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Natural conformal extensions of the Standard Model

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Based on: arxiv 1310.0957 [hep-ph]
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Firenze- March 5, 2014

Outline

- Degrees of (un)naturality of theories with fundamental scalars
- Perturbative natural conformality
- Examples and predictions:
  Standard Model + scalars
- Conclusions

Degrees of (un)naturality

Recall the definitions of the renormalized perturbation theory

Unrenormalized (bare parameters)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_B)^2 - \frac{1}{2} m_0^2 \phi_B^2 - \frac{\lambda_0}{4!}\phi_B^4$$

Renormalized (measurable parameters)

Define: $\phi_B \equiv \sqrt{Z}\phi_r \quad \delta_Z \equiv Z - 1 \quad m^2 \equiv m_0^2 Z - \delta_m \quad \delta_\lambda \equiv \lambda_0 Z^2 - \lambda$

$$\Rightarrow \quad \mathcal{L} = \frac{1}{2}(\partial_\mu \phi_r)^2 - \frac{1}{2} m^2 \phi_r^2 - \frac{\lambda}{4!}\phi_r^4 \underbrace{+ \frac{\delta_Z}{2}(\partial_\mu \phi_r)^2 - \frac{\delta_m}{2}\phi_r^2 - \frac{\delta_\lambda}{4!}\phi_r^4}_{\text{counterterms}}$$

The leading divergencies: $Z = 1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2} + \ldots \quad \delta_m = f_2(\lambda, g_i)\Lambda^2 + \ldots$

↑ In a cutoff scheme

$$m^2 = m_0^2\left(1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2}\right) - f_2(\lambda, g_i)\Lambda^2$$

Degrees of (un)naturality

The only quadratically divergent parameter of the theory:

$$m^2 = m_0^2\left(1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2}\right) - f_2(\lambda, g_i)\Lambda^2$$

- SM tuning = no predictions for the BSM physics
- Tuning via "classical conformality": $m_0 = 0, \quad \Lambda$ is dropped
- Delayed naturality (Veltman conditions): $f_2 = 0$ in P.T.
- Perturbative natural conformality (PNC) =
  delayed naturality + "classical conformality"
- Natural theories : SUSY , Technicolor

Perturbative natural conformality

delayed naturality* + "classical conformality"

- Veltman conditions** are imposed order-by-order in P.T. in every scalar field direction of the potential
- This postpones the energy scale where unnaturality re-emerges

- Bare mass of the scalar is set to zero* (conformal invariance at tree-level)
- EW symmetry is broken radiatively via Coleman-Weinberg mechanism

*Delayed naturality does not require conformality

** Veltman '81

* Theory renormalized in e.g. dimensional regularization where explicit cutoff does not appear

PNC conditions 1: CW mechanism

Idea : In a massless theory, radiative corrections could lead to a SSB

Consider a scalar sector of a generic renormalized gauge theory:

$$V_0(\phi_i) = \frac{\lambda_{ijkl}}{24}\phi_i\phi_j\phi_k\phi_l + \text{fermionic } (y) \text{ and vectorial contributions}(g) \; + c.t.$$

Consistent P.T. requires: $\lambda_{ijkl} \sim g^2 \sim y^2 \ll 1$

⇨ Tree-level potential dominates unless it's zero (flat direction) or very small

Solve: $\min(\lambda_{ijkl}u_iu_ju_ku_l)\big|_{u_iu_i=1} = 0$

If a solution $u_i = n_i$ exists, then $\phi_i = n_i\phi$ is a flat direction and CW analysis can be performed

PNC conditions 1: CW mechanism

Now consider one-loop corrections to the effective potential:

$$V_1(\phi_c) = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\; Str\left[\,\ln(k^2 + M^2(\phi_c))\right] + c.t.$$

$$Str \equiv \sum_{scalars} - 2 \sum_{Weyl fermions} + 3 \sum_{vectors}$$

background field dependent mass matrix

For $M^2(\phi_c) \ll \Lambda^2$:

$$V_1(\phi_c) = \frac{1}{64\pi^2} Str\left[\Lambda^4\left(\ln\Lambda^2 - \frac{1}{2}\right) + 2M^2(\phi_c)\Lambda^2 + M^4(\phi_c)\left(\ln\frac{M^2(\phi_c)}{\Lambda^2} - \frac{1}{2}\right)\right] + c.t.$$

cosmological const

quadratic divergence

one-loop phi^4 correction

subtract away

can be tuned away by choice of c.t.
or
vanish for symmetry reasons (SUSY)

PNC conditions 2:Veltman Conditions

Even without symmetry, naturality can be "delayed"
by imposing (at some specific RG scale μ_0) :

$$\frac{1}{2}\frac{\partial^2 Str[M^2(\phi_i)]}{\partial\phi_i^2}\Bigg|_{\mu_0} = 0$$

for masses of the
non-Goldstone scalars

this leads to extra constraints on the couplings of the theory

➡

$$V(\phi_c) = V_1(\phi_c) = \frac{1}{64\pi^2} Str\left[M^4(\phi_c)\ln\frac{M^2(\phi_c)}{\mu_0^2}\right] \equiv A\phi_c^4 + B\phi_c^4 \ln\frac{\phi_c^2}{\mu_0^2}$$

In a classically scale-invariant theory: $M^2(\phi_c) \equiv W^2\phi_c^2$

$$A = \frac{1}{64\pi^2} StrW^4 \ln W^2 \ , \quad B = \frac{1}{64\pi^2} StrW^4$$

PNC conditions 2: CW mass

$$V(\phi_c) = A\phi_c^4 + B\phi_c^4 \ln \frac{\phi_c^2}{\mu_0^2}$$

The non-trivial stationary point of the potential:

$$\log \frac{\langle \phi_c^2 \rangle}{\mu_0^2} = -\frac{1}{2} - \frac{A}{B}$$

A and B both start at one-loop
thus P.T. is valid since log is small
This is why we needed flat
direction in the tree-level potential

⇒ $< \phi_c >= \mu_0 \times \mathcal{O}(1)$

Coleman-Weinberg one-loop mass:

$$m_{CW}^2 = 8B\langle \phi_c^2 \rangle$$

⇒ in practice we will only need to calculate B in a given theory

CW scalar as a dilaton

The scalar along the flat direction is a dilaton of the theory

need to show that our scalar state D is created from the vacuum by the spontaneously broken dilatation current which is related to the trace anomaly of the improved energy-momentum tensor

$$\partial_\mu D^\mu = \Theta^\mu_\mu = \sum_c^{g,y,\lambda} \beta(c) \frac{\partial \mathcal{L}}{\partial c}$$

$$\Theta^\mu_\mu = \frac{\beta(g)}{g}(F_{\mu\nu})^2 + \beta(y)\bar{q}Hq + \beta_{ijkl}\phi_i\phi_j\phi_k\phi_l$$

The dilaton is a pseudo-NG boson and its mass is defined via matrix element (analogy with the pion):

$$\langle 0 \mid \Theta^\mu_\mu \mid D\rangle_{x=0} = -f_D m_D^2 = -8B < \phi_c >^3$$

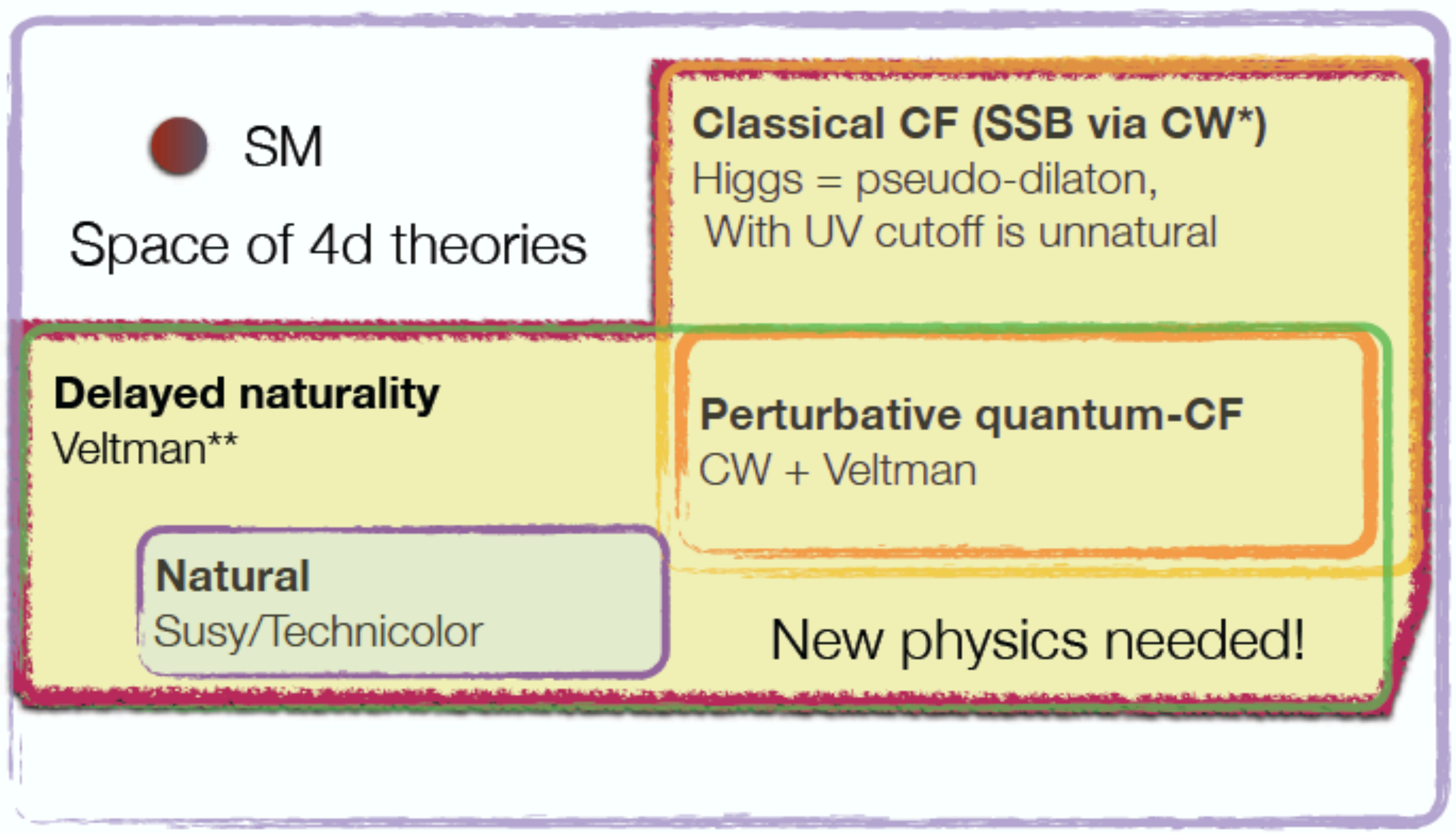
$$f_D = < \phi_c > \; , \quad \Theta^\mu_\mu = -\beta_i < \phi_c >^3 \phi + \cdots$$

Interested in the scalar sector only

need to show that

$$\boxed{8B = \beta_i}$$

Degrees of (un)naturality



SM

Space of 4d theories

Classical CF (SSB via CW*)
Higgs = pseudo-dilaton,
With UV cutoff is unnatural

Delayed naturality
Veltman**

Perturbative quantum-CF
CW + Veltman

Natural
Susy/Technicolor

New physics needed!

* CW = Coleman-Weinberg

**Perturbative cancellation of quadratic divergences

in the rest of the talk we concentrate on PNC...

PNC examples

PNC examples : Standard Model

One-loop potential and mass matrix read : $H = \frac{1}{\sqrt{2}}(\pi_2 + i\pi_1, v + h - i\pi_3)$

$$V_0^{SM} = \lambda\left(H^\dagger H\right)^2 - \frac{1}{2}\left(g^2 W_\mu^+ W^{-\mu} + \frac{g^2 + g'^2}{2} Z_\mu Z^\mu\right) H^\dagger H + y_t(\bar{t}_L, 0)\left(i\sigma^2 H^*\right) t_R + c.t.$$

$$\frac{M^2(h)}{h^2} = \text{diag}\left\{3\lambda,\ \overset{\text{Goldstones}}{\boxed{\lambda,\ \lambda,\ \lambda,}}\ \frac{1}{4}g^2,\ \frac{1}{4}g^2,\ \frac{1}{4}(g^2 + g'^2),\ \frac{1}{2}y_t^2,\ \frac{1}{2}y_t^2\right\}$$

Flat direction condition: $\lambda(\mu_0) = 0$

Veltman condition: $\frac{1}{2}\frac{\partial^2 Str[M^2(h)]}{\partial h^2}\bigg|_{\mu_0} = \cancel{6\lambda(\mu_0)} + \frac{9}{4}g^2(\mu_0) + \frac{3}{4}g'^2(\mu_0) - 6y_t^2(\mu_0) = 0$

➡ $4m_t^2 = m_Z^2 + 2m_W^2 \implies m_t \approx 73\ \text{GeV}$

wrong!

CW Higgs mass: $m_h^2 = \frac{3}{8\pi^2}\left[\underbrace{\frac{1}{16}(3g^4 + 2g^2g'^2 + g'^4) + 4\lambda^2 - y_t^4}_{\text{the quartic beta function}}\right]v^2 \implies m_h \approx 5\ \text{GeV}$

wrong!

PNC examples: Standard Model + scalars

Next simplest possibility: add to the SM a real scalar S

$$V_0 = V_0^{SM} + \lambda_{HS} H^\dagger H S^2 + \frac{\lambda_S}{4} S^4 + c.t.$$

Want a bounded potential: $\lambda \geq 0\ , \quad \lambda_S \geq 0\ , \quad \text{and if}\ \ \lambda_{HS} < 0 : \quad \lambda \lambda_S \geq \lambda_{HS}^2$

Veltman conditions:

Direction along the S-axis:

$$\frac{1}{2} \frac{\partial^2 Str[M^2(S)]}{\partial S^2} \bigg|_{\mu_0} = 3\lambda_S(\mu_0) + \boxed{4\lambda_{HS}(\mu_0)} = 0$$

$$\Rightarrow \quad \boxed{\lambda_{HS} < 0}$$

Direction along the H-axis:

$$\frac{1}{2} \frac{\partial^2 Str[M^2(h)]}{\partial h^2} \bigg|_{\mu_0} = 6\lambda(\mu_0) + \frac{9}{4} g^2(\mu_0) + \frac{3}{4} g'^2(\mu_0) - 6 y_t^2(\mu_0) + \boxed{\lambda_{HS}(\mu_0)} = 0$$

PNC examples: Standard Model + scalars

Now add Coleman-Weinberg analysis... first we find flat direction

Parametrize: $H = \frac{r}{\sqrt{2}} \begin{pmatrix} 0 \\ \cos\omega \end{pmatrix} \, , \quad S = r \sin\omega$

⇨ $$V_0 = \frac{r^4}{4} \left(\lambda \cos^4\omega + \lambda_S \sin^4\omega + 2\lambda_{HS} \sin^2\omega \cos^2\omega \right) + c.t.$$

The minima will be along the ω-direction with the unit vector: $n = (\cos <\omega>, \sin <\omega>)$

The results of minimization are:

$0 \leq \lambda < \min\{\lambda_S, \lambda_{HS}\} \ : \quad <\omega>= 0 \, ,$ Only Higgs gets a vev

$0 \leq \lambda_S < \min\{\lambda, \lambda_{HS}\} \ : \quad <\omega>= \frac{\pi}{2} \, ,$ Only “S” gets a vev

$-\sqrt{\lambda\lambda_S} \leq \lambda_{HS} < \min\{\lambda, \lambda_S\} \ : \quad \tan^2 <\omega>= \frac{\lambda - \lambda_{HS}}{\lambda_S - \lambda_{HS}}$ Both get a vev

PNC examples: Standard Model + scalars

$\lambda_{HS} < 0$ not satisfied

No EWSB

$$0 \leq \lambda < \min\{\lambda_S, \lambda_{HS}\} \; : \quad <\omega> = 0 \, ,$$

Only Higgs gets a vev

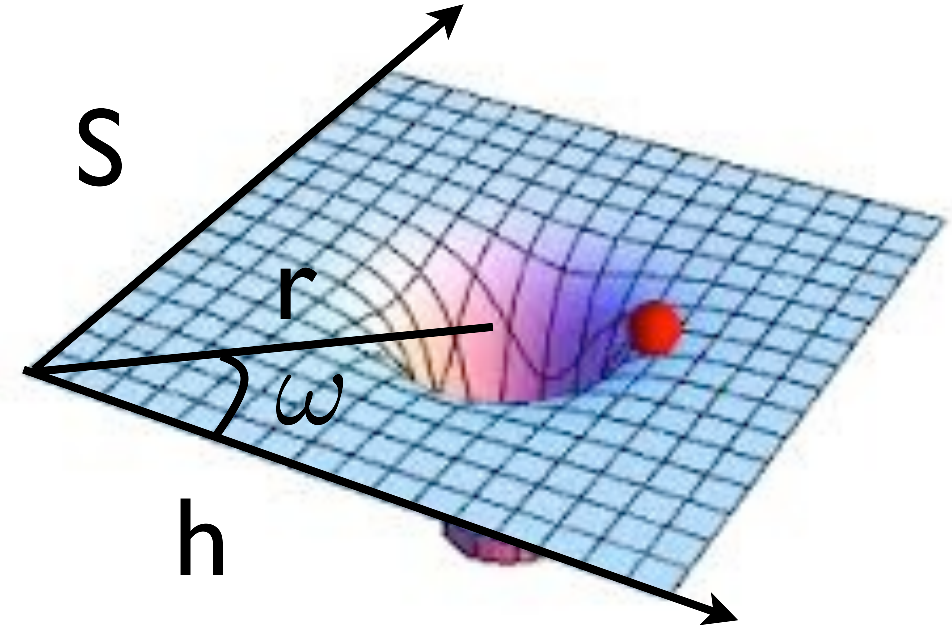
$$0 \leq \lambda_S < \min\{\lambda, \lambda_{HS}\} \; : \quad <\omega> = \frac{\pi}{2} \, ,$$

Only "S" gets a vev

$$-\sqrt{\lambda\lambda_S} \leq \lambda_{HS} < \min\{\lambda, \lambda_S\} \; : \quad \tan^2 <\omega> = \frac{\lambda - \lambda_{HS}}{\lambda_S - \lambda_{HS}}$$

Both get a vev

For all these reasons we are forced to the case # 3...



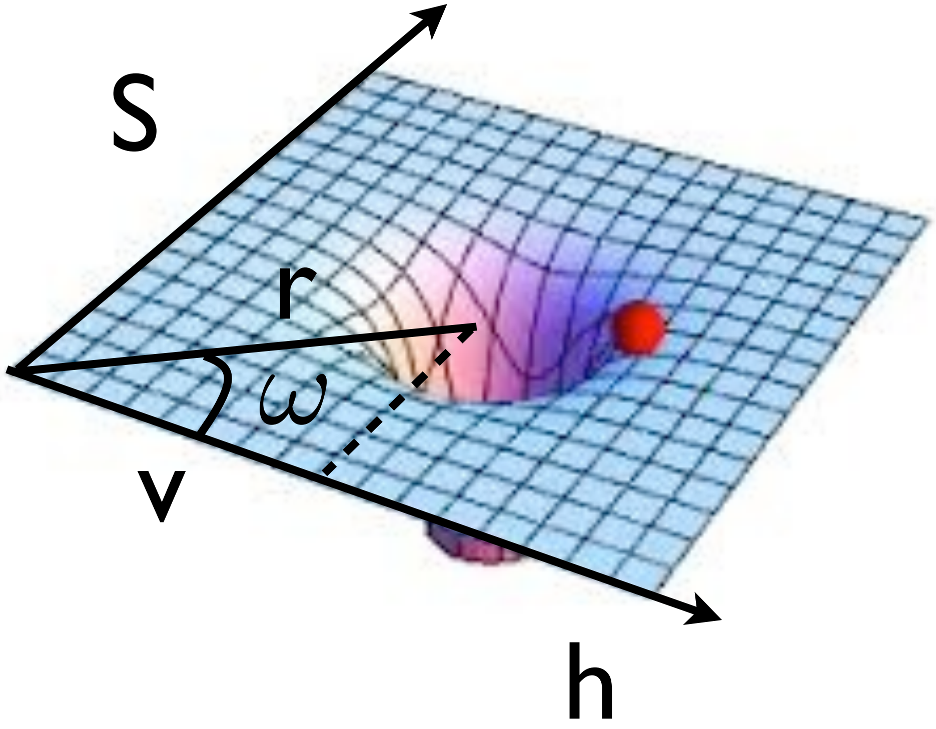
PNC examples: Standard Model + scalars

From CW analysis we get

<r> which we fix via:

$$\log \frac{\langle r^2 \rangle}{\mu_0^2} = -\frac{1}{2} - \frac{A}{B}$$

$$< r > \cos < \omega > = v \approx 246 \text{ GeV}$$



Rewriting $r \cos < \omega > = (v + h)$ and $r \sin < \omega > = v \tan < \omega > + s$

and introducing light and heavy fields

$$\phi = h \cos < \omega > + s \sin < \omega > \; ,$$

$$\Phi = s \cos < \omega > - h \sin < \omega >$$

we get tree-level masses: $m^2_{0,\phi} = 0 \; , \quad m^2_{0,\Phi} = 2(\lambda - \lambda_{HS}) v^2$

$\uparrow$ field along the flat direction!

PNC examples: Standard Model + scalars

One-loop CW (Higgs) mass reads:

$$m_{1,\phi}^2 = \frac{1}{8\pi^2}\frac{StrM(<r>)^4}{<r>^4}<r>^2 = \frac{\cos^2 <\omega>}{8\pi^2 v^2}[6m_W^4 + 3m_Z^4 + m_\Phi^4 - 12m_t^4]$$

$$= \cos^2 <\omega> \frac{v^2}{8\pi^2}\left[\frac{6}{16}g^4 + \frac{3}{16}(g^2 + {g'}^2)^2 + 4(\lambda - \lambda_{HS})^2 - \frac{12}{4}y_t^4\right]$$

After imposing flatness condition: $\lambda_{HS}^2 = \lambda\lambda_S$

+ Veltman conditions we obtain :

$$m_\phi \approx 95 \text{ GeV} \ , \quad m_\Phi \approx 541 \text{ GeV}$$

NLO corrections are important to consider

Summary: we had 3 conditions (2 Veltman + 1 flatness) which allowed to fix all three quartic couplings

PNC examples: Intriguing candidate

$$V_0 = V_0^{SM} + \lambda_{HS} H^\dagger H S^2 + \frac{\lambda_S}{4} S^4 + c.t.$$

Want a bounded potential: $\lambda \geq 0\,, \quad \lambda_S \geq 0\,, \quad \text{and if } \lambda_{HS} < 0 : \quad \lambda \lambda_S \geq \lambda_{HS}^2$

Veltman conditions:

Direction along the S-axis:

$$\frac{1}{2}\frac{\partial^2 Str[M^2(S)]}{\partial S^2}\Bigg|_{\mu_0} = 3\lambda_S(\mu_0) + \boxed{4\lambda_{HS}(\mu_0)} = 0$$

$\boxed{\lambda_{HS} < 0}$ did not allowed to study case I before

Let’s add some fermionic matter to allow $\boxed{\lambda_{HS} > 0}$:

Interesting DM candidates and/or neutrino masses

$$V_0 = V_0^{SM} + \lambda_{HS} H^\dagger H S^2 + \frac{\lambda_S}{4} S^4 + \boxed{y_\chi S(\chi\chi + \bar{\chi}\bar{\chi})} + c.t.$$

Direction along the S-axis:

$$\frac{1}{2}\frac{\partial^2 Str[M^2(S)]}{\partial S^2}\Bigg|_{\mu_0} = 3\lambda_S(\mu_0) + 4\lambda_{HS}(\mu_0) - \boxed{8y_\chi^2} = 0$$

PNC examples: Intriguing candidate

Solving Veltman conditions:

Now $\lambda_{HS} > 0$ because of $\chi-$field

$$\lambda_{HS}(\mu_0) = 6y_t^2(\mu_0) - \frac{9}{4}g^2(\mu_0) - \frac{3}{4}g'^2(\mu_0) \overset{\mu_0 \approx v}{\approx} 4.84\,,$$

$$\lambda_S(\mu_0) = \frac{8}{3}y_\chi^2(\mu_0) - \frac{4}{3}\lambda_{HS}(\mu_0) \overset{\mu_0 \approx v}{\approx} \frac{8}{3}y_\chi^2(\mu_0) - 6.45$$

One-loop CW (Higgs) mass reads:

$$m_h^2 = \frac{3}{8\pi^2}\left[\frac{1}{16}(3g^4 + 2g^2g'^2 + g'^4) - y_t^4 + \frac{\lambda_{HS}^2}{3}\right]v^2 \qquad \Longrightarrow \qquad \boxed{m_h \approx 126 \text{ GeV}}\,, \ ✓$$

$$m_S^2 = \lambda_{HS}v^2 \qquad \Longrightarrow \qquad m_S \approx 541 \text{ GeV}$$

Predictions: one real scalar with mass 541 GeV + extra fermionic matter

Comments: 1. Quartic coupling is quite large (though still perturbative) so NLO corrections are important to consider
2. The exact structure of the fermionic sector is not predicted (we only needed to satisfy Veltman condition)

Conclusions

- We classified degrees of naturality of various BSM approaches
- We introduced a new PNC class of theories
- We studied various "SM + scalars" examples and have shown that PNC theories are highly predictive
- Generic feature of PNC models is to predict some new states within the LHC reach
- The simplest PNC candidate lead to a correct Higgs mass prediction
- In PNC models Higgs self-coupling emerges radiatevely and differs from the SM prediction