An analytical approach to space charge distortions for Time Projection Chambers

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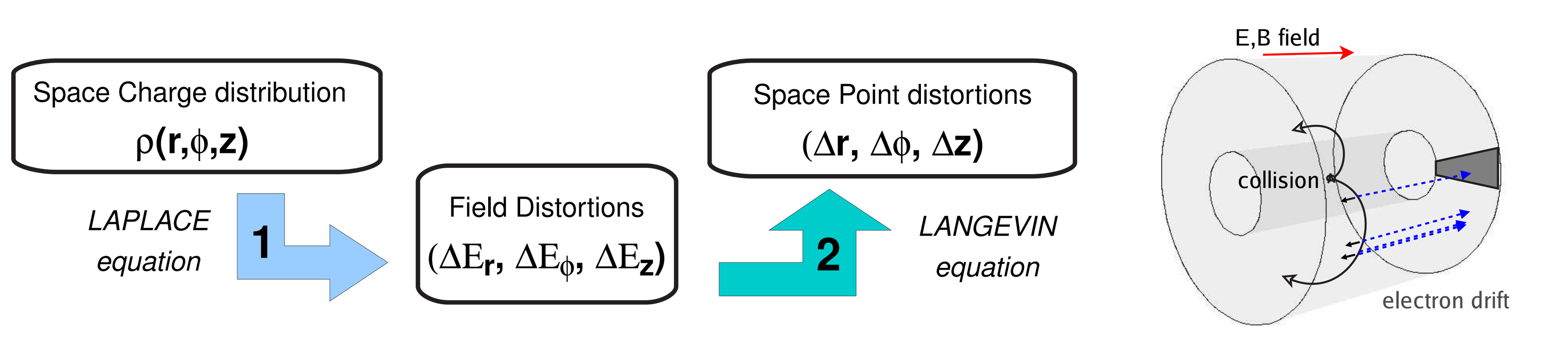
Abstract

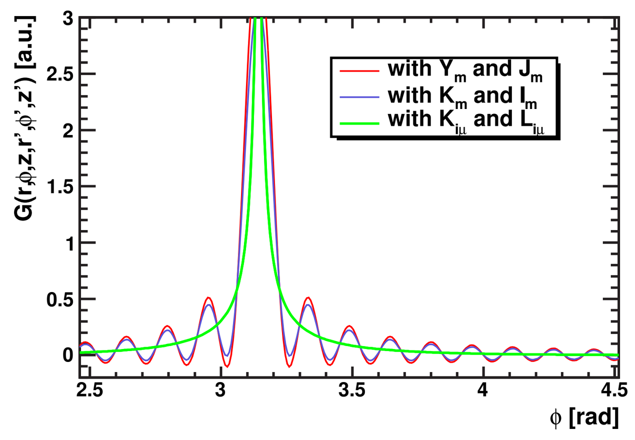
In a Time Projection Chamber (TPC), the possible ion feedback and also the primary ionization of high multiplicity events result in accumulation of static charge inside the gas volume (space charge). This charge introduces electrical field distortions and modifies the cluster trajectory and shape along the drift path, affecting the tracking performance of the detector. In order to calculate the track distortions due to an arbitrary space charge distribution in the TPC, the Green's function for a TPC geometry was worked out. This analytical approach finally permits accurate predictions of track distortions due to an arbitrary space charge distribution by solving the Langevin equation.

Basic concept



1. Analytical solution of the LAPLACE equation: Green's function for a coaxial cavity

Three representations of the Green's function with different convergence regions were derived. Each one is a sum over two of the three labels of the eigenfunctions of the homogeneous boundary value problem. The coefficients correspond to one-dimensional Green's functions in the third variable, which are found by the method of particular integrals. Another method to derive solution (2), which leads to the same result, is given in [1]. The third representation is an innovative one using modified Bessel function of purely imaginary order, which can also be derived by a Sommerfeld-Watson transformation. These Green's functions lead to fast converging representations for every electric field component ($\boldsymbol{E_r}$, $\boldsymbol{E_\phi}$, $\boldsymbol{E_z}$) for any 3D space charge distribution $\rho(r,\phi,z)$ within a TPC field cage, and not only radially symmetric ones. The detailed derivations can be found in [2].



$G(r,\phi,z;r',\phi',z') =$

$$= \frac{1}{2\pi}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}(2-\delta_{m0})\cos[m(\phi-\phi')]\frac{\bar{R}_{mn}(r)\bar{R}_{mn}(r')}{\bar{N}^2_{mn}}\frac{\sinh(\beta_{mn}z_<)\sinh(\beta_{mn}(L-z_>))}{\beta_{mn}\sinh(\beta_{mn}L)} \quad (1)$$

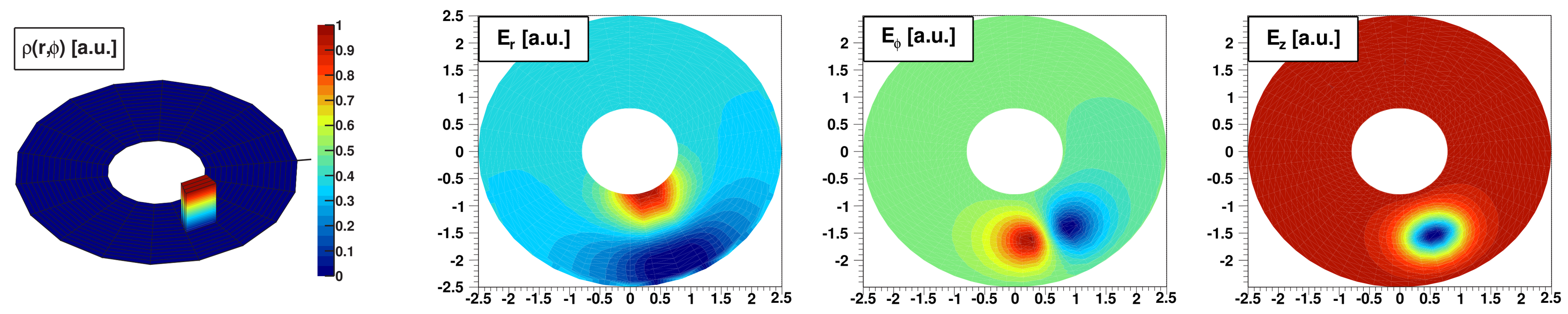
with $\bar{R}_{mn}(r) = Y_m(\beta_{mn}a)J_m(\beta_{mn}r) - J_m(\beta_{mn}a)Y_m(\beta_{mn}r)$

$$= \frac{1}{\pi L}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}(2-\delta_{m0})\cos[m(\phi-\phi')]\sin(\beta_n z)\sin(\beta_n z')\frac{R_{mn\,1}(r_<)R_{mn\,2}(r_>)}{I_m(\beta_n a)K_m(\beta_n b)-I_m(\beta_n b)K_m(\beta_n a)} \quad (2)$$

with $R_{mn\,1}(r) = K_m(\beta_n a)I_m(\beta_n r) - I_m(\beta_n a)K_m(\beta_n r)$ and $R_{mn\,2}(r) = K_m(\beta_n b)I_m(\beta_n r) - I_m(\beta_n b)K_m(\beta_n r)$

$$= \frac{1}{L}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cosh[\mu_{nk}(\pi-|\phi-\phi'|)]}{\mu_{nk}\sinh(\pi\mu_{nk})}\sin(\beta_n z)\sin(\beta_n z')\frac{R_{nk}(r)R_{nk}(r')}{N^2_{nk}} \quad (3)$$

with $R_{nk}(\mu_{nk};a,b) = L_{i\mu}(\beta_n a)K_{i\mu}(\beta_n b) - K_{i\mu}(\beta_n a)L_{i\mu}(\beta_n b) = 0$ and $L_{i\mu}(\beta r) := \frac{1}{2}(I_{-i\mu}(\beta r) + I_{i\mu}(\beta r))$



$\epsilon \approx 10^{-7}$
for $\lesssim$ 30 terms

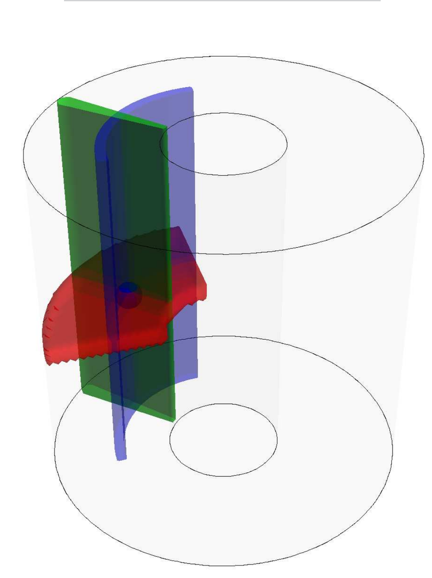


Figure: Regions of slow convergence

Simulated space charge distributions and resulting potentials (E field components)

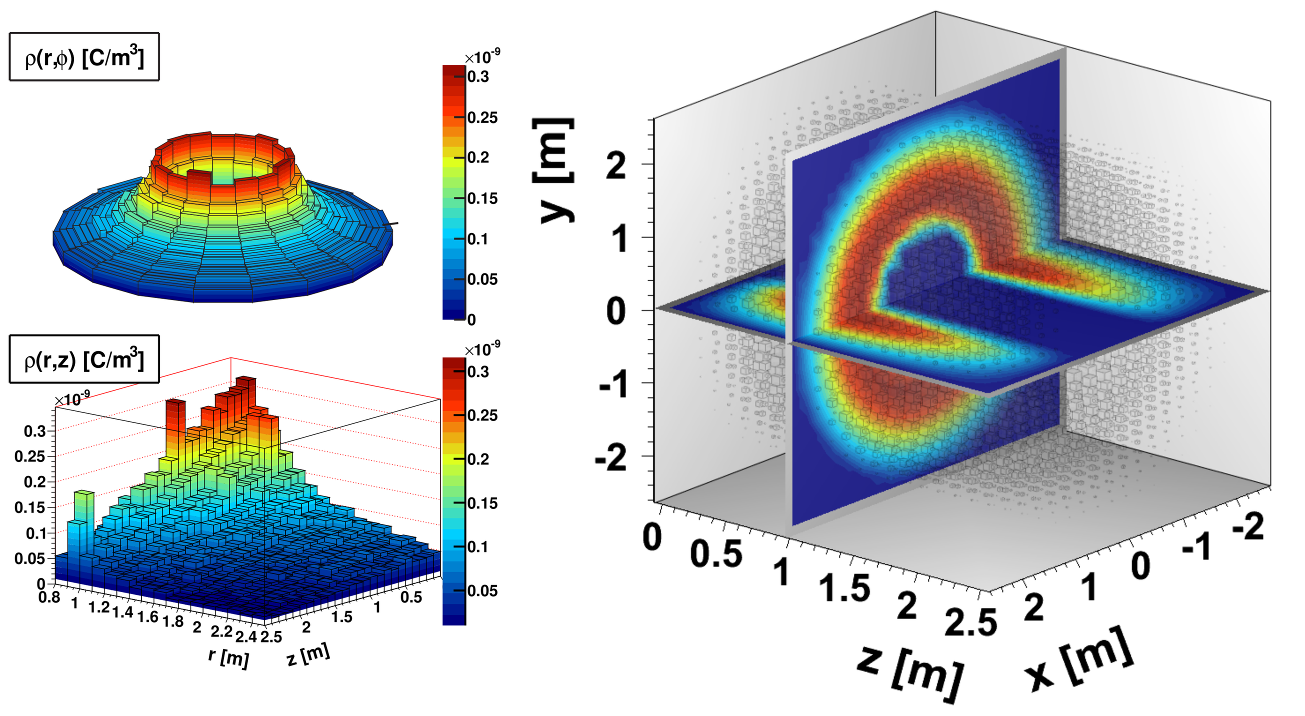


Figure: Expected scenario; left: space charges; right: resulting potential

Pb-Pb collisions were simulated (HiJing) in order to estimate the charge distributions from single events and the pile up within the gas volume.

Possible Ion sources:

1. **PI**: **Primary Ionization** from the tracks within the gas volume
2. **ROC-IFB**: **Ion Feedback** from tracks within the read out chamber (from the prev. event)
3. **ILK**: Ion leakage — typically suppressed by the gating grid

Numbers and possible scenarios for the Alice TPC:
Ion clearing time (drift volume): $\sim$ 0.156s
Pile-Up: $\sim$ 480 min.bias (at a event rate of $\sim$ 3000Hz)

- **Expected**: PI plus ROC-IFB: (basically radialsymetric) $\rho(r,z) \approx (3-0.9\cdot z)/r^2 \times 10^{-10} C/m^3$
- **Unlikely**: Additional ILK due to unexpected problems with the gating grid

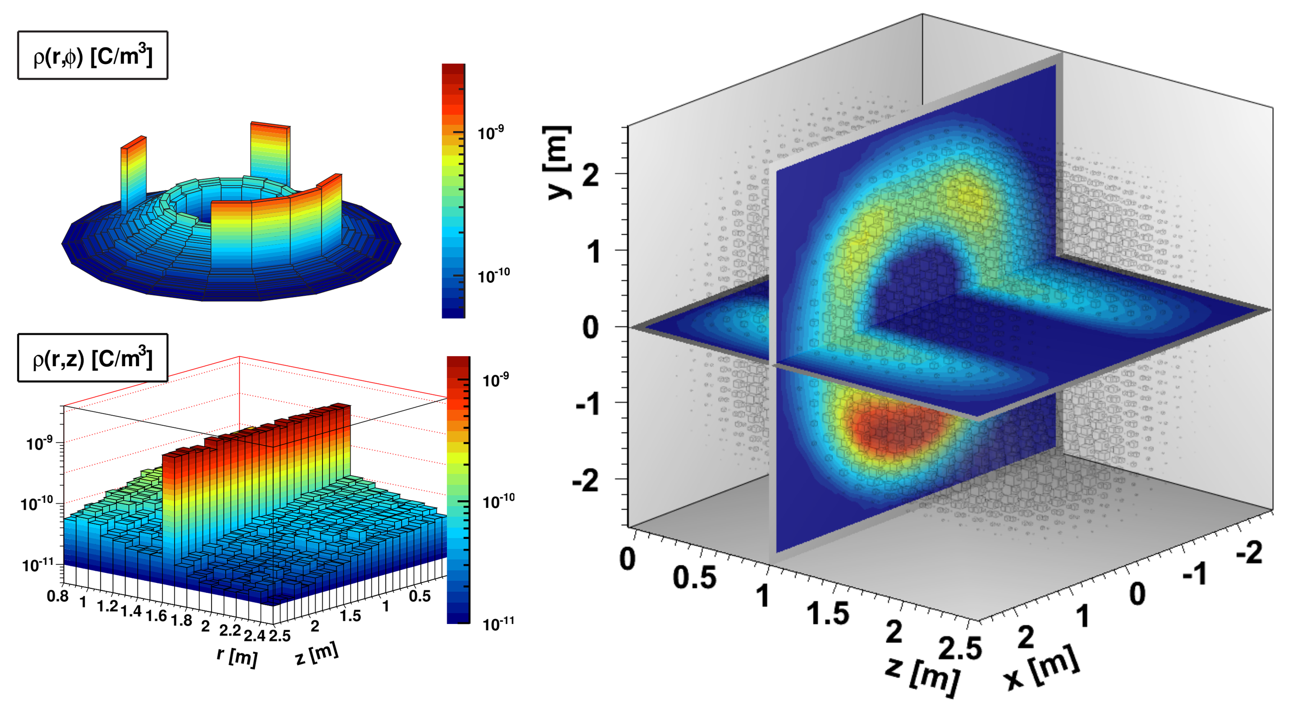


Figure: Unlikely scenario; left: space charges; right: resulting potential

2. Solving the LANGEVIN equation: Lorenz angle calculations

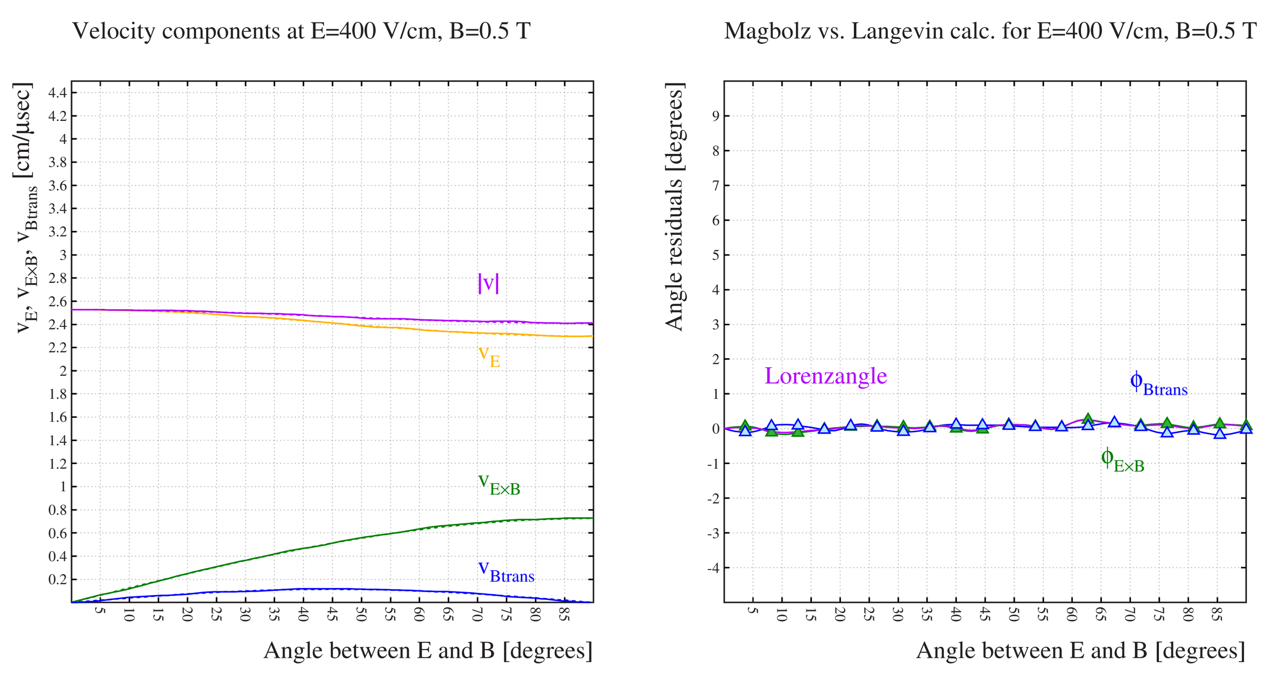


Figure: $NeCO_2N_2$ gas - solid lines: Magboltz; dotted lines: Langevin

- Lorenz angle depends on the gas composition and the angle between the E and B field
- It can be calculated by means of MC techniques (see Magboltz [3]) or by solving the Langevin equation (for const. mobility μ)
- Sizable differences between the Magboltz and Langevin velocity components are possible

Two example gases:

1. **$NeCO_2N_2$ (90/10/5)** (e.g. Alice TPC): $\omega\tau\sim$**0.31** $\longrightarrow$ good agreement ($\boldsymbol{\Delta\phi_{max}}\sim$**0.1°**)
2. **P10 - $ArCH_3$ (90/10)** (e.g. Star TPC): $\omega\tau\sim$**2.01** $\longrightarrow$ good agreement at small angles, but not for large ones ($\boldsymbol{\Delta\phi}\sim$**1°** at $\angle(\boldsymbol{E},\boldsymbol{B})\sim$**10°**)

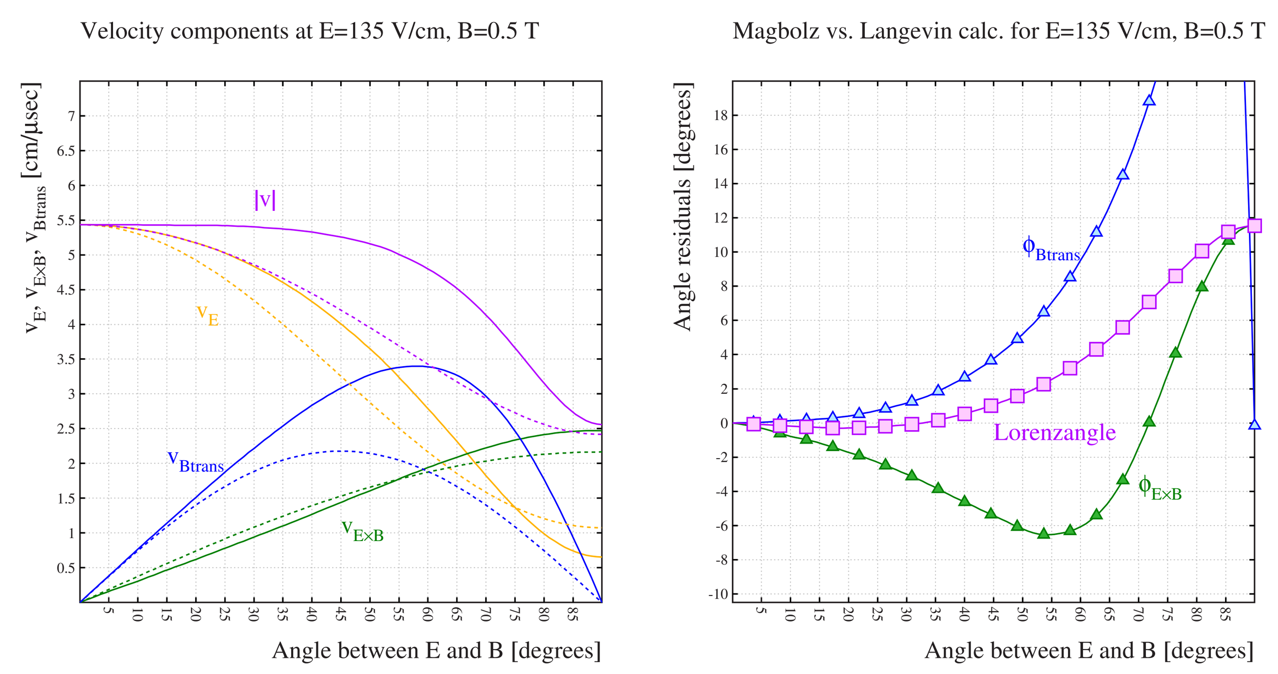


Figure: P10 gas ($ArCH_4$) - solid lines: Magboltz; dotted lines: Langevin

Expected space point distortions in the Alice TPC

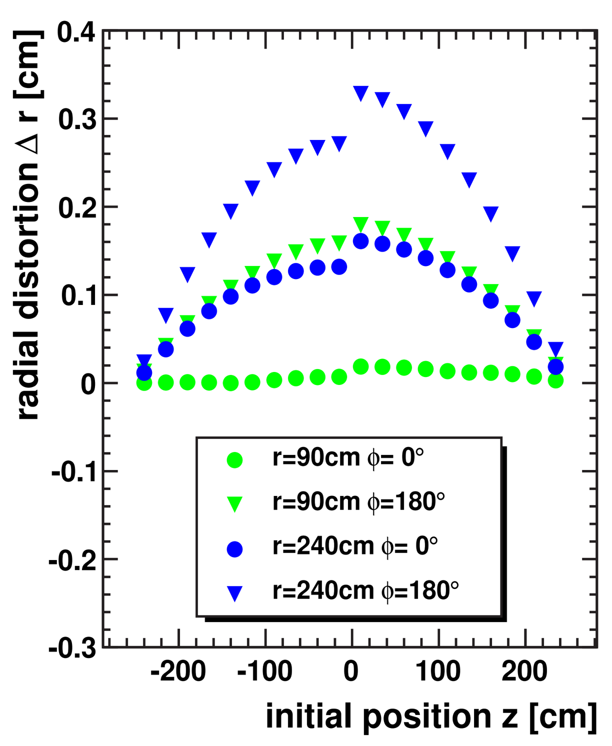


Figure: Space point distortions due to B field inhomogeneities

By solving the Langevin equation e.g. within Garfield [4] we can include nearly every effect which disturbs the drifting electron:

- inhom. B field
- inhom. E field due to space charges or field cage imperfections
- gas density properties like drift velocity and diffusion (and P/T variations)

$$(\boldsymbol{\Delta r}, \boldsymbol{\Delta\phi}, \boldsymbol{\Delta z}) \propto (\boldsymbol{\Delta B}, \boldsymbol{\Delta E}, \boldsymbol{\Delta N})$$

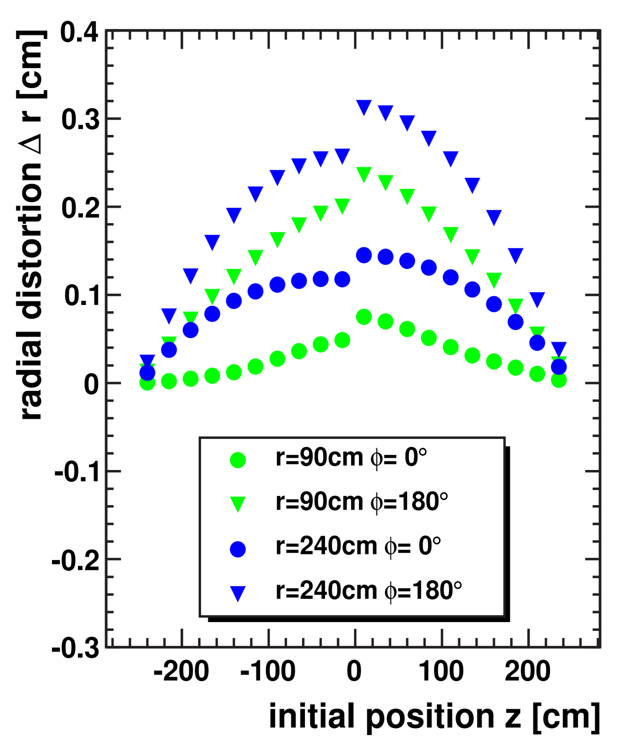


Figure: combined effect of B field and expected space charges

Summary & Outlook

- We present a fast converging analytical solution of the Laplace equation for a typical TPC geometry which can be used to calculate the electric field inhomogeneities due to arbitrary space charge configurations
- We show that the Langevin approximation of the Lorenz angle is sufficient for a gas composition of $NeCO_2N_2$ as used in the Alice TPC
- *Now possible:* Fast and accurate predictions of space point distortions due to space charges and more ...
- *FUTURE PLANS:* Locating space charge clouds within the gas volume through laser measurements and an inverse model which can be used for an event-by-event space charge correction within TPCs

References

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