Large-x resummation and impact on parton densities

Gennaro Corcella

INFN - Laboratori Nazionali di Frascati

1. Introduction
2. Soft resummation in the DIS coefficient function
3. Large-x CC and NC data
4. Resummed and NLO parton densities
5. Outlook and conclusions

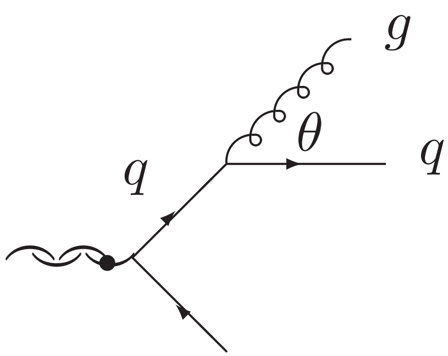
Quite old work (with L. Magnea for the HERA-LHC workshop), but new interest due to JLAB and LHC measurements

For reliable measurements at present and future high-energy colliders, precise QCD calculations are necessary

Fixed-order calculations (NLO, NNLO) suitable to predict total cross sections or widths

Differential distributions present large terms $\sim \alpha_S^k L^p$ which one needs to resum

L: large logarithms for soft ($E_g \to 0$) and (massless partons) collinear emission ($\theta \to 0$)



With heavy quarks: no collinear divergence, but $\alpha_S \ln(m^2/Q^2)$

Soft and collinear logarithms can be summed up to all orders as they exponentiate:

$$\sum_k \alpha_S^k \sum_p^{2k} c_{kp} L^p \longrightarrow \exp\left[L g_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S g_3(\alpha_S L) + \ldots\right]$$

Structure functions $\mathcal{F}_i$ in DIS as convolution of $\overline{\mathrm{MS}}$ coefficient functions and parton distribution functions

$$\nu_\mu(k)N(P) \to \mu(k')X(p_X)$$

Hard subprocesses at NLO (charged current):

$$q(p_q)W(q) \to q'(p_{q'})(g(p_g)) \;\; ; \;\; g(p_g)W(q) \to \bar{q}(p_{\bar{q}})q\prime(p_{q\prime})$$

$$Q^2 = -q^2 \;\; ; \;\; x = \frac{Q^2}{2P\cdot q} \;\; ; \;\; W^2 = (q+P)^2 \;\; ; \;\; y = \frac{P\cdot q}{P\cdot k} \; ; \; \mu_R^2 = \mu_F^2 = Q^2$$

$$\mathcal{F}_i(x,Q^2) = \int_x^1 \frac{d\xi}{\xi} \sum_{q,q\prime} |V_{qq\prime}|^2 \left[C_i^q(\xi,Q^2) q\left(\frac{x}{\xi},Q^2\right) + C_i^g(\xi,Q^2) g\left(\frac{x}{\xi},Q^2\right)\right]$$

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 ME}{\pi(1+Q^2/m_W^2)^2}\left\{y^2 xF_1 + \left[1 - \left(1+\frac{Mx}{2E}\right)y\right]F_2 \pm y\left(1-\frac{y}{2}\right)xF_3\right\}$$

$$F_1 = \mathcal{F}_1 \qquad F_2 = \frac{2x}{\rho^2}\mathcal{F}_2 \qquad F_3 = \frac{2}{\rho}\mathcal{F}_3 \qquad \rho = \sqrt{1+\left(\frac{2Mx}{Q}\right)^2}$$

Soft- and collinear-gluon radiation

Quark-initiated $\overline{\mathrm{MS}}$ coefficient function contains terms which get large for $x \to 1$

$$C^{\mathrm{soft}}(x,\mu_F^2) = \delta(1-x) + \frac{C_F \alpha_S}{\pi} \left\{ \left[\frac{\ln(1-x)}{1-x} \right]_+ + \frac{1}{(1-x)_+} \left(\ln \frac{Q^2}{\mu_F^2} - \frac{3}{4} \right) \right\}$$

$$f_N = \int_0^1 dx x^{N-1} f(x) \qquad \frac{1}{(1-x)_+} \to \ln N \qquad \left[\frac{\ln(1-x)}{1-x} \right]_+ \to \ln^2 N \quad \text{for } N \to \infty$$

$$C_N^{\mathrm{soft}} = 1 + \frac{\alpha_S C_F}{\pi} \left\{ \frac{1}{2} \ln^2 N + \left[\gamma_E + \frac{3}{4} - \ln \frac{Q^2}{\mu_F^2} \right] \ln N \right\}$$

Resummed coefficient function (Catani, Marchesini, Webber, NPB 349 (1991) 635)

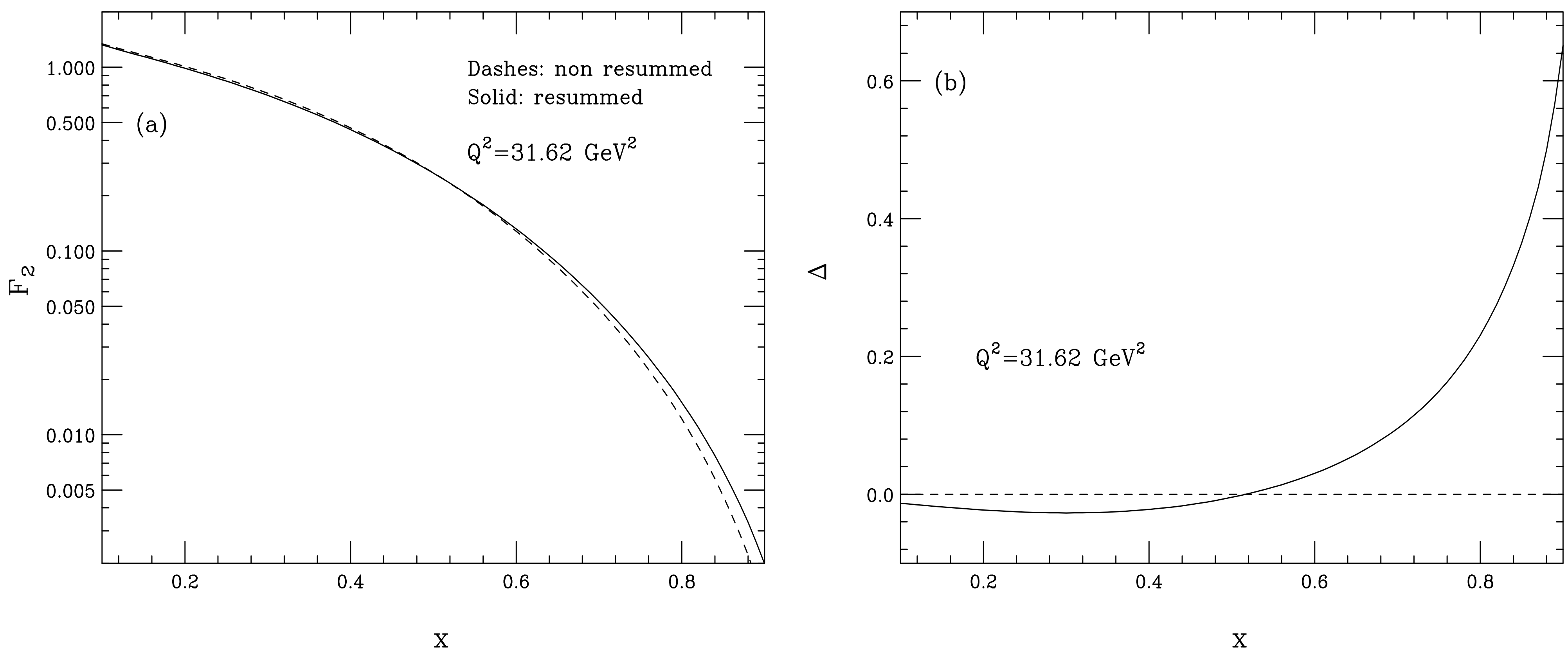
$$\Delta_N = \exp \left\{ \int_0^1 dx \frac{x^{N-1} - 1}{1-x} \int_{\mu_F^2}^{Q^2(1-x)} \left[\frac{dk^2}{k^2} A\left[\alpha_S(k^2)\right] + \frac{1}{2} B\left[\alpha_S\left(Q^2(1-x)\right)\right] \right] \right\}$$

$$A(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n A^{(n)} \ ; \ B(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n B^{(n)}$$

LL $\alpha_S^n \ln^{n+1} N$: $A^{(1)}$; NLL $\alpha_S^n \ln^n N$: $A^{(1)}$, $A^{(2)}$, $B^{(1)}$

CC F_2 with resummed coefficient function and NLO pdf (CTEQ6M) (G.C. and A.D.Mitov)

$Q^2 = 31.62$ GeV^2 (NuTeV), $\quad \Delta = (F_2^{\rm res} - F_2^{\rm NLO})/F_2^{\rm NLO}$



Remarkable impact of large-x resummation for $x > 0.6$

Change of sign of Δ about $x \simeq 0.5$ due to same NLO normalization

A toy model for large-x resummed parton densities

- NuTeV data on F_2 and F_3 (CC), NMC/BCDMS on $F_2^{\rm NS}$ (NC)

- Parametrization of data at fixed values of Q^2

- Power corrections are neglected

- Resummation is performed in N space: extraction of moments of linear combinations of parton densities

- Determination of NLO and resummed valence quark distributions in N-space, with assumptions on gluon and sea densities

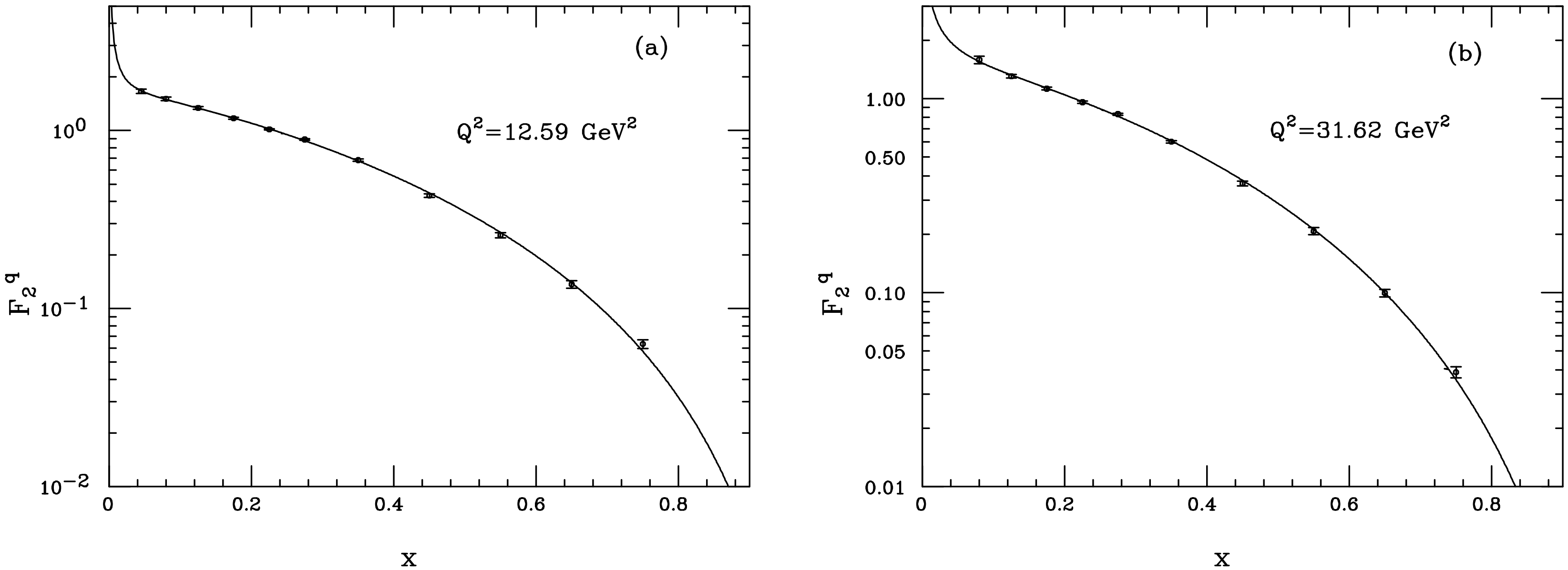
- Inversion to x-space and check of consistency with DGLAP

Fit of NuTeV F_2 at $Q^2 = 31.62$ and 12.59 GeV^2 (correlations are neglected)

$$F_2 = \frac{1}{2}(F_2^{\nu} + F_2^{\bar{\nu}}) = 2x \sum_{q,q'} |V_{qq'}|^2 \left[(q+\bar{q}) \otimes C_2^q + g \otimes C_2^g\right] = F_2^q + F_2^g$$

We take the gluon pdf from CTEQ6M, compute F_2^g and fit F_2^q (nuclear corrections included as in W. Seligman, Ph.D. Thesis)

$$F_2^q(x) = F_2(x) - F_2^g(x) = Ax^{-\alpha}(1-x)^{\beta}(1+bx)$$



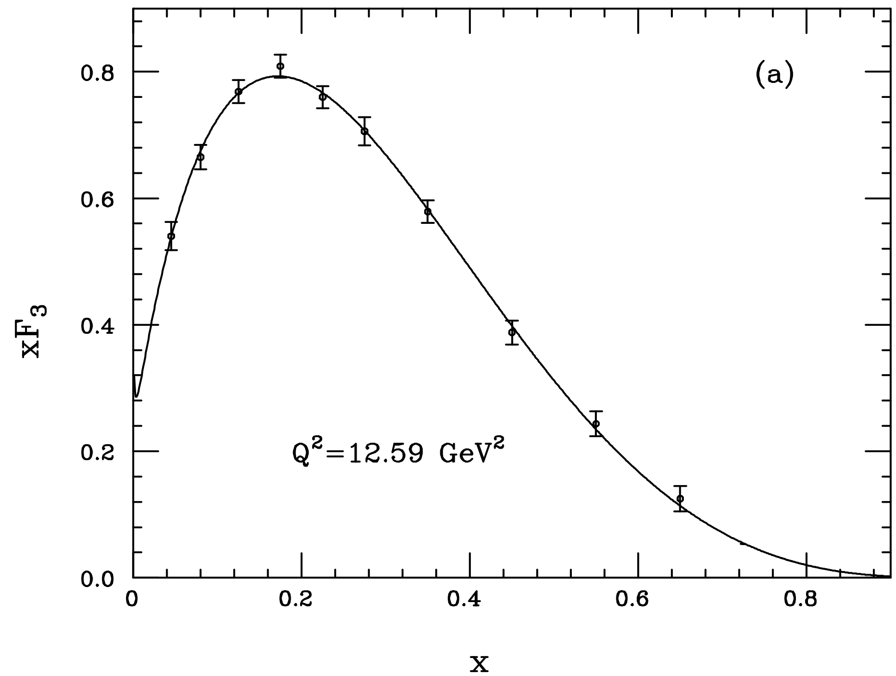
$Q^2 = 12.59$ GeV^2: $A = 0.038 \pm 0.005$, $\alpha = 0.816 \pm 0.021$, $\beta = 2.697 \pm 0.050$, $b = 66.804 \pm 7.583$, $\chi^2/\mathrm{dof} = 9.55/6$

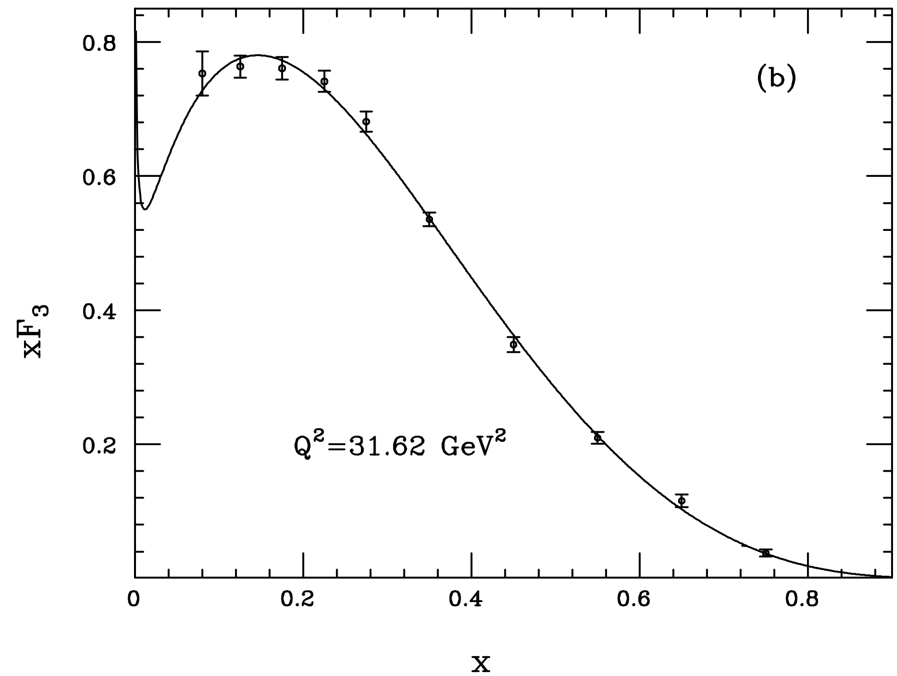
$Q^2 = 31.62$ GeV^2: $A = 0.240 \pm 0.002$, $\alpha = 0.562 \pm 0.020$, $\beta = 3.211 \pm 0.065$, $b = 17.20 \pm 1.37$ $\chi^2/\mathrm{dof} = 9.99/6$

Likewise, for F_3 (NuTeV, CC, $Q^2 = 12.59$ and 31.62 GeV^2):

$$xF_3(x) = \frac{1}{2}\left(xF_3^{\nu} - xF_3^{\bar{\nu}}\right) = x\left[\sum_{q,q'} |V_{qq'}|^2 (q - \bar{q}) \otimes C_3^q\right]$$

$$F_3(x) = Cx^{-\rho}(1-x)^{\sigma}(1+kx)$$





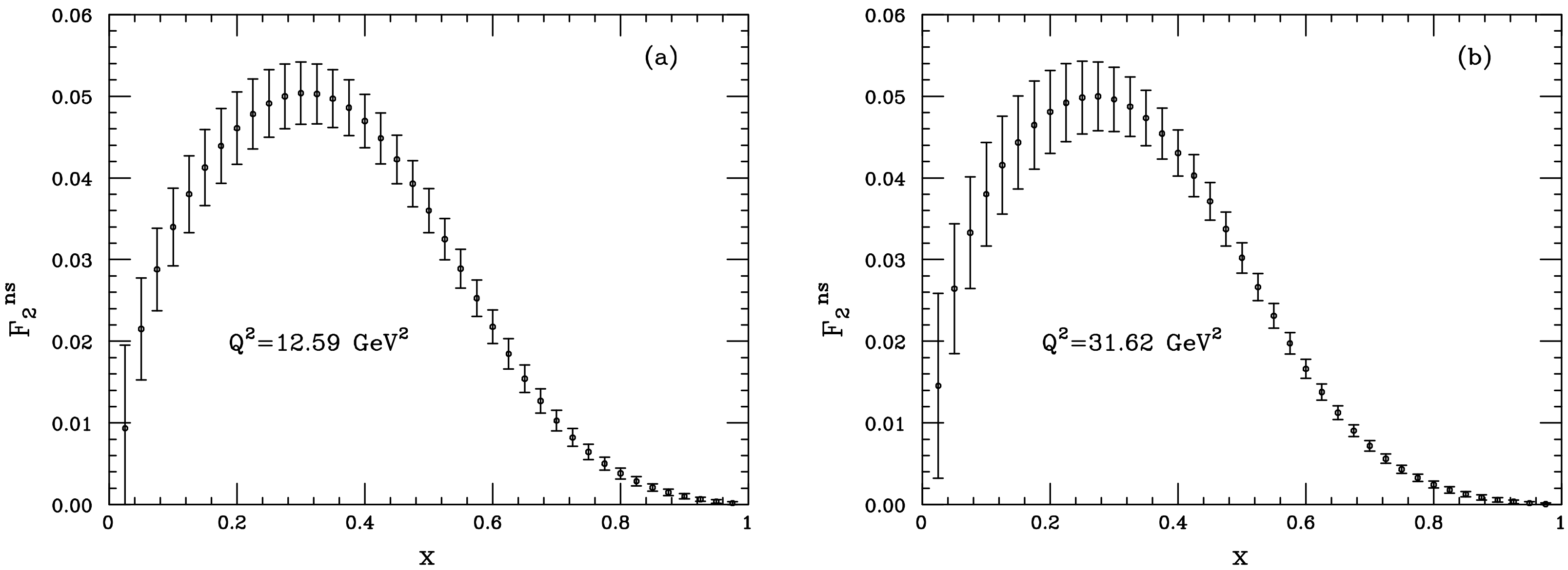
$Q^2 = 12.59$ GeV^2: $C = 0.054 \pm 0.005$, $\rho = 0.245 \pm 0.038$, $\sigma = 3.374 \pm 0.145$, $k = 99.719 \pm 0.247$
$\chi^2/\mathrm{dof} = 2.06/6$

$Q^2 = 31.62$ GeV^2: $C = 0.103 \pm 0.012$, $\rho = 0.294 \pm 0.034$, $\sigma = 3.325 \pm 0.089$, $k = 42.972 \pm 4.700$
$\chi^2/\mathrm{dof} = 7.20/6$

NC: nonsinglet F_2 at the same Q^2 from NMC and BCDMS data

$$F_2^{\rm NS} = F_2^p - F_2^d = x(u-d) \otimes C_2^q = x f_2^{\rm NS} \otimes C_2^q$$

The NNPDF Collaboration, R.D. Ball et al, arXiv:1410.8849:



$$f_i^{\rm NLO}(N+1,Q^2) = \frac{F_i\left(N,Q^2\right)}{C_i^{\rm NLO}\left(N+1,Q^2\right)} \quad ; \quad f_i^{\rm res}\left(N+1,Q^2\right) = \frac{F_i\left(N,Q^2\right)}{C_i^{\rm res}\left(N+1,Q^2\right)}$$

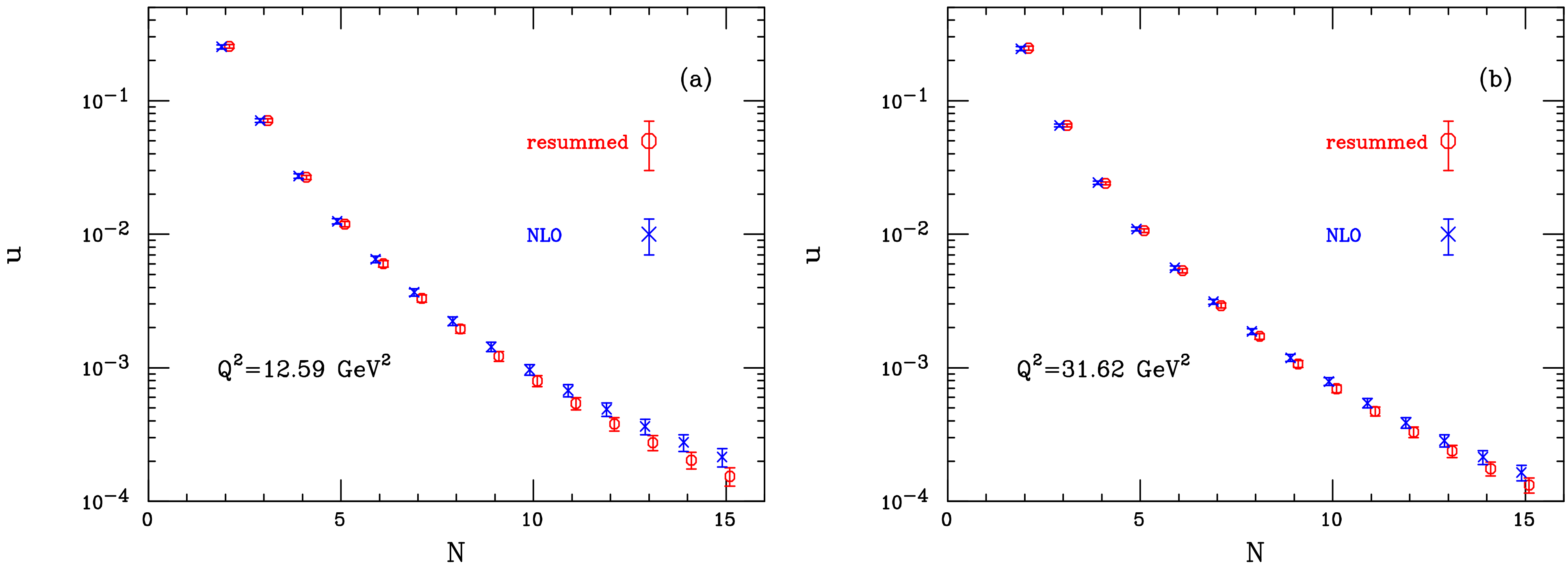
$$f_2^q({\rm CC}) = |V_{ud}|^2(u+d+\bar{u}+\bar{d}) + |V_{us}|^2(u+s+\bar{u}+\bar{s}) + |V_{cs}|^2(c+\bar{c}+s+\bar{s}) + |V_{cd}|^2(d+\bar{d}+c+\bar{c})$$

$$f_3({\rm CC}) = |V_{ud}|^2(u+d-\bar{u}-\bar{d}) + |V_{us}|^2(u+s-\bar{u}-\bar{s}) + |V_{cs}|^2(c-\bar{c}+s-\bar{s}) + |V_{cd}|^2(d-\bar{d}+c-\bar{c})$$

$$f_2^{\rm NS}({\rm NC}) = u-d$$

$u = u_V + \bar{u}$, $d = d_V + \bar{d}$ $\quad s = \bar{s}$, $\bar{u} = \bar{d} = 2s$, $c = \bar{c} = 0$

Moments of the u_V distributions



Visible effect of large-x resummation for $N > 7$

d- and s-quark density: the errors are too large for the resummation to have an impact

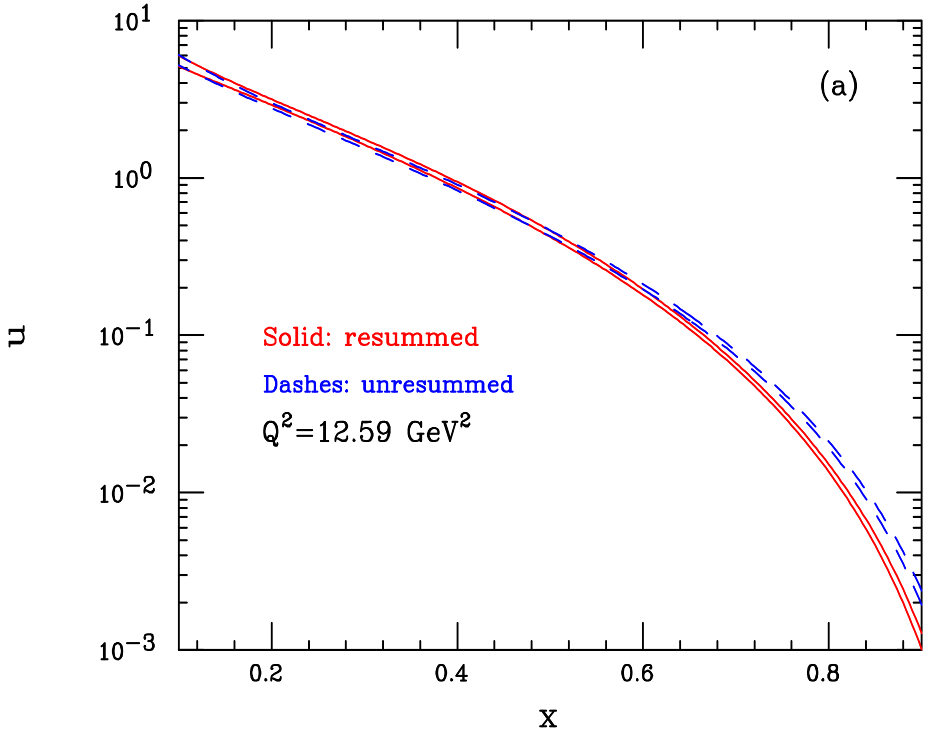
Inversion to x-space: $q(x) = Dx^{-\gamma}(1-x)^{\delta}$

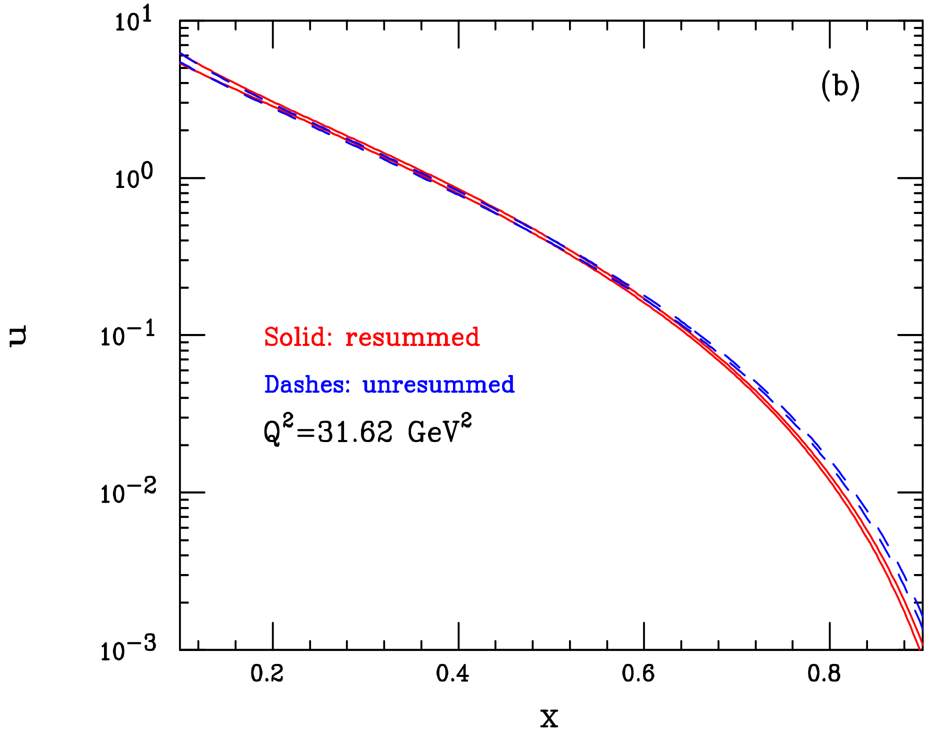
$Q^2 = 12.59$ GeV^2: $D = 3.025 \pm 0.534$, $\gamma = 0.418 \pm 0.101$, $\delta = 3.162 \pm 0.116$, $\chi^2/\mathrm{dof} = 1.62/11$ (NLO);

$D = 4.647 \pm 0.881$, $\gamma = 0.247 \pm 0.109$, $\delta = 3.614 \pm 0.128$, $\chi^2/\mathrm{dof} = 0.64/11$ (resummed)

$Q^2 = 31.62$ GeV^2: $D = 2.865 \pm 0.420$, $\gamma = 0.463 \pm 0.086$, $\delta = 3.301 \pm 0.098$, $\chi^2/\mathrm{dof} = 1.10/11$ (NLO);

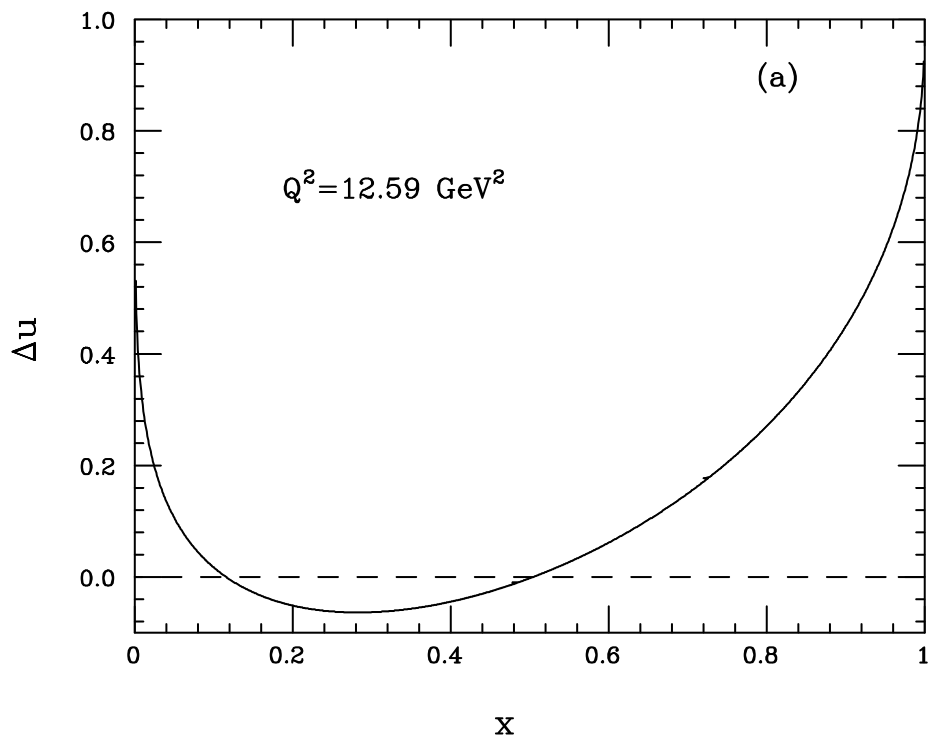
$D = 3.794 \pm 0.583$, $\gamma = 0.351 \pm 0.090$, $\delta = 3.598 \pm 0.104$, $\chi^2/\mathrm{dof} = 0.53/11$ (resummed)

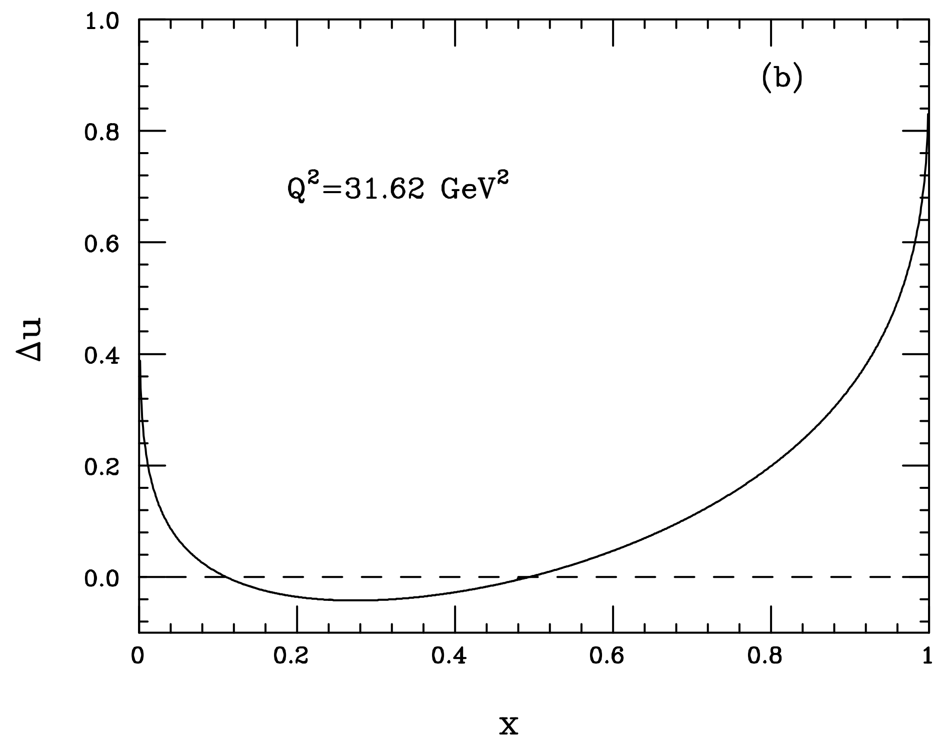




Relative change in the up-quark distribution (central values)

$$\Delta u = \frac{u_{\mathrm{NLO}} - u_{\mathrm{res}}}{u_{\mathrm{NLO}}}$$

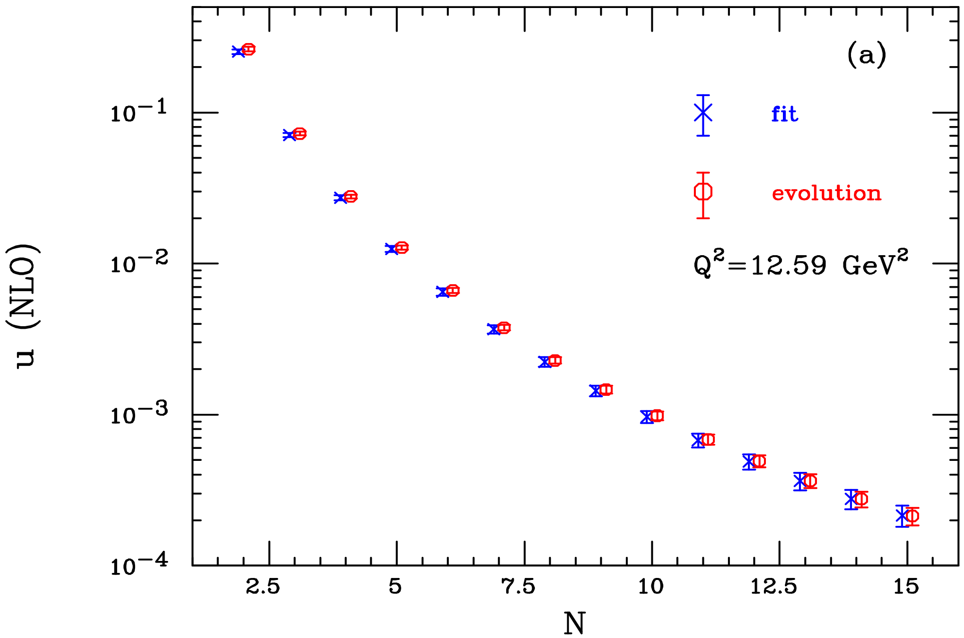


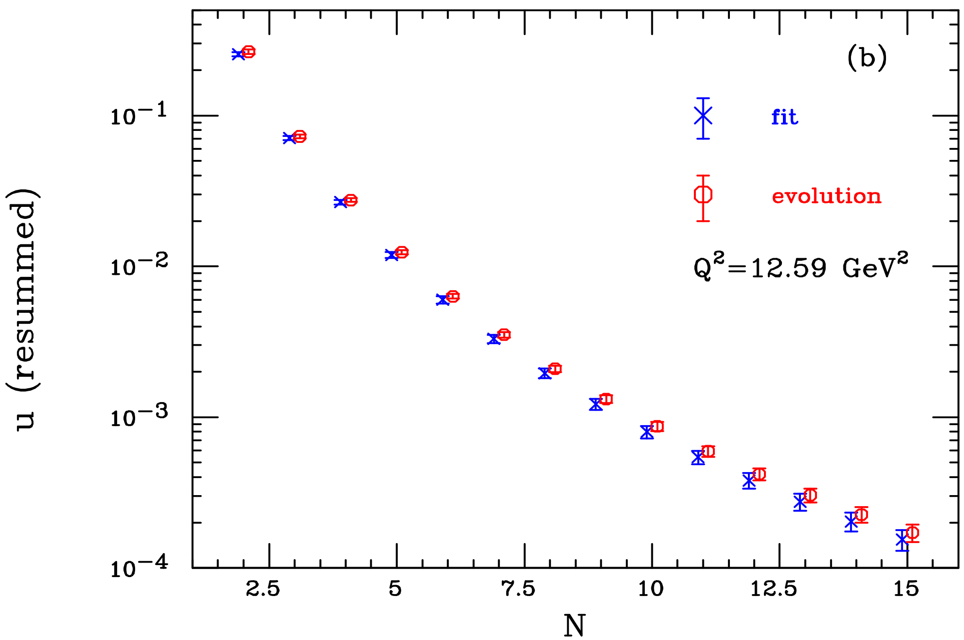


$Q^2 = 12.59$ GeV^2 $\quad x = 0.58$: 5%, $\;x = 0.65$: 10%, $\;x = 0.75$: 20%

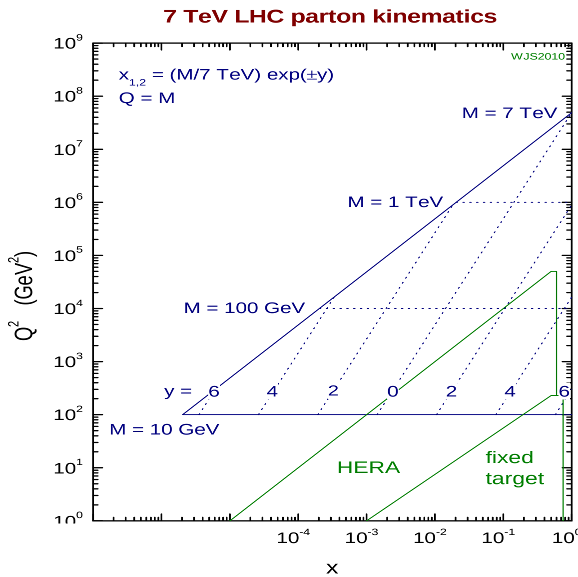
$Q^2 = 31.62$ GeV^2 $\quad x = 0.61$: 5%, $\;x = 0.69$: 10%, $\;x = 0.80$: 20%

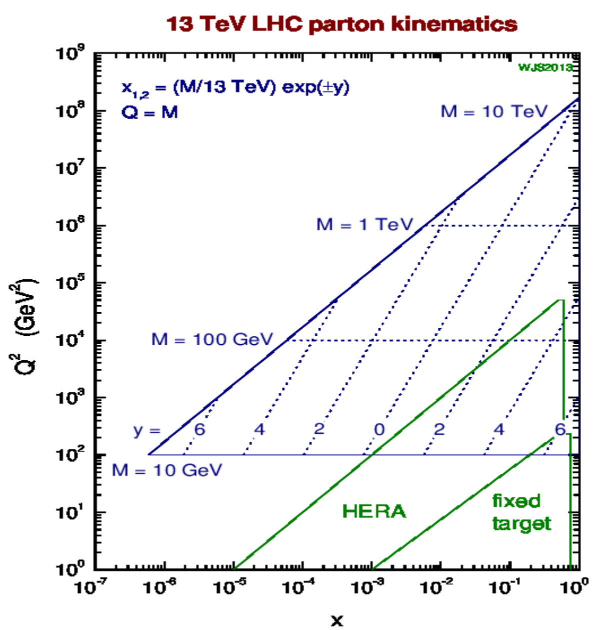
Check of consistency with DGLAP evolution equations

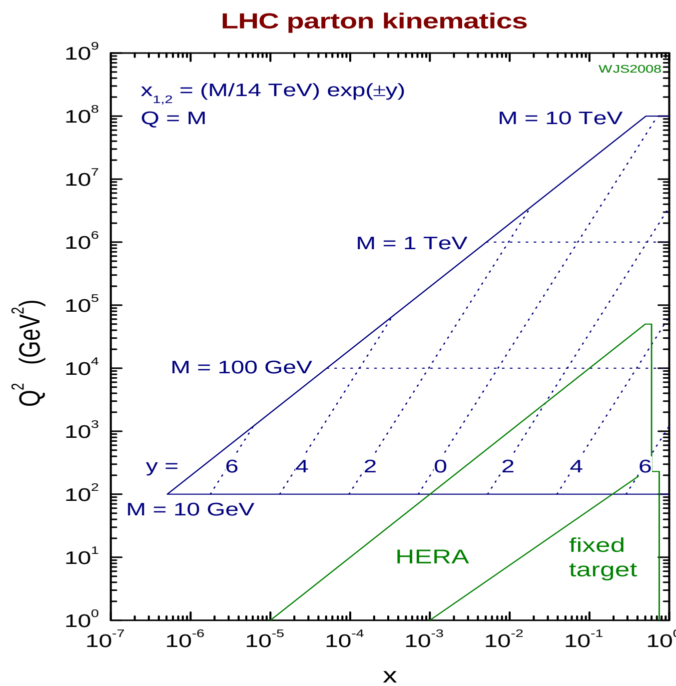




Kinematic reach of the LHC at 7, 13 and 14 TeV in terms of x, Q^2 and y (W.J.Stirling)







Global fits do not include the resummation, but rather set cuts on the variable W^2:

$$W^2 = (q+P)^2 = M^2 + \frac{1-x}{x}\,Q^2$$

Neglecting the proton mass:

$$W^2 > W^2_{\rm min} \quad \Rightarrow \quad x_{\rm max} = 1 - \frac{W^2_{\rm min}}{W^2_{\rm min} + Q^2}$$

Typical cuts (MSTW): $W^2 > 15$ GeV^2 (NLO and NNLO) ; $W^2 > 20$ GeV^2 (LO)

$$Q^2 = 10\ {\rm GeV}^2 \Rightarrow x_{\rm max} = 0.4 \ \ ; \ \ Q^2 = 100\ {\rm GeV}^2 \Rightarrow x_{\rm max} = 0.85$$

At small Q^2 a part of the large-x spectrum is discarded; at large Q^2 one is sensitive to x where the resummation should be taken into account for the sake of precision measurements

At the moment, partonic resummed calculations (jet observables, transverse momentum, etc.) use fixed-order pdfs (NLO or NNLO)

Interest to account for resummation in the global fits, e.g., NNPDF should have it in 2015 (S. Forte, private communication)

Conclusions

Large-x resummation in DIS coefficient function with NLL accuracy

Relevant effect of resummation on structure functions at large x

CC structure functions from NuTeV and NC from NMC/BCDMS

Extraction of resummed and NLO parton densities in moment and x-space

Visible impact (10-20%) of resummation on u-quark distribution for $0.55 < x < 0.8$

Agreement with DGLAP evolution at one standard deviation

In progress:

Application to LHC, e.g. $t\bar{t}$ or Z' production, or JLAB processes in conjunction with resummed calculation

Possible application within parton shower generators (LL resummation with some NLLs)

A quantitative analysis requires more data in the context of a global fit