Generalised form factors and charges from lattice QCD

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Outline

- Introduction: what do we calculate?
- Aspects of lattice calculations - recent progress.
- Iso-vector quark momentum fraction: $\langle x \rangle_{u-d}$.
- Axial charge: g_A^{u-d}.
- Strangeness contribution to the spin of the nucleon $\Delta s + \Delta \bar{s}$.
- Outlook

What do we calculate?

Matrix elements of local operators $\langle N, p+q, s' | \mathcal{O} | N, p, s \rangle$:

$$\mathcal{O}_q^{\{\mu_1 \cdots \mu_j\}} = \bar{q}\gamma^{\mu_1} \overset{\leftrightarrow}{D}^{\mu_2} \ldots \overset{\leftrightarrow}{D}^{\mu_j} q \qquad \mathcal{O}_{5q}^{\{\mu_1 \cdots \mu_j\}} = \bar{q}\gamma^{\mu_1}\gamma_5 \overset{\leftrightarrow}{D}^{\mu_2} \ldots \overset{\leftrightarrow}{D}^{\mu_j} q$$

$$\mathcal{O}_g^{\{\mu_1 \cdots \mu_j\}} = \mathrm{Tr} F^{\mu_1 \nu} \overset{\leftrightarrow}{D}^{\mu_2} \ldots \overset{\leftrightarrow}{D}^{\mu_{j-1}} F_\nu^{\mu_j}$$

Lorentz decomposition gives generalised form factors. $t = q^2$

$$\text{Unpolarised} : \langle N, p+q | \mathcal{O}_q^{\mu_1} | N, p \rangle = \bar{u}_N \left[\gamma^{\mu_1} A_{10}(t) + \frac{i}{2m} \sigma^{\mu_1 \alpha} q_\alpha B_{10}(t) \right] u_N$$

$$\text{Polarised} : \langle N, p+q, s' | \mathcal{O}_{5q}^{\mu_1} | N, p, s \rangle = \bar{u}_N \left[\gamma^{\mu_1}\gamma_5 \tilde{A}_{10}(t) + \frac{1}{2m} q^{\mu_1}\gamma_5 \tilde{B}_{10}(t) \right] u_N$$

$\mathcal{O}_q^{\{\mu_1 \mu_2\}} \to A_{20}(t),\ B_{20}(t),\ C_{20}(t)$, $\mathcal{O}_{5q}^{\{\mu_1 \mu_2\}} \to \tilde{A}_{20}(t),\ \tilde{B}_{20}(t),\ \tilde{C}_{20}(t)$, etc.

$A_{10} = F_1,\ B_{10} = F_2$ and $\tilde{A}_{10} = F_A,\ \tilde{B}_{10} = F_P$

Also transition matrix elements, meson matrix elements.

Generalised form factors are moments of generalised parton distributions:

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\, \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z^+=0} =$$

$$\frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

Similarly $\tilde{H}^q(x,\xi,t)$, $\tilde{E}^q(x,\xi,t)$ and gluon distributions.

$$\int_{-1}^{1} dx\, H(x,\xi,t) = A_{10}(t), \qquad \int_{-1}^{1} dx\, E(x,\xi,t) = B_{10}(t)$$

$$\int_{-1}^{1} dx\, x H(x,\xi,t) = A_{20}(t) + (2\xi)^2 C_{20}(t), \qquad \int_{-1}^{1} dx\, x E(x,\xi,t) = B_{20}(t) - (2\xi)^2 C_{20}(t)$$

Forward limit: $(\xi = t = 0)$, $\Delta q(-x) \equiv \Delta\bar{q}(x)$

$$\langle \mathbb{1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, \Delta q(x) = \tilde{A}_{10}(0) = F_A(0), \qquad \langle x\rangle_{\Delta q} = \int_{-1}^{1} dx\, x\, \Delta q(x) = \tilde{A}_{20}(0)$$

$$\langle x\rangle_q = \int_{-1}^{1} dx\, x\, q(x) = A_{20}(0)$$

High moments are dominated by high x.

Difficulty in calculating higher moments

- Statistics: higher n require matrix elements of operators with more derivatives. Signal/noise is low, at present $n = 1, 2$ calculated.
- Renormalisation: mixing of operators under renormalisation due to reduced symmetry on the lattice (rotational sym.$\rightarrow$ hypercubic). Worse for higher n.

Nonetheless wide variety of quantities can be calculated

- momentum fractions $\langle x \rangle_q + \langle x \rangle_g = 1$.
- decomposition of spin contributions $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$.
- vector, axial form factors and charges ($Q^2 = 0$).
- estimate charge radii r_1 and r_2.
- similar quantities for other baryons and mesons (pion and ρ).
- nucleon and pion distribution amplitudes.
- scalar, pseudoscalar, tensor form factors $\rightarrow$ new physics processes.

Very active field: 45 talks/posters at Lattice 2014 conference.

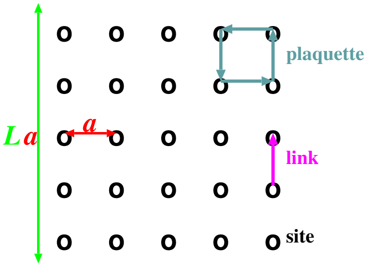
Ideally calculate parton distributions from first principles on the lattice:

$$q(x,\mu) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p|\, \bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z^+=0}$$

- Ji [1305.1539] calculate spatial (quasi) parton distributions $\tilde{q}(x,\mu,p_z)$ where N has momentum p_z, which $\to q(x,\mu)$ as $p_z \to \infty$ when properly renormalised.

  Exploratory studies by Lin et al. [1402.1462] and ETMC [1411.0891]. Talk by F. Steffens
- Transverse momentum distributions, talk by M. Engelhardt.

Lattice QCD



continuum limit: $a \to 0$, La fixed

infinite volume: $La \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\,[d\psi][d\bar{\psi}]\; O[U] e^{-S[U,\psi,\bar{\psi}]}$$

"Measurement": average over a *representative* ensemble of gluon configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad\qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

Input: $\mathcal{L}_{QCD} = -\frac{1}{16\pi\alpha_L} FF + \bar{q}_f(\not{D} + m_f) q_f$

$$\begin{aligned} m_N^{\text{latt}} = m_N^{\text{phys}} \quad &\longrightarrow \quad a \\ m_\pi^{\text{latt}}/m_N^{\text{latt}} = m_\pi^{\text{phys}}/m_N^{\text{phys}} \quad &\longrightarrow \quad m_u \approx m_d \\ \ldots \end{aligned}$$

Output: hadron masses, matrix elements, decay constants, etc...

Required:

- **Large enough volume**: ideally use several volumes. FSE suppressed with $\exp(-Lm_\pi)$, want $Lm_\pi \geq 4$.
- **Extrapolation to physical quark masses**: typically quark masses such that $m_\pi > 200$ MeV, requiring (chiral) extrapolation of results.
- **Continuum extrapolation**: need results for several a (> 3) in order to perform $a \to 0$.

Cost estimate: fixed Lm_π

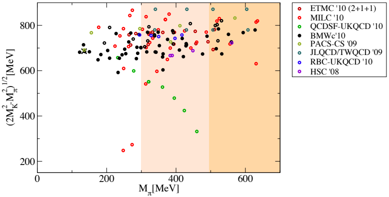
$$\text{cost} \propto \frac{1}{a^{\geq 6}\, m_\pi^{7.5}}$$

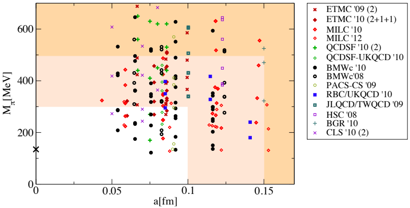
additional systematic uncertainties ...

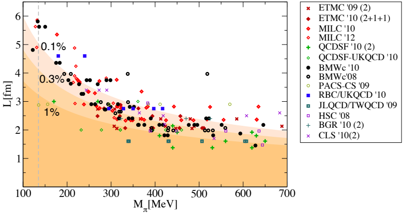
- **Sea quarks**: present simulations with $N_f = 2$ (u/d), $N_f = 2 + 1$ ($u/d + s$), $N_f = 2 + 1 + 1$ ($u/d + s + c$).
- **Excited state contamination**: all states with same QNs are created.
- **Renormalisation constants**: needed to relate lattice matrix elements to continuum results in the $\overline{\mathrm{MS}}$ scheme.

However, balance between statistical and systematic error, for some quantities (stat.)$>$(sys.).

Landscape of recent lattice simulations







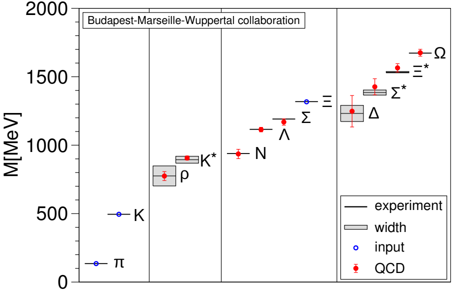
Figures taken from

C Hoelbling, arXiv:1410.3403

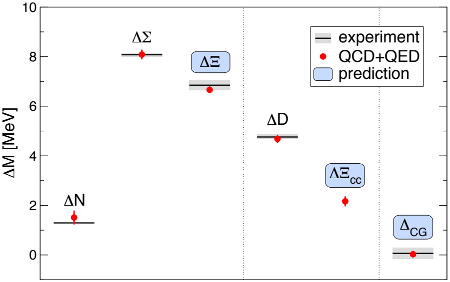
Precision calculations now possible

Huge progress in algorithms as well as hardware.

Durr et al.[0906.3599], $N_f = 2 + 1$

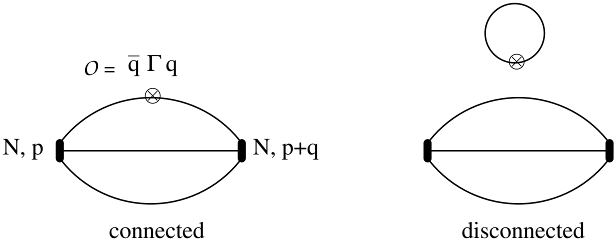


Borsanyi et al. [1406.4088], $N_f = 1 + 1 + 1 + 1$



Iso-vector/Iso-scalar matrix elements

Unpolarised, $\langle N, p+q|\mathcal{O}|N, p\rangle$ or polarised, $\langle N, s', p+q|\mathcal{O}|N, s, p\rangle$ matrix elements



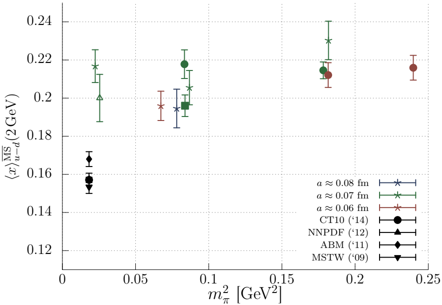
Γ contains combinations of γ-matrices and D_μ.

Disconnected terms computationally very expensive to calculate - need to solve matrix equations with $M = 12V \times 12V$ and $V \sim 10^7$ (all-to-all propagators).

Previously isovector operators considered: $\bar{u}\Gamma u - \bar{d}\Gamma d$.
Progress using stochastic methods: $12V \times N_{stoch}$. Now, iso-scalar quantities also computed.

Iso-vector quark momentum fraction: $\langle x\rangle^{\overline{\mathrm{MS}}}_{u-d}(2\,\mathrm{GeV})$

RQCD [arXiv:1408.6850] $N_f = 2$



- △ $Lm_\pi < 3.4$
- × $Lm_\pi < 4.0$
- • $Lm_\pi < 6.0$
- □ $Lm_\pi > 6.0$

Mild dependence on V, m_π. Renormalised nonperturbatively. $O(a)$ leading errors, a varied from 0.08 to 0.06 fm.

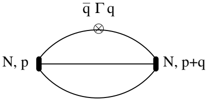
Results near the physical point.

Improvement on earlier calculations which suffered from excited state contamination $\langle x\rangle^{\overline{\mathrm{MS}}}_{u-d}(2\,\mathrm{GeV}) \sim 0.25$.

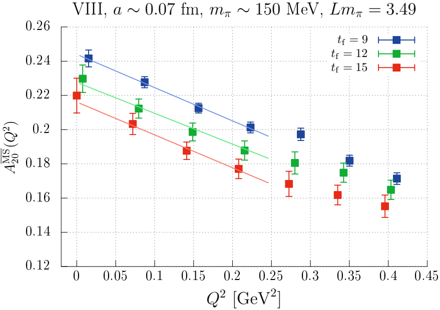
More work needs to be done - lattice spacing dependence.

Excited state contributions: $A_{20}^{\overline{\mathrm{MS}}}(t)$

Ground state dominates in the limit of large time separations (source-sink, source-operator).



Reduce excited state contamination by optimising $\mathcal{O}_{\mathcal{N}}$.

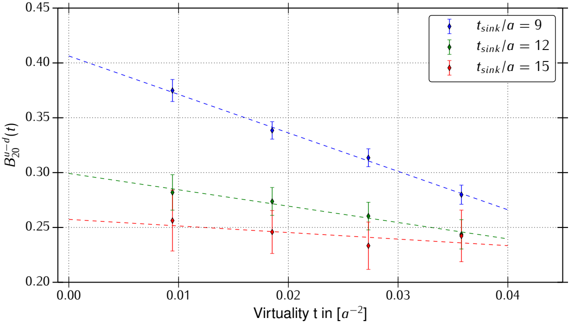


If $\mathcal{O}_{\mathcal{N}}$ not optimised then significant excited state contributions also for larger time separations.

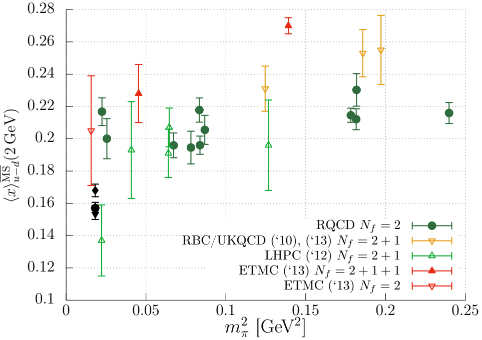
Excited state analysis performed in forward limit, to be repeated for $Q^2 > 0$.

$B_{20}(t)$

$a \sim 0.07$ fm, $m_\pi = 150$ MeV, $Lm_\pi = 3.49$



$\langle x\rangle^{\overline{\mathrm{MS}}}_{u-d}(2\,\mathrm{GeV})$: summary of recent lattice results



Consistency from different actions, N_f, volumes, lattice spacings - different systematics.

Several collaborations with near physical point results.

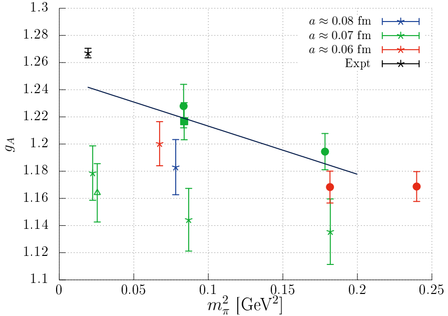
Big improvements but precision calculation requires more work.

ETMC [arXiv:1410.8761] disconnected contributions small $\Rightarrow$ predictions for $\langle x\rangle^{\overline{\mathrm{MS}}}_{q}(2\,\mathrm{GeV})$ soon $\Rightarrow$ $\langle x\rangle_g$.

Iso-vector quark spin contribution

$$g_A = \Delta u - \Delta d = \tilde{A}^u_{10}(0) - \tilde{A}^d_{10}(0) = F^u_A(0) - F^d_A(0)$$

RQCD 2014 $N_f = 2$



- △ $Lm_\pi < 3.4$
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- □ $Lm_\pi > 6.0$

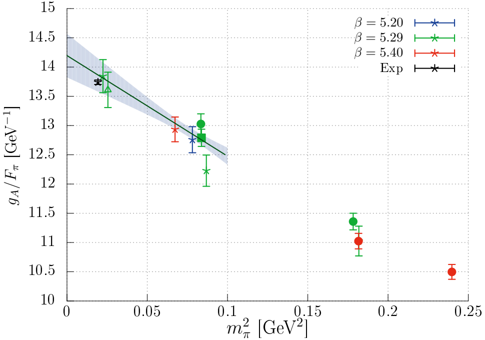
Renormalised nonperturbatively.
$O(a^2)$ leading errors.

With optimised $\mathcal{O}_\mathcal{N}$ excited states are under control.

Significant dependence on V and m_π.

Finite volume effects similar between g_A and f_π (seen from chiral pert. theory). Cancellation in the ratio. QCDSF [1302.2233]

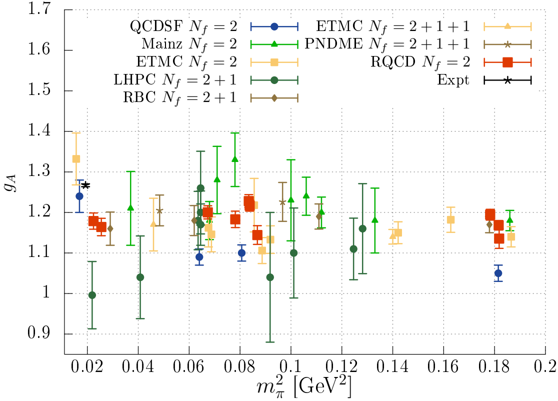
RQCD 2014 $N_f = 2$



Extrapolation to physical point: $g_A/F_\pi = 13.88(29)$ GeV^{-1} cf. Expt$= 13.797(34)$ GeV^{-1}.

Using $F_\pi(\mathrm{Expt}) = 92.21$ MeV we obtain $g_A = 1.280(27)(35)$ cf. Expt$= 1.2670(35)$.

g_A: summary of recent lattice results



Spin content of the nucleon

Spin of the nucleon:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

where

$$\begin{aligned}
\Delta\Sigma &= \sum_{q=u,d,s} \Delta q, \quad \text{and} \quad \Delta q = \tilde{A}^q_{10}(0) = F^q_A(0) \\
J_q &= \frac{1}{2} \sum_{q=u,d,s} \left(A^q_{20}(0) + B^q_{20}(0)\right) \\
L_q &= J_q - \Delta\Sigma
\end{aligned}$$

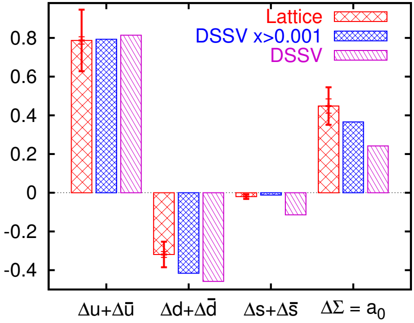
Notation: Δq contains both the spin of the quarks q and of the antiquarks $\bar{q}$.

$\Delta\Sigma \sim 0.30$, individual Δq not well known.

Δs: model fits to DIS and SIDIS data not well constrained.

Δs

QCDSF [1112.3354] $N_f = 2$, $m_\pi = 289$ MeV
g_A underestimated by 13% and therefore allow for 20% systematic errors.



We obtain ($\overline{\mathrm{MS}}$ at $\mu = \sqrt{7.4}$ GeV):

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.45(4)(9)$$
$$\Delta s = -0.020(10)(4)$$

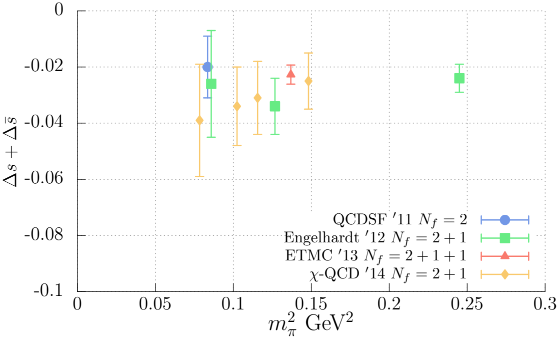
Comparison with DSSV fits to DIS and SIDIS: de Florian et al. [0904.3821]

$$\int_{x_{min}}^{1} dx\, g_1^{p,n}(x, Q^2) = \frac{1}{36}[(a_8 \pm 3a_3)C_{NS} + 4a_0 C_S].$$

$$a_3 = g_A = \Delta u - \Delta d,\ a_8 = \Delta u + \Delta d - 2\Delta s,\ a_0(Q^2) = \Delta u + \Delta d + \Delta s = \Delta\Sigma$$

Significant difference if $x_{min} = 0.001$ or 0, a_3 from neutron β decay, a_8 from hyperon β decay + $SU(3)$ flavour symmetry.

Comparison of recent lattice calculations



Consistency between different determinations, small $\Delta s + \Delta\bar{s}$ favoured.

ETMC result shows statistical accuracy that can now be achieved.
Systematics!

$$J_q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right)$$

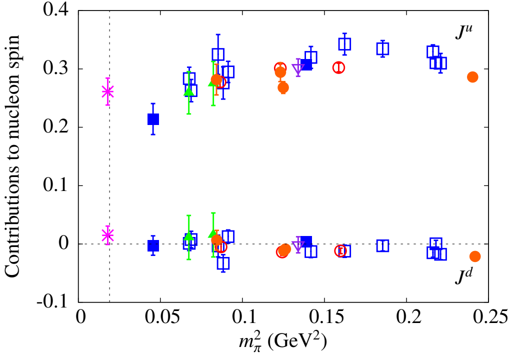


Figure taken from Lattice 2014 review by M. Constantinou [1411.0078]

LHPC $N_f = 2+1$ [1111.0718], QCDSF/UKQCD $N_f = 2$ [1203.6579], ETMC $N_f = 2$ [1104.1600,Lattice2014] and $N_f = 2+1+1$ [1303.5979].

Outlook

- Enormous progress in lattice calculations in last 10 years.
- Nucleon structure is a very active area.
- Moving towards precision lattice calculations of iso-vector $n \leq 2$ generalised form factors.
- Results with near physical m_π.
- Improvements in $\langle x \rangle_{u-d}$ but requires more work.
- g_A sensitive to finite size effects.
- Techniques have been developed to determine iso-scalar quantities - strangeness in the nucleon, Δs. More results likely soon.
- Future studies of $SU(3)$ flavour symmetry breaking.
- Exploratory studies of new approach to directly extract parton distribution functions.