Comment on evolution of Sum Rules for TMDs

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- Cover of whole K_T region required
- Yet another TMD evolution (Bassetto, Ciafaloni Marchesini, 80; Ceccopieri, Trentadue,2006): DGLAP with controlled TM

$$\frac{\partial}{\partial \ln Q^2} \mathcal{F}_P^i(x_B, Q^2, \boldsymbol{k}_\perp) = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^3} P_{ji}(u, \alpha_s(Q^2)) \int \frac{d^2\boldsymbol{q}_\perp}{\pi} \delta((1-u)Q^2 - q_\perp^2)\, \mathcal{F}_P^j\Big(\frac{x_B}{u}, Q^2, \frac{\boldsymbol{k}_\perp - \boldsymbol{q}_\perp}{u}\Big)$$

- Collins and Sivers: Ratcliffe, OT, 1406.1604

$$\frac{\partial}{\partial \ln Q^2} f_S^i(x_B, Q^2, k_\perp^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^4} P_{ji}(u, \alpha_s(Q^2)) \times \int \frac{d^2\boldsymbol{q}_\perp}{\pi} \delta((1-u)Q^2 - q_\perp^2)\, \frac{k_\perp^2 - q_\perp^2 + (\boldsymbol{k}_\perp - \boldsymbol{q}_\perp)^2}{2k_\perp^2} f_S^j\Big(\frac{x_B}{u}, Q^2, \frac{(\boldsymbol{k}_\perp - \boldsymbol{q}_\perp)^2}{u^2}\Big).$$

Sum Rules

- Moment – like xq(x)

$$\frac{\partial}{\partial \ln Q^2}\int d^2\boldsymbol{k}_\perp k_\perp^2 f_S^i(x_B,Q^2,k_\perp^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_{x_B}^1 \frac{du}{u}\Big[uP_{ji}(u,\alpha_s(Q^2))\Big]\int d^2\boldsymbol{p}_\perp p_\perp^2\, f_S^j\Big(\frac{x_B}{u},Q^2,p_\perp^2\Big).$$

- Burkardt ~ Longitudinal momentum SR
- Asymptotically singlet quarks and gluons → 0
- Collins:

$$\frac{\partial}{\partial \ln Q^2} D_T^i(z,Q^2,k_\perp^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_z^1 \frac{du}{u}\,\delta P(u,\alpha_s(Q^2))$$

$$\times \int \frac{d^2\boldsymbol{q}_\perp}{\pi}\,\delta(u(1-u)Q^2-q_\perp^2)\,\frac{k_\perp^2-(\frac{z}{u})^2q_\perp^2+(\boldsymbol{k}_\perp-\frac{z}{u}\boldsymbol{q}_\perp)^2}{2k_\perp^2}\,D_T^i\Big(\frac{z}{u},Q^2,(\boldsymbol{k}_\perp-\frac{z}{u}\boldsymbol{q}_\perp)^2\Big).$$

- Moment

$$\frac{\partial}{\partial \ln Q^2}\int d^2\boldsymbol{k}_\perp k_\perp^2 D_T^i(z,Q^2,k_\perp^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_z^1 \frac{du}{u^2}\Big[u\,\delta P(u,\alpha_s(Q^2))\Big]\int d^2\boldsymbol{p}_\perp p_\perp^2\, D_T^i\Big(\frac{z}{u},Q^2,p_\perp^2\Big).$$

Discussion

- SR for Collins functions (Schaefer,OT,2000, explains sign difference of favoured and unfavoured) –due to boundary condition like for g2
- Direct test from twist-3 evolution (for Sivers – T(x,x))
- Variety of evolution schemes may be non-occasional: limited region of applicability, matching with other schemes and extrapolation to the new region