Chiral Odd GPDs and Generalizations

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These ideas were developed in Trento ECT*, INT, Jlab, DIS2011, SPIN, Frascati INF, Transversity 2011-2013, PANIC, POETIC, Transversity2014 & in consultation with many of you

Collaborators

GPDs Fit

Aurore Courtoy, GRG, Osvaldo Gonzalez Hernandez, Simonetta Liuti, Jon Poage, Abha Rajan, Silvia Pisano

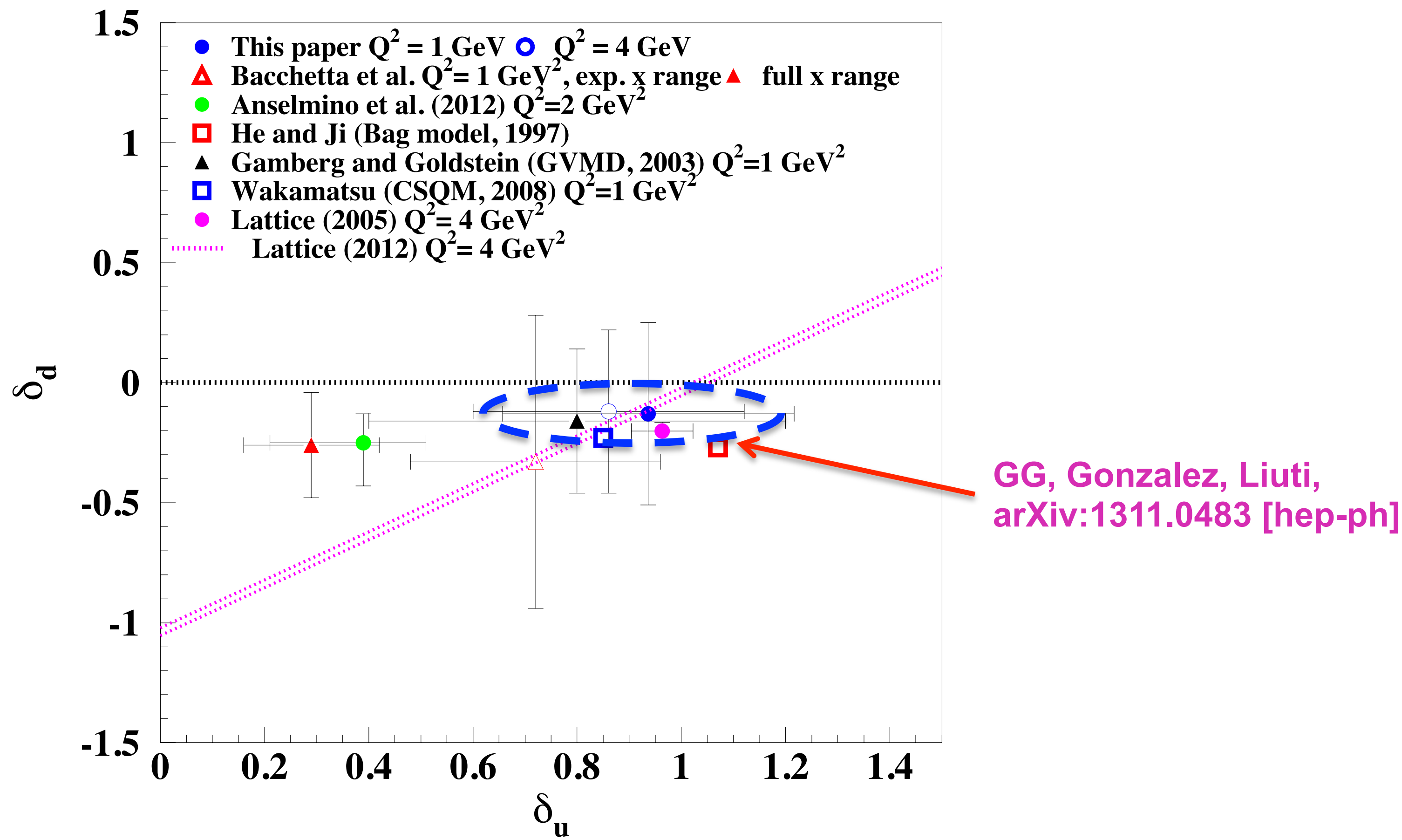
Angular Momentum/OAM

Aurore Courtoy, GRG, Osvaldo Gonzalez Hernandez, Simonetta Liuti, Abha Rajan

Extension to Chiral Odd Sector

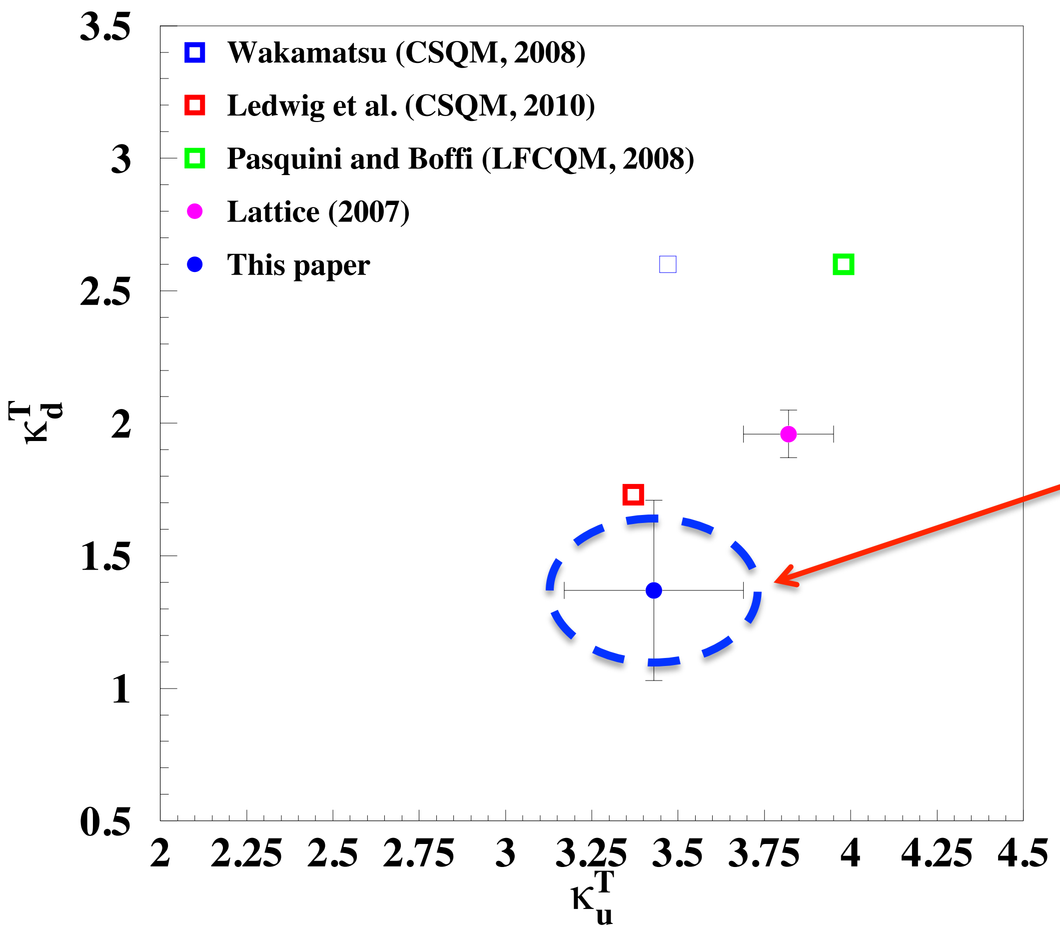
GRG, Osvaldo Gonzalez Hernandez, Simonetta Liuti

Why look for Chiral odd GPDs? → Transversity → tensor charges δ_q



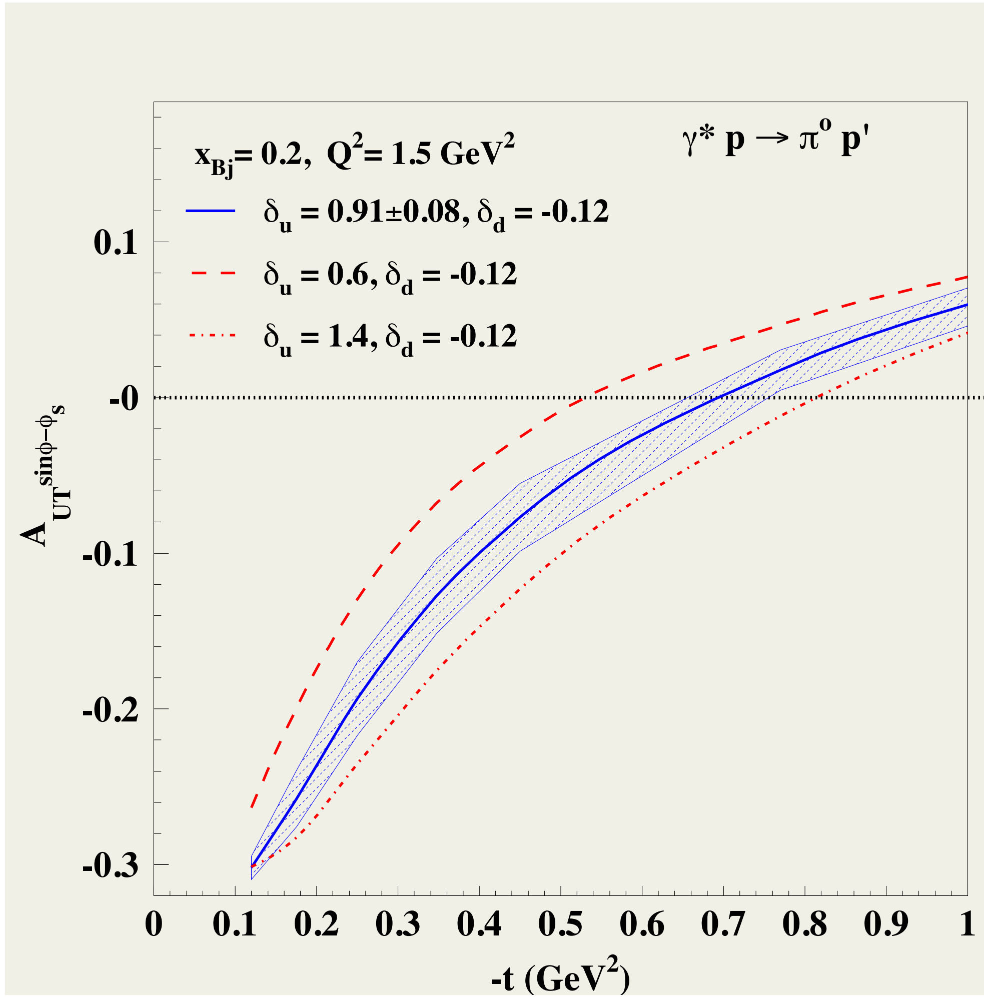
Chiral odd GPDs → transverse spin-flavor **dipole moments κ_T^q**

defined by M. Burkardt, PRD72,094020(2005)



GG, Gonzalez, Liuti, arXiv:1311.0483 [hep-ph]

Asymmetry sensitive to tensor charge



Outline
How to measure transversity & tensor charge

- Word about OAM
- Hadron Spin & Transversity Structure from GPDs →
- "Flexible" parameterization for Chiral Even GPDs
- Extend to Chiral Odd GPDs via diquark parity & spin relations

  → Transversity

  - Model relations between Chiral even & odd helicity amps
  - π^0 & η production & flavor separation
  - Tensor charge δ_q
  - Transverse spin-flavor dipole moments κ_T^q

- Observables: Cross sections & Asymmetries

Quark Orbital Angular Momentum

Recent work in interpreting OAM (Elliot Leader's introductory talk) has led us to consider higher twist GPDs & GTMDs.

A.Courtoy, GRG, J.O.Gonzalez Hernandez, S.Liuti, A.Rajan, "On the observability of the quark orbital angular momentum distribution", Physics Letters B731, 141 (2014).

Ji sum rule yields OAM – at twist 3, 1st moment of $\gamma^{\perp}$ distribution function $G_2(x,\xi,\Delta)$ (see Kiptily & Polyakov '04) satisfies sum rule.

$$\int dx\, x G_2^q(x,0,0) = \frac{1}{2}\left[-\int dx\, x\left(H^q(x,0,0) + E^q(x,0,0)\right) + \int dx\, \tilde{H}^q(x,0,0)\right] = -L_q$$

Observing OAM in DVCS

$G_2 = \tilde{E}_{2T}$

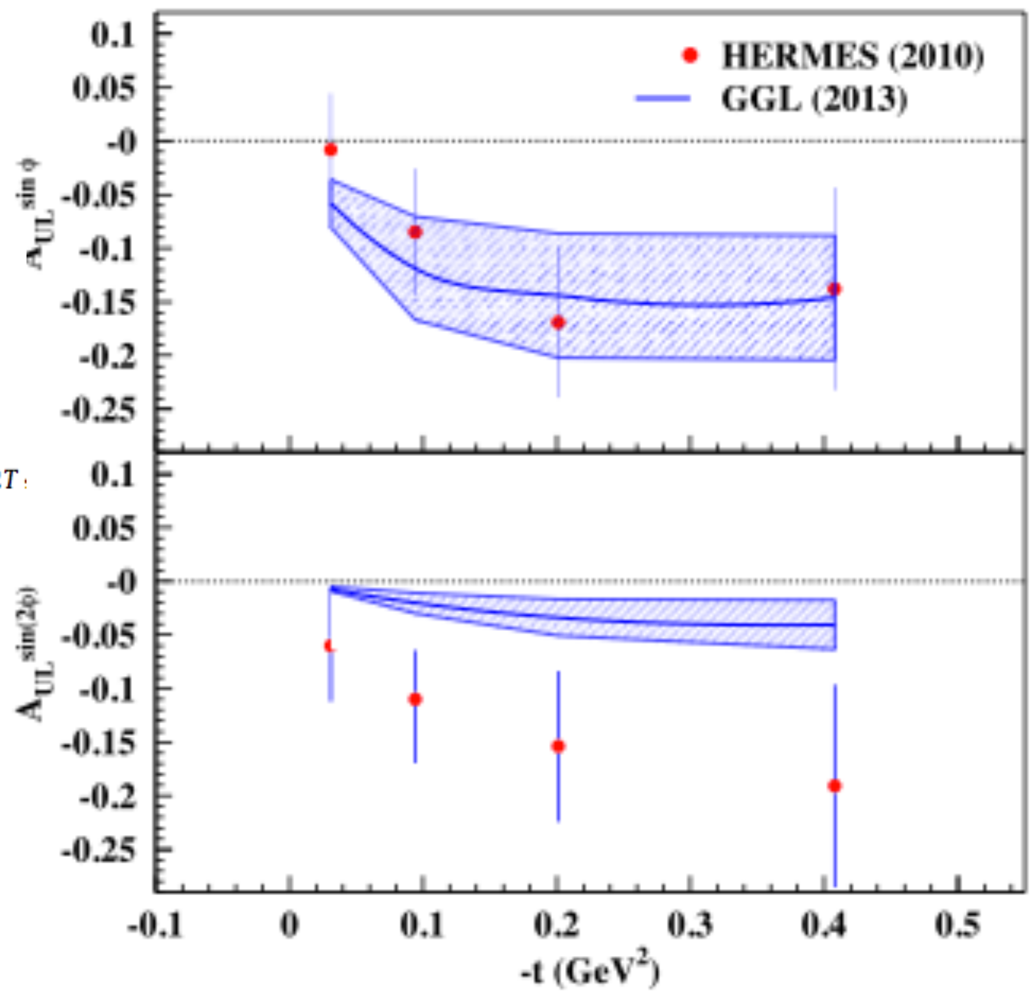
in Balitsky, Kirchener & Mueller ‘02.

$$\tilde{\mathcal{H}}^{eff} = -2\xi\left(\frac{1}{1+\xi}\tilde{\mathcal{H}} + \tilde{\mathcal{H}}_3^{+} - \tilde{\mathcal{H}}_3^{-}\right),$$

where (Table 1),

$$\tilde{\mathcal{H}} = C^{+} \otimes \tilde{H}, \qquad \tilde{\mathcal{H}}_3^{+} = C^{+} \otimes \tilde{E}'_{2T}, \qquad \tilde{\mathcal{H}}_3^{-} = C^{-} \otimes \tilde{E}_{2T},$$

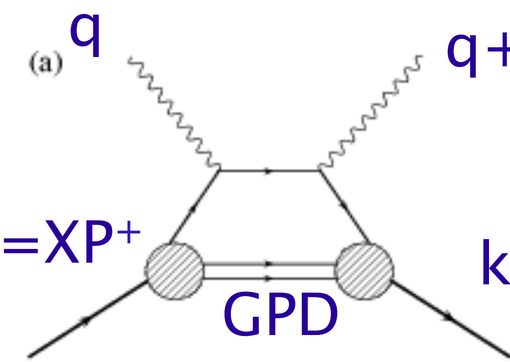
Appears in $A_{UL}^{\sin 2\phi}$ →



quark GPDs

DVCS & DVMP $\gamma^*(Q^2)+P \rightarrow (\gamma \text{ or meson})+P'$

partonic picture – leading twist



{ k_T, $k^+=XP^+$ }

{ $k'_T=k_T-\Delta$, $k'^+=(X-\zeta)P^+$ }

P^+

{ $P'^+=(1-\zeta)P^+$, $P'_T=-\Delta$ }

$\zeta \rightarrow 0$
Regge

Factorized
"handbag"
picture

Quark–spectator
quark+diquark

$X>\zeta$ DGLAP $\quad \Delta_T$ -> b_T transverse spatial

$X<\zeta$ ERBL $\quad x=(X-\zeta/2)/(1-\zeta/2)$; $x=\zeta/(2-\zeta)$

see Ahmad, GG, Liuti, PRD79, 054014, (2009) for first chiral odd GPD parameterization Gonzalez, GG, Liuti PRD84, 034007 (2011)

GPD definitions – 8 quark + 8 gluon

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar{\psi}(-\tfrac{1}{2}z)\, \gamma^+ \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar{u}(p',\lambda') \left[H^q \gamma^+ + E^q \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} \right] u(p,\lambda),$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar{\psi}(-\tfrac{1}{2}z)\, \gamma^+\gamma_5\, \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar{u}(p',\lambda') \left[\tilde{H}^q \gamma^+\gamma_5 + \tilde{E}^q \frac{\gamma_5 \Delta^+}{2m} \right] u(p,\lambda),$$

Chiral even GPDs
-> Ji sum rule

$$\left\langle J_q^x \right\rangle = \tfrac{1}{2}\int dx \left[H(x,0,0) + E(x,0,0) \right] x$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar{\psi}(-\tfrac{1}{2}z)\, i\sigma^{+i}\, \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar{u}(p',\lambda') \left[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q \frac{P^+\Delta^i - \Delta^+ P^i}{m^2} \right.$$

$$\left. + E_T^q \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q \frac{\gamma^+ P^i - P^+\gamma^i}{m} \right] u(p,\lambda$$

Chiral odd GPDs
-> transversity
How to measure
and/or
parameterize them?

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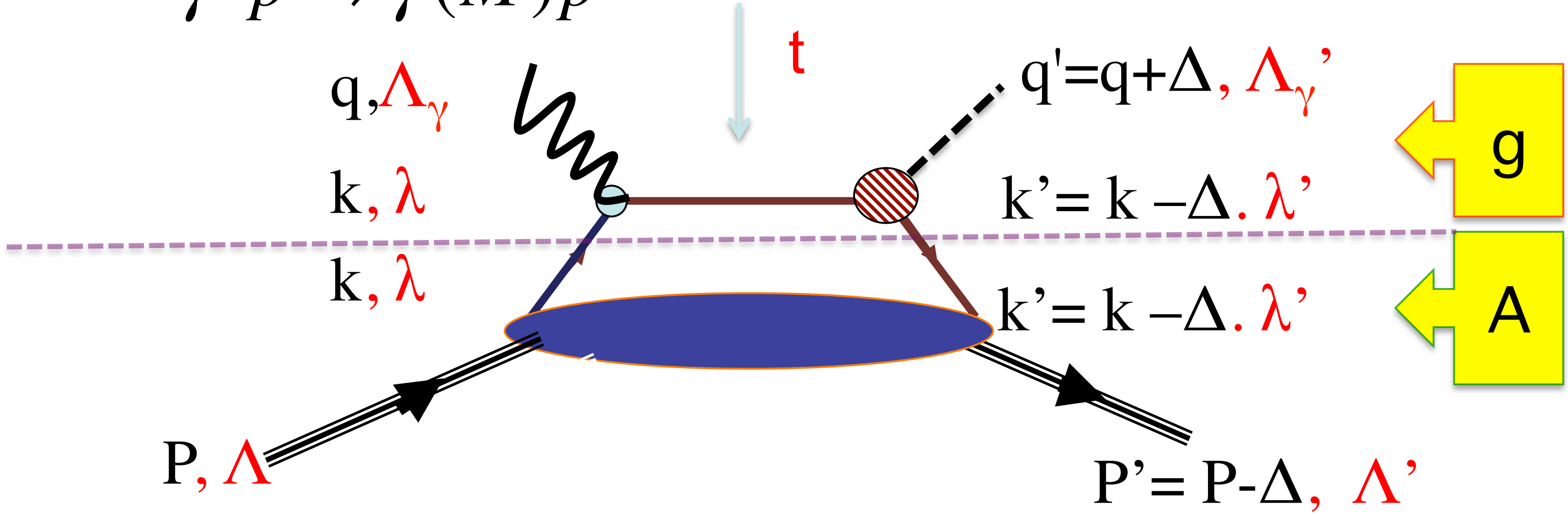
Factorization in exclusive processes (DVCS, DVMP…)

$$\gamma^* p \rightarrow \gamma(M) p'$$



Convolution of "hard part" with quark-proton **Helicity** amplitudes

$$f_{\Lambda_\gamma,\Lambda;\Lambda'_\gamma,\Lambda'} = \sum_{\lambda,\lambda'} g_{\lambda,\lambda'}^{\Lambda_\gamma,\Lambda'_{\gamma(M)}}(x,k_T,\zeta,t;Q^2) \otimes A_{\Lambda',\lambda';\Lambda,\lambda}(x,k_T,\zeta,t),$$

Normalizing GPDs - Chiral even

Form factor, Forward limit

$$\int_0^1 H_q(x,\xi,t)\,dx = F_1^q(t), \quad H_q(x,0,0) = q(x)$$

Integrates to charge

$$\int_0^1 E_q(x,\xi,t)\,dx = F_2^q(t)$$

→Anomalous magnetic moments

$$\int_0^1 \tilde{H}_q(x,\xi,t)\,dx = g_A^q(t), \quad \tilde{H}_q(x,0,0) = \Delta q(x) = q^{\rightarrow}_{\Rightarrow}(x) - q^{\leftarrow}_{\Rightarrow}(x)$$

Integrates to axial charge

$$\int_0^1 \tilde{E}_q(x,\xi,t)\,dx = g_P^q(t)$$

The question is: how do we normalize chiral-odd GPDs?

Only Physical constraints on the various chiral-odd GPDs are

Forward limit

$$H_T(x,0,0) = q_{\Uparrow}^{\uparrow}(x) - q_{\Uparrow}^{\downarrow}(x) = h_1(x)$$ Transversity

Integrates to tensor charge δ_q

Form Factors

$$\int H_T^q(x,\xi,t)\,dx = \delta q(t)$$

$$\int \bar{E}_T^q(x,\xi,t)\,dx = \int \left(2\tilde{H}_T^q + E_T^q\right) dx = \kappa_T^q(t)$$

Integrates to "transverse moment" κ_T^q

$$\int \tilde{E}_T(x,\xi,t)\,dx = 0$$

No direct interpretation of E_T .

The Model – Reggeized Diquarks

Chiral Even

Observables

Chiral Odd

$$A_{\Lambda'\pm,\Lambda\pm} \Leftrightarrow H, E, \tilde{H}, \tilde{E}$$

$$A_{\Lambda'\pm,\Lambda\mp} \Leftrightarrow H_T, E_T, \tilde{H}_T, \tilde{E}_T$$

Compton Form Factors

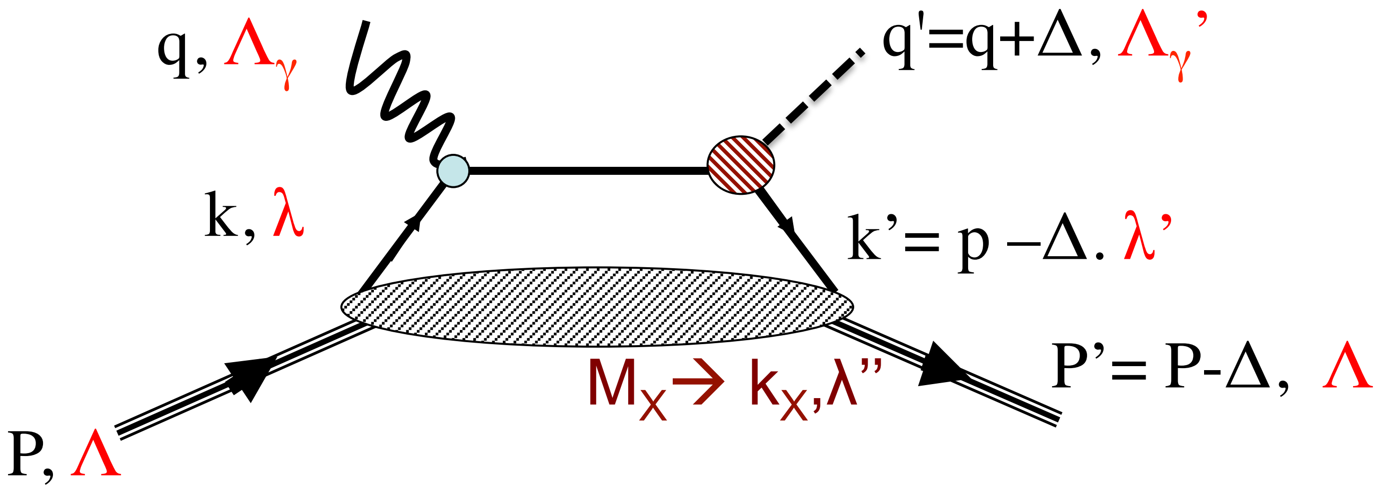
$$\mathcal{H}(\xi,t;Q^2) = \int dx \left[\frac{1}{x-\xi-i\varepsilon} \mp \frac{1}{x+\xi-i\varepsilon} \right] H(x,\xi,t;Q^2)$$

$$\rightarrow \left(P.V.\int dx \frac{H(x,\xi,t;Q^2)}{x-\xi} + i\pi H(\xi,\xi,t;Q^2) \right) \mp (symm.term)$$

Re $\mathcal{H}$ Im $\mathcal{H}$

The Model – first for Chiral Even – Reggeized Diquarks

Reggeization



$$A = \mathcal{N} \int \frac{dk_X^2 dk^2}{(k^2 - m^2 - i\epsilon)(k'^2 - m^2 - i\epsilon)} \frac{\rho(k_X^2, k^2) \times (spin\ structure)}{(k_X^2 - M_X^2 - i\epsilon)}$$

Landshoff, Polkinghorn, Short '71
Brodsky, Close, Gunion '71 Regge behavior required for Compton
Ahmad, Honkanen,Liuti,Taneja '07, '09
Gorshteyn & Szczepaniak (PRD, 2010)
Brodsky, Llanes-Estrada '07
Brodsky, Llanes, Szczepaniak '08

Gonzalez, GG, Liuti, **arXiv:1201.6088 [hep-ph]** *J. Phys. G: Nucl. Part. Phys.* **39** 115001 (2012)

Our method: Recursive fit

Advantage: control over the number of parameters to be fitted at different stages so that it can be optimized

Functional form:

From DIS

$$q(x,Q_o^2) = A_q\, x^{-\alpha_q}(1-x)^{\beta_q} F(x,c_q,d_q,...)$$

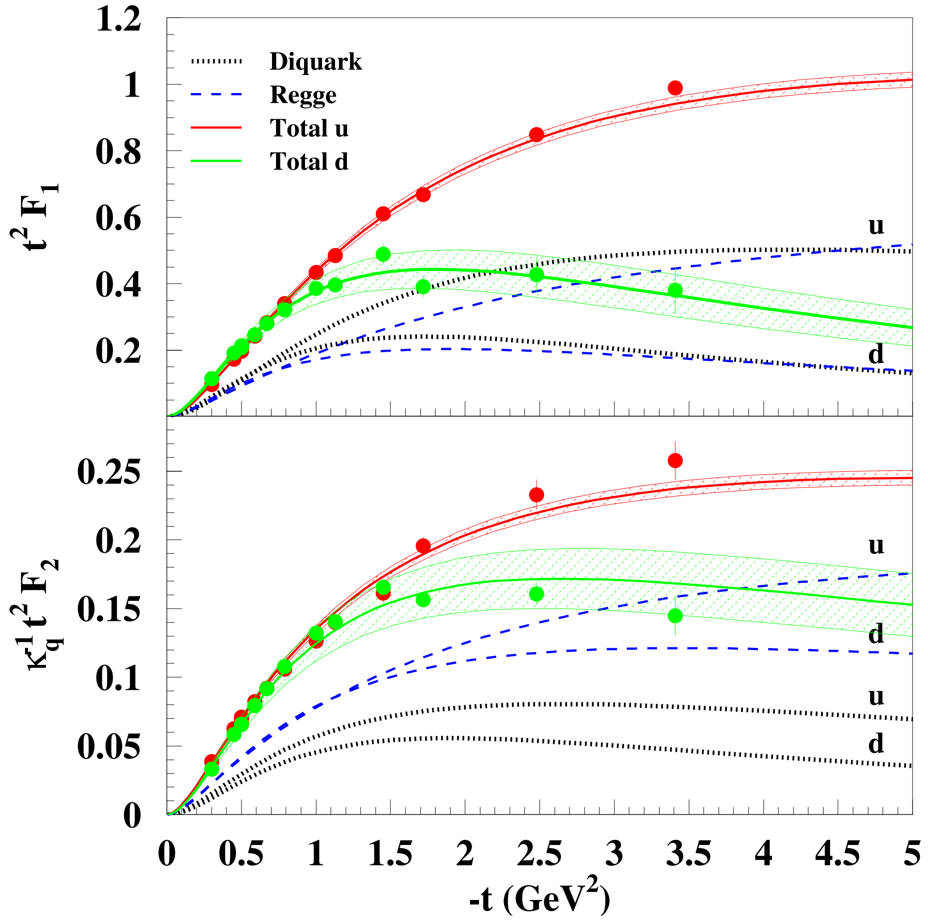
to Form factors, DVCS, DVMP

$$H_q(x,\xi,t;Q_o^2) = N_q\, x^{-\left[\alpha_q+\alpha'_q(1-x)^p t\right]} G^{a_1 a_2 a_3 ..}(x,\xi,t)$$

$$a_1 = m_q,\ a_2 = M_X^q,\ a_3 = M_\Lambda^q, ...$$

"Flexible" parameterization based on the Reggeized quark-diquark model.

EM Form Factors (t dependence)



O.Gonzalez, GRG, S.Liuti, K.Kathuria PRC88, 065206(2013)
data: G.D. Cates, et al. PRL106,252003 (2011).

Parametric Form

$$F(X,\zeta,t) = \mathcal{N} G^{M_\Lambda}_{M_X,m}(X,\zeta,t)\, R_p^{\alpha,\alpha'}(X,\zeta,t)$$

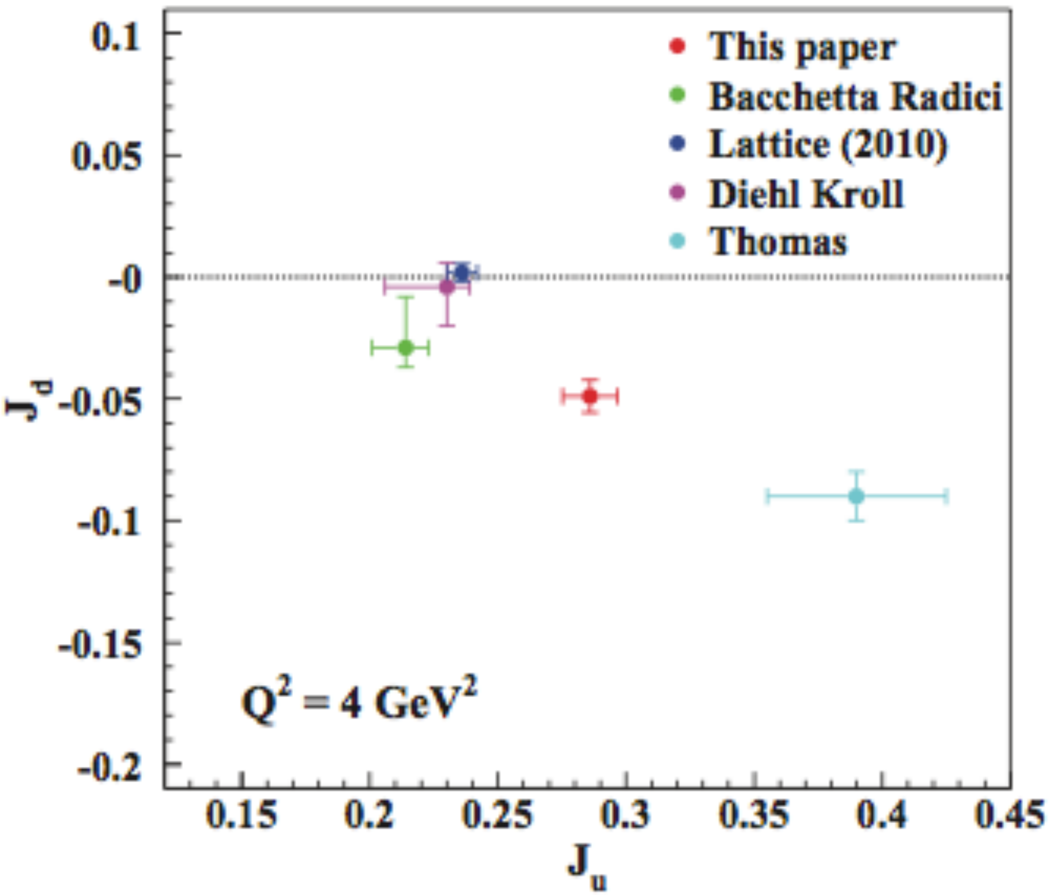
R✖Dq

Fit via DVCS dσ & asymmetries

see GRG, Gonzalez Hernandez, Liuti, PRD84, 034007 (2011)

Valence quark angular momenta

PHYSICAL REVIEW C **88**, 065206 (2013)



Valence quark angular momenta – scale dependence

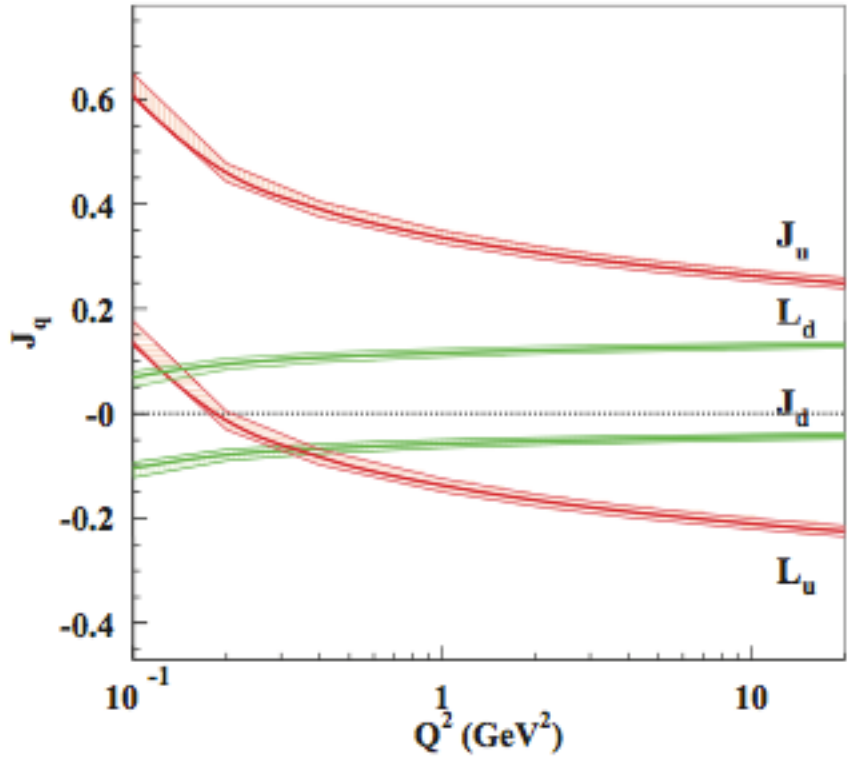


FIG. 18. (Color online) Quarks angular momentum, J_q, and orbital angular momentum, L_q, plotted vs the scale Q^2.

Valence quark angular momenta

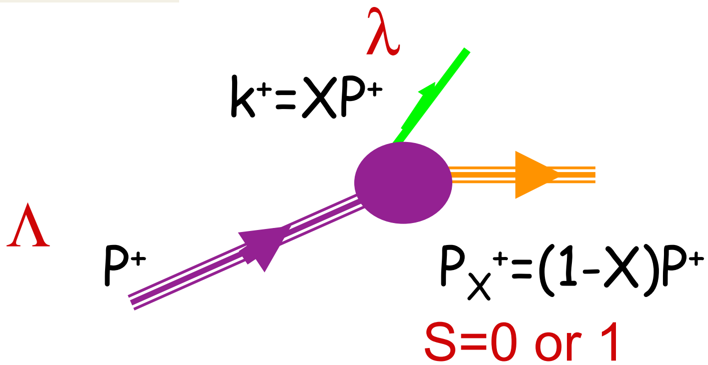
GONZALEZ-HERNANDEZ, LIUTI, GOLDSTEIN, AND KATHURIA PHYSICAL REVIEW C **88**, 065206 (2013)

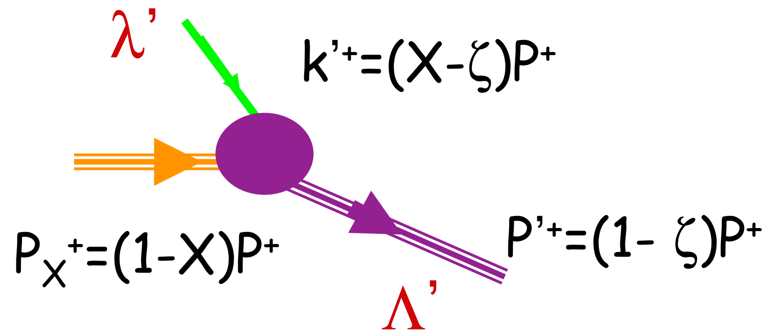
TABLE V. Values of angular momentum, J_u and J_d, at $Q^2 = 4$ GeV^2, obtained in various approaches: Our parametrization which is constrained by the flavor separated Dirac and Pauli form factors, compared to other determinations including theoretical uncertainty: from a similar analysis in Ref. [58], from a model calculation [59], from a model dependent analysis including transverse momentum distributions (TMDs) data [61], and from the most recent lattice QCD evaluation [62,63].

Reference	This paper	LHPC [62]	Thomas [59]	TMDs [61]	Diehl & Kroll [58]
u	0.286 ± 0.011	0.236 ± 0.0018	0.390 ± 0.035	$0.214^{+0.009}_{-0.013}$	$0.230^{+0.009}_{-0.024}$
d	-0.049 ± 0.007	0.006 ± 0.0037	-0.09 ± 0.01	$-0.029^{+0.021}_{-0.008}$	$-0.004^{+0.010}_{-0.016}$

The Model for Chiral Odd – Reggeized Diquarks

Vertex Structures with Diquark Spectator





First focus on S=0 pure spectator – beginning

$$H \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) + \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$E \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) + \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

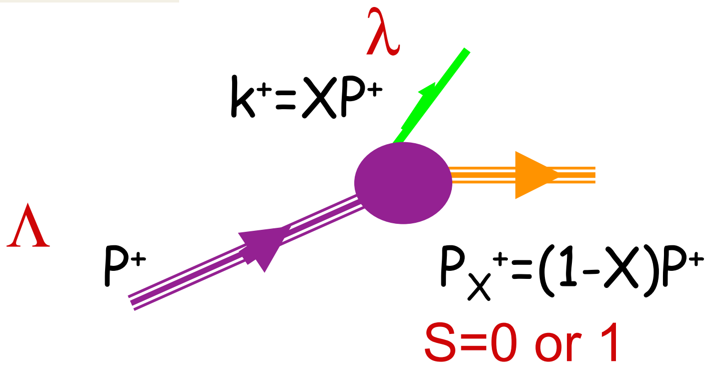
$$\tilde{H} \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) - \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

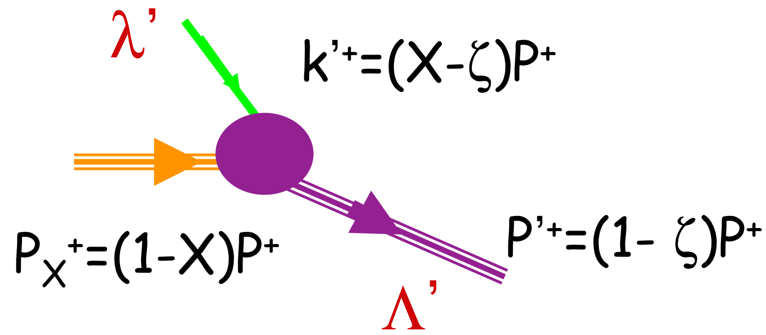
$$\tilde{E} \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) - \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

Vertex function

$$\phi(k^2,\lambda) = \frac{k^2 - m^2}{|k^2 - \lambda^2|^2}.$$

Vertex Structures with Diquark Spectator





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$$H \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) + \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$E \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) + \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

$$\tilde{H} \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) - \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$\tilde{E} \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) - \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

Parity at vertices:
Note that by switching λ➔- λ & Λ ➔ -Λ (Parity) will have chiral evens go to ± chiral odds giving relations – before k integrations
A(Λ'λ';Λλ)➔
±A(Λ',λ';-Λ,-λ)*

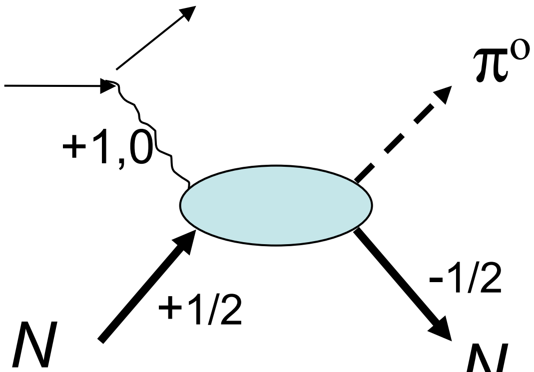
Vertex function

$$\phi(k^2,\lambda) = \frac{k^2 - m^2}{|k^2 - \lambda^2|^2}.$$

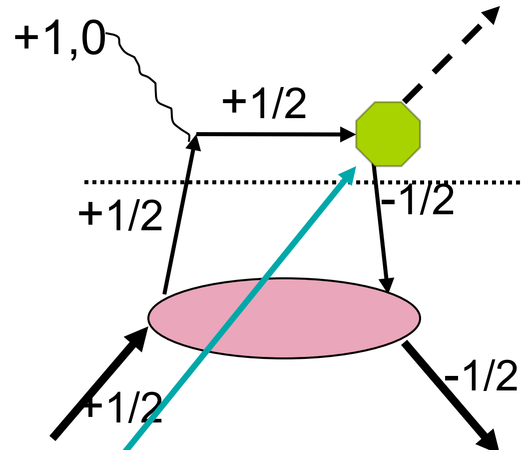
but then (Λ'-λ')-(Λ-λ) ≠(Λ'-λ')+(Λ-λ) unless Λ=λ

How to single out chiral odd GPDs?

Exclusive Lepto-production of π^o or η, η' to measure **chiral odd GPDs & Transversity**



e.g. $f_{+1+,0-}(s,t,Q^2)$

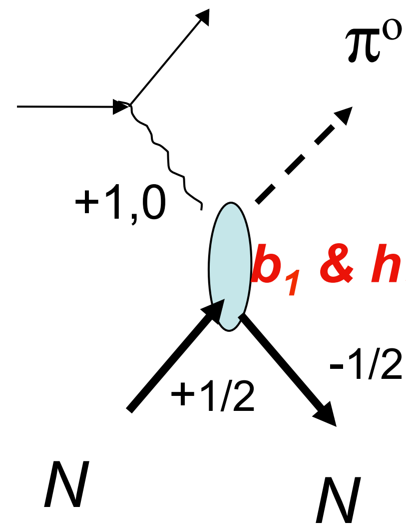


q nos of C-odd 1^{--} exchange
ρ & ω
1^{+-} exchange
b_1 & h_1

$g_{+1+,0-}\ A_{++,--}$

↓

H_T



$$\mathcal{P} = K f_\pi \left\{ \gamma_5 \, \not{q}' \phi_\pi(\tau) + \gamma_5 \mu_\pi \phi_\pi^{(3)}(\tau) \right\}$$

t-channel J^{PC} quantum numbers enhance chiral odd

6 helicity amps for π^0

f_1

$$f_{10}^{++} = g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{4M}\left[2\widetilde{\mathcal{H}}_T + (1+\xi)\left(\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right)\right]$$

$$= g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{\mathcal{H}}_T + \frac{1}{2-\zeta}E_T + \frac{1}{2-\zeta}\widetilde{\mathcal{E}}_T\right],$$

f_2

$$f_{10}^{+-} = \frac{g_\pi^{V,odd}(Q) + g_\pi^{A,odd}(Q)}{2}\sqrt{1-\xi^2}\left[\mathcal{H}_T + \frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]$$

$$= \frac{g_\pi^{V,odd}(Q) + g_\pi^{A,odd}(Q)}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}\left[\mathcal{H}_T + \frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T + \frac{\zeta^2/4}{1-\zeta}\mathcal{E}_T + \frac{\zeta/2}{1-\zeta}\widetilde{E}_T\right]$$

f_3

$$f_{10}^{-+} = -\frac{g_\pi^{A,odd}(Q) - g_\pi^{V,odd}(Q)}{2}\sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T$$

$$= -\frac{g_\pi^{A,odd}(Q) - g_\pi^{V,odd}(Q)}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}\,\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T$$

f_4

$$f_{10}^{--} = g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{4M}\left[2\widetilde{\mathcal{H}}_T + (1-\xi)\left(\mathcal{E}_T - \widetilde{\mathcal{E}}_T\right)\right]$$

$$= g_\pi^{V,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{\mathcal{H}}_T + \frac{1-\zeta}{2-\zeta}\mathcal{E}_T + \frac{1-\zeta}{2-\zeta}\widetilde{\mathcal{E}}_T\right]$$

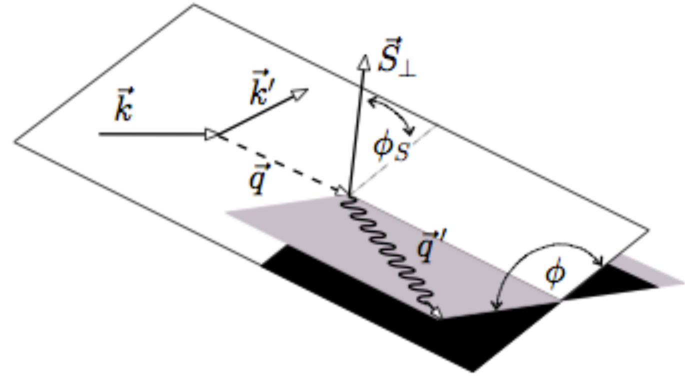
f_5

$$f_{00}^{+-} = g_\pi^{A,odd}(Q)\sqrt{1-\xi^2}\left[\mathcal{H}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]\sqrt{t_0-t}$$

f_6

$$f_{00}^{++} = -g_\pi^{A,odd}(Q)\frac{\sqrt{t_0-t}}{2M}\left[\xi\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right]\sqrt{t_0-t}$$

Cross sections for π^0



$$
\begin{aligned}
\frac{d^4\sigma}{dx_{Bj}dyd\phi dt} &= \Gamma\Big\{\Big[F_{UU,T}+\epsilon_L F_{UU,L}+\epsilon\cos 2\phi F_{UU}^{\cos 2\phi}+\sqrt{2\epsilon_L(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi}+h\sqrt{2\epsilon_L(1-\epsilon)}\,\sin\phi F_{LU}^{\sin\phi}\Big] \\
&+ S_{||}\Big[\sqrt{2\epsilon_L(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}+\epsilon\sin 2\phi F_{UL}^{\sin 2\phi}+h\Big(\sqrt{1-\epsilon^2}\,\sin\phi\,F_{LL}+\sqrt{2\epsilon_L(1-\epsilon)}\,\cos\phi\,F_{LL}^{\cos\phi}\Big)\Big] \\
&+ S_\perp\Big[\sin(\phi-\phi_S)\Big(F_{UT,T}^{\sin(\phi-\phi_S)}+\epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\Big)+\epsilon\Big(\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)}+\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}\Big) \\
&+ \sqrt{2\epsilon(1+\epsilon)}\Big(\sin\phi_S F_{UT}^{\sin\phi_S}+\sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\Big)\Big] \\
&+ S_\perp h\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)}+\sqrt{2\epsilon(1-\epsilon)}\Big(\cos\phi_S F_{LT}^{\cos\phi_S}+\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big)\Big]\Big\}
\end{aligned} \tag{55}
$$

$$
\begin{aligned}
F_{UU,T} &= \frac{d\sigma_T}{dt}, \quad F_{UU,L}=\frac{d\sigma_L}{dt}, \quad F_{UU}^{\cos\phi}=\frac{d\sigma_{LT}}{dt}, \\
F_{UU}^{\cos 2\phi} &= \frac{d\sigma_{TT}}{dt}, \quad F_{LU}^{\sin\phi}=\frac{d\sigma_{LT'}}{dt}
\end{aligned}
$$

$\mathcal{H}_T, \mathcal{E}_T, \tilde{\mathcal{E}}_T, \bar{\mathcal{E}}_T$

$$\frac{d\sigma_T}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\left[\mid \mathcal{H}_T \mid^2 + \tau \left(\mid \bar{\mathcal{E}}_T \mid^2 + \mid \tilde{\mathcal{E}}_T \mid^2 \right) \right] \tag{11}$$

$$\frac{d\sigma_L}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4} \frac{2M^2\tau}{Q^2} \mid \mathcal{H}_T \mid^2 \tag{12}$$

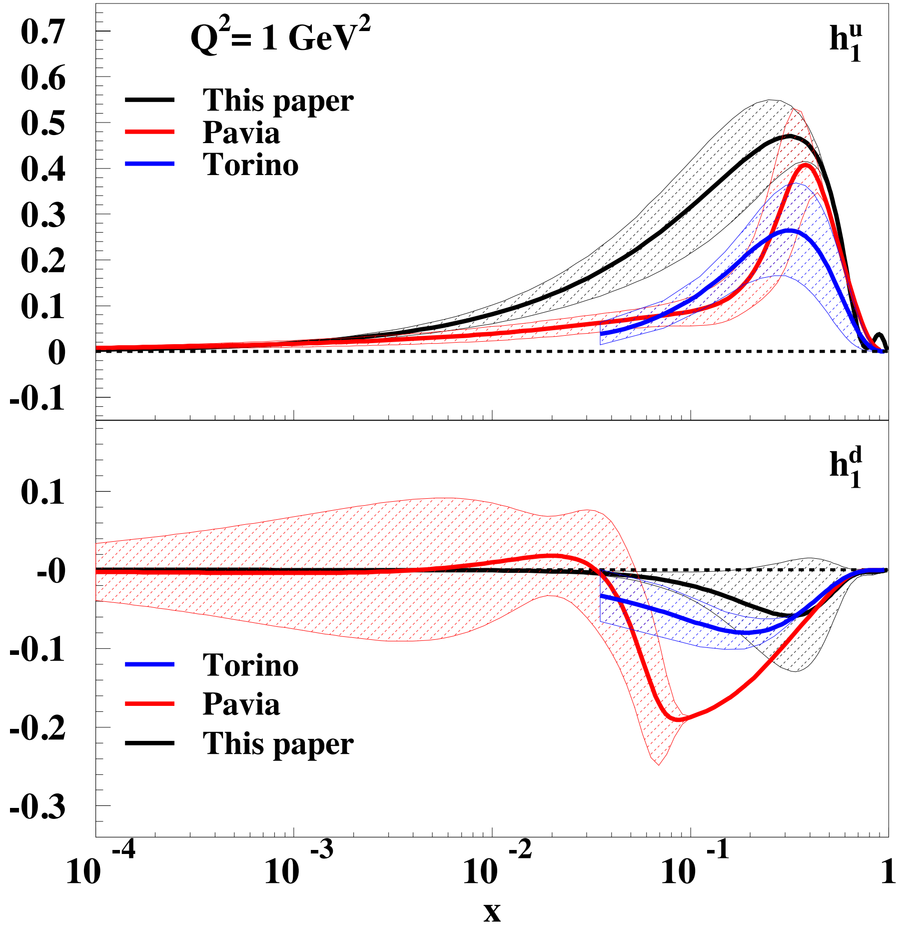
$$\frac{d\sigma_{TT}}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4} \tau \left[\mid \bar{\mathcal{E}}_T \mid^2 - \mid \tilde{\mathcal{E}}_T \mid^2 + \Re e \mathcal{H}_T \frac{\Re e(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2} + \Im m \mathcal{H}_T \frac{\Im m(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2} \right] \tag{13}$$

$$\frac{d\sigma_{LT}}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\, 2\sqrt{\frac{2M^2\tau}{Q^2}} \mid \mathcal{H}_T \mid^2 \tag{14}$$

$$\frac{d\sigma_{L'T}}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4} \tau \sqrt{\frac{2M^2\tau}{Q^2}} \left[\Re e \mathcal{H}_T \frac{\Im m(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2} - \Im m \mathcal{H}_T \frac{\Re e(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2} \right] \tag{15}$$

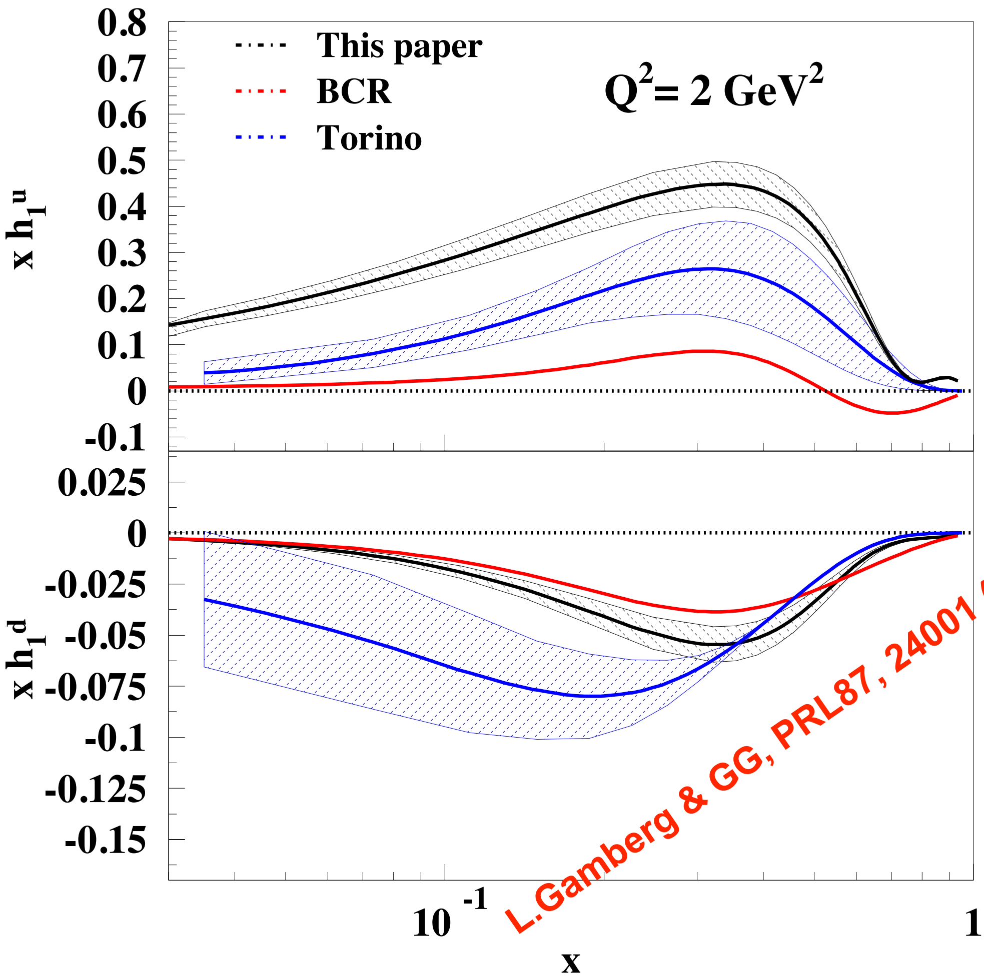
$(t_0-t)/2M^2 = \tau$

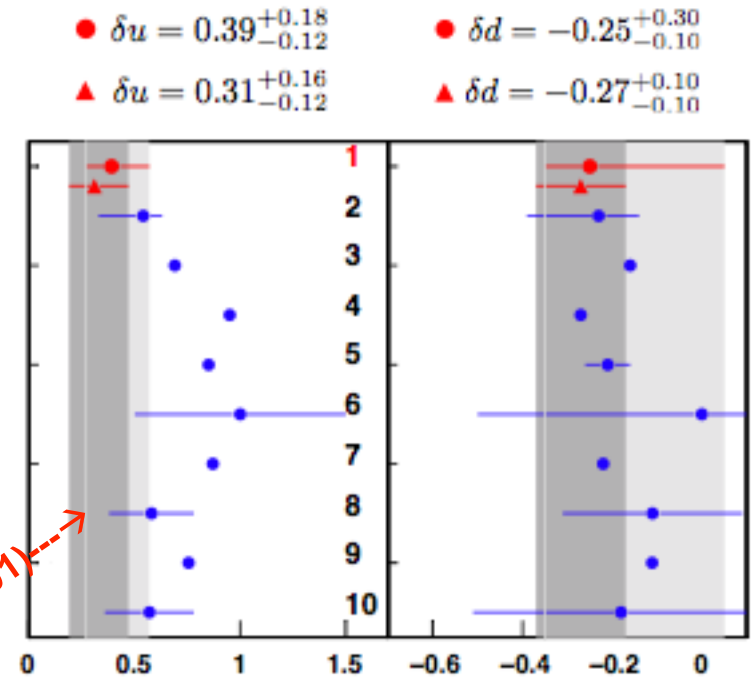
Chiral odd GPDs → Transversity → pdf's: $h_1^q(x,Q^2)$



GG, Gonzalez, Liuti, arXiv:1311.0483 [hep-ph]

Extraction of tensor charge

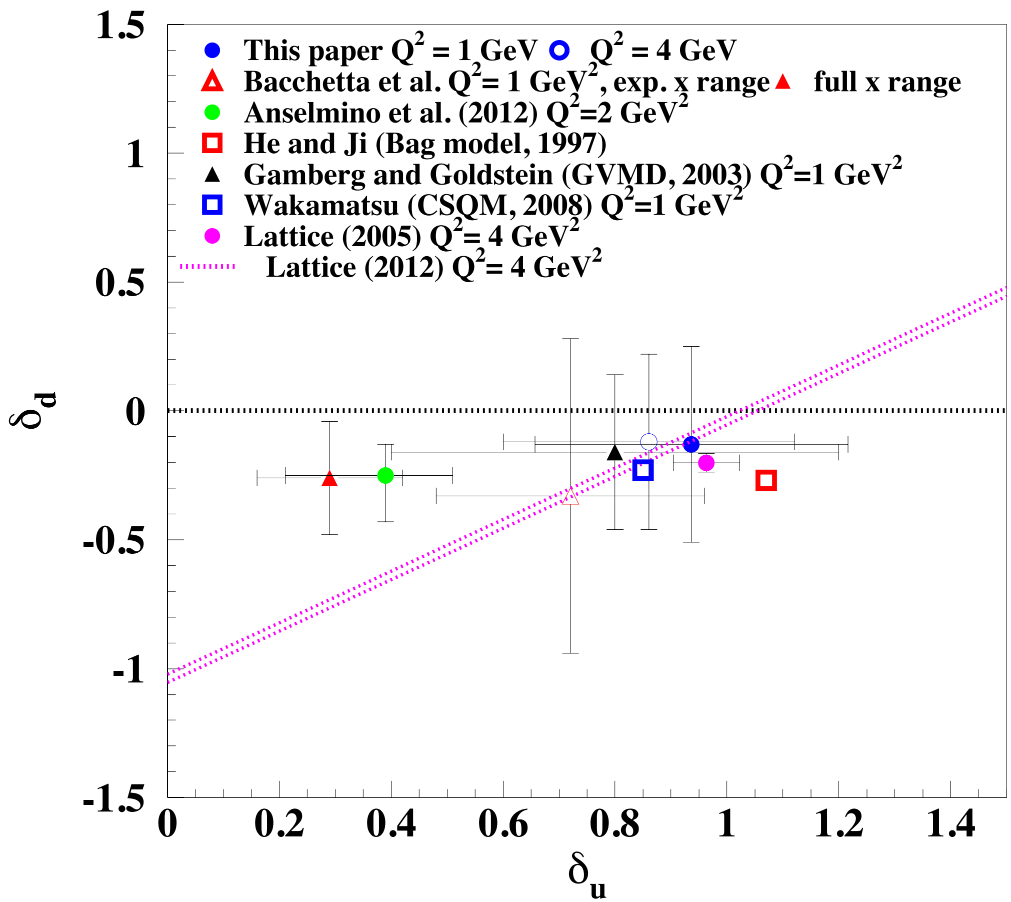




Anselmino, Boglione, et al., Phys.Rev. D87 (2013) 094019
$\delta u = 0.31_{-0.12}^{+0.16}$ $\delta d = -0.27_{-0.10}^{+0.10}$

From our Reggeized form
$\delta u \approx 1.2$ $\delta d \approx -0.08$
Closer to QCD sum rule values

Chiral odd GPDs → Transversity → tensor charges δ_q



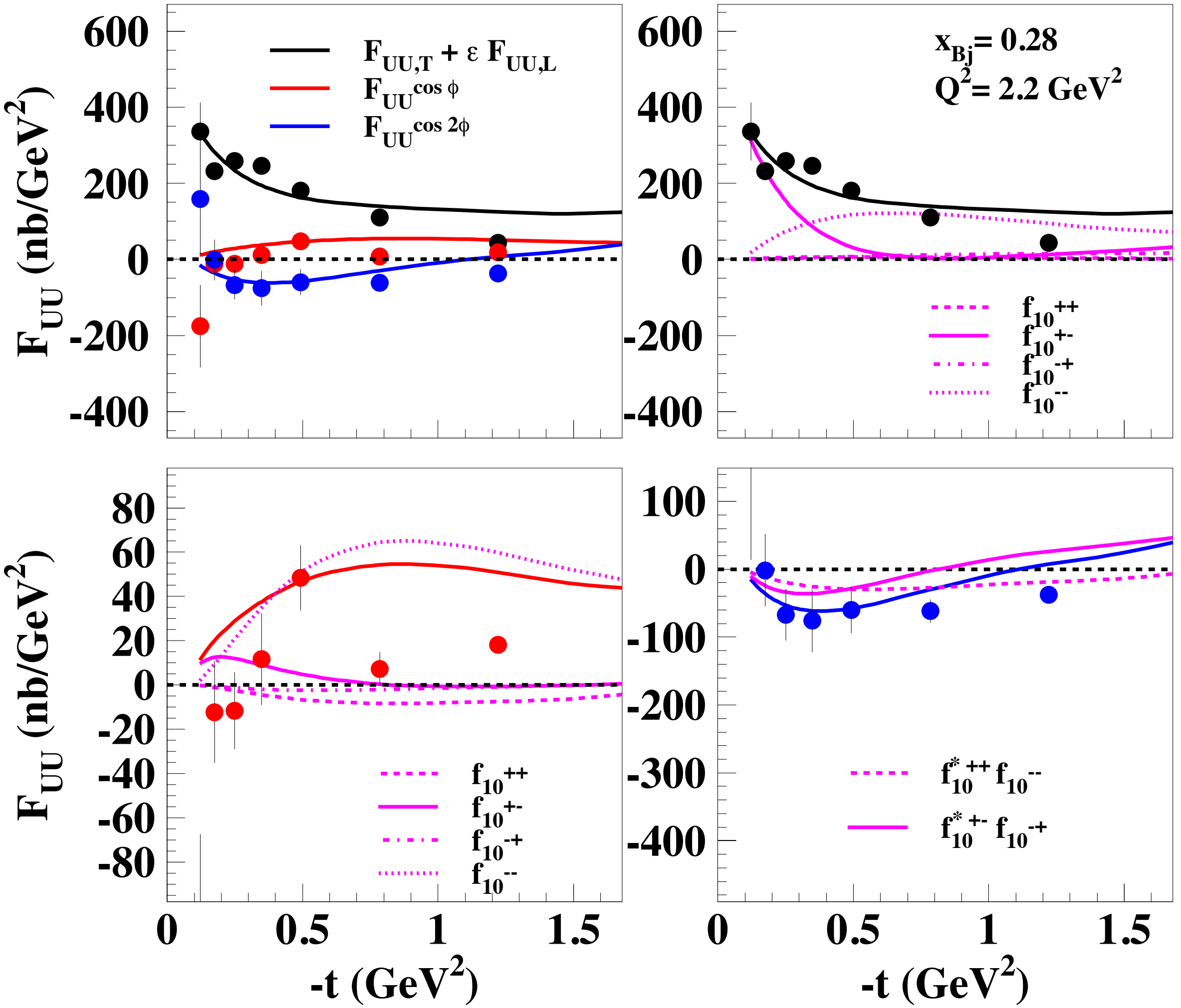
GG, Gonzalez, Liuti, arXiv:1311.0483 [hep-ph]

Observables

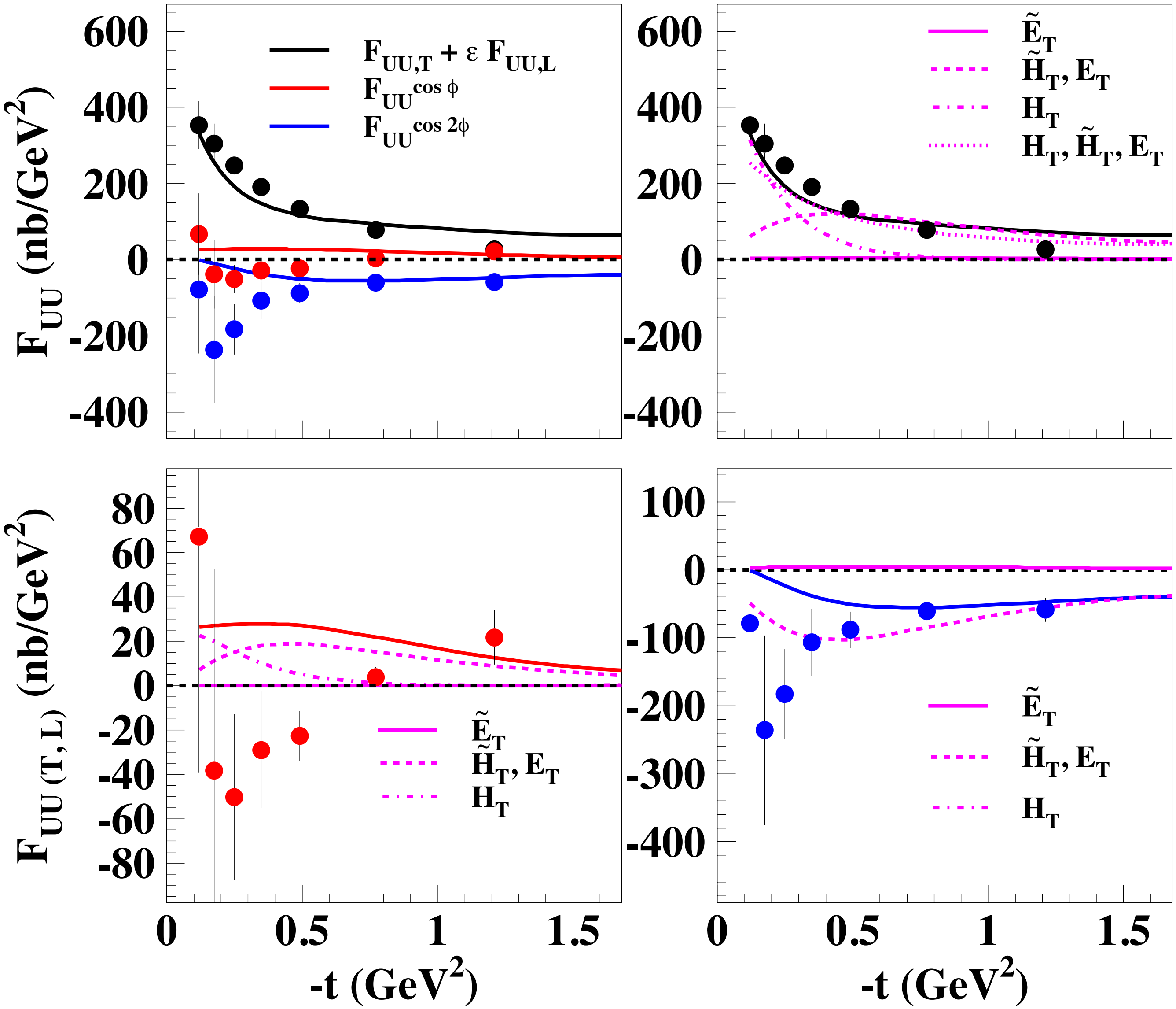
- Cross sections
- Asymmetries

How well do the parameters fixed with DVCS data reproduce π^o electroproduction data?

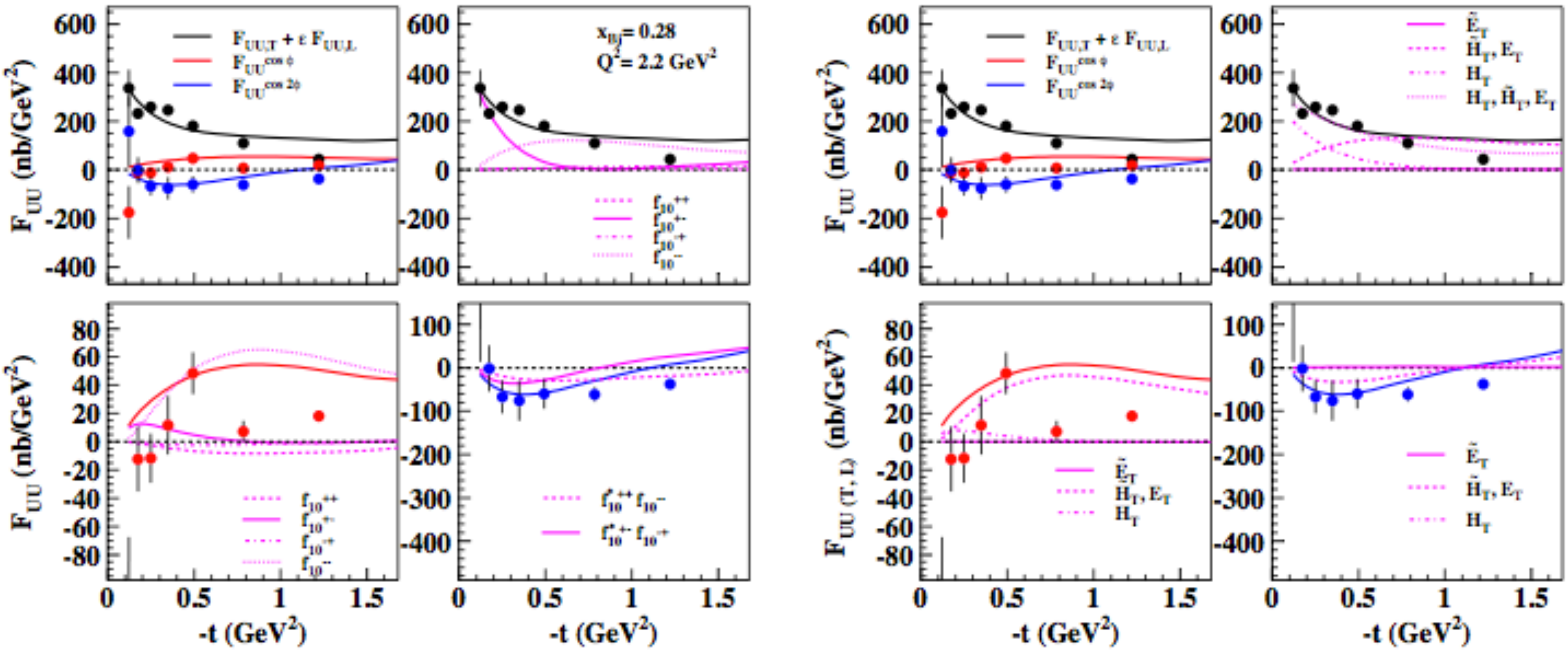
Hall B data, Kubarovsky& Stoler, PoS ICHEP 2010 & PRL 109, 112001 (2012)



Same, separating the GPDs contribution



Same, at x_{Bj} = 0.28, Q^2=2.2 GeV^2



Asymmetries:
Longitudinal polarizations

$$F_{UL}^{\sin\phi} = \frac{1}{\sqrt{2}}\mathcal{N}\Im m\left[(f_{00}^{+-})^*(f_{10}^{+-} - f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} + f_{10}^{--})\right]$$

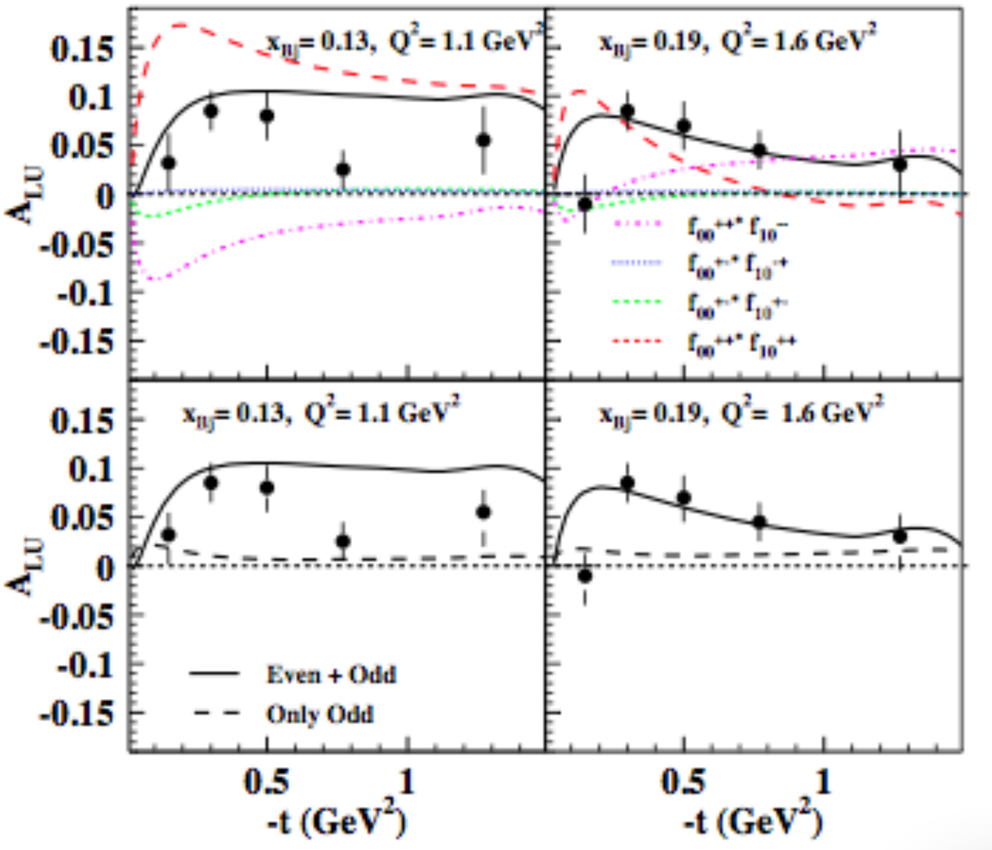
$$F_{UL}^{\sin 2\phi} = -\mathcal{N}\Im m\left[(f_{10}^{++})^*(f_{10}^{--}) - (f_{10}^{+-})^*(f_{10}^{-+})\right]$$

$$F_{LL}^{\cos\phi} = \frac{1}{\sqrt{2}}\mathcal{N}\Re e\left[(f_{00}^{+-})^*(f_{10}^{+-} - f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} + f_{10}^{--})\right]$$

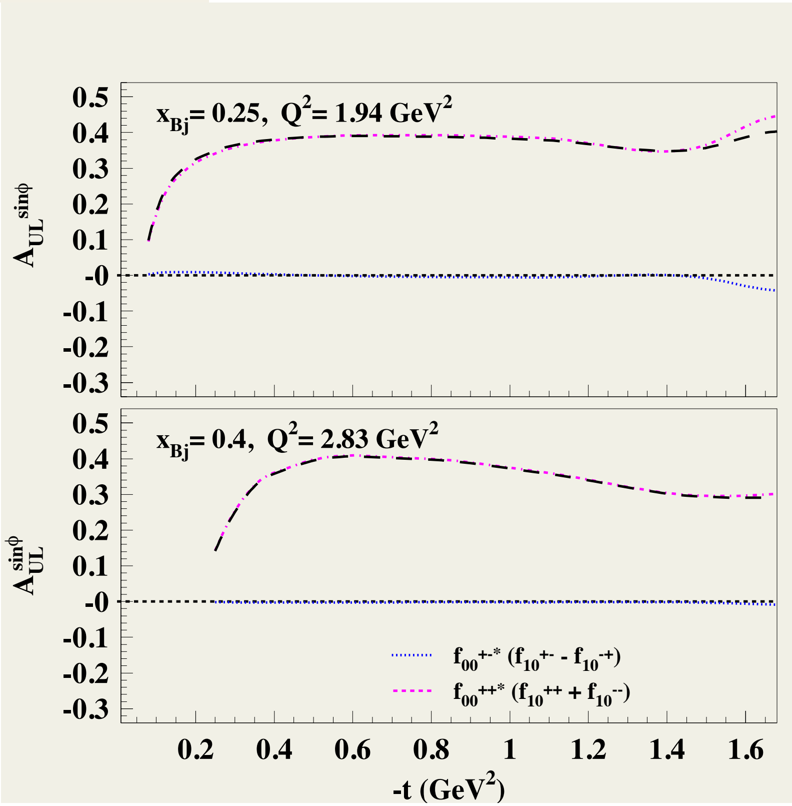
$$F_{LL} = \frac{1}{2}\mathcal{N}\left[\mid f_{10}^{++}\mid^2 + \mid f_{10}^{+-}\mid^2 - \mid f_{10}^{-+}\mid^2 - \mid f_{10}^{--}\mid^2\right]$$

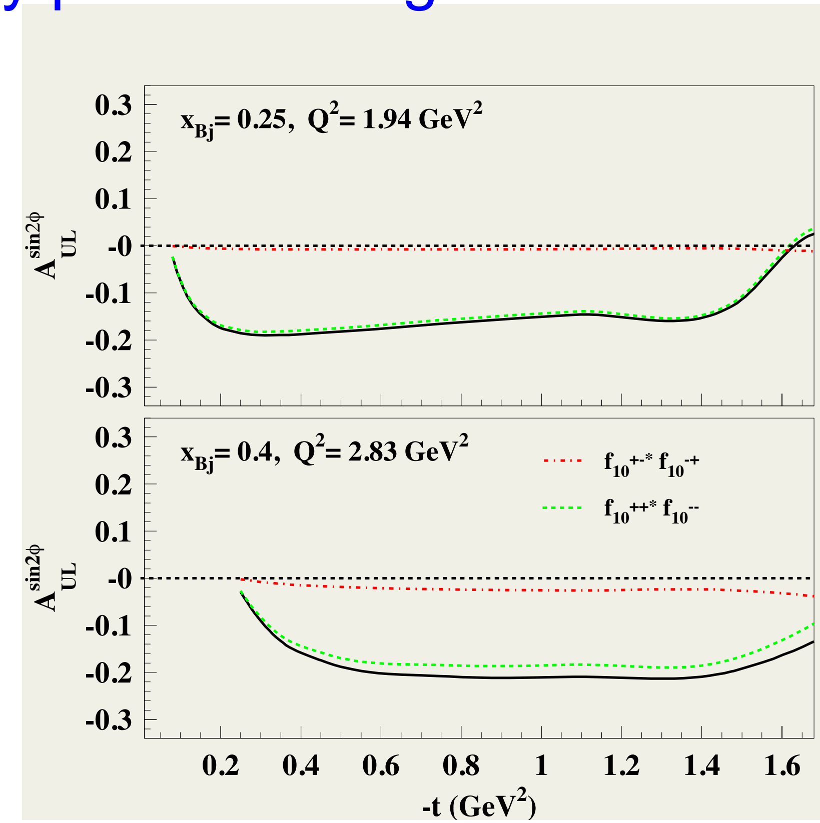
$$A_{LL} = \frac{\sqrt{1-\epsilon^2}\,F_{LL} + \sqrt{2\epsilon_L(\epsilon-1)}\,\cos\phi\,F_{LL}^{\cos\phi}}{F_{UU,T} + \epsilon_L F_{UU,L}}$$

Beam spin asymmetry shows importance of $\tilde{H}$ chiral even



Longitudinally polarized target

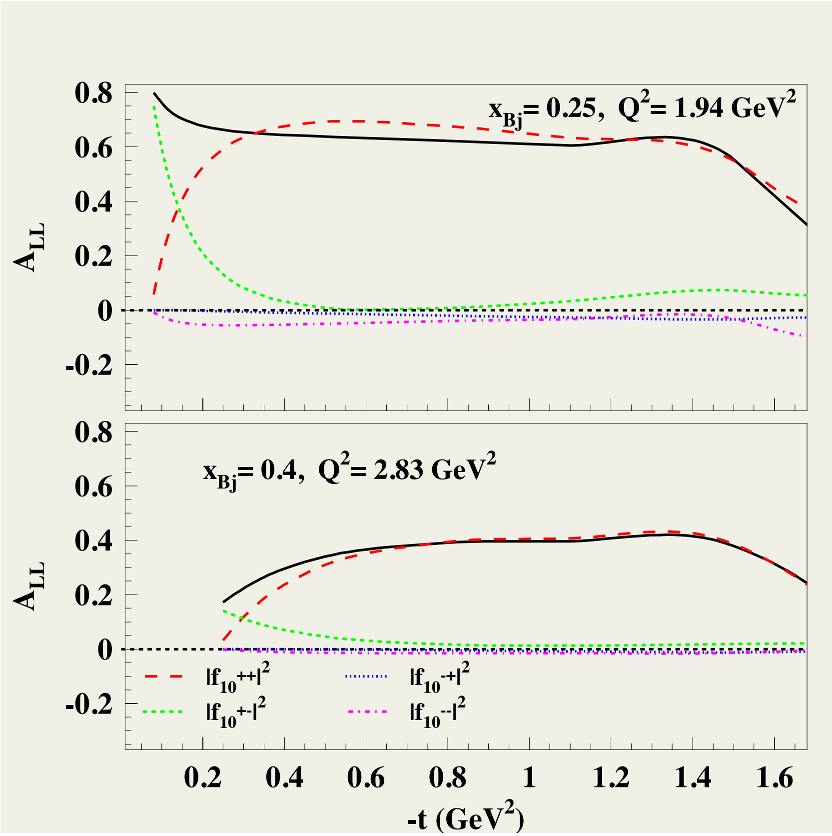


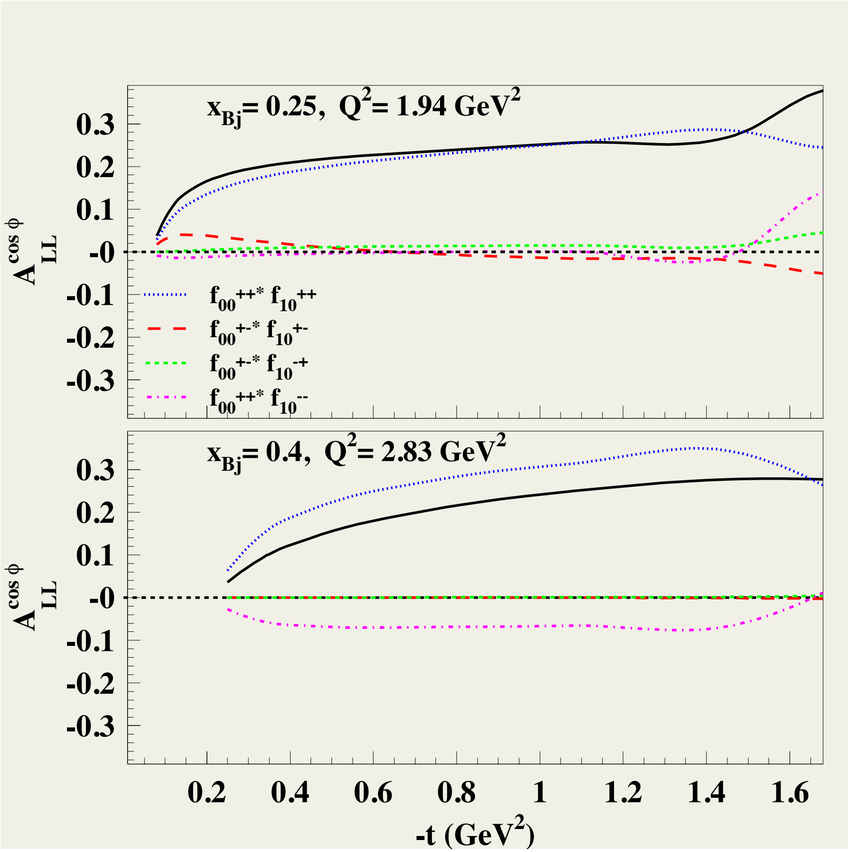


Look for tensor charge in f_{10}^{+-}

Transverse dipole moment in f_{10}^{++}, f_{10}^{--}

Longitudinally polarized beam and target

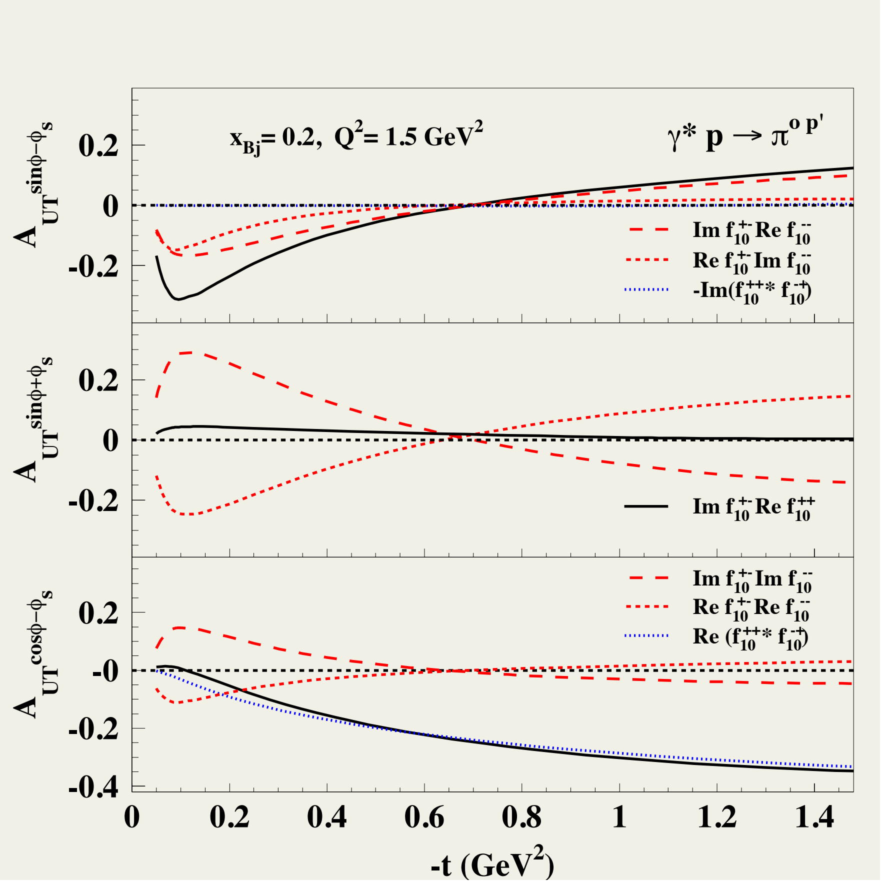


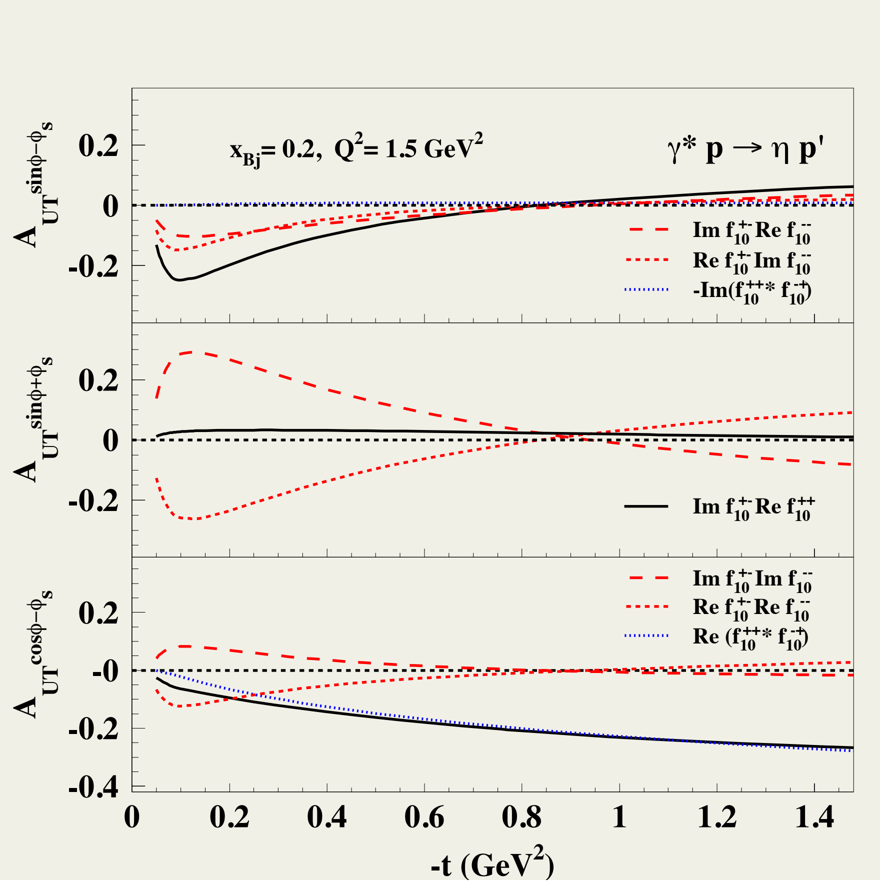


Look for tensor charge in f_{10}^{+-}

Transverse dipole moment in f_{10}^{++}, f_{10}^{--}

Transverse target

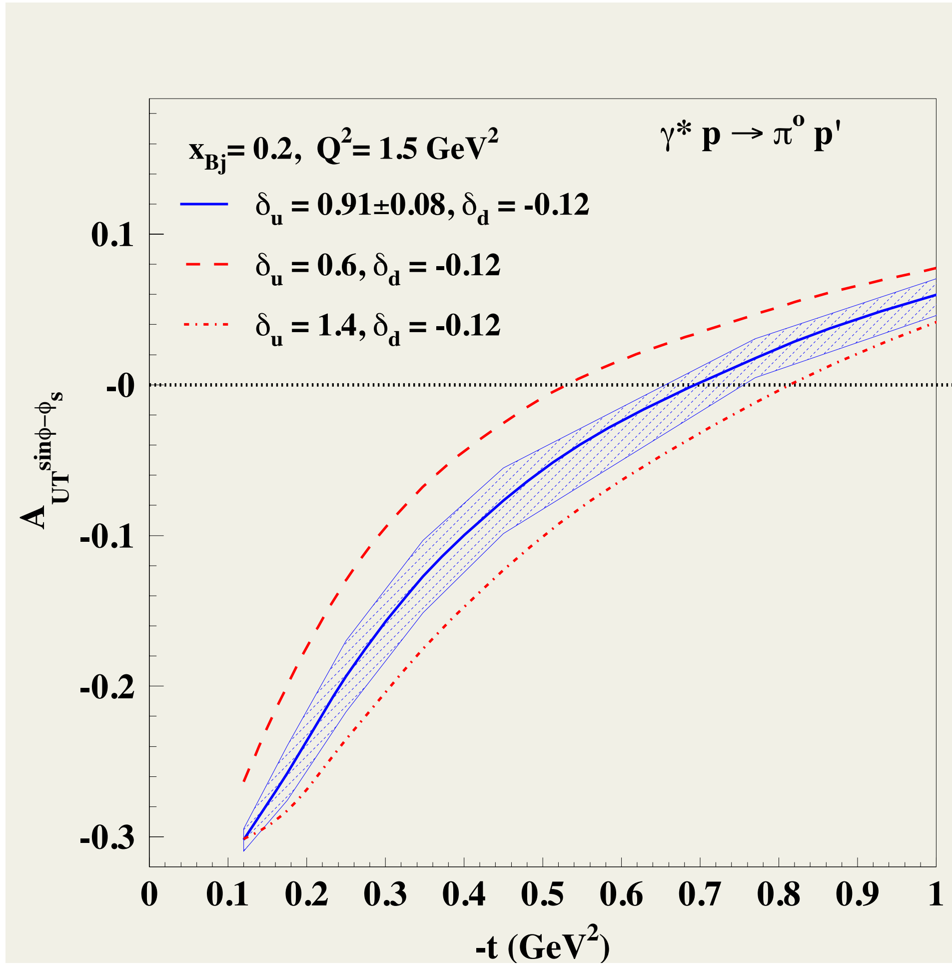




Look for tensor charge in f_{10}^{+-}

Transverse dipole moment in f_{10}^{++}, f_{10}^{--}

Asymmetry sensitive to tensor charge

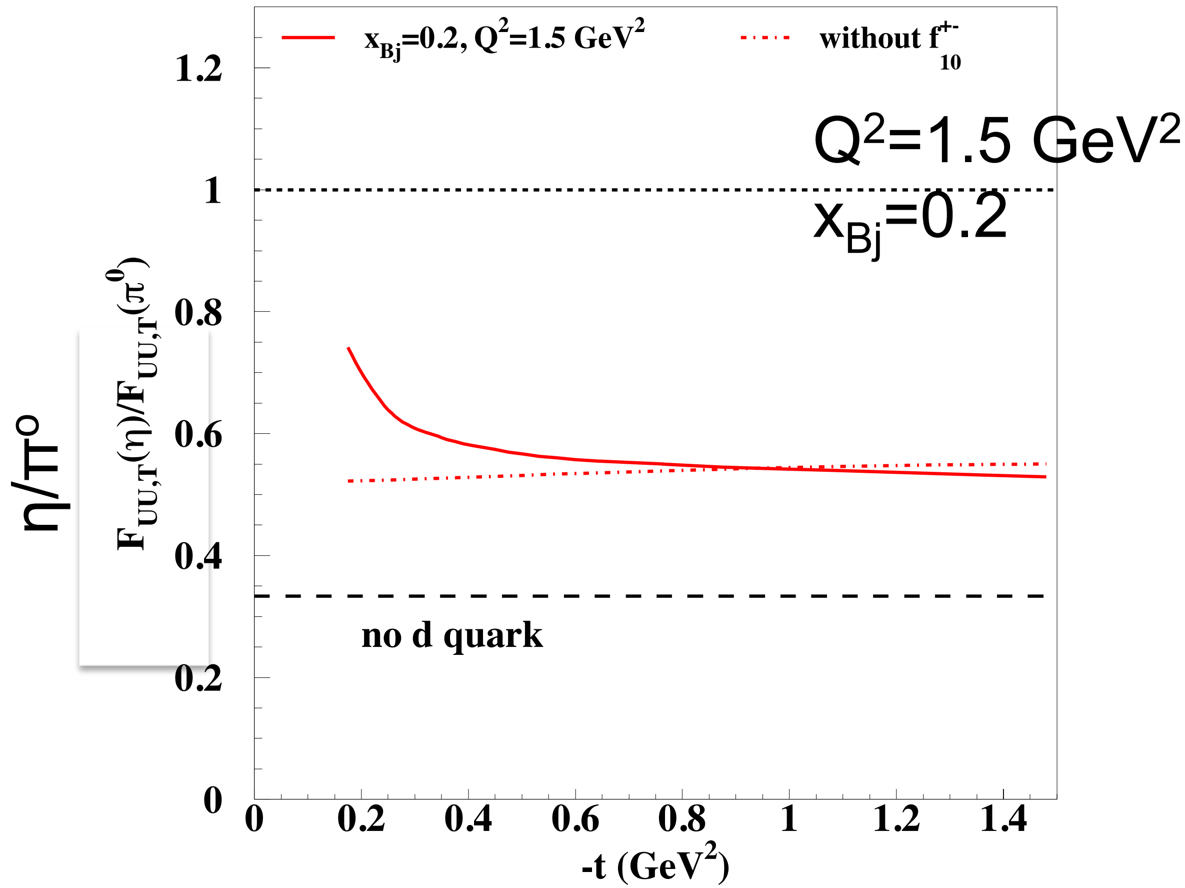


$$A_{UT}^{\sin(\phi+\phi_S)} = -\frac{\epsilon}{2}\frac{F_{UT}^{\sin(\phi+\phi_S)}}{F_{UU,T}+\epsilon F_{UU,L}} = -\epsilon\,\frac{\Re e f_{10}^{+-}\,\Im m f_{10}^{++} - \Re e f_{10}^{++}\,\Im m f_{10}^{+-}}{d\sigma/dt}$$

Comparing to other models

- The t-> 0 feature for us is that f_{10}^{+-} dominates & it is driven by H_T. But f_{10}^{++} & f_{10}^{--} also contribute as $\sim\sqrt{(t_0-t)}$, however weaker.
- f_{10}^{++} & f_{10}^{--} are not equal in magnitude, especially vs. ζ or ξ. ➔$\tilde{E}_T$ is significant.
- In $A_{LL} \sim |f_{10}^{++}|^2 + |f_{10}^{+-}|^2 - |f_{10}^{-+}|^2 - |f_{10}^{--}|^2$ sensitive to differences
- Our normalization is set by Chiral-odd<-->even
- c.f. Goloskokov & Kroll – different dominant amps.

Ratio of unpolarized η / π^0



State of the art

After these studies we are returning to a global fit

A. DVCS

Unpolarized scattering cross section

$$d^4\sigma = F_{UU,T} = c_0 + c_1 \cos\phi + c_2 \cos 2\phi \tag{1}$$

BSA

$$A_{LU} = \sqrt{\epsilon(1-\epsilon)}\,\frac{F_{LU}^{\sin\phi}}{F_{UU,T}} = \frac{a_1 \sin\phi}{c_0 + c_1\cos\phi + c_2\cos 2\phi} \tag{2}$$

TSA

$$\begin{aligned} A_{UL} &= \frac{\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}}{F_{UU,T} + \epsilon\, F_{UU,L}} + \frac{\epsilon\sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T}} \\ &= \frac{a_2\sin\phi + a_3\sin 2\phi}{c_0 + c_1\cos\phi + c_2\cos 2\phi} \end{aligned} \tag{3}$$

Double TSA

$$\begin{aligned} A_{LL} &= \frac{\sqrt{1-\epsilon^2}\; F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)}\;\cos\phi\, F_{LL}^{\cos\phi}}{F_{UU,T} + \epsilon F_{UU,L}} \\ &= \frac{a_4 + a_5\cos\phi}{c_0 + c_1\cos\phi + c_2\cos 2\phi} \end{aligned} \tag{4}$$

Summarizing chiral odd for π^0, η & charged pseudoscalars

Based on our analysis we expect the following behaviors to approximately appear in the data.

(i) The order of magnitude of the various terms approximately follows a sequence determined by the inverse powers of Q and the powers of $\sqrt{t_o - t}$: $d\sigma_T/dt \geqslant d\sigma_{TT}/dt \geqslant d\sigma_{LT/L'T}/dt \geqslant d\sigma_L/dt$.

(ii) $d\sigma_T/dt$ is dominated by $\mathcal{H}_T$ at small t, and governed by the interplay of $\mathcal{H}_T$ and $\overline{\mathcal{E}}_T$ at larger t.

(iii) $d\sigma_L/dt$ and $d\sigma_{LT}/dt$ are directly sensitive to $\mathcal{H}_T$.

(iv) $d\sigma_{TT}/dt$ and $d\sigma_{L'T}/dt$ contain a mixture of GPDs. They will play an important role in singling out the less known terms, $\overline{E}_T$, E_T and $\widetilde{E}_T$.

The interplay of the various GPDs can already be seen by comparing to the Hall B data [4] shown in figure 2.

One can see, for instance, that the ordering predicted in (i) is followed, and that $d\sigma_T/dt$ exhibits a form factor-like fall off of $\mathcal{H}_T$ with $-t$.

Summary

- Flexible parameterization for chiral even from form factors, pdfs & DVCS **R✖Dq**
- Extended **R✖Dq** to chiral odd sector
- DVMP – π^0 many dσ ‘s & Asymmetries

  measure ***Transversity***

Backup slides

Quark Orbital Angular Momentum

- Other ansätze using GTMDs or Wigner distributions invoke F_{14}, leading twist, but odd expectation values that vanish in TMD or GPD limits → not observable.
- Appropriate twist 3 GTMD has the kinematic factor $i\sigma^{ji}\, k_T^{\,j}/M$.

$$-\frac{4}{P^+}\left[\frac{\bar{\mathbf{k}}_T\cdot\mathbf{\Delta}_T}{\Delta_T}F_{27}+\Delta_T F_{28}-\left(\frac{\bar{\mathbf{k}}_T\cdot\mathbf{\Delta}_T}{\Delta_T}G_{27}+\Delta_T G_{28}\right)\right] = A^{tw3}_{++,++}+A^{tw3}_{+-,+-}-A^{tw3}_{-+,-+}-A^{tw3}_{--,--}$$

$G_2 \Rightarrow \sigma_{ij}\,\Delta^j \Rightarrow \underline{S}_L \times \underline{\Delta}$ expectation even at twist 3

This can be measured in DVCS *sin2ϕ* asymmetry $A_{UL}^{\sin 2\phi} \propto$

$$b \approx s^{\mathcal{I}}_{2,LP} \propto F_1(t)\,\Im m\,\tilde{\mathcal{H}}^{\mathrm{eff}}$$

$$\tilde{\mathcal{H}}^{\mathrm{eff}} = -2\xi\left(\frac{1}{1+\xi}\tilde{\mathcal{H}}+\tilde{\mathcal{H}}_3^{+}-\tilde{\mathcal{H}}_3^{-}\right),$$

where (Table 1),

$$\tilde{\mathcal{H}} = C^{+}\otimes\tilde{H}, \qquad \tilde{\mathcal{H}}_3^{+} = C^{+}\otimes\tilde{E}'_{2T}, \qquad \tilde{\mathcal{H}}_3^{-} = C^{-}\otimes\tilde{E}_{2T}.$$

Reggeization via spectator diquark mass formulation
Where does the Regge behavior come from?

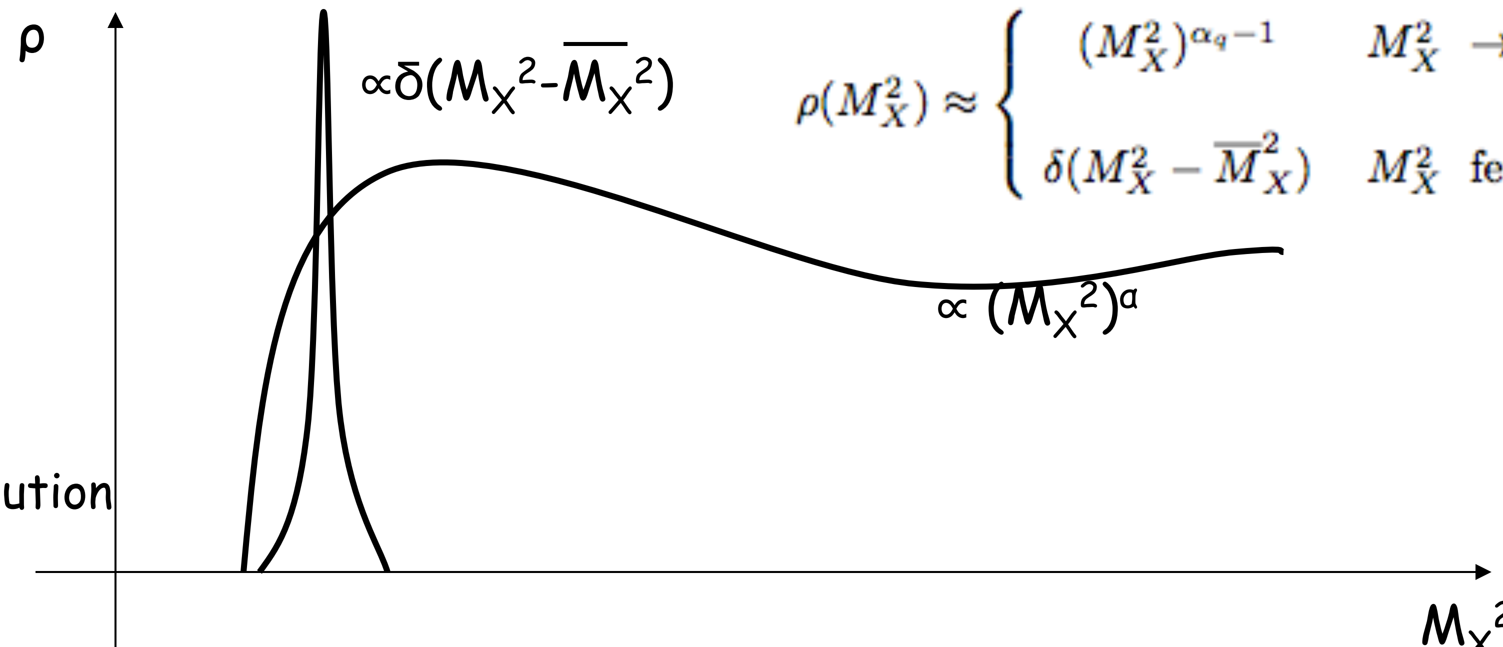
$$F_T^q(X,\zeta,t) \;=\; \mathcal{N}_q \int_0^\infty dM_X^2 \rho(M_X^2) F_T^{(m_q, M_\Lambda^q)}(X,\zeta,t;M_X),$$

Diquark spectral function

$$F(X,\zeta,t) \approx \mathcal{N} G_{M_X,m}^{M_\Lambda}(X,\zeta,t)\, R_p^{\alpha,\alpha'}(X,\zeta,t)$$

"Regge"

$$\rho(M_X^2) \approx \begin{cases} (M_X^2)^{\alpha_q - 1} & M_X^2 \to \infty \\ \delta(M_X^2 - \overline{M}_X^2) & M_X^2 \text{ few GeV}^2 \end{cases}$$



+ Q^2 Evolution

Following DIS work by Brodsky, Close, Gunion (1973)

✓ $H_q(x,0,0;Q^2) = f_1^q(x,Q^2)$, ...

DIS data

$$F_2^{p,d}, G_1, \ldots$$

Total number of parameters = N

fix $n_1 < N$ parameters

$t = \xi = 0$

$$\phi_{\Lambda\lambda}^{a_1,a_2,a_3,\ldots}(k,P) \rightarrow A_{\Lambda\pm\Lambda\pm}^{S=0,S=1} \rightarrow f_1^{S=0,S=1},\ g_1^{S=0,S=1} \rightarrow f_1^{u,d},\ g_1^{u,d}$$

Q^2 evol

Q^2 evol

✓ Switch on t: $H_q(x,0,t;Q^2)$

fix n_2 parameters $n_1 + n_2 < N$

form factor data

$$F_1(t), F_2(t), G_A, G_P$$

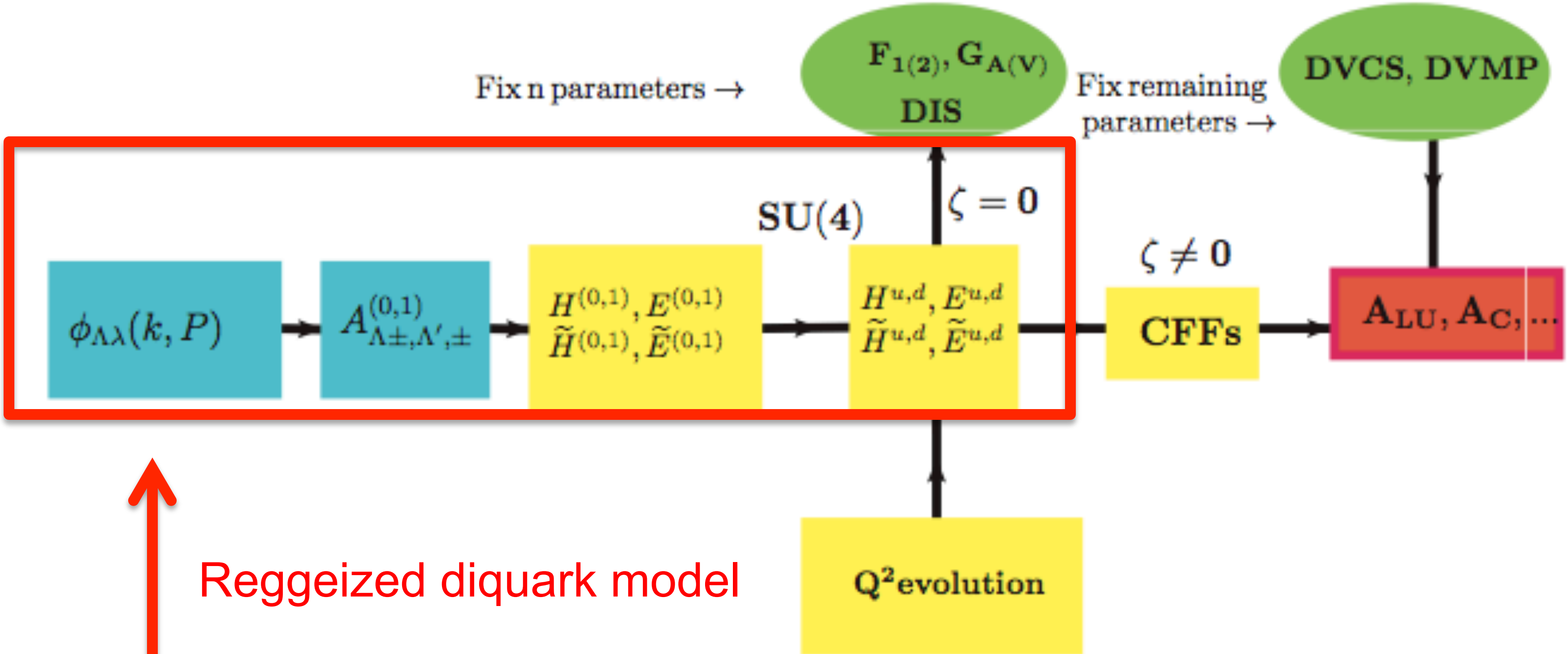
✓ Switch on ξ: $H_q(x,\xi,t;Q^2)$

fix remaining $N-(n_1+n_2)$ parameters

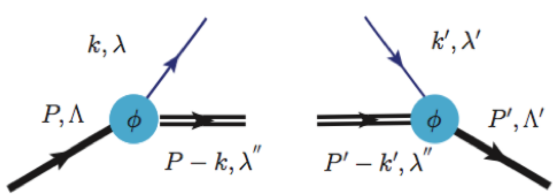
DVCS data

$$A_{UL}(\xi,t), A_{LU}(\xi,t), A_{LL}(\xi,t), \ldots$$

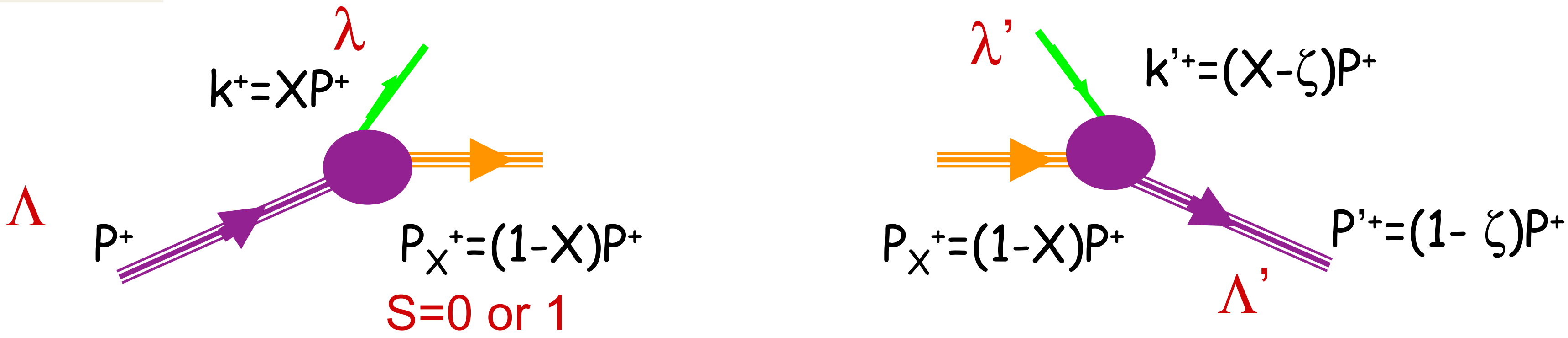
Summary so far



Reggeized diquark model



Vertex Structures with Diquark Spectator



First focus on S=0 pure spectator - beginning

$$H \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P)+\varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$E \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P)+\varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

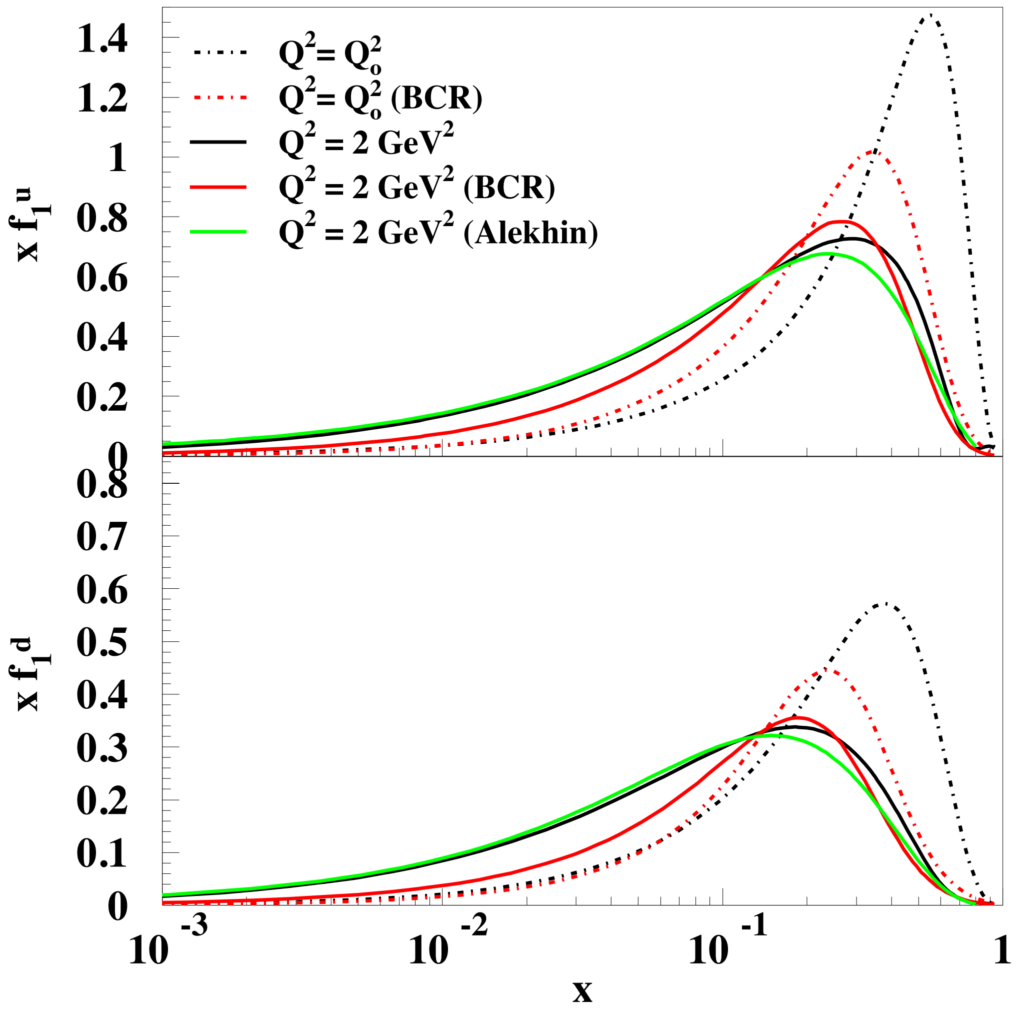
$$\tilde{H} \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P)-\varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$\tilde{E} \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P)-\varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

Vertex function

$$\phi(k^2,\lambda) = \frac{k^2-m^2}{|k^2-\lambda^2|^2}.$$

Fit to pdf's (x dependence & evolution)



Having fit other data we predict Hermes data

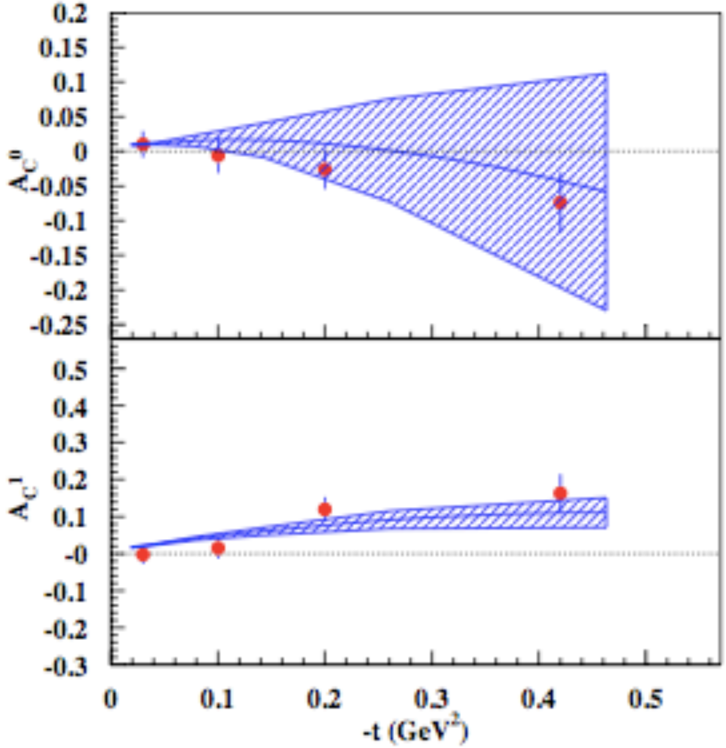


FIG. 20: Coefficients of the beam charge asymmetry, A_C, extracted from experiment [52, 53]. The lower panel is the coefficient for the $\cos\phi$ dependent term in Eq.(82), while the upper panel is the $\cos\phi$ independent term.

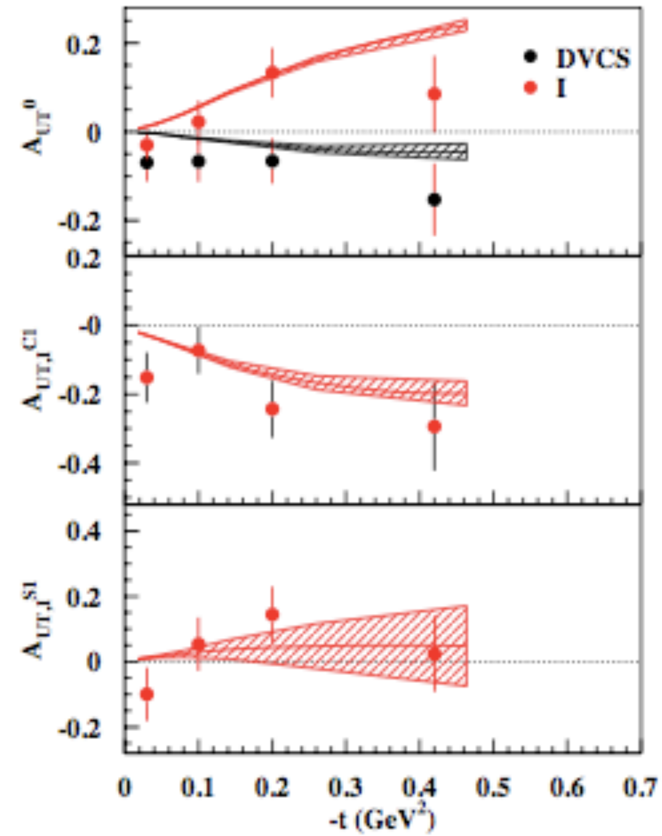
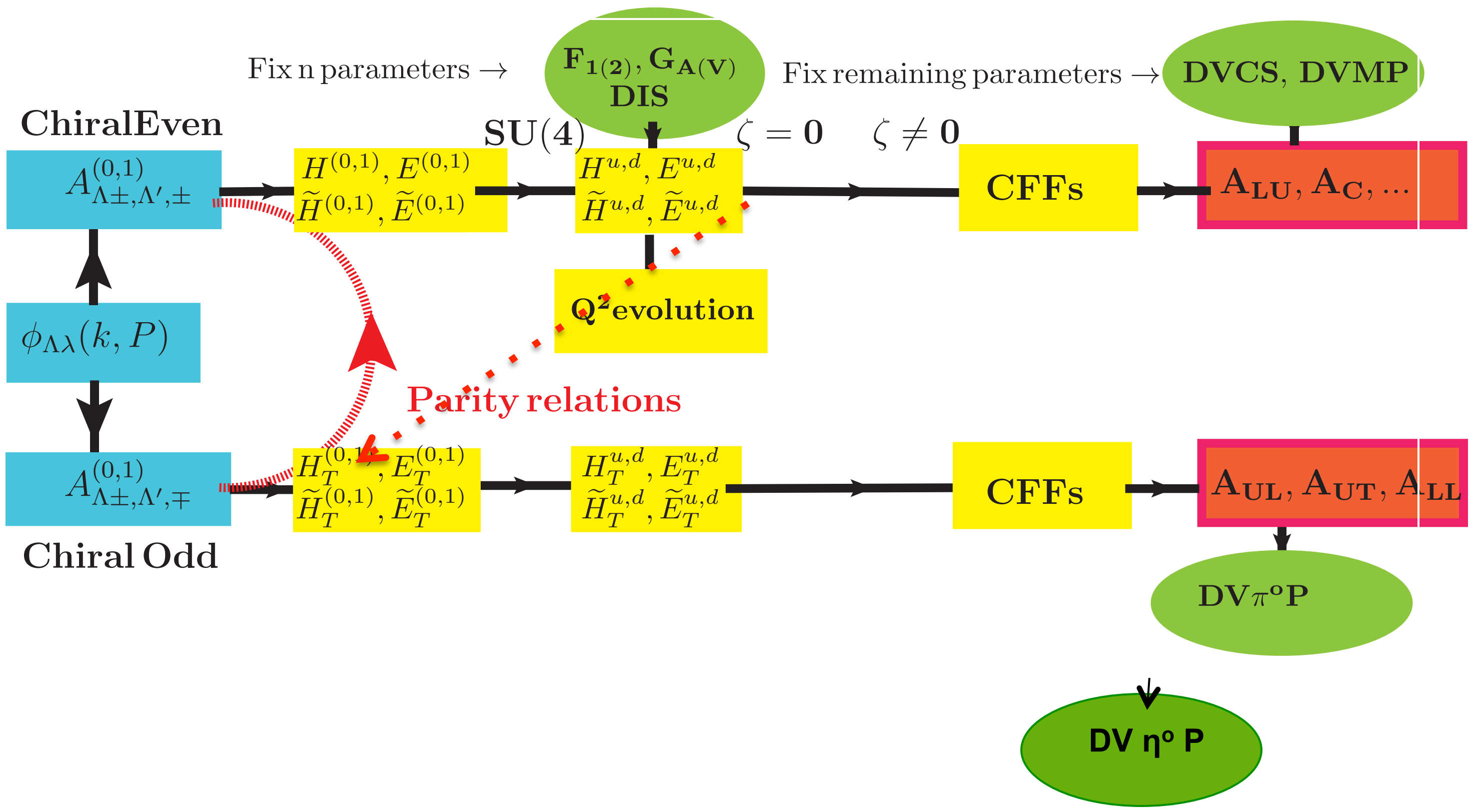
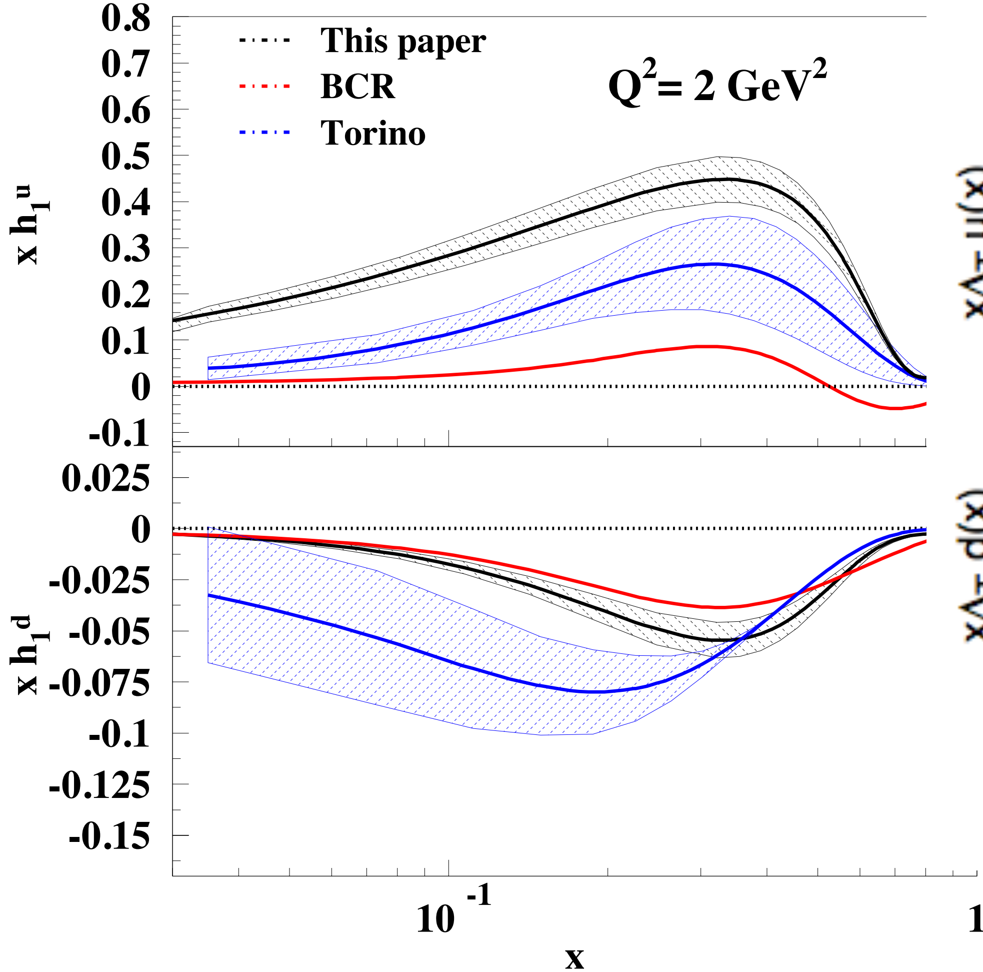


FIG. 21: Coefficients of the b[illegible]etry, A_{UT}, extracted from experiment [52, 53]. The upper panel shows the terms E and F from Eqs.(83) and (84), respectively; the middle panel shows G, and the lower panel H, both in Eq.(84). The curves are predictions obtained extending our quantitative fit of Jefferson lab data to the Hermes set of observables.

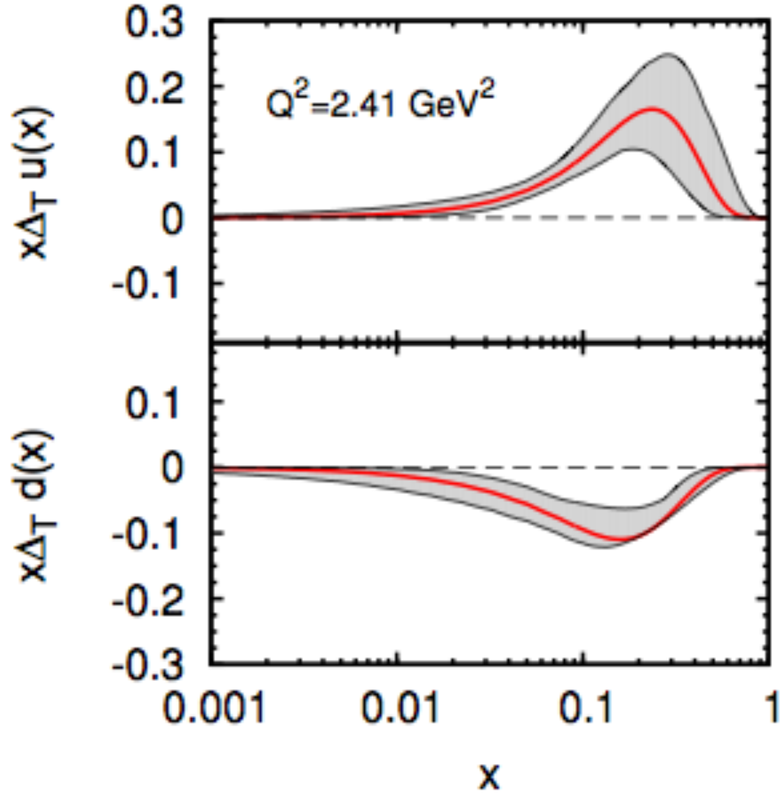
Recursive GPD fitting procedure → Flexible Parameterization



Extraction of transversity after using DVCS data via chiral even←→odd



BCR=Bacchetta, Conte, Radici



Anselmino, Boglione, et al., Phys.Rev. D87 (2013) 094019
$\delta u = 0.31_{-0.12}^{+0.16}$ $\delta d = -0.27_{-0.10}^{+0.10}$

S=0 Chiral even <-> odd helicity amps (+ S=1)

$$A^{(0)}_{++,--} = A^{(0)}_{++,++}$$
$$A^{(0)}_{++,+-} = -A^{(0)}_{++,-+}$$
$$A^{(0)}_{+-,++} = -A^{(0)}_{-+,++},$$

Invert to get GPDs – same helicity amp sets

$$\widetilde{H}^0_T = -(1-\zeta)^2 \frac{M(1-x)}{m+Mx'}\left[E^0 - \frac{\zeta}{2}\widetilde{E}^0\right]$$

$$E^0_T = -\frac{(1-\zeta/2)^2}{1-\zeta}\left[2\widetilde{H}^0_T - E^0 + \left(\frac{\zeta/2}{1-\zeta/2}\right)^2 \widetilde{E}^0\right]$$

$$\widetilde{E}^0_T = -\frac{\zeta/2(1-\zeta/2)}{1-\zeta}\left[2\widetilde{H}^0_T - E^0 + \widetilde{E}^0\right]$$

$$H^0_T = \frac{H^0+\widetilde{H}^0}{2} - \frac{\zeta^2/4}{1-\zeta}\frac{E^0+\widetilde{E}^0}{2} - \frac{\zeta^2/4}{(1-\zeta/2)(1-\zeta)}E^0_T + \frac{\zeta/4(1-\zeta/2)}{1-\zeta}\widetilde{E}^0_T + \widetilde{H}^0_T,$$

$S=0$ double helicity flip amplitude was calculated directly from Eq.(16),

$$A^{(0)}_{+-,-+} = \frac{t_0-t}{4M}\frac{1}{\sqrt{1-\zeta}}\frac{1}{(1-\zeta/2)}\frac{\tilde{x}}{m+Mx'}\left[E^0 - \frac{\zeta}{2}\widetilde{E}^0\right].$$

Unpolarized cross section components

$$F_{UU,T} = \frac{d\sigma_T}{dt}, \quad F_{UU,L} = \frac{d\sigma_L}{dt}, \quad F_{UU}^{\cos\phi} = \frac{d\sigma_{LT}}{dt},$$
$$F_{UU}^{\cos 2\phi} = \frac{d\sigma_{TT}}{dt}, \quad F_{LU}^{\sin\phi} = \frac{d\sigma_{LT'}}{dt}$$

$$F_{UU,T} = \mathcal{N}\left[\mid f_{10}^{++}\mid^2 + \mid f_{10}^{+-}\mid^2 + \mid f_{10}^{-+}\mid^2 + \mid f_{10}^{--}\mid^2\right]$$

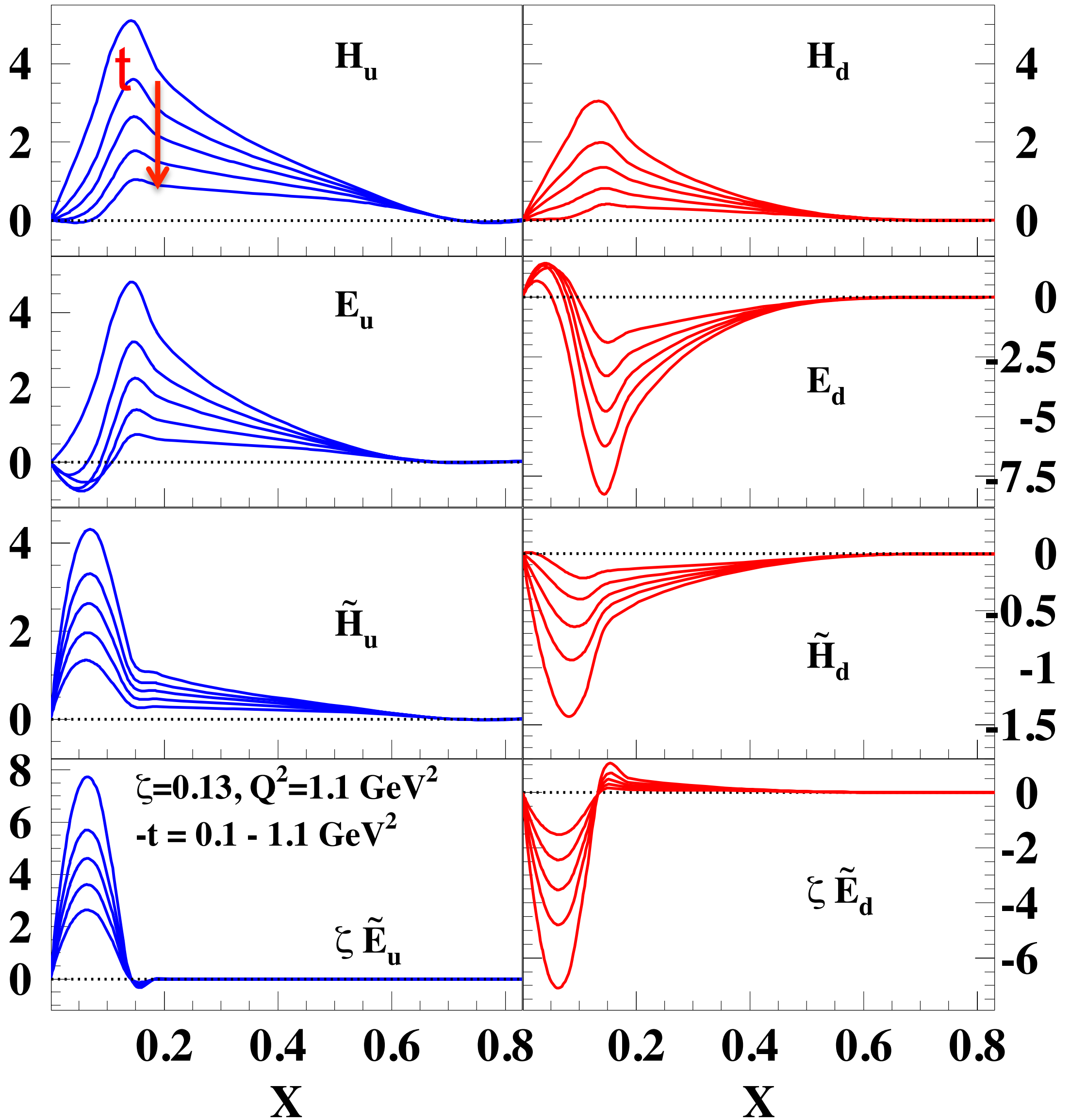
$$F_{UU,L} = \mathcal{N}\left[\mid f_{00}^{++}\mid^2 + \mid f_{00}^{+-}\mid^2\right]$$

$$F_{UU}^{\cos 2\phi} = -\mathcal{N}\, 2\Re e\left[(f_{10}^{++})^*(f_{10}^{--}) - (f_{10}^{+-})^*(f_{10}^{-+})\right]$$

$$F_{UU}^{\cos\phi} = -\mathcal{N}\Re e\left[(f_{00}^{+-})^*(f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} - f_{10}^{--})\right]$$

$$F_{LU}^{\sin\phi} = \mathcal{N}\Im m\left[(f_{00}^{+-})^*(f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} - f_{10}^{--})\right]$$

RESULT: determined the chiral even GPDs



Helicity amps (q'+N->q+N') are linear combinations of GPDs

$$A_{+,+;+,+} = \sqrt{1-\xi^2}\left[\frac{H+\tilde{H}}{2} - \frac{\xi^2}{1-\xi^2}\frac{E+\tilde{E}}{2}\right]$$

$$A_{-,+;-,+} = \sqrt{1-\xi^2}\left[\frac{H-\tilde{H}}{2} - \frac{\xi^2}{1-\xi^2}\frac{E-\tilde{E}}{2}\right]$$

$$A_{+,+;-,+} = -\frac{\sqrt{t_0-t}}{4M}(E-\xi\tilde{E})$$

$$A_{-,+;+,+} = \frac{\sqrt{t_0-t}}{4M}(E+\xi\tilde{E})$$

for chiral even GPDs and

$$A_{+-,++} = -\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1+\xi}{2}E_T - \frac{1+\xi}{2}\widetilde{E}_T\right]$$

$$A_{++,--} = \sqrt{1-\xi^2}\left[H_T + \frac{t_0-t}{4M^2}\widetilde{H}_T - \frac{\xi^2}{1-\xi^2}E_T + \frac{\xi}{1-\xi^2}\widetilde{E}_T\right]$$

$$A_{+-,-+} = -\sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\,\widetilde{H}_T$$

$$A_{++,+-} = \frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1-\xi}{2}E_T + \frac{1-\xi}{2}\widetilde{E}_T\right],$$

for chiral odd GPDs, where for consistency with previous literature we have

T-reversal at $\xi=0$

In diquark spectator models $A_{++;++}$, etc. are calculated directly. Inverted -> GPDs

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S=1 Chiral even <-> odd

$$A^{(1)}_{++,--} = -\frac{x+x'}{1+xx'} A^{(1)}_{++,++}$$

$$A^{(1)}_{+-,-+} = 0$$

$$A^{(1)}_{++,+-} = -\sqrt{\frac{\langle \tilde{k}_\perp^2 \rangle}{x'^2 + \langle \tilde{k}_\perp^2 \rangle / P^{+\,2}}}\, A^{(1)}_{++,-+}$$

$$A^{(1)}_{+-,++} = -\sqrt{\frac{\langle k_\perp^2 \rangle}{x^2 + \langle k_\perp^2 \rangle / P^{+\,2}}}\, A^{(1)}_{-+,++},$$

Invert to get GPDs

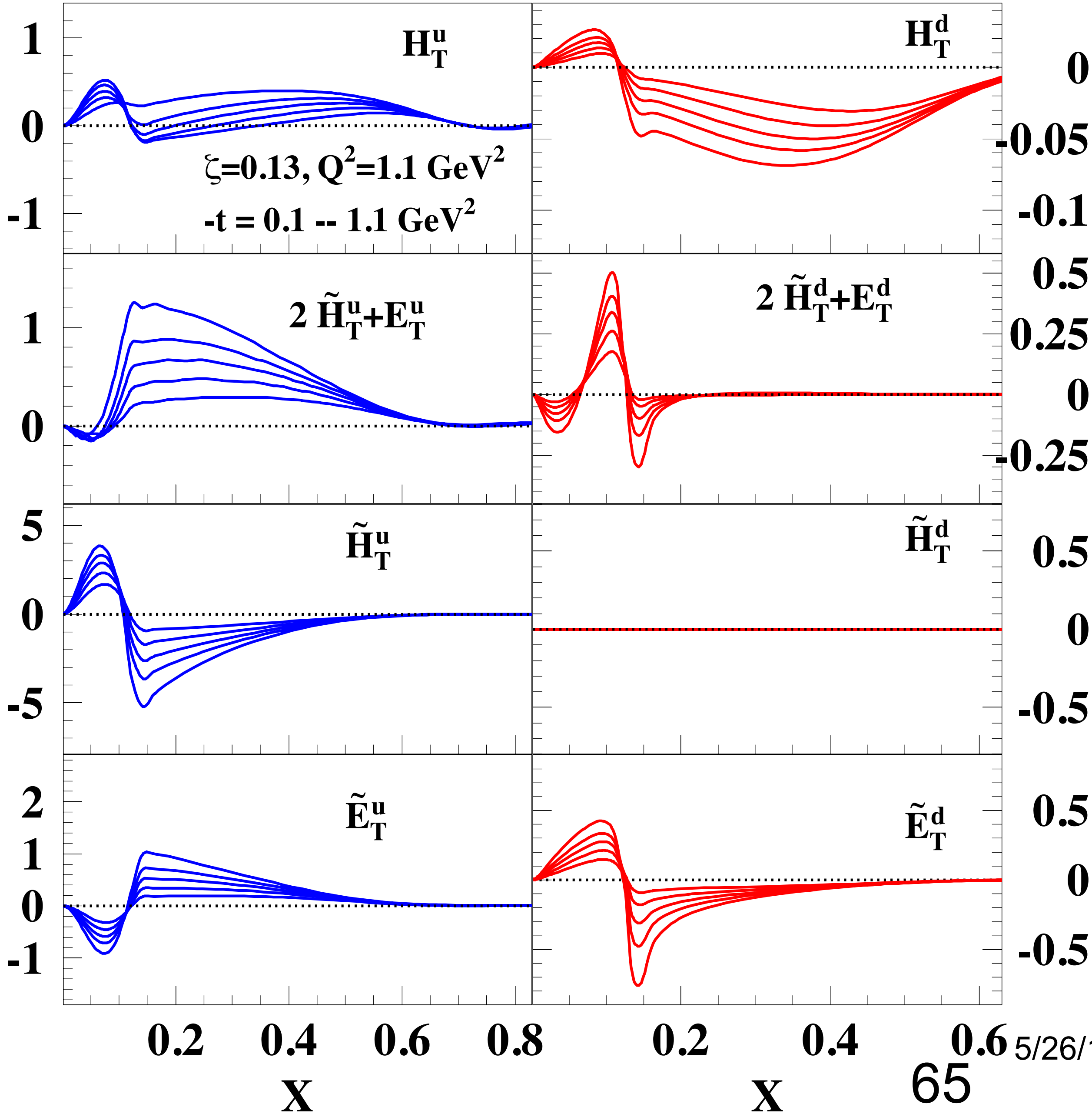
$$\widetilde{H}_T^{(1)} = 0$$

$$E_T^{(1)} = \frac{1-\zeta/2}{1-\zeta}\left[\tilde{a}\left(E^{(1)} - \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right) + a\left(E^{(1)} + \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right)\right]$$

$$\widetilde{E}_T^{(1)} = \frac{1-\zeta/2}{1-\zeta}\left[\tilde{a}\left(E^{(1)} - \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right) - a\left(E^{(1)} + \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right)\right]$$

$$H_T^{(1)} = -\frac{x+x'}{1+xx'}\left[\frac{H^{(1)}+\widetilde{H}^{(1)}}{2} - \frac{\zeta^2/4}{1-\zeta}\,\frac{E^{(1)}+\widetilde{E}^{(1)}}{2}\right] - \frac{\zeta^2/4}{1-\zeta}E_T^{(1)} + \frac{\zeta/4}{1-\zeta}\widetilde{E}_T^{(1)}$$

RESULT: Chiral odd GPDs



Invert to obtain model parameterization for GPDs

S=0 diquark

Spectator model

$A_{++,-+} = - A_{++,+-}^{*}$

$A_{-+,++} = - A_{+-,++}^{*}$

$A_{++,++} = A_{++,--}$

$$H(x,\xi,t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;+,+} + A_{-,+;-,+}) - \frac{2M\xi^2}{\Delta(1-\xi^2)}(A_{+,+;-,+} - A_{-,+;+,+})$$

$$E(x,\xi,t) = -\frac{2M}{\Delta}(A_{+,+,-,+} - A_{-,+;+,+})$$

$$\tilde{H}(x,\xi,t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;+,+} - A_{-,+;-,+}) + \frac{2M\xi}{\Delta(1-\xi^2)}(A_{+,+,-,+} + A_{-,+;+,+})$$

$$\tilde{E}(x,\xi,t) = \frac{2M}{\Delta\xi}(A_{+,+,-,+} + A_{-,+;+,+})$$

for chiral even GPDs and

$$H_T(x,\xi,t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;-,-} + A_{-,+;+,-}) + \frac{2M\xi}{\Delta(1-\xi^2)}(A_{+,+;+,-} - A_{-,+;-,-})$$

$$\xi E_T(x,\xi,t) - \tilde{E}_T(x,\xi,t) = \frac{2M}{\Delta}(A_{+,+;+,-} - A_{-,+;-,-})$$

$$E_T(x,\xi,t) + \tilde{E}_T(x,\xi,t) = \frac{\Delta}{2M(1-\xi)}[2A_{+,+;+,-} + \frac{4M}{\Delta\sqrt{1-\xi^2}}A_{-,+;+,-}]$$

double flip

$$\tilde{H}_T(x,\xi,t) = \frac{4M^2}{\Delta^2\sqrt{1-\xi^2}}A_{-,+;+,-}$$

66

E_u & E_d , etc.
c.f. A.Bacchetta, et al.
& Ji sum rule
Disp.Rel'n

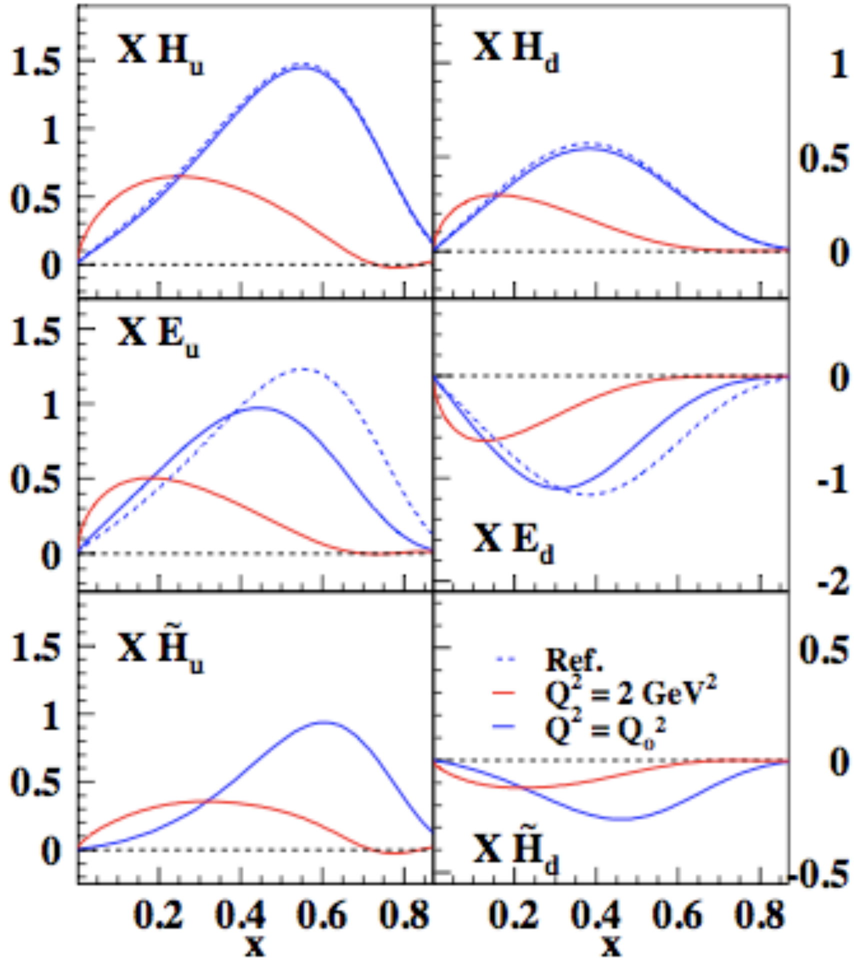


FIG. 6: (color online) GPDs $F_q(X,0,0) \equiv \{H_q, E_q, \tilde{H}_q\}$, for $q = u$ (left) and $q = d$ (right), evaluated at the initial scale, $Q_o^2 = 0.0936$ GeV^2, and at $Q^2 = 2$ GeV^2, respectively. The dashed lines were calculated using the model in Refs.[24, 25] at the initial scale.

Spectral function leads to small x

$$\rho(k_X^2, k^2) = (k_X^2)^{\alpha-1}\beta(k^2),$$

↓

$$H(X,0,0) = \mathcal{N}\int_{M^2-m^2}^{\infty} dk_X^2 (k_X^2)^{\alpha-1} \int_{-\infty}^{\mathcal{M}^2(X,k_X^2)+M_\Lambda^2} dk^2 \beta(k^2)\overline{H}(X, M_\Lambda^2, k_X^2, k^2).$$

Regge behavior

X≈0

$$H(X,0,0) \;=\; \mathcal{N}X^{-\alpha}\left[\int_0^\infty dz z^{\alpha-1}\int_{-\infty}^{-z} dk^2 \frac{m^2-k^2-z}{(k^2-M_\Lambda^2)^4}\right]_{X\to 0},$$

~ Factorization with non Regge behavior

X≠0

$$\begin{aligned} H(X,0,0) \;&=\; \mathcal{N}X^{-\alpha}\int_0^\infty dz \frac{z^{\alpha+\beta_1-1}}{[1+\beta_2(z-\beta_3)^2]^3}\int_{-\infty}^{\mathcal{M}^2(X,z)+M_\Lambda^2} dk^2 (1-X)^3 \frac{(m+XM)^2+(1-X)(\mathcal{M}^2(X,z)+M_\Lambda^2-k^2)}{(k^2-M_\Lambda^2)^4} \\ &= \mathcal{N}X^{-\alpha}\int_0^\infty dz \frac{z^{\alpha+\beta_1-1}}{[1+\beta_2(z-\beta_3)^2]^3}(1-X)^4 \frac{2(m+XM)^2+\mathcal{M}^2(X,z)}{\mathcal{M}^6(X,z)} \\ &\approx \mathcal{N}X^{-\alpha}(1-X)^4 \frac{2(m+XM)^2+\mathcal{M}^2(X,\overline{M}_X^2)}{\mathcal{M}^6(X,\overline{M}_X^2)} = \mathcal{N}X^{-\alpha}H_{\frac{M_\Lambda}{\overline{M}_X},m}(X,0,0) \end{aligned} \quad (49)$$

Compton Form Factors → Real & Imaginary Parts

$$\mathcal{H}_q(\zeta, t, Q^2) = \int_{-1+\zeta}^{+1} dX H_q(X, \zeta, t, Q^2) \times \left(\frac{1}{X - \zeta + i\epsilon} + \frac{1}{X - i\epsilon} \right)$$

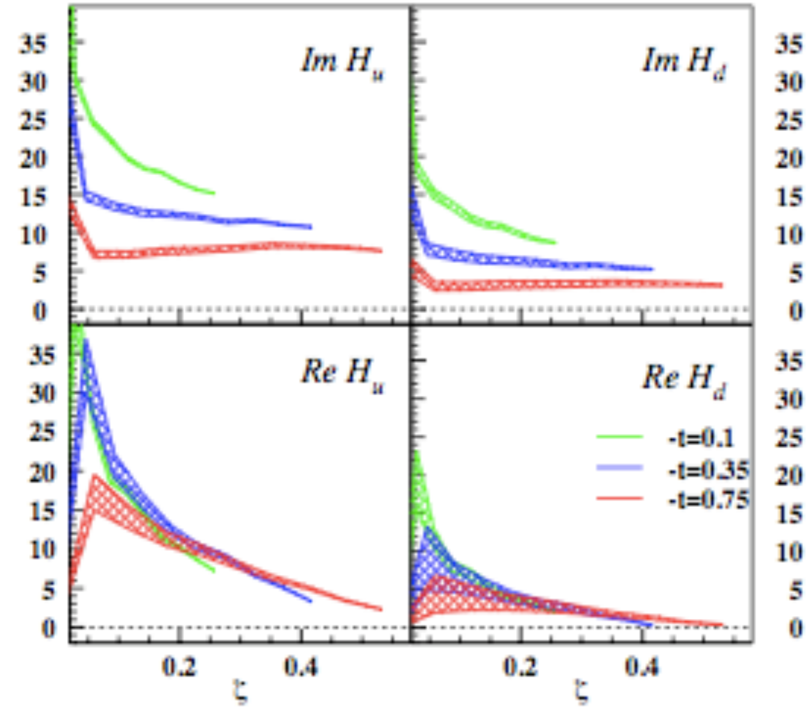
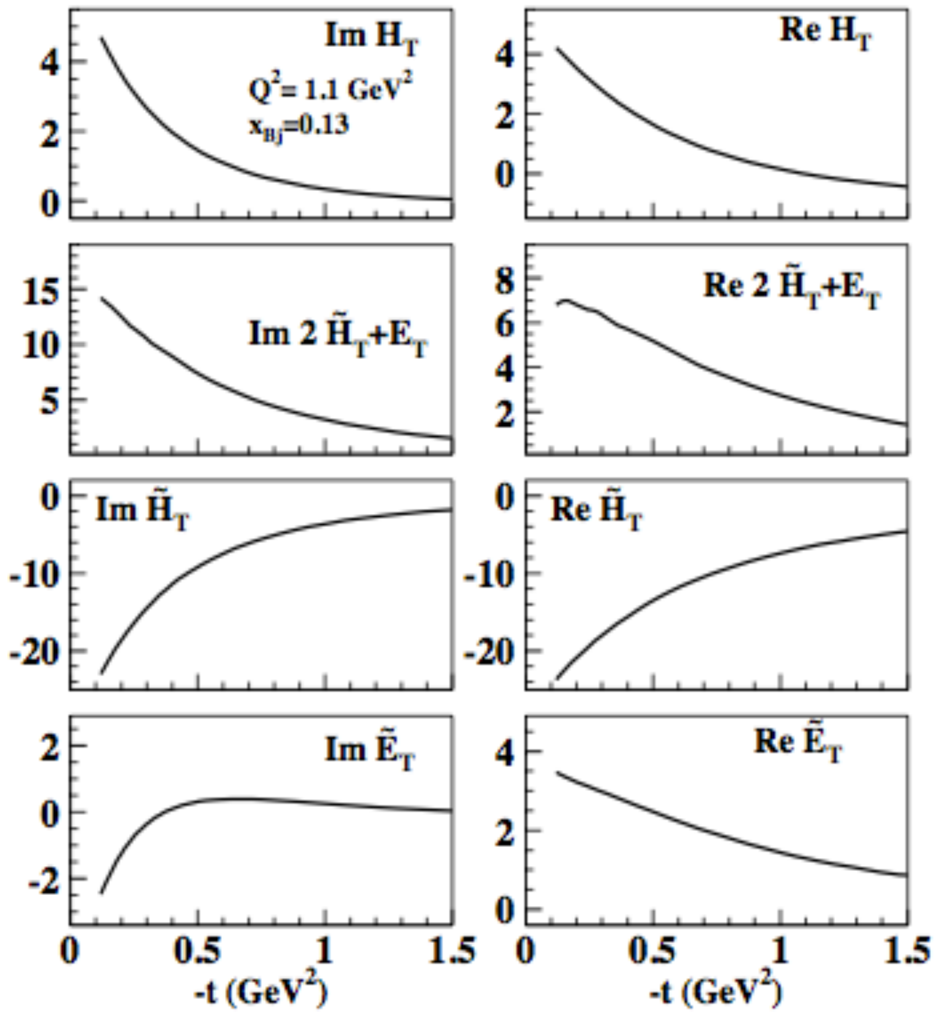


FIG. 9: (color online) Real and imaginary parts of the CFFs, $\mathcal{H}_i(\zeta, t)$, entering Eqs.(85). The CFFs are plotted vs. $x_{Bj} \equiv \zeta$, for different values of t, at $Q^2 = 2$ GeV^2. They are shown with the theoretical uncertainty from the parameters in TableI. Similar results are obtained for E and $\tilde{H}$.

Chiral odd GPD Compton form factors



First Chiral Even: Recursive Fitting Procedure

e.g. for H and E

- Fit at ζ=0, t=0 => $H_q(x,0,0)=q(X)$ c.f. well known data

  3 parameters per quark flavor (M_X^q, Λ_q, α_q) + initial Q_o^2

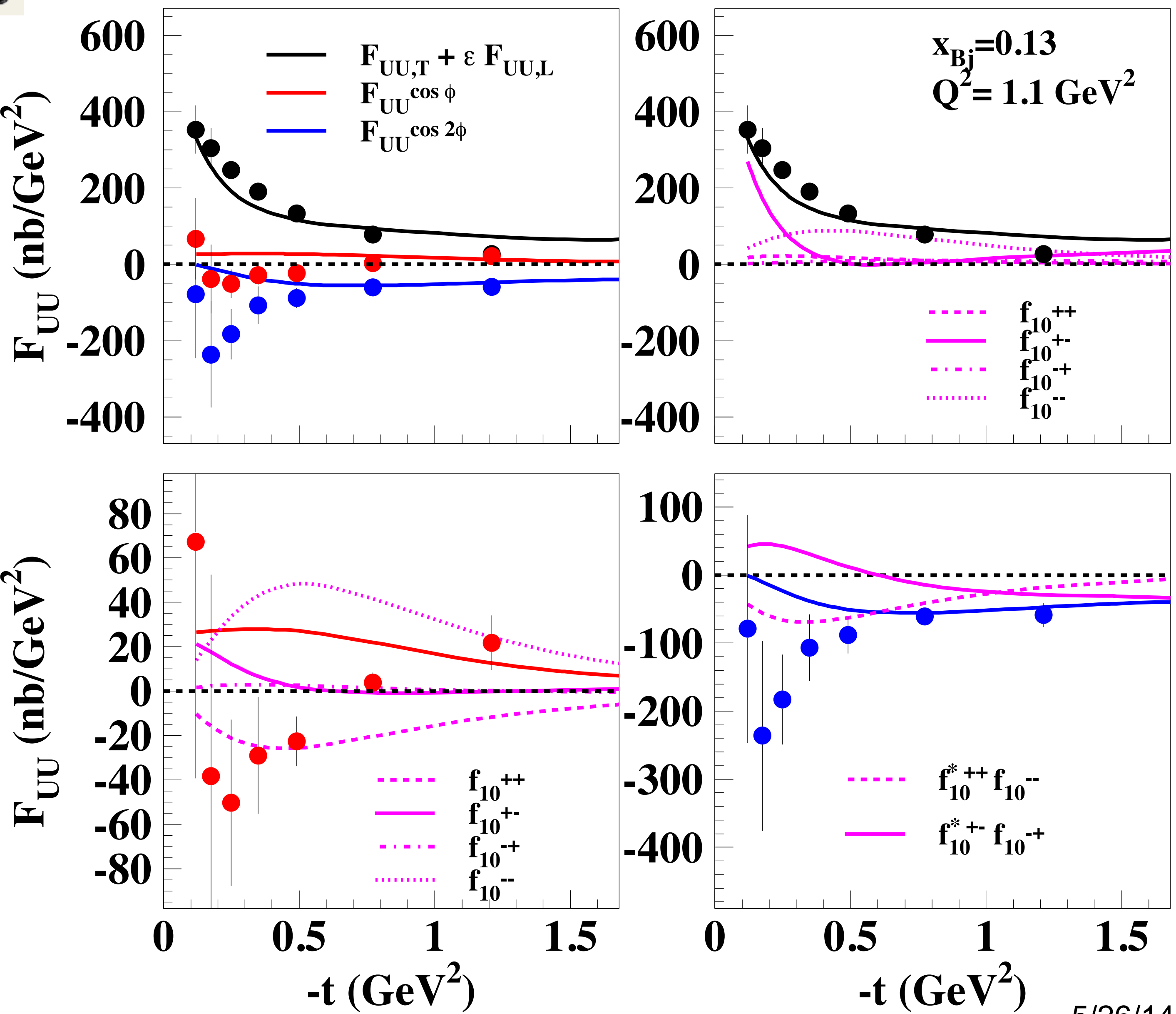
- Fit at ζ=0, t≠0 ⇒
$$\int_0^1 dX\, H^q(X,t) = F_1^q(t)$$
$$\int_0^1 dX\, E^q(X,t) = F_2^q(t),$$

  - 2 parameters per quark flavor (β, p)
    - Regge factor $R \sim X^{-\alpha(t)}$

$$G^{\lambda}_{M_X}(X,t) = \mathcal{N}\frac{X}{1-X}\int d^2\mathbf{k}_\perp \frac{\phi(k^2,\lambda)}{D(X,\mathbf{k}_\perp)}\frac{\phi(k'^2,\lambda)}{D(X,\mathbf{k}_\perp+(1-X)\mathbf{\Delta}_\perp)}$$ Quark-Diquark

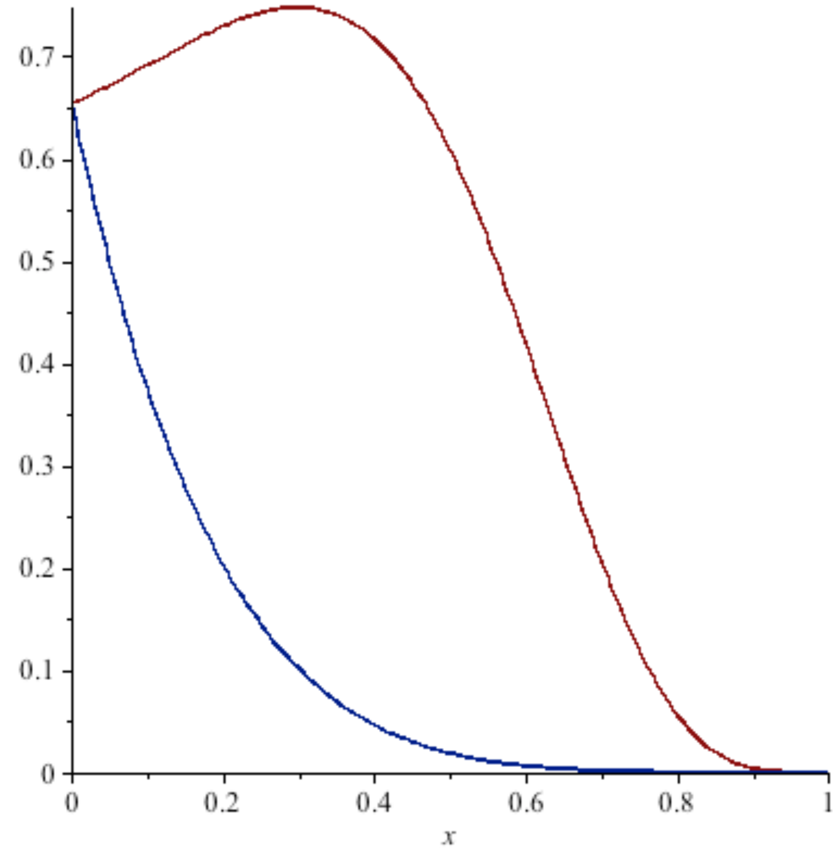
- Fit at ζ≠0, t≠0 ⇒ DVCS, DVMP,… data (convolutions of GPDs with Wilson coefficient functions) + lattice results (Mellin Moments of GPDs) Evolution
- Note! This is a multivariable analysis ⇒ see e.g. Moutarde, Kumericki and D. Mueller, Guidal and Moutarde

Unpolarized Helicity Amplitudes



Scalar diquark contribution to H(X,0,0) → G(X,0,0) for M_X =0.6 & 1.7

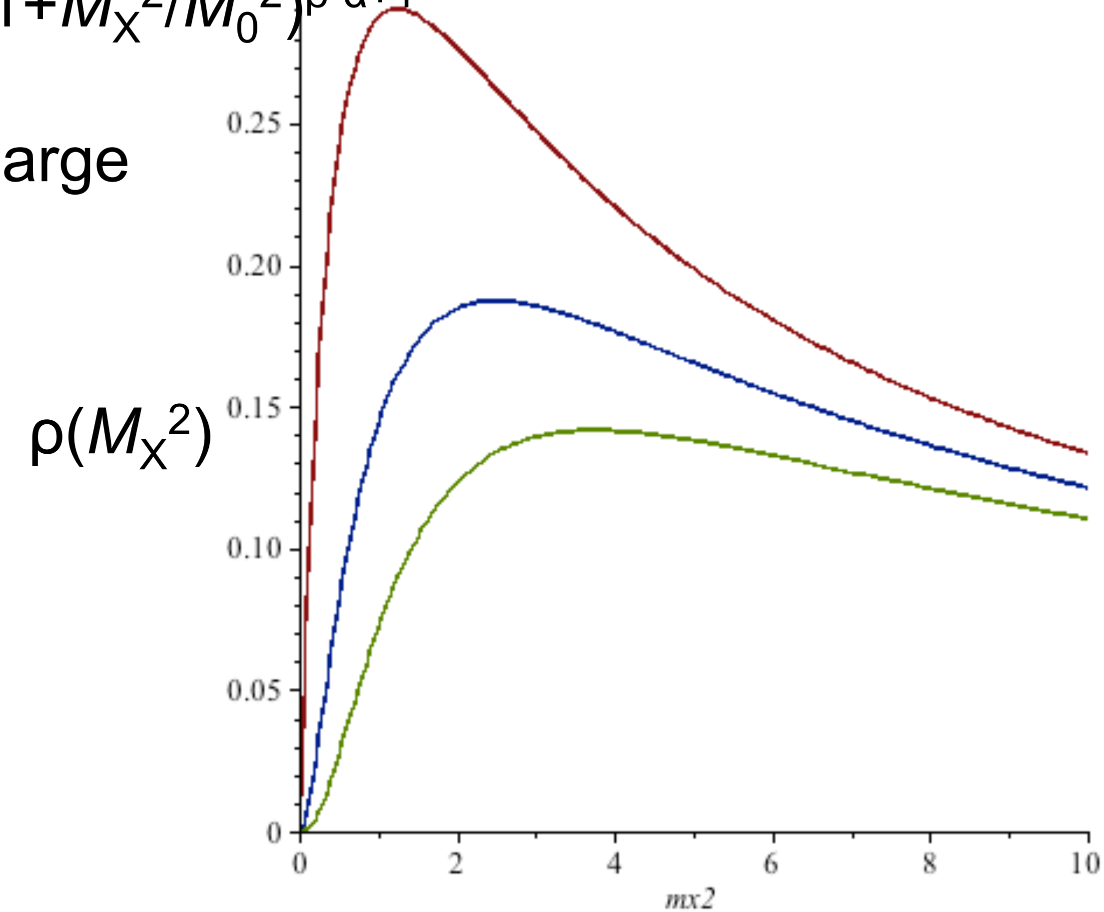
How to “Reggeize” for small x while keeping large x spectator behavior?



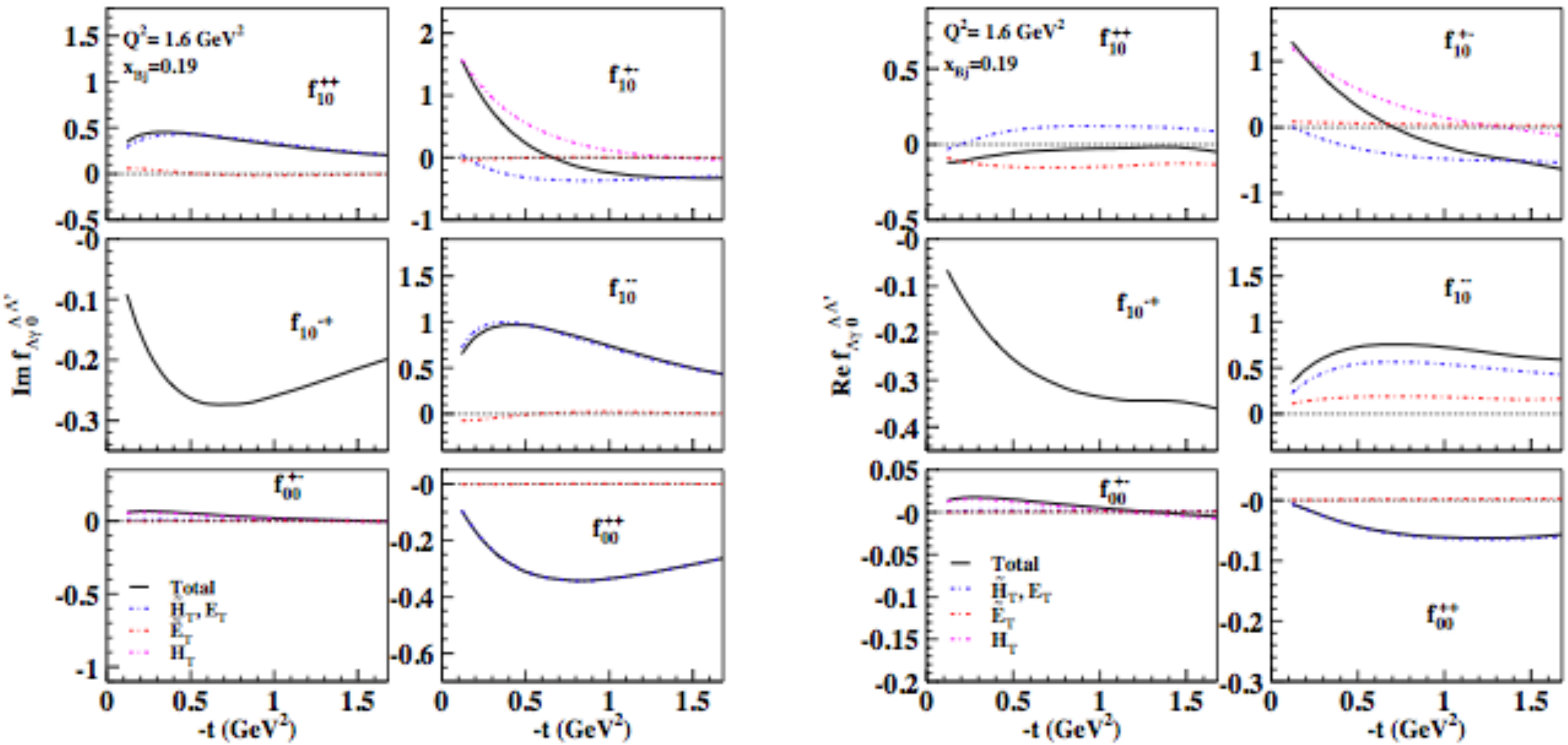
Spectral distribution
of form $\rho(M_X^2, k^2) \approx \rho(M_X^2)\, \beta(k^2)$

$\rho(M_X^2) = (M_X^2/M_0^2)^{\beta}/(1+M_X^2/M_0^2)^{\beta-\alpha+1}$

$\beta(k^2)$ chosen to give large
k_T^2 falloff behavior

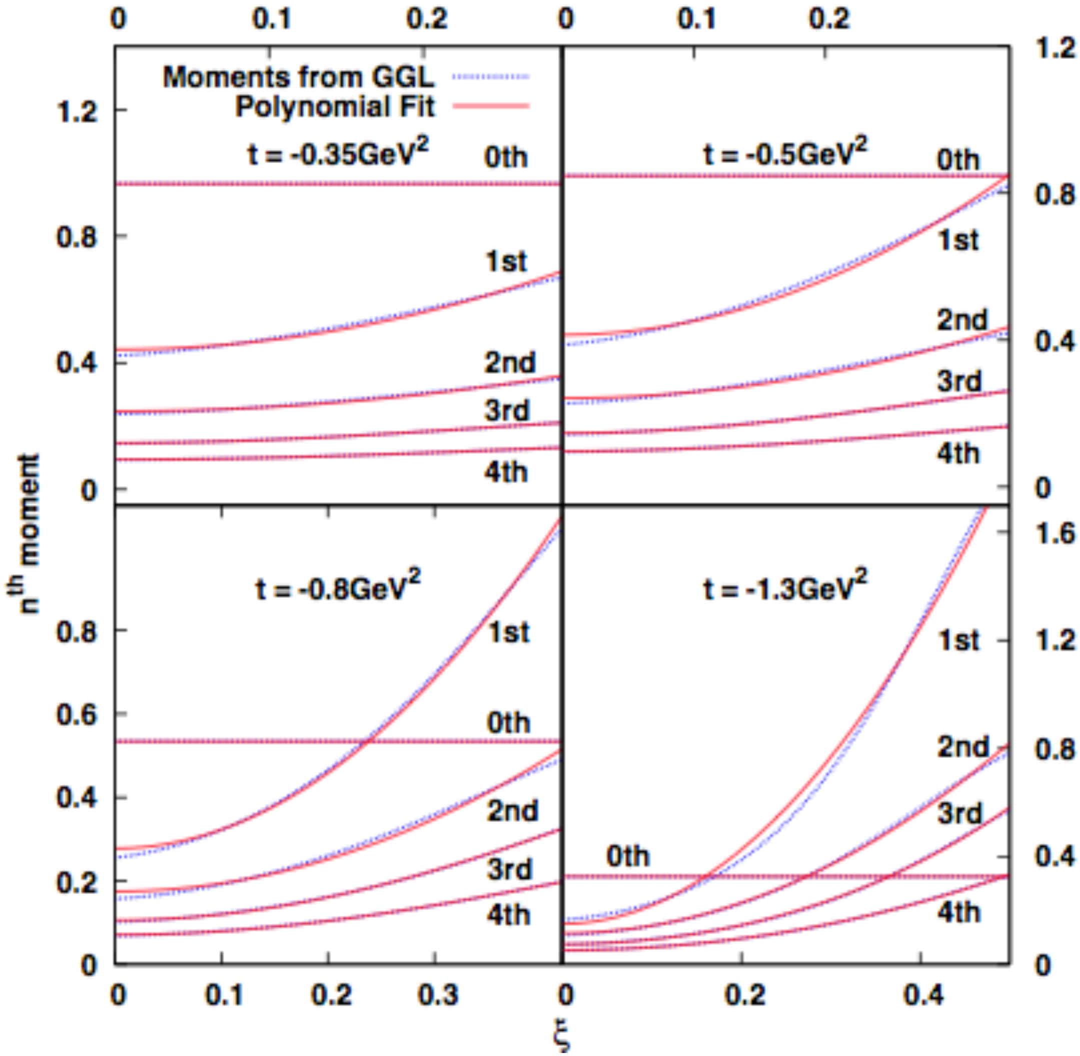


6 helicity amps for π^0 after Compton Form Factors



Polynomiality!

Goldstein et al. arXiv:1012.3776



the minimal number of parameters
necessary to fit X and t ?" Results of Recursive Fit

Parameters	H	E	$\widetilde{H}$	$\widetilde{E}$
m_u (GeV)	0.420	0.420	2.624	2.624
M_X^u (GeV)	0.604	0.604	0.474	0.474
M_Λ^u (GeV)	1.018	1.018	0.971	0.971
α_u	0.210	0.210	0.219	0.219
α'_u	2.448 ± 0.0885	2.811 ± 0.765	1.543 ± 0.296	5.130 ± 0.101
p_u	0.620 ± 0.0725	0.863 ± 0.482	0.346 ± 0.248	3.507 ± 0.054
$\mathcal{N}_u$	2.043	1.803	0.0504	1.074
χ^2	0.773	0.664	0.116	1.98
m_d (GeV)	0.275	0.275	2.603	2.603
M_X^d (GeV)	0.913	0.913	0.704	0.704
M_Λ^d (GeV)	0.860	0.860	0.878	0.878
α_d	0.0317	0.0317	0.0348	0.0348
α'_d	2.209 ± 0.156	1.362 ± 0.585	1.298 ± 0.245	3.385 ± 0.145
p_d	0.658 ± 0.257	1.115 ± 1.150	0.974 ± 0.358	2.326 ± 0.137
$\mathcal{N}_d$	1.570	-2.800	-0.0262	-0.966
χ^2	0.822	0.688	0.110	1.00

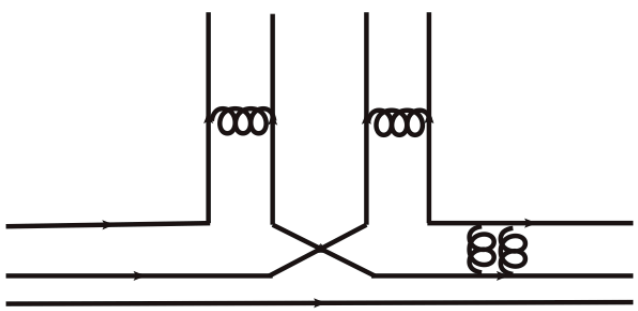
$$R_p^{\alpha,\alpha'} = X^{-[\alpha+\alpha'(X)t+\beta(\zeta)t]},$$

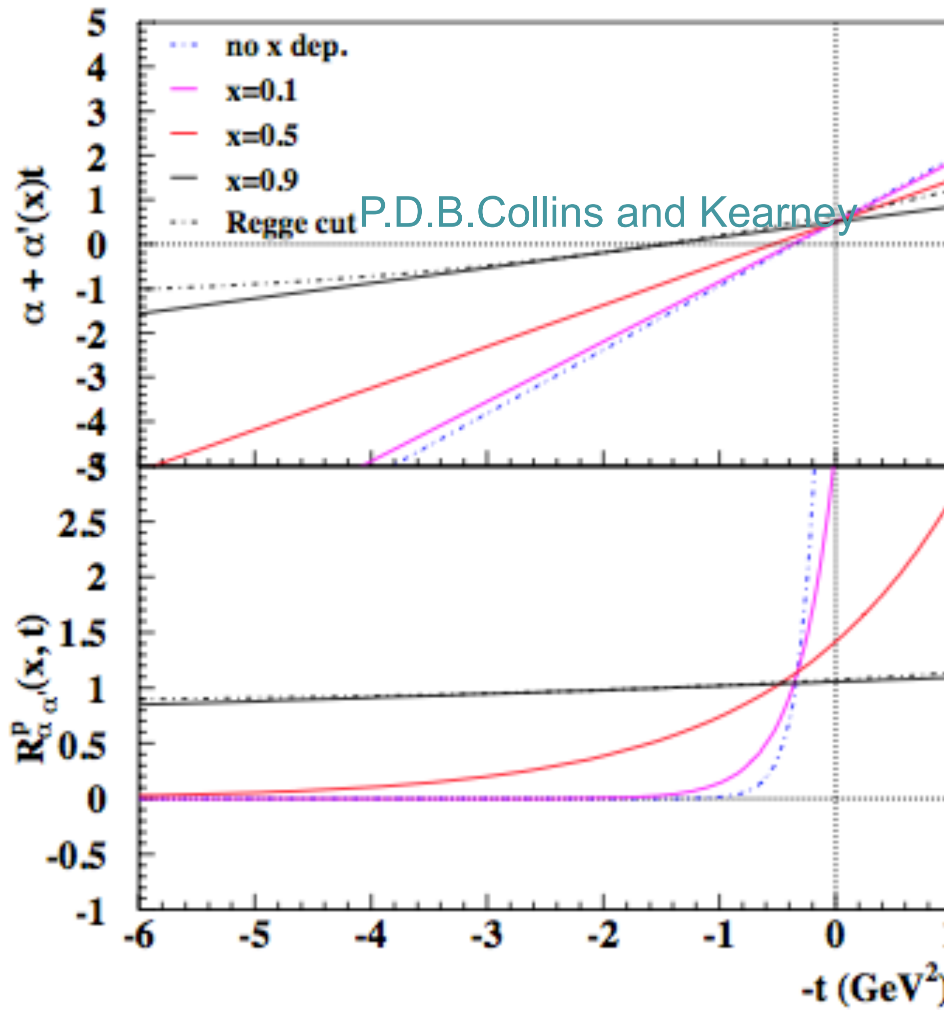
~Initially introduced by Radyushkin, Burkardt, … to account for coordinate space behavior

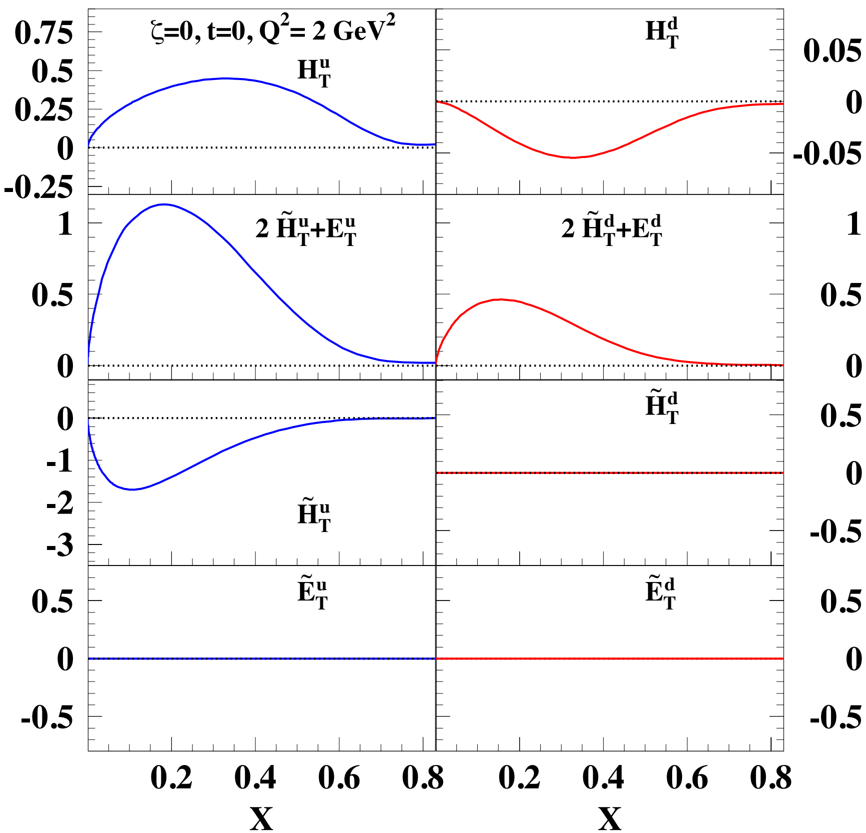
$$\alpha'(X) \equiv \alpha'(1-X)^p \qquad \beta = 0.$$

Now see as effectively taking into account Regge cuts

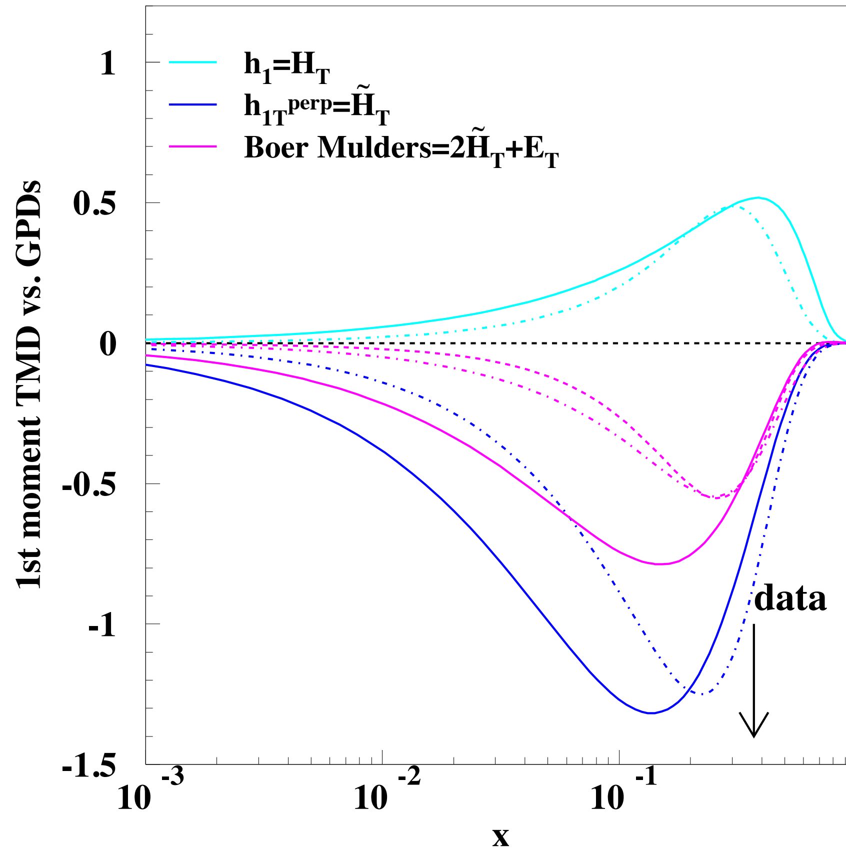
O. Gonzalez Hernandez, GG, S. Liuti arXiv 1206.1876







ζ=0, t=0, Q²= 2 GeV²
H_T^u
H_T^d
2 H̃_T^u+E_T^u
2 H̃_T^d+E_T^d
H̃_T^u
H̃_T^d
Ẽ_T^u
Ẽ_T^d
0.75
0.5
0.25
0
-0.25
1
-1
-2
-3
0.05
-0.05
-0.5
0.2
0.4
0.6
0.8
X



h_1=H_T
h_1T^perp=H̃_T
Boer Mulders=2H̃_T+E_T
1st moment TMD vs. GPDs
data
1
0.5
0
-0.5
-1
-1.5
10^-3
10^-2
10^-1
1
x