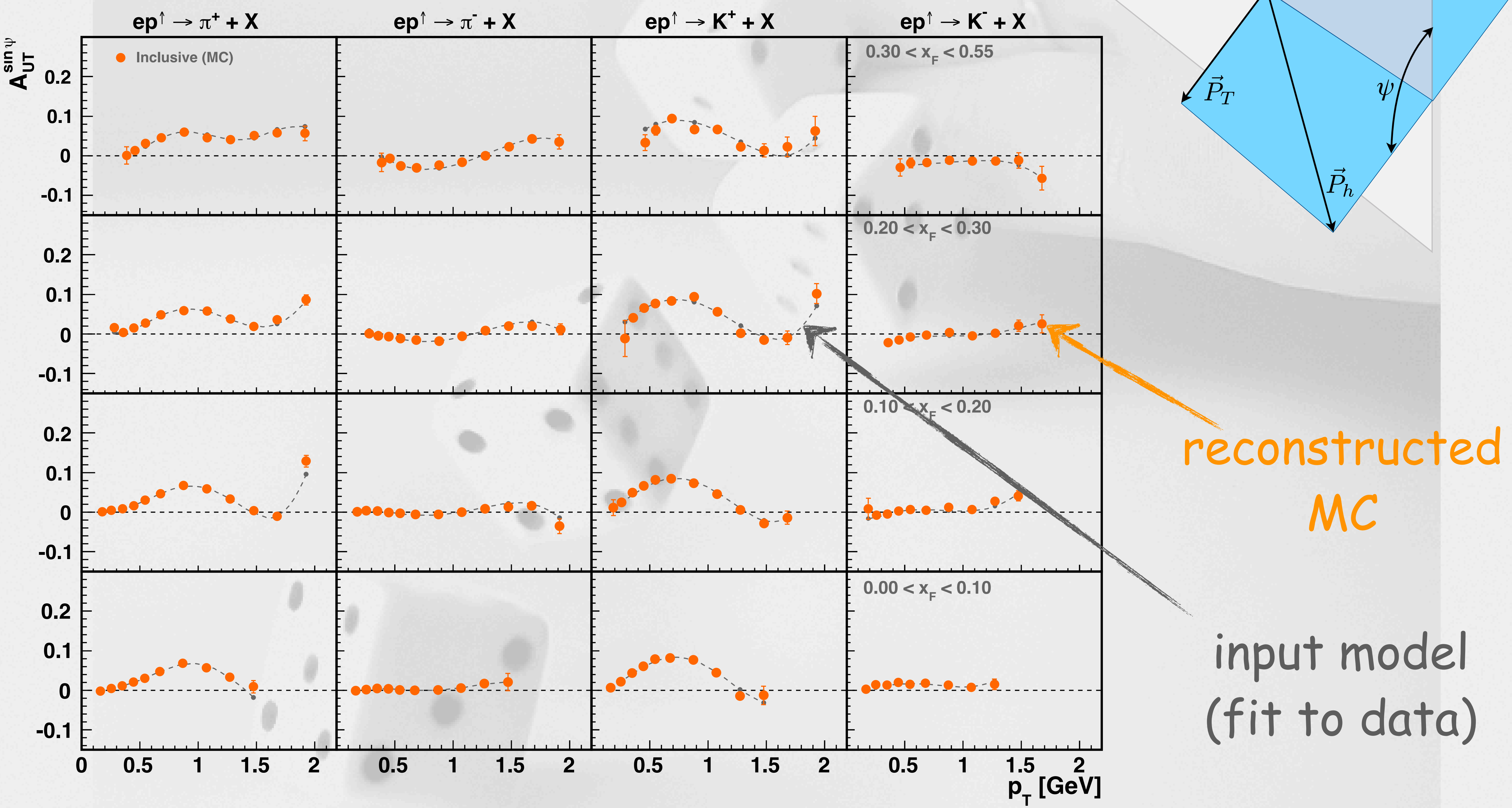


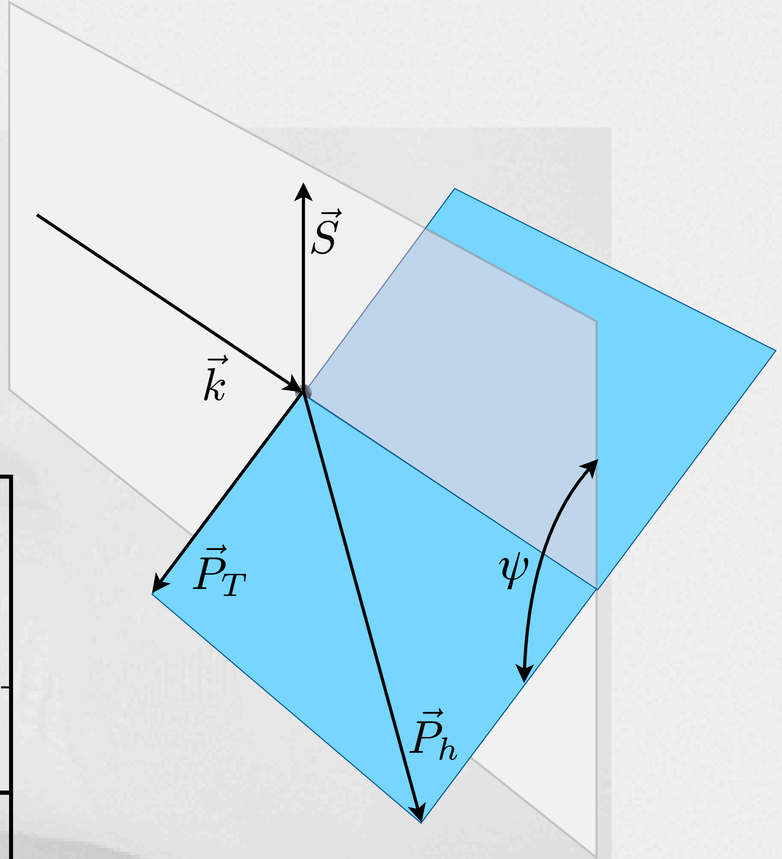


reconstructed MC

input model (fit to data)

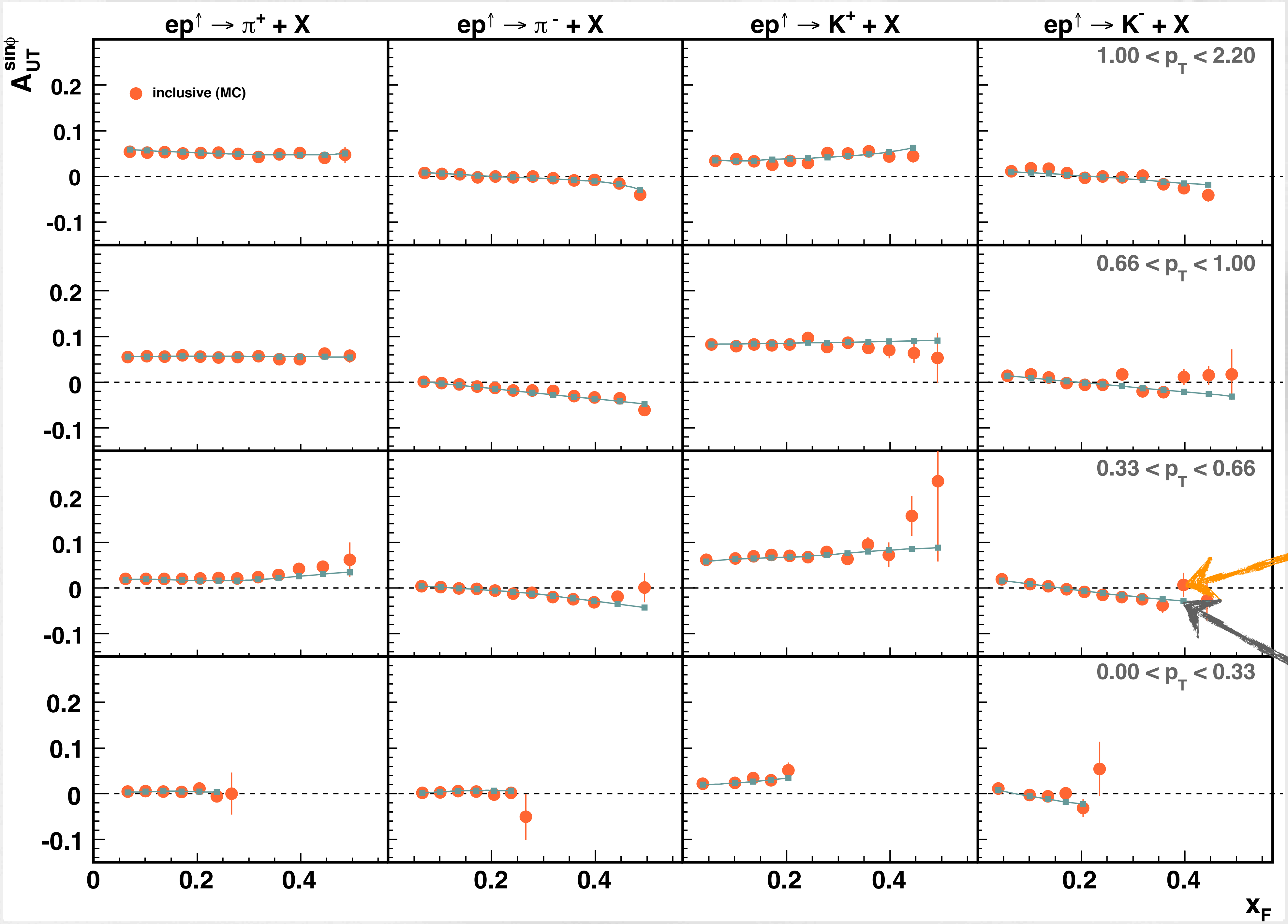
Another example: A_{UT} in inclusive hadron production

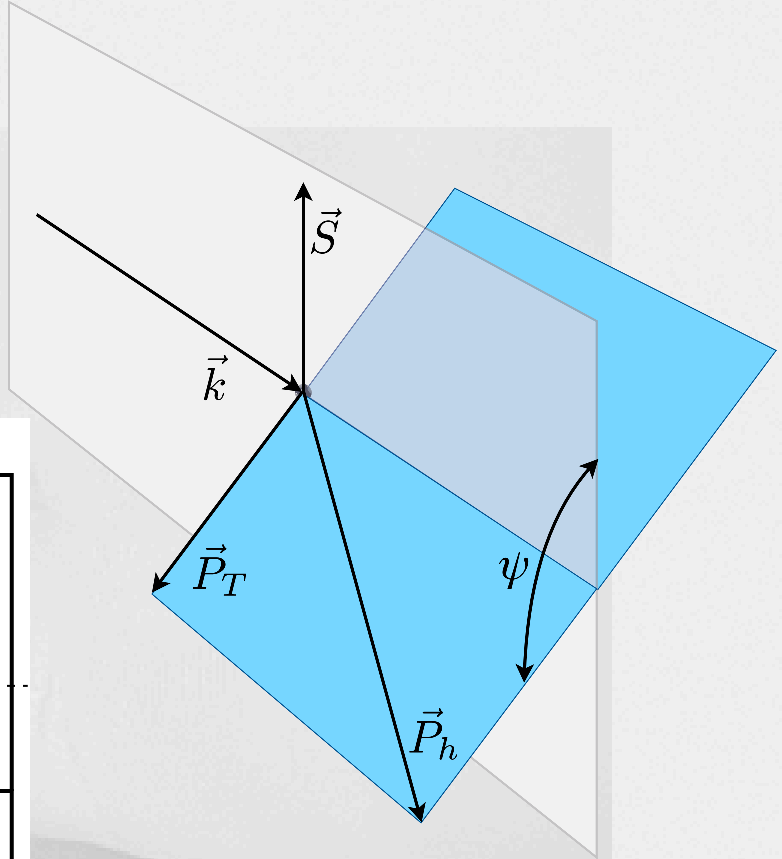




small detector effects in fully differential analysis

Another example: A_{UT} in inclusive hadron production



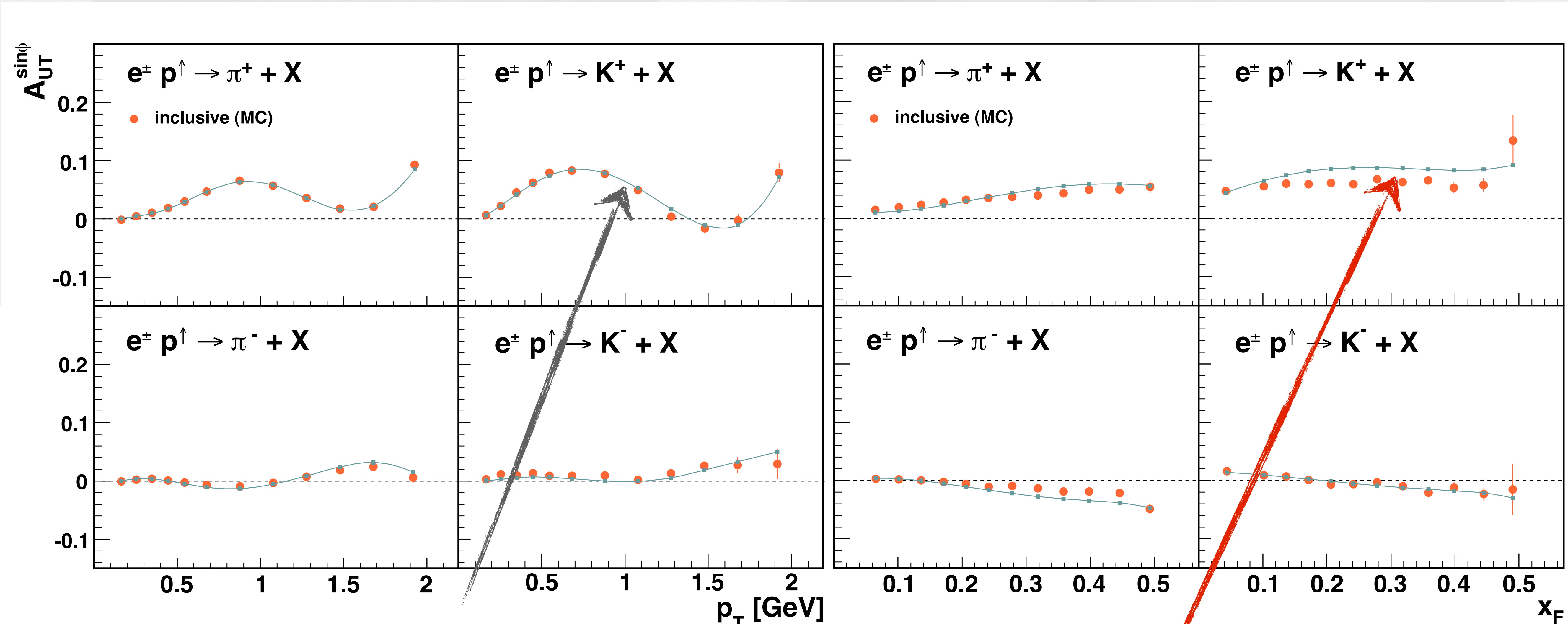


reconstructed MC

input model (fit to data)

small detector effects in fully differential analysis

Another example: A_{UT} in inclusive hadron production



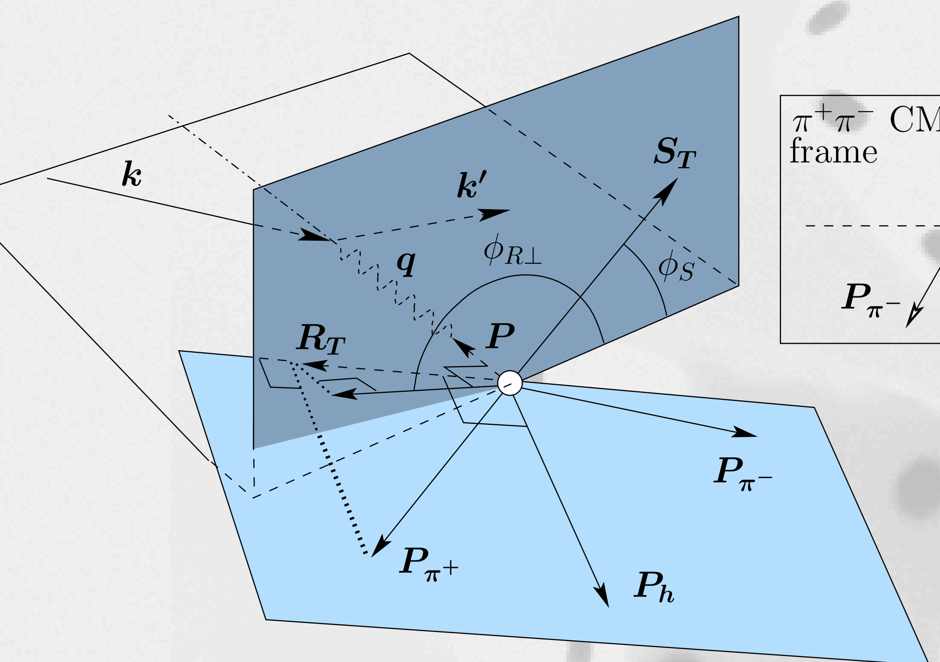
strong kinematic dependence can lead to large systematic effects if integrated over

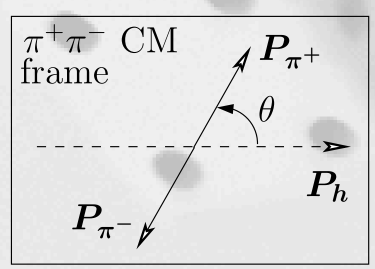
not so small detector effects in 1D analysis

similar problematics: di-hadron A_{UT}

- many kinematic variables needed to describe process

$$N^{\uparrow(\downarrow)}(\phi_{R\perp},\phi_S,\theta,M_{\pi\pi}) \propto \int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P_{h\perp}}\,\epsilon(x,y,z,\boldsymbol{P_{h\perp}},\phi_{R\perp},\phi_S,\theta,M_{\pi\pi}) \times \times\ \sigma_{U\uparrow(\downarrow)}(x,y,z,\boldsymbol{P_{h\perp}},\phi_{R\perp},\phi_S,\theta,M_{\pi\pi}),$$



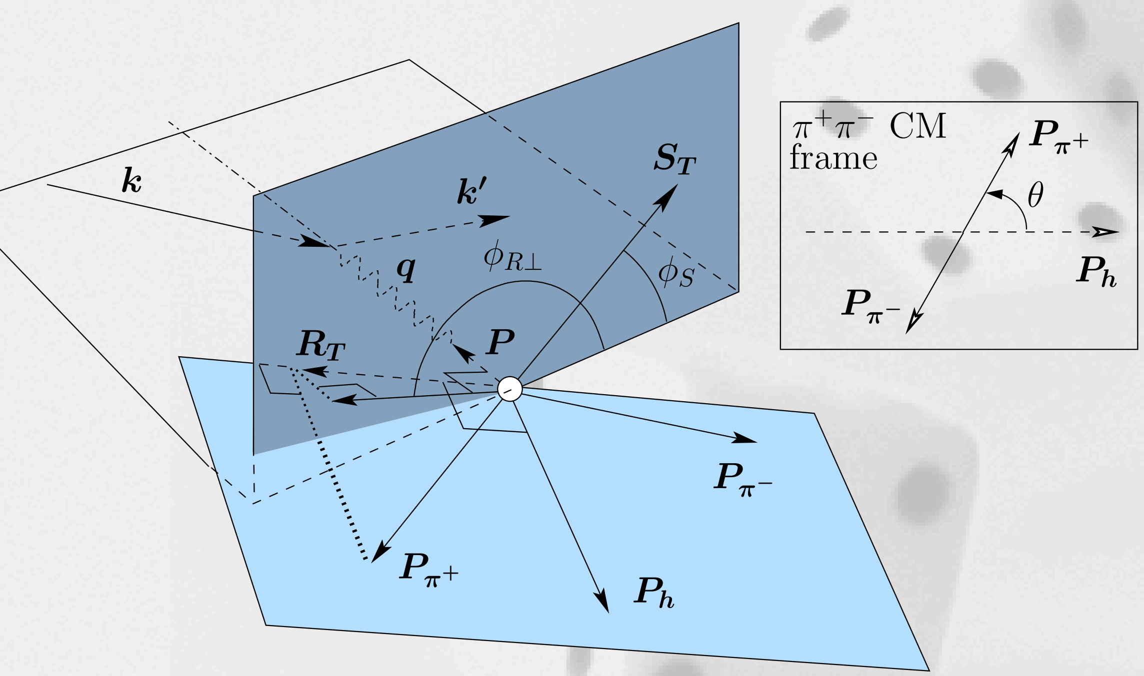


similar problematics: di-hadron A_{UT}

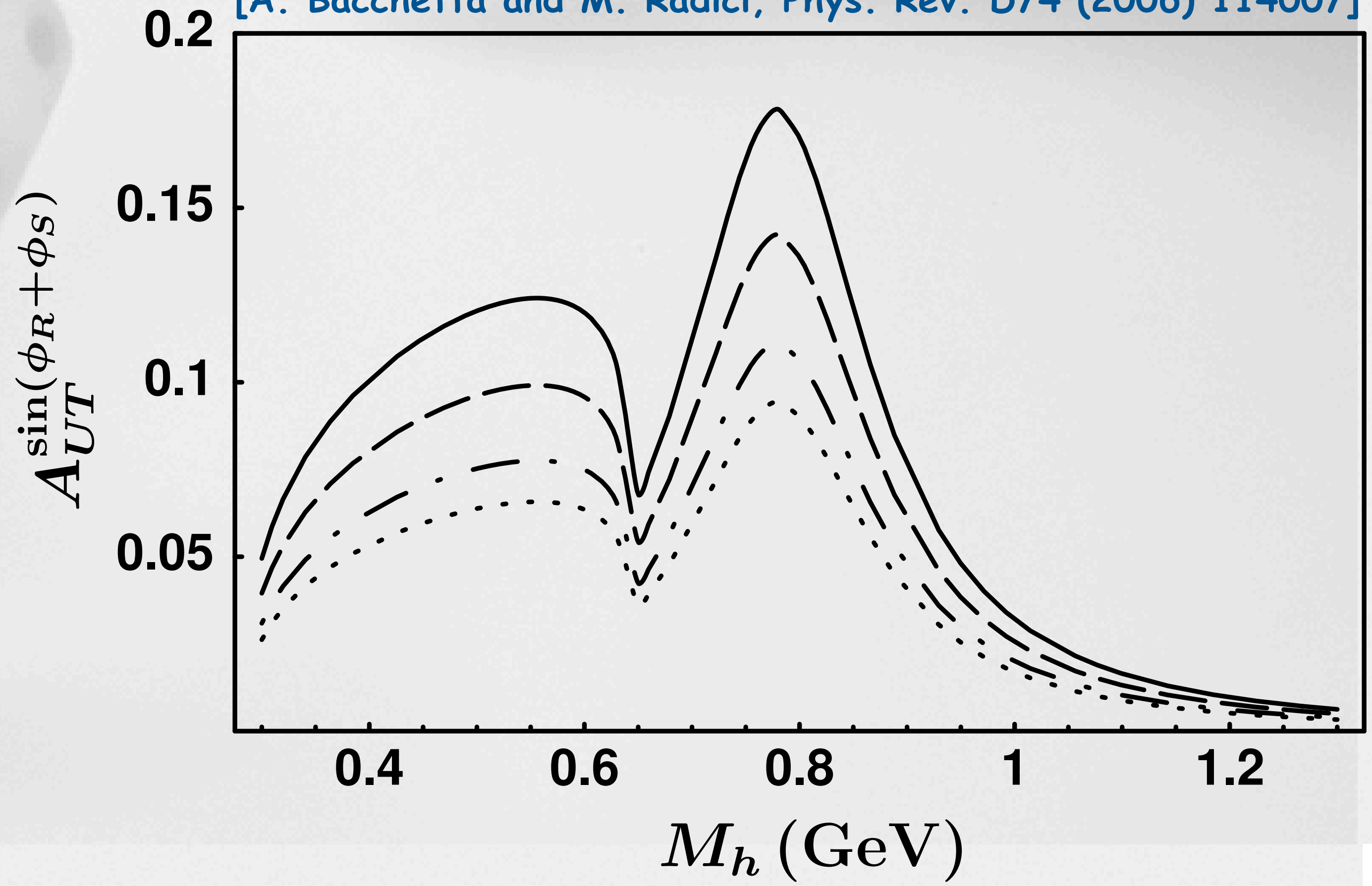
- many kinematic variables needed to describe process

$$N^{\uparrow(\downarrow)}(\phi_{R\perp},\phi_S,\theta,M_{\pi\pi}) \propto \int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P_{h\perp}}\,\epsilon(x,y,z,\boldsymbol{P_{h\perp}},\phi_{R\perp},\phi_S,\theta,M_{\pi\pi}) \times$$
$$\times\ \sigma_{U\uparrow(\downarrow)}(x,y,z,\boldsymbol{P_{h\perp}},\phi_{R\perp},\phi_S,\theta,M_{\pi\pi}),$$

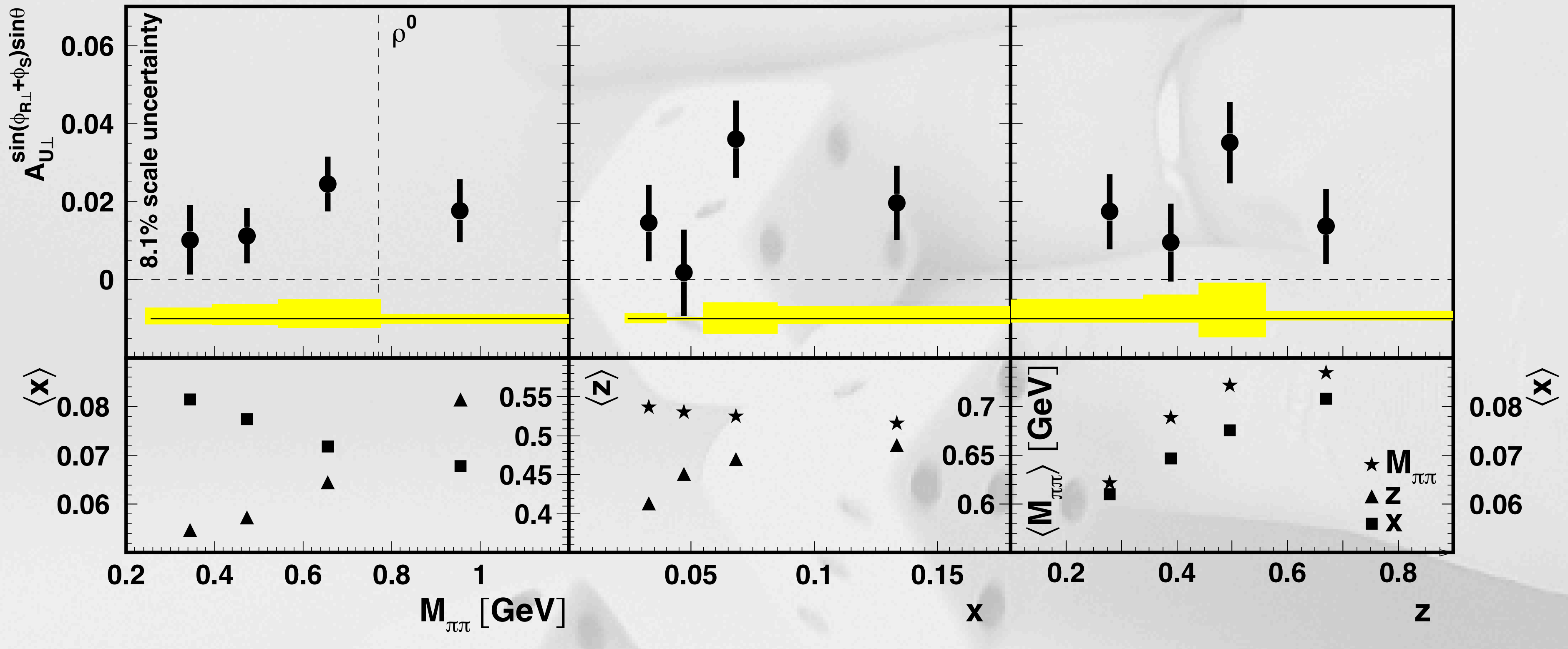
- at least for one of them strong dependence expected:



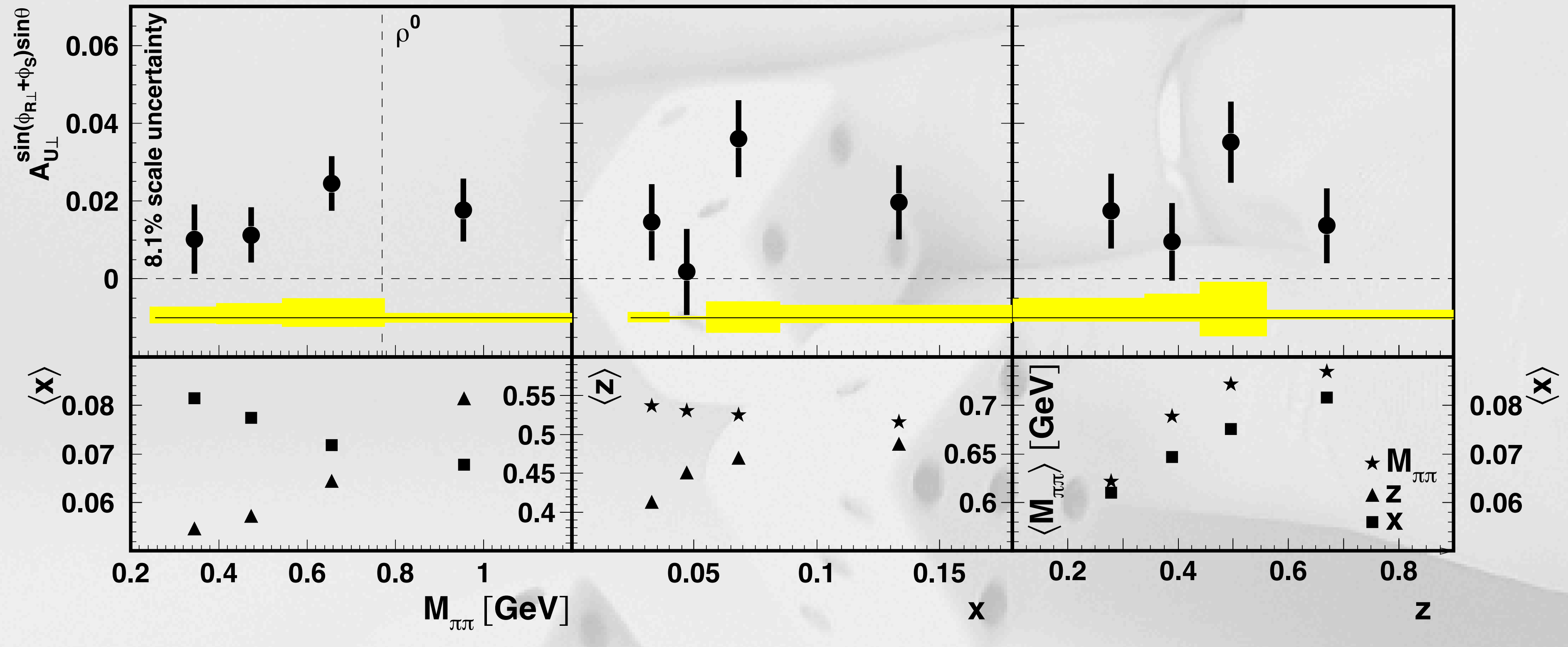
[A. Bacchetta and M. Radici, Phys. Rev. D74 (2006) 114007]



define your measurement wisely!



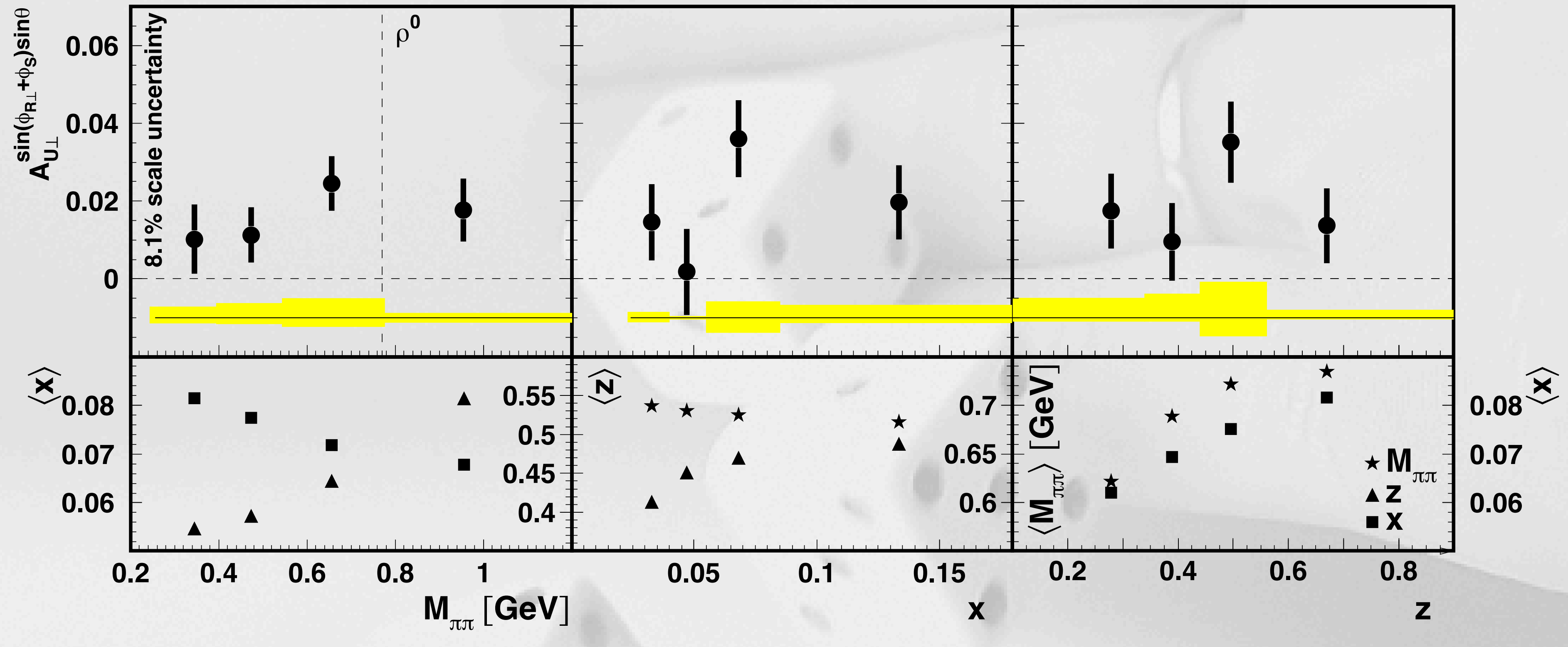
define your measurement wisely!



"possible sources of systematic uncertainties have been examined: the difference in the modulation amplitude of interest extracted as done for real data in the experimental acceptance and similarly in 4π acceptance" [JHEP 06 (2008) 017]

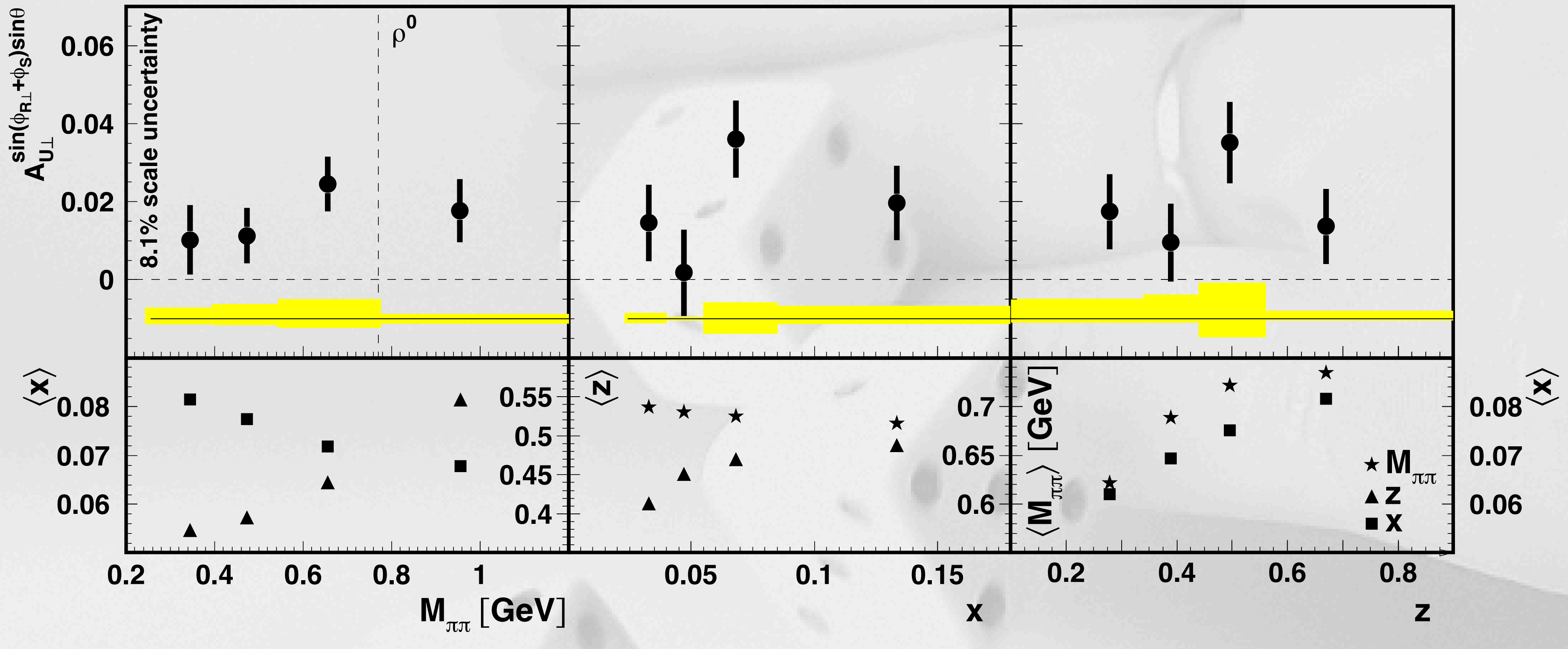
read the fineprint

define your measurement wisely!



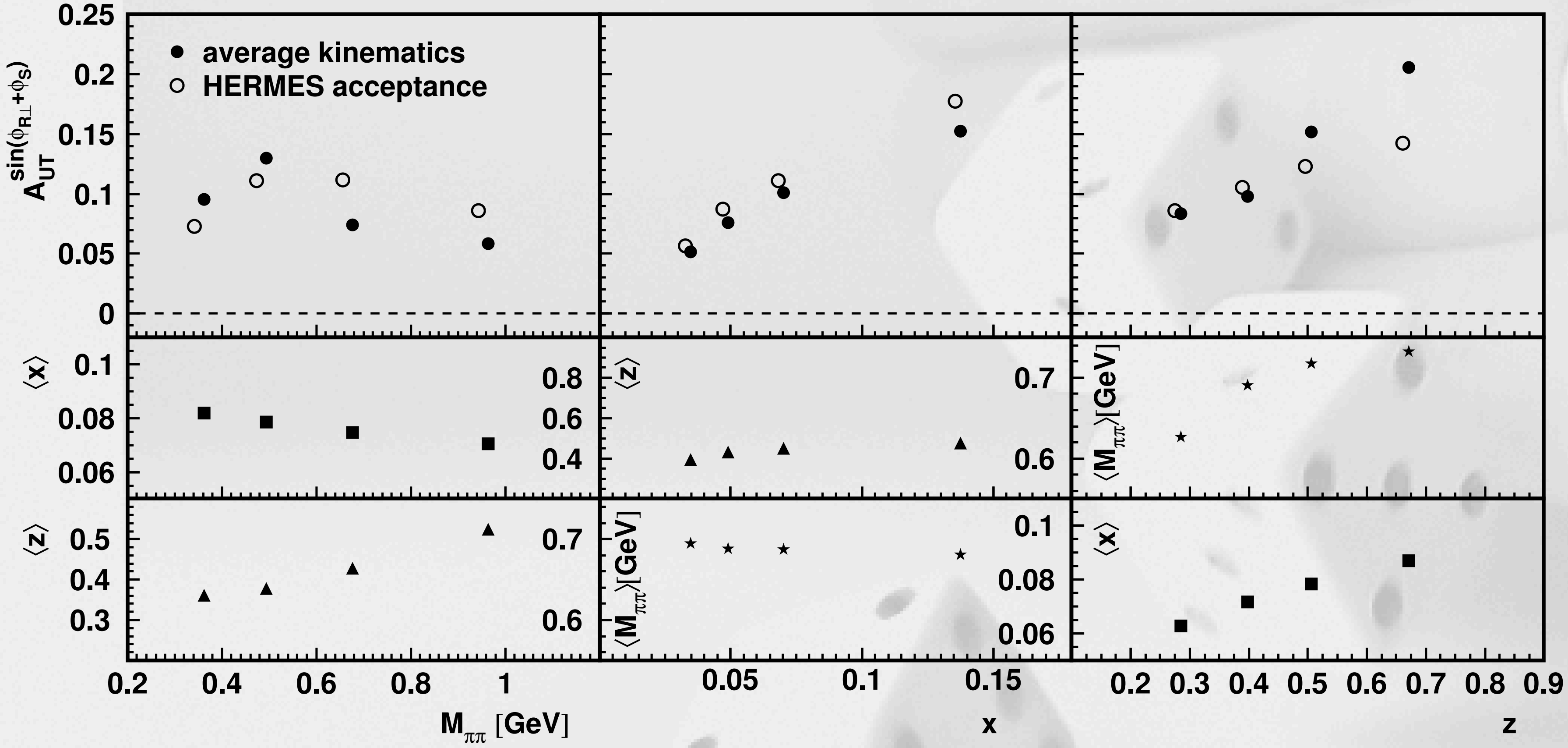
- when plotting data points they can be interpreted as asymmetry
  - at the average kinematics given
  - integrated over kinematic ranges of bin

define your measurement wisely!



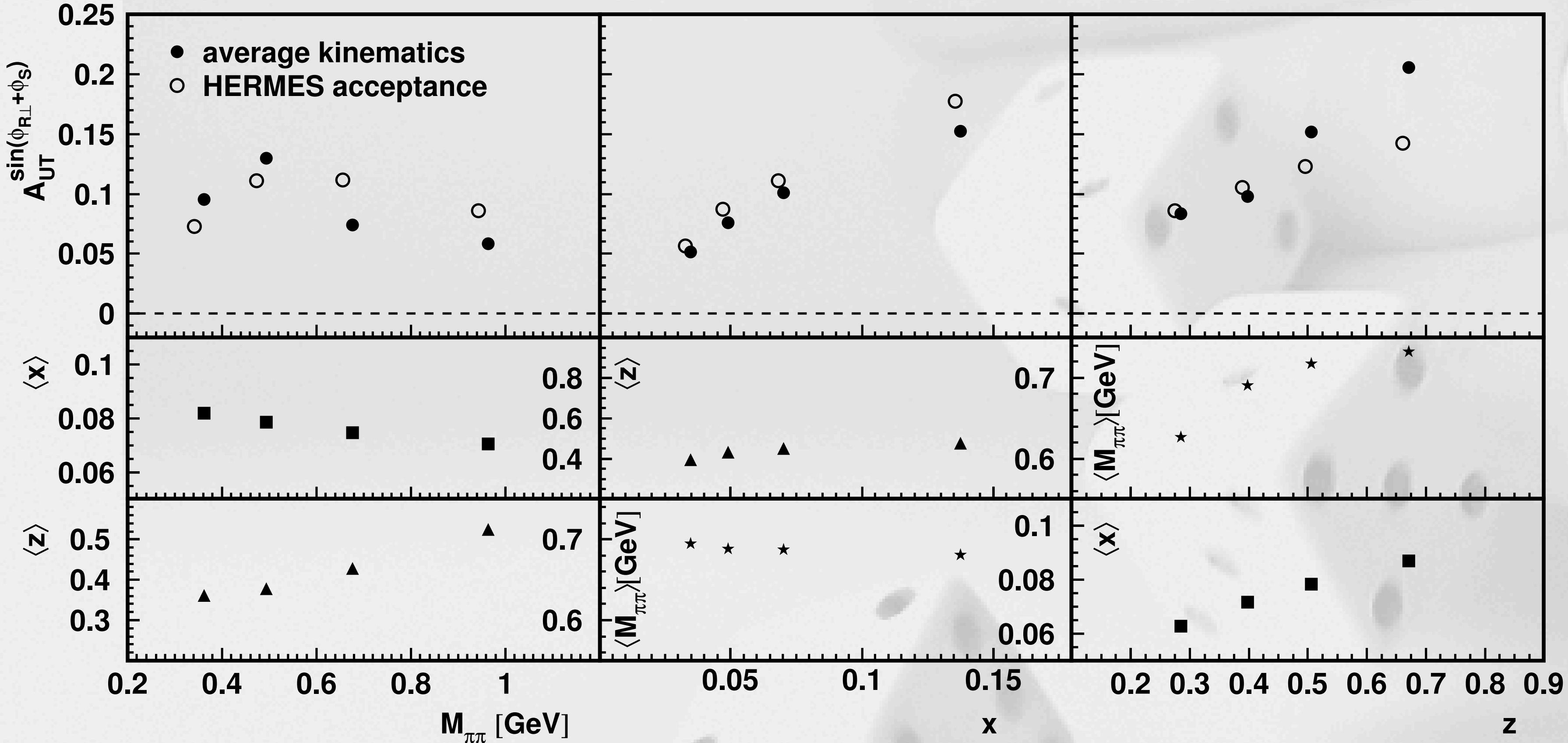
- when plotting data points they can be interpreted as asymmetry
    - at the average kinematics given
    - integrated over kinematic ranges of bin
- results in different systematics -> ideally select the one with smallest systematics

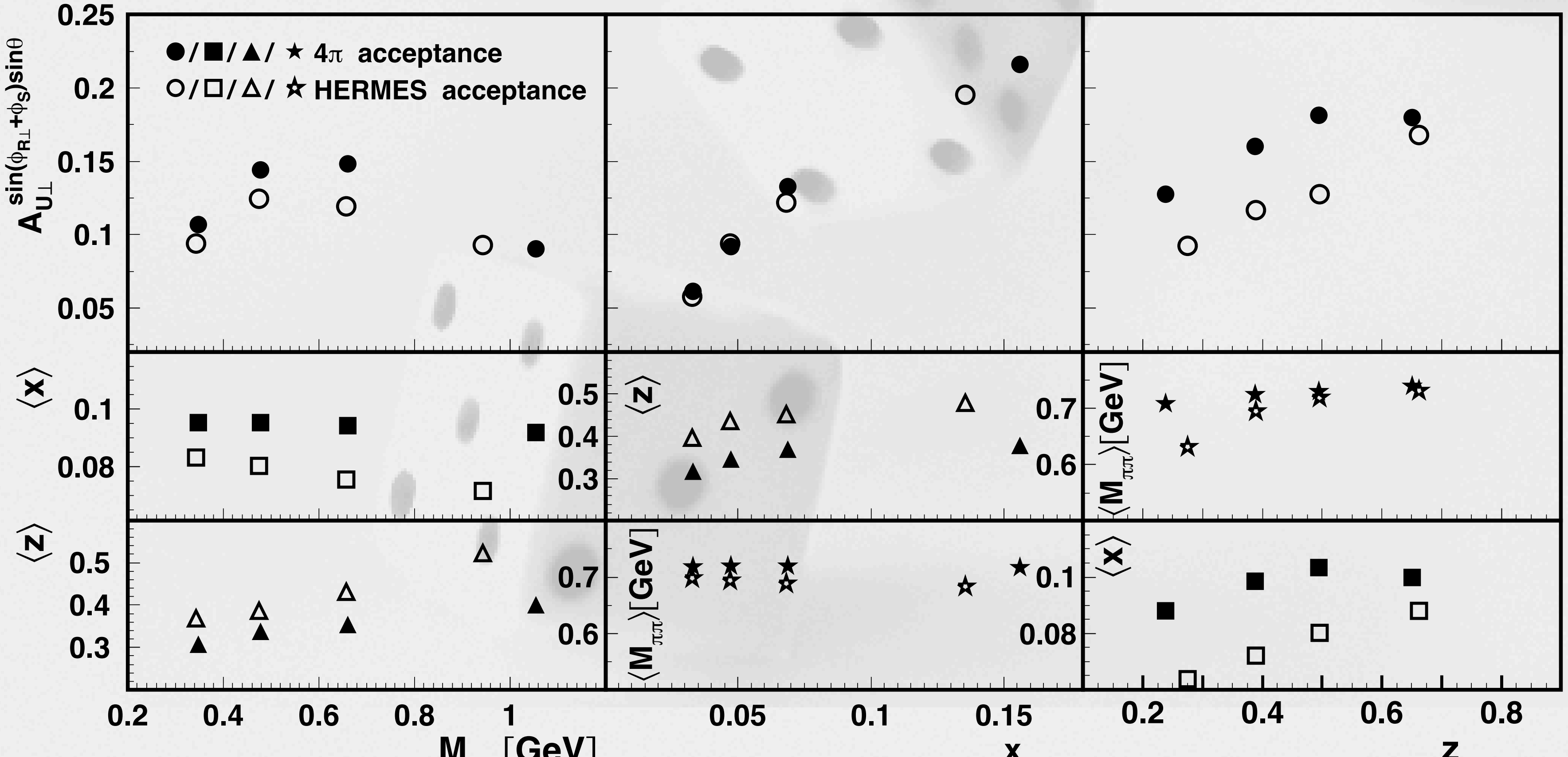
back to di-hadron production



- asymmetries at average kinematics -> large effects with strong model dependence

back to di-hadron production





- asymmetries at average kinematics -> large effects with strong model dependence
- integrated over kinematic range -> still large effects but less model dependent

Unpolarized SIDIS

SIDIS cross section

$$\frac{d^5\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\{F_{UU,T}+\epsilon F_{UU,L}$$

$$+\sqrt{2\epsilon(1-\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h+\epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h\}$$

SIDIS cross section

hadron multiplicity:

normalize to inclusive DIS cross section

$$\frac{d^2\sigma^{\text{incl.DIS}}}{dx\,dy} \propto F_T + \epsilon F_L$$

$$\frac{d^4\mathcal{M}^h(x,y,z,P_{h\perp}^2)}{dx\,dy\,dz\,dP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\frac{F_{UU,T}+\epsilon F_{UU,L}}{F_T+\epsilon F_L}$$

$$\approx \frac{\sum_q e_q^2\, f_1^q(x,p_T^2)\otimes D_1^{q\to h}(z,K_T^2)}{\sum_q e_q^2\, f_1^q(x)}$$

$$\frac{d^5\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\epsilon F_{UU,L}$$

$$+\sqrt{2\epsilon(1-\epsilon)}\,F_{UU}^{\cos\phi_h}\cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h\Big\}$$

SIDIS cross section

hadron multiplicity:
normalize to inclusive DIS cross section

$$\frac{d^2\sigma^{\text{incl.DIS}}}{dxdy} \propto F_T + \epsilon F_L$$

$$\frac{d^4\mathcal{M}^h(x,y,z,P_{h\perp}^2)}{dxdydzdP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\frac{F_{UU,T}+\epsilon F_{UU,L}}{F_T+\epsilon F_L}$$

$$\approx \frac{\sum_q e_q^2\, f_1^q(x,p_T^2)\otimes D_1^{q\to h}(z,K_T^2)}{\sum_q e_q^2\, f_1^q(x)}$$

$$\frac{d^5\sigma}{dxdydzd\phi_h dP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\{F_{UU,T}+\epsilon F_{UU,L}$$

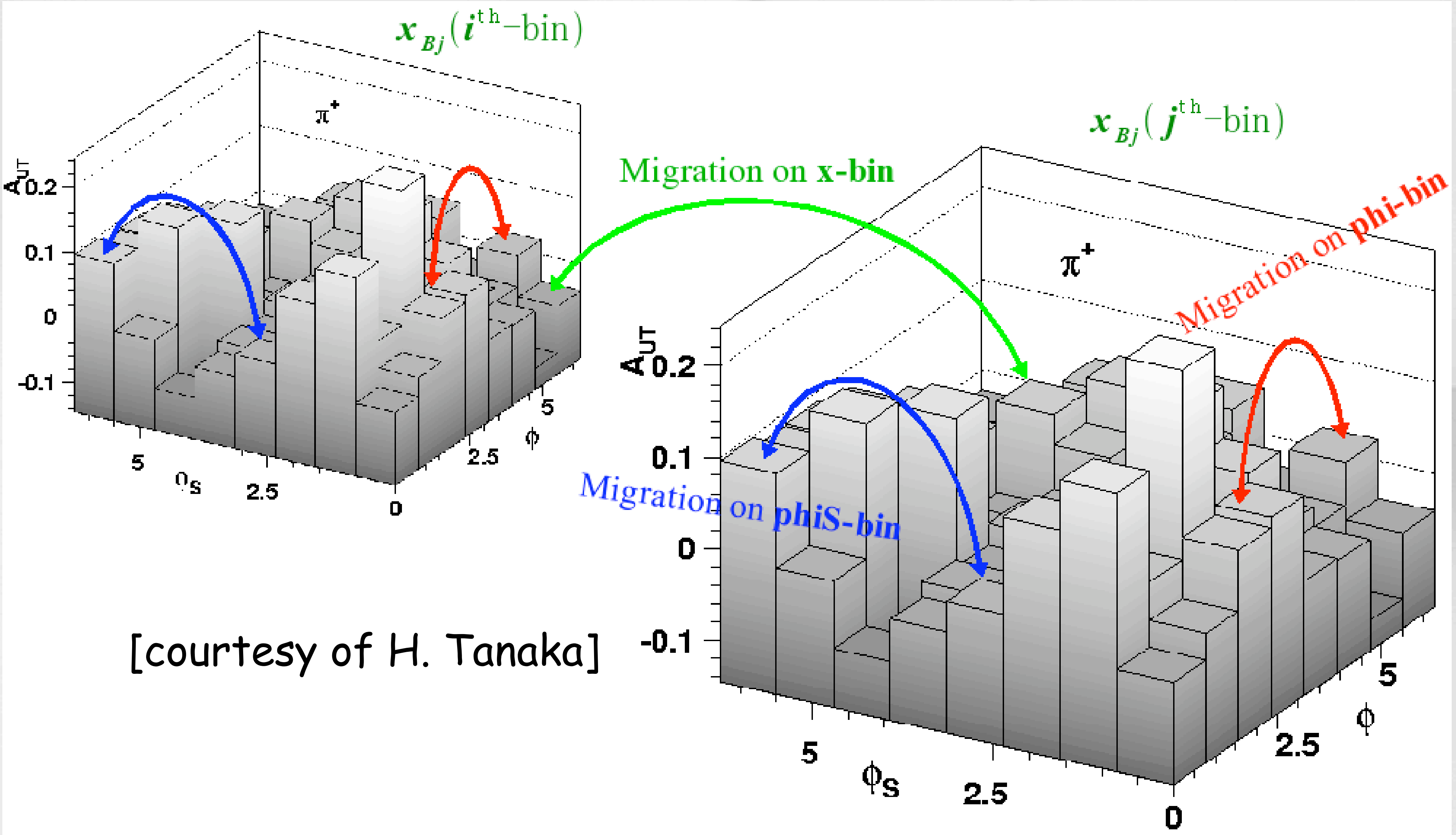
$$+\sqrt{2\epsilon(1-\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h\}$$

$$2\langle\cos 2\phi\rangle_{UU} \equiv 2\frac{\int d\phi_h \cos 2\phi\, d\sigma}{\int d\phi_h d\sigma} = \frac{\epsilon F_{UU}^{\cos 2\phi}}{F_{UU,T}+\epsilon F_{UU,L}}$$

$$\approx \epsilon\frac{\sum_q e_q^2\, h_1^{\perp,q}(x,p_T^2)\otimes_{\text{BM}} H_1^{\perp,q\to h}(z,K_T^2)}{\sum_q e_q^2\, f_1^q(x,p_T^2)\otimes D_1^{q\to h}(z,K_T^2)}$$

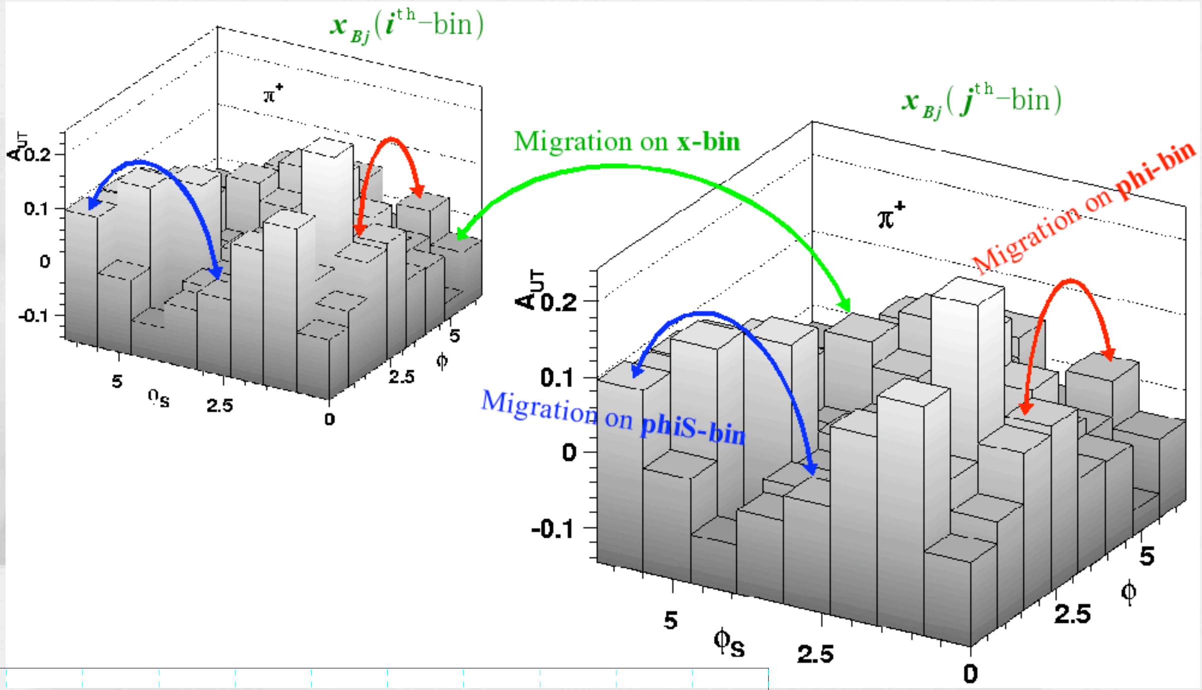
moments:
normalize to azimuth-independent cross-section

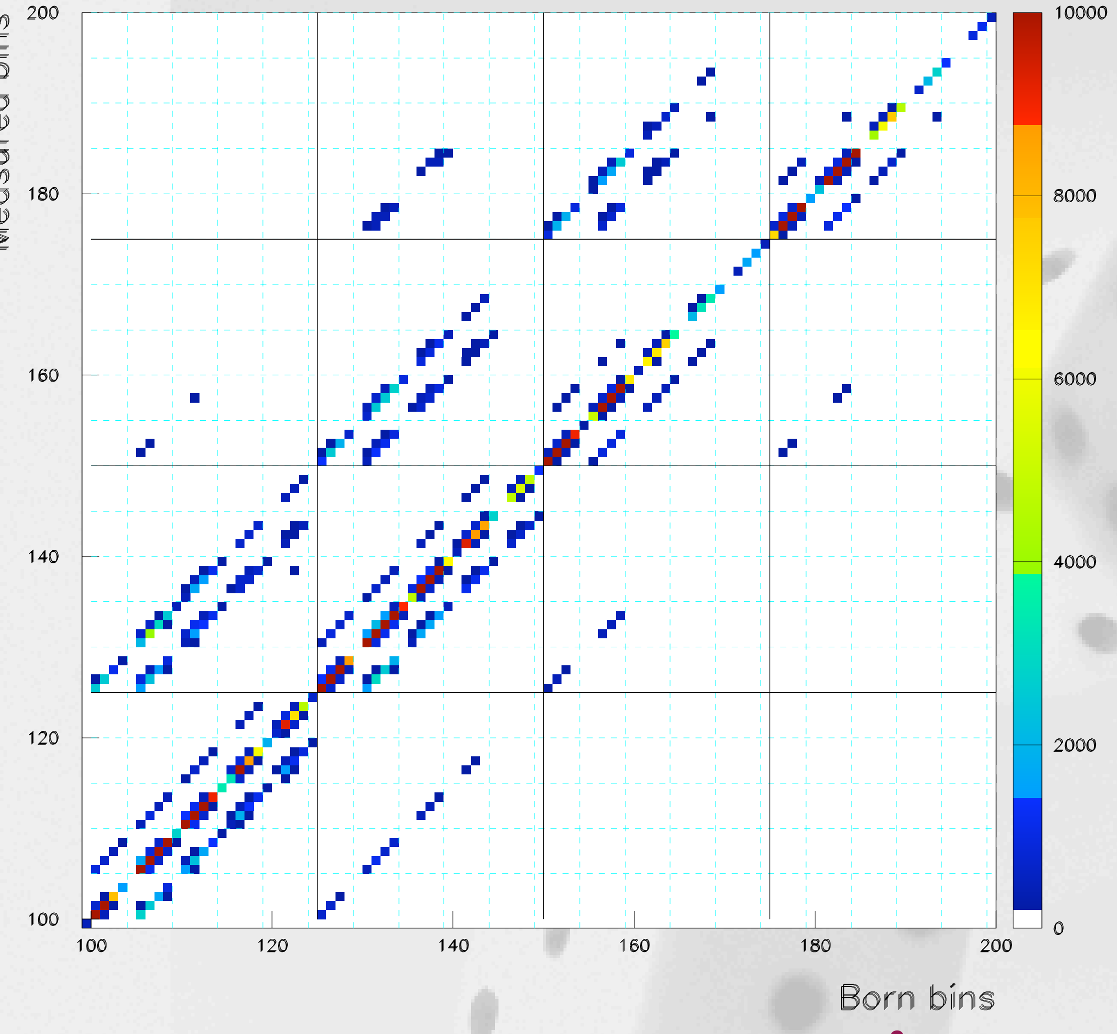
… event migration …

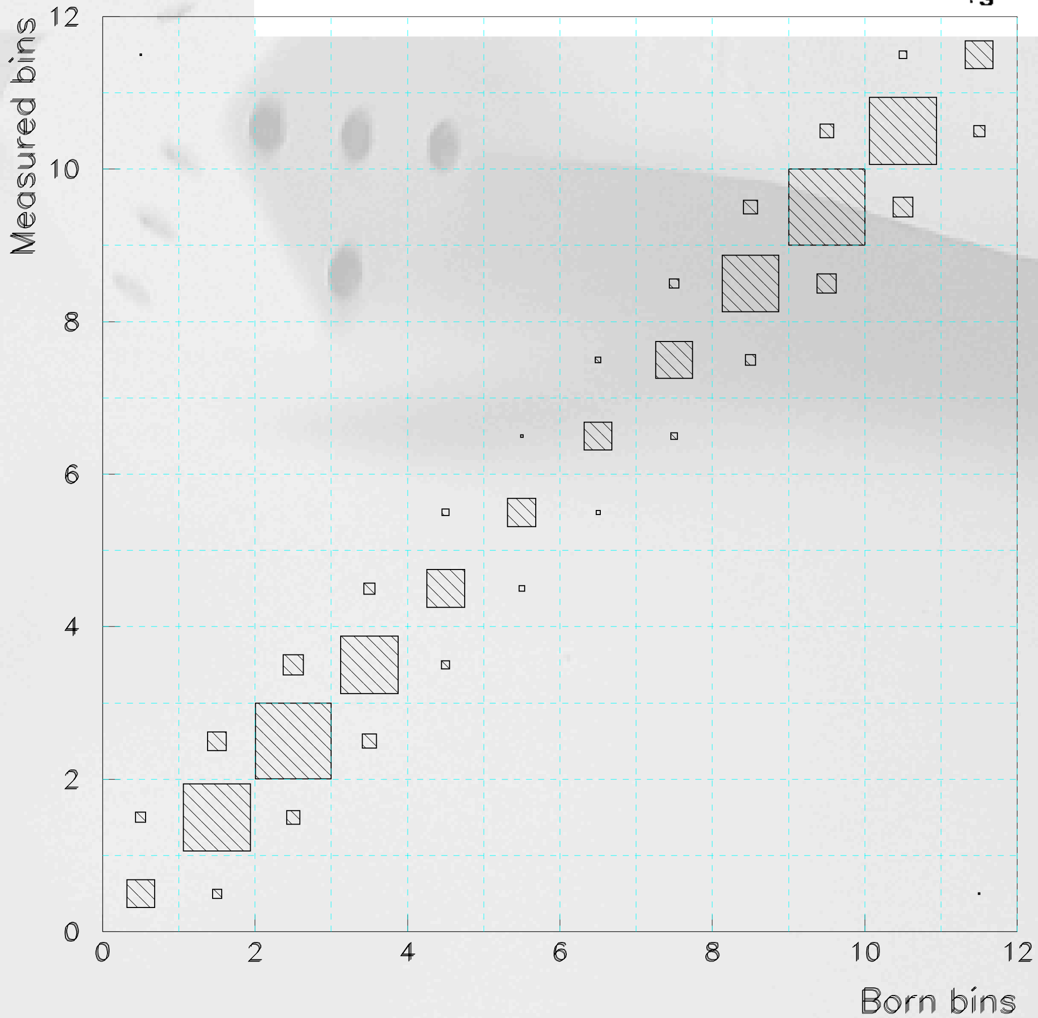


[courtesy of H. Tanaka]

… event migration …

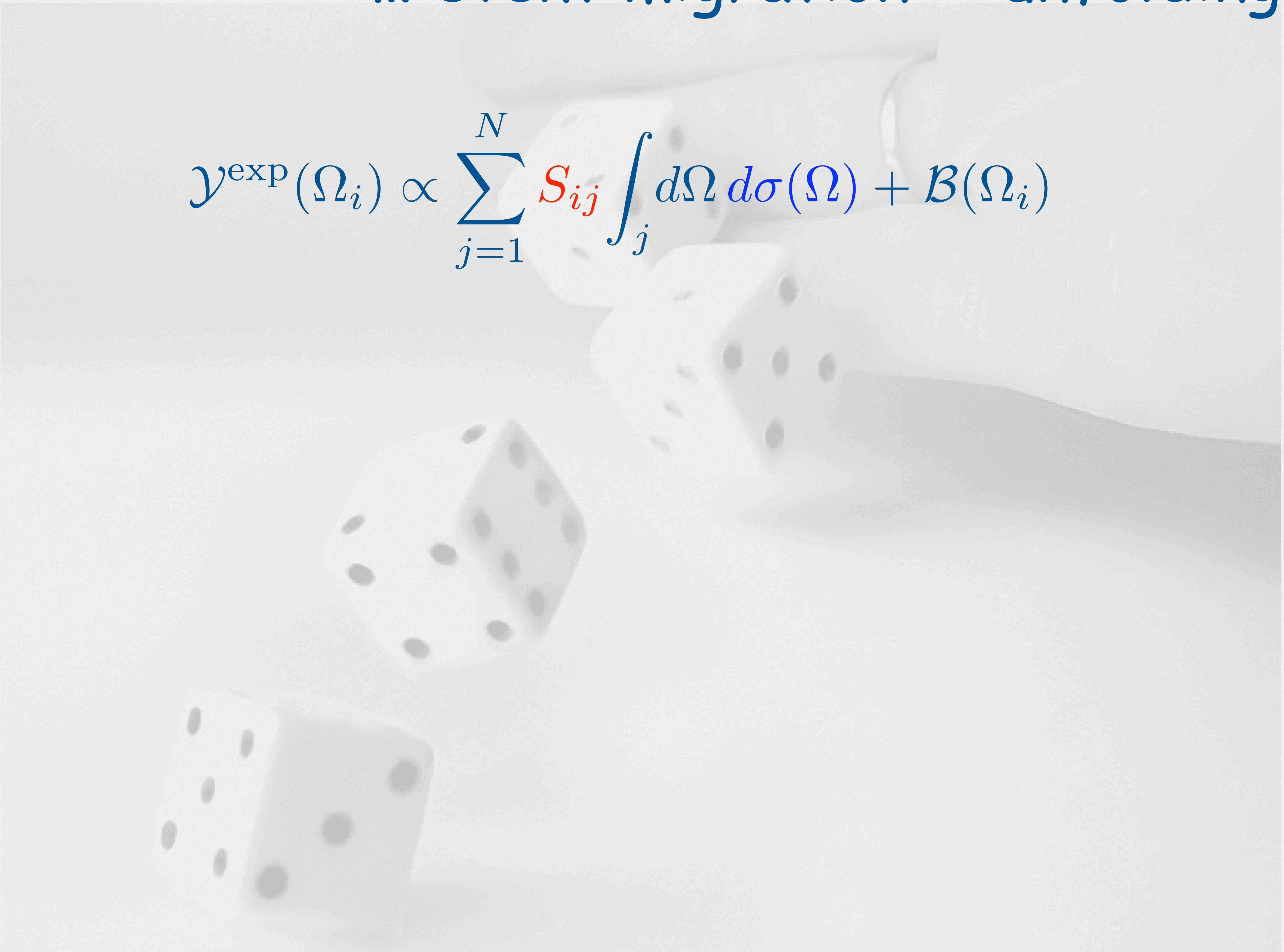






- migration correlates yields in different bins
- can't be corrected properly in bin-by-bin approach

… event migration -> unfolding



$$\mathcal{Y}^{\exp}(\Omega_i) \propto \sum_{j=1}^{N} S_{ij} \int_j d\Omega \, d\sigma(\Omega) + \mathcal{B}(\Omega_i)$$

… event migration -> unfolding

$$\mathcal{Y}^{\text{exp}}(\Omega_i) \propto \sum_{j=1}^{N} S_{ij} \int_j d\Omega \, d\sigma(\Omega) + \mathcal{B}(\Omega_i)$$

- experimental yield in i^{th} bin depends on all Born bins j …

… event migration -> unfolding

$$\mathcal{Y}^{\mathrm{exp}}(\Omega_i) \propto \sum_{j=1}^{N} S_{ij} \int_j d\Omega\, d\sigma(\Omega) + \mathcal{B}(\Omega_i)$$

- experimental yield in i^{th} bin depends on all Born bins j …
- … and on BG entering kinematic range from outside region

… event migration -> unfolding

$$\mathcal{Y}^{\exp}(\Omega_i) \propto \sum_{j=1}^{N} S_{ij} \int_j d\Omega\, d\sigma(\Omega) + \mathcal{B}(\Omega_i)$$

- experimental yield in i^{th} bin depends on all Born bins j …
- … and on BG entering kinematic range from outside region
- smearing matrix S_{ij} embeds information on migration
  - determined from Monte Carlo - independent of physics model in limit of infinitesimally small bins and/or flat acceptance/cross-section in every bin
  - in real life: dependence on BG and physics model due to finite bin sizes

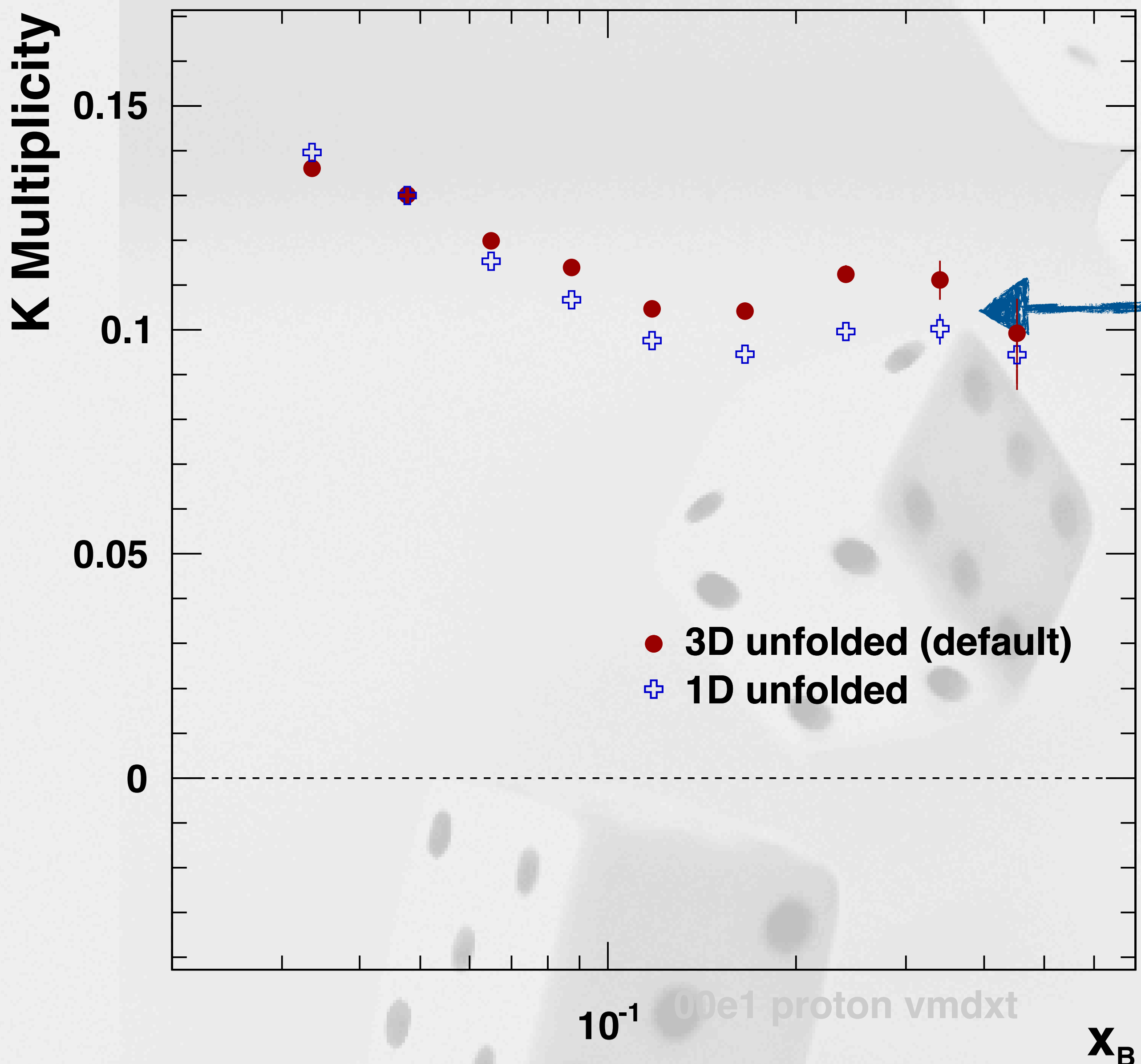
… event migration -> unfolding

$$\mathcal{Y}^{\mathrm{exp}}(\Omega_i) \propto \sum_{j=1}^{N} S_{ij} \int_j d\Omega\, d\sigma(\Omega) + \mathcal{B}(\Omega_i)$$

- experimental yield in i[th] bin depends on all Born bins j …
- … and on BG entering kinematic range from outside region
- smearing matrix S_{ij} embeds information on migration
  - determined from Monte Carlo - independent of physics model in limit of infinitesimally small bins and/or flat acceptance/cross-section in every bin
  - in real life: dependence on BG and physics model due to finite bin sizes
- inversion of relation gives Born cross section from measured yields

Multi-D vs. 1D unfolding at work

[S.J. Joosten, PhD thesis UIUC (2013)]

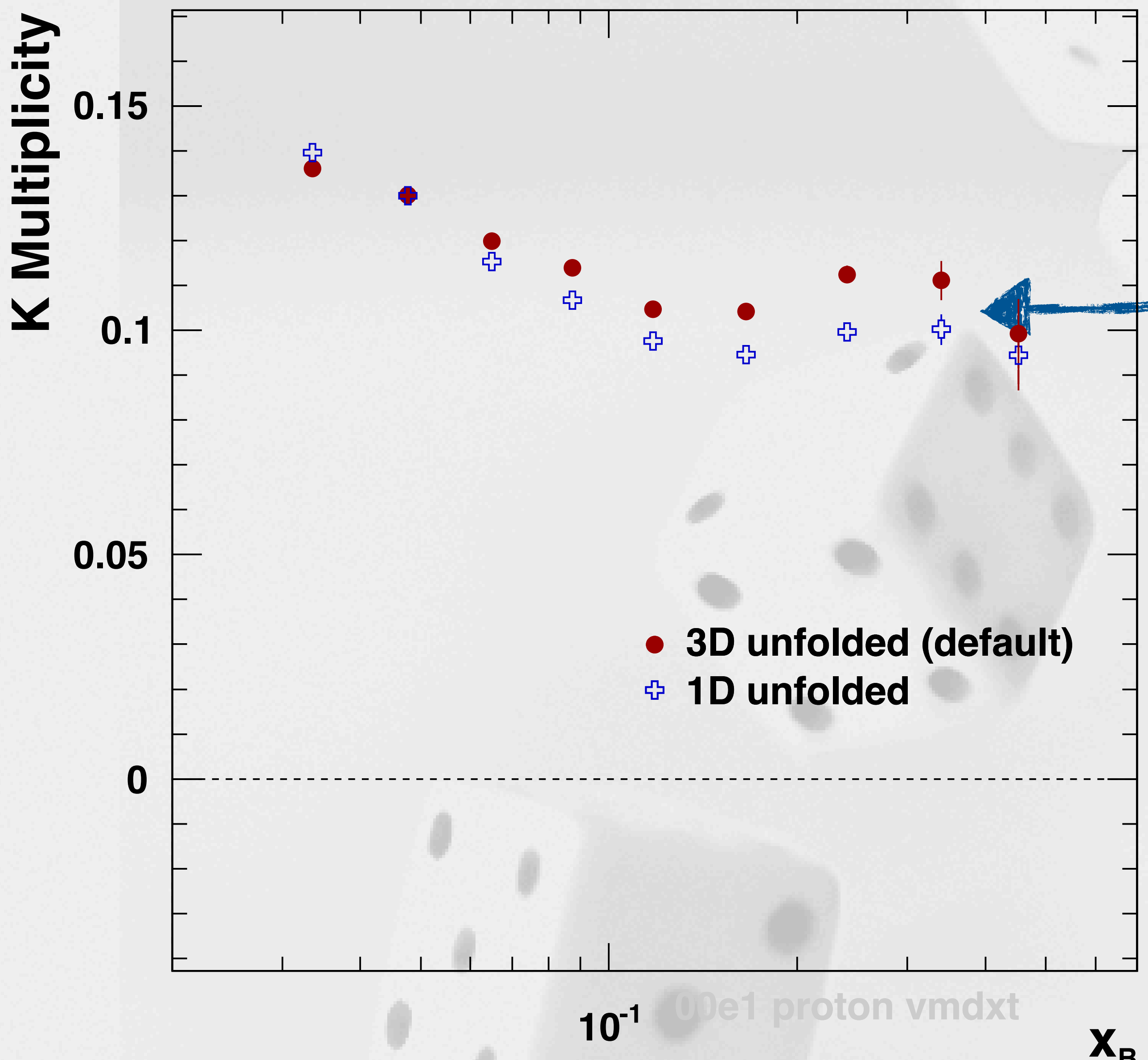


Neglecting to unfold in z changes x dependence dramatically

➡ 1D unfolding clearly insufficient

Multi-D vs. 1D unfolding at work

[S.J. Joosten, PhD thesis UIUC (2013)]

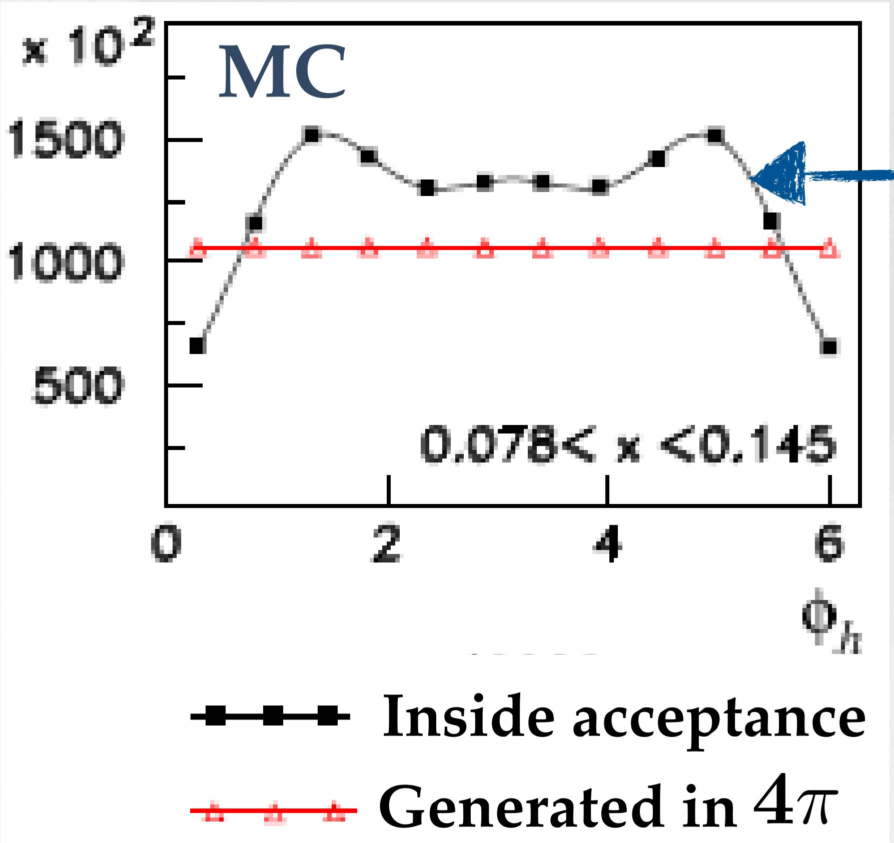


Neglecting to unfold in z changes x dependence dramatically

➡ 1D unfolding clearly insufficient

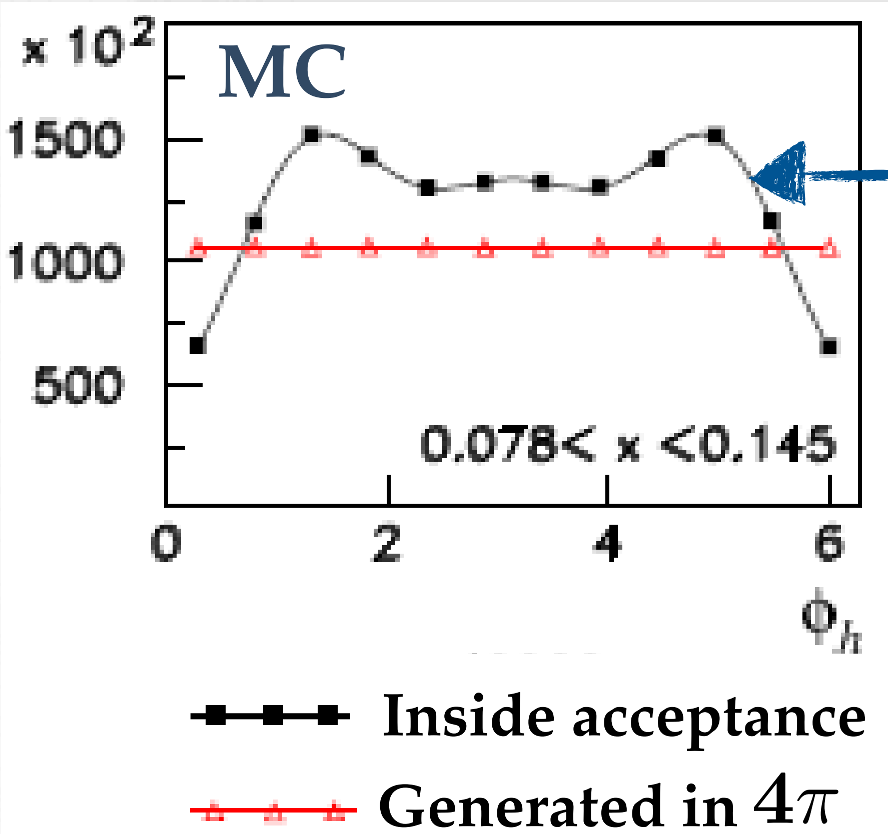
even though only interested in collinear observable, need to carefully consider transverse d.o.f.

Multi-D vs. 1D unfolding at work

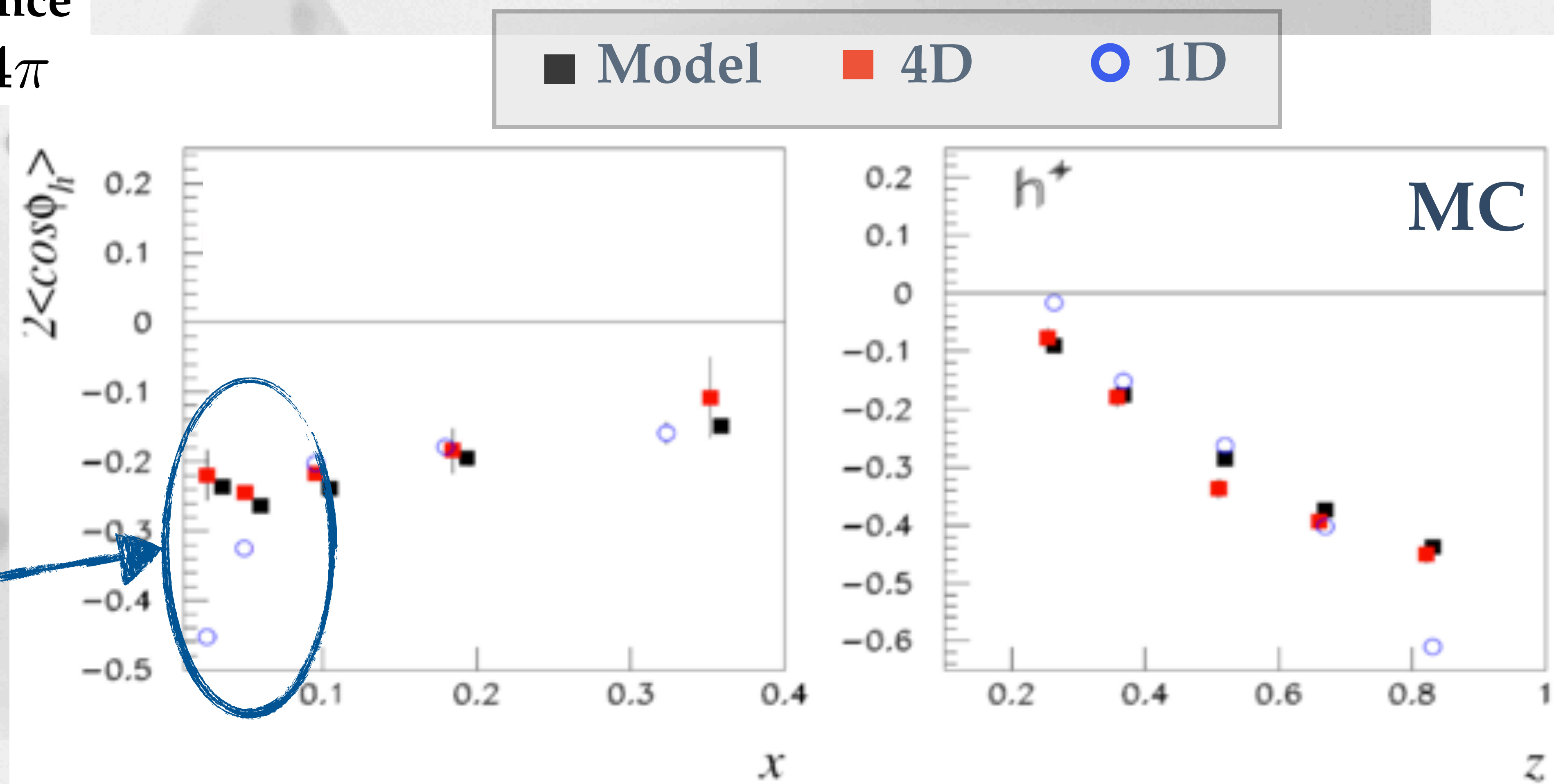


simulated yield with clear cosine modulations from migration and acceptance

Multi-D vs. 1D unfolding at work



simulated yield with clear cosine modulations from migration and acceptance



1D clearly not sufficient

summary

- acceptance plays crucial part, especially in analysis of multi-particle final states, and that even for asymmetries
- acceptance studies and/or corrections (e.g., unfolding) require realistic Monte Carlo simulation of underlying physics
- $\mathrm{GMC}_{\mathrm{TRANS}}$ provides reasonably realistic description of Collins and Sivers amplitudes for pions and kaons based on Gaussian Ansatz
- reshuffling PYTHIA events, guided by, e.g., real data, provides a powerful tool to study systematics
  - still relies on good description of unpolarized cross section
- make a careful choice of how data points are to be interpreted (at average kinematics or average over kinematic range)
  - evaluate systematics accordingly
- fully differential analyses clearly preferred, though more challenging
- need realistic MC for reliable corrections/systematics

summary

- acceptance plays crucial part, especially in analysis of multi-particle final states, and that even for asymmetries
- acceptance studies and/or corrections (e.g., unfolding) require realistic Monte Carlo simulation of underlying physics
- GMC$_{\text{TRANS}}$ provides reasonably realistic description of Collins and Sivers amplitudes for pions and kaons based on Gaussian Ansatz
- reshuffling PYTHIA events, guided by, e.g., real data, provides a powerful tool to study systematics
  - still relies on good description of unpolarized cross section
- make a careful choice of how data points are to be interpreted (at average kinematics or average over kinematic range)
  - evaluate systematics accordingly
- fully differential analyses clearly preferred, though more challenging
- need realistic MC for reliable corrections/systematics -> **discussion**