A check-up for the statistical parton model.

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Summary

1) PHENOMENOLOGICAL MOTIVATIONS TO INTRODUCE THE QUANTUM STATISTICAL PARTON MODEL

2) THE PARTON DISTRIBUTIONS PROPOSED IN 2002 BY CLAUDE BOURRELY, F. B. AND JACQUES SOFFER

3) THE EXTENSION TO THE TRANSVERSE ENERGY SUM RULE

4) COMPARISON WITH THE HERA FIT FOR THE LIGHT FERMION AND GLUON DISTRIBUTION

5) CONCLUSIONS

Phenomenological Motivations For The Statistical Parton Distributions. The Statistical Parton distributions.

1) The isospin asymmetry in the sea of the proton ;

$\bar{d}(x) > \bar{u}(x)$

advocated many years ago by Niegawa and Sisiki and by Feynman and Field as a consequence of Pauli principle and confirmed by the defect in the Gottfried sum rule and by the larger Drell-Yan production of muon pairs in pn scattering than in pp scattering.

2) The correlation between the first moments of the valence partons and the shapes of their x distributions is the one expected for a quantum gas:

broader shapes for higher first moments,

as it is shown by the dramatic decrease at high x of the ratio $\frac{F_2^n(x)}{F_2^p(x)}$ as a consequence of a similar behavior of the ratio $\frac{d(x)}{u(x)}$ the increasing with x of the positive ratio $\frac{\Delta u(x)}{u(x)}$ and the decreasing with x of the ratio $\frac{\Delta d(x)}{d(x)}$.

The quantum statistical parton distributions

The role of Pauli principle suggests to write Fermi-Dirac functions for the quarks in the variable x, which is the one appearing in the parton model sum rules.

Remember that the usual choice of the energy as the variable appearing in statistical mechanics follows from its appearing in the constraint, which fixes the total energy available.

To reproduce data to get

$xq(x)$

one had to multiply the Fermi-Dirac function :

$\frac{1}{(\exp \frac{x-\tilde{X}_q}{\bar{x}}+1)}$

where $\bar{x}$ plays the role of the"temperature" and $\tilde{X}_q$ is the potential of the parton depending on its flavor and its helicity for the factor :

$A\tilde{X}_q x^b$

and add the diffractive contribution :

$\frac{\tilde{A}x^{\tilde{b}}}{(e^{x/\bar{x}}+1)}$

The quantum statistical parton distributions

For the light antiquarks we have the same diffractive contribution:

$$\frac{\tilde{A}x^{\tilde{b}}}{(e^{x/\bar{x}}+1)}$$

to be added to:

$$\frac{\bar{A}x^{2b}}{\tilde{X}_q} \times \frac{1}{\exp\left(\frac{x-\tilde{X}_{\bar{q}}}{\bar{x}}\right)+1}$$

where q and $\bar{q}$ have opposite helicities

Finally for the gluon we have the Planck form, namely a Bose-Einstein formula with vanishing potential:

$$xG(x) = \frac{A_G x^{b_G}}{\exp(x/\bar{x})-1}$$

The QCD equilibrium conditions

By requiring equilibrium for the two elementary QCD processes, the emission of a gluon by a fermion parton and the conversion of the gluon into a $q\bar{q}$ pair with opposite helicity, one has the important consequence to have a vanishing potential for the gluons of both helicities and opposite values for the potentials for a quark and its antiparticle with opposite helicity.

So the Bose-Einstein expression for the gluons $xG(x)$ turns into a Planck form :

$$\frac{1}{\exp(x/\bar{x})-1}$$

and $\Delta G(x) = 0$

while the relation:

$$\tilde{X}_q^h + \tilde{X}_{\bar{q}}^{(-h)} = 0$$

allows to disentangle the quark and antiquark contributions in the e. m. DIS.

While for the unpolarized distributions the disentangling is obtained from the obvious conditions :

$$u - \bar{u} = 2$$

$$d - \bar{d} = 1$$

for the polarized distributions the equilibrium conditions allow to determine the polarization of the light antiquarks from the knowledge of the shapes of the valence quark distributions.

The determination of the parameters in the 2002 paper

From a selected choice of Deep Inelastic Scattering we have been able to determine the small number of parameters introduced, getting the following values :

2002	2014	$\frac{A'}{A}\ln(1+\exp\tilde{Y}_q)$
$\bar{x} =$ 0.099	0.102	
$\tilde{X}^{\uparrow}{}_u =$ 0.461	0.446	0.465
$\tilde{X}^{\downarrow}{}_d =$ 0.301	0.320	0.3115
$\tilde{X}^{\downarrow}{}_u =$ 0.298	0.297	0.2975
$\tilde{X}^{\uparrow}{}_d =$ 0.228	0.222	0.235
$b =$ 0.41	0.43	
$\tilde{b} =$ -0.25	-0.25	
$b_G =$ 0.75	1	
$\tilde{A} =$ 0.083	0.070	
$\underline{A} =$ 1.497	0.615	
$\bar{A} =$ 1.93	3.5	

As we expected, the biggest potential is $\tilde{X}_u{}^{\uparrow}$ and the smallest is $\tilde{X}_d{}^{\uparrow}$

The equilibrium conditions imply.

$\Delta\bar{u}(x) > 0$

$\Delta\bar{d}(x) < 0$

which is confirmed by the asymmetries in the production of $W^{\pm}$ and implies a positive contribution to the Bjorken sum rule.

The Extension to the Transverse Degrees of Freedom

THE TRANSVERSE ENERGY SUM RULE

In 2002 to comply with data one introduced the ”ad hoc” factors $\tilde{X}_q$ for the valence partons and guessed opposite factors for their antiparticles with opposite helicity.

To explain these factors the statistical model has been extended to the transverse degrees of freedom.

A crucial role to fix the p_T dependance is the sum rule on the transverse energy, the difference between the energy and the momentum.

The r. h. s. is $P_0 - P_z$, which for M ¡¡ P_z, is $\frac{M^2}{2P_z}$

The contribution of a light parton to the sum rule multiplied by $2P_z$ is

$$\frac{2P_z P_T^2}{p_0+p_z} = \frac{2P_T^2}{x+\sqrt{x^2+\frac{p^2}{P_Z^2}}}$$

This implies the following dependance on p_T^2 :

$$\frac{1}{\exp\left[\frac{2P_T^2}{\mu^2(x+\sqrt{x^2+\frac{p^2}{P_Z^2}})}-\tilde{Y}_q\right]+1}$$

which with the transformation:

$$P_T^2 = \frac{\mu^2\eta(x+\sqrt{x^2+\frac{p^2}{P_Z^2}})}{2}$$

gives rise to the integral in η of:

$$\frac{1+\frac{2\mu^2(1-x)}{Q^2}}{e^{(\eta-\tilde{Y}_q)}+1}$$

which gives rise to:

$$\ln[1+\exp\tilde{Y}_q] + \frac{(1-x)2\mu^2}{Q^2} Poly(-2, -\exp\tilde{Y}_q)$$

The Statistical Parton Distributions

So we have:

$$xu(x) = \frac{A'x^b}{(e^{\frac{x-\tilde{X}_{u\uparrow}}{\bar{x}}}+1)}[\ln(1+e^{\tilde{Y}_{u\uparrow}}) + \frac{(1-x)2\mu^2}{Q^2}Poly(-2,-e^{\tilde{Y}_{u\uparrow}})] + \frac{A'x^b}{(e^{\frac{x-\tilde{X}_{u\downarrow}}{\bar{x}}}+1)}[\ln(1+e^{\tilde{Y}_{u\downarrow}}) + \frac{(1-x)2\mu^2}{Q^2}Poly(-2,-e^{\tilde{Y}_{u\downarrow}})]$$

$$x\Delta u(x) = \frac{A'x^b}{(e^{\frac{x-\tilde{X}_{u\uparrow}}{\bar{x}}}+1)}\ln(1+e^{\tilde{Y}_{u\uparrow}}) - \frac{A'x^b}{(e^{\frac{x-\tilde{X}_{u\downarrow}}{\bar{x}}}+1)}\ln(1+e^{\tilde{Y}_{u\downarrow}})$$

where the second term is absent in the polarized distribution as a consequence of the Wigner-Melosh rotation.

By the substitution u to d one gets the distributions for d, while to get the ones for the antiquarks one has to write $\bar{A'}$ instead of A' and relate their potentials to the ones for the valence partons by the equilibrium conditions:

$\tilde{X}_q^h$ + $\tilde{X}_{\bar{q}}^{(-h)}$ = 0

$\tilde{Y}_q^h$ + $\tilde{Y}_{\bar{q}}^{(-h)}$ = 0

For the diffractive contributions one has the same form of the 2002 paper:

$$\frac{\tilde{A}x^{\tilde{b}}}{(e^{\frac{x}{\bar{x}}}+1)}$$

The Comparison with the Hera Fit for the Unpolarized Distributions

Some years ago a joint analysis of the DIS data measured in the H_1 and $ZEUS$ experiments has been performed to give the unpolarized parton distributions and Jacques Soffer immediately realized the similarity with the statistical distributions.

To perform a check for the quantum statistical parton distributions, we determine the parameters introduced in order to reproduce the Hera result for the unpolarized distributions of the light parton fermions and of the gluon, while for the polarized ones we require to reproduce the expressions found in 2002, which have been successful to describe the polarized structure functions $g_1^N(x)$ and the production of the $W^{\pm}$ weak bosons.

The comparison with the Hera fit and with the 2002 polarized distributions are shown in the following figures and the parameters are compared with the ones determined in that paper.

The agreement is very good for both the comparisons for the fermions, while for gluons we have to multiply for the slowing changing factor.

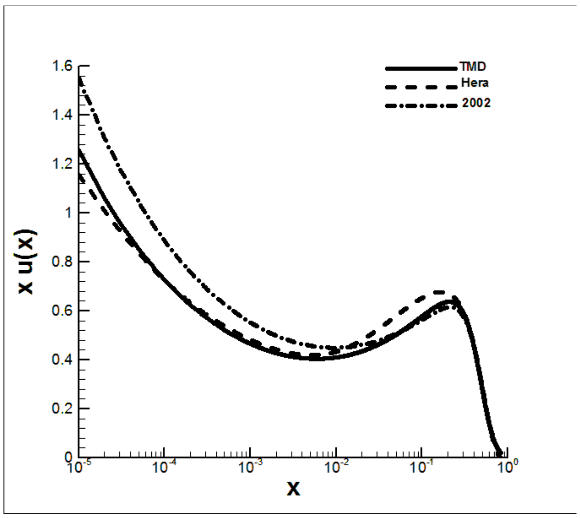
$(1+\ln\frac{x+r}{x+u})e^{[-C_G(x-c_G)^2]}$

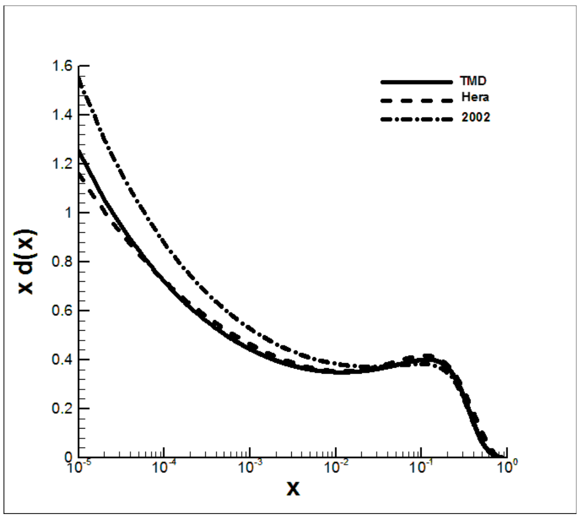
Conclusions

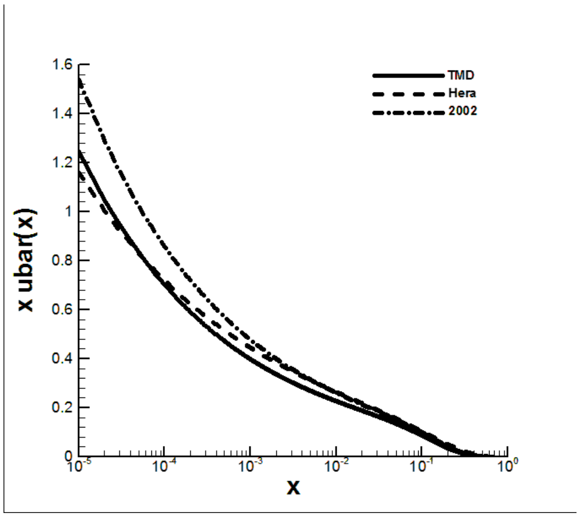
The agreement with the Hera distributions with the form dictated by the quantum statistical approach is an impressive confirm of the validity of the proposal in the 2002 paper with the improved theoretical foundation achieved with the extension of the transverse degrees of freedom and with the consideration of the Melosh-Wigner rotation. The similarity of the values of the parameters with the ones found in the previous work is another point in favor of the statistical approach. As long as the p_T dependance in the classical limit, neglecting the power dependance and with the gaussian approximation for the exponential we get the behavior:

$$\sqrt{p_T} e^{\left(\frac{-2p_T}{\mu\sqrt{x}}\right)}$$

with an "effective temperature" $49 MeV$, smaller than the range proposed in the paper by Cleymans, Lykasov, Sorin and Teryaev, $120 - 150 MeV$, but the important quantum effect gives rise to a harder p_T distribution.



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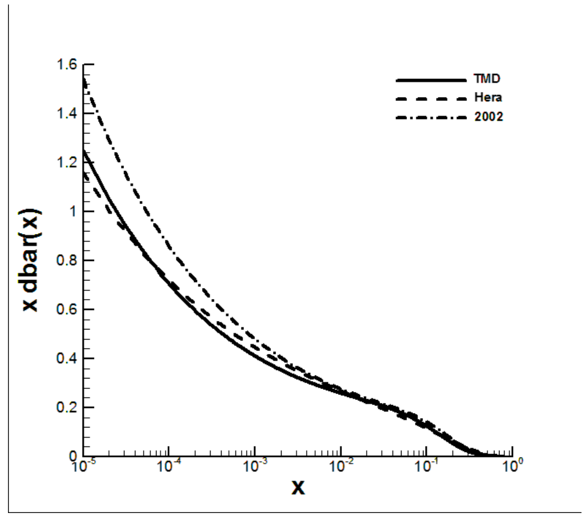
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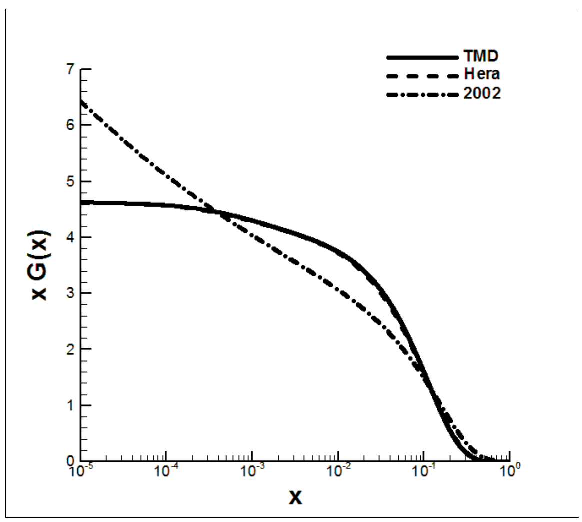
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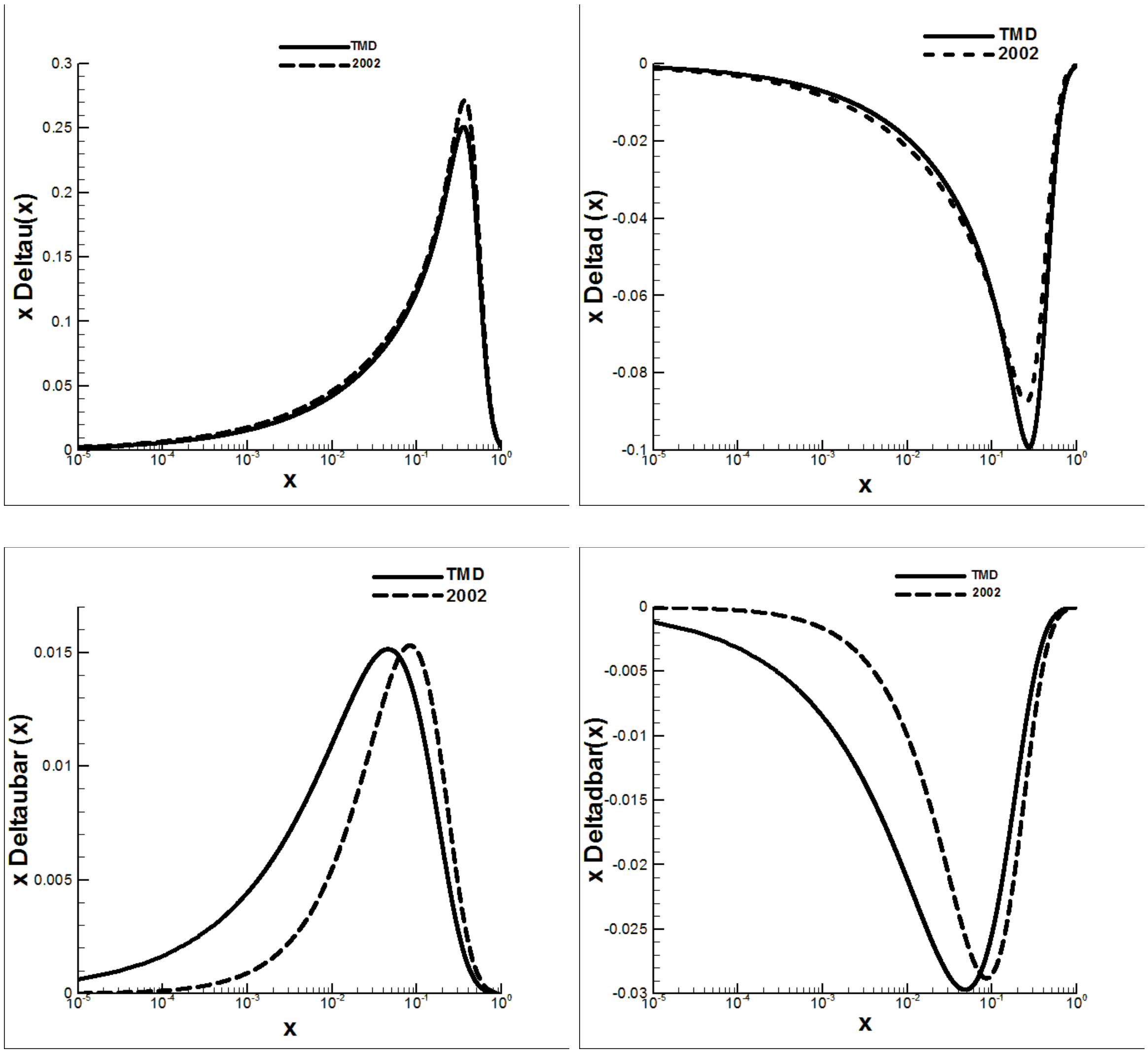
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