Transverse Force on Quarks in DIS

Matthias Burkardt

New Mexico State University

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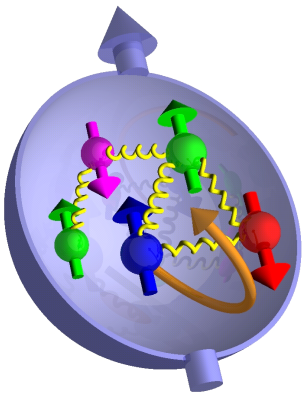
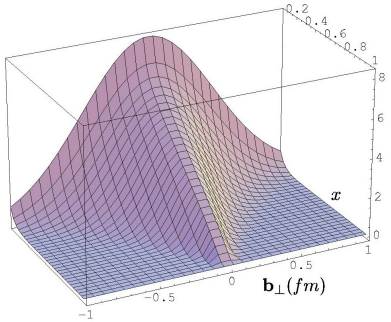
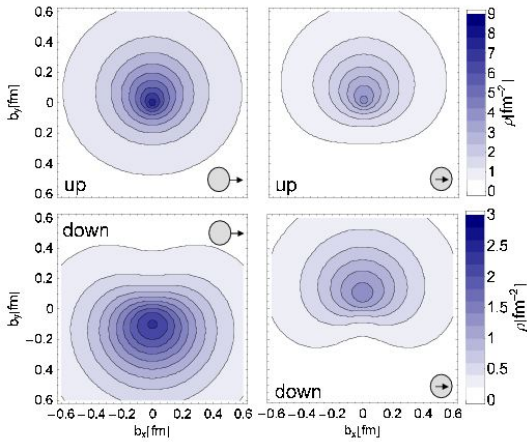
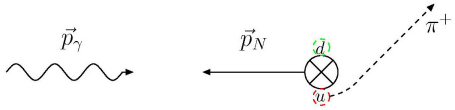
Outline

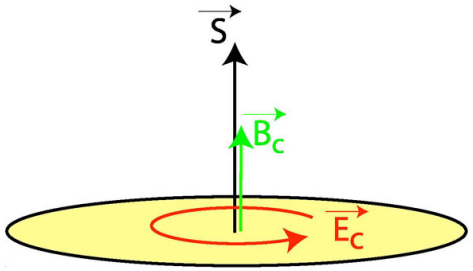






- what do we learn from d_2?
- Single-Spin Asymmetries (SSAs)
- angular momentum decompositions (Jaffe v. Ji)
- Aharonov Bohm effect
- Summary





Average $\perp$ force on quarks in DIS

$d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

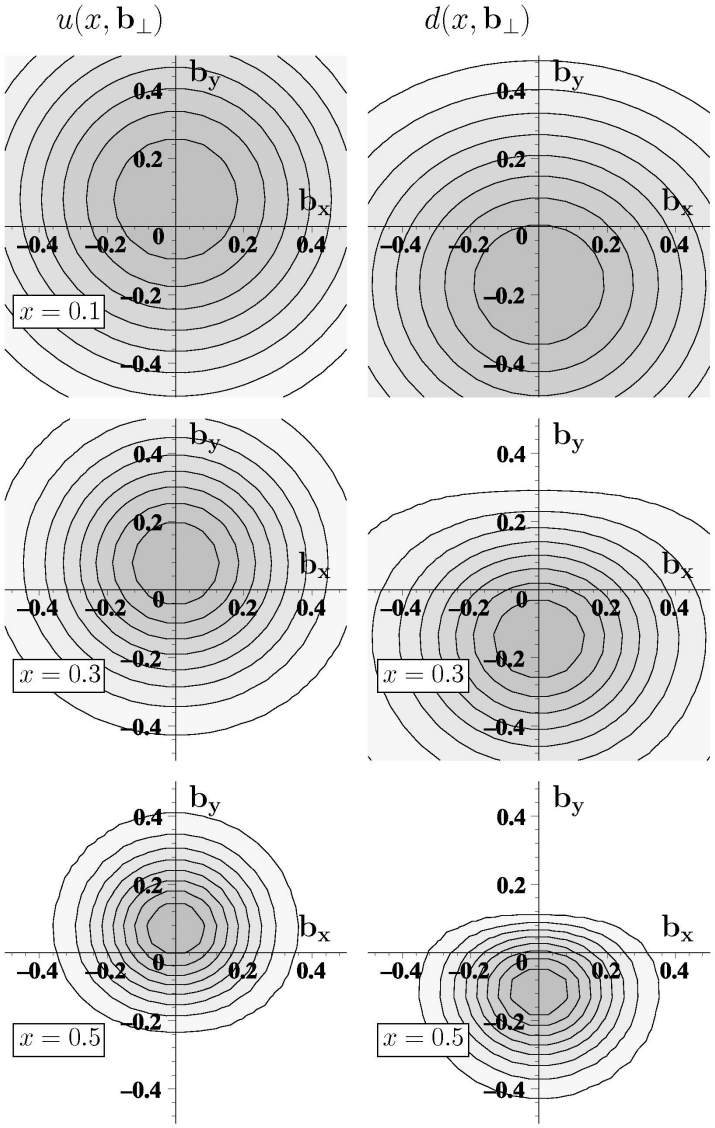
- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \langle P, S \,|\bar{q}(0)\gamma^+ gF^{+y}(0)q(0)|\, P, S\rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0, 0, -1)$$

matrix element defining d_2	$\leftrightarrow$	1st integration point in QS-integral $\hookrightarrow$ Sivers
$\int x^2 e(x)$ (scalar twist-3 PDF)	$\leftrightarrow$	'Boer-Mulders force': $\perp$ pol. quarks; unpol. target



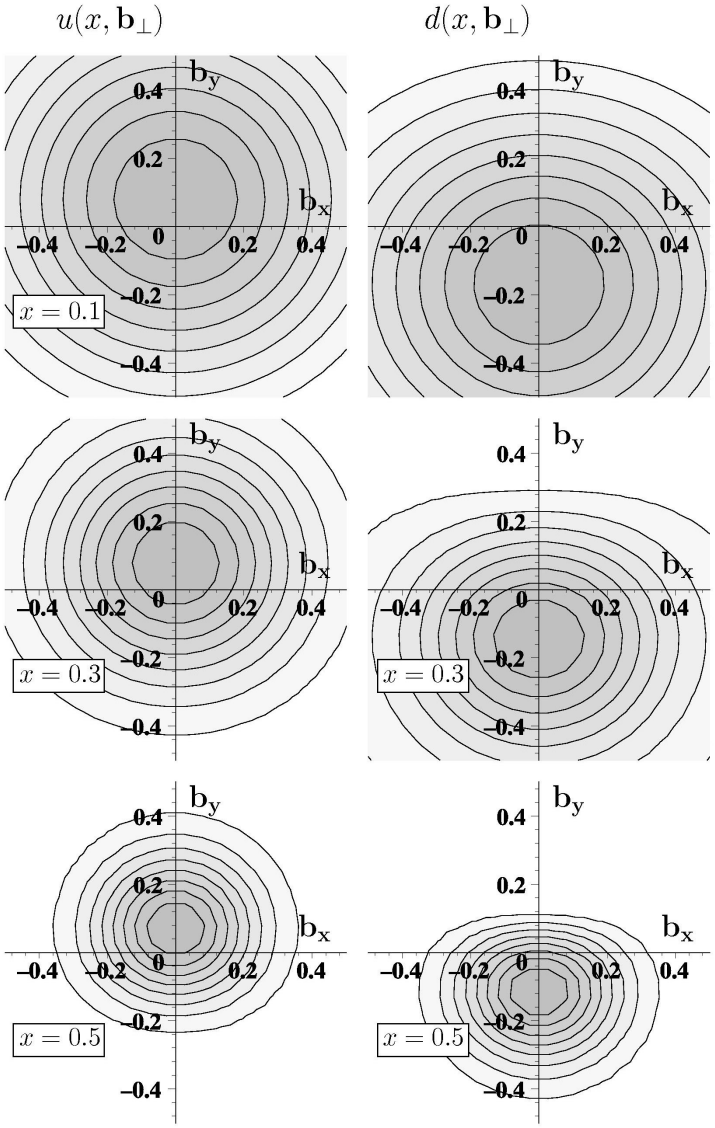
proton polarized in $+\hat{x}$ direction

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$
$$- \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y$$
$$= \frac{1}{2M} \int dx E_q(x, 0) = \frac{\kappa_q}{2M}$$



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$$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + \ldots$$

- u-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

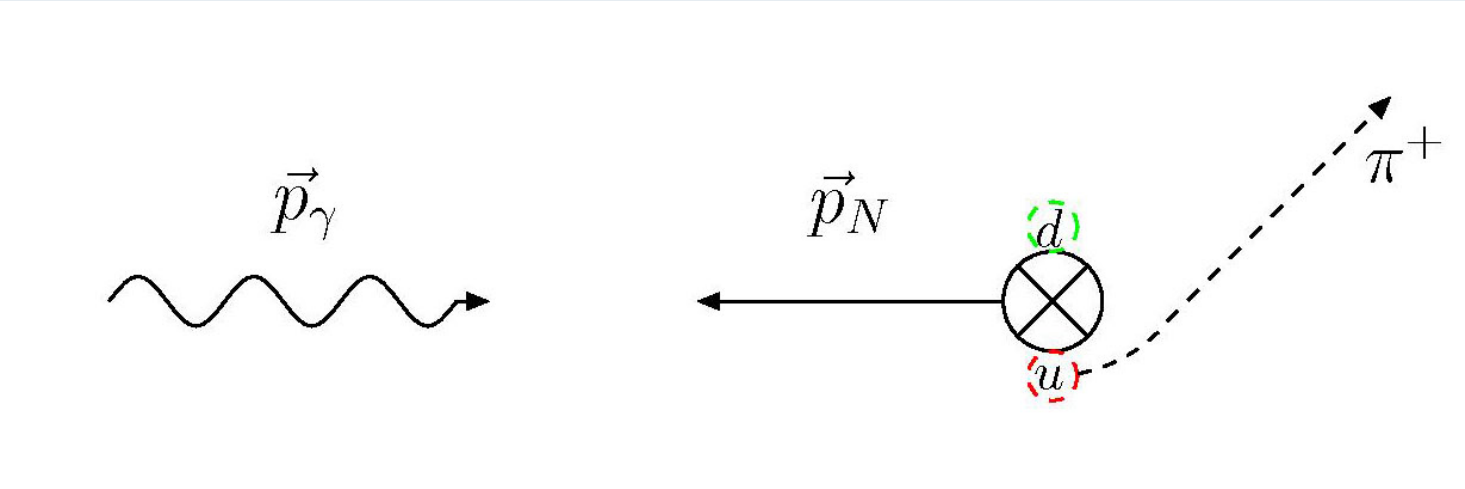
$\hookrightarrow$ shift in $+\hat{y}$ direction

- d-quarks: $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$

$\hookrightarrow$ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}(\pm 0.2 fm)$!!!!

example: $\gamma p \to \pi X$ (chromodynamic lensing)



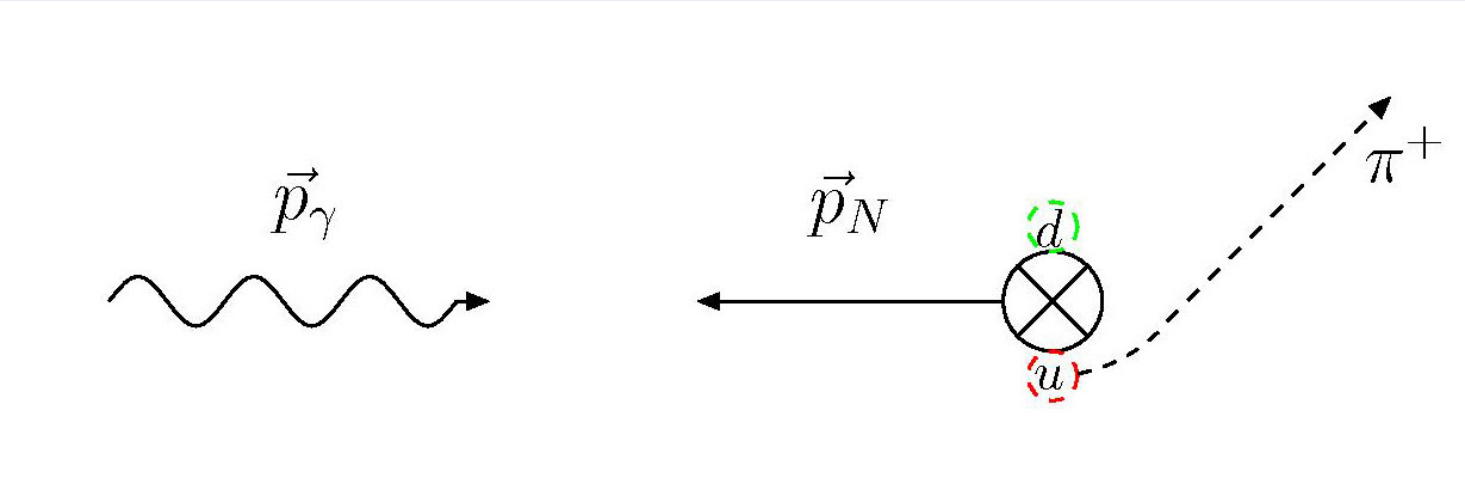
- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive final state interaction (FSI) deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction$\to$ chromodynamic lensing

$\Rightarrow$ $\kappa_p\,,\kappa_n \quad \longleftrightarrow \quad$ sign of SSA/d_2!!!!!!!! (MB,2004)

- sign of SSA confirmed by HERMES (& COMPASS) data

chromodynamic lensing



$\Rightarrow$ $\kappa_p\,,\kappa_n \quad \longleftrightarrow \quad$ sign of SSA/d_2!!!!!!!! (MB,2004)

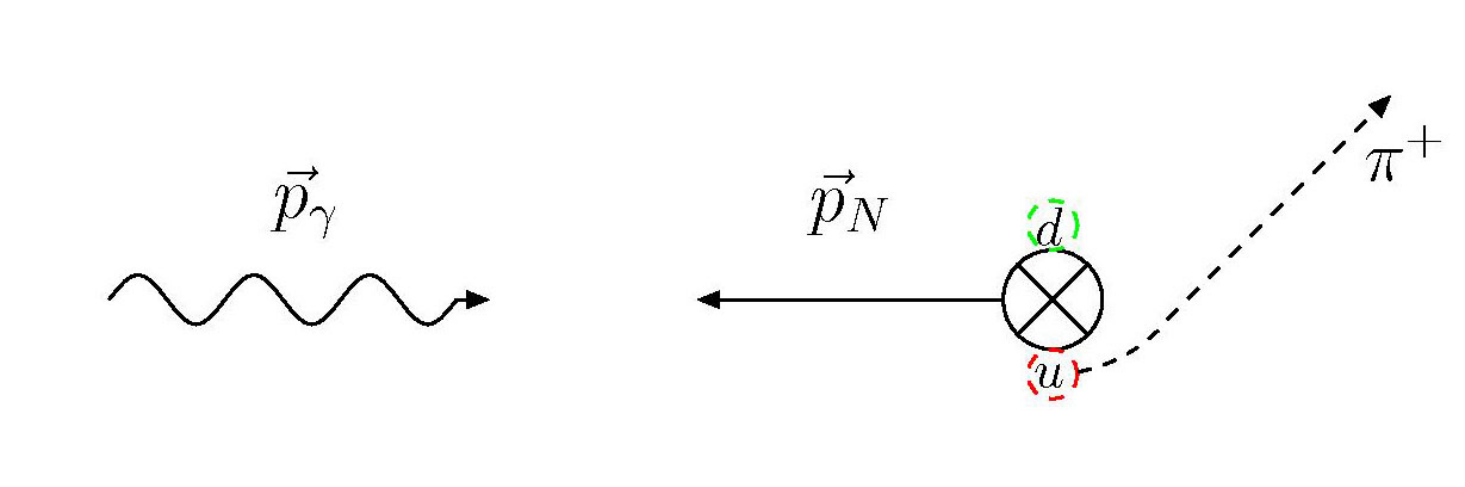
proton target

- $\hookrightarrow$ $f_{1T}^{\perp u} < 0 \quad \& \quad f_{1T}^{\perp d} > 0$
- $\hookrightarrow$ $d_2^u > 0 \quad \& \quad d_2^d < 0$
  - $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$
  - u quarks dominate
- $\hookrightarrow$ $d_2 = \frac{4}{9} d_2^u + \frac{1}{9} d_2^d > 0$

neutron target

- $\hookrightarrow$ $f_{1T}^{\perp u} > 0 \quad \& \quad f_{1T}^{\perp d} < 0$
- $\hookrightarrow$ $d_2^u = d_2^{d/p} < 0 \quad \& \quad d_2^d = d_2^{u/p} > 0$
  - $d_2 = \frac{4}{9} d_2^{d/p} + \frac{1}{9} d_2^{u/p} < 0$

chromodynamic lensing



transverse force

- string tension $\sigma \sim 1\frac{GeV}{fm}$ (all spectators pull in same direction)
- partial cancellation of forces in SSA

$\hookrightarrow$ $\sigma \gg F_\perp \sim 0.1\frac{GeV}{fm}$

- $F_\perp^q = M^2 d_2^q = 5\frac{GeV}{fm}$

$\hookrightarrow$ $d_2^q \sim \pm 0.02$; $d_2^d \sim -d_2^u$

$\hookrightarrow$ $d_2 = \frac{1}{2}\left[\frac{4}{9}d_2^u + \frac{1}{9}d_2^d\right] < 0.01$

QED with electrons

$$\begin{aligned}\vec{J}_\gamma &= \int d^3r\, \vec{r}\times\left(\vec{E}\times\vec{B}\right) = \int d^3r\, \vec{r}\times\left[\vec{E}\times\left(\vec{\nabla}\times\vec{A}\right)\right] \\ &= \int d^3r\, \left[E^j\left(\vec{r}\times\vec{\nabla}\right)A^j - \vec{r}\times(\vec{E}\cdot\vec{\nabla})\vec{A}\right] \\ &= \int d^3r\, \left[E^j\left(\vec{r}\times\vec{\nabla}\right)A^j + \left(\vec{r}\times\vec{A}\right)\vec{\nabla}\cdot\vec{E} + \vec{E}\times\vec{A}\right]\end{aligned}$$

- replace 2nd term (eq. of motion $\vec{\nabla}\cdot\vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

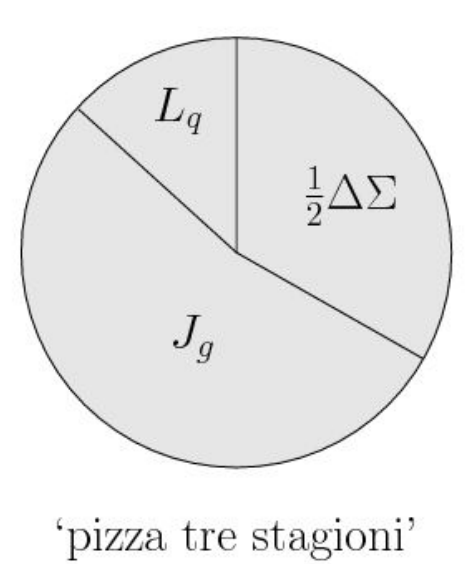
$$\vec{J}_\gamma = \int d^3r\, \left[\psi^\dagger\vec{r}\times e\vec{A}\psi + E^j\left(\vec{r}\times\vec{\nabla}\right)A^j + \vec{E}\times\vec{A}\right]$$

- $\psi^\dagger\vec{r}\times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger\vec{r}\times(\vec{p} - e\vec{A})\psi$

$\hookrightarrow$ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

- can also be done for only part of $\vec{A}$ $\rightarrow$ Chen/Goldman, Wakamatsu

Ji decomposition



$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

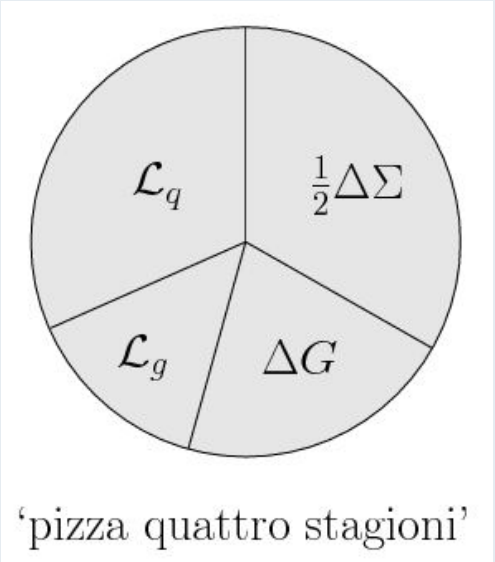
$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S| \, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \, |P,S\rangle$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x})\left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe-Manohar decomposition



light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S| \, \bar{q}(\vec{r})\gamma^+\left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) |P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij}\int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S| \mathrm{Tr} F^{+j}\left(\vec{x} \times i\vec{\partial}\right)^z A^j |P,S\rangle$$

manifestly gauge invariant definition for each term exists ($\rightarrow$ Hatta)

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manifestly gauge invariant definitions for each term exist ($\rightarrow$ Hatta)

- GPDs $\longrightarrow L^q$
- $\overrightarrow{p}\overleftarrow{p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i \in q,g} \mathcal{L}^i$
- QED: $\mathcal{L}^e \neq L^e$ [M.B. + Hikmat BC, PRD **79**, 071501 (2009)]
- $\mathcal{L}^q - L^q = ?$
  - can we calculate/predict/measure the difference?
  - what does it represent?

Wigner Functions (Belitsky, Ji, Yuan; Metz et al.; Lorcé, Pasquini)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ q(\xi)|PS\rangle.$$

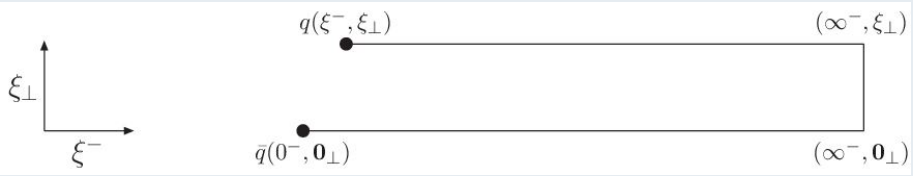
- TMDs: $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- GPDs: $q(x,\mathbf{b}_\perp) = \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$
- need to include Wilson-line gauge link $\mathcal{U}_{\prime\xi}$ to connect 0 and ξ (Ji, Yuan; Hatta; Lorcé;...)

straight line (Ji et al.)

straigth Wilson line from 0 to ξ yields Ji-OAM:

$$L^q = \int d^3x \langle P,S|q^\dagger(\vec{x})\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

Light-Cone Staple for $\mathcal{U}_{0\xi}^{\pm LC}$ (Hatta)



'light-cone staple' yields $\mathcal{L}_{Jaffe-Manohar}$

Connection with Jaffe-Manohar-Bashinsky

PT (Hatta)

- PT$\longrightarrow \mathcal{L}_+ = \mathcal{L}_-$

(different from SSAs due to factor $\vec{x}$ in OAM)

antisymm. boundary condition

- $A^+ = 0$
- fix residual gauge inv. $A^\mu \to A^\mu + \partial^\mu \phi(\vec{x}_\perp)$ by imposing $\vec{A}_\perp(\infty, \vec{x}_\perp) = -\vec{A}_\perp(-\infty, \vec{x}_\perp)$
- $\vec{A}_\perp(\infty, \vec{x}_\perp) - \vec{A}_\perp(-\infty, \vec{x}_\perp) = \int dx^- F^{+\perp}$ gauge inv.
- $\mathcal{L}_+$ involves $i\vec{\mathcal{D}}_+ = i\vec{\partial} - g\vec{A}(\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_-$ involves $i\vec{\mathcal{D}}_- = i\vec{\partial} - g\vec{A}(-\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_+ = \mathcal{L}_- \to$ no contribution from $\vec{A}(\infty, \mathbf{x}_\perp)$

$\hookrightarrow$ 'naive' JM OAM $\mathcal{L}_{JM} = \mathcal{L}_+ = \mathcal{L}_-$

alternative: Bashinsky-Jaffe

- $A^+ = 0$
- $\vec{x} \times i\vec{\partial} \quad \to \quad \vec{x} \times \left[i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp) \right]$
- $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-}$

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- $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-} = \frac{1}{2}\left(\vec{A}_\perp(\infty, \vec{x}_\perp) + \vec{A}_\perp(-\infty, \vec{x}_\perp) \right)$

$\hookrightarrow$ $\mathcal{L}_{JB} = \frac{1}{2}\left(\mathcal{L}_+ + \mathcal{L}_-\right) = \mathcal{L}_+ = \mathcal{L}_-$

straight line (→Ji)

$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$

$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (→ Jaffe-Manohar)

$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$

$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$

$i\mathcal{D}^j = i\partial^j - gA^j(x^-,\mathbf{x}_\perp) - g\int_{x^-}^{\infty} dr^- F^{+j}$

difference $\mathcal{L}^q - L^q$

$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 014014)

$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y$ for $\vec{v} = (0,0,-1)$

Torque along the trajectory of q

$T^z = \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$

Change in OAM

$\Delta L^z = \int_{x^-}^{\infty} dr^- \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$

straight line (→Ji)

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$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

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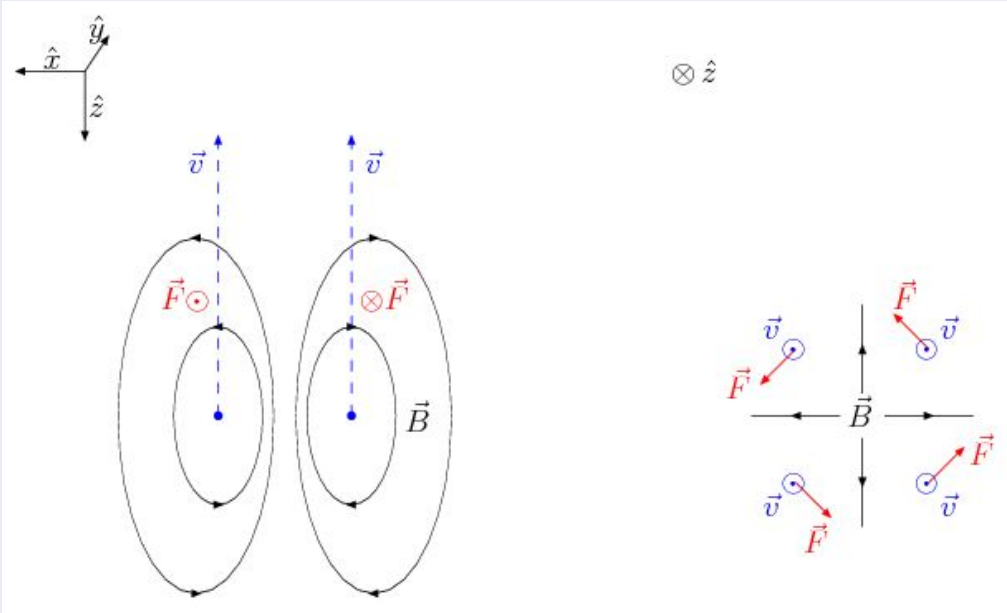
$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

difference $\mathcal{L}^q - L^q$ (MB, PRD 88 (2013) 014014)

$\mathcal{L}^q - L^q = \Delta L^q_{FSI}$ = change in OAM as quark leaves nucleon

example: torque in magnetic dipole field



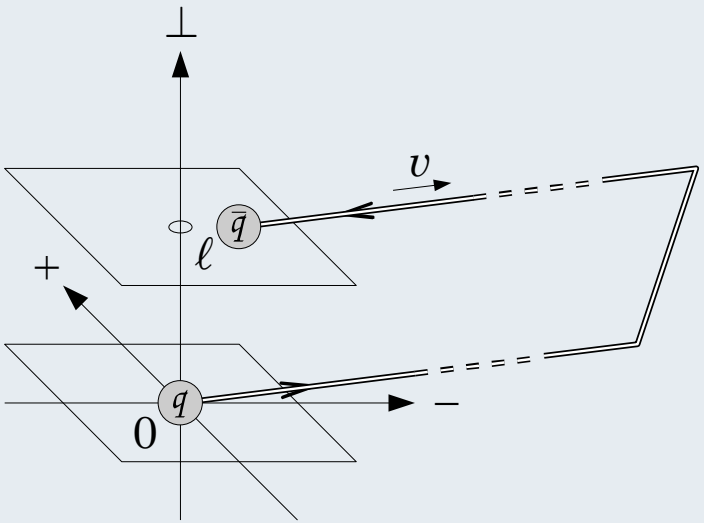
MB+A.Miller+W.-D.Nowak, Rept.Prog.Phys. 73 (2010) 016201

In decomposition (103), each term has a partonic interpretation. The gluon spin contribution ΔG appears explicitly. It is experimentally accessible (see section 2.8) and can be defined as the expectation value of a (nonlocal) manifestly gauge invariant operator. In light-cone gauge, this operator collapses to a local operator (and its expectation value has a partonic interpretation). No direct experimental access to the parton orbital angular momentum $\mathcal{L}$ has been identified. Its value can be obtained only by subtracting the quark and gluon spin contributions from the nucleon spin. Both ΔG and $\mathcal{L}$ can be defined through matrix elements of local operators only in light-cone gauge $A^+ = 0$. Explicit definitions for the operators appearing in both decompositions can be found in [242]. Since neither one can be represented as the matrix element of a manifestly gauge invariant local operator, they cannot be analytically continued to Euclidean space and are thus inaccessible for lattice QCD.

Jaffe-Manohar Decomposition

$$\frac{1}{2} = J_z = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}. \tag{103}$$

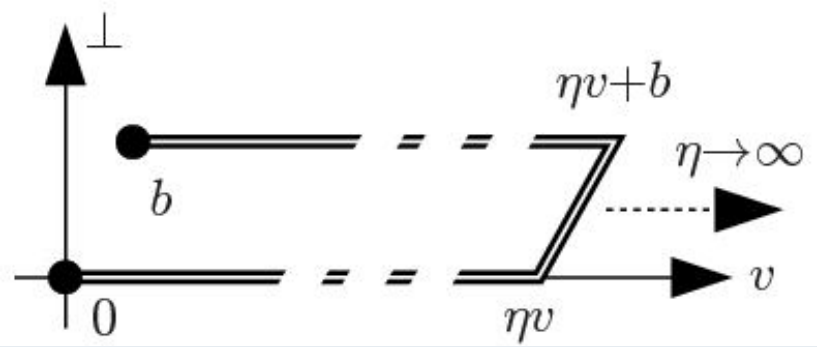
challenge



- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

TMDs in lattice QCD

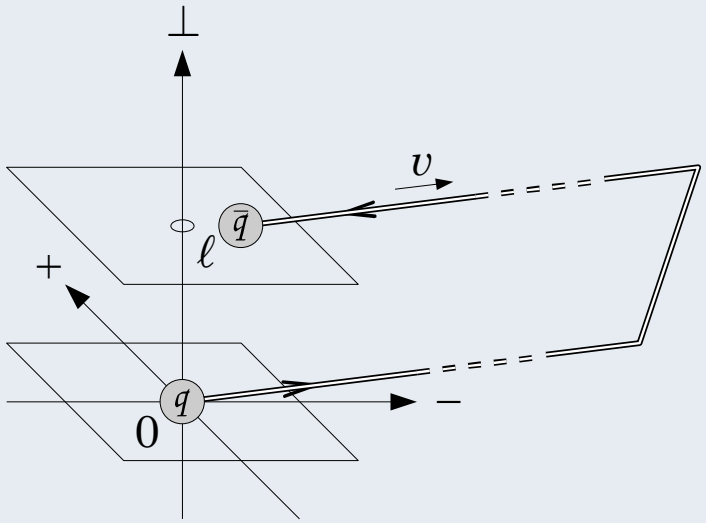
B. Musch, P. Hägler, M. Engelhardt



- calculate space-like staple-shaped Wilson line pointing in $\hat{z}$ direction; length $L \to \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios

$\hookrightarrow$ extrapolate/evolve to $P_z \to \infty$

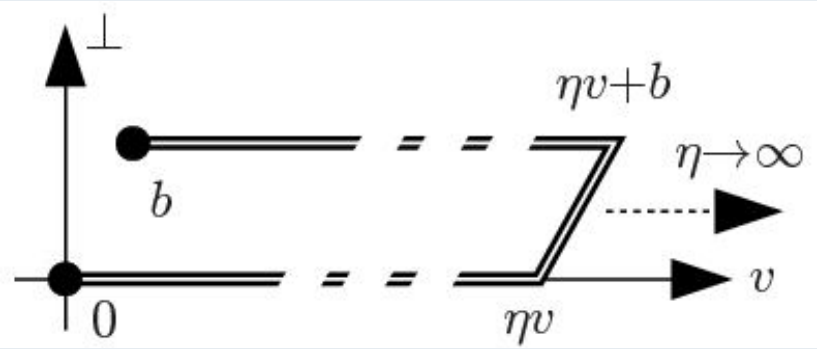
challenge



- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

Jaffe-Manohar OAM in lattice QCD

M. Engelhardt & S. Liutti



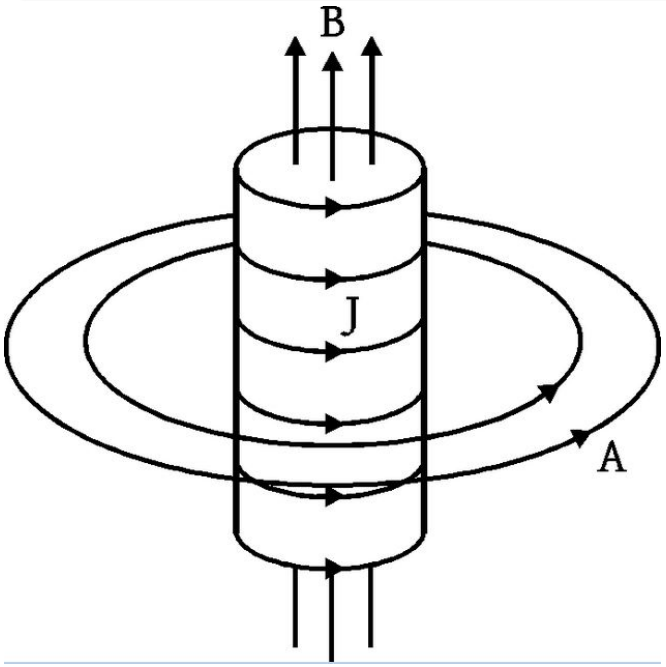
- calculate form factor for space-like staple-shaped Wilson line pointing in $\hat{z}$ direction; length $L \to \infty$
- evaluate slope for zero momentum transfer
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios

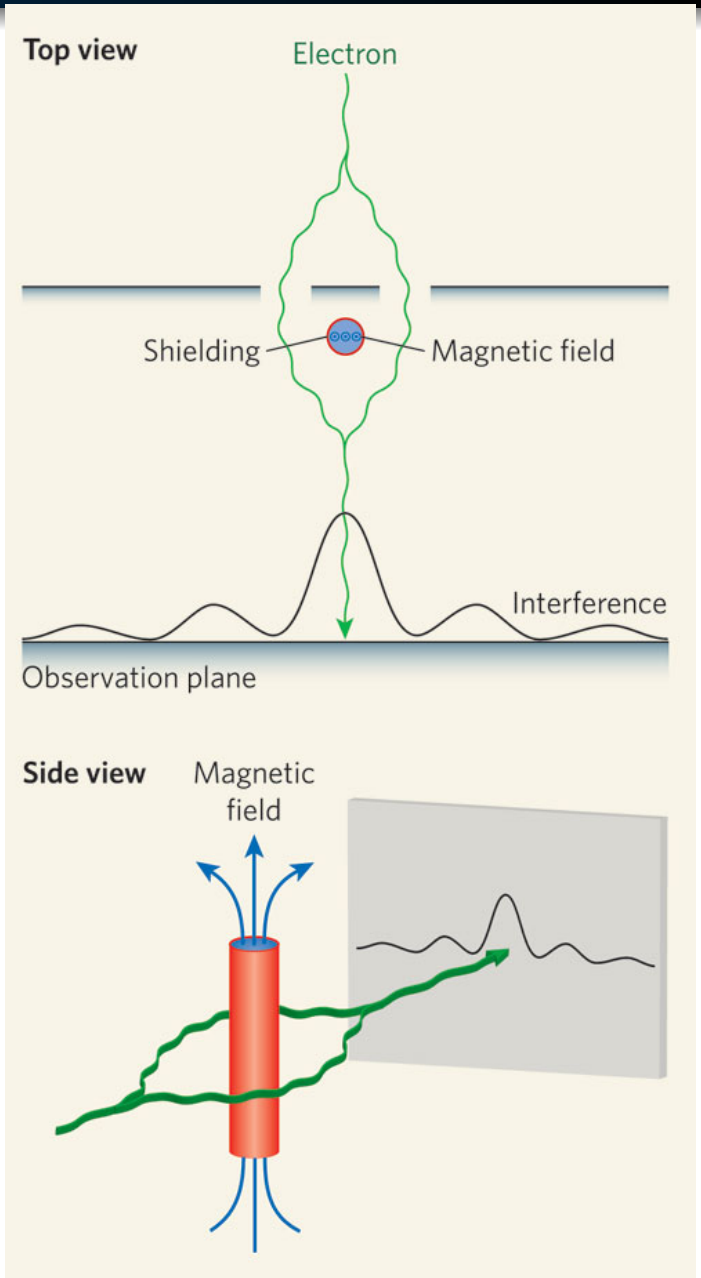
$\hookrightarrow$ extrapolate/evolve to $P_z \to \infty$

Aharonov Bohm effect

- double slit experiment with solenoid between beams
- no magnetic field at beam
- vector potential $\vec{A}$ modifies QM phase $\sim \exp\left(i\frac{e}{\hbar}\oint \vec{A}\cdot d\vec{r}\right) = \exp\left(i\frac{e}{\hbar}\iint \vec{B}\cdot d\vec{S}\right)$

$\hookrightarrow$ $B \neq 0 \Rightarrow$ shifts interference pattern





Aharonov Bohm Effect in SSAs?

Aharonov Bohm effect

- double slit experiment with solenoid between beams
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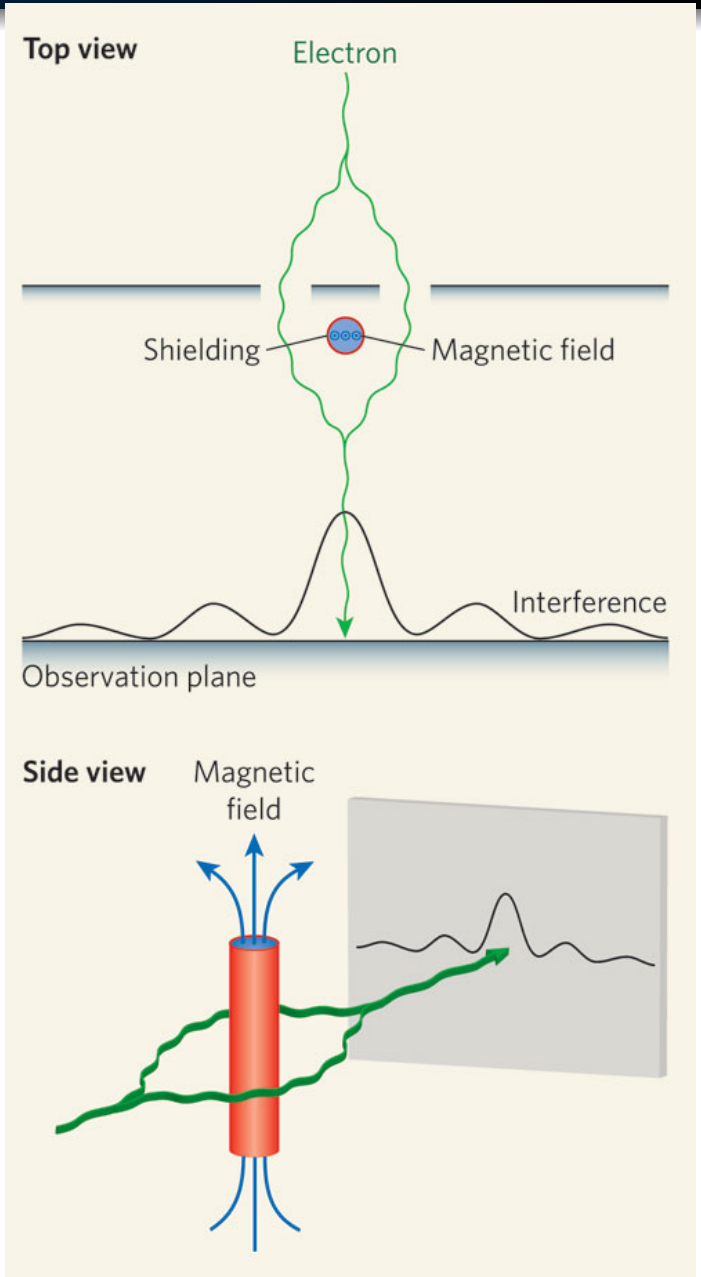
transverse SSA

- Wilson line phase factor $\sim \exp\left(i\frac{g}{\hbar}\int \vec{A}\cdot d\vec{r}\right)$ crucial for SSAs in SIDIS et al.
- However, contrary to AB-effect, $\perp$ momentum in SIDIS generated when quark travels through regions of nonzero color magnetic/eletcric fields



Summary

- $\perp$ force in DIS
- TMDs and OAM from Wigner distributions
- straight-(Wilson)line gauge link $\longrightarrow L^q$ ('Ji-OAM')
- light-cone staple- gauge link $\longrightarrow \mathcal{L}^q_+$ ('JM-OAM')
- $\mathcal{L}^q_+ - L^q$ = change in OAM as quark leaves nucleon (torque due to FSI)
- $A^+ = 0$ gauge (with anti-symmetric boundary condition) $\mathcal{L}^q_+ \to$ canonical OAM (Jaffe-Manohar-OAM)
- JM-OAM from lattice QCD
- Aharonov Bom effect $\overset{?}{\to} SSA$