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# $A_N$ in proton-proton collisions and the role of twist-3 fragmentation


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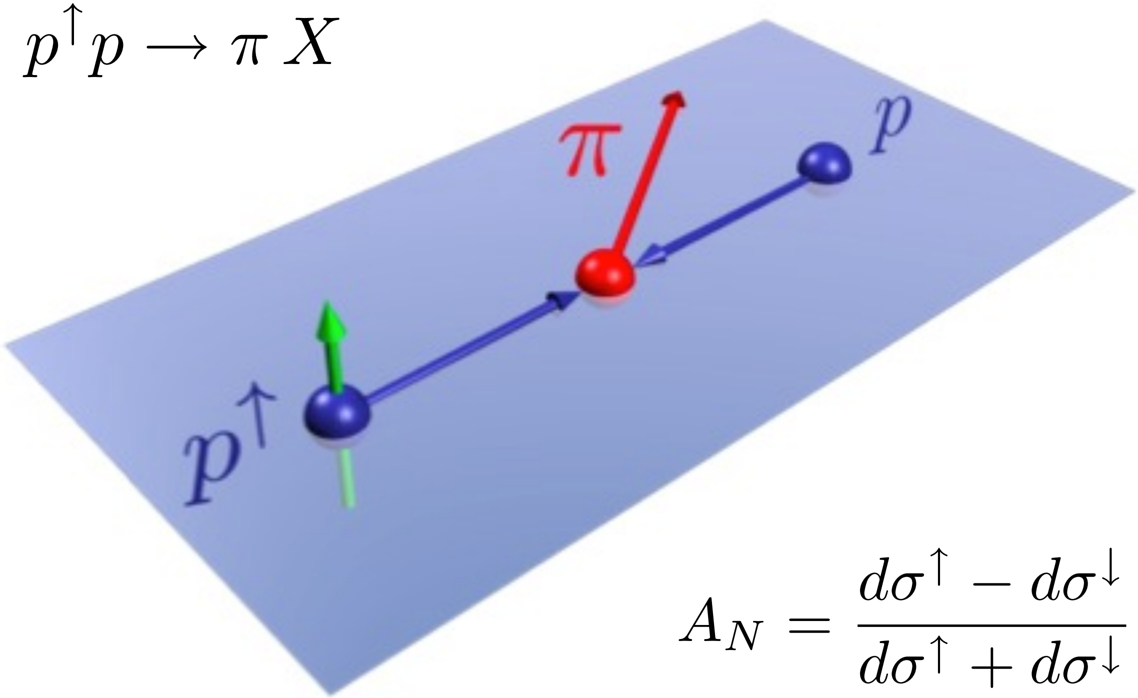
# Outline

- Motivation
  - What are transverse single-spin asymmetries (TSSAs)?
  - Collinear twist-3 formalism
    (Note: also work done in TMD approach – see, e.g., Anselmino, et al., PRD **86** (2012), PRD **88** (2013))
- A puzzle with TSSAs
  - "Sign mismatch" between the Qiu-Sterman function and the Sivers function
  - Insight from TSSAs in inclusive DIS
  - The role of twist-3 fragmentation in TSSAs
- Summary and outlook

# Motivation

➢ TSSAs in proton-proton collisions

$$p^{\uparrow}p \to \pi\, X$$




$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

Data available from RHIC (BRAHMS, PHENIX, STAR), FNAL (E704, E581), and AGS

(Figure thanks to K. Kanazawa)

## ➢ Collinear twist-3 formalism

$$
\begin{aligned}
d\sigma = \; & H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
& + H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\
& + H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
\end{aligned}
$$

Collinear twist-3 approach

(Efremov and Teryaev (1982, 1985); Qiu and Sterman (1992, 1999))

$P_{hT} >> \Lambda_{QCD}$

- Collinear twist-3 formalism

$$d\sigma = H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)}$$
$$+ H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}$$
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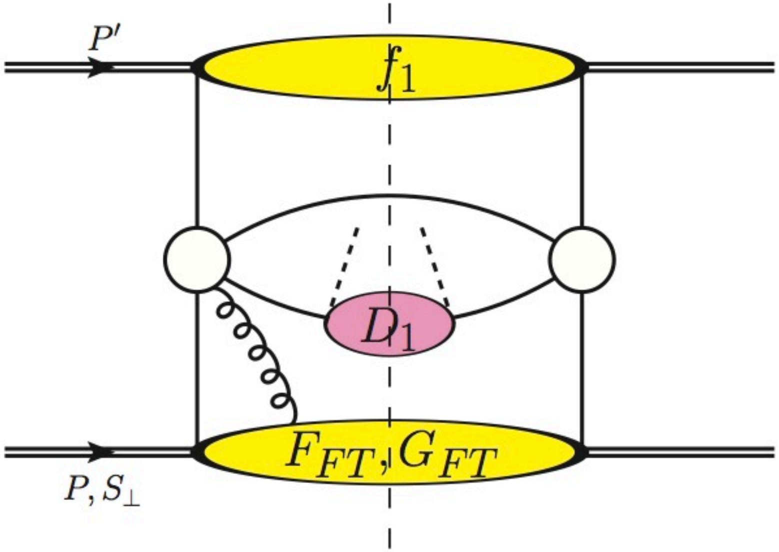
Collinear twist-3 approach

(Efremov and Teryaev (1982, 1985); Qiu and Sterman (1992, 1999))

$P_{hT} >> \Lambda_{QCD}$




– T-odd effect ➡ need to generate an imaginary part ➡ soft-gluon pole (SGP) or soft-fermion pole (SFP) ➡ internal particle goes on-shell

– One can also have SGPs with tri-gluon correlations

- SGP term (Qiu and Sterman (1999), Kouvaris, et al. (2006)):

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^1 \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x')$$
$$\times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n\bar{n}}}{z\hat{u}}\right) \frac{1}{x} \left[ T_{a,F}(x,x) - x\left(\frac{d}{dx} T_{a,F}(x,x)\right)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

Qiu-Sterman function

- SFP term (Koike and Tomita (2009); Kanazawa and Koike (2011)):

$$E_h \frac{d^3\Delta\sigma^{\rm SFP}}{dP_h^3} = \frac{\alpha_s^2}{S} \frac{M_N \pi}{2} \epsilon^{pnP_hS_\perp} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'} \int \frac{dx}{x} \frac{1}{x'S + T/z} \delta\left(x - \frac{-x'U/z}{x'S + T/z}\right)$$
$$\times \Bigg[ \sum_{a,b,c} \left(G_F^a(0,x) + \tilde{G}_F^a(0,x)\right) \{ q^b(x') \left(D^c(z)\hat{\sigma}_{ab\to c} + D^{\bar{c}}(z)\hat{\sigma}_{ab\to\bar{c}}\right)$$
$$+ q^{\bar{b}}(x') \left(D^c(z)\hat{\sigma}_{a\bar{b}\to c} + D^{\bar{c}}(z)\hat{\sigma}_{a\bar{b}\to\bar{c}}\right) \}$$
$$+ \sum_{a,b} \left(G_F^a(0,x) + \tilde{G}_F^a(0,x)\right) \left(q^b(x') D^g(z)\hat{\sigma}_{ab\to g} + q^{\bar{b}}(x') D^g(z)\hat{\sigma}_{a\bar{b}\to g}\right)$$
$$+ \sum_{a,c} \left(G_F^a(0,x) + \tilde{G}_F^a(0,x)\right) G(x') \left(D^c(z)\hat{\sigma}_{ag\to c} + D^{\bar{c}}(z)\hat{\sigma}_{ag\to\bar{c}}\right)$$
$$+ \sum_{a} \left(G_F^a(0,x) + \tilde{G}_F^a(0,x)\right) G(x') D^g(z)\hat{\sigma}_{ag\to g} \Bigg]$$

$$T_F \sim G_F \sim F_{FT}$$
$$\tilde{T}_F \sim \tilde{G}_F \sim G_{FT}$$

- Tri-gluon correlators (Beppu, Kanazawa, Koike, Yoshida (2013)): (see talk by Y. Koike)

$$E_{P_h}\frac{d^3\Delta\sigma}{d^3P_h} = \frac{2\pi M_N\alpha_s^2}{S}\epsilon^{P_h p n S_\perp}\sum_{i,j}\int\frac{dx}{x}\int\frac{dx'}{x'}f_i(x')\int\frac{dz}{z^2}D_j(z)\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{z\hat{u}}$$

$$\times\Big[\zeta_{ij}\Big(\frac{d}{dx}O(x)-\frac{2O(x)}{x}\Big)\hat{\sigma}^{(O)}_{gi\to j}+\Big(\frac{d}{dx}N(x)-\frac{2N(x)}{x}\Big)\hat{\sigma}^{(N)}_{gi\to j}\Big]$$

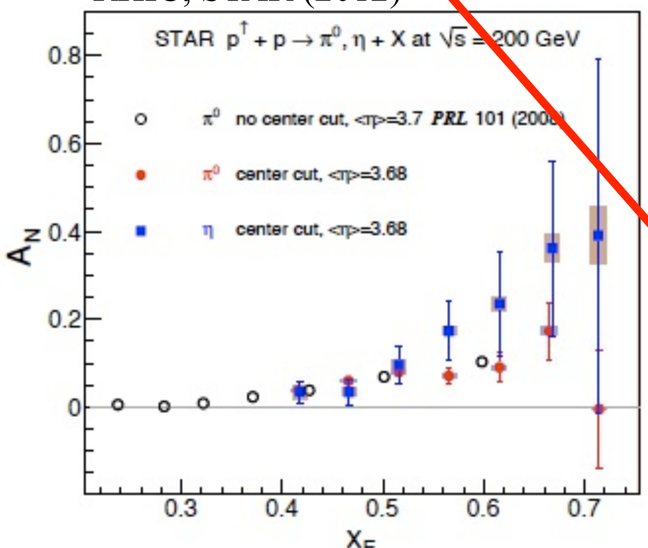
➡ For many years the SGP term involving the Qiu-Sterman function was thought to be the dominant contribution to TSSAs in $p^\uparrow p \to hX$

## ➢ A puzzle with TSSAs (the "sign mismatch" issue)

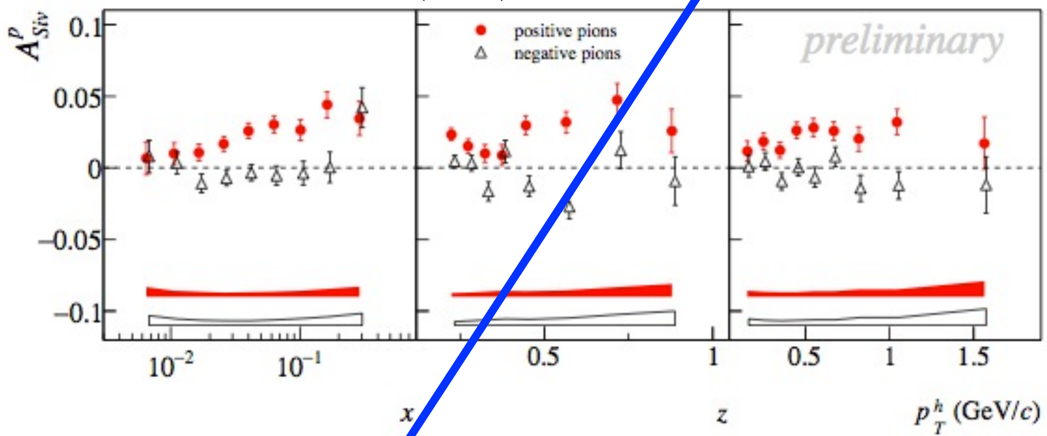
$$p^{\uparrow}p \to h\,X$$

$$\ell N^{\uparrow} \to \ell'\,h\,X$$

RHIC, STAR (2012)



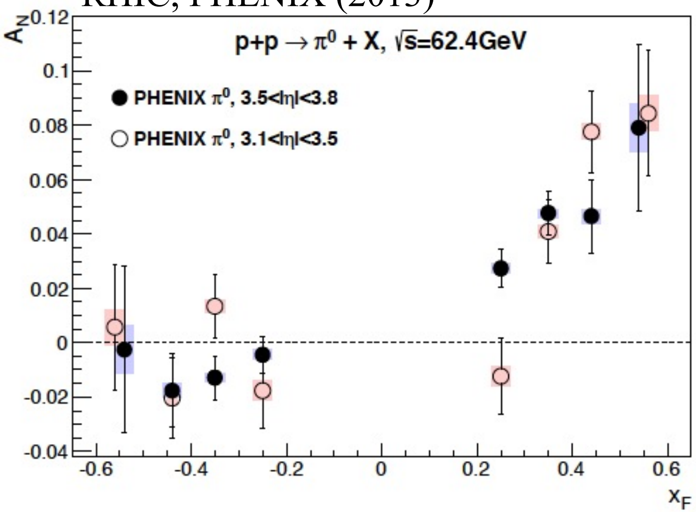

CERN, COMPASS (2013)




$$\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

$$F_{FT} \sim T_F$$

RHIC, PHENIX (2013)



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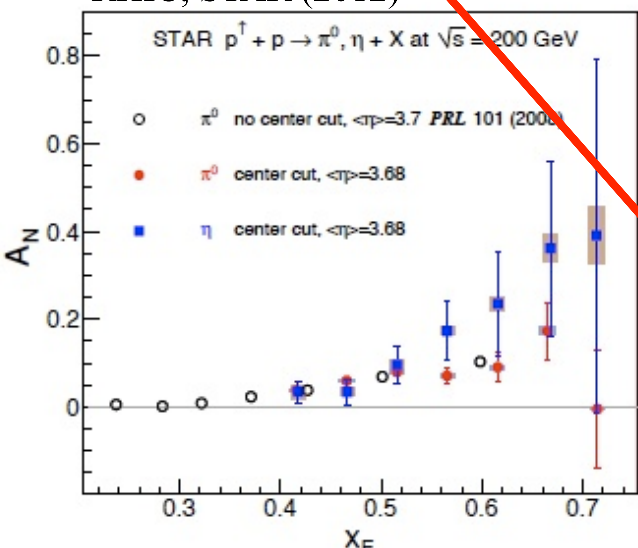
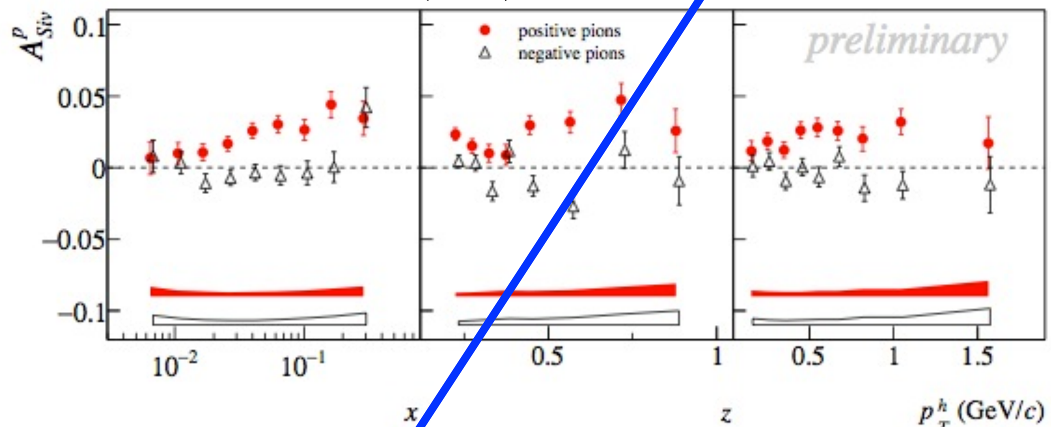



CERN, COMPASS (2013)

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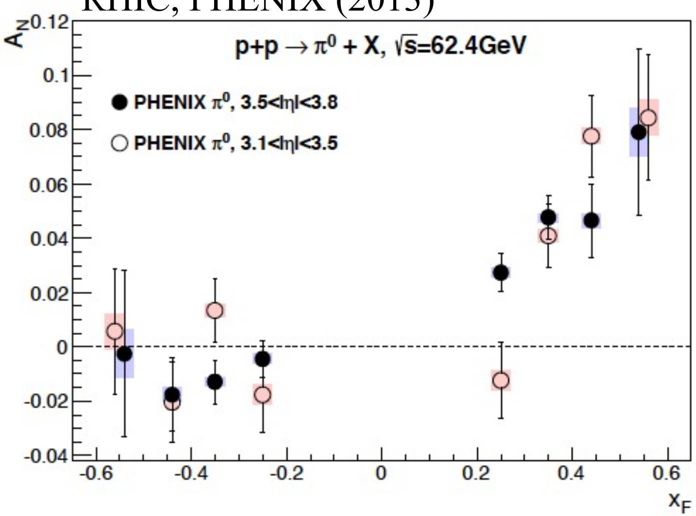
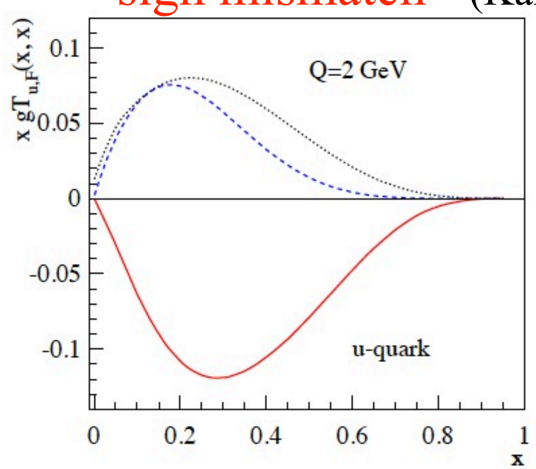
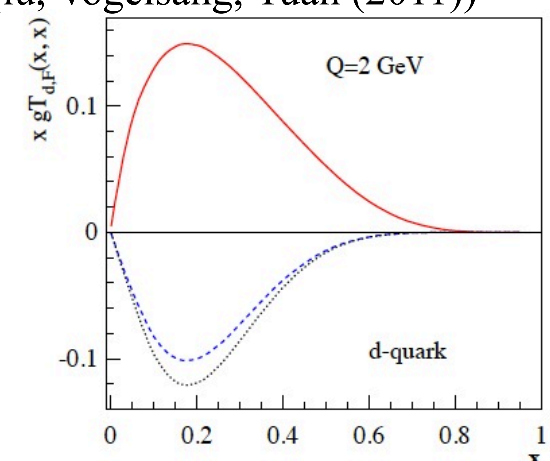
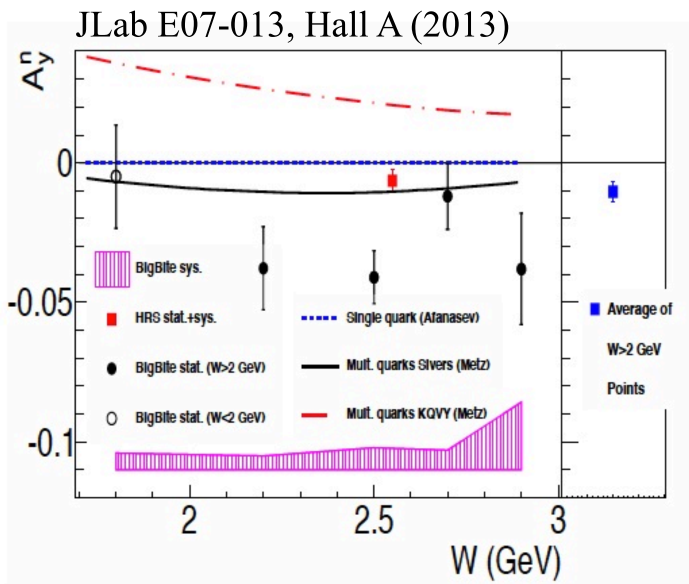



"sign mismatch" (Kang, Qiu, Vogelsang, Yuan (2011))



- TSSA in inclusive DIS (Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou - PRD **86** (2012))




Neutron TSSA

**Sivers** input agrees reasonably well with the JLab data

➡ Node in $k_T$ for the Sivers function can be ruled out/Also node in $x$ is disfavored from proton data from HERMES (see also Kang and Prokudin (2012))

➡ FIRST INDICATION that the Sivers effect is intimately connected to the re-scattering of the active parton with the target remnants (PROCESS DEPENDENT)

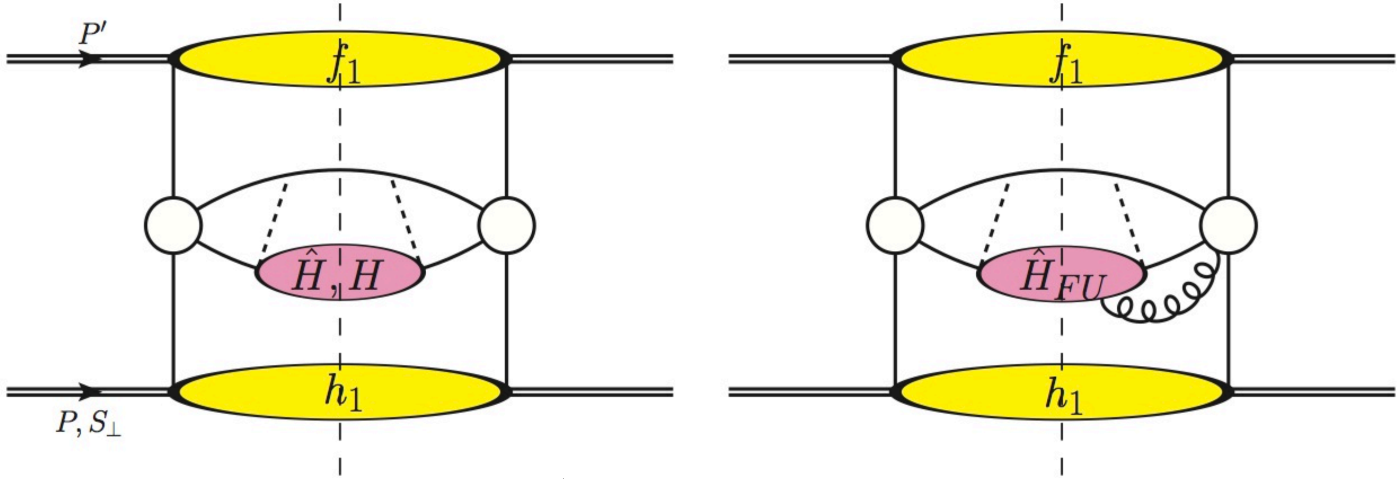
**KQVY** input gives the wrong sign ➡ SGP contribution on the side of the transversely polarized incoming proton cannot be the main cause of the large TSSAs seen in pion production (i.e., $T_F(x,x)$ term)

$$d\sigma = H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)}$$
$$+ \, H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}$$
$$+ \, H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)}$$

Negligible (Kanazawa and Koike (2000))

$$+ H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)}$$

- Collinear twist-3 fragmentation term:




$$\hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^{\,2}}{2M_h^2} H_1^{\perp h/q}(z, z^2\vec{k}_\perp^{\,2})$$ Collins-type function

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q,\Im}(z, z_1) = H^{h/q}(z) + 2z\hat{H}^{h/q}(z)$$ 3-parton correlator

➡ There are 2 independent (unpolarized) collinear twist-3 FFs

**Collinear twist-3 fragmentation structure is richer than that for the TMD formalism**

– Calculation of twist-3 fragmentation term (Metz and DP - PLB **723** (2013))

$$\frac{P_h^0 d\sigma_{pol}}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\,\epsilon_{\perp\mu\nu}\,S_\perp^\mu P_{h\perp}^\nu \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'} \frac{1}{x'S+T/z} \frac{1}{-x\hat{u}-x'\hat{t}}$$
$$\times \frac{1}{x}\,h_1^a(x)\,f_1^b(x') \left\{ \left( \hat{H}^{C/c}(z) - z\frac{d\hat{H}^{C/c}(z)}{dz} \right) S_{\hat{H}}^i + \frac{1}{z}\,H^{C/c}(z)\,S_H^i \right.$$
$$\left. + 2z^2 \int \frac{dz_1}{z_1^2}\,PV\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{C/c,\Im}(z,z_1)\,\frac{1}{\xi}\,S_{\hat{H}_{FU}}^i \right\}$$

➡ First time we have a complete pQCD result for this term in *pp* within the collinear twist-3 approach

➡ Also has been studied for TSSA in SIDIS (Kanazawa and Koike (2013))

➡ "Derivative term" has been calculated previously (Kang, Yuan, Zhou (2010))

➡ Derivative and non-derivative piece combine into a "compact" form as on the distribution side

➡ Must determine numerical significance of 3-parton fragmentation correlator

**Unpolarized FF (DSS)**

**Unpolarized PDF (GRV98)**

Distribution term (SGP)

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int_{z_{\min}}^1 \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x'S+T/z}\phi_{b/B}(x')$$
$$\times \sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{\ell s_T n\bar{n}}}{z\hat{u}}\right)\frac{1}{x}\left[T_{a,F}(x,x) - x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

Fragmentation term

$$\frac{P_h^0 d\sigma_{pol}}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\epsilon_{\perp\mu\nu}S_\perp^\mu P_{h\perp}^\nu \sum_i \sum_{a,b,c}\int_{z_{min}}^1 \frac{dz}{z^3}\int_{x'_{min}}^1 \frac{dx'}{x'}\frac{1}{x'S+T/z}\frac{1}{-x\hat{u}-x'\hat{t}}$$
$$\times \frac{1}{x} h_1^a(x) f_1^b(x') \left\{\left(\hat{H}^{C/c}(z) - z\frac{d\hat{H}^{C/c}(z)}{dz}\right)S_{\hat{H}}^i + \frac{1}{z}H^{C/c}(z) S_H^i \right.$$
$$\left. + 2z^2\int\frac{dz_1}{z_1^2} PV\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{C/c,\Im}(z,z_1)\frac{1}{\xi}S_{\hat{H}_{FU}}^i\right\}$$

**Transversity PDF (Torino13)**

**Recall:** $H^{h/q}(z) = -2z\hat{H}^{h/q}(z) + 2z^3\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{h/q,\Im}(z,z_1)$

## ➢ The role of twist-3 fragmentation in TSSAs
(Kanazawa, Koike, Metz, DP - PRD **89**(RC) (2014))

- Numerical study (Note: we only use $\sqrt{S} = 200$ GeV data ➔ higher $P_T$ values)

Distribution term

➡ SGP: $\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$, Sivers function taken from Torino group (2009/2013)

$F_{FT} \sim T_F$

➡ SFP/Tri-gluon: neglect for now

Fragmentation term

➡ Transversity: taken from Torino group (2013), but allow $\beta$ parameters to be free

➡ $\hat{H}^{h/q}(z)$: use Collins function extracted by the Torino group (2013)

$$\hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^{\,2}}{2M_h^2} H_1^{\perp\, h/q}(z, z^2\vec{k}_\perp^{\,2})$$

➡ $\hat{H}_{FU}^{h/q,\Im}(z,z_1)$ ➔ use the following ansatz:

$$\frac{\hat{H}_{FU}^{\pi^+/(u,\bar{d}),\Im}(z,z_1)}{D^{\pi^+/(u,\bar{d})}(z)\, D^{\pi^+/(u,\bar{d})}(z/z_1)} = \frac{N_{\rm fav}}{2I_{\rm fav}J_{\rm fav}}\, z^{\alpha_{\rm fav}}(z/z_1)^{\alpha'_{\rm fav}}(1-z)^{\beta_{\rm fav}}(1-z/z_1)^{\beta'_{\rm fav}}$$

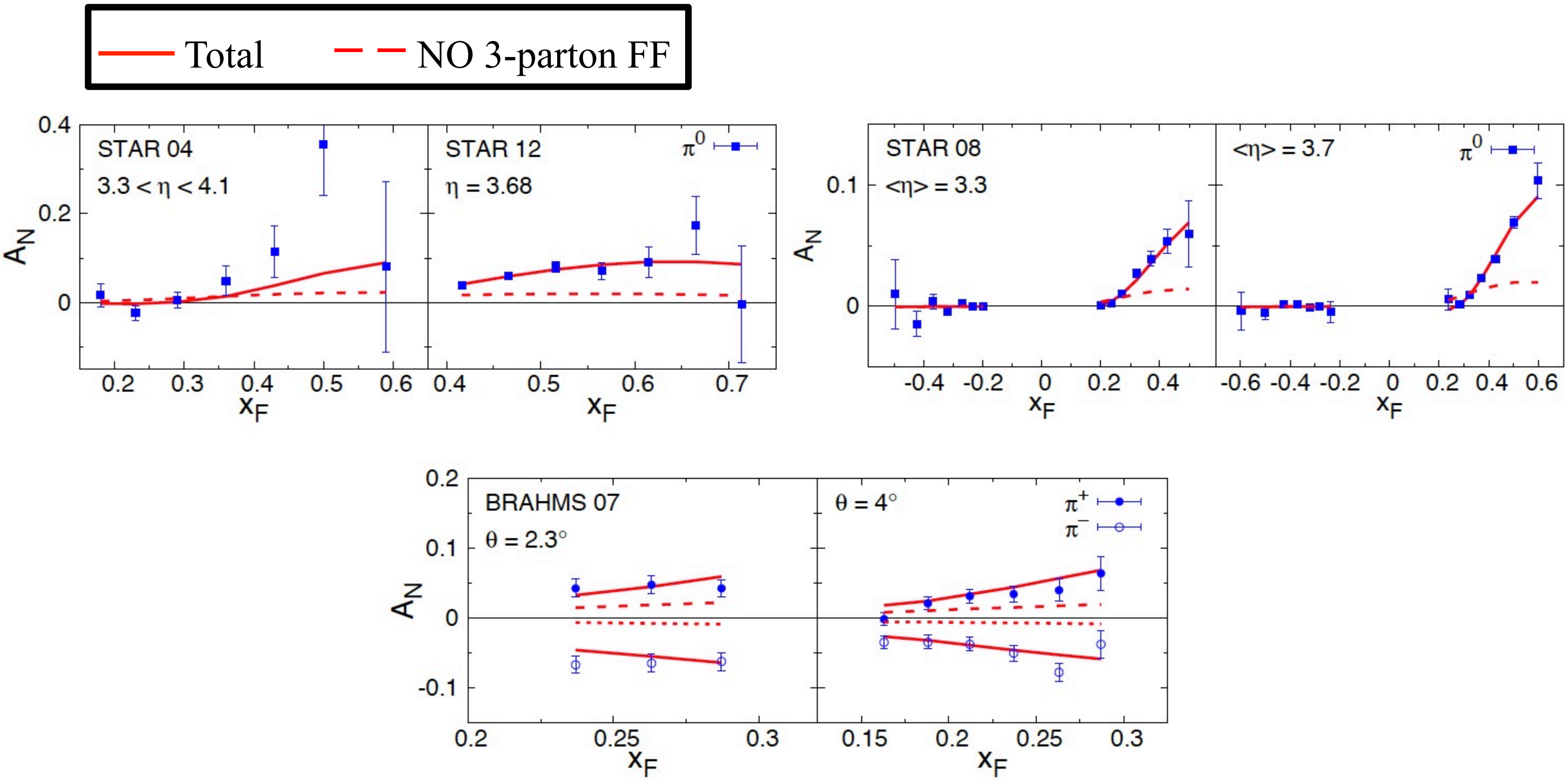
(similar for disfavored, $\pi^-$ defined through c.c., $\pi^0$ defined as average of $\pi^+$ and $\pi^-$)

8 free parameters: $N_{fav},\ \alpha_{fav} = \alpha'_{fav},\ \beta_{fav},\ \beta'_{fav} = \beta'_{dis}$

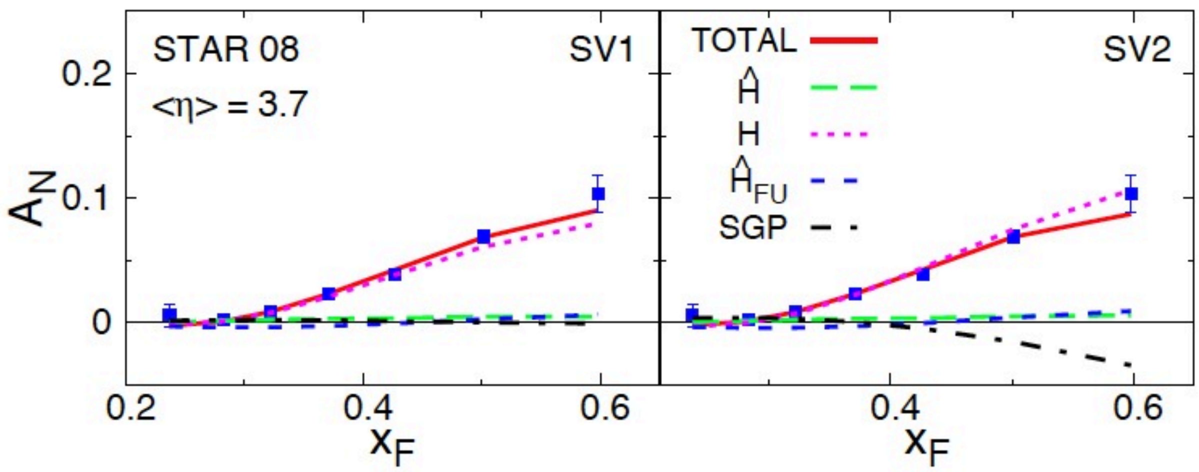
$N_{dis},\ \alpha_{dis} = \alpha'_{dis},\ \beta_{dis},\ \beta_u^T = \beta_d^T$

| $\chi^2/\text{d.o.f.} = 1.03$ | |
|---|---|
| $N_{\text{fav}} = -0.0338$ | $N_{\text{dis}} = 0.216$ |
| $\alpha_{\text{fav}} = \alpha'_{\text{fav}} = -0.198$ | $\beta_{\text{fav}} = 0.0$ |
| $\beta'_{\text{fav}} = \beta'_{\text{dis}} = -0.180$ | $\alpha_{\text{dis}} = \alpha'_{\text{dis}} = 3.99$ |
| $\beta_{\text{dis}} = 3.34$ | $\beta_u^T = \beta_d^T = 1.10$ |

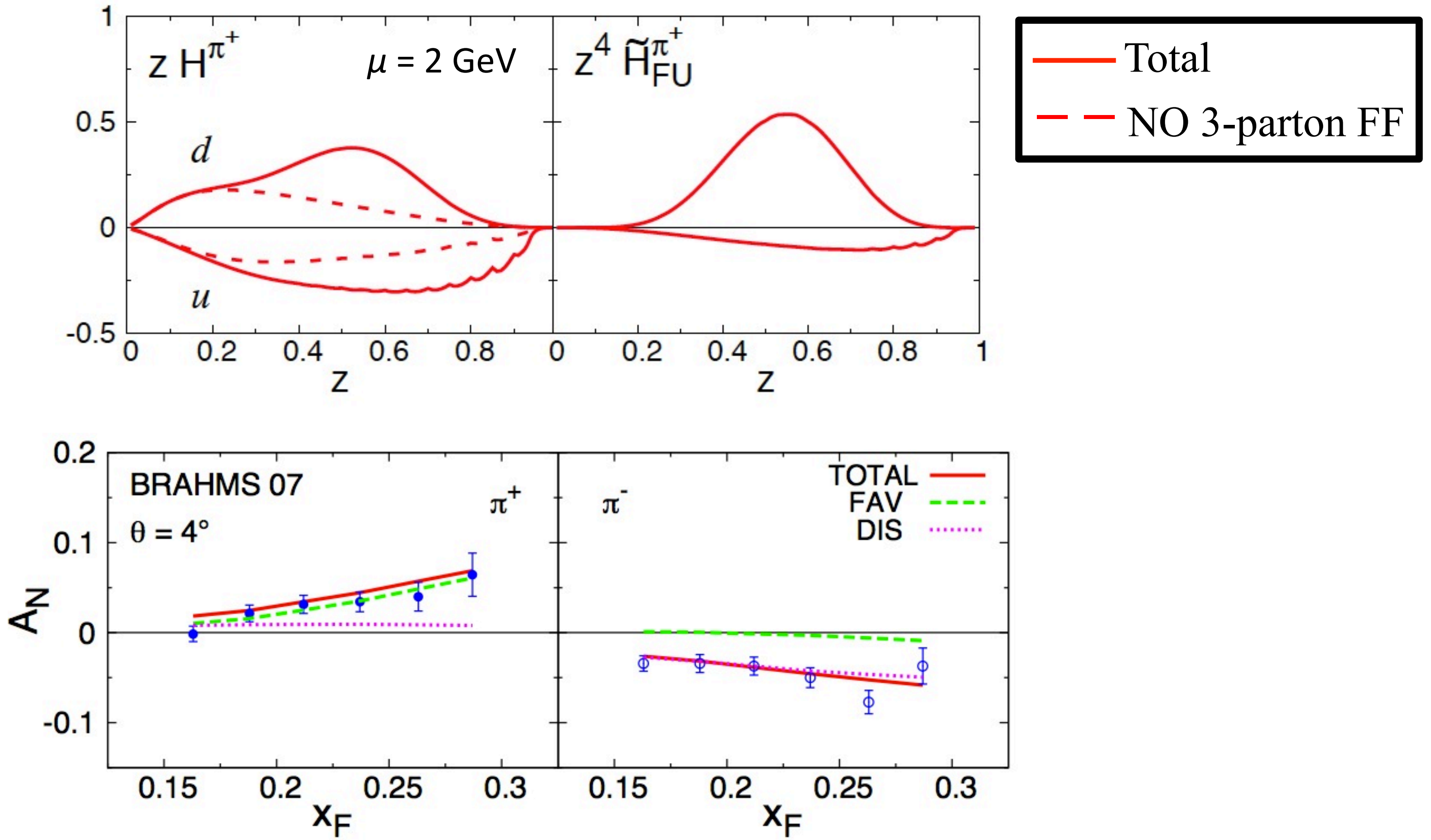
➡ Above parameters are from using 2009 Sivers function (SV1). Using 2013 Sivers function (SV2) gives similar values and $\chi^2/d.o.f. = 1.10$




- Including the (total) fragmentation term leads to very good agreement with the RHIC data, especially with its characteristic rise towards large $x_F$
- Without the 3-parton FF, one has difficulty describing the RHIC data




- $H$ term is dominant; Sivers-type, Collins-type, and $\hat{H}_{FU}$ terms are negligible
- SV1 – 2009 Sivers function from Torino group ➔ flavor-*independent* large-*x* behavior
- SV2 – 2013 Sivers function from Torino group ➔ flavor-*dependent* large-*x* behavior and slower decrease at large-*x* than SV1
  - Including 3-parton FF, one can accommodate such a Sivers function through the $H$ term
  - Without the 3-parton FF, one would have serious issues handling such a (negative) SGP contribution to obtain a (large) positive $A_N$

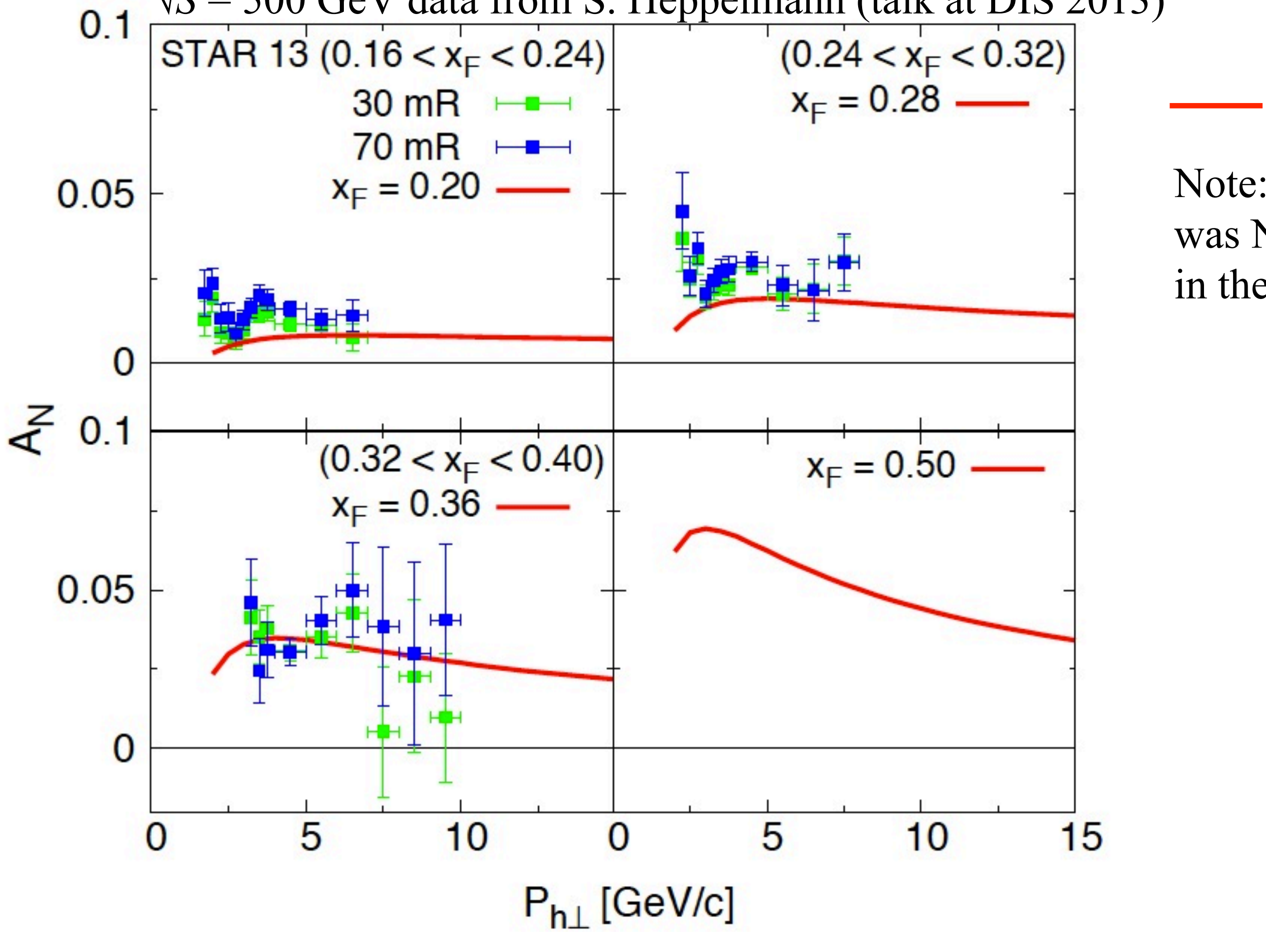



➡ Favored and disfavored (chiral-odd) collinear twist-3 FFs are roughly equal in magnitude but opposite in sign ➔ similar to Collins FF

➡ $A_N$ for $\pi^+$ ($\pi^-$) dominated by favored (disfavored) fragmentation

- Flat $P_T$ dependence thought to be an issue for collinear twist-3 approach ➔ $A_N \sim 1/P_T$
- First argued by Qiu and Sterman (1998) and later shown by Kanazawa and Koike (2011) that this does not have to be the case

$\sqrt{S}$ = 500 GeV data from S. Heppelmann (talk at DIS 2013)




Note: 500 GeV data was NOT included in the fit

- Our analysis also shows a flat $P_T$ dependence for $A_N$ seen so far at RHIC ➔ remains flat even to larger $P_T$ values

# Summary and outlook

- For many years it was unclear what mechanism causes large TSSAs in hadron production from *pp* collisions
- Twist-3 fragmentation could finally give us an explanation

➡ Full analytical pQCD result now available

➡ Including this term allows for a very good description of the RHIC data, in particular the rise in $A_N$ towards large $x_F$ and flat $P_T$ dependence

➡ Our analysis provides a consistency between spin/azimuthal asymmetries in *pp* (collinear) and SIDIS, $e^+e^-$ (TMD); In particular, "sign mismatch" is NOT an issue (DO NOT need Qiu-Sterman function to be dominant mechanism causing $A_N$)

➡ Future work: include SFPs (can help with charged hadrons), proper evolution of the 3-parton FF; analyze kaons and etas

- Global analysis involving several reactions will be needed in order to extract all the collinear twist-3 distribution and fragmentation functions in $p^\uparrow p \to hX$

➡ Measurement of $p^\uparrow p \to jet\ X$ by the AnDY Collaboration (Bland, et al. (2013))

➡ Measurements of Drell-Yan in $p^\uparrow p$ and $p^\uparrow p \to \gamma\ X$ at RHIC (also DY experiment planned at COMPASS for $\pi p^\uparrow$)

➡ Large $P_{h\perp}$ measurement of Sivers and Collins asymmetries in SIDIS should also be possible at JLab12, COMPASS, or a future EIC

➡ HERMES (Airapetian, et al. (2013)) / JLab (Allada, et al. (2013)) have recently published data on $ep^\uparrow \to hX$ / $en^\uparrow \to hX$; should be measured at an EIC (see talk by A. Prokudin)

➡ Can one consistently describe all of these reactions?

# Backup slides

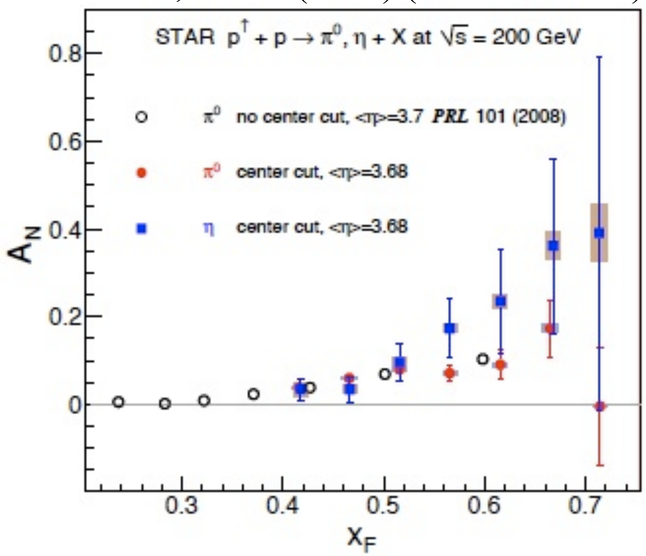
- Large TSSAs observed in the mid-1970s in the detection of hyperons from proton-beryllium collisions (Bunce, et al. (1976))
- Initially thought to contradict pQCD (Kane, Pumplin, Repko (1978)) – within the naïve collinear parton model:

$$A_N \sim \alpha_s m_q / P_{h\perp}$$

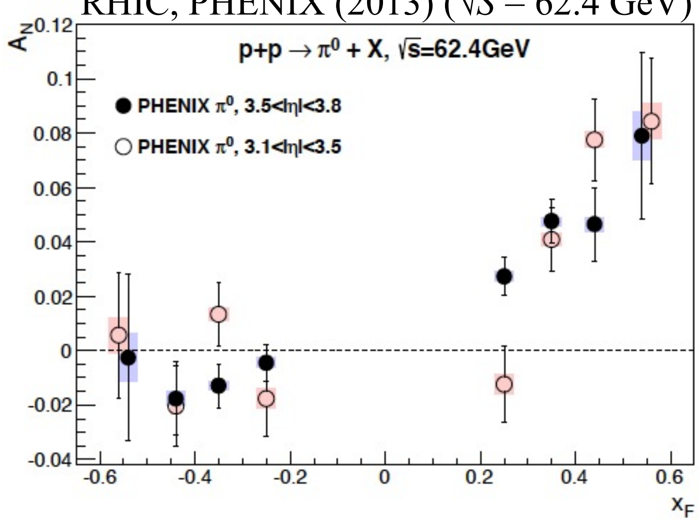
- Higher-twist approach to calculating TSSAs in *pp* collisions introduced in the 1980s (Efremov and Teryaev (1982, 1985))
- Benchmark calculations performed starting in the early 1990s (Qiu and Sterman (1992, 1999); Kouvaris, et al. (2006); Koike and Tomita (2009), etc.)
- RHIC (BRAHMS, STAR, PHENIX) has provided the most recent experimental data on proton-proton TSSAs (also FNAL (E704) in the 1990s)

## ➢ Experimental data

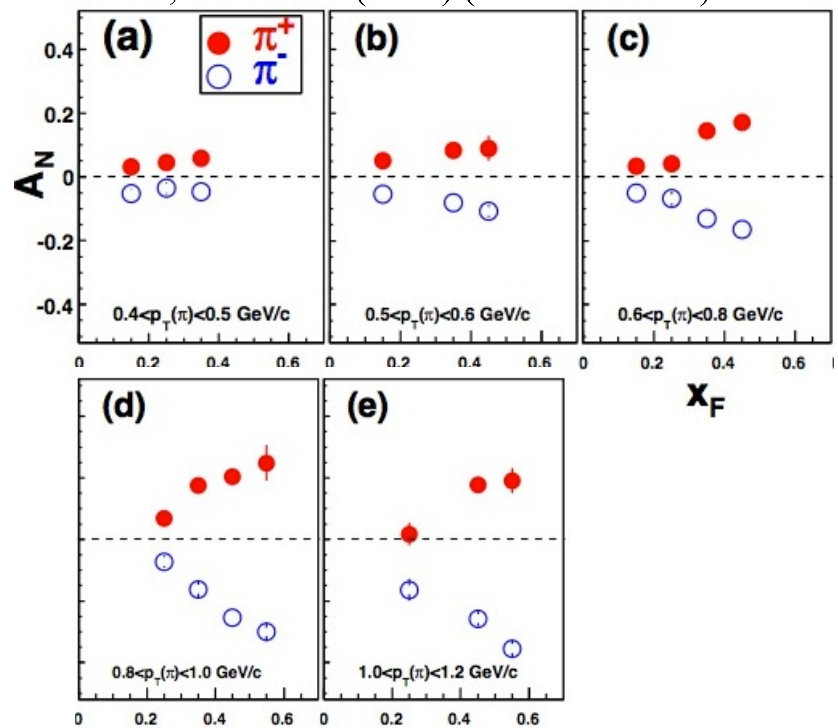
RHIC, STAR (2012) ($\sqrt{S} = 200$ GeV)




RHIC, PHENIX (2013) ($\sqrt{S} = 62.4$ GeV)

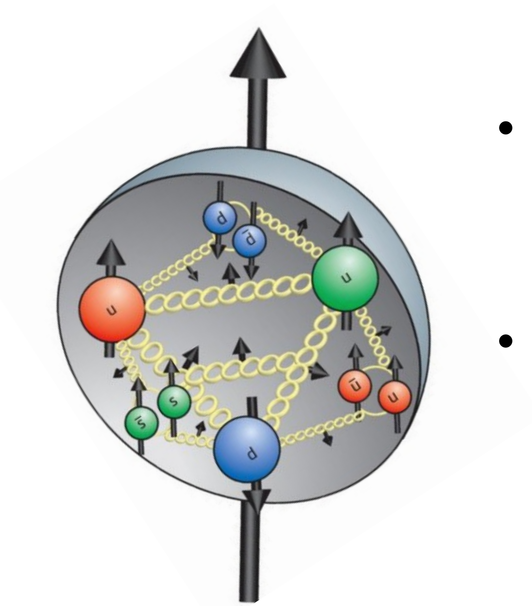



RHIC, BRAHMS (2008) ($\sqrt{S} = 62.4$ GeV)




Also preliminary data from BRAHMS at $\sqrt{S} = 200$ GeV

$$x_F = 2p_z/\sqrt{S}$$

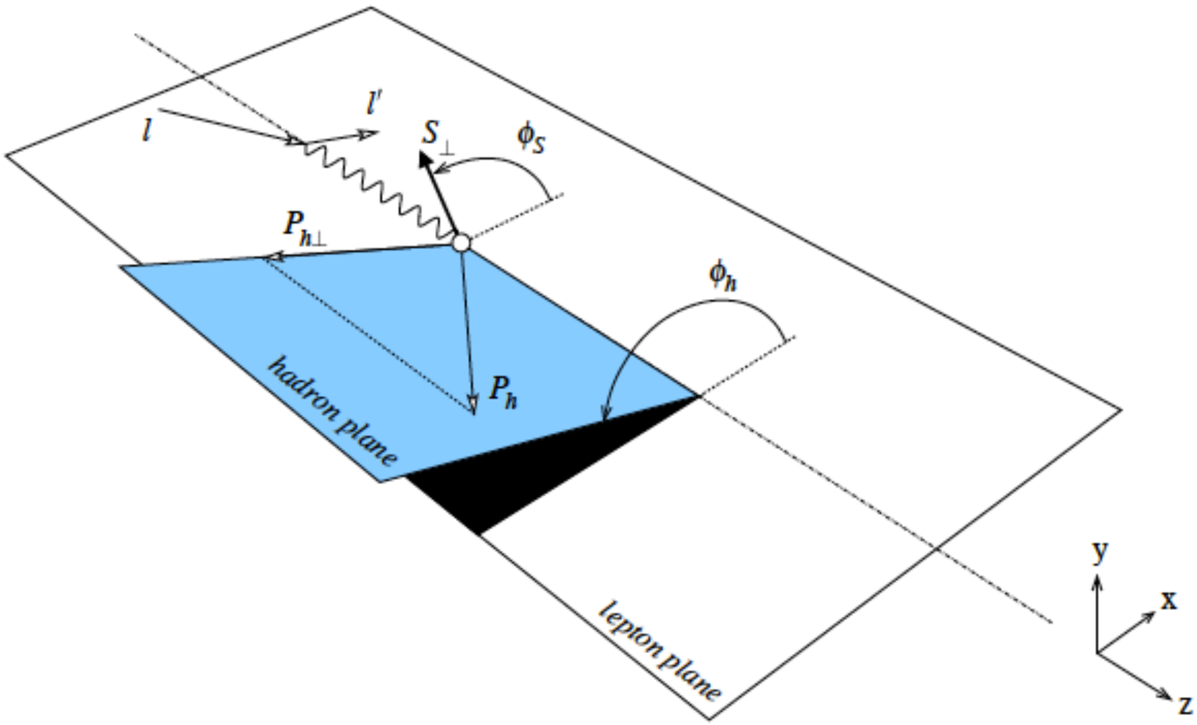


$$A_N = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + \sigma_R}$$

- Data tells us (if fragmentation mechanism dominates) that the pions care about the transverse spin of the fragmenting quark ➔ fragment in a particular direction (left or right)
- Small and negative $x_F$ ➔ probe sea quarks and gluons in $p^\uparrow$
  - ➡ $gg \to gg$ channel gives large contribution to unpolarized cross section, but NO gluon "transversity" ➔ no such channel in spin-dependent cross section
  - ➡ Little information on sea quark "transversity" ➔ might speculate sea quarks, on average, are less likely to emerge from $p^\uparrow$ with a transverse spin in a certain direction
- Large $x_F$ ➔ probe valence quarks in $p^\uparrow$
  - ➡ From SIDIS we know $u$ quarks ($d$ quarks) are more likely emerge from $p^\uparrow$ with their transverse spin aligned (anti-aligned) with $p^\uparrow$ ➔ pions more likely to fragment in a particular direction (left or right)
  - ➡ $gg \to gg$ channel dies out in this region ➔ unpolarized cross section becomes smaller

- An aside: TSSAs in SIDIS and the TMD formalism

$$A_{UT}^{\sin(\phi_h-\phi_s)} = \frac{\int d\phi_h d\phi_S \, \sin(\phi_h - \phi_S)\, d\sigma}{\int d\phi_h d\phi_S \, d\sigma}$$




(Figure from Bacchetta, et al. (2007))

Sivers function

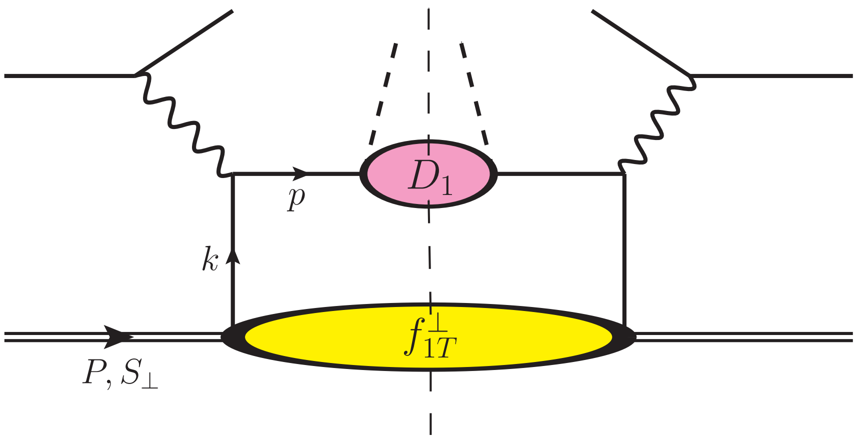
$$A_{UT}^{\sin(\phi_h-\phi_s)} = \frac{w(k_\perp)\, f_{1T}^{\perp,q}(x,\vec{k}_\perp) \otimes D_1^{h/q}(z,\vec{p}_\perp)}{f_1^q(x,\vec{k}_\perp) \otimes D_1^{h/q}(z,\vec{p}_\perp)}$$

Sivers asymmetry

TMD approach

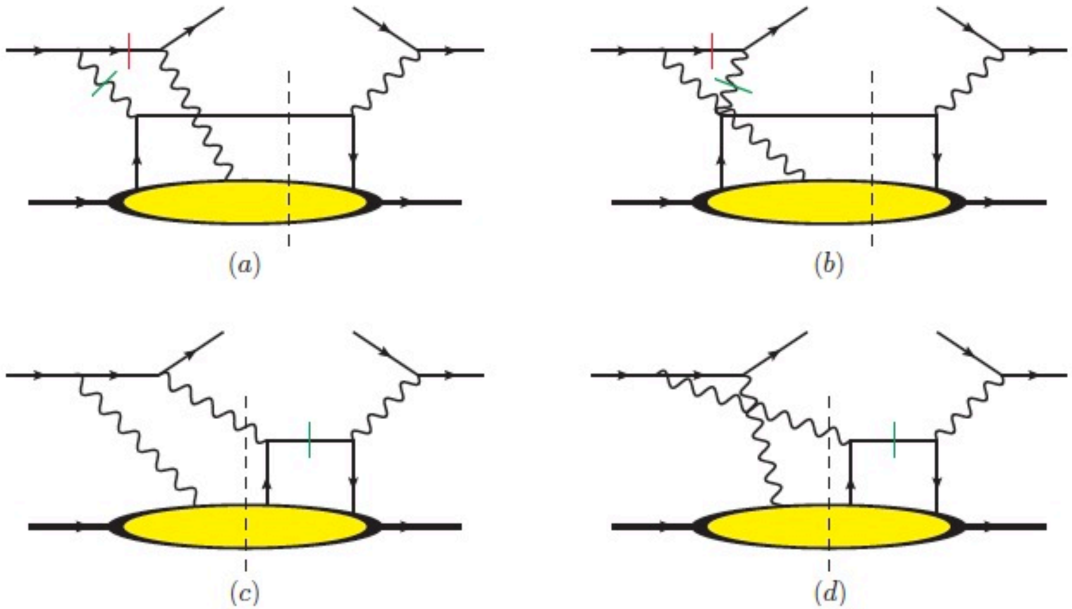
(Sivers (1990, 1991); Collins (1993))

$Q >> P_{hT} \geq \Lambda_{QCD}$




- T-odd effect ➡ imaginary phase is generated by "Wilson line"
  ➡ multiple re-interactions of the quark with the target remnants
- Process dependence: $f_{1T}^\perp(x,\vec{k}_\perp^2)\big|_{SIDIS} = -f_{1T}^\perp(x,\vec{k}_\perp^2)\big|_{DY}$ (Collins (2002))

- TSSA in inclusive DIS (Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou, PRD **86** (2012))



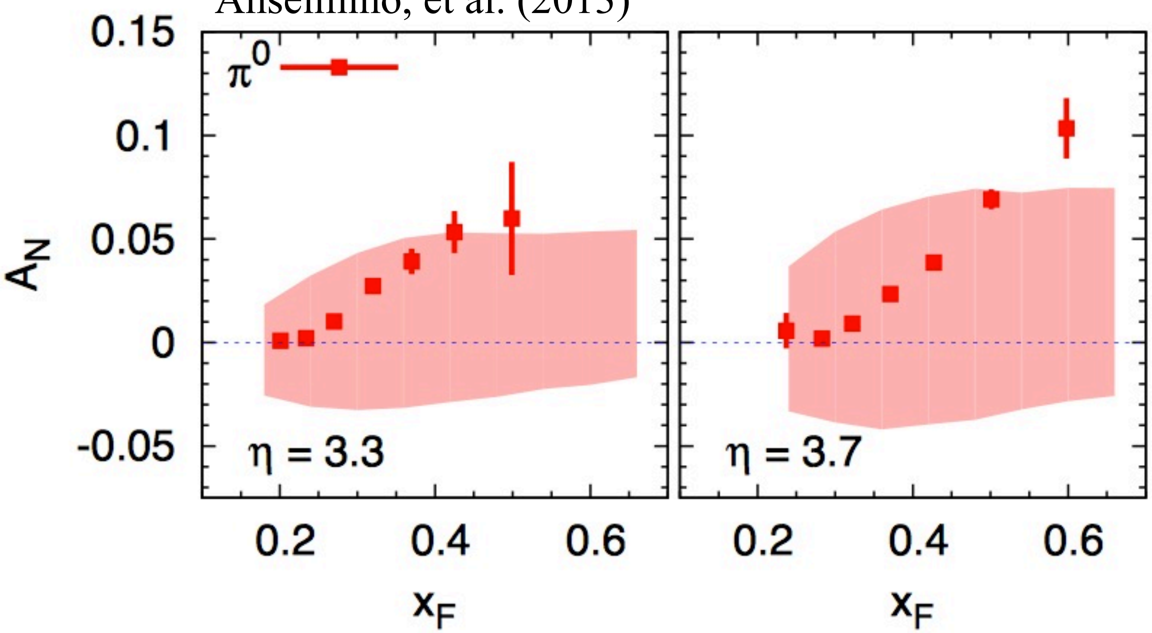

$$k'^0 \frac{d\sigma^N_{pol}}{d^3\vec{k}'} = \frac{8\pi\alpha_{em}^2\, x y^2\, M}{Q^8}\, \frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\left(2+\frac{\hat{u}}{\hat{t}}\right)\varepsilon^{S_N P k k'} \sum_q e_q^2\, x \tilde{F}_{FT}^{q/N}(x,x)$$

with $\tilde{F}_{FT}(x,x) = F_{FT}(x,x) - x\frac{d}{dx}F_{FT}(x,x)$

(Work has also been done on both photons coupling to the same quark: Metz, Schlegel, Goeke (2006); Afanasev, Strikman, Weiss (2007); Schlegel (2012))

- A note on the TMD approach to TSSAs in *pp* collisions
  - ➡ Only a phenomenological model, since there is no proof such a formalism holds in processes with only one (large) scale
  - ➡ Use Sivers function extracted from SIDIS ➔ large uncertainties due to unknown large *x* behavior ➔ cannot draw any definite conclusions

Anselmino, et al. (2013)




  - ➡ NO sign mismatch problem, but if one takes the re-scattering picture seriously then the issue cannot be avoided

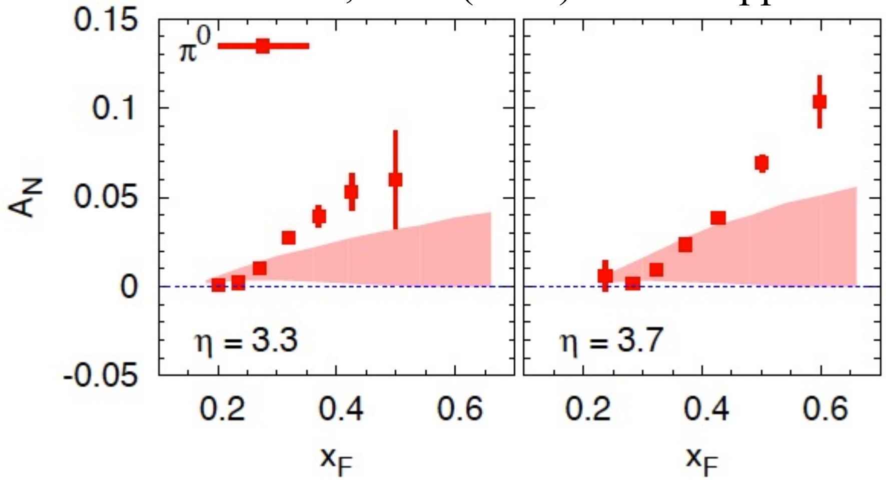
$$d\sigma = H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)}$$
$$+ H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}$$

→ Negligible (Kanazawa and Koike (2000))

- Collinear twist-3 fragmentation term:

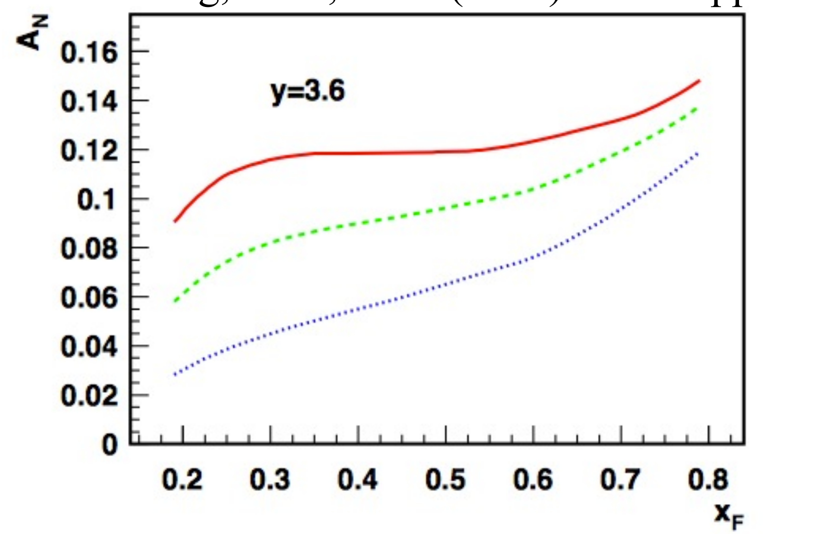
$$+ H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)}$$

Anselmino, et al. (2012) – TMD approach



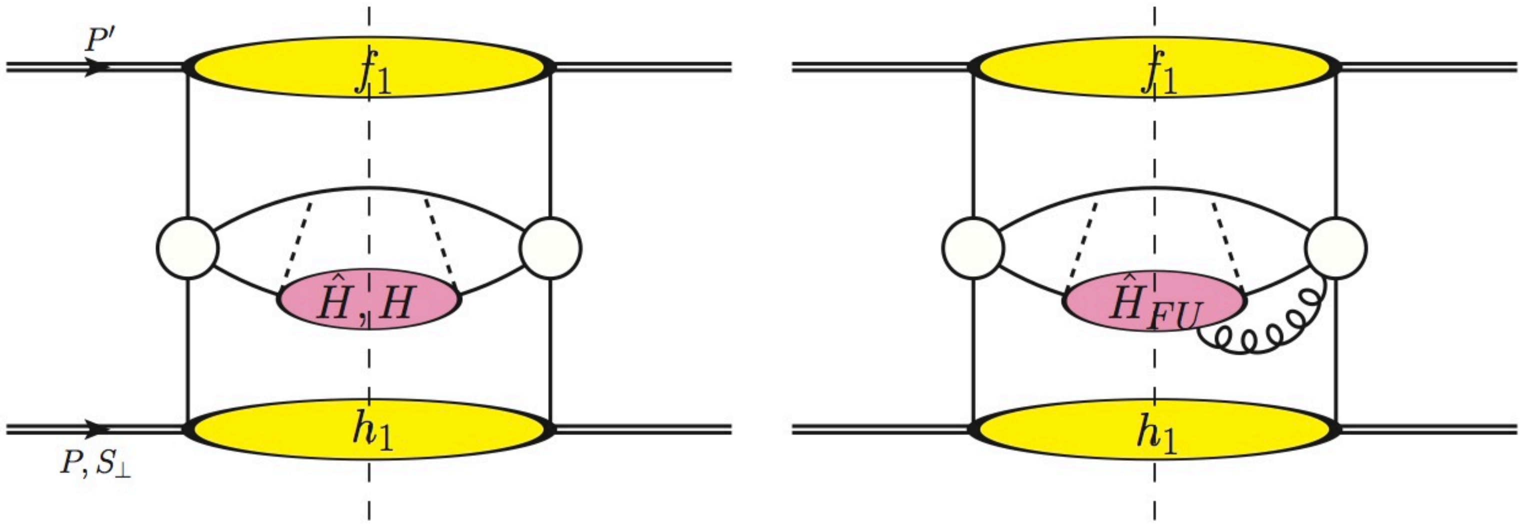

- Uses Collins function extracted from $e^+e^-$ and SIDIS

Kang, Yuan, Zhou (2010) – CT3 approach




- Only looks at "derivative term" using simple parameterization

– Could at the very least give a contribution comparable to SGP term




$$\hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_\perp \, \frac{\vec{k}_\perp^{\,2}}{2M_h^2} \, H_1^{\perp h/q}(z, z^2\vec{k}_\perp^{\,2})$$

**Collins-type function**

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q,\Im}(z, z_1) = H^{h/q}(z) + 2z\hat{H}^{h/q}(z)$$

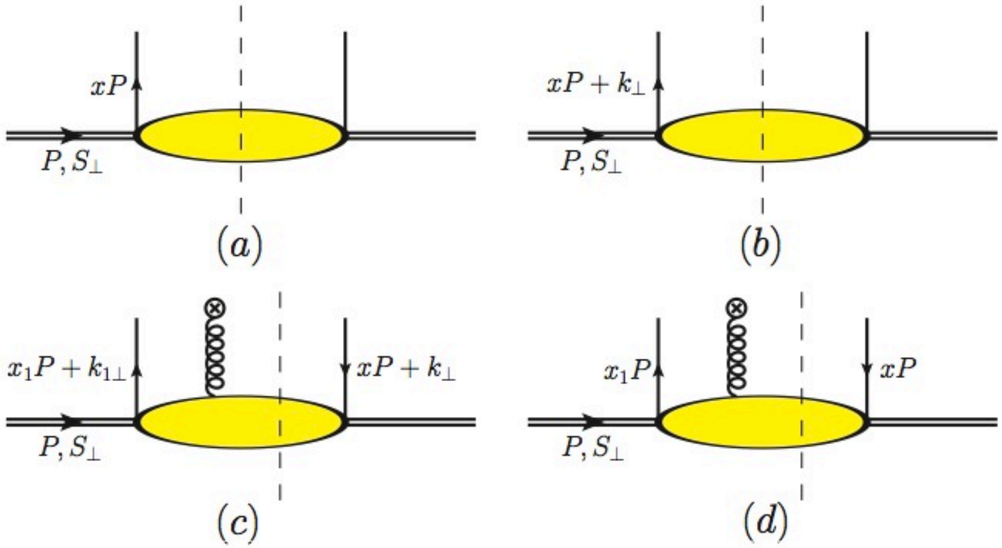
**3-parton correlator**

➡ There are 2 independent (unpolarized) collinear twist-3 FFs

Collinear twist-3 fragmentation structure is richer than that for the TMD formalism

## ➢ Theoretical description: collinear twist-3 formalism

Lightcone gauge




(a) ➡ $\Phi^q_{ij}(x; P, S_\perp) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, S_\perp|\bar{\psi}^q_j(0)\psi^q_i(\xi^-)|P, S_\perp\rangle \xrightarrow{\gamma^i\gamma_5} g^q_T(x)$

(b) ➡ $\Phi^{q,\mu}_{\partial_\perp,ij}(x; P, S_\perp) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, S_\perp|\bar{\psi}^q_j(0)\partial^\mu_\perp\psi^q_i(\xi^-)|P, S_\perp\rangle \xrightarrow{\gamma^+\gamma_5} \tilde{g}^q(x) \left(= g^{q(1)}_{1T}(x)\right)$

(d) ➡ $\Phi^{q,\mu}_{A,ij}(x, x_1; P, S_\perp) = \int \frac{d\xi^-}{2\pi} \int \frac{d\zeta^-}{2\pi} e^{ix_1P^+\xi^-} e^{i(x-x_1)P^+\zeta^-} \times \langle P, S_\perp|\bar{\psi}^q_j(0)A^\mu_\perp(\zeta^-)\psi^q_i(\xi^-)|P, S_\perp\rangle \xrightarrow{\gamma^+/\gamma^+\gamma_5} \begin{cases} F^q_{FT}(x, x_1) \\ G^q_{FT}(x, x_1) \\ F^q_{DT}(x, x_1) \\ G^q_{DT}(x, x_1) \end{cases}$

Rewrite in terms of $F$ or $D$

Twist-3 collinear PDFs for a transversely polarized *p*

(c) gives a twist-4 contribution

(see, e.g., Zhou, Yuan, Liang (2010))

- Symmetry properties

$$F^q_{FT}(x,\, x_1) = F^q_{FT}(x_1,\, x) \;\text{ and }\; G^q_{FT}(x,\, x_1) = -G^q_{FT}(x_1,\, x)$$

$$F^q_{DT}(x,\, x_1) = -F^q_{DT}(x_1,\, x) \;\text{ and }\; G^q_{DT}(x,\, x_1) = G^q_{DT}(x_1,\, x)$$

- Relations between F-type and D-type functions (see, e.g., Eguchi, et al. (2006))

$$F^q_{DT}(x, x_1) = PV \frac{1}{x - x_1} F^q_{FT}(x, x_1)$$

$$G^q_{DT}(x, x_1) = PV \frac{1}{x - x_1} G^q_{FT}(x, x_1) + \delta(x - x_1)\, \tilde{g}^q(x)$$

- $g_T$ can be related to D-type functions through the EOM (see, e.g., Efremov and Teryaev (1985); Jaffe and Ji (1992); Boer, Mulders, Teryaev (1998)):

$$x\, g^q_T(x) = \int dx_1 \left[G^q_{DT}(x,\, x_1) - F^q_{DT}(x,\, x_1)\right]$$

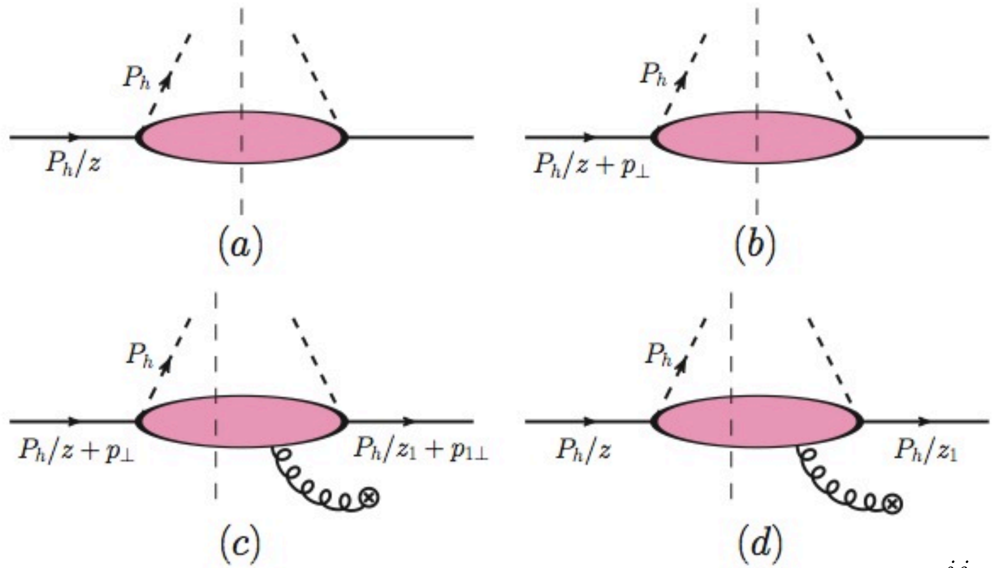
There are 3 independent collinear twist-3 functions relevant for a transversely polarized *p*

$\tilde{g},\; F_{FT},\; G_{FT}$

*or*

$\tilde{g},\; F_{DT},\; G_{DT}$

Lightcone gauge




(a) ➡ $$\Delta_{ij}^{h/q}(z;P_h) = \sum_X z \int \frac{d\xi^+}{2\pi} e^{i\frac{P_h^-}{z}\xi^+} \langle 0|\psi_i^q(\xi^+)|P_h;X\rangle\langle P_h;X|\bar{\psi}_j^q(0)|0\rangle \xrightarrow{i\sigma^{ij}\gamma_5/\mathbb{1}} \begin{cases} H^{h/q}(z) \\ E^{h/q}(z) \end{cases}$$

(b) ➡ $$\Delta_{\partial_\perp,ij}^{h/q,\mu}(z;P_h) = \sum_X z \int \frac{d\xi^+}{2\pi} e^{i\frac{P_h^-}{z}\xi^+} \langle 0|\partial_\perp^\mu \psi_i^q(\xi^+)|P_h;X\rangle\langle P_h;X|\bar{\psi}_j^q(0)|0\rangle \xrightarrow{i\sigma^{i-}\gamma_5} \hat{H}^{h/q}(z) = \left(H_1^{\perp\, h/q(1)}(z)\right)$$

(d) ➡ $$\Delta_{A,ij}^{h/q,\mu}(z,z_1;P_h) = \sum_X \frac{1}{z} \int \frac{d\xi^+}{2\pi} \int \frac{d\zeta^+}{2\pi} e^{i\frac{P_h^-}{z_1}\xi^+} e^{i\left(\frac{1}{z}-\frac{1}{z_1}\right)P_h^-\zeta^+} \times \langle 0|A_\perp^\mu(\zeta^+)\psi_i^q(\xi^+)|P_h;X\rangle\langle P_h;X|\bar{\psi}_j^q(0)|0\rangle \xrightarrow{i\sigma^{i-}\gamma_5} \begin{cases} \hat{H}_{FU}^{h/q}(z,z_1) \\ \hat{H}_{DU}^{h/q}(z,z_1) \end{cases}$$

Rewrite in terms of *F* or *D*

Twist-3 collinear FFs for an unpolarized *h*

Note: $\hat{H}_{FU}$ and $\hat{H}_{DU}$ have real and imaginary parts.

(c) gives a twist-4 contribution

- Relations between F-type and D-type function

$$\hat{H}_{DU}^{h/q,\Im}(z,z_1) = PV\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{h/q,\Im}(z,z_1) - \frac{1}{z^2}\hat{H}^{h/q}(z)\,\delta\!\left(\frac{1}{z}-\frac{1}{z_1}\right)$$

$$\hat{H}_{DU}^{h/q,\Re}(z,z_1) = PV\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{h/q,\Re}(z,z_1)$$

- *H* (*E*) can be related to the imaginary (real) part of the D-type function through the EOM:

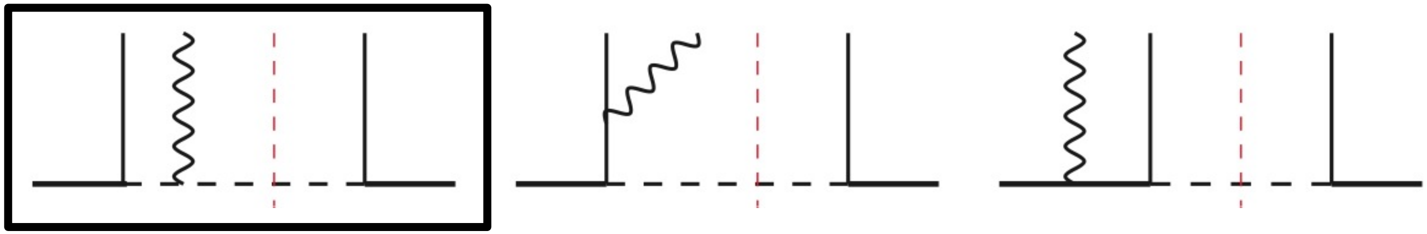
$$H^{h/q}(z) = 2z^3\int\frac{dz_1}{z_1^2}\hat{H}_{DU}^{h/q,\Im}(z,z_1)$$

$$E^{h/q}(z) = -2z^3\int\frac{dz_1}{z_1^2}\hat{H}_{DU}^{h/q,\Re}(z,z_1)$$

→ There are 2 independent collinear twist-3 functions relevant for the fragmentation of a quark into an unpolarized *h*

$\hat{H},\ \hat{H}_{FU}$
or
$\hat{H},\ \hat{H}_{DU}$

– Involves $F_{FT}$ in a QED process ($q\gamma q$ correlator) ➡ relate to $F_{FT}$ in a QCD process ($qgq$ correlator) through a diquark model



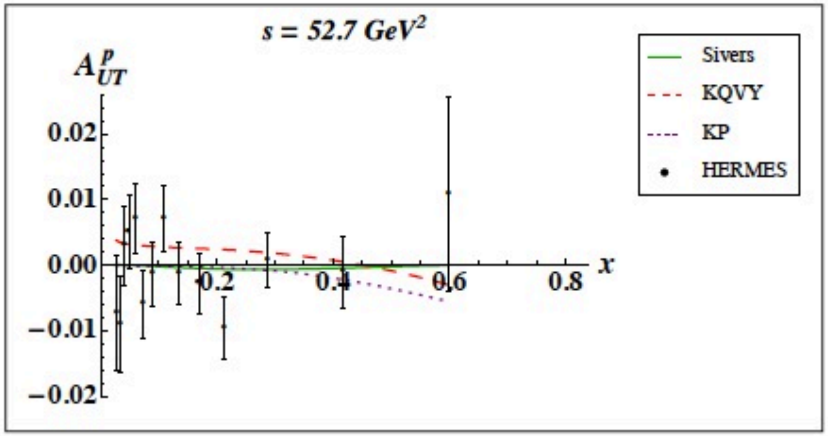
$$(F_{FT}^{u/p})_{QED} = \frac{\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{u/p})_{QCD} \qquad (F_{FT}^{d/p})_{QED} = \frac{4\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{d/p})_{QCD}$$

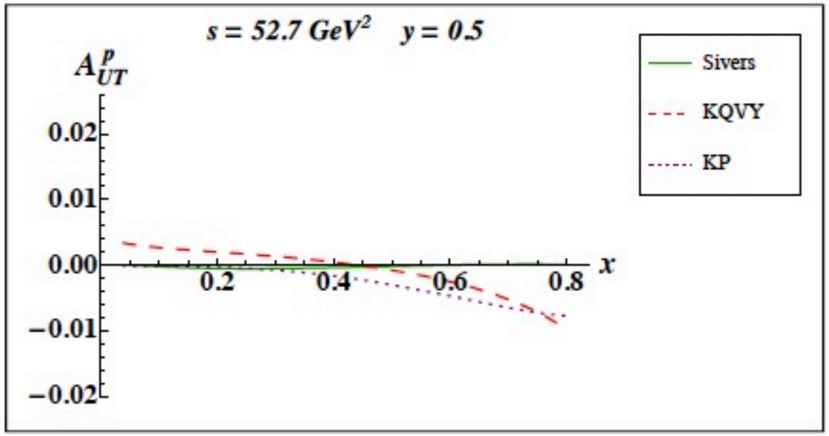
$$(F_{FT}^{u/n})_{QED} = -\frac{2\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{d/p})_{QCD} \qquad (F_{FT}^{d/n})_{QED} = \frac{\alpha_{em}}{3C_F\alpha_s}(F_{FT}^{u/p})_{QCD}$$

– Use 3 different inputs for $F_{FT}$ in a QCD process:

1) Sivers: fit from Anselmino, et al. (2008) of Sivers asymmetry from SIDIS data
2) KQVY: fit from Kouvaris, et al. (2006) for SSAs in *pp* collisions
3) KP: simultaneous fit from Kang and Prokudin (2012) of *pp* and SIDIS data

- Proton SSA:



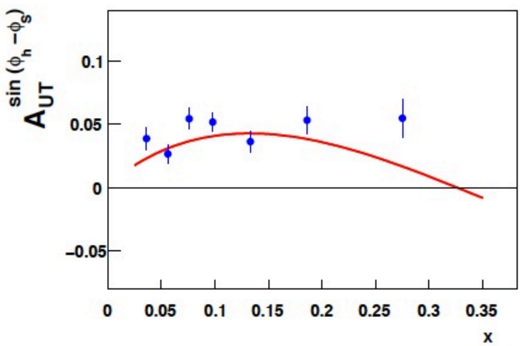

Sivers input agrees exactly with the HERMES data (Airapetian, et al. (2009))

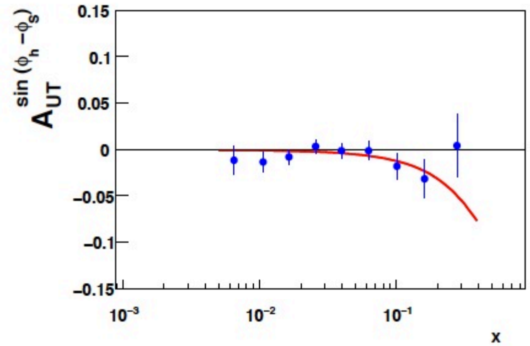
KP input appears to become too large at large $x$ (result of the node in $x$ for the up quark Sivers function)

➡ Node in $x$ in the Sivers function is not preferred, although it cannot be definitively excluded by the current data → need more accurate data at larger $x$

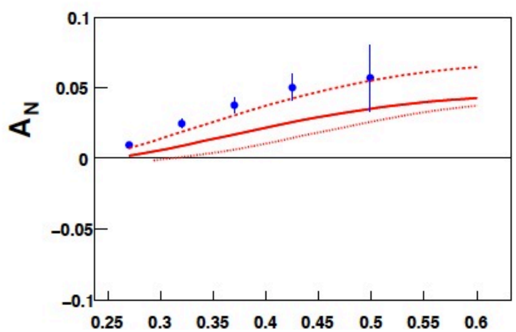
KQVY input also appears to become too large at large $x$ and actually diverges as $x \to 1$

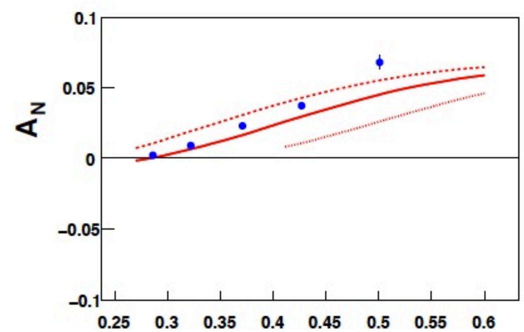
- Node in $x$ or $k_T$ in the Sivers function:
  - Attempt to simultaneously fit SIDIS and $pp$ data (Kang and Prokudin (2012))



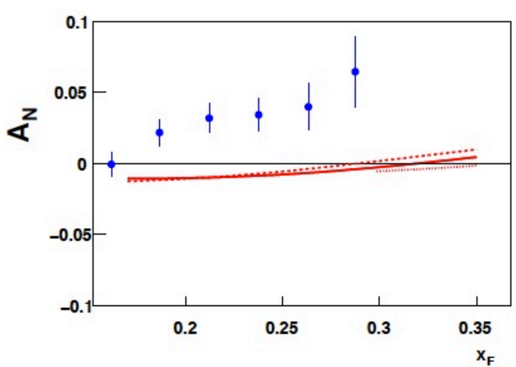



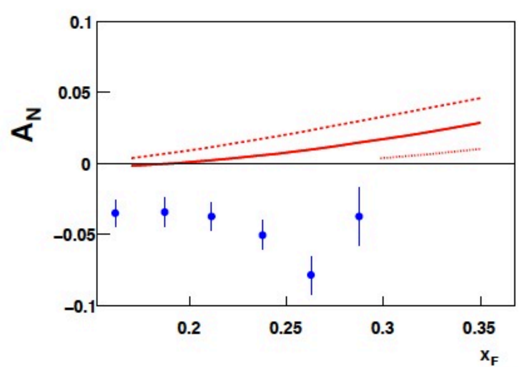

SIDIS data from HERMES (left) and COMPASS (right)




Proton-proton data from STAR at $y$ = 3.3 (left) and $y$ = 3.7 (right)




Proton-proton data from BRAHMS for $\pi^+$ (left) and $\pi^-$ (right)