Multiplicities and Phenomenology (Part II)

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11-Jun-2014

Transversity 2014

OUTLINE

- **Multiplicities HERMES and COMPASS.**
- **Extracting information from Multiplicites.**
- **Other observables**
- **Conclusions**

In collaboration with

M. Anselmino (Torino)

M. Boglione (Torino)

S. Melis (Torino)

V. Barone (Alessandria)

A. Prokudin (JLab)

OUTLINE

Multiplicities HERMES and COMPASS.

Extracting information from Multiplicites.

Other observables

Conclusions

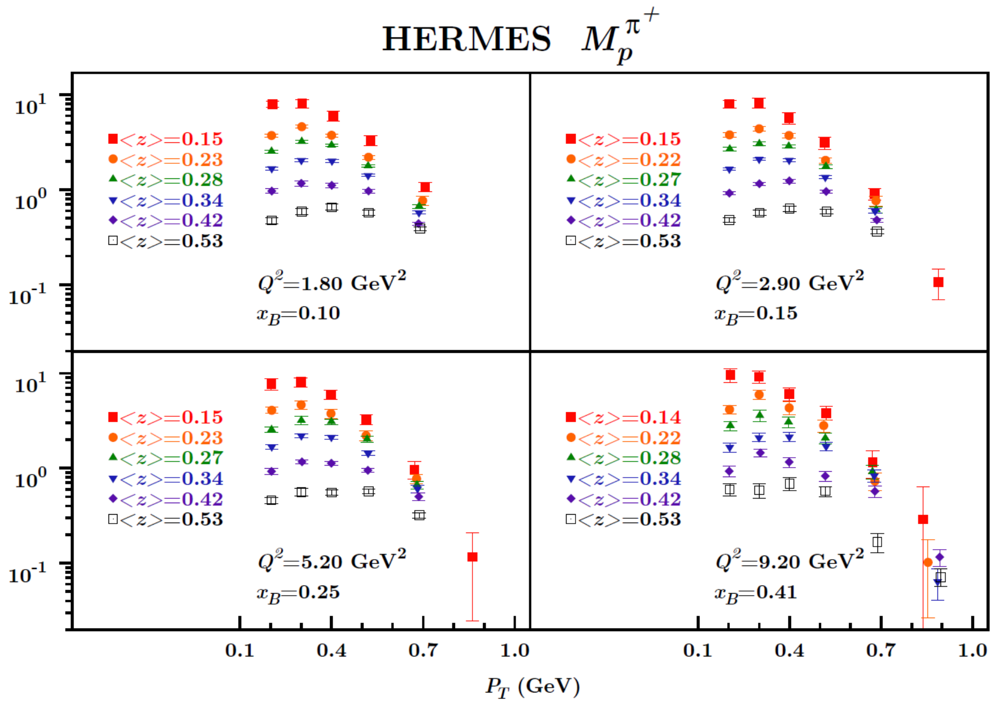
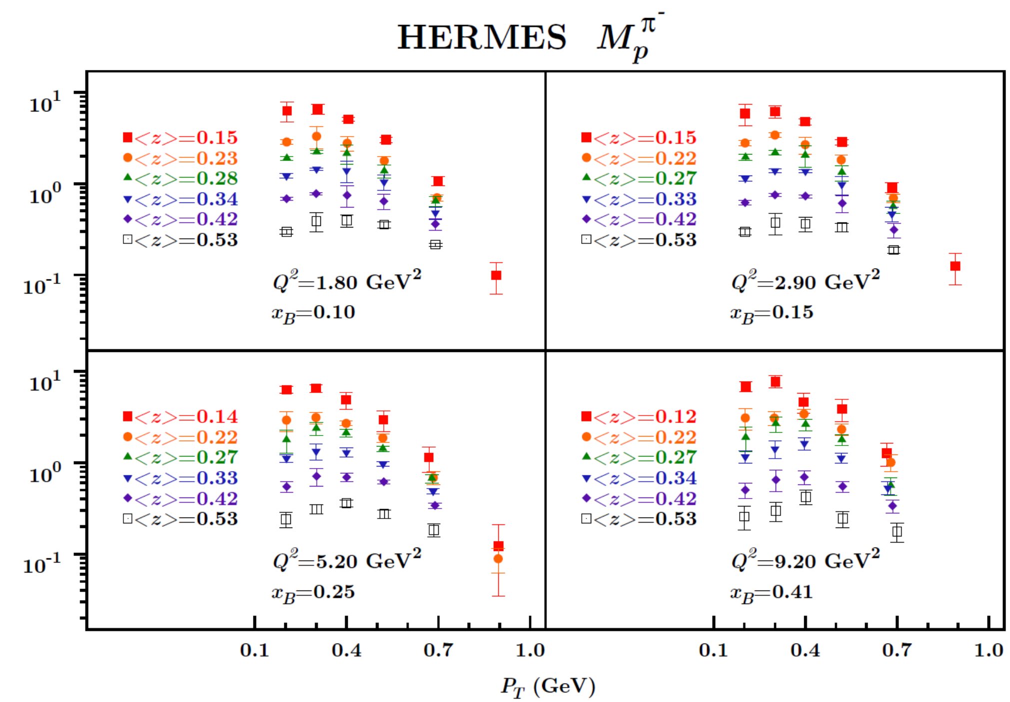
We are discussing Multiplicies from

HERMES

Airapetian, A. et al. Phys.Rev. D87 (2013) 074029

COMPASS

Adolph, C. et al. Eur.Phys.J. C73 (2013) 2531


HERMES $M_p^{\pi^+}$
<z>=0.15 <z>=0.23 <z>=0.28 <z>=0.34 <z>=0.42 <z>=0.53
Q^2=1.80 GeV^2 x_B=0.10
<z>=0.15 <z>=0.22 <z>=0.27 <z>=0.34 <z>=0.42 <z>=0.53
Q^2=2.90 GeV^2 x_B=0.15
<z>=0.15 <z>=0.23 <z>=0.27 <z>=0.34 <z>=0.42 <z>=0.53
Q^2=5.20 GeV^2 x_B=0.25
<z>=0.14 <z>=0.22 <z>=0.28 <z>=0.34 <z>=0.42 <z>=0.53
Q^2=9.20 GeV^2 x_B=0.41
P_T (GeV)

HERMES $M_p^{\pi^-}$
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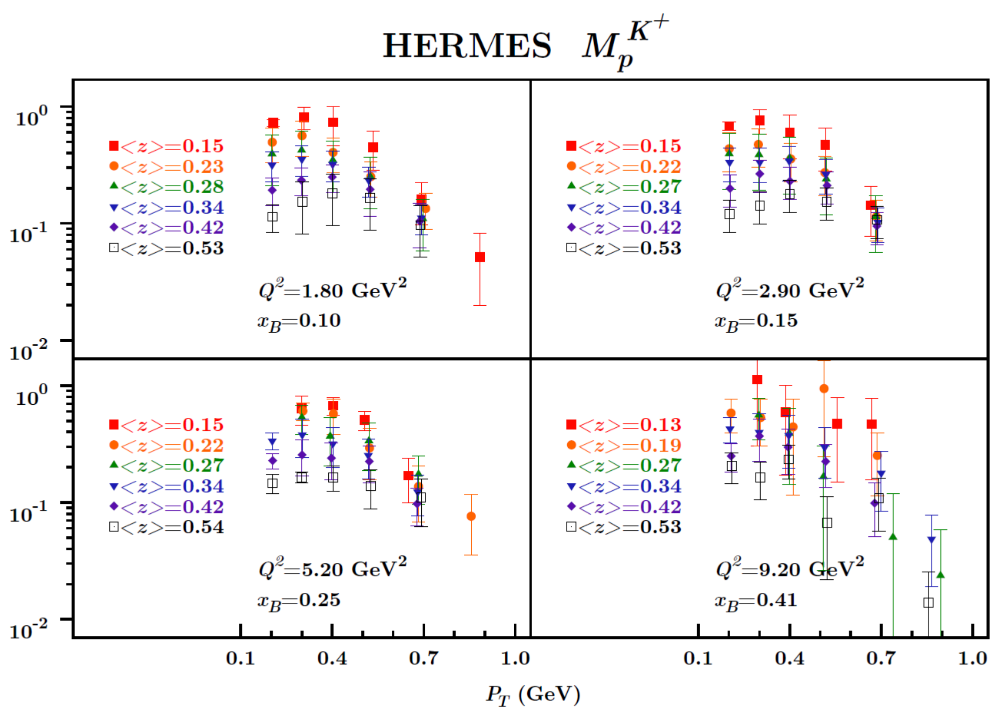
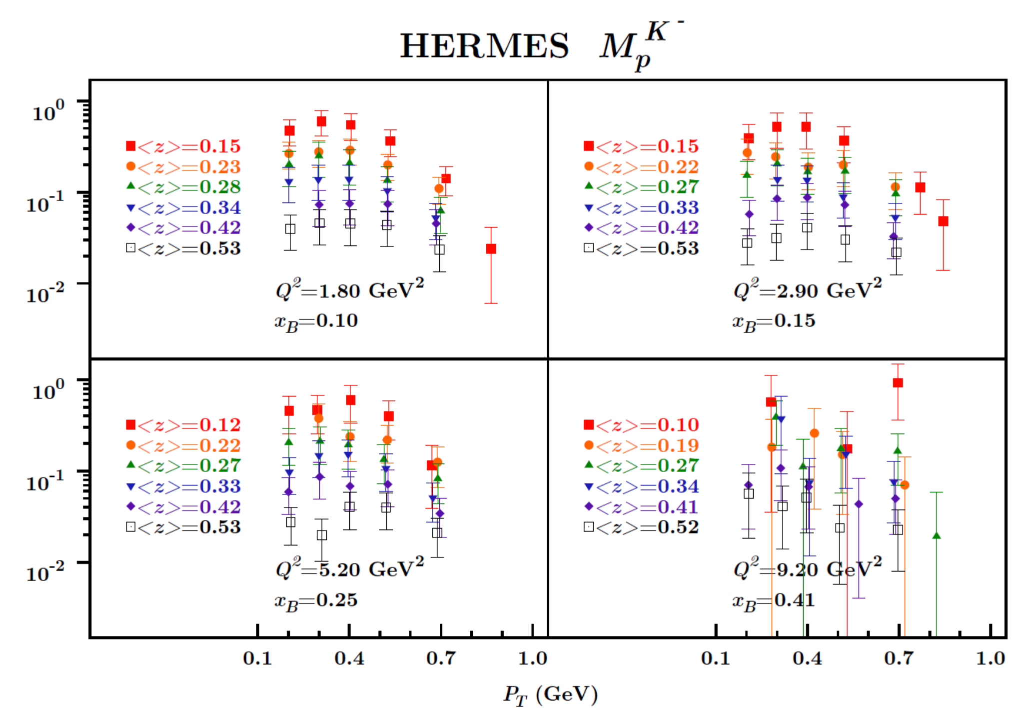
Q^2=1.80 GeV^2 x_B=0.10
<z>=0.15 <z>=0.22 <z>=0.27 <z>=0.33 <z>=0.42 <z>=0.53
Q^2=2.90 GeV^2 x_B=0.15
<z>=0.14 <z>=0.22 <z>=0.27 <z>=0.33 <z>=0.42 <z>=0.53
Q^2=5.20 GeV^2 x_B=0.25
<z>=0.12 <z>=0.22 <z>=0.27 <z>=0.34 <z>=0.42 <z>=0.53
Q^2=9.20 GeV^2 x_B=0.41
P_T (GeV)

HERMES $M_p^{K^+}$
<z>=0.15 <z>=0.23 <z>=0.28 <z>=0.34 <z>=0.42 <z>=0.53
Q^2=1.80 GeV^2 x_B=0.10
<z>=0.15 <z>=0.22 <z>=0.27 <z>=0.34 <z>=0.42 <z>=0.53
Q^2=2.90 GeV^2 x_B=0.15
<z>=0.15 <z>=0.22 <z>=0.27 <z>=0.34 <z>=0.42 <z>=0.54
Q^2=5.20 GeV^2 x_B=0.25
<z>=0.13 <z>=0.19 <z>=0.27 <z>=0.34 <z>=0.42 <z>=0.53
Q^2=9.20 GeV^2 x_B=0.41
P_T (GeV)

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<z>=0.15 <z>=0.22 <z>=0.28 <z>=0.34 <z>=0.42 <z>=0.53
Q^2=1.80 GeV^2 x_B=0.10
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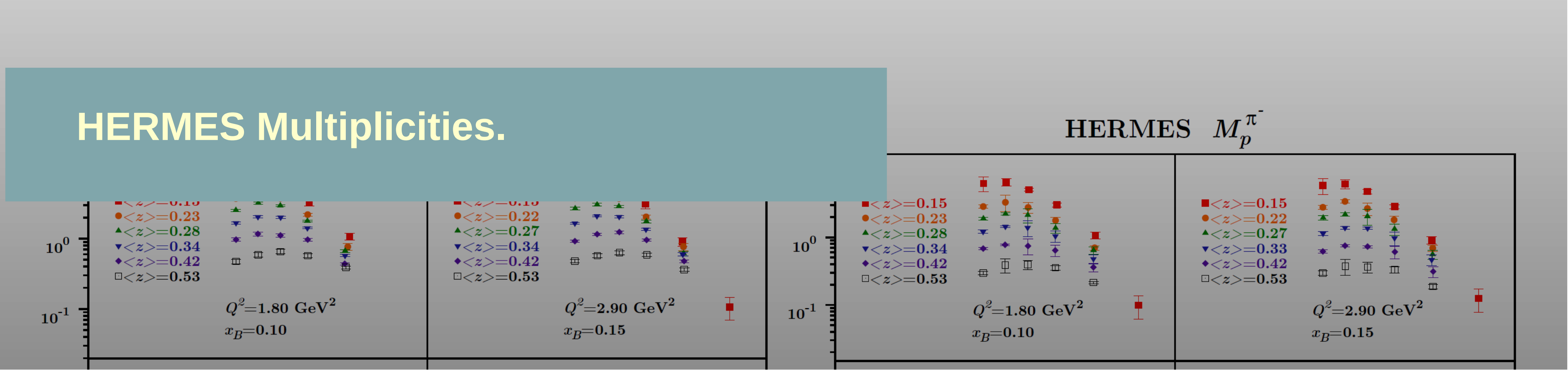
<z>=0.12 <z>=0.22 <z>=0.27 <z>=0.33 <z>=0.42 <z>=0.53
Q^2=5.20 GeV^2 x_B=0.25
<z>=0.10 <z>=0.19 <z>=0.27 <z>=0.34 <z>=0.41 <z>=0.52
Q^2=9.20 GeV^2 x_B=0.41
P_T (GeV)

HERMES Multiplicities.

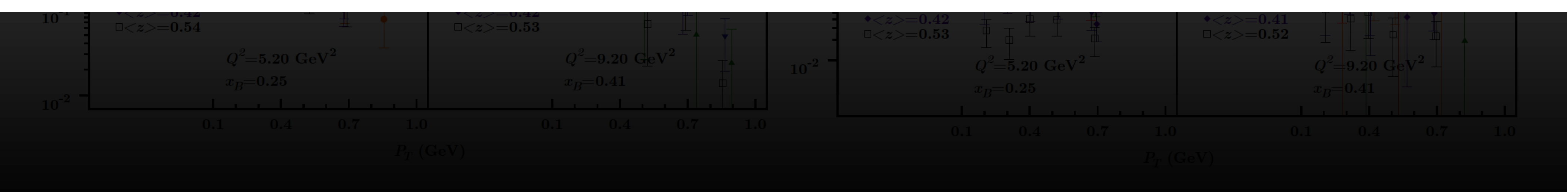


From P and D targets.

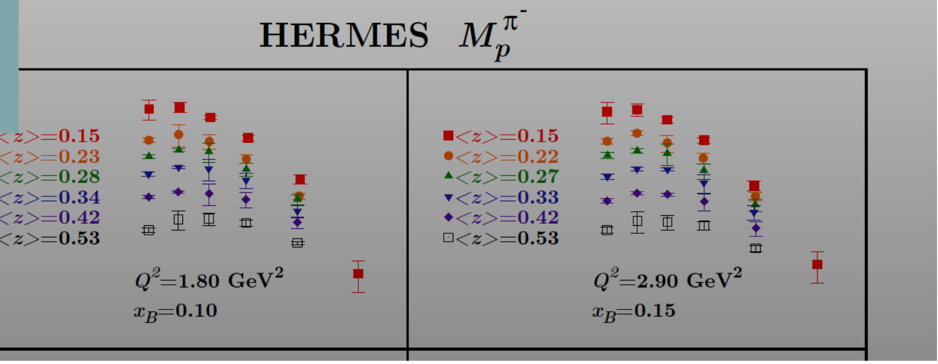
Hadron separation

3D-binning: (Q^2 x_B), z , P_T

Total number of points: 1341



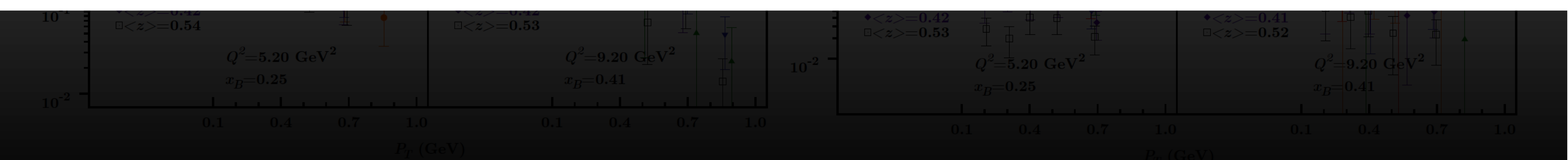
HERMES Multiplicities.



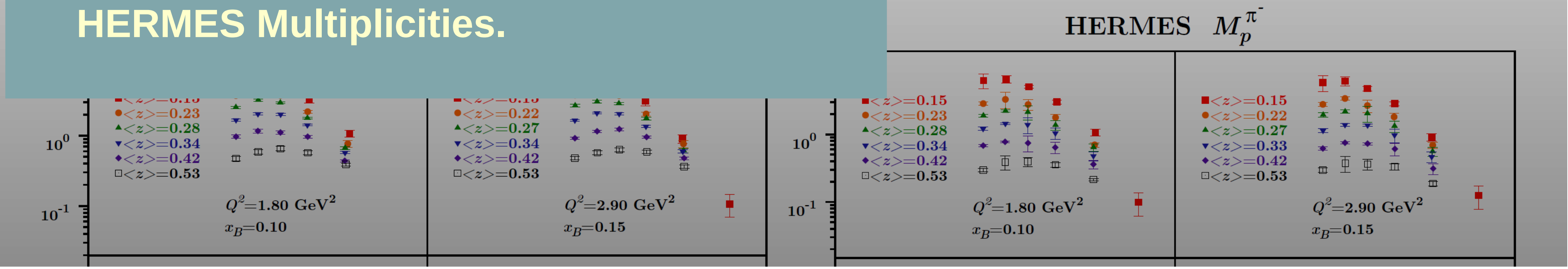
Particularly suitable for

flavor-dependence studies

(Previous talk A. Signori).



HERMES Multiplicities.



Particularly suitable for

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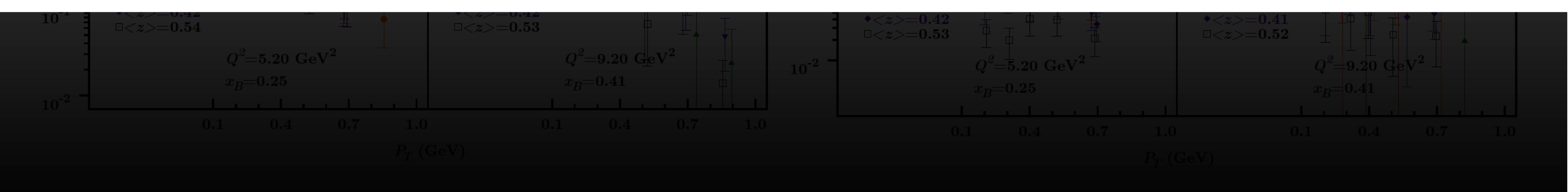
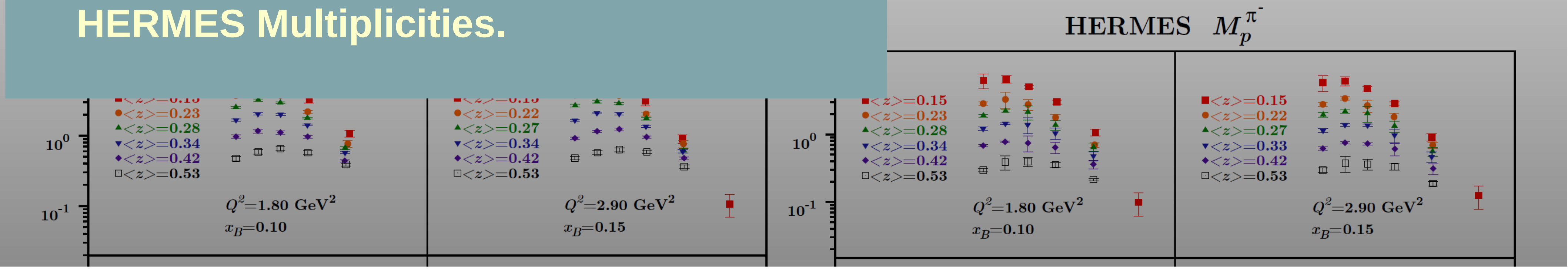
TMD evolution?



HERMES Multiplicities.



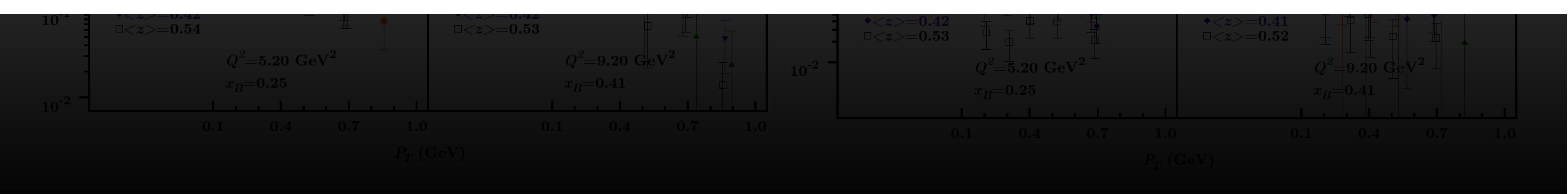
Particularly suitable for

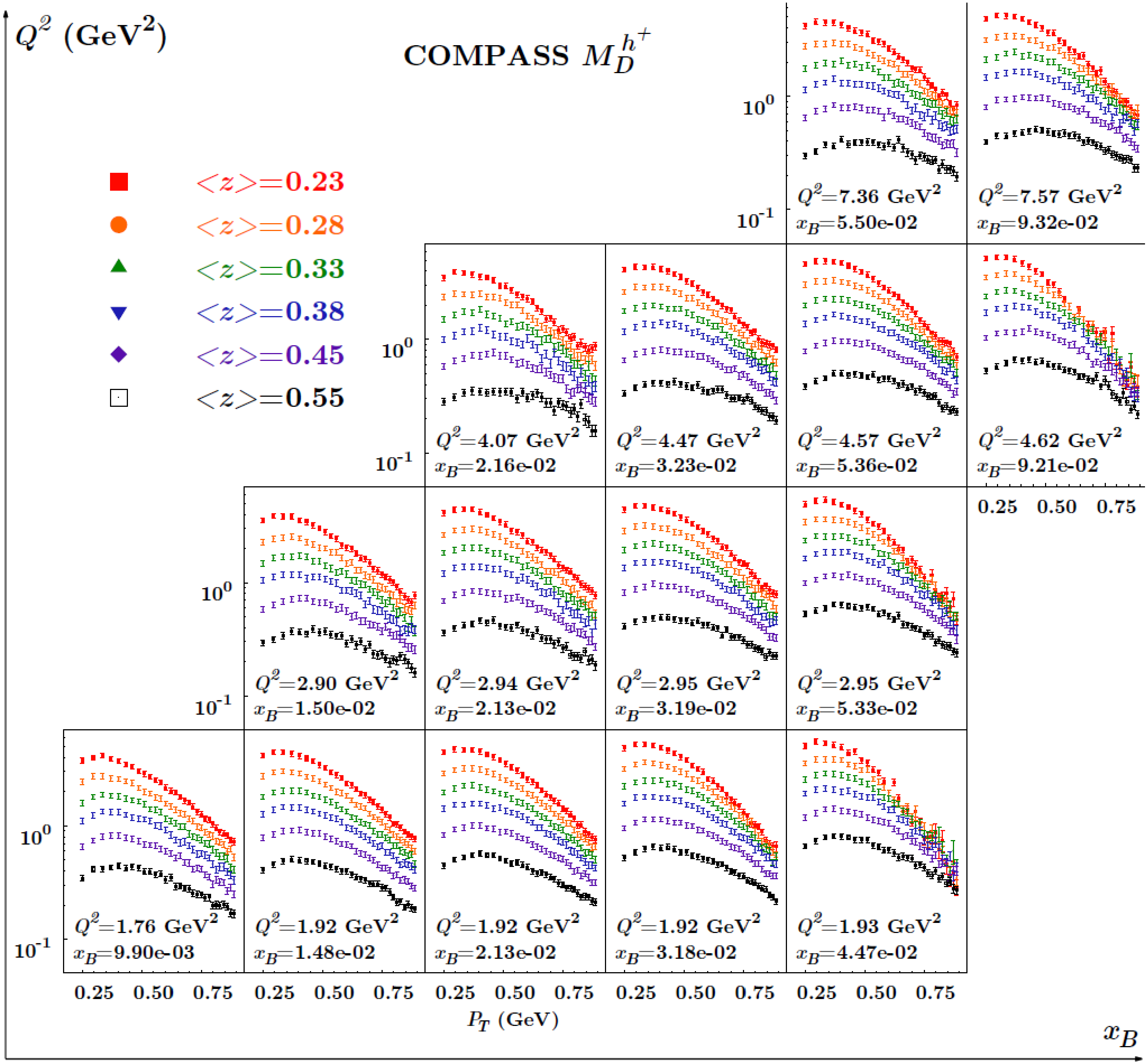
flavor-dependence studies

(Previous talk A. Signori).

TMD evolution?

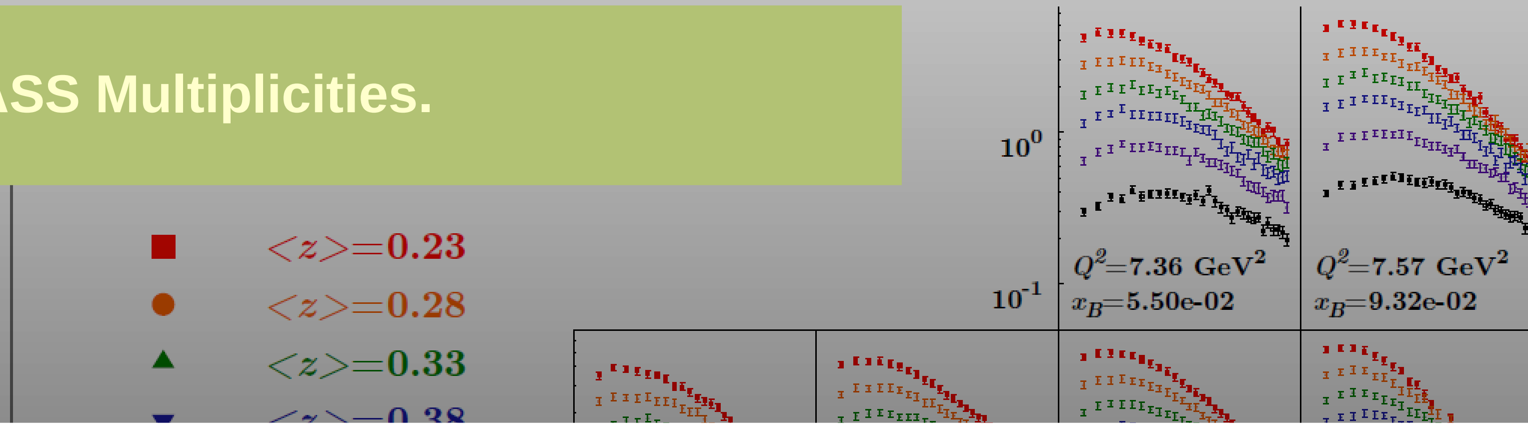
- **It is not possible to decouple the x_B and Q^2 dependences.**
- **1.25 GeV^2 < Q^2 < 9.20 GeV^2**





COMPASS $M_D^{h^+}$
Q^2 (GeV^2)
$<z>=0.23$
$<z>=0.28$
$<z>=0.33$
$<z>=0.38$
$<z>=0.45$
$<z>=0.55$
Q^2=7.36 GeV^2 x_B=5.50e-02
Q^2=7.57 GeV^2 x_B=9.32e-02
Q^2=4.07 GeV^2 x_B=2.16e-02
Q^2=4.47 GeV^2 x_B=3.23e-02
Q^2=4.57 GeV^2 x_B=5.36e-02
Q^2=4.62 GeV^2 x_B=9.21e-02
Q^2=2.90 GeV^2 x_B=1.50e-02
Q^2=2.94 GeV^2 x_B=2.13e-02
Q^2=2.95 GeV^2 x_B=3.19e-02
Q^2=2.95 GeV^2 x_B=5.33e-02
Q^2=1.76 GeV^2 x_B=9.90e-03
Q^2=1.92 GeV^2 x_B=1.48e-02
Q^2=1.92 GeV^2 x_B=2.13e-02
Q^2=1.92 GeV^2 x_B=3.18e-02
Q^2=1.93 GeV^2 x_B=4.47e-02
10^0
10^{-1}
0.25 0.50 0.75
P_T (GeV)
x_B

COMPASS Multiplicities.



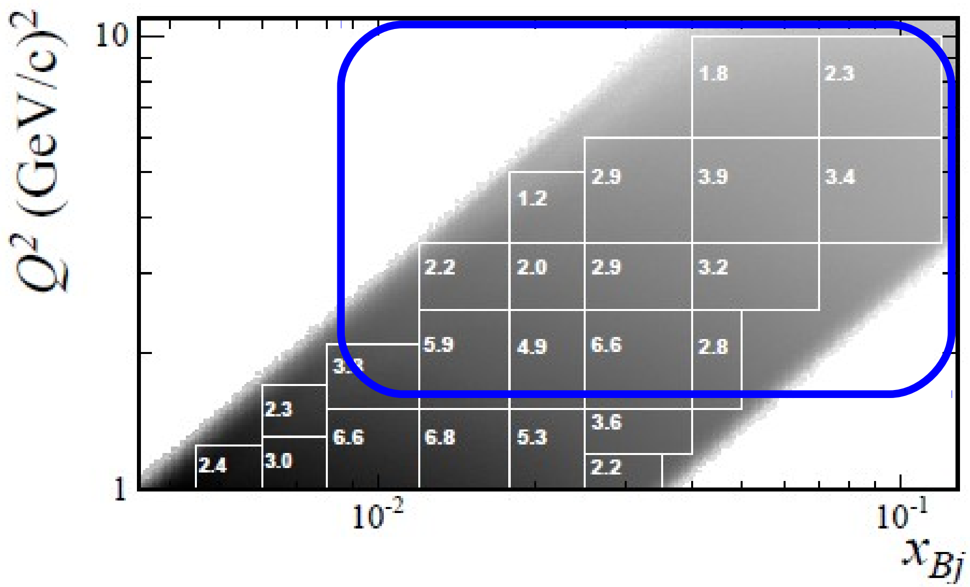
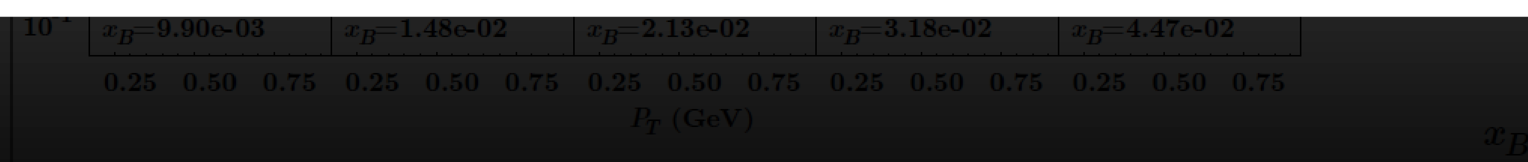
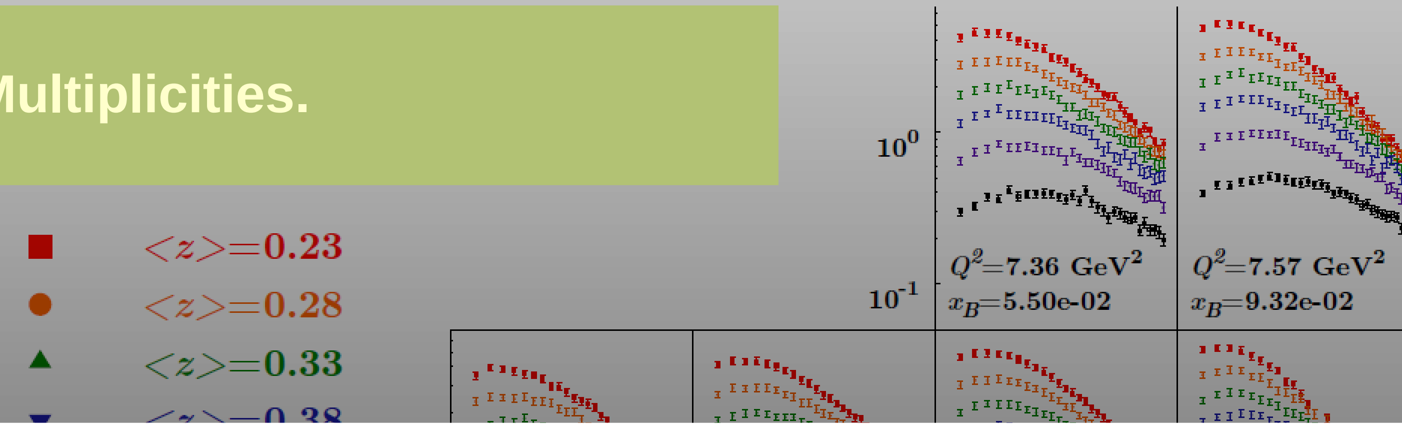


Figure from: Adolph, C. et al. Eur.Phys.J. C73 (2013) 2531

- From **Deuteron** only
- No hadron separation
- **4D**-binning: Q^2, x_B , z , P_T
- Total number of points: **18624**



COMPASS Multiplicities.



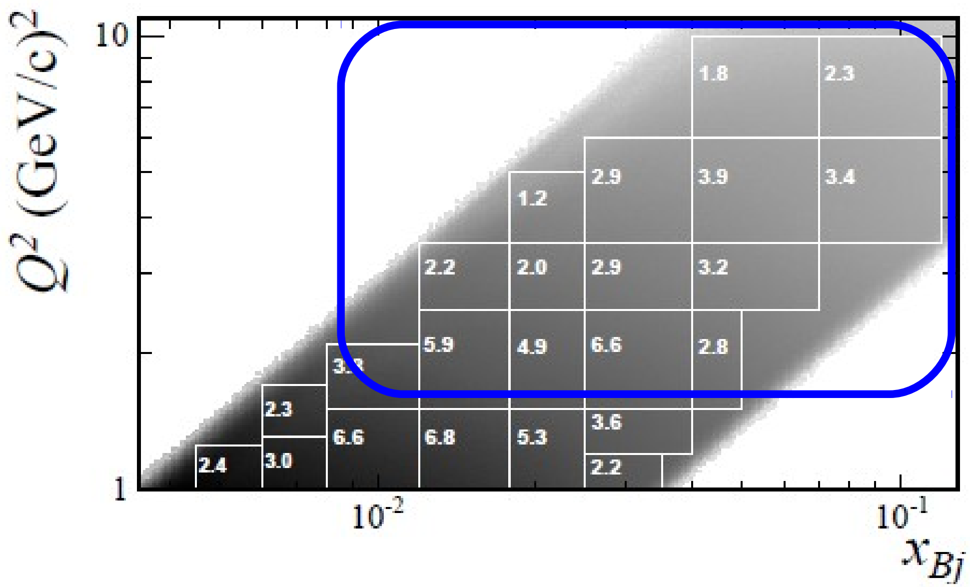


Figure from: Adolph, C. et al. Eur.Phys.J. C73 (2013) 2531

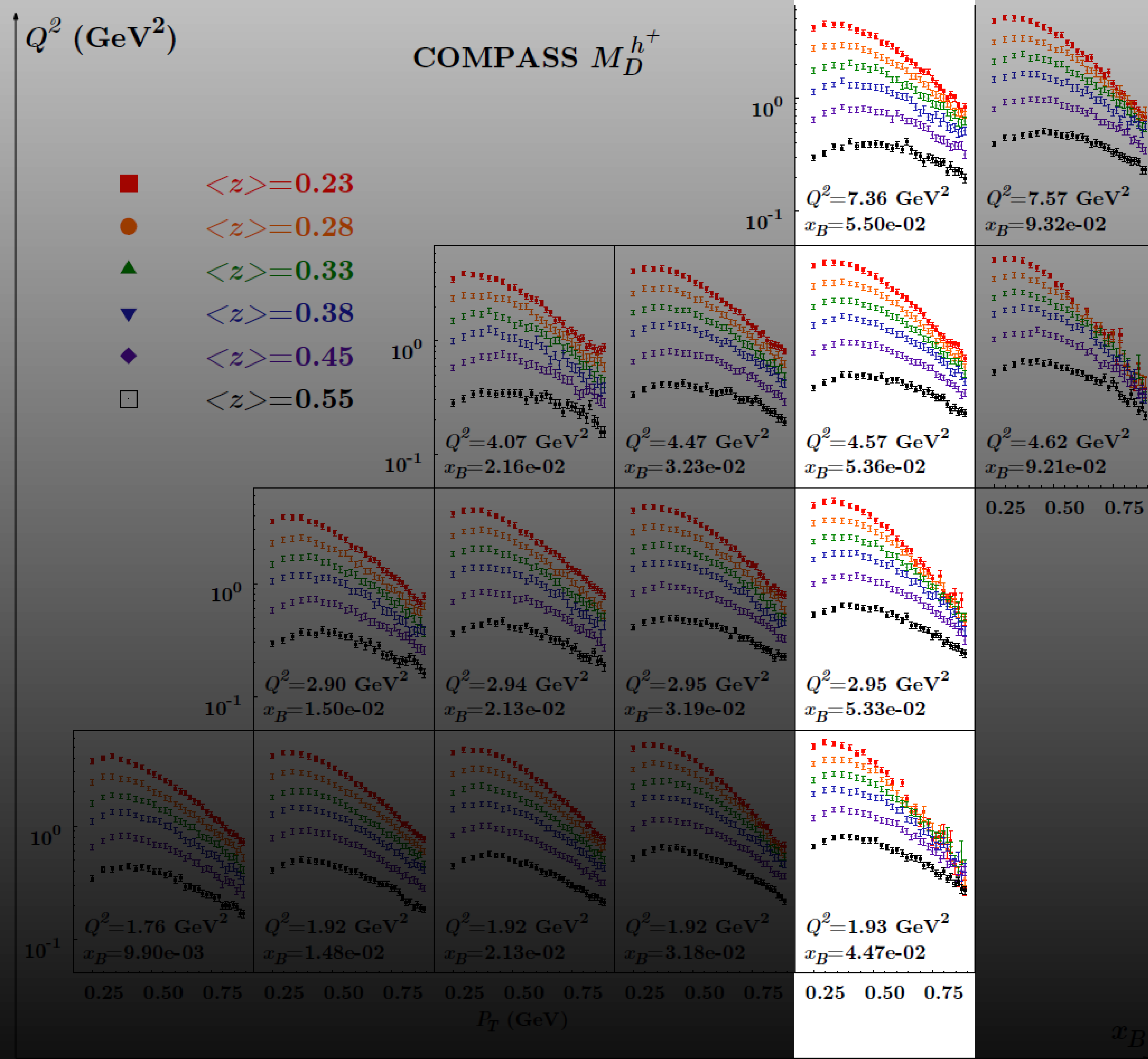
TMD evolution?

x_B=9.90e-03 x_B=1.48e-02 x_B=2.13e-02 x_B=3.18e-02 x_B=4.47e-02

0.25 0.50 0.75 0.25 0.50 0.75 0.25 0.50 0.75 0.25 0.50 0.75 0.25 0.50 0.75

P_T (GeV)

x_B



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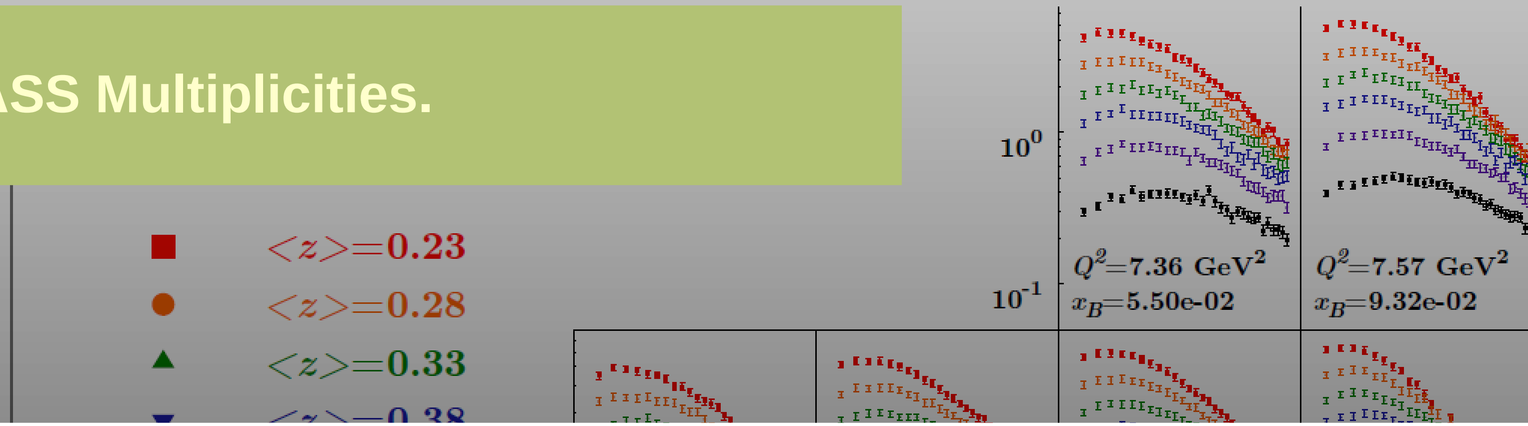
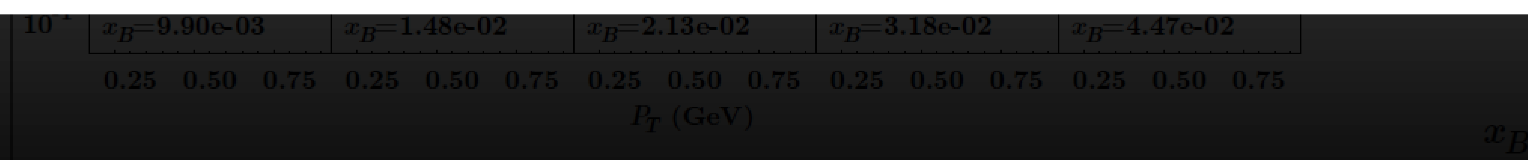
Q^2=4.62 GeV^2 x_B=9.21e-02
Q^2=2.90 GeV^2 x_B=1.50e-02
Q^2=2.94 GeV^2 x_B=2.13e-02
Q^2=2.95 GeV^2 x_B=3.19e-02
Q^2=2.95 GeV^2 x_B=5.33e-02
Q^2=1.76 GeV^2 x_B=9.90e-03
Q^2=1.92 GeV^2 x_B=1.48e-02
Q^2=1.92 GeV^2 x_B=2.13e-02
Q^2=1.92 GeV^2 x_B=3.18e-02
Q^2=1.93 GeV^2 x_B=4.47e-02
10^0
10^{-1}
0.25 0.50 0.75
P_T (GeV)
x_B

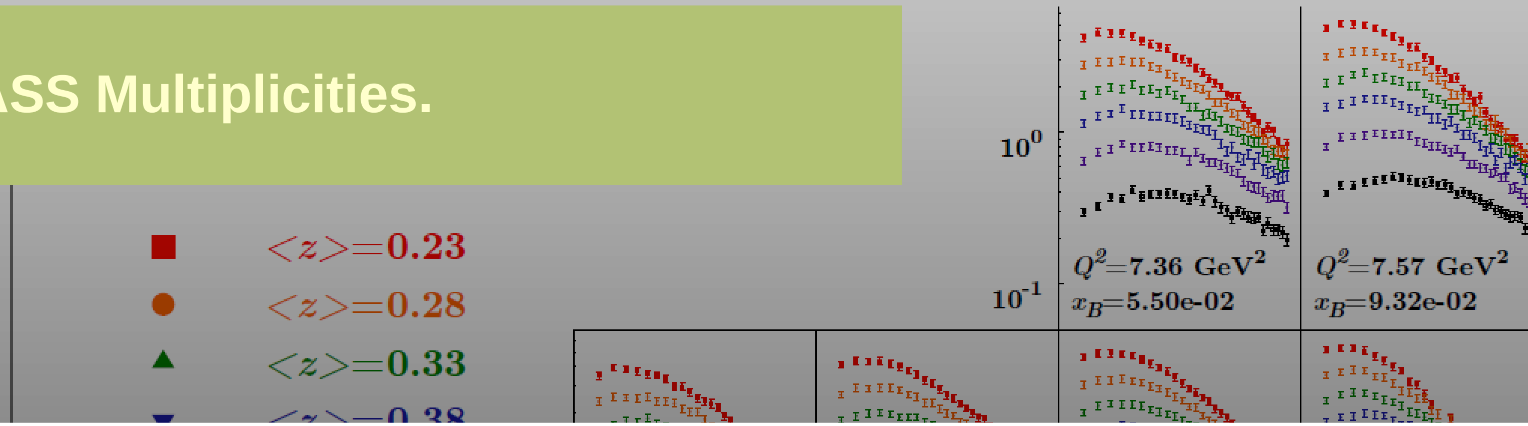
COMPASS Multiplicities.



- **Multidimensional data! Q2, xB, z, PT dependence**
- **All of these dependences must be understood for TMDs.**



COMPASS Multiplicities.



- **Multidimensional data! Q2, xB, z, PT dependence**
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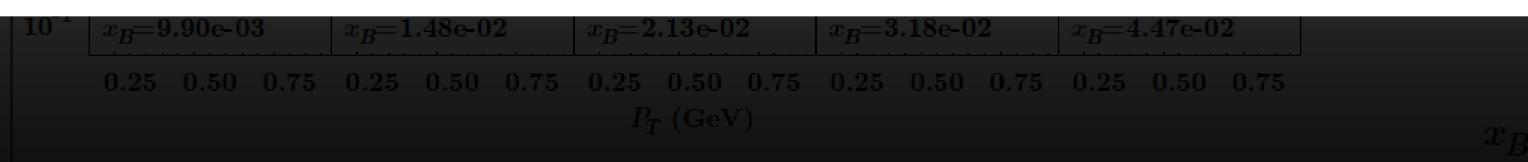
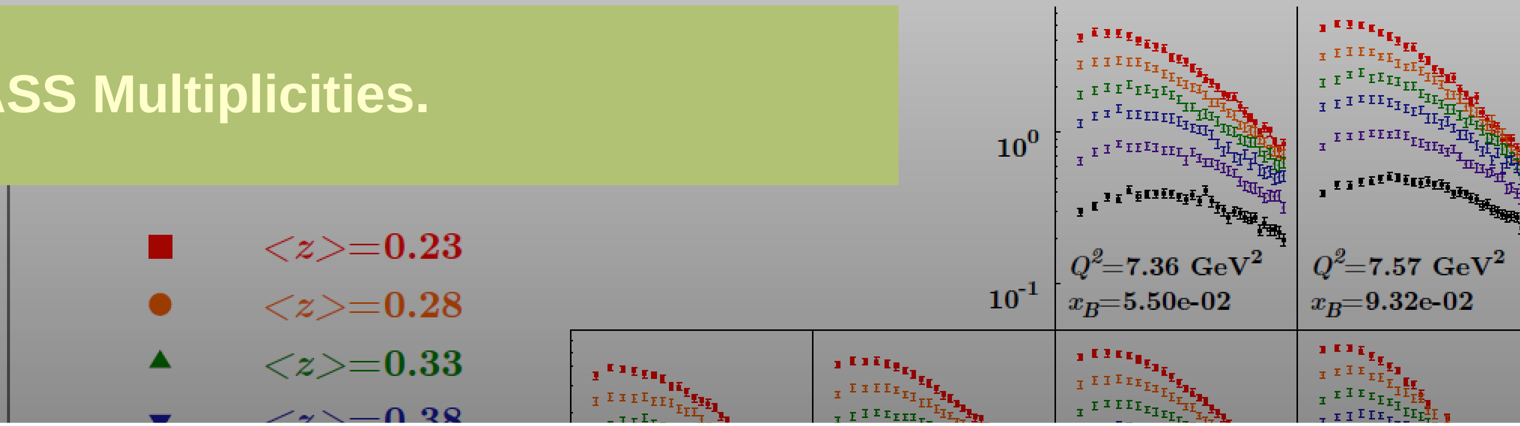
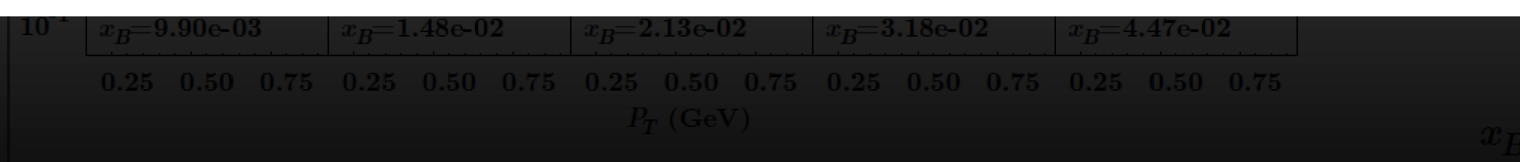
- **1.11 GeV^2 < Q^2 < 7.57 GeV^2 Might be hard to see TMD-evolution.**



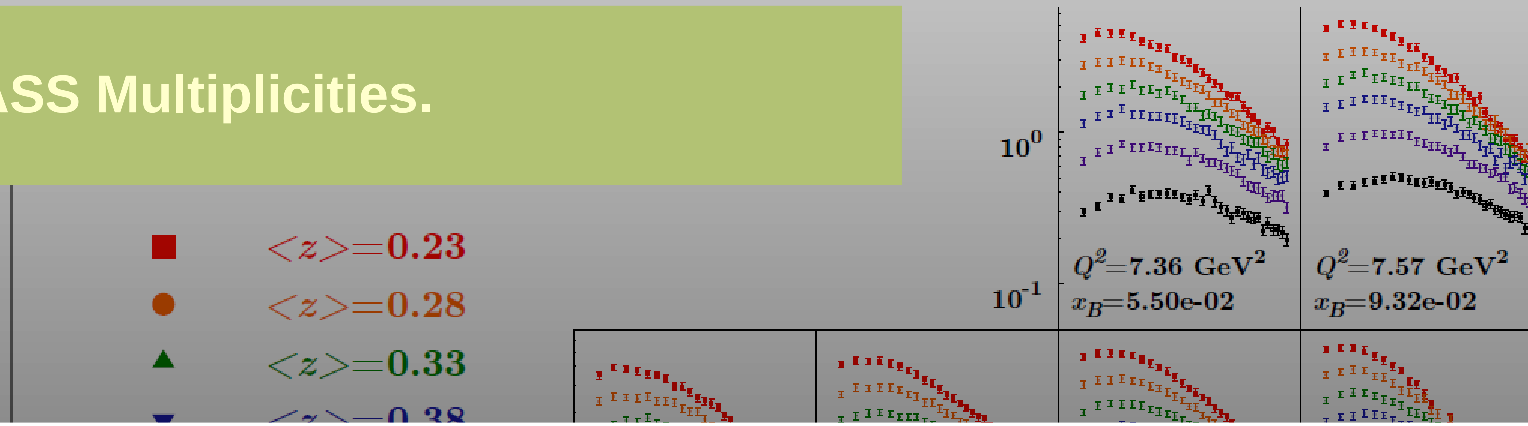
COMPASS Multiplicities.



- **Multidimensional data! Q2, xB, z, PT dependence**
- **All of these dependences must be understood for TMDs.**
- **1.11 GeV^2 < Q^2 < 7.57 GeV^2 Might be hard to see TMD-evolution.**
- **A control analysis or “Benchmark” helps to understand the data. That is step 1.**

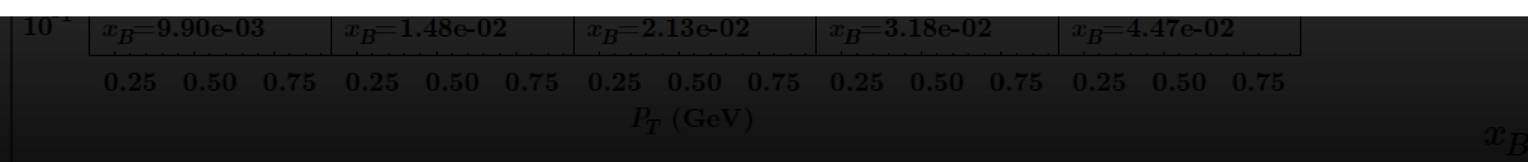


COMPASS Multiplicities.



Results presented here have been published in:

DOI: 10.1007/JHEP04(2014)005

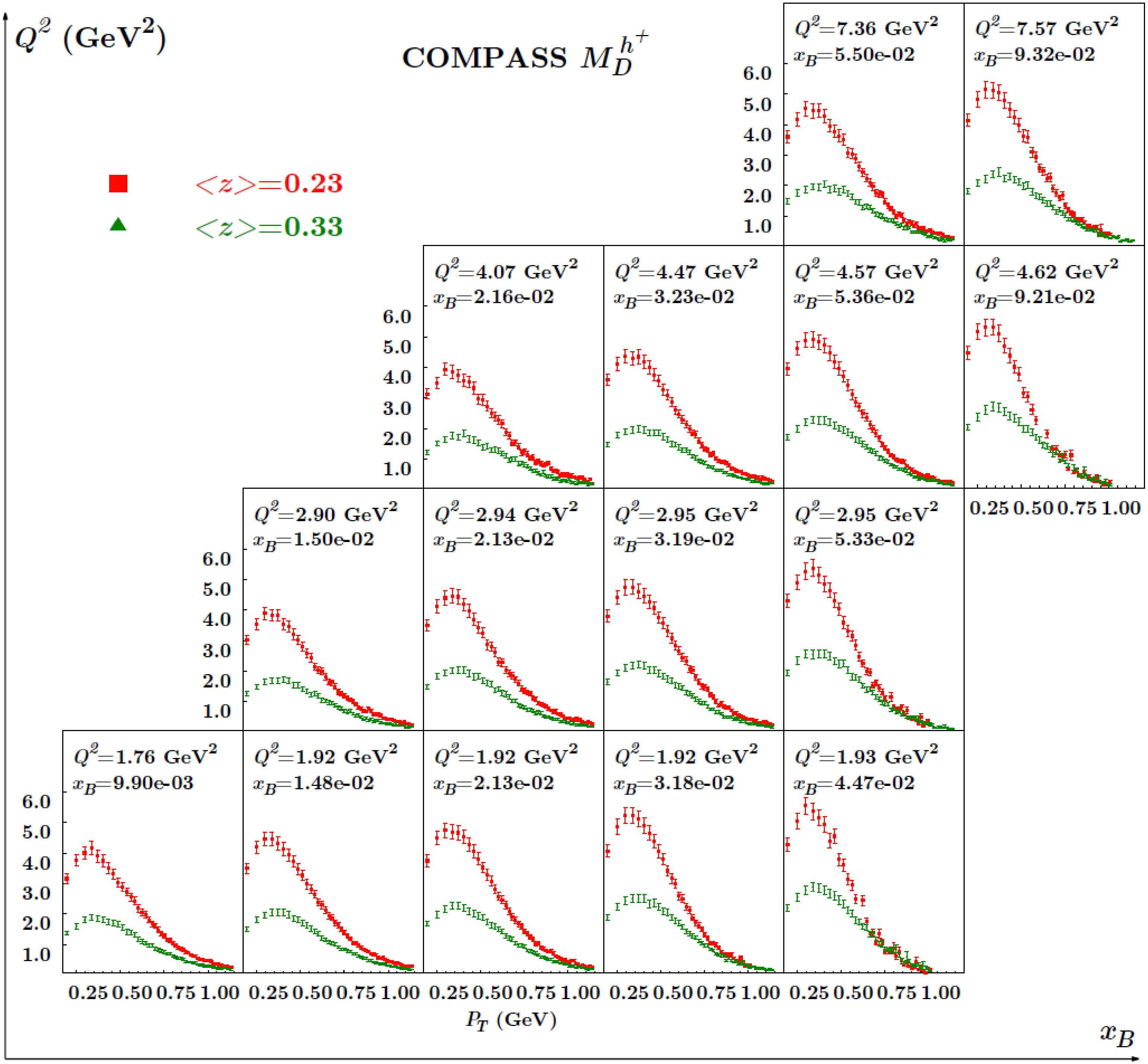


OUTLINE

- Multiplicities HERMES and COMPASS.
- **Extracting information from Multiplicites.**
- Other observables
- Conclusions

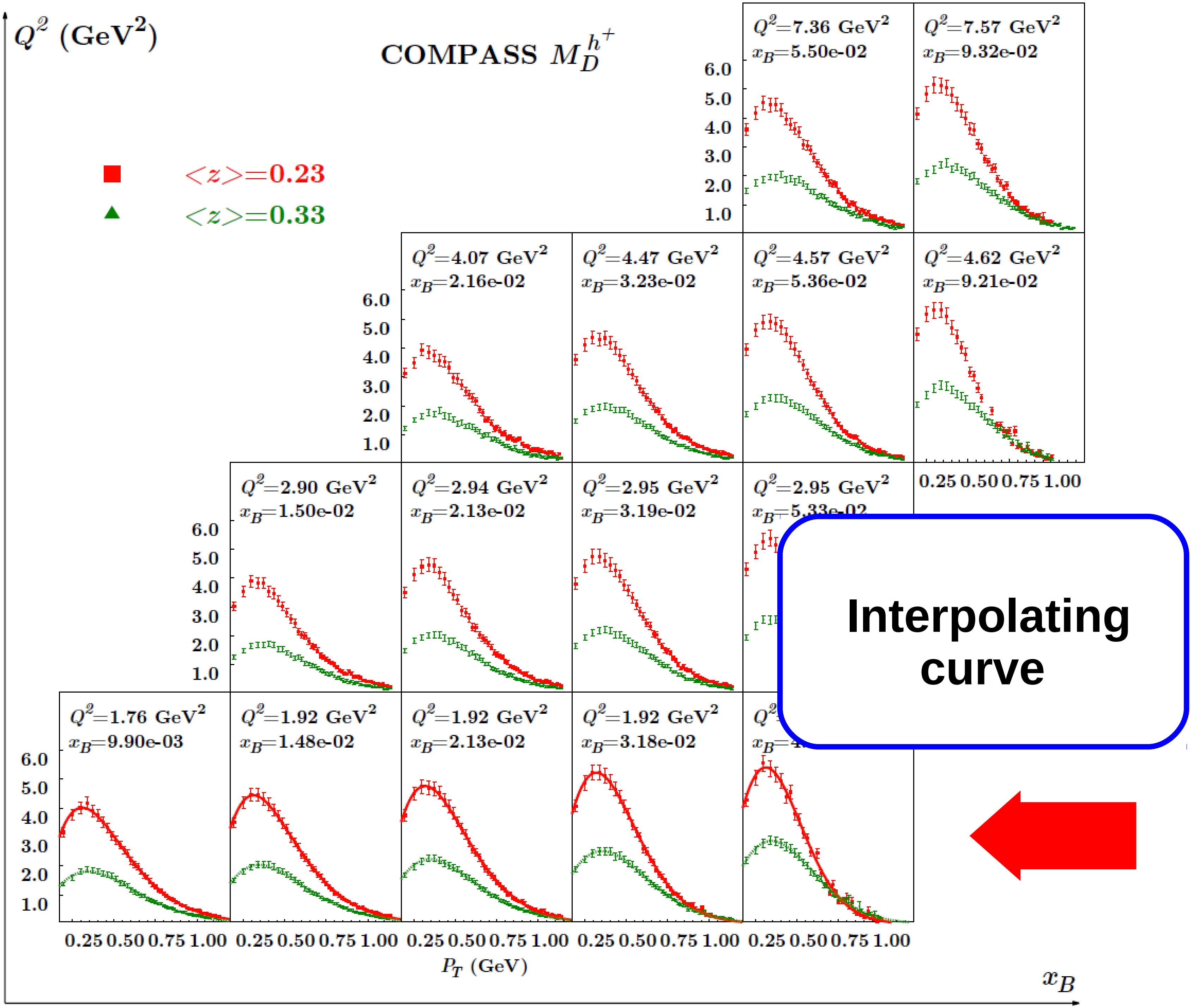
Extraction from COMPASS data.

Qualitatively.



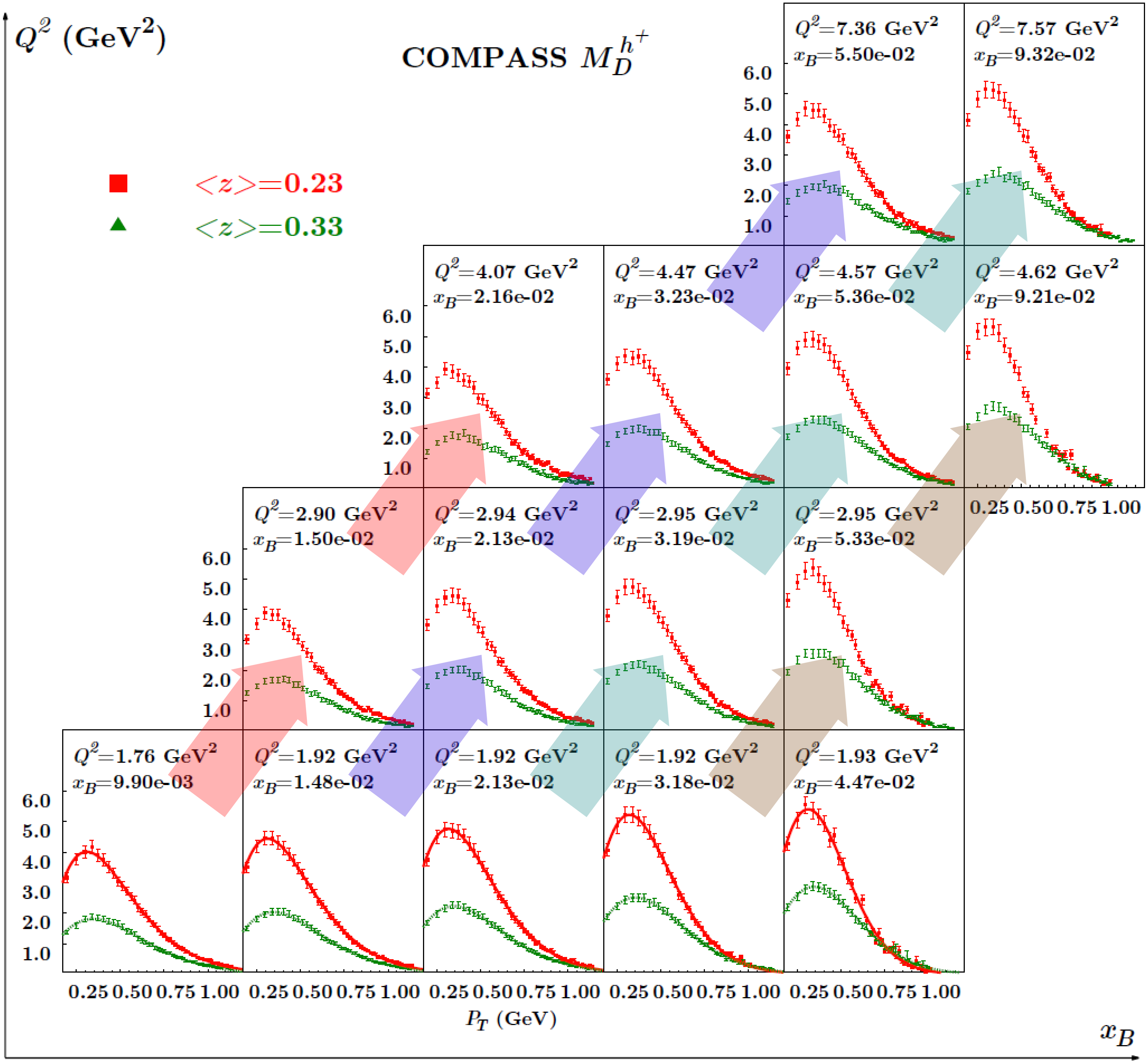
Extraction from COMPASS data.

Qualitatively.



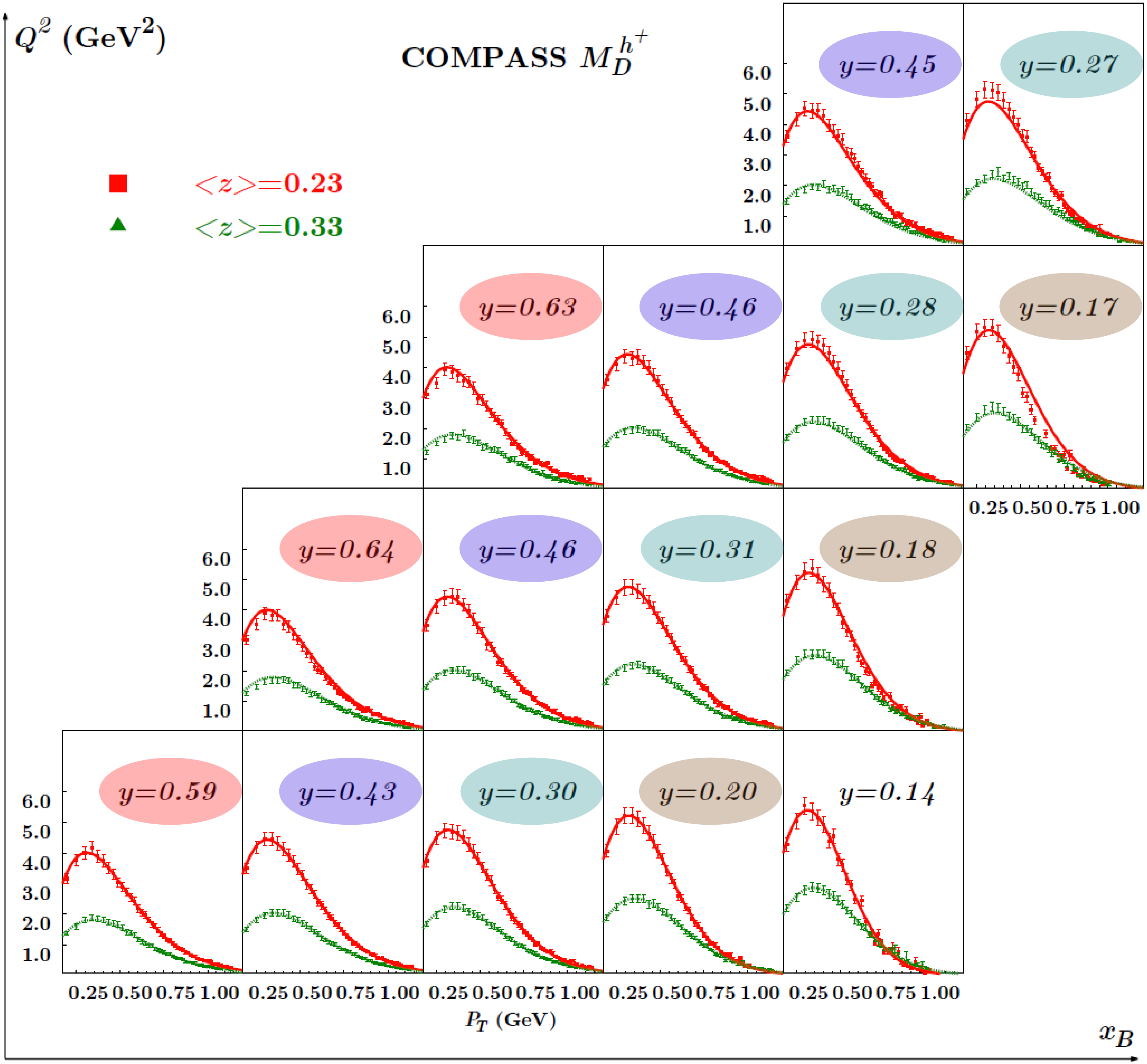
Extraction from COMPASS data.

Qualitatively.



Extraction from COMPASS data.

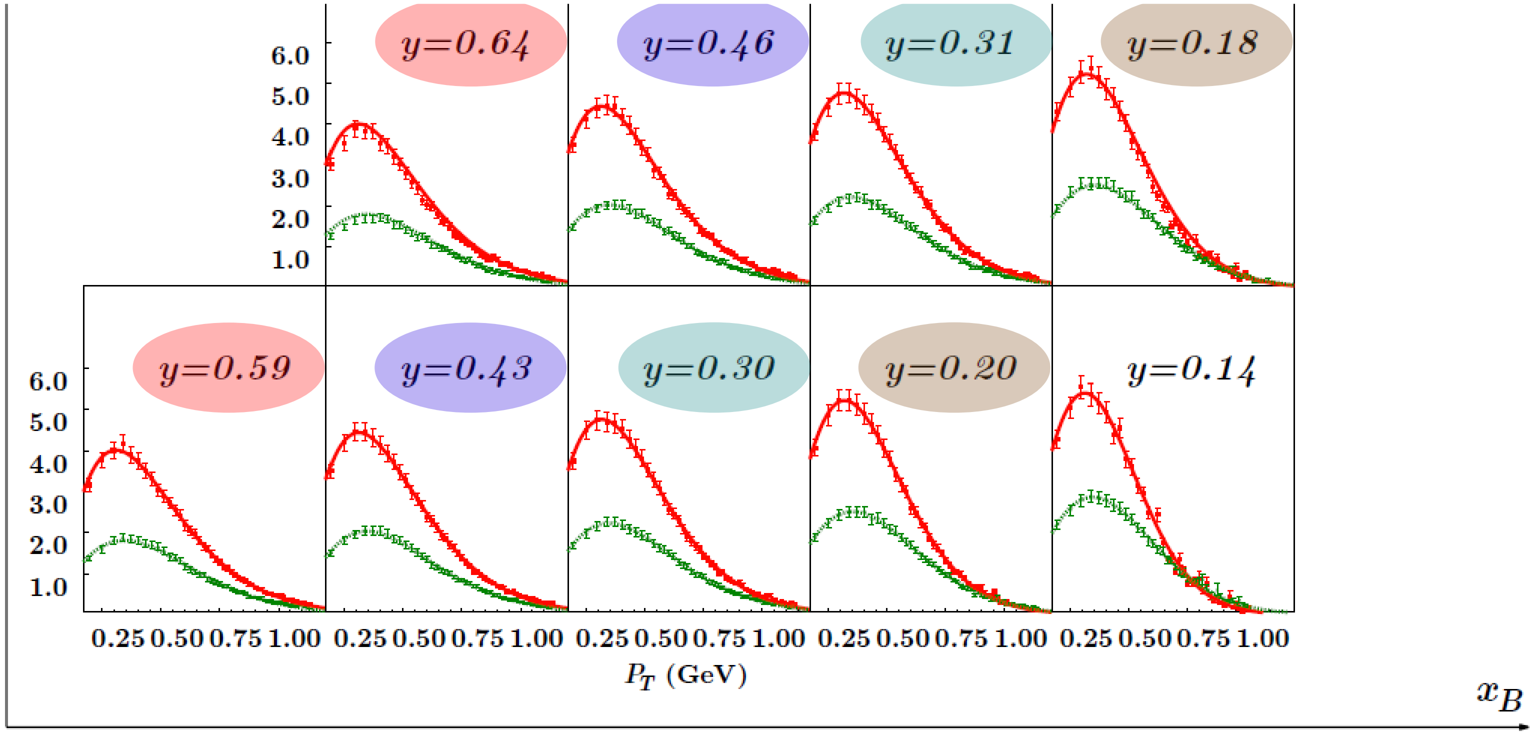
Qualitatively.



Extraction from COMPASS data.

Qualitatively.

Two important points

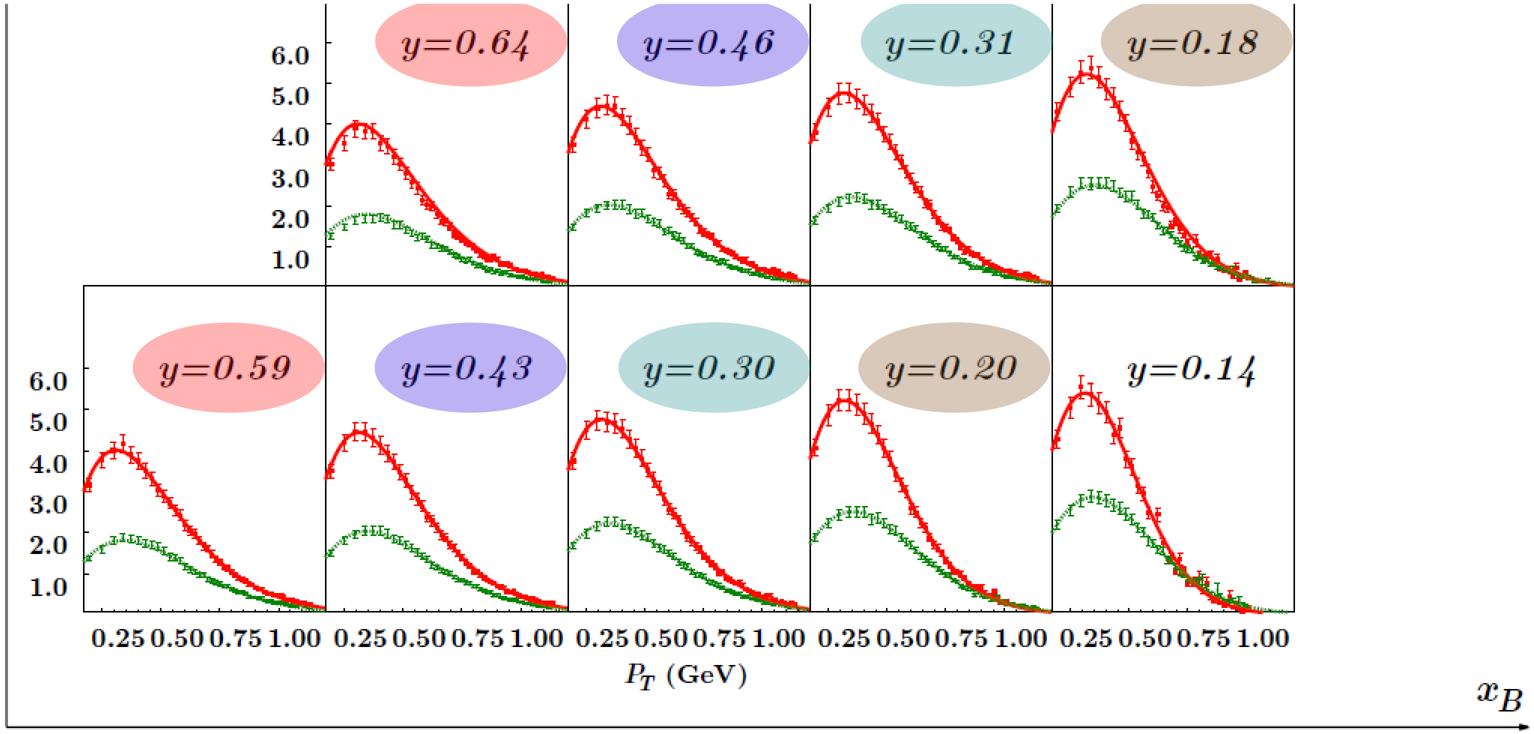


Extraction from COMPASS data.

Qualitatively.

Two important points

- **At fixed y, Width & Normalization roughly constant.**

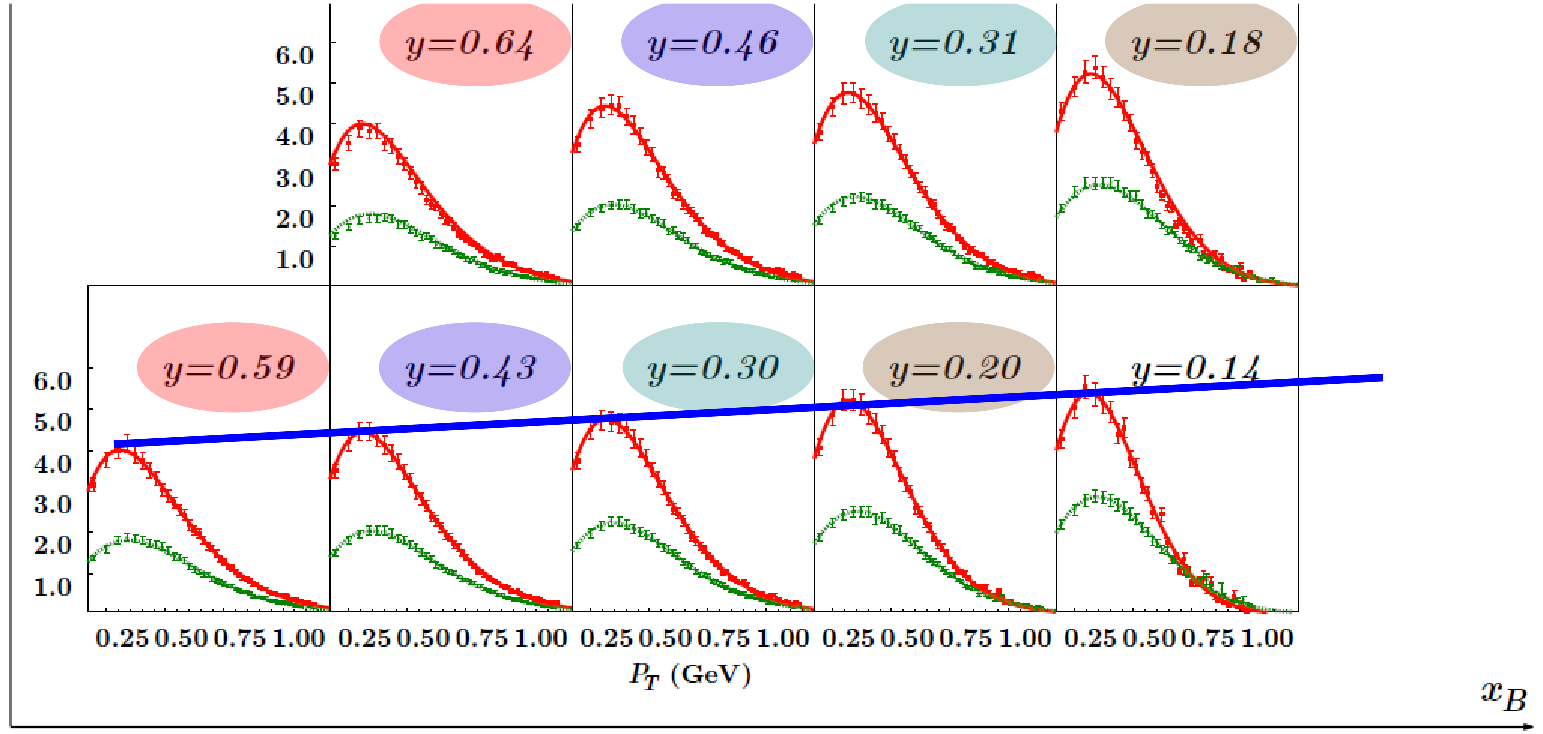


Extraction from COMPASS data.

Qualitatively.

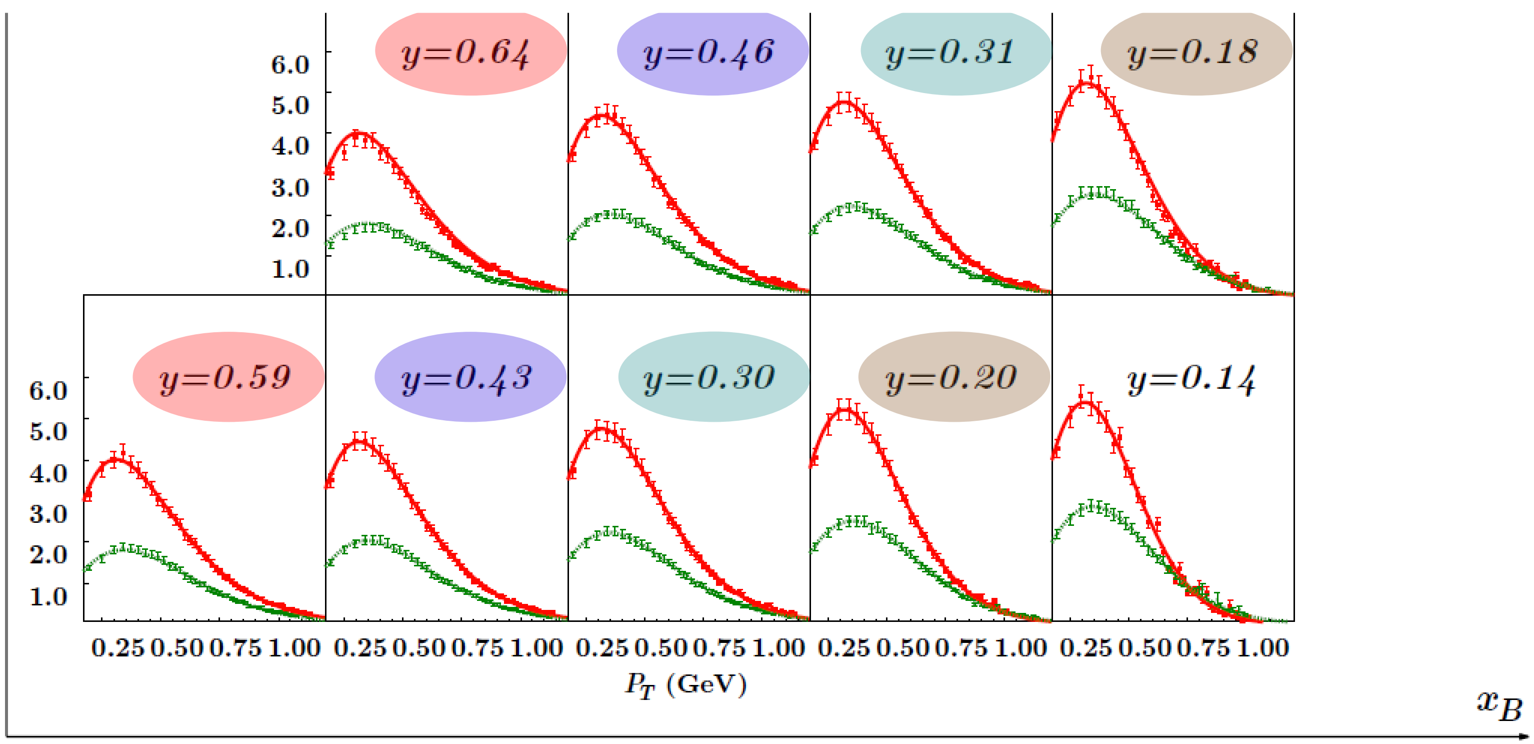
Two important points

- **At fixed y, Width & Normalization roughly constant.**
- **Normalization resembles a straight line.**



Quantitatively

(a.k.a. Step 1)



Extraction from HERMES

and COMPASS data.

Model

$$f_q(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

$$D_q^h(z, p_\perp) = D_q^h(z) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-p_\perp^2/\langle p_\perp^2 \rangle}$$

Kinematical Cuts

$$Q^2 > 1.69 \text{ GeV}^2$$
$$0.2 < P_T < 0.9 \text{ GeV}$$
$$z < 0.6$$

Processes included

π^+ and π^- production from both P and D targets.

h^+ and h^- production from D.

Extraction from HERMES data.

Cuts	χ^2_{pts}	n. points	$[\chi^2_{pts}]^{\pi^+}$	$[\chi^2_{pts}]^{\pi^-}$	Parameters
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$	1.69	497	1.93	1.45	$\langle k_\perp^2 \rangle = 0.57 \pm 0.08$ GeV^2 $\langle p_\perp^2 \rangle = 0.12 \pm 0.01$ GeV^2

Extraction from HERMES data.

Cuts	χ^2_{pts}	n. points	$[\chi^2_{pts}]^{\pi^+}$	$[\chi^2_{pts}]^{\pi^-}$	Parameters
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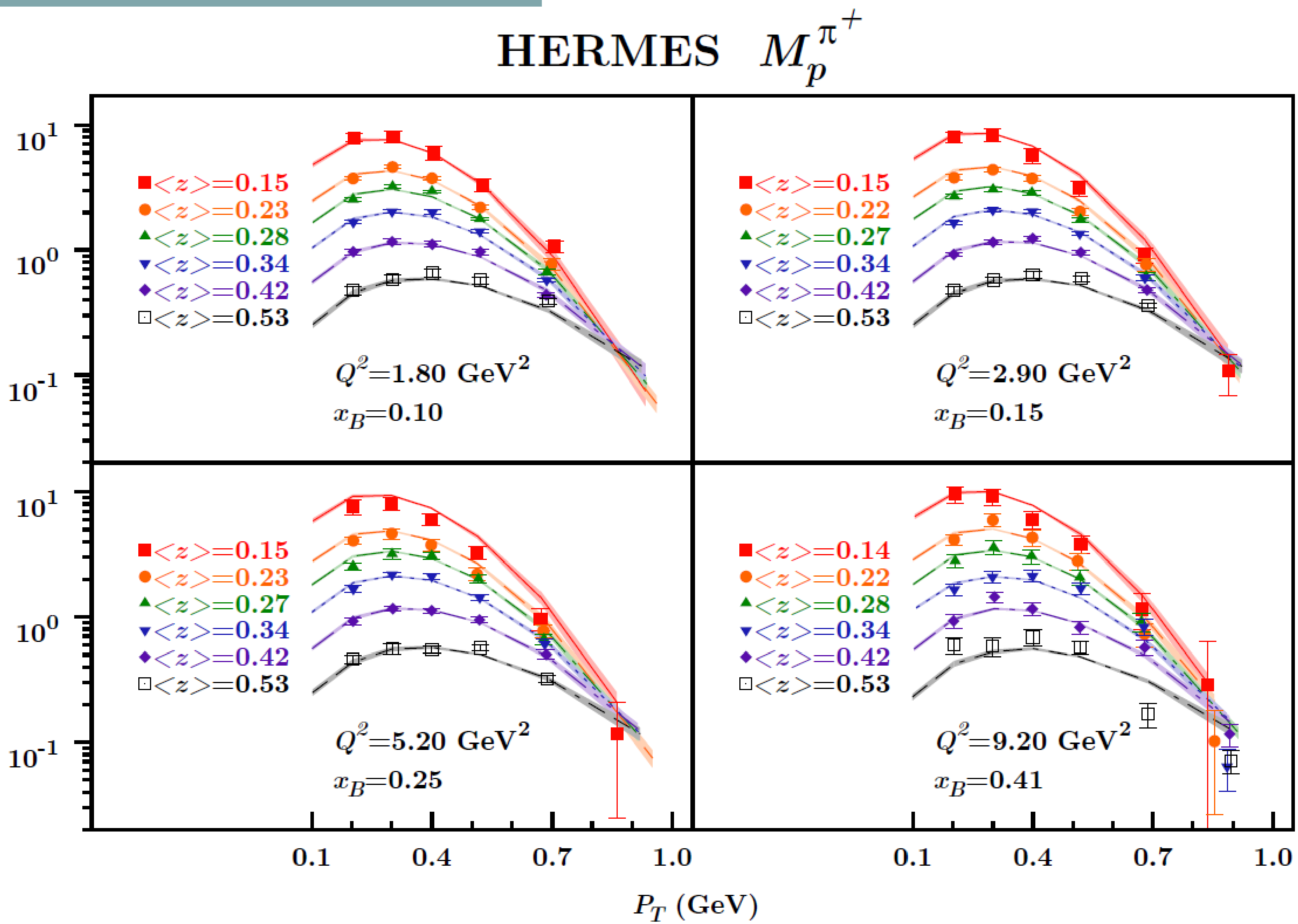
Consideration of flavor-dependence on the fragmentation slightly improves the quality of the fit.

It is not possible to resolve additional z-dependence.

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle.$$

Gaussian model.

Extraction from HERMES data.

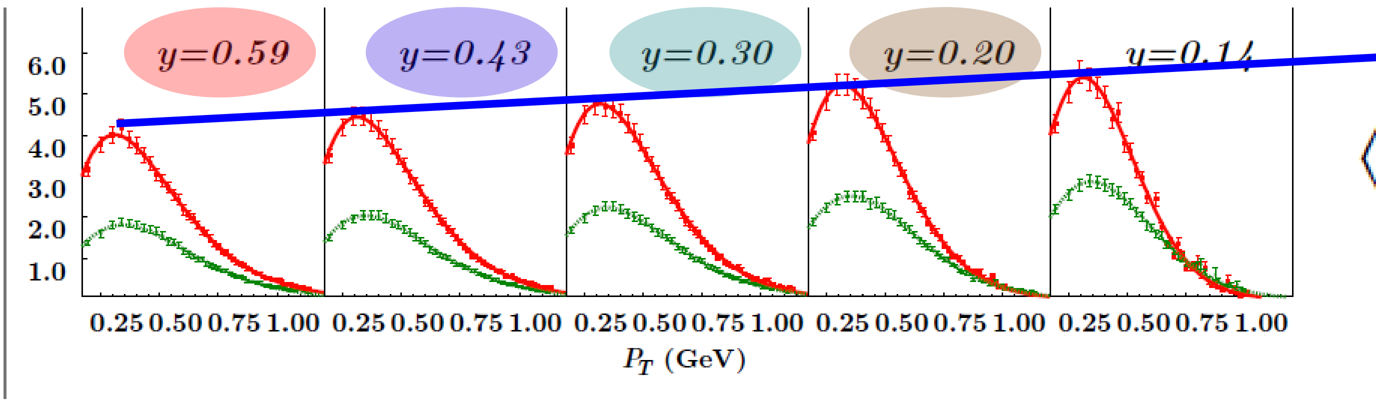


Extraction from COMPASS data.

Cuts	$\chi^2_{\rm dof}$	n. points	$[\chi^2_{\rm dof}]^{h^+}$	$[\chi^2_{\rm dof}]^{h^-}$	Parameters
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$	8.54	5385	8.94	8.15	$\langle k_\perp^2 \rangle = 0.61 \pm 0.20$ GeV^2 $\langle p_\perp^2 \rangle = 0.19 \pm 0.02$ GeV^2

In the Gaussian model.

$$F_{UU} = \sum_q \int d^2\boldsymbol{k}_\perp\, f_q(x,k_\perp) D_q(z,p_\perp) \quad \propto \quad \frac{1}{\pi\langle P_T^2\rangle} e^{-P_T^2/\langle P_T^2\rangle}$$



$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle.$$

Extraction from COMPASS data.

Cuts	$\chi^2_{\rm dof}$	n. points	$[\chi^2_{\rm dof}]^{h^+}$	$[\chi^2_{\rm dof}]^{h^-}$	Parameters
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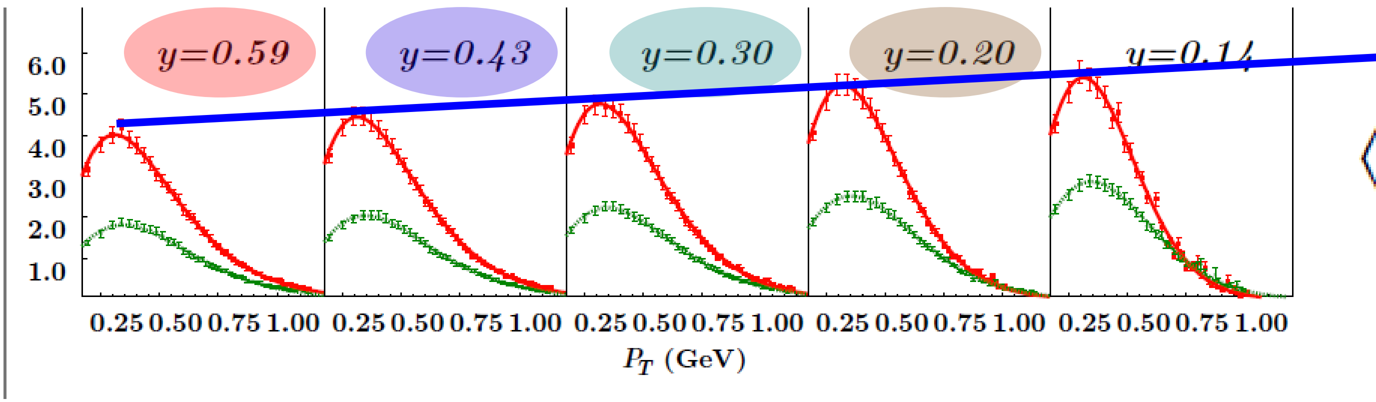


$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle.$$

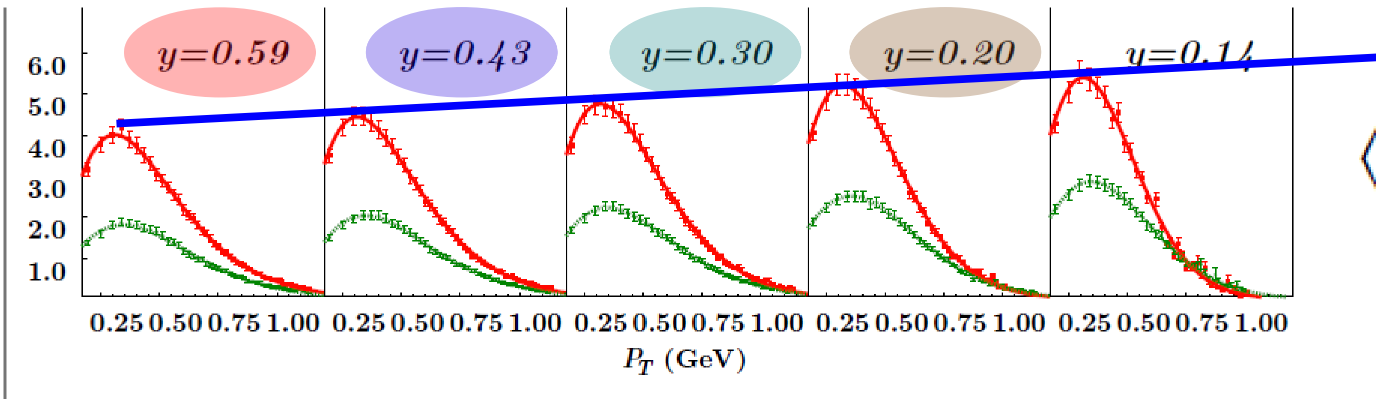
Extraction from COMPASS data.

Cuts	$\chi^2_{\rm dof}$	n. points	$[\chi^2_{\rm dof}]^{h^+}$	$[\chi^2_{\rm dof}]^{h^-}$	Parameters
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$	8.54	5385	8.94	8.15	$\langle k_\perp^2\rangle = 0.61 \pm 0.20$ GeV^2 $\langle p_\perp^2\rangle = 0.19 \pm 0.02$ GeV^2

$$N_y = A + B\,y$$

In the Gaussian model.

$$F_{UU} = \sum_q \int d^2\boldsymbol{k}_\perp\, f_q(x, k_\perp) D_q(z, p_\perp) \quad \propto \quad \frac{1}{\pi\langle P_T^2\rangle} e^{-P_T^2/\langle P_T^2\rangle}$$



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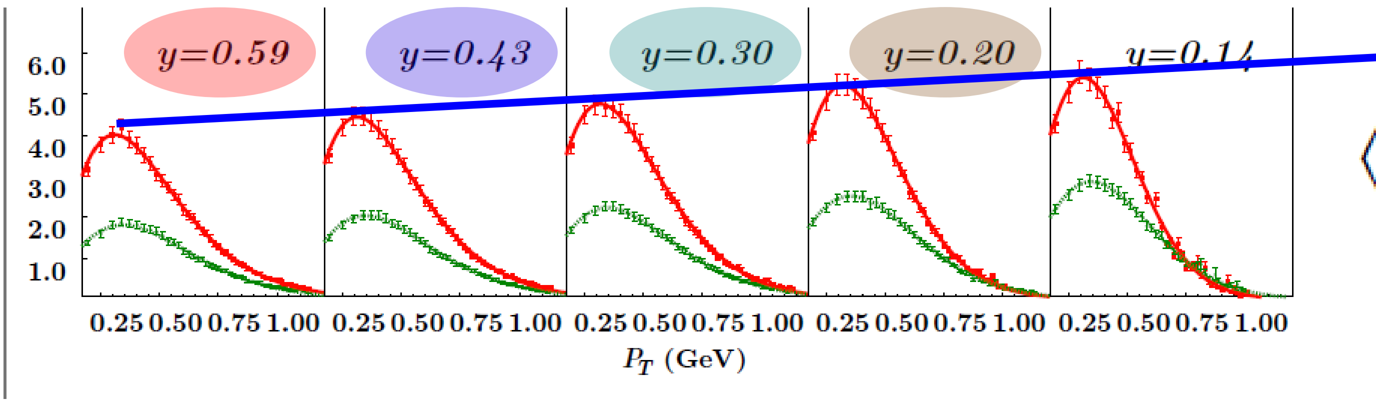
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$ $N_y = A + B\,y$	3.42	5385	3.25	3.60	$\langle k_\perp^2\rangle = 0.60 \pm 0.14$ GeV^2 $\langle p_\perp^2\rangle = 0.20 \pm 0.02$ GeV^2 $A = 1.06 \pm 0.06$ $B = -0.43 \pm 0.14$

In the Gaussian model.

$$F_{UU} = \sum_q \int d^2\boldsymbol{k}_\perp\, f_q(x,k_\perp) D_q(z,p_\perp) \quad \propto \quad \frac{1}{\pi\langle P_T^2\rangle} e^{-P_T^2/\langle P_T^2\rangle}$$



$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle.$$

Extraction from COMPASS data.

Cuts	χ^2_{dof}	n. points	$[\chi^2_{\text{dof}}]^{h^+}$	$[\chi^2_{\text{dof}}]^{h^-}$	Parameters
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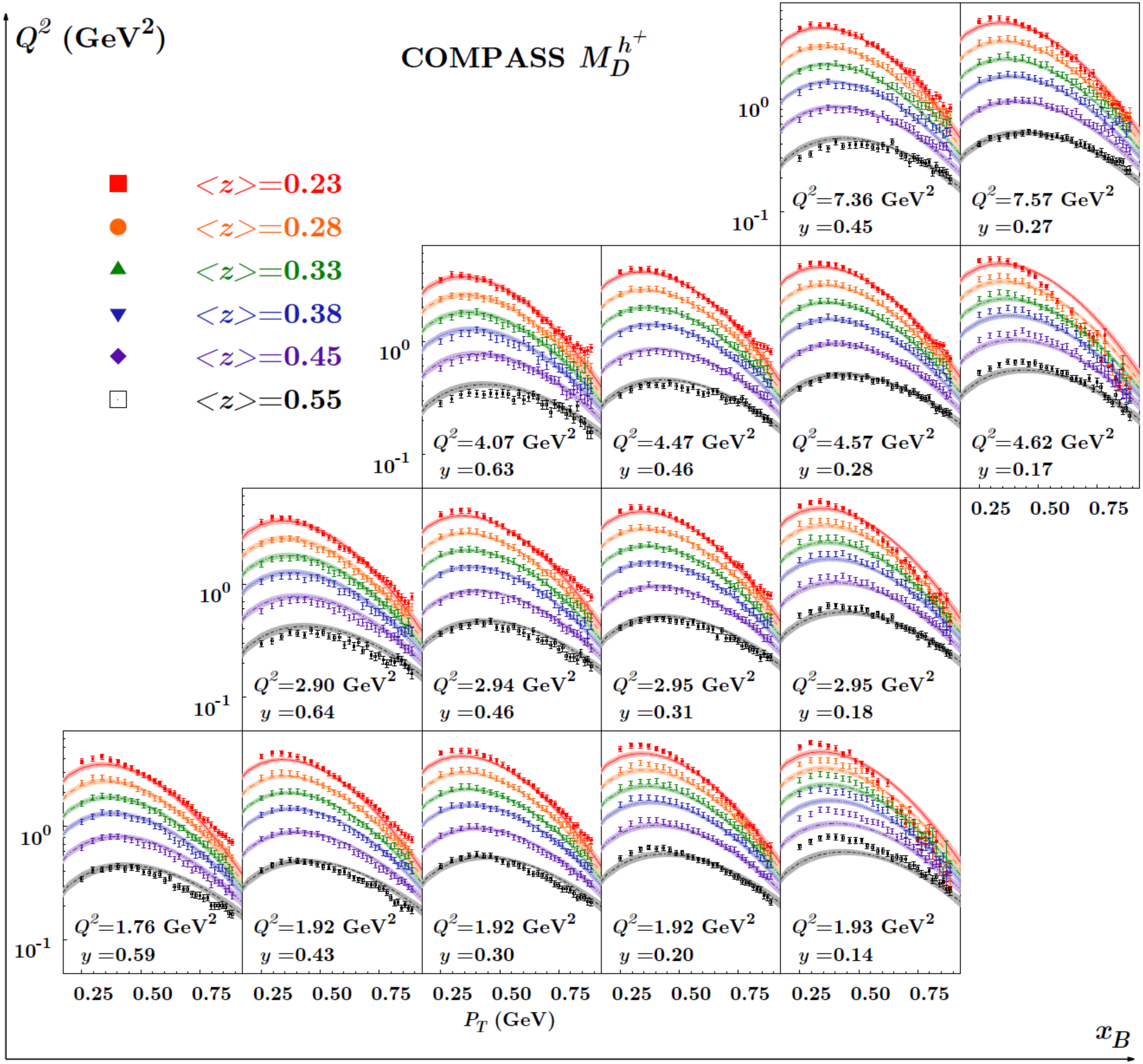
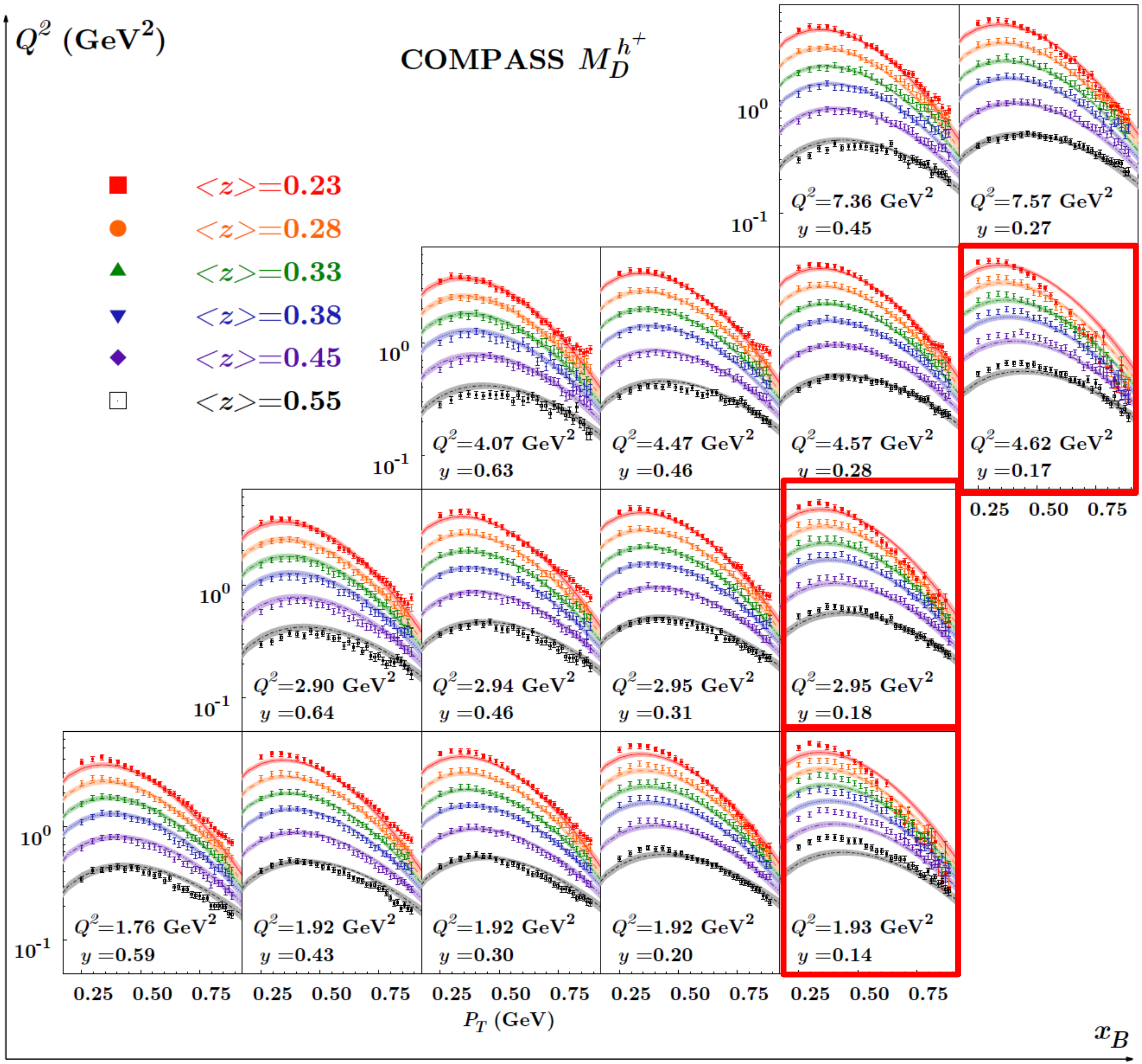
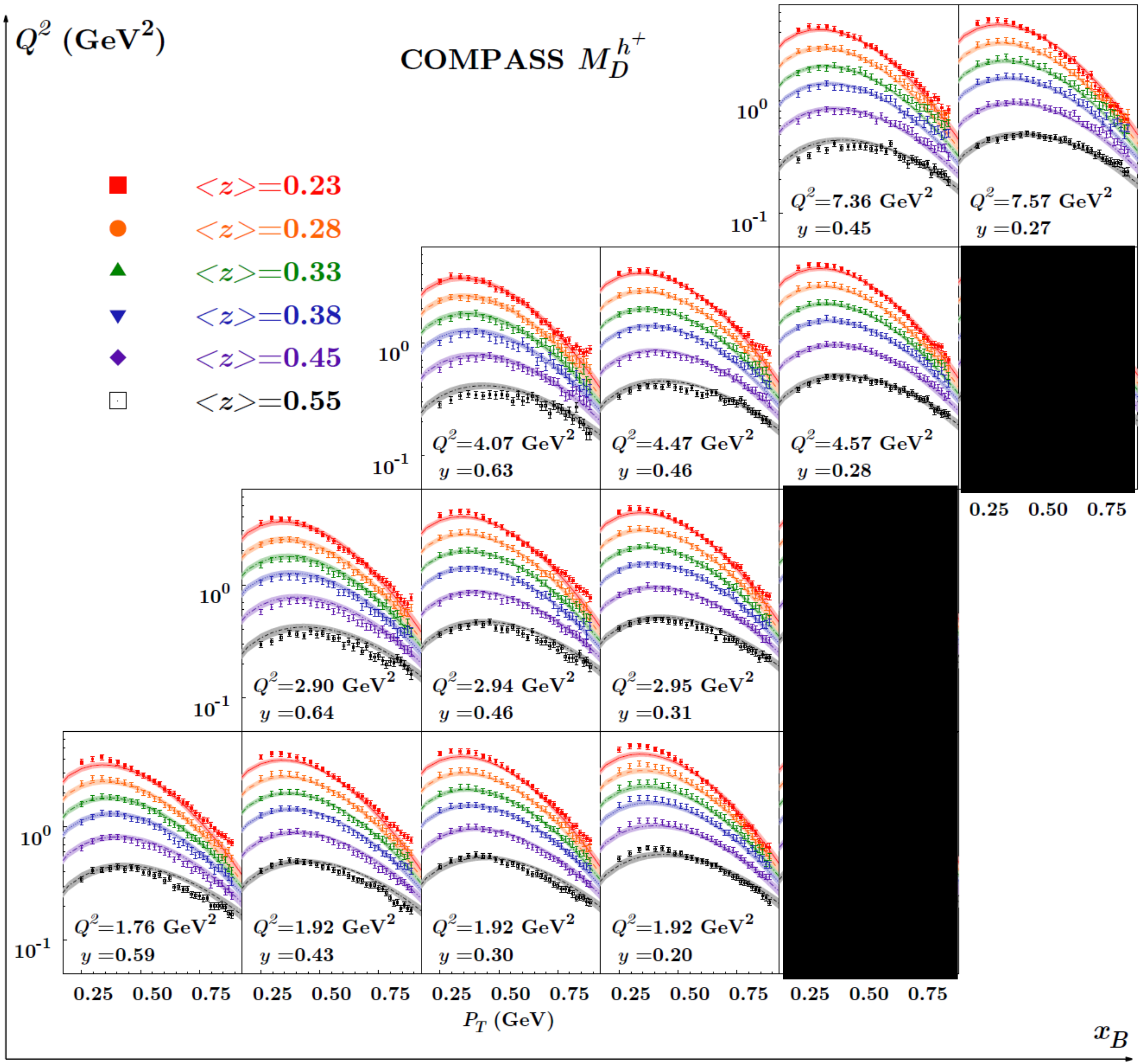
Extraction from COMPASS data.



Extraction from COMPASS data.



Extraction from COMPASS data.



OUTLINE

- Multiplicities HERMES and COMPASS.
- Extracting information from Multiplicites.
- **Other observables**
- Conclusions

Multiplicities → **Azimuthal Asymmetries**

(Both COMPASS and HERMES available.)

Azimuthal Asymmetries

$$F_{UU}^{\cos\phi_h}|_{Cahn} = -2\sum_q \int d^2\boldsymbol{k}_\perp \, \frac{(\boldsymbol{k}_\perp \cdot \boldsymbol{h})}{Q} f_q(x, k_\perp) D_q(z, p_\perp)$$

$$F_{UU}^{\cos\phi_h}|_{BM} = \sum_q \int d^2\boldsymbol{k}_\perp \, \frac{k_\perp}{Q} \frac{\Delta^N f_{q^\uparrow/p}(x, k_\perp) \Delta^N D_{h/q^\uparrow}(z, p_\perp)}{p_\perp} [P_T - z_h(\boldsymbol{k}_\perp \cdot \boldsymbol{h})]$$

$$F_{UU}^{\cos 2\phi_h}|_{Cahn} = 2\sum_q \int d^2\boldsymbol{k}_\perp \, \frac{2(\boldsymbol{k}_\perp \cdot \boldsymbol{h})^2 - k_\perp^2}{Q^2} f_q(x, k_\perp) D_q(z, p_\perp)$$

$$F_{UU}^{\cos 2\phi_h}|_{BM} = \sum_q \int d^2\boldsymbol{k}_\perp \, \frac{-\Delta^N f_{q^\uparrow/p}(x, k_\perp) \Delta^N D_{h/q^\uparrow}(z, p_\perp)}{2k_\perp p_\perp} \left\{P_T(\boldsymbol{k}_\perp \cdot \boldsymbol{h}) + z_h \left[k_\perp^2 - 2(\boldsymbol{k}_\perp \cdot \boldsymbol{h})^2\right]\right\}$$

Must be careful when interpreting

Parameters!!!

Multiplicities → $$\langle P_\perp^2 \rangle = \langle p_\perp^2 \rangle + \langle k_\perp^2 \rangle \; z^2$$

Azimuthal Asymmetries → $\langle p_\perp^2 \rangle \quad \langle k_\perp^2 \rangle$

$$\langle p_\perp^2 \rangle = A \quad \Longleftrightarrow \quad \langle P_\perp^2 \rangle = A + \langle k_\perp^2 \rangle \; z^2$$

Multiplicities $\Rightarrow$ $\langle P_\perp^2 \rangle = \langle p_\perp^2 \rangle + \langle k_\perp^2 \rangle \; z^2$

Azimuthal Asymmetries $\Rightarrow$ $\langle p_\perp^2 \rangle \quad \langle k_\perp^2 \rangle$

$$\langle p_\perp^2 \rangle = A \quad \Longleftrightarrow \quad \langle P_\perp^2 \rangle = A + \langle k_\perp^2 \rangle \; z^2$$

$$\langle p_\perp^2 \rangle = A + B \; z^2 \quad \Longleftrightarrow \quad \langle P_\perp^2 \rangle = A + (\langle k_\perp^2 \rangle + B) \; z^2$$

Multiplicities $\Rightarrow$ $\langle P_\perp^2 \rangle = \langle p_\perp^2 \rangle + \langle k_\perp^2 \rangle \; z^2$

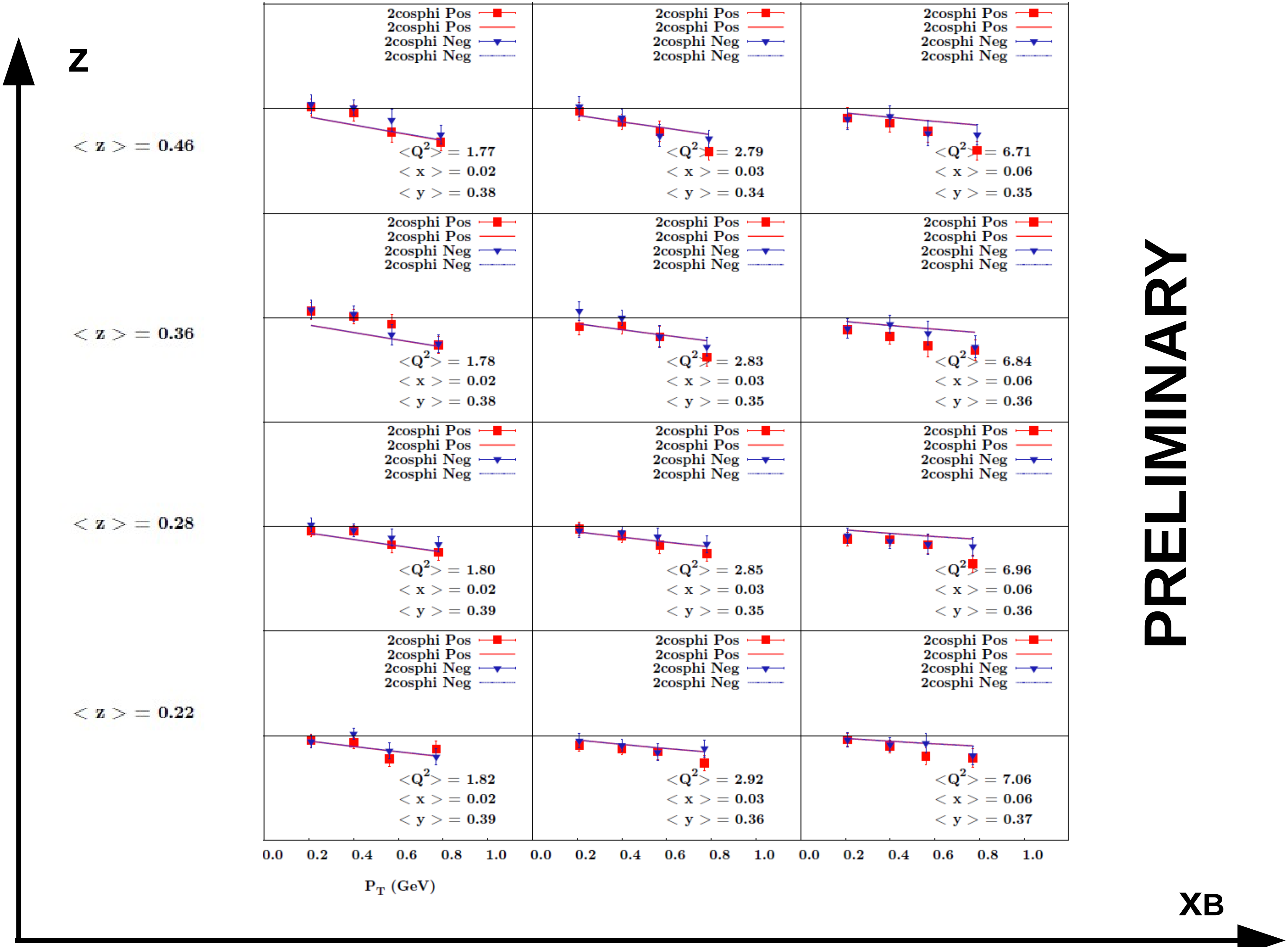
Azimuthal Asymmetries $\Rightarrow$ $\langle p_\perp^2 \rangle \quad \langle k_\perp^2 \rangle$

A fit on Multiplicities + cos<φ> asymmetry using only Cahn effect

$$\langle p_\perp^2 \rangle = A + B\ z^2 \quad \Longleftrightarrow \quad \langle P_\perp^2 \rangle = A + (\langle k_\perp^2 \rangle + B)\ z^2$$

Multiplicities ⟹ $\langle P_\perp^2 \rangle = \langle p_\perp^2 \rangle + \langle k_\perp^2 \rangle\ z^2$

Azimuthal Asymmetries ⟹ $\langle p_\perp^2 \rangle \quad \langle k_\perp^2 \rangle$



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< z > = 0.28
< z > = 0.22
2cosphi Pos
2cosphi Pos
2cosphi Neg
2cosphi Neg
<Q²> = 1.77
< x > = 0.02
< y > = 0.38
<Q²> = 2.79
< x > = 0.03
< y > = 0.34
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< x > = 0.02
< y > = 0.38
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< x > = 0.03
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<Q²> = 1.82
< x > = 0.02
< y > = 0.39
<Q²> = 2.92
< x > = 0.03
< y > = 0.36
<Q²> = 7.06
< x > = 0.06
< y > = 0.37
0.0 0.2 0.4 0.6 0.8 1.0
P_T (GeV)
PRELIMINARY
XB

OUTLINE

- Multiplicities HERMES and COMPASS.
- Extracting information from Multiplicites.
- Other observables
- **Conclusions**

Conclusions

- We conducted an analysis of both COMPASS and HERMES Multiplicities.
- Very simple approach works well for a portion of the (MULTIDIMENSIONAL) data.
- Different results (widths) for COMPASS and HERMES.
- HERMES : no much room for Q2-dependence.
- COMPASS: surprising and NOT subtle (seemingly) y-dependence
  In the normalization.
- This analysis serves as a step 1 to understand the information in these multidimensional sets.
- One MUST look at Azimuthal asymmetries. (careful with parameter interpretation.)

Grazie Mille.

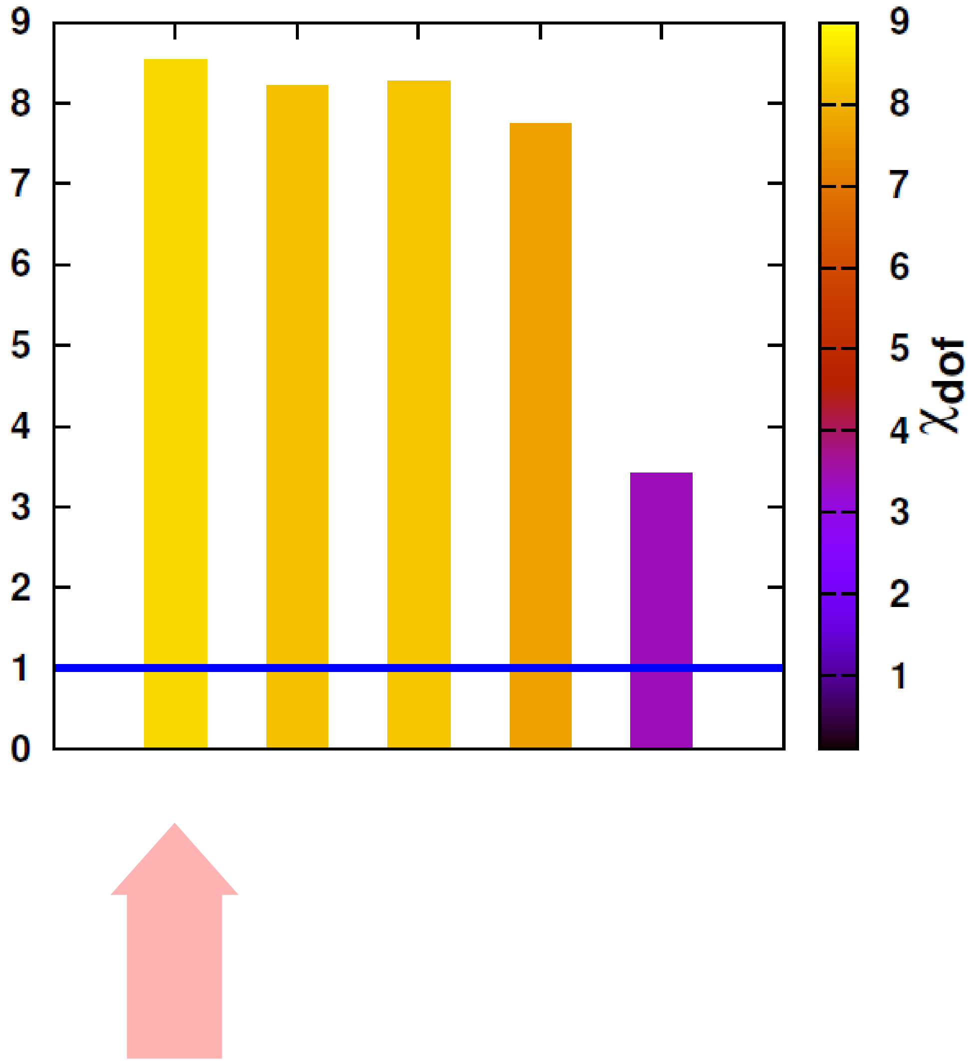
Grazie Mille.

Grazie Mille.

Extraction from COMPASS data.

$$\langle k_\perp^2 \rangle = g_1$$

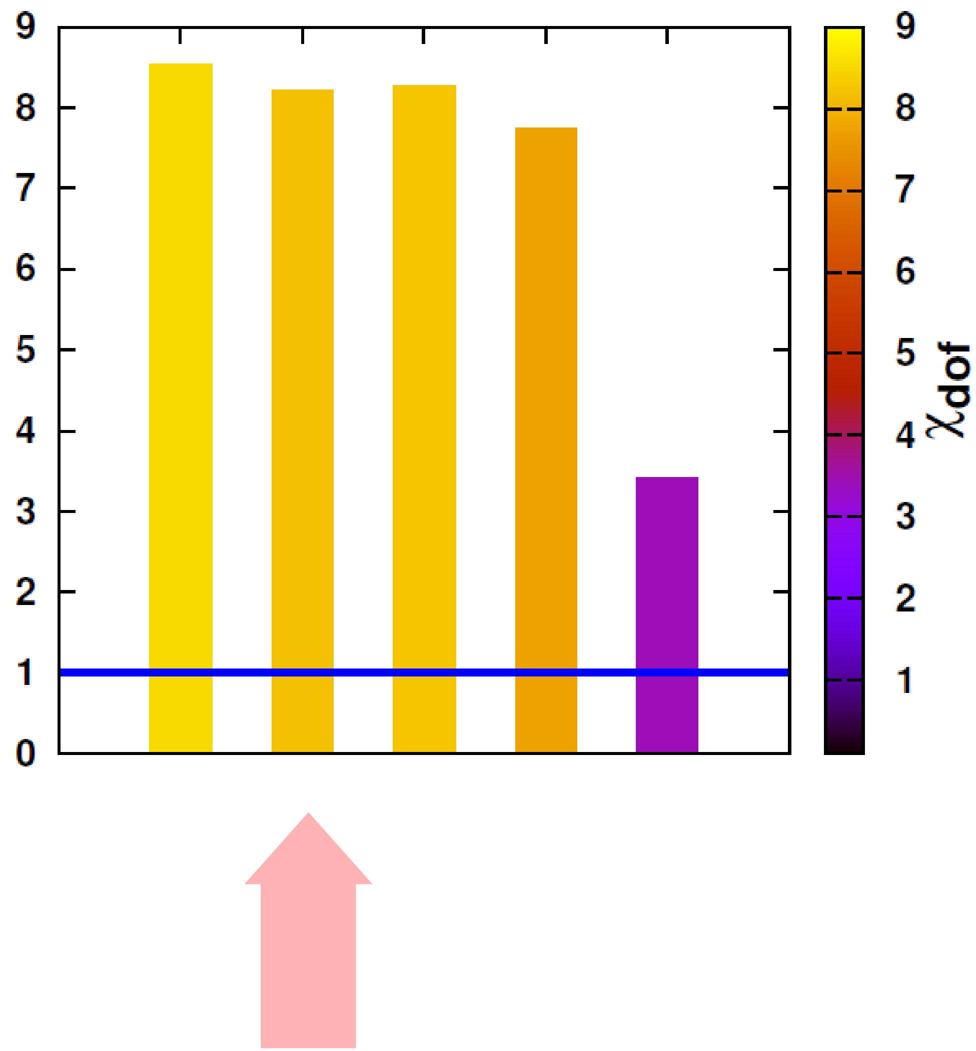
$$\langle p_\perp^2 \rangle = g_1'$$



Extraction from COMPASS data.

$$\langle k_\perp^2 \rangle = g_1 + g_2 \ln\left(Q^2/Q_0^2\right) + g_3 \ln\left(10ex\right)$$

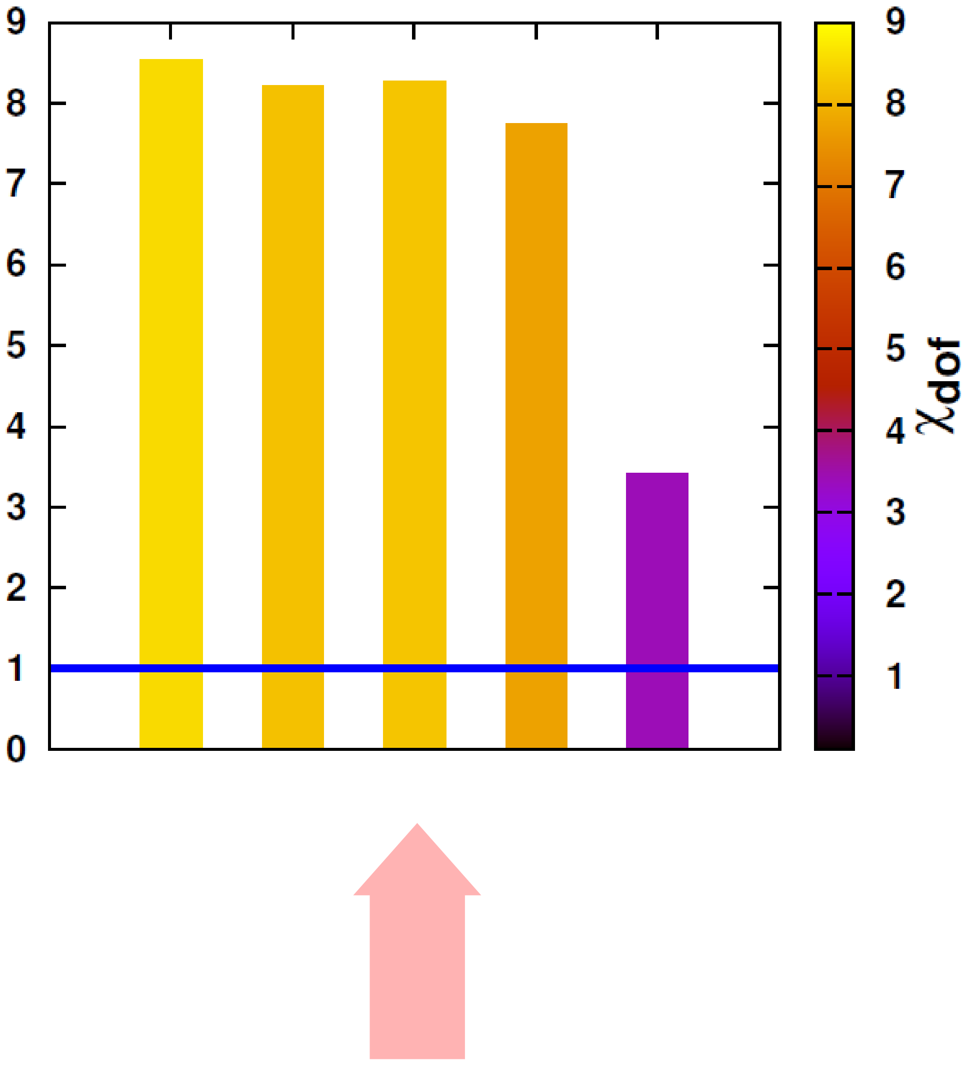
$$\langle p_\perp^2 \rangle = g_1' + z^2 g_2' \ln\left(Q^2/Q_0^2\right)$$



Extraction from COMPASS data.

$$\langle k_\perp^2 \rangle = a_1 + a_2 \ln(10y)$$

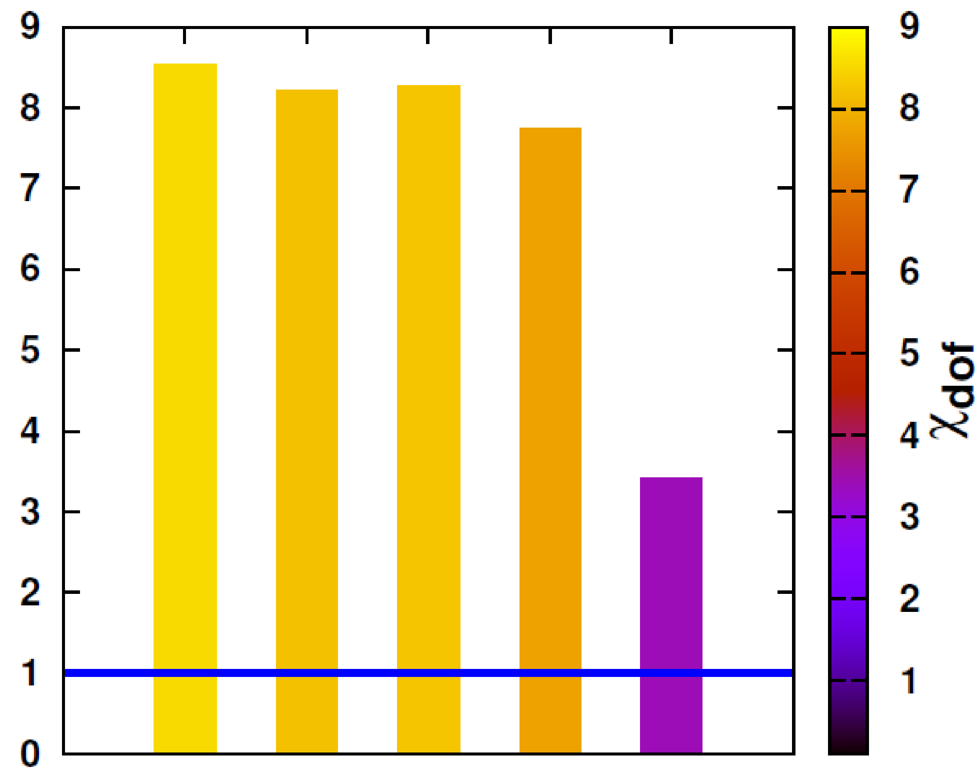
$$\langle p_\perp^2 \rangle = a_1' + a_2' \ln(10y)$$



Extraction from COMPASS data.

$$\langle k_\perp^2 \rangle = a_1 + a_2 \ln(10y)$$

$$\langle p_\perp^2 \rangle = a_1' + a_2' \ln(10y) + a_3' \sqrt{y}$$

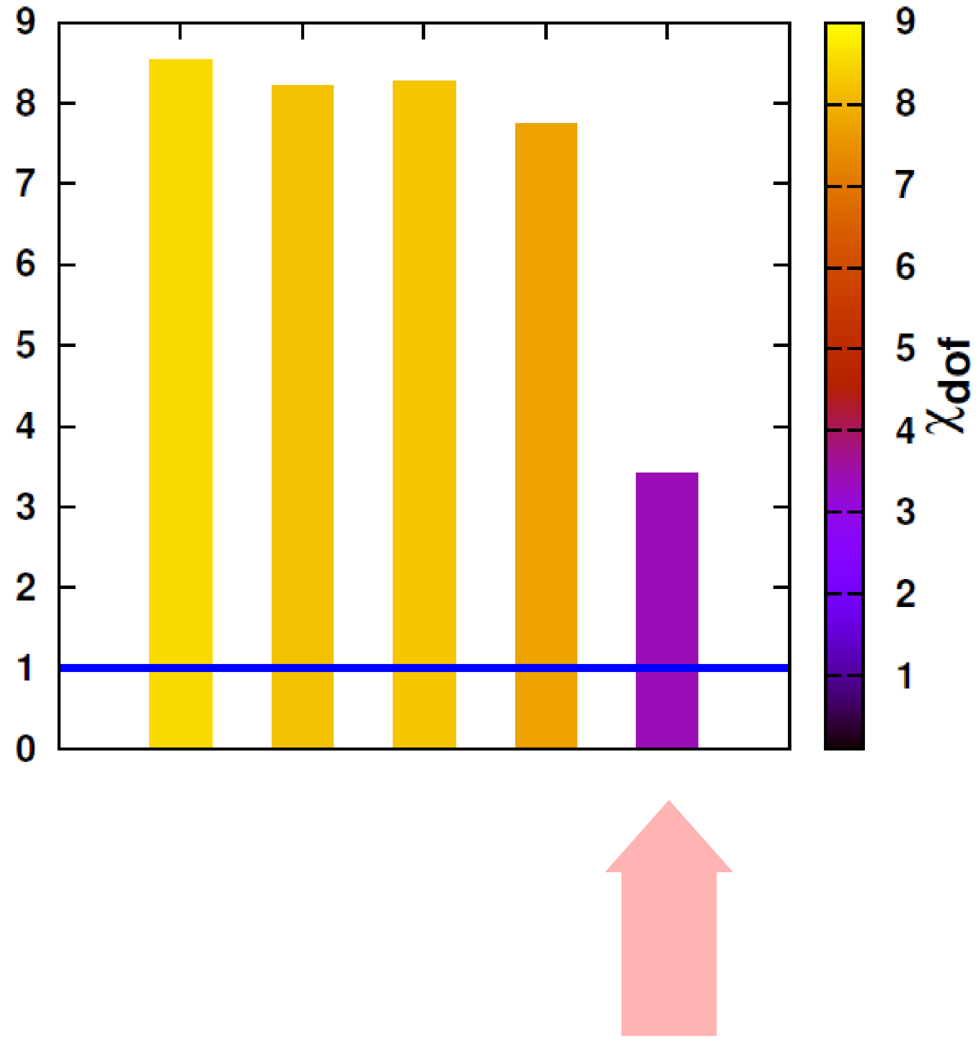


Extraction from COMPASS data.

$$\langle k_\perp^2 \rangle = g_1$$

$$\langle p_\perp^2 \rangle = g_1'$$

$$N = A + By$$



COMPASS		
	$\chi^2_{d.o.f}$	
	$N = 1.0$	$N = A + By$
$\langle k_\perp^2 \rangle = g_1$ $\langle p_\perp^2 \rangle = g_1'$	8.54	3.42 $A = 1.06 \quad B = -0.43$
$\langle k_\perp^2 \rangle = g_1 + g_2 \ln(Q^2/Q_0^2) + g_3 \ln(10ex)$ $\langle p_\perp^2 \rangle = g_1' + z^2 g_2' \ln(Q^2/Q_0^2)$	8.21	2.74 $A = 1.10 \quad B = -0.53$
$\langle k_\perp^2 \rangle = a_1 + a_2 \ln(10y)$ $\langle p_\perp^2 \rangle = a_1' + a_2' \ln(10y)$	8.27	2.00 $A = 1.13 \quad B = -0.62$
$\langle k_\perp^2 \rangle = a_1 + a_2 \ln(10y)$ $\langle p_\perp^2 \rangle = a_1' + a_2' \ln(10y) + a_3' \sqrt{y}$	7.75	1.81 $A = 1.12 \quad B = -0.59$

Extraction from HERMES data.

π only, simplest model

$\langle k_\perp^2 \rangle = 0.57 \pm 0.08\ \mathrm{GeV}^2$

$\langle p_\perp^2 \rangle = 0.12 \pm 0.01\ \mathrm{GeV}^2$

$\chi^2_{pt} = 1.69$

π only, z dependence

$$\langle p_\perp^2 \rangle \rightarrow A\,(1-z)^B\, z^C$$

$\langle k_\perp^2 \rangle = 0.48 \pm 0.54\ \mathrm{GeV}^2$

$A = 0.21 \pm 0.60\ \mathrm{GeV}^2$

$B = 0.34 \pm 6.42$

$C = 0.27 \pm 0.73$

$\chi^2_{pt} = 1.63$

Comparing extractions

Extraction from EMC data (2005)

$$\langle k_\perp^2 \rangle = 0.25 \ \text{GeV}^2 \qquad \langle p_\perp^2 \rangle = 0.20 \ \text{GeV}^2$$

Extraction from HERMES data (2013)

$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \ \text{GeV}^2 , \qquad \langle p_\perp^2 \rangle = 0.124 \pm 0.008 \ \text{GeV}^2$$

Extraction from COMPASS data (2013)

$$\langle k_\perp^2 \rangle = 0.61 \pm 0.20 \ \text{GeV}^2 \qquad \langle p_\perp^2 \rangle = 0.19 \pm 0.02 \ \text{GeV}^2$$

In order to compare, one needs to take into account correlations between parameters.

$$\sigma \propto \frac{1}{\pi \langle P_T^2 \rangle} e^{-P_T^2/\langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle .$$

z dependence?

Other kinematical dependences.

π only, simplest model

$\langle k_\perp^2 \rangle = 0.57 \pm 0.08$ GeV^2

$\langle p_\perp^2 \rangle = 0.12 \pm 0.01$ GeV^2

$\chi^2_{pt} = 1.69$

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$A = 0.21 \pm 0.60$ GeV^2

$B = 0.34 \pm 6.42$

$C = 0.27 \pm 0.73$

$\chi^2_{pt} = 1.63$

Flavor Dependence. HERMES.

#pts = 497 chi2pt = 1.60 chi2 = 794.89

#optimal parameters: YES

#TMDPDF version 0 : k2avg = a

#TMDFF version 0 : pt2avg = A

#name	free	val	err	lim	min	max
a	1	5.91e-01	3.79e-02	1	0.00e+00	1.00e+00
b	0	0.00e+00	0.00e+00	0	0.00e+00	1.00e+00
c	1	1.16e-01	4.92e-03	1	0.00e+00	1.00e+00
A	1	1.36e-01	6.35e-03	1	0.00e+00	1.00e+00

Flavor Dependence. COMPASS.

#pts = 5385 chi2pt = 3.42 chi2 = 18436.98

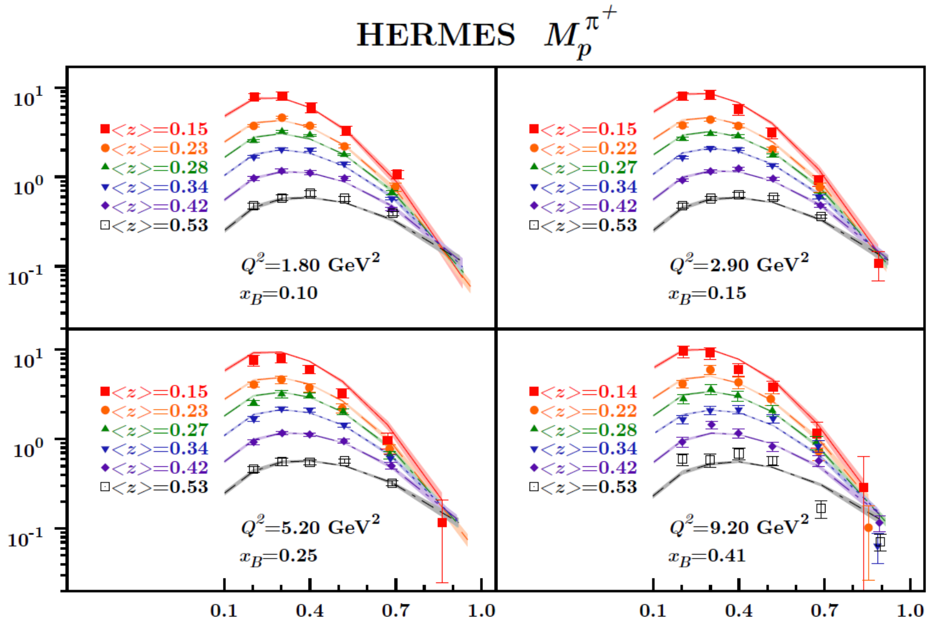
#optimal parameters: YES

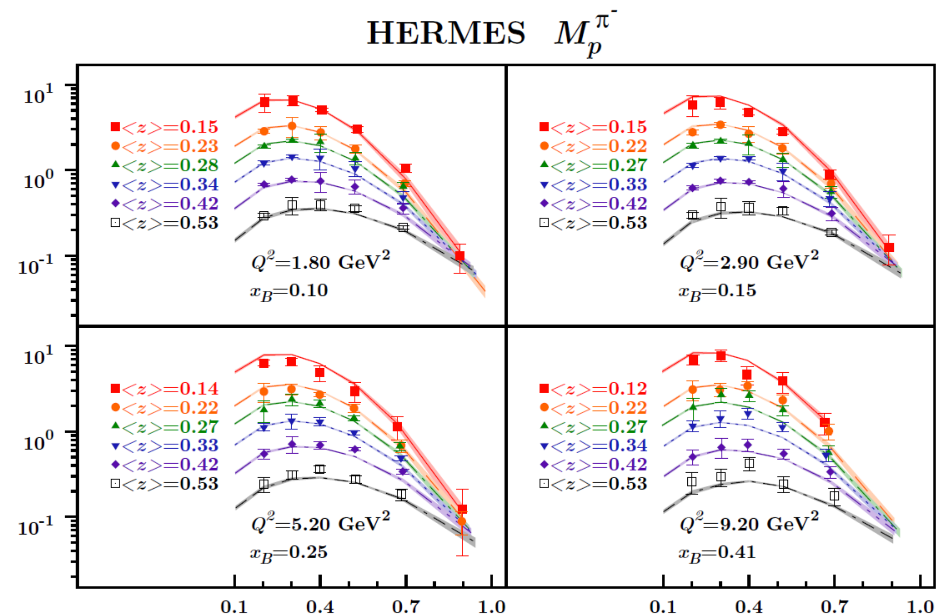
#TMDPDF version 0 : k2avg = a

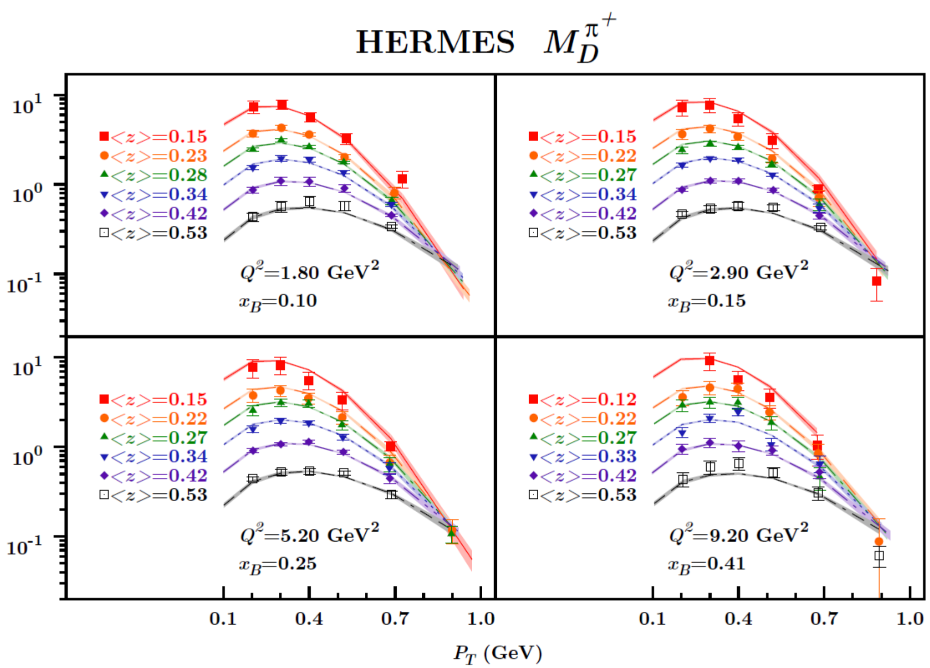
#TMDFF version 0 : pt2avg = A

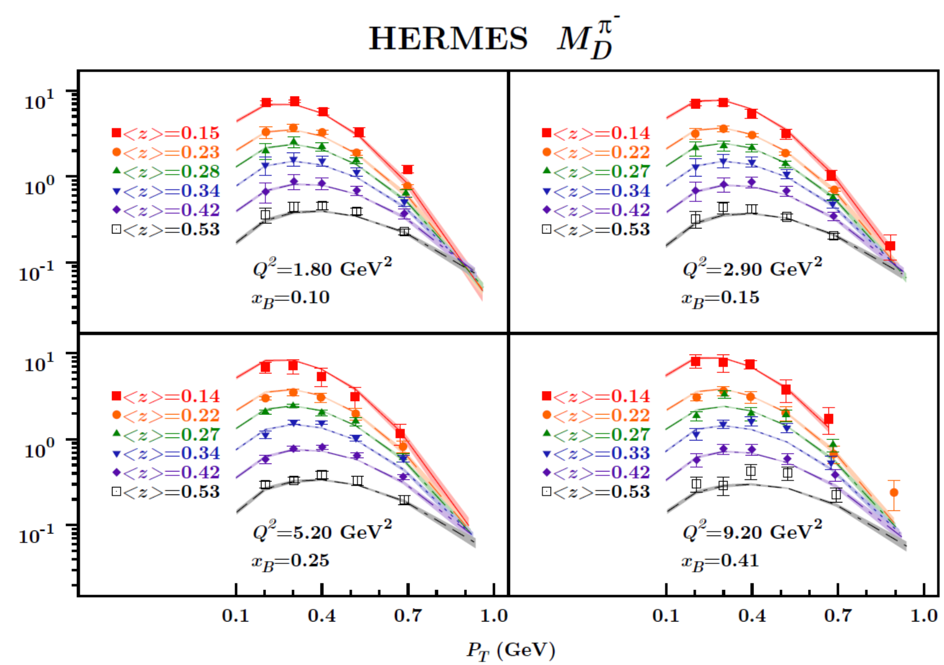
#name	free	val	err	lim	min	max
a	1	6.04e-01	1.68e-02	1	0.00e+00	1.00e+00
b	0	0.00e+00	0.00e+00	0	0.00e+00	1.00e+02
c	1	1.98e-01	4.31e-03	1	0.00e+00	1.00e+00
A	1	2.02e-01	5.40e-03	1	0.00e+00	1.00e+00

Extraction from HERMES data.

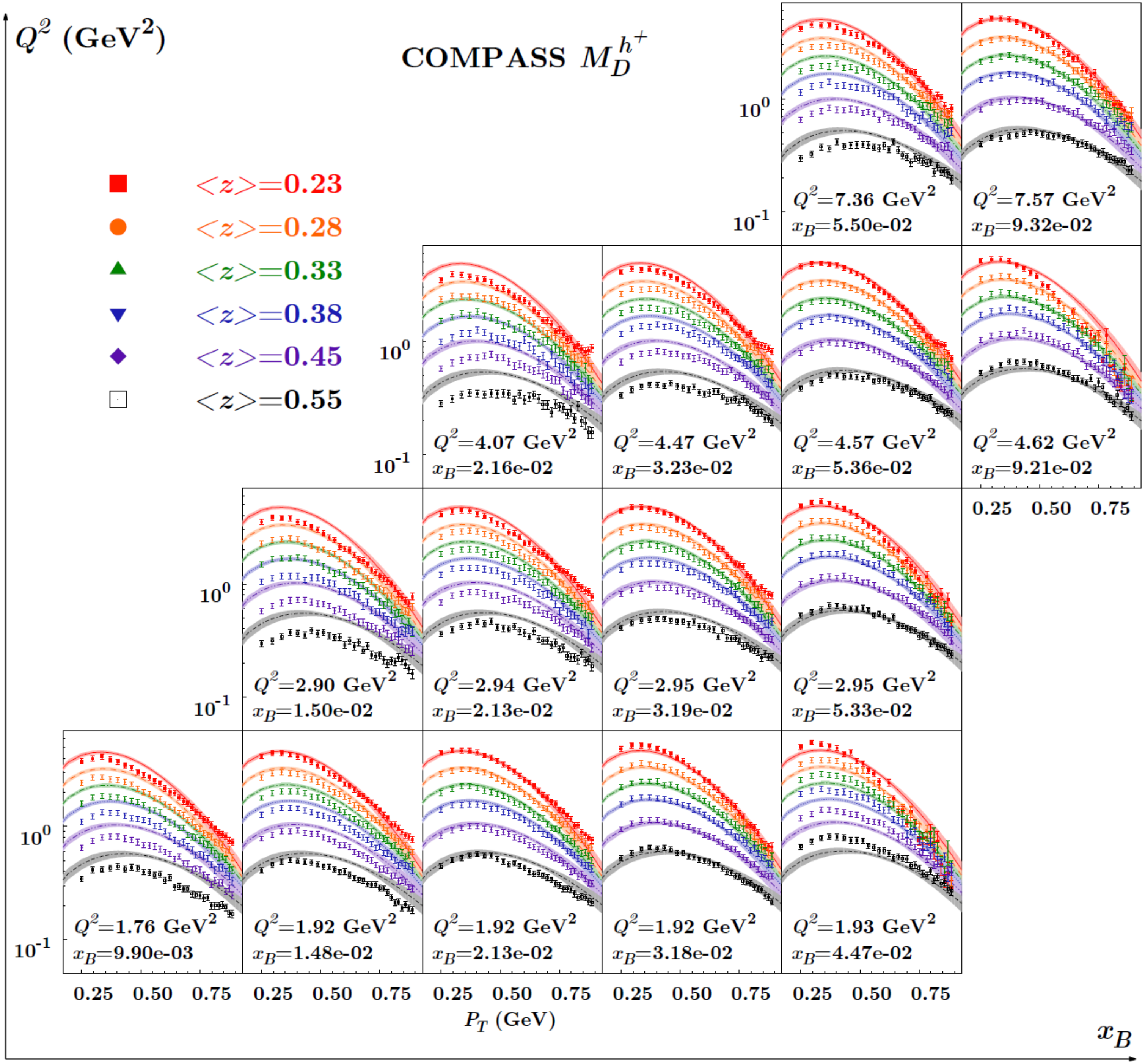




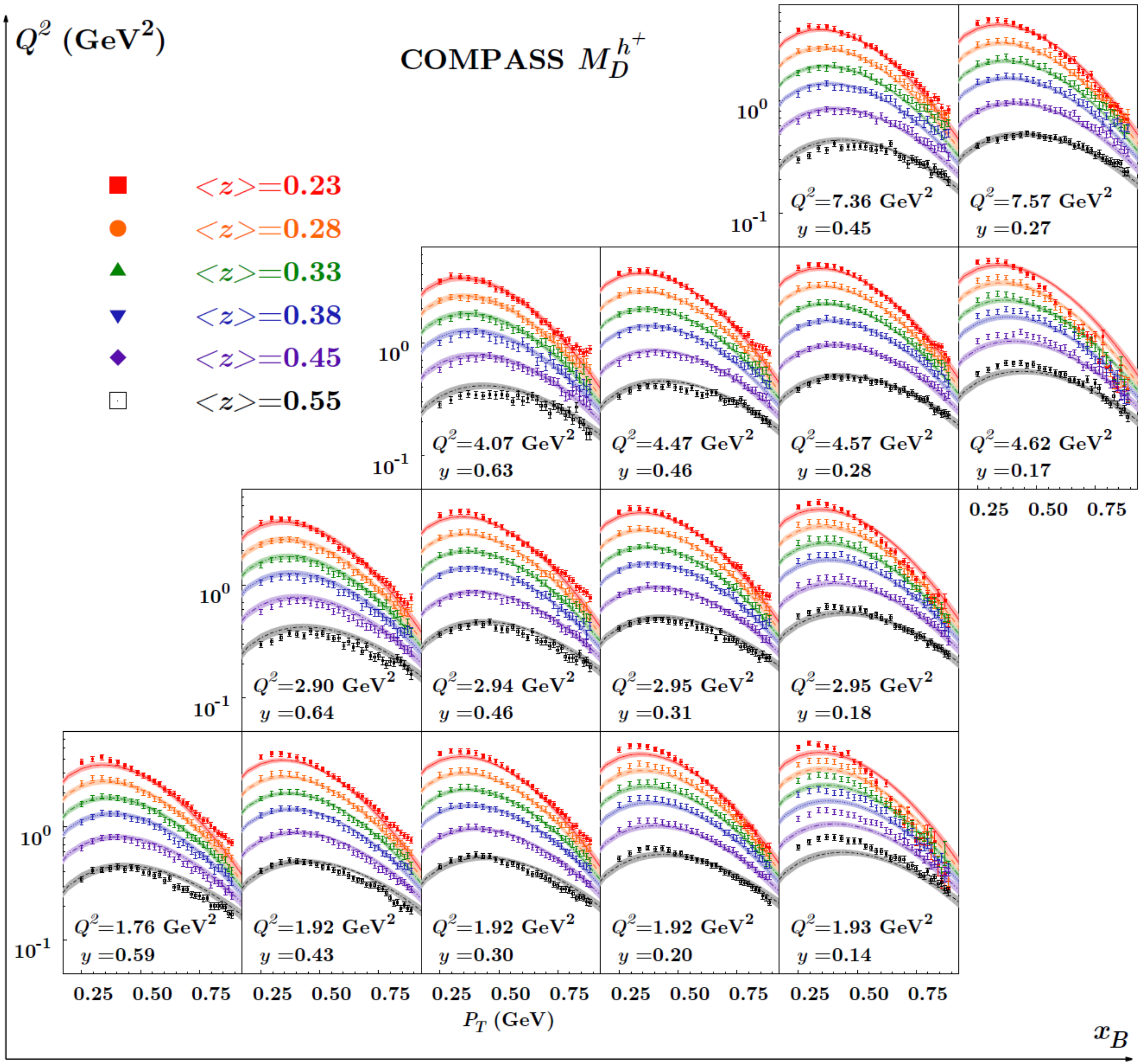




Extraction from COMPASS data.



Extraction from COMPASS data.



Jlab SIDIS data (2012).