Polarized parton distributions

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HiX, Frascati, 11/17/2014

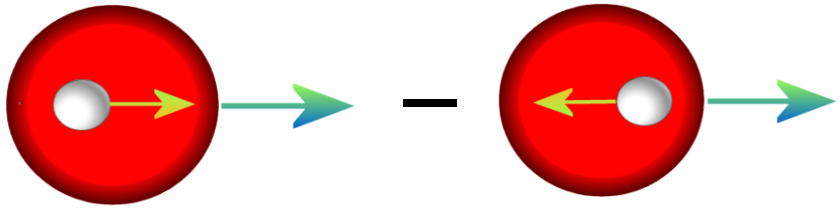
Outline:

- Brief introduction
- Overview of global analyses
- (Some) news and highlights
- Toward high x
- Conclusions

*focus entirely on helicity structure

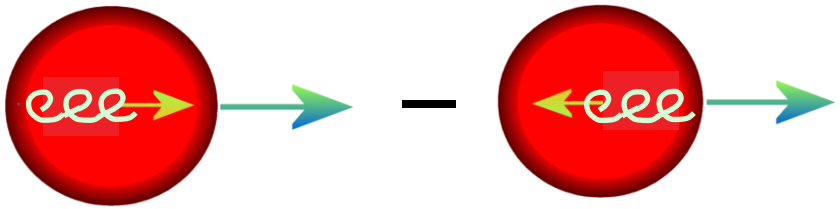
Introduction

$$\Delta q(x) \;=$$



$$\Delta q(x) = \frac{1}{4\pi}\int dy^- \, \mathrm{e}^{-iy^- x P^+} \, \langle P,S \,|\, \bar{\psi}\left(0, y^-, \mathbf{0}_\perp\right) \gamma^+ \gamma_5 \, \psi(0) \,|\, P,S \rangle$$

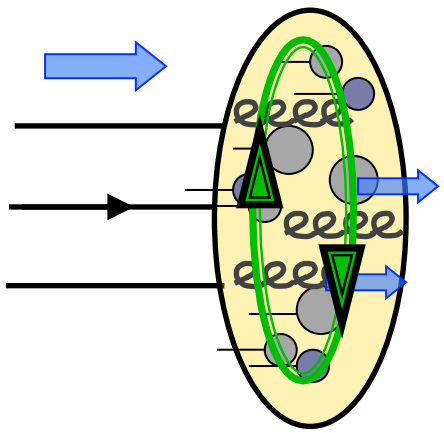
$$\Delta g(x) \;=$$



- in QCD: $\Delta q(x,\mu^2)$, $\Delta g(x,\mu^2)$
- DGLAP evolution:

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \begin{pmatrix} \Delta q(x,\mu^2) \\ \Delta g(x,\mu^2) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg} \\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg} \end{pmatrix} \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix} \left(\frac{x}{z}, \mu^2\right)$$

- proton helicity sum rule:



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

Jaffe, Manohar; Ji, Hoodbhoy, Lu; Brodsky; Ji, Yuan; Wakamatsu; Chen et al.; Burkardt; Leader et al.; Ji, Zhang, Zhao; Lorce; Hatta; …

$$\Delta\Sigma = \int_0^1 dx \left[\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right](x)$$

$$\Delta G = \int_0^1 dx\, \Delta g(x)$$

- known for past ~25 years: $\Delta\Sigma \sim 0.25 \ll 1$

Δq, Δg "beyond the proton spin sum rule"

Models of nucleon structure, e.g.:

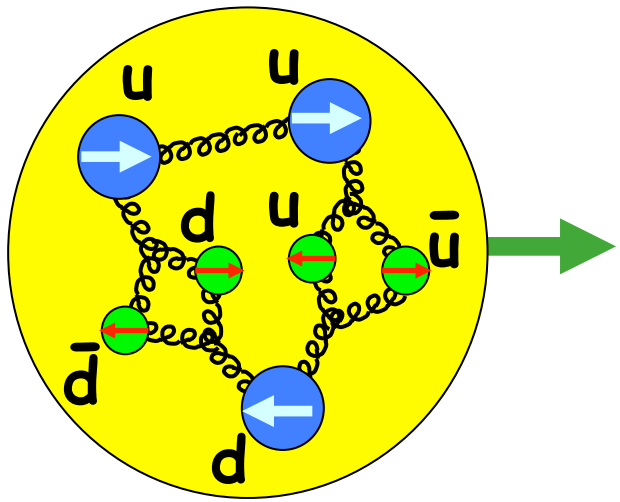
- valence region

$$\frac{\Delta d}{d} \xrightarrow{x \to 1} \begin{cases} 1 & \text{counting rules/pQCD} \\ -1/3 & \text{constituent quark model} \end{cases}$$

- flavor / sea structure

$$\Delta\bar{u} \ \text{ vs. } \ \Delta\bar{d}$$

large-N_c,
chiral quark models,
meson cloud,…



- lattice (→ recent work by X. Ji et al.)

- connection to hyperon β-decays, SU(3)

$$\Delta\Sigma_q \equiv \int_0^1 dx\,(\Delta q + \Delta\bar{q})\,(x, Q^2) \;\propto\; \langle\, P, s \,|\, \bar{\psi}_q\, \gamma^\mu \gamma_5\, \psi_q \,|\, P, s \,\rangle$$

(axial charges)

Bjorken;
Ellis, Jaffe;
Sehgal;
Karliner, Lipkin;
Ratcliffe;…

$$\Delta\Sigma_u - \Delta\Sigma_d = g_A = 1.257 \pm \ldots$$

$$\Delta\Sigma_u + \Delta\Sigma_d - 2\Delta\Sigma_s = 3F - D = 0.58 \pm 0.03 \quad ?$$

Savage, Walden; …

- strangeness?

$$\Delta\Sigma = \Delta\Sigma_u + \Delta\Sigma_d + \Delta\Sigma_s = 3F - D + 3\Delta\Sigma_s$$

Overview of global analyses

Key players over past ~4 years:

DSSV: De Florian, Sassot, Stratmann, WV, PRL 113 (2014) 012001

NNPDF: Nocera, Ball, Forte, Ridolfi, Rojo, NPB 887 (2014) 276

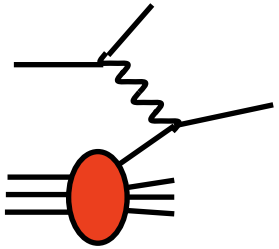
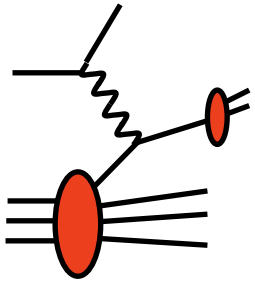
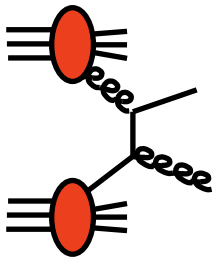
LSS: Leader, Stamenov, Sidorov, PRD 82 (2010) 114018;
arXiv:1410.1657

BB: Blümlein, Böttcher, NPB 841 (2010) 205

JAM: Jimenez-Delgado, Accardi/Avakian, Melnitchouk, (+Sato),
PRD 89 (2014) 034025; PLB 738 (2014) 263

BBS: Bourrely, Buccella, Soffer, PRD 83 (2011) 074008,
arXiv:1408.7057

(also: COMPASS (Andrieux et al.), Arbabifar et al.
"older generation" GRSV, AAC,…)

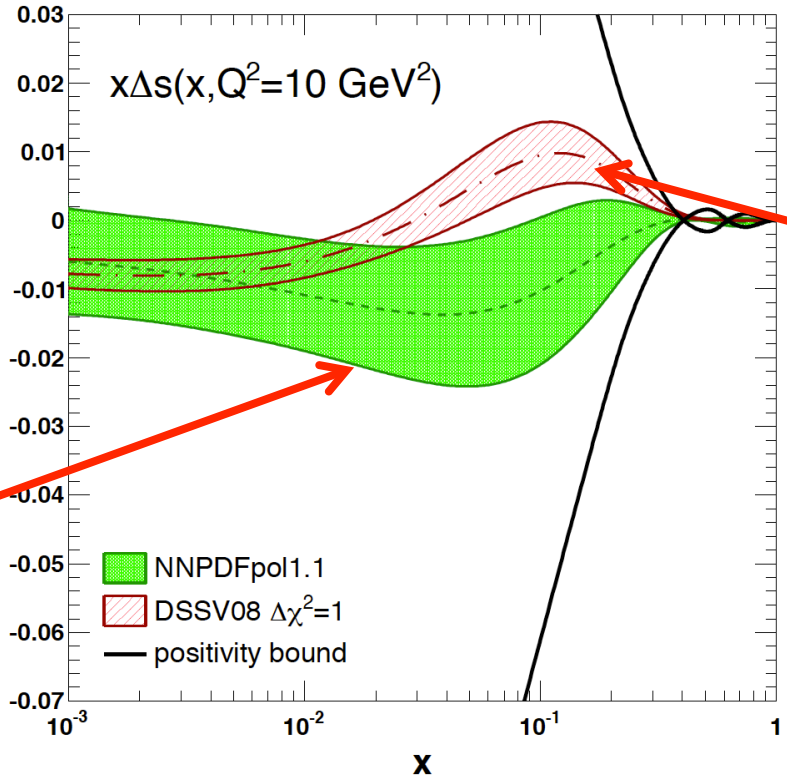
				features
DSSV	✔	✔	✔ jets, π^0	full fit
NNPDF	✔	✗	✔ jets, **W**	reweighting, replicas
LSS	✔	✔	✗	higher twist
BB	✔	✗	✗	α_s
JAM	✔	✗	✗	large-x
BBS	✔	✗	✗	statistical PDF

(Some) news and highlights

Not so new:

Strangeness "puzzle"

(DSSV 2008)



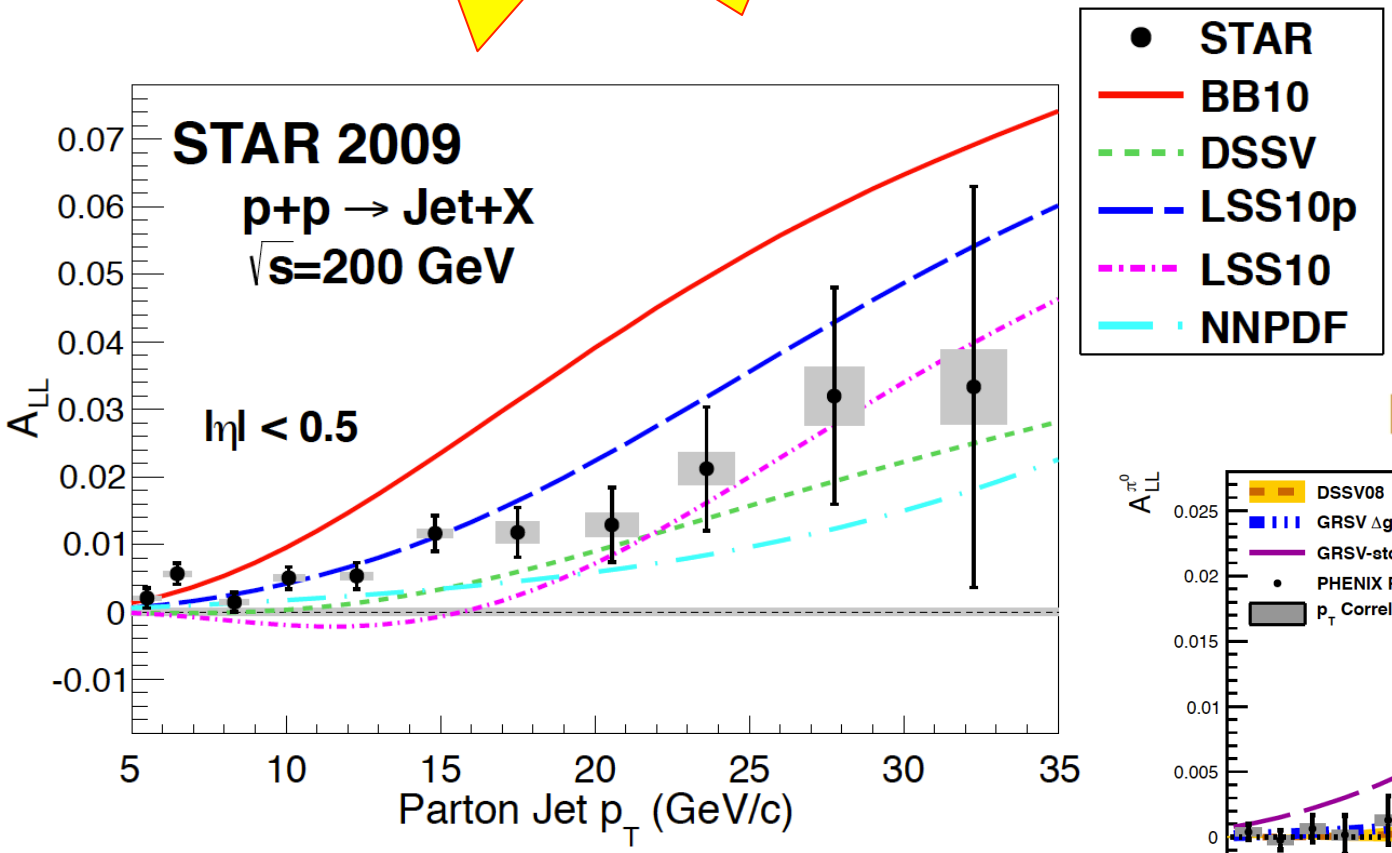
SIDIS

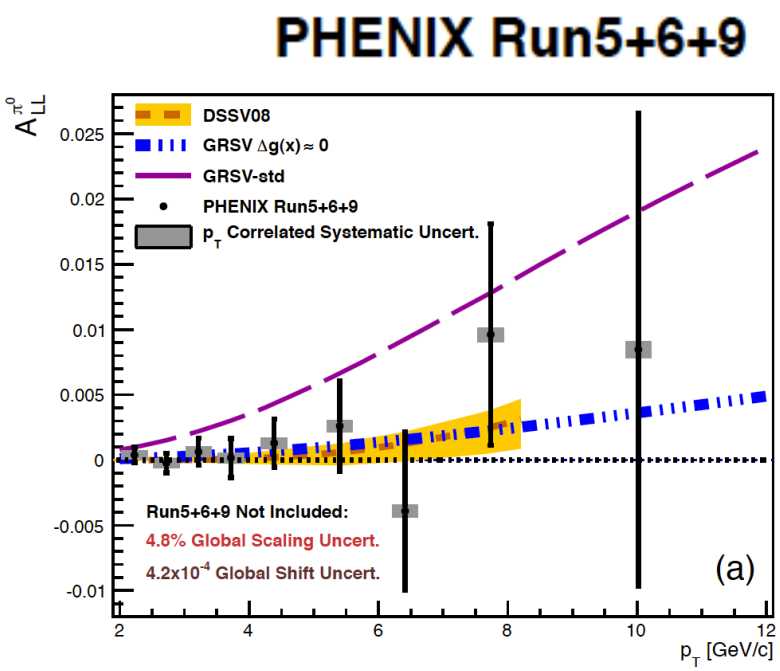
inclusive
DIS &
3F-D

(NNPDF)

(see also LSS)

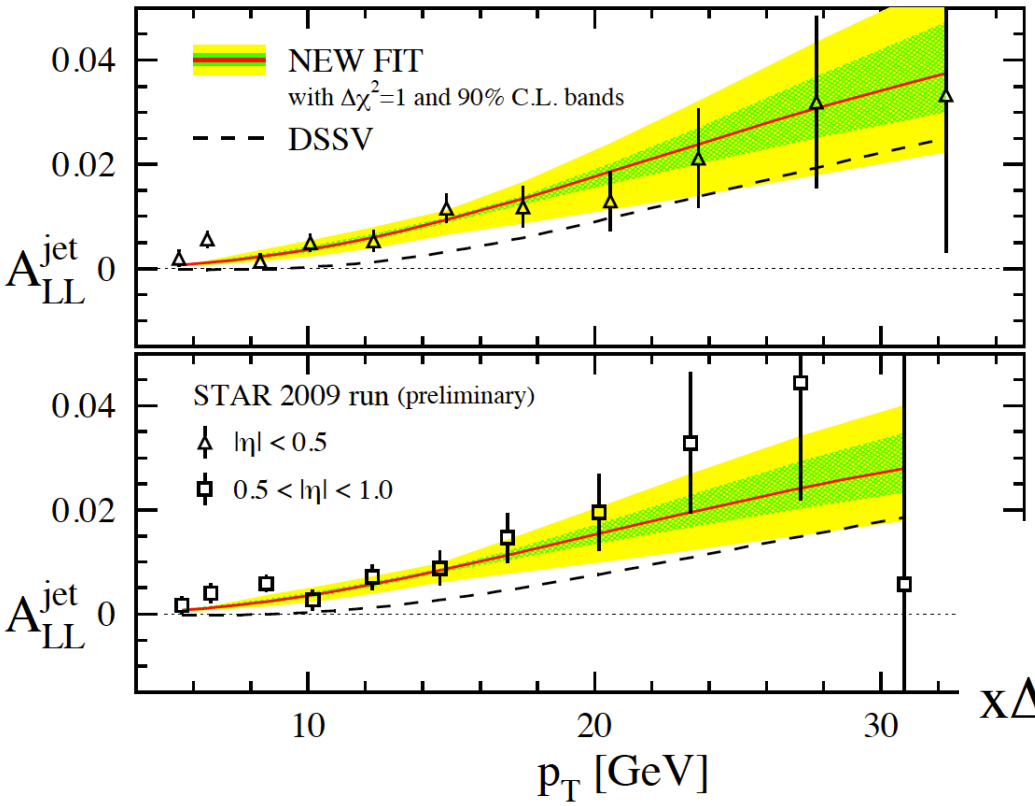
Evidence for $\Delta g \neq 0$

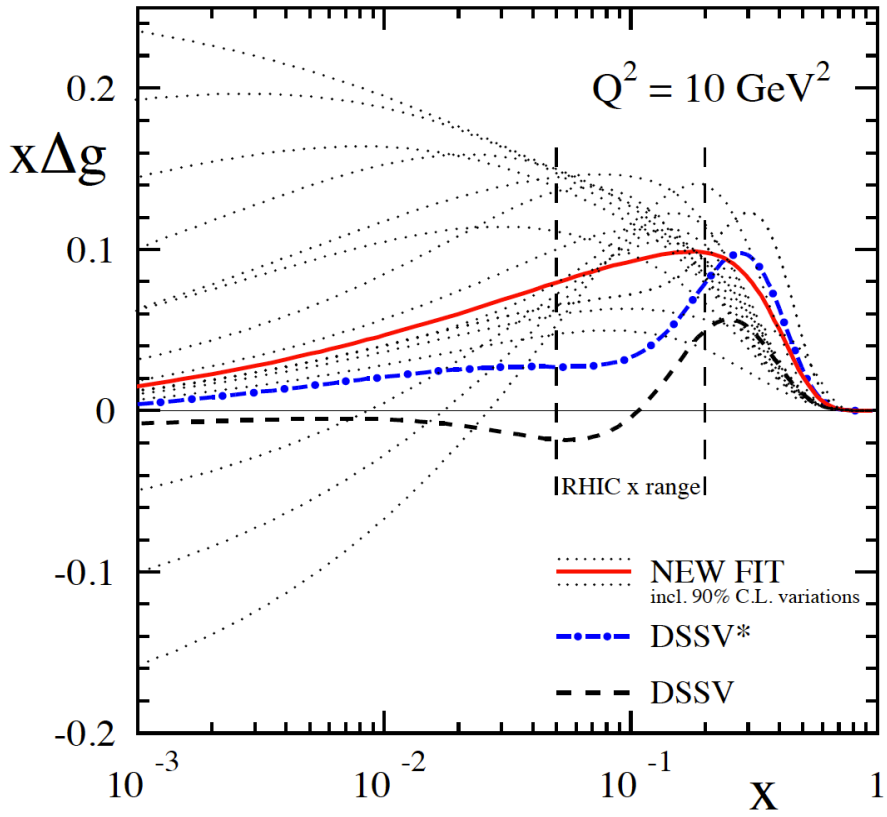




→ see talk C. Aidala

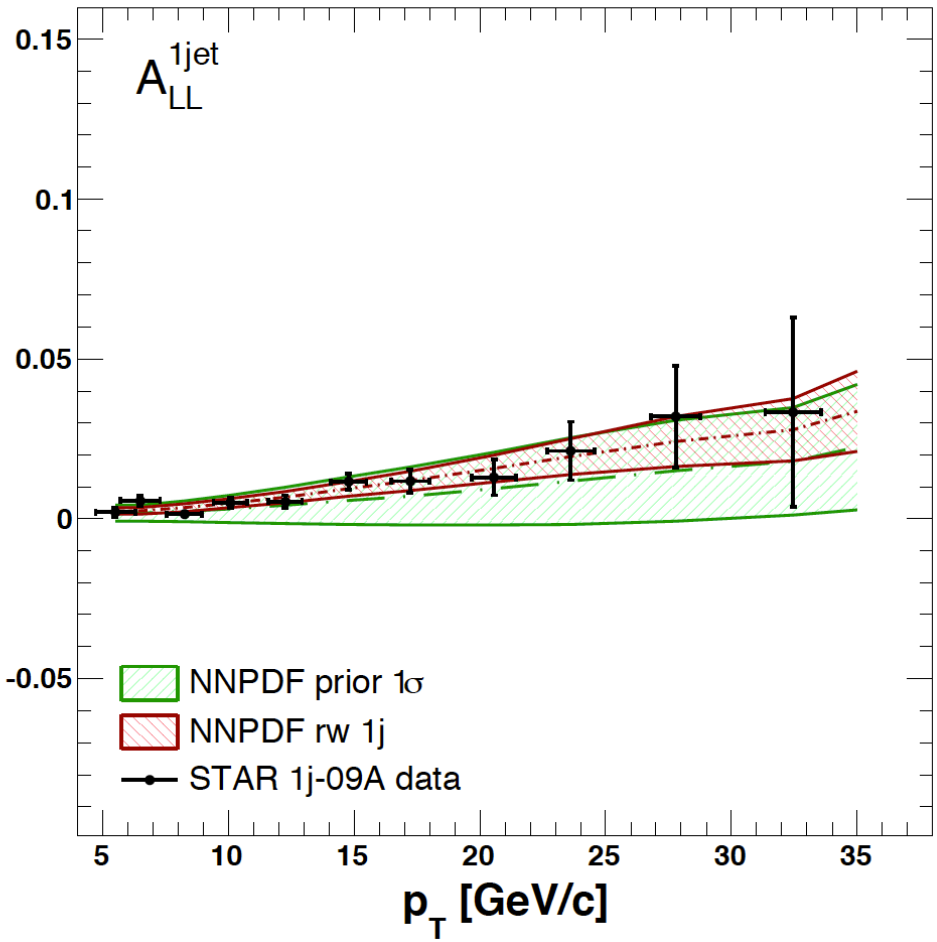
DSSV





→ see talk M. Stratmann

NNPDF

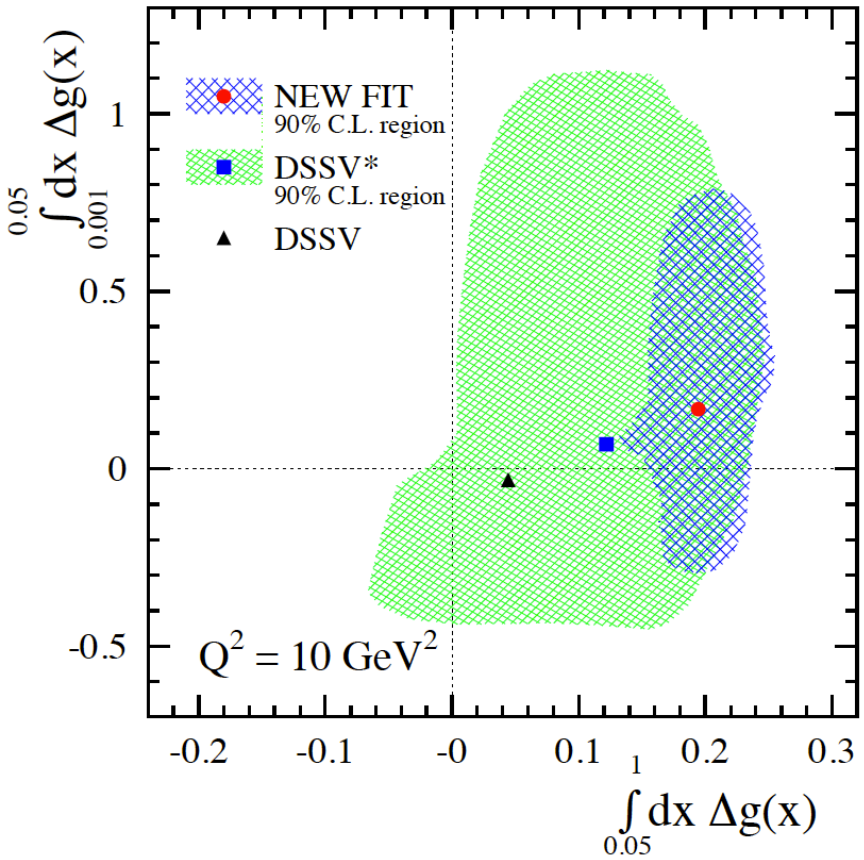


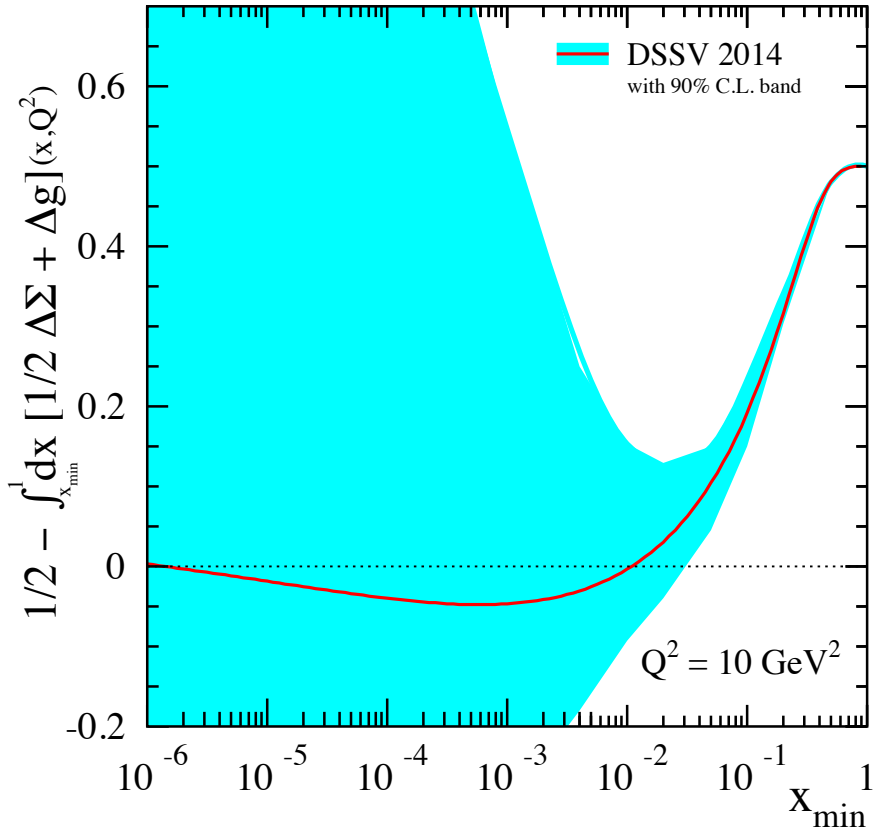
reweighting based on 1000 replicas; about 250 survive

→ see talk E. Nocera

Implications:

DSSV



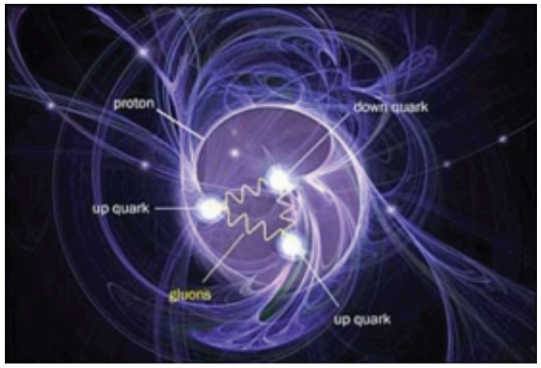


$$\frac{1}{2} \approx \frac{1}{2}\Delta\Sigma + \Delta G + 0\ ?$$

Proton Spin Mystery Gains a New Clue

Physicists long assumed a proton's spin came from its three constituent quarks. New measurements suggest particles called gluons make a significant contribution

Jul 21, 2014 | By Clara Moskowitz



Protons have a constant spin that is an intrinsic particle property like mass or charge. Yet where this spin comes from is such a mystery it's dubbed the "proton spin crisis." Initially physicists thought a proton's spin was the sum of the spins of its three constituent quarks. But a 1987 experiment showed that quarks can account for only a small portion of a proton's spin,

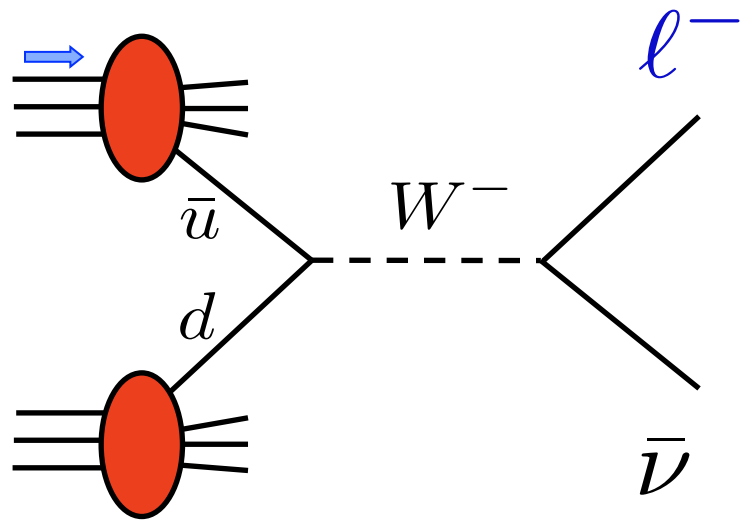
Mystery of 'proton spin' solved? Particle collider reveals that quarks AND gluons may hold answer to great subatomic puzzle

- Researchers using a collider in New York say they have solved 'spin' mystery
- Since an experiment in 1987 the origins of proton spin have been unknown
- It had once been thought to be cause exclusively by quarks
- But this was proved to be wrong in the failed experiment 27 years ago
- Now a new study says gluons play an important role in proton spin
- Could bring to a close one of the greatest mysteries of subatomic physics

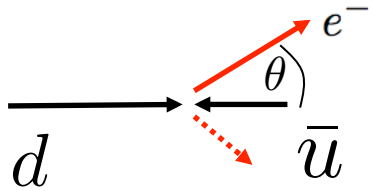
By JONATHAN O'CALLAGHAN

PUBLISHED: 09:23 EST, 22 July 2014 | **UPDATED:** 09:37 EST, 22 July 2014

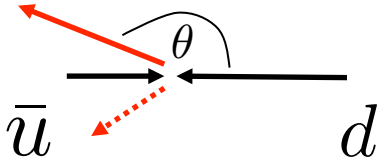
W bosons at RHIC



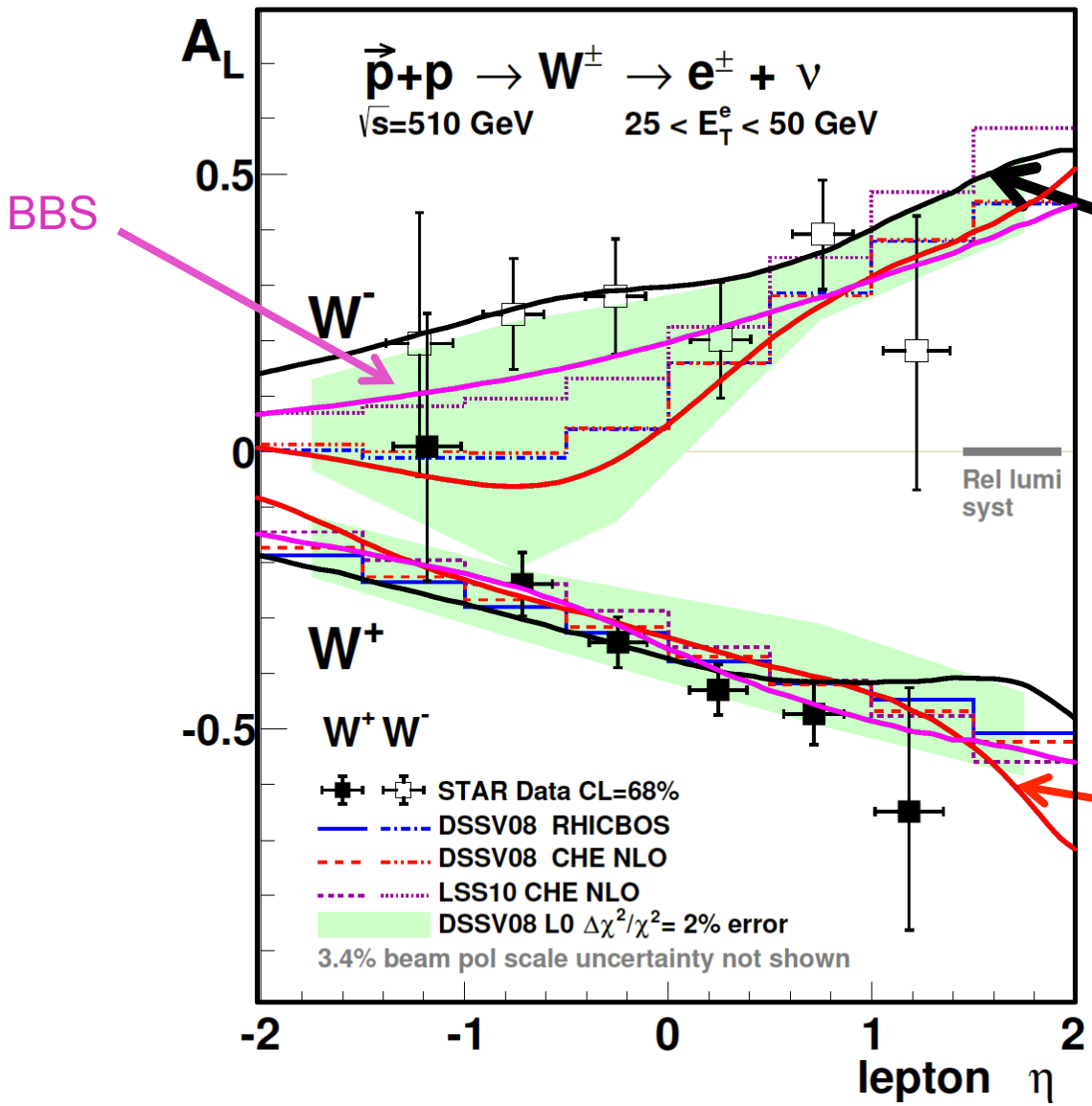
$$A_L^{e^-} \sim \frac{\Delta\bar{u}(x_1)d(x_2)(1-\cos\theta)^2 - \Delta d(x_1)\bar{u}(x_2)(1+\cos\theta)^2}{\bar{u}(x_1)d(x_2)(1-\cos\theta)^2 + d(x_1)\bar{u}(x_2)(1+\cos\theta)^2}$$



$$\sim (1+\cos\theta)^2$$



$$\sim (1-\cos\theta)^2$$



BBS

GRSV 2000
Based on:

$$\Delta u_v + \Delta d_v = 3F - D$$

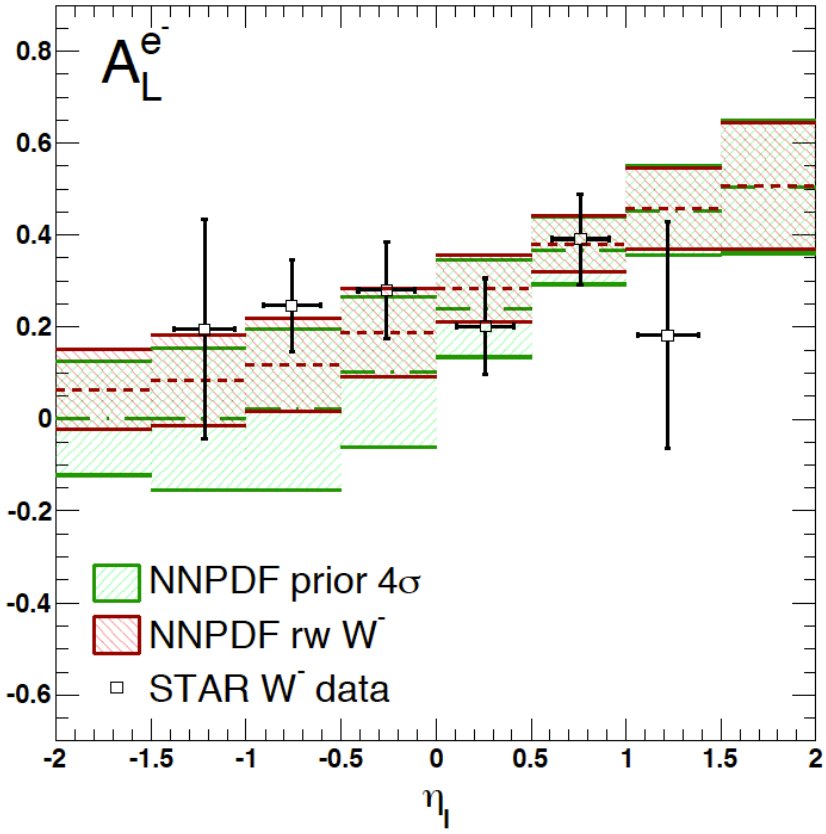
$$\frac{\Delta \bar{d}}{\Delta \bar{u}} = \frac{\Delta u}{\Delta d}$$

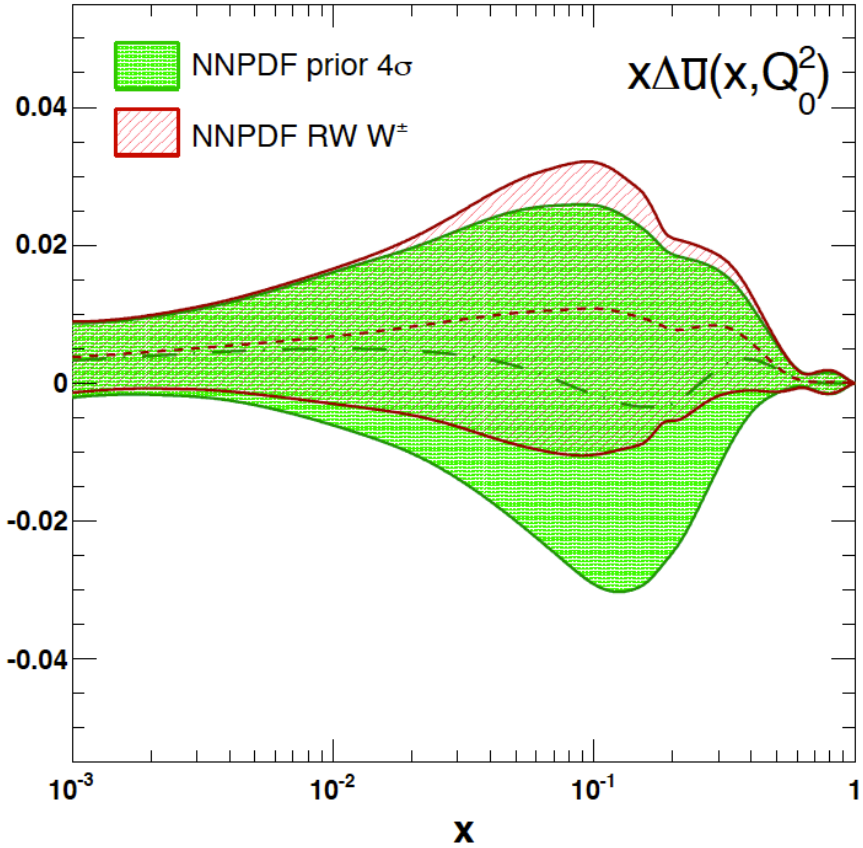
(Glück, Reya)

DSSV '14

(new NLO calc.
by F. Ringer)

NNPDF


$A_L^{e^-}$
NNPDF prior 4σ
NNPDF rw W⁻
STAR W⁻ data
η_l


NNPDF prior 4σ
NNPDF RW W±
$x\Delta\bar{u}(x,Q_0^2)$
x

NNLO evolution

Moch, Rogal, Vermaseren, Vogt 2008
Moch, Vermaseren, Vogt 2014

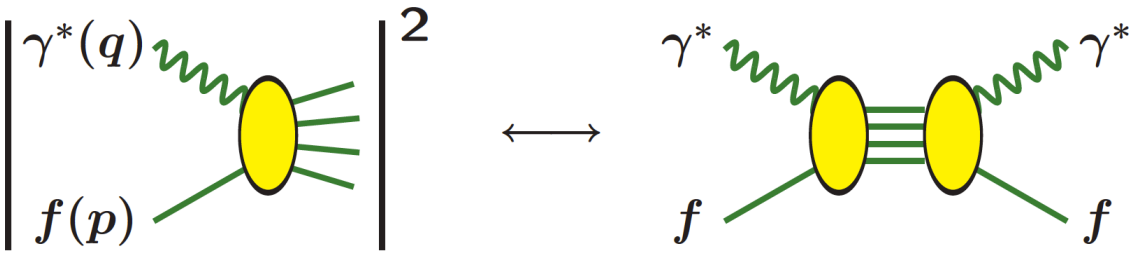
- DGLAP evolution:

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \begin{pmatrix} \Delta q(x,\mu^2) \\ \Delta g(x,\mu^2) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg} \\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg} \end{pmatrix} \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix} \left(\frac{x}{z}, \mu^2\right)$$

$$\Delta\mathcal{P}_{ij} = \frac{\alpha_s}{2\pi} \Delta P_{ij}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \Delta P_{ij}^{(2)} + \ldots$$

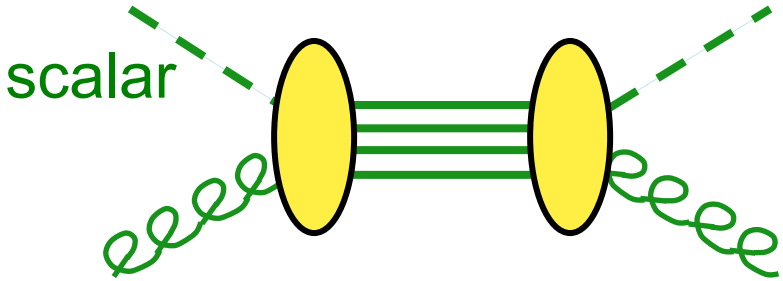
Ahmed,Ross
Altarelli,Parisi,…
1977

Mertig, van Neerven
WV 1995



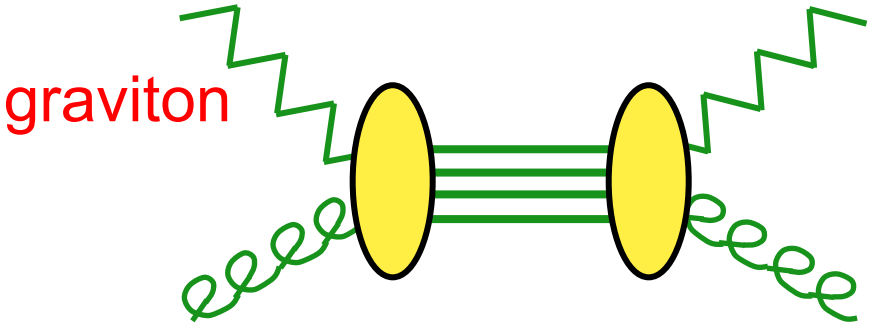
$\longrightarrow (\Delta)P^N_{qq},\ (\Delta)P^N_{qg}$

Gluons in unpol. case:



$\longrightarrow P^N_{gq},\ P^N_{gg}$

Pol. case:



$\longrightarrow \Delta P^N_{gq},\ \Delta P^N_{gg}$

- however, tensor-structure → huge number of integrals

- one can prove that final answer contains only harmonic sums, e.g.,

$$S_m(N) = \sum_{i=1}^{N} \frac{1}{i^m}$$

with integer coefficients (in suitable overall normalization)

- → if enough fixed moments are known, one can reconstruct full result for arbitrary N

C_A^3 part of $\Delta P_{gq}^{(2)}$:

$$N = 25: \; -\frac{189047325528380293767883074510292186993863 7}{23^4 19^4 17^4 13^5 11^4 7^4 5^{10} 3^5 2^{12}}$$

$$
\begin{aligned}
\Delta P_{gg}^{(2)}(x) \;=\; & 16 C_A^3 \Big(4\Delta p_{gg}(-x)\,(-11/8\,\zeta_2^2 + \mathrm{H}_{-3,0} - 4\mathrm{H}_{-2}\zeta_2 - 2\mathrm{H}_{-2,-1,0} + 3\mathrm{H}_{-2,2} \\
& + 9/2\,\mathrm{H}_{-2,0,0} - 3\mathrm{H}_{-1}\zeta_3 - 2\mathrm{H}_{-1,-2,0} + 4\mathrm{H}_{-1,-1}\zeta_2 - 6\mathrm{H}_{-1,-1,0,0} - 4\mathrm{H}_{-1,-1,2} \\
& - 9/2\,\mathrm{H}_{-1,0}\zeta_2 + 4\mathrm{H}_{-1,0,0,0} + \mathrm{H}_{-1,2,0} + 4\mathrm{H}_{-1,3} + 5/4\,\mathrm{H}_0\zeta_3 + 2\mathrm{H}_{0,0}\zeta_2 - \mathrm{H}_{0,0,0,0} \\
& - 1/2\,\mathrm{H}_2\zeta_2 - 1/2\,\mathrm{H}_{3,0} - 2\mathrm{H}_4 + 11/24\,\mathrm{H}_0\zeta_2 + 67/36\,(\zeta_2 + 2\mathrm{H}_{-1,0} - \mathrm{H}_{0,0})) \\
& + 4\Delta p_{gg}(x)\,(245/96 - 3/40\,\zeta_2^2 - \mathrm{H}_{-3,0} + 3/2\,\mathrm{H}_{-2}\zeta_2 + \mathrm{H}_{-2,-1,0} - \mathrm{H}_{-2,0,0} - \mathrm{H}_{-2,2} \\
& - 7/4\,\mathrm{H}_0\zeta_3 - 2\mathrm{H}_{0,0}\zeta_2 + \mathrm{H}_{0,0,0,0} - 3/2\,\mathrm{H}_1\zeta_3 - \mathrm{H}_{1,-2,0} - 3/2\,\mathrm{H}_{1,0}\zeta_2 + 2\mathrm{H}_{1,0,0,0} \\
& + 2\mathrm{H}_{1,1,0,0} + 2\mathrm{H}_{1,2,0} + 2\mathrm{H}_{1,3} - \mathrm{H}_2\zeta_2 + 5/2\,\mathrm{H}_{2,0,0} + 2\mathrm{H}_{2,1,0} + 2\mathrm{H}_{2,2} + 5/2\,\mathrm{H}_{3,0} \\
& + 2\mathrm{H}_{3,1} + 2\mathrm{H}_4 + 11/12\,\zeta_3 + 11/12\,\mathrm{H}_{-2,0} + 11/24\,\mathrm{H}_{1,0,0} + 11/24\,\mathrm{H}_3 \\
& - 67/36\,(\zeta_2 - \mathrm{H}_{0,0} - 2\mathrm{H}_{1,0} - 2\mathrm{H}_2) + 1/24\,\mathrm{H}_0) - 1/3\,(72 - 185x - 22x^2)\,\mathrm{H}_0\zeta_2 \\
& - 1/3\,(32 - 161x - 11x^2)\,\mathrm{H}_{-2,0} + 4\,(1 - 5x)\,\mathrm{H}_{-3,0} - 1/6\,(312 - 393x - 55x^2)\,\zeta_3 \\
& + (1-x)\,(5579/18 + 4\mathrm{H}_{-2}\zeta_2 + 8\mathrm{H}_{-2,-1,0} + 12\mathrm{H}_{-2,0,0} - 21/2\,\mathrm{H}_1\zeta_2 + 37\mathrm{H}_{1,0,0} \\
& + 1/18\,\mathrm{H}_1 - 19/2\,\mathrm{H}_{1,0}) - 1/5\,(43 + 33x)\,\zeta_2^2 - 8\,(1 + 3x)\,\mathrm{H}_0\zeta_3 - 2\,(11 + 13x)\,\mathrm{H}_{0,0}\zeta_2 \\
& + (1+x)\,(21\mathrm{H}_{-1,-1,0} - 25/2\,\mathrm{H}_{-1}\zeta_2 + 65\mathrm{H}_{-1,0,0} + 23\mathrm{H}_{-1,2} - 4\mathrm{H}_2\zeta_2 + 10\mathrm{H}_{2,0,0} \\
& + 16\mathrm{H}_{3,0} + 26\mathrm{H}_4 - 215/3\,\mathrm{H}_{-1,0}) - 1/9\,(74 - 97x)\,\mathrm{H}_2 + 1/3\,(77 - 115x)\,\mathrm{H}_{2,0} \\
& + 1/3\,(40 - 185x - 11x^2)\,\mathrm{H}_3 - 1/9\,(571 + 97x)\,\zeta_2 + 1/3\,(158 - 87x - 11x^2)\,\mathrm{H}_{0,0,0} \\
& + 1/12\,(1019 - 1489x)\,\mathrm{H}_{0,0} + 1/216\,(24625 + 40069x)\,\mathrm{H}_0 - 11/6\,(x^{-1} - x^2)\,\mathrm{H}_1\zeta_2 \\
& + 28\mathrm{H}_{0,0,0,0} - 11/2\,(x^{-1} + x^2)\,(\mathrm{H}_{-1}\zeta_2 + 2/3\,\mathrm{H}_{-1,-1,0} - 2/3\,\mathrm{H}_{-1,0,0} - 2/3\,\mathrm{H}_{-1,2}) \\
& + \delta(1-x)\,(79/32 - 5\zeta_5 + 67/6\,\zeta_3 + 1/6\,\zeta_2 - \zeta_2\zeta_3 + 11/24\,\zeta_2^2)\Big) \\
& + 8 C_A^2 n_f \Big(2/3\,\Delta p_{gg}(x)\,(10/3\,\zeta_2 - 10/3\,\mathrm{H}_{0,0} - 20/3\,\mathrm{H}_{1,0} - 20/3\,\mathrm{H}_2 - 209/36 - 8\zeta_3 \\
& - 2\mathrm{H}_{-2,0} - \mathrm{H}_{1,0,0} - \mathrm{H}_3 - 1/2\,\mathrm{H}_0) + 2/9\,\Delta p_{gg}(-x)\,(10\mathrm{H}_{0,0} - 10\zeta_2 - 20\mathrm{H}_{-1,0} \\
& - 3\mathrm{H}_0\zeta_2) - 1/6\,(51 - 61x - 16x^2)\,\mathrm{H}_0\zeta_2 - 1/18\,(146 + 227x + 36x^2)\,\mathrm{H}_{0,0} \\
& - 1/3\,(23 + 43x - 4x^2)\,\mathrm{H}_{-2,0} - 1/3\,(1 - 12x + 4x^2)\,\mathrm{H}_{0,0,0} - 2\,(1 - 5x)\,\mathrm{H}_{-3,0} \\
& + 2\,(1-x)\,(512/9 + 3\mathrm{H}_{-2}\zeta_2 + 6\mathrm{H}_{-2,-1,0} - 3\mathrm{H}_{-2,0,0} - 11/2\,\mathrm{H}_1\zeta_2 + 11/4\,\mathrm{H}_{1,0,0} \\
& + 1087/72\,\mathrm{H}_1 - 2\mathrm{H}_{1,0}) + (1+x)\,(7\mathrm{H}_{-1}\zeta_2 + 22\mathrm{H}_{-1,-1,0} - 9\mathrm{H}_{-1,0,0} + 4\mathrm{H}_{-1,2} \\
& - 4/3\,\mathrm{H}_{2,0} - 6\mathrm{H}_2\zeta_2 + 3\mathrm{H}_{2,0,0} + 3\mathrm{H}_4 - 19\mathrm{H}_{-1,0}) - 2/39\,(507 - 195x - 65x^2)\,\zeta_3 \\
& - 1/18\,(499 + 301x - 36x^2)\,\zeta_2 + 3/10\,(13 + 23x)\,\zeta_2^2 + 1/6\,(5 - 61x - 8x^2)\,\mathrm{H}_3 \\
& - (5 + 3x)\,\mathrm{H}_{0,0}\zeta_2 + 1/18\,(157 + 301x)\,\mathrm{H}_2 + 1/108\,(2422 + 7609x)\,\mathrm{H}_0 - 12\mathrm{H}_0\zeta_3 \\
& - 2/3\,(x^{-1} - x^2)\,\mathrm{H}_1\zeta_2 - 2\,(x^{-1} + x^2)\,(\mathrm{H}_{-1}\zeta_2 + 2/3\,\mathrm{H}_{-1,-1,0} - 2/3\,\mathrm{H}_{-1,0,0} \\
& - 2/3\,\mathrm{H}_{-1,2} - \mathrm{H}_{-1,0}) + 2\mathrm{H}_{0,0,0,0} - 1/3\,\delta(1-x)\,(233/48 + 10\zeta_3 + \zeta_2 + 1/2\,\zeta_2^2)\Big) \\
& + 8/3\, C_A C_F n_f \Big(4\Delta p_{gg}(x)\,(3\zeta_3 - 55/16) + 3\,(1-x)\,(8\mathrm{H}_{-2,0,0} - 7507/27 - 16\mathrm{H}_{-2}\zeta_2 \\
& - 32\mathrm{H}_{-2,-1,0} + 30\mathrm{H}_1\zeta_2 - 29\mathrm{H}_{1,0,0} - 10\mathrm{H}_{1,1,0} - 10\mathrm{H}_{1,1,1} - 26/3\,\mathrm{H}_{1,0} - 65/6\,\mathrm{H}_{1,1} \\
& - 1127/18\,\mathrm{H}_1) + 6\,(1+x)\,(61/6\mathrm{H}_{-1,0} - 11\mathrm{H}_{-1}\zeta_2 - 30\mathrm{H}_{-1,-1,0} + 3\mathrm{H}_{-1,0,0} \\
& - 4\mathrm{H}_{-1,2} + 6\mathrm{H}_{0,0}\zeta_2 + 8\mathrm{H}_2\zeta_2 - 7\mathrm{H}_{2,0,0} - 2\mathrm{H}_{2,1,0} - 2\mathrm{H}_{2,1,1} - 4\mathrm{H}_{3,0} - \mathrm{H}_{3,1} - 6\mathrm{H}_4) \\
& + (125 + 38x - 20x^2)\,\zeta_3 + 1/6\,(848 + 341x - 108x^2)\,\zeta_2 - 1/18\,(8363 + 3362x)\,\mathrm{H}_0 \\
& - (181 + 88x - 8x^2)\,\mathrm{H}_{0,0,0} - 1/6\,(1723 - 692x - 108x^2)\,\mathrm{H}_{0,0} - 3/5\,(43 + 83x)\,\zeta_2^2 \\
& - (32 - 43x - 8x^2)\,\mathrm{H}_3 - 24\,(3 - 2x)\,\mathrm{H}_{0,0,0,0} + 6\,(9 - x)\,\mathrm{H}_0\zeta_3 - (19 - 11x)\,\mathrm{H}_{2,1} \\
& + 8\,(3 + 12x - x^2)\,\mathrm{H}_{-2,0} + (56 - 43x - 16x^2)\,\mathrm{H}_0\zeta_2 - 1/6\,(482 + 341x)\,\mathrm{H}_2 \\
& - (38 - 37x)\,\mathrm{H}_{2,0} + 4\,(x^{-1} - x^2)\,\mathrm{H}_1\zeta_2 + 4\,(x^{-1} + x^2)\,(3\mathrm{H}_{-1}\zeta_2 + 2\mathrm{H}_{-1,-1,0} \\
& - 2\mathrm{H}_{-1,0,0} - 2\mathrm{H}_{-1,2} - 9/2\,\mathrm{H}_{-1,0}) - 48x\mathrm{H}_{-3,0} - 241/48\,\delta(1-x)\Big) \\
& + 8 C_F^2 n_f \Big(8\,(1-x)\,(\mathrm{H}_{-2}\zeta_2 + 1 + 2\mathrm{H}_{-2,-1,0} - \mathrm{H}_{-2,0,0} - 2\mathrm{H}_1\zeta_2 + 11/8\,\mathrm{H}_{1,0,0} \\
& + 5/4\,(\mathrm{H}_{1,1,0} + \mathrm{H}_{1,1,1}) - 7/8\,\mathrm{H}_{1,0} + 13/16\,\mathrm{H}_{1,1} + 41/16\,\mathrm{H}_1) + 4\,(1+x)\,(4\mathrm{H}_{-1}\zeta_2 \\
& + 8\mathrm{H}_{-1,-1,0} - 4\mathrm{H}_{-1,0,0} + \mathrm{H}_{0,0}\zeta_2 - \mathrm{H}_{0,0,0,0} - 2\mathrm{H}_2\zeta_2 + 3/2\,\mathrm{H}_{2,0,0} + \mathrm{H}_{2,1,0} + \mathrm{H}_{2,1,1} \\
& + 1/2\,\mathrm{H}_{3,1} - \mathrm{H}_4 + 5/2\,\mathrm{H}_{-1,0}) + (8 - 19/2x + 4x^2)\,\zeta_2 - (23 + 3/2x + 4x^2)\,\mathrm{H}_{0,0} \\
& + (9 + 13x)\,\zeta_2^2 - 2\,(1 - 7x)\,\mathrm{H}_0\zeta_3 + 2\,(2 - 3x)\,\mathrm{H}_{2,1} + 2\,(4 - x)\,(\mathrm{H}_0\zeta_2 - \mathrm{H}_3) \\
& - 2\,(3 + 4x)\,\mathrm{H}_{2,0} + (2 + 19/2x)\,\mathrm{H}_2 - 5/2\,(5 - 2x)\,\mathrm{H}_0 - 2\,(7 - 3x)\,\mathrm{H}_{0,0,0} \\
& - 2\,(5 + 21x)\,\zeta_3 + 4\,(x^{-1} + x^2)\,\mathrm{H}_{-1,0} - 16x\,(2\mathrm{H}_{-2,0} - \mathrm{H}_{-3,0}) + 1/8\,\delta(1-x)\Big) \\
& + 2/27\, C_A n_f^2 \Big(-8\,\Delta p_{gg}(x) + 48\,(1+x)\,(\zeta_2 - 1/2\,\mathrm{H}_{0,0} - \mathrm{H}_2) - 3\,(1-x)\,(33 + 41\,\mathrm{H}_1) \\
& - (56 - 67x)\,\mathrm{H}_0 + 87/4\,\delta(1-x)\Big) \\
& + 2/27\, C_F n_f^2 \Big(-4\,(1-x)\,(146 + 90\mathrm{H}_{1,0} + 45\mathrm{H}_{1,1} + 78\mathrm{H}_1) - 72\,(1+x)\,(\zeta_3 - 2\mathrm{H}_0\zeta_2 \\
& + \mathrm{H}_{0,0,0} + 2\mathrm{H}_{2,0} + \mathrm{H}_{2,1} + 2\mathrm{H}_3) + 24\,(13 - 8x)\,(\zeta_2 - \mathrm{H}_2) - 12\,(7 - 23x)\,\mathrm{H}_{0,0} \\
& - 52\,(5 - x)\,\mathrm{H}_0 + 33/2\,\delta(1-x)\Big)\,.
\end{aligned}
$$

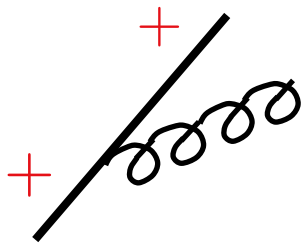
$$\Delta P_{gg}^{(2)}(x)$$

- another important aspect: (dimensional regularization) $\gamma^5, \ \varepsilon^{\mu\nu\rho\sigma}$

- ‘t Hooft-Veltman or Larin treatments

- already an issue at NLO WV 1995

LO:



$$P_{qq}^{(0)}(x) \; = \; \Delta P_{qq}^{(0)}(x) - 4C_F\,\epsilon(1-x)$$

($d = 4 - 2\epsilon$ dimensions)

- requires one to do scheme transformation at NLO to avoid problems w/ non-singlet axial current and Bj sum rule

Moch, Rogal, Vermaseren, Vogt

- discovery of interesting large-x feature:

LO: $$P_{ij}^{(0)}(x) - \Delta P_{ij}^{(0)}(x) \sim (1-x)^2 + \ldots$$

- in accordance with large-x helicity arguments

Brodsky, Burkardt, Schmidt

NLO:

$$P_{qq}^{(1)}(x) - \Delta P_{qq}^{(1)}(x) \sim (1-x) + \ldots$$

$$P_{gg}^{(1)}(x) - \Delta P_{gg}^{(1)}(x) \sim (1-x) + \ldots$$

$$P_{qg}^{(1)}(x) - \Delta P_{qg}^{(1)}(x) \sim (1-x)^2 + \ldots$$

$$P_{gq}^{(1)}(x) - \Delta P_{gq}^{(1)}(x) \sim \log(1-x) + \mathrm{const.} + \ldots \quad ?!$$

- can be “repaired” by simple fact. scheme transformation:

$$\Delta\tilde{P}^{(1)}_{qq} = \Delta P^{(1)}_{qq} - \beta_0\, z^{(1)}_{qq} - \Delta P^{(0)}_{qg}\, z^{(1)}_{gq}$$

$$\Delta\tilde{P}^{(1)}_{qg} = \Delta P^{(1)}_{qg} + \Delta P^{(0)}_{qg}\, z^{(1)}_{qq}$$

$$\Delta\tilde{P}^{(1)}_{gq} = \Delta P^{(1)}_{gq} - \Delta P^{(0)}_{gq}\, z^{(1)}_{qq} + \left(\Delta P^{(0)}_{qq} - \Delta P^{(0)}_{gg} - \beta_0\right) z^{(1)}_{gq}$$

$$\Delta\tilde{P}^{(1)}_{gg} = \Delta P^{(1)}_{gg} + \Delta P^{(0)}_{qg}\, z^{(1)}_{gq}$$

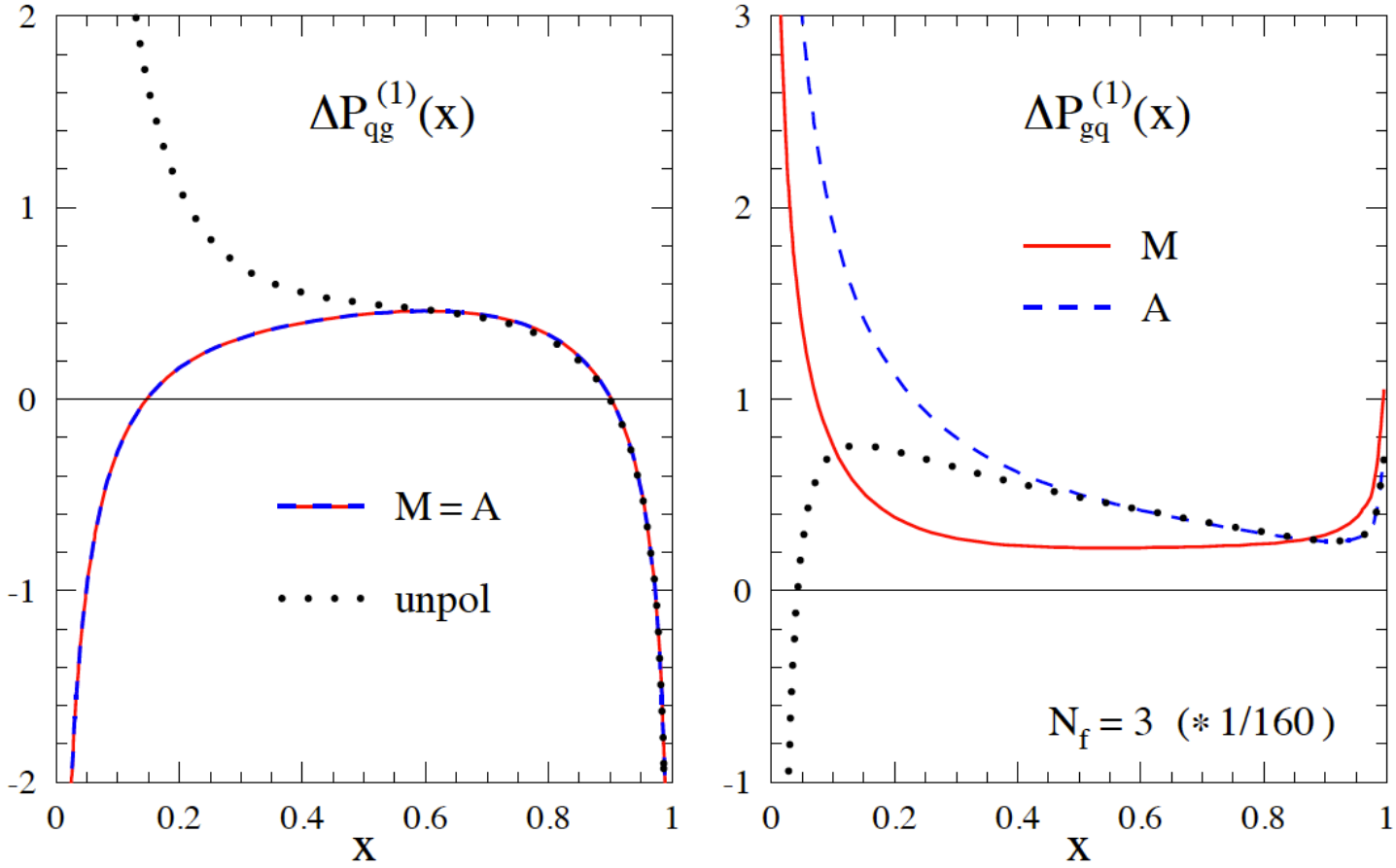
where $z^{(1)}_{qq} = -4C_F(1-x)$ $\quad z^{(1)}_{gq} = -C_F(2-x)$

- after this:

$$P^{(1)}_{gq}(x) - \Delta P^{(1)}_{gq}(x) \sim (1-x)^2$$

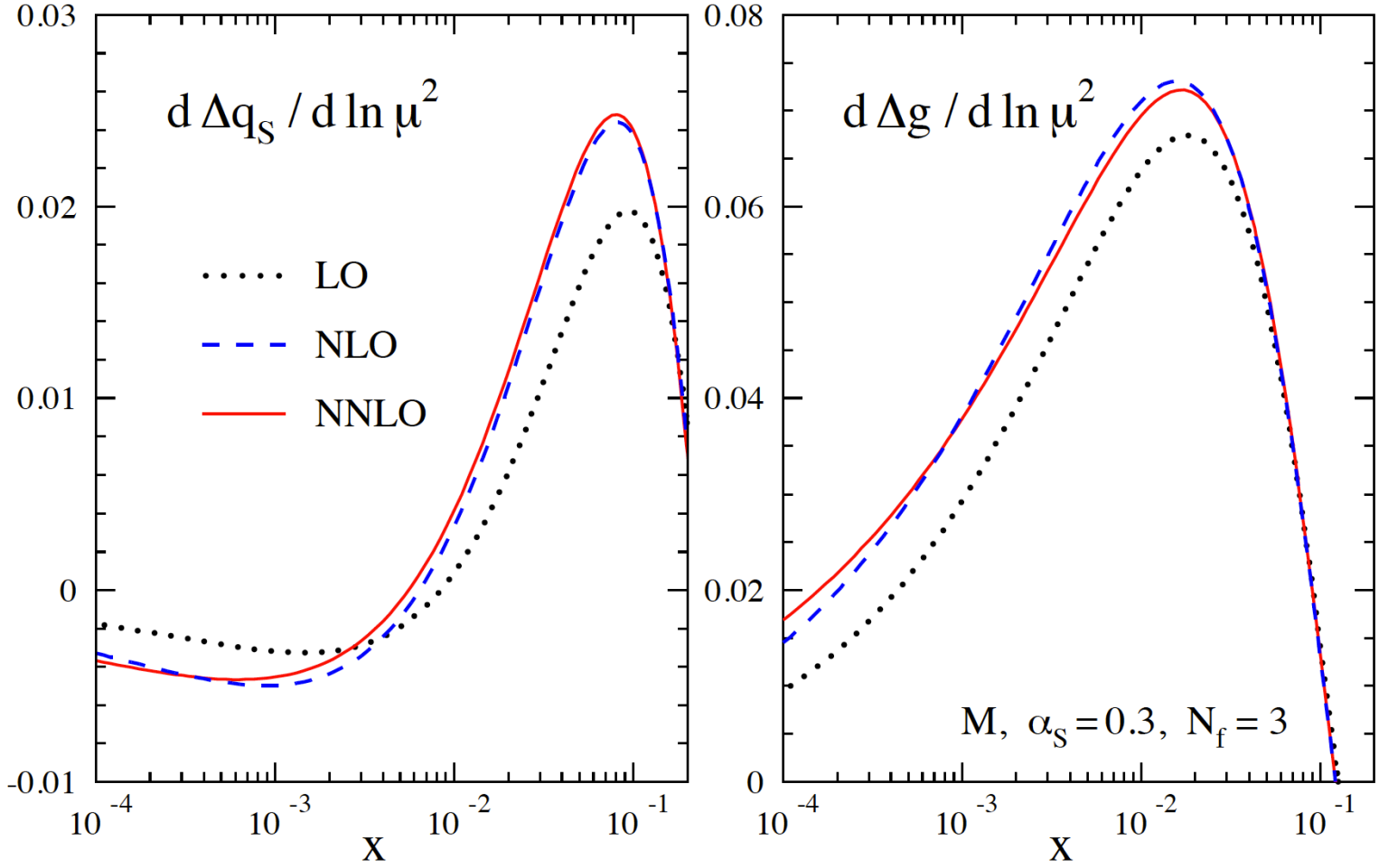
- can be extended to NNLO

Moch, Vermaseren, Vogt 2014



NLO

effects of NNLO evolution

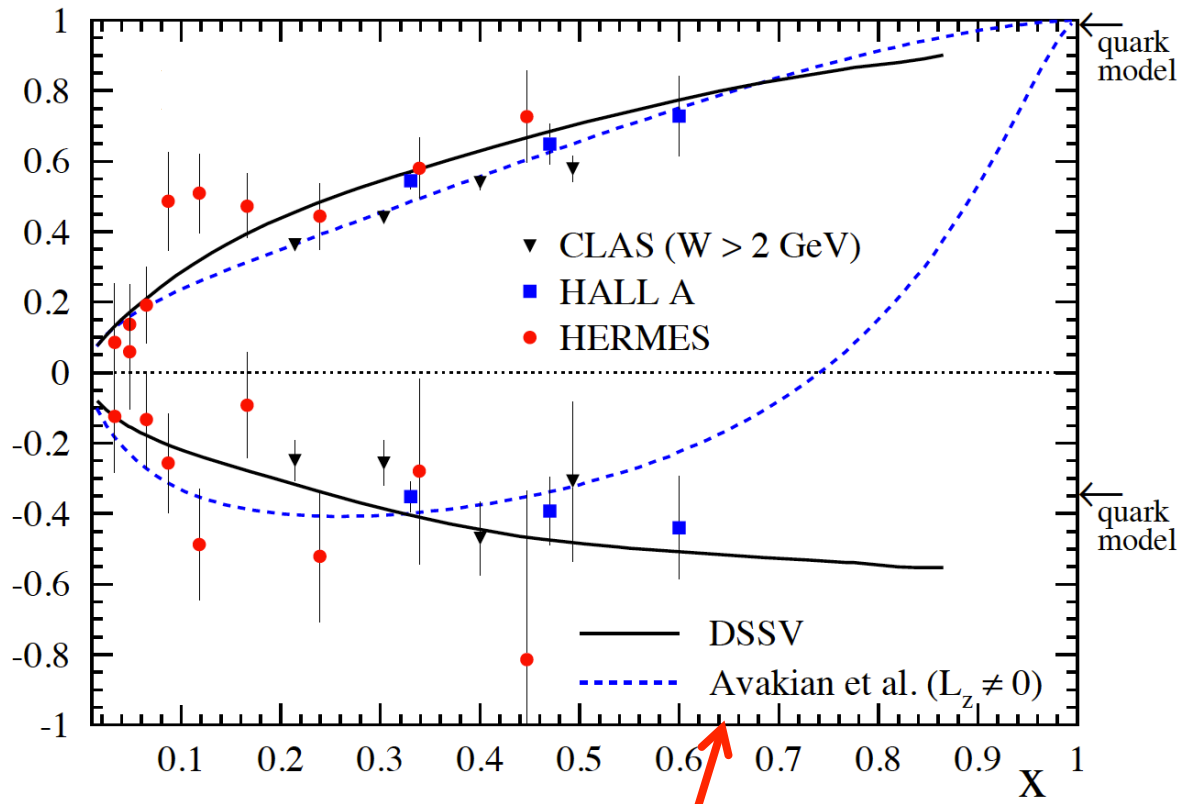


Toward high x

DSSV '08

$$\frac{\Delta u + \Delta \bar{u}}{u + \bar{u}}$$

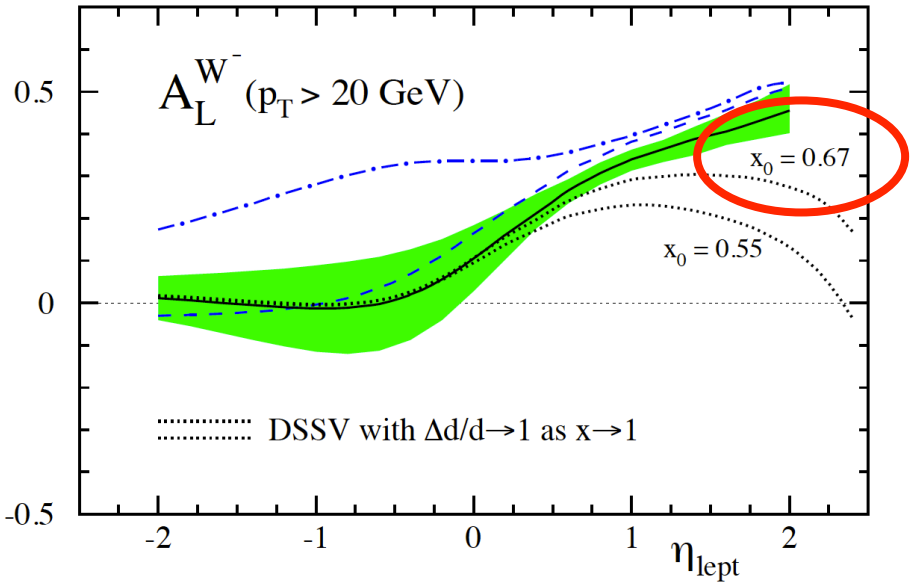
$$\frac{\Delta d + \Delta \bar{d}}{d + \bar{d}}$$



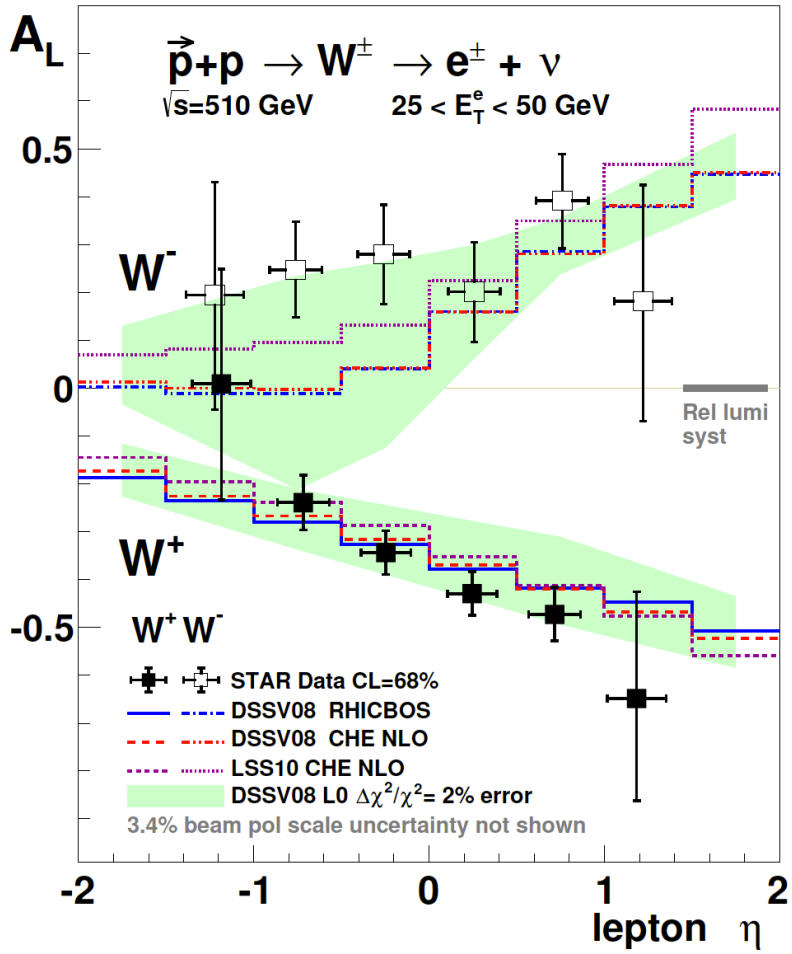
$$d^+(x) = \frac{1}{x^\alpha}\left[A_d(1-x)^3 + B_d(1-x)^4\right]$$

$$d^-(x) = \frac{1}{x^\alpha}\left[C_d(1-x)^5 + C'_d(1-x)^5 \log^2(1-x) + D_d(1-x)^6\right]$$

Avakian, Brodsky, Deur, Yuan

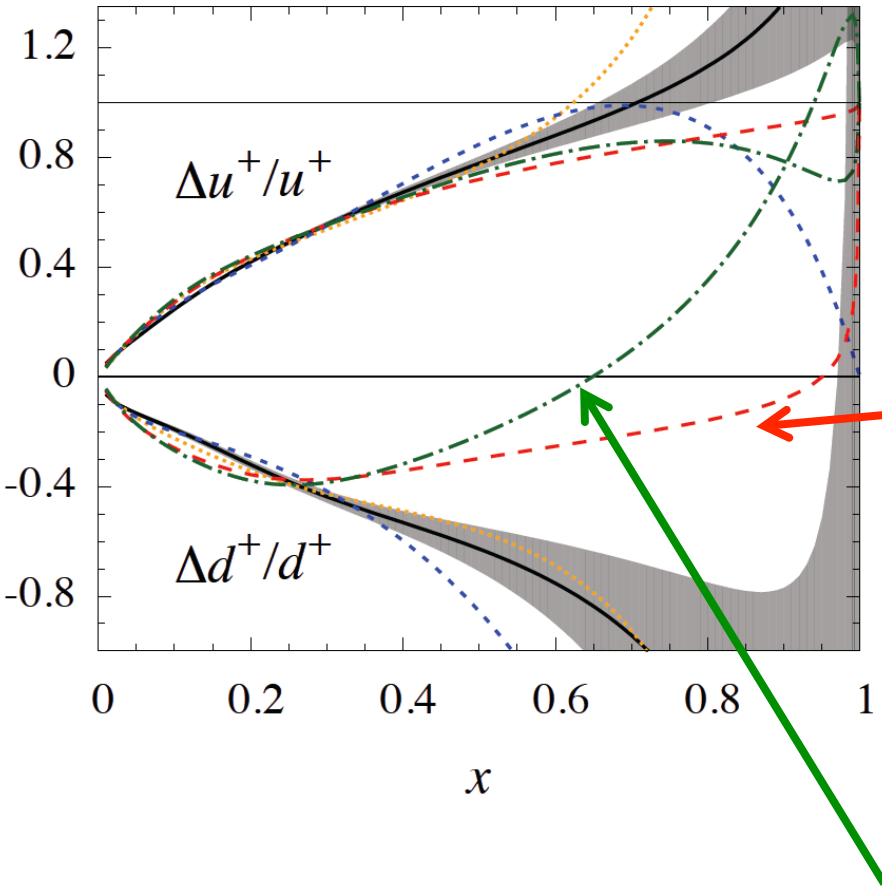


~4 units in χ^2



→ bright future also at JLab12

- recent refined analysis: Jimenez-Delgado, Avakian, Melnitchouk



force
$\Delta d/d \rightarrow 1$

$$\Delta q + \Delta\bar{q} = N x^{\alpha}(1-x)^{\beta} + N' x^{\alpha'}(1-x)^{\beta'} \log^2(1-x)$$

- however: perturbation theory also generates $\log^2(1-x)$:

$$\mathcal{F}_i(x,Q^2) = \sum_f \int_x^1 \frac{d\hat{x}}{\hat{x}} f\left(\frac{x}{\hat{x}},\mu^2\right) \mathcal{C}_f^i\left(\hat{x},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)$$

NLO:

$$\mathcal{C}_q^i = e_q^2 \left[\delta(1-\hat{x}) + \frac{\alpha_s}{2\pi} C_F \left\{(1+\hat{x}^2)\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - \frac{3}{2}\frac{1}{(1-\hat{x})_+} + \ldots\right\}\right]$$

where

$$\int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ f\left(\frac{x}{\hat{x}}\right) = \int_x^1 d\hat{x}\, \frac{\ln(1-\hat{x})}{1-\hat{x}} \left(\frac{1}{\hat{x}} f\left(\frac{x}{\hat{x}}\right) - f(x)\right) + \frac{1}{2}\log^2(1-x)\, f(x)$$

Same logarithms for unpol. and pol. cases

Logarithms recur at all orders of perturbation theory

- Mellin moments: $$\alpha_s \left(\frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ \longleftrightarrow \alpha_s \log^2(N)$$

- k^{th} order of perturbation theory:

$$\alpha_s^k \log^{2k}(N), \quad \alpha_s^k \log^{2k-1}(N), \quad \alpha_s^k \log^{2k-2}(N), \ \ldots$$

- can be resummed to all orders: Sterman; Catani, Trentadue; …

Fixed Order

Resummation

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
…	…	…	…	…	…	
N^kLO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	…

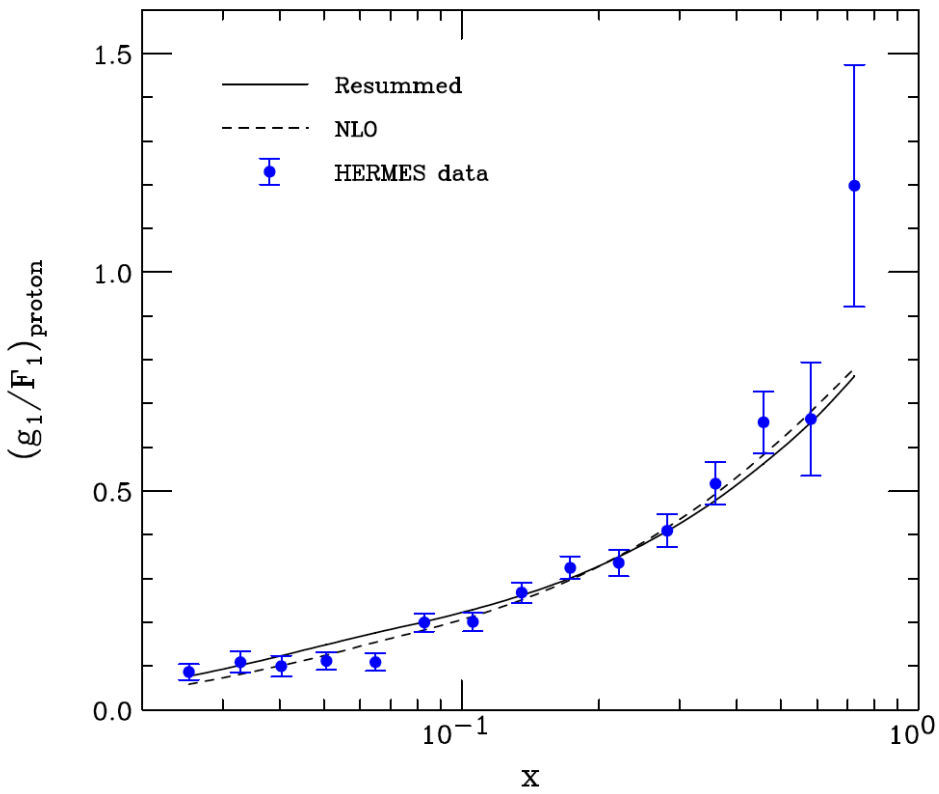
LL NLL NNLL

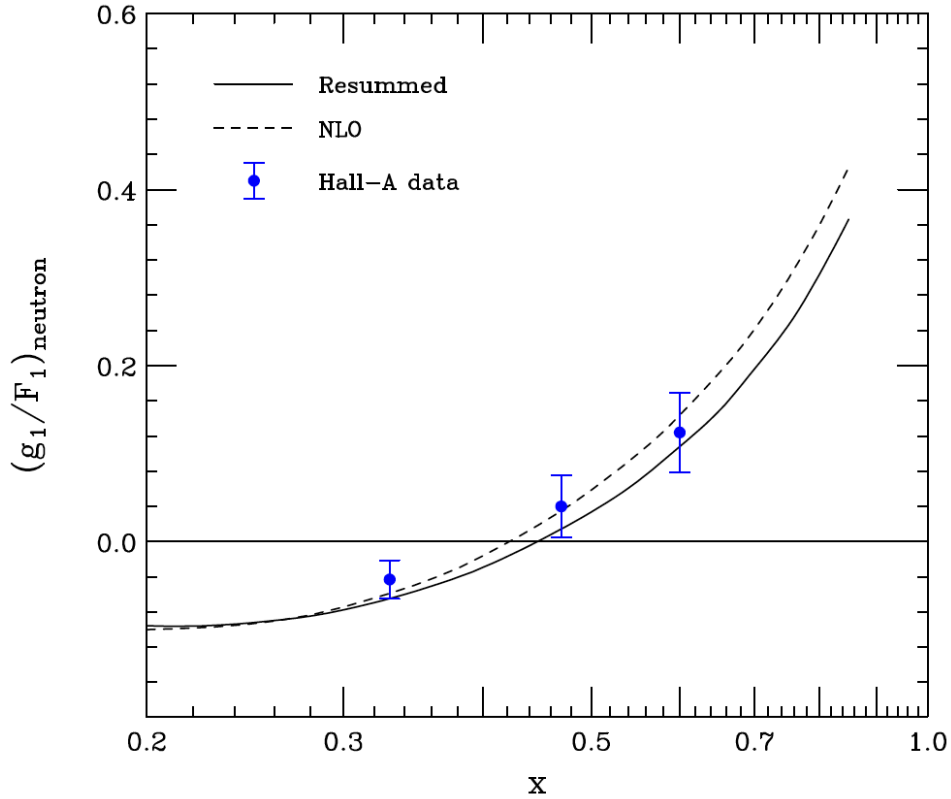
$$\mathcal{C}_q^{\text{resum}} \sim \exp\left[\int_0^1 d\xi \frac{\xi^N - 1}{1-\xi} \left\{ \int_{Q^2}^{(1-\xi)Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q(\alpha_s(k_\perp^2)) + \frac{1}{2} B_q\left(\alpha_s((1-\xi)Q^2)\right) \right\}\right]$$

$$A_q(\alpha_s) = \frac{\alpha_s}{\pi} C_F + \left(\frac{\alpha_s}{\pi}\right)^2 \ldots \qquad B_q(\alpha_s) = \frac{\alpha_s}{\pi}\left(-\frac{3}{2} C_F\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \ldots$$

- recent development: resummation for spin asymmetry

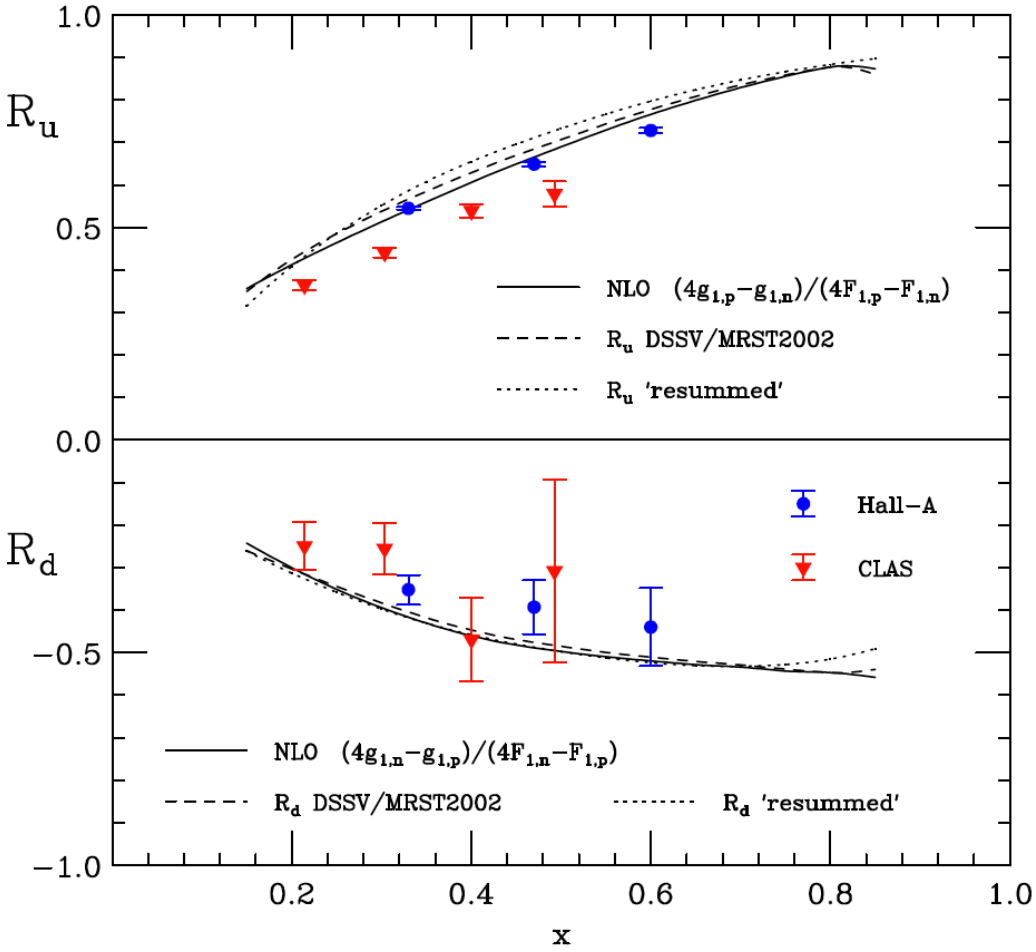
Anderle, Ringer, WV ’13;


Resummed
NLO
HERMES data
$(g_1/F_1)_{proton}$
x
1.5
1.0
0.5
0.0
10^{-1}
10^{0}


Resummed
NLO
Hall−A data
$(g_1/F_1)_{neutron}$
x
0.6
0.4
0.2
0.0
0.2
0.3
0.5
0.7
1.0

$$R_u \equiv \frac{\Delta u + \Delta \bar{u}}{u + \bar{u}}(x, Q^2) = \frac{4g_{1,\mathrm{p}} - g_{1,\mathrm{n}}}{4F_{1,\mathrm{p}} - F_{1,\mathrm{n}}}(x, Q^2)$$

$$R_d \equiv \frac{\Delta d + \Delta \bar{d}}{d + \bar{d}}(x, Q^2) = \frac{4g_{1,\mathrm{n}} - g_{1,\mathrm{p}}}{4F_{1,\mathrm{n}} - F_{1,\mathrm{p}}}(x, Q^2)$$



"resummed PDFs":

$$\tilde{q}^{N,\mathrm{res}}(Q^2) \equiv \frac{\tilde{\mathcal{C}}_q^{\mathrm{NLO}}(N, \alpha_s(Q^2))}{\tilde{\mathcal{C}}_q^{\mathrm{res}}(N, \alpha_s(Q^2))} \tilde{q}^{N,\mathrm{NLO}}(Q^2)$$

- another recent development: resummation for SIDIS

Anderle, Ringer, WV '13;
Sterman, WV

$$\mathcal{F}_i^h(x,z,Q^2) = \sum_{f,f'} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} f\left(\frac{x}{\hat{x}},\mu^2\right) D_{f'}^h\left(\frac{z}{\hat{z}},\mu^2\right) \mathcal{C}_{f'f}^i\left(\hat{x},\hat{z},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)$$

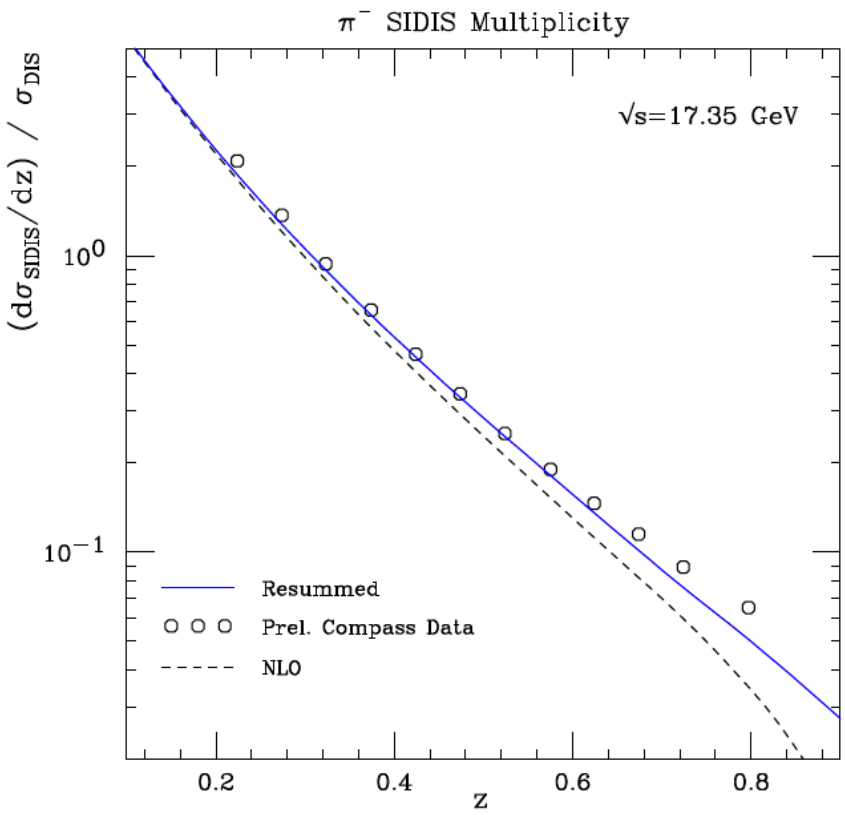
$$\alpha_s^k \left(\frac{\ln^m(1-\hat{x})}{1-\hat{x}}\right)_+ \left(\frac{\ln^n(1-\hat{z})}{1-\hat{z}}\right)_+ \qquad (m+n \leq 2k-2)$$

- → double Mellin moments:

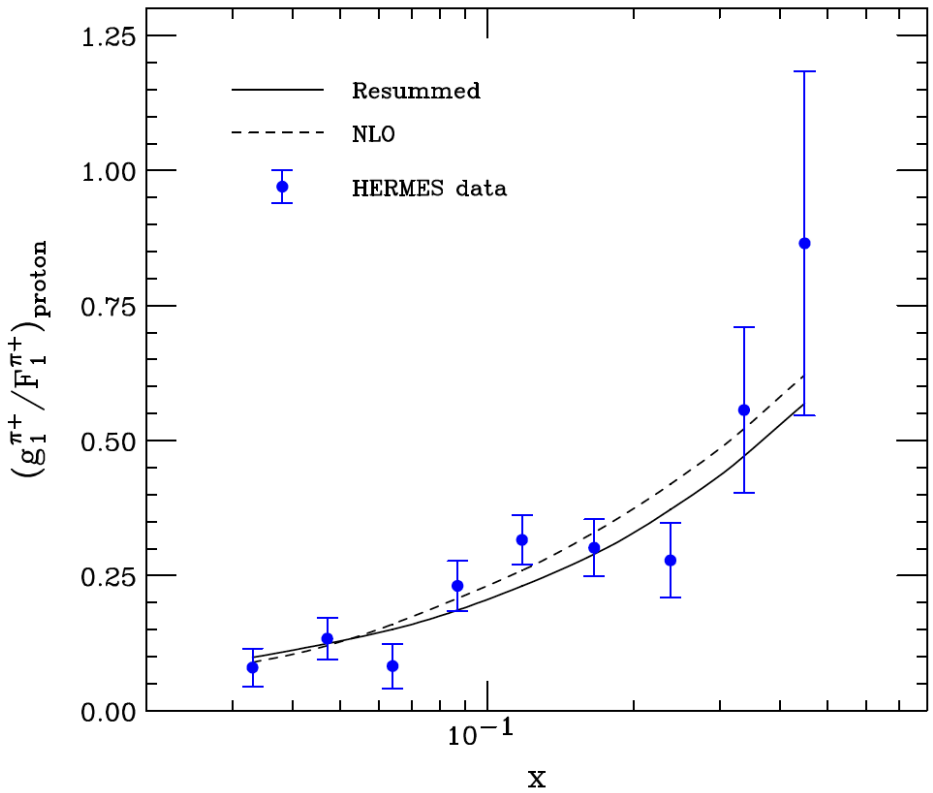
$$\mathcal{C}_{qq}^{T,\mathrm{res}}(N,M,\alpha_s(Q^2))$$

$$\propto \exp\left[\int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) \left\{\int_{\frac{k_\perp^2}{Q^2}}^1 \frac{d\xi}{\xi}\left[\mathrm{e}^{-N\xi - M\frac{k_\perp^2}{\xi Q^2}} - 1\right] + \ln\bar{N} + \ln\bar{M}\right\}\right]$$

SIDIS multiplicity



spin asymmetry:



Conclusions:

Many exciting new developments:

- state of the art **global** analyses: DSSV, NNPDF, …
- gluons may contribute (significantly) to proton spin!

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

25-30% 70% ??

- new information on nucleon sea (SIDIS vs $W^{\pm}$?)
- new frontiers of perturbation theory: NNLO, resummation