

Phenomenology of TMD Extraction & TMD evolution issues

HiX2014
Nucleon Structure at Large Bjorken x
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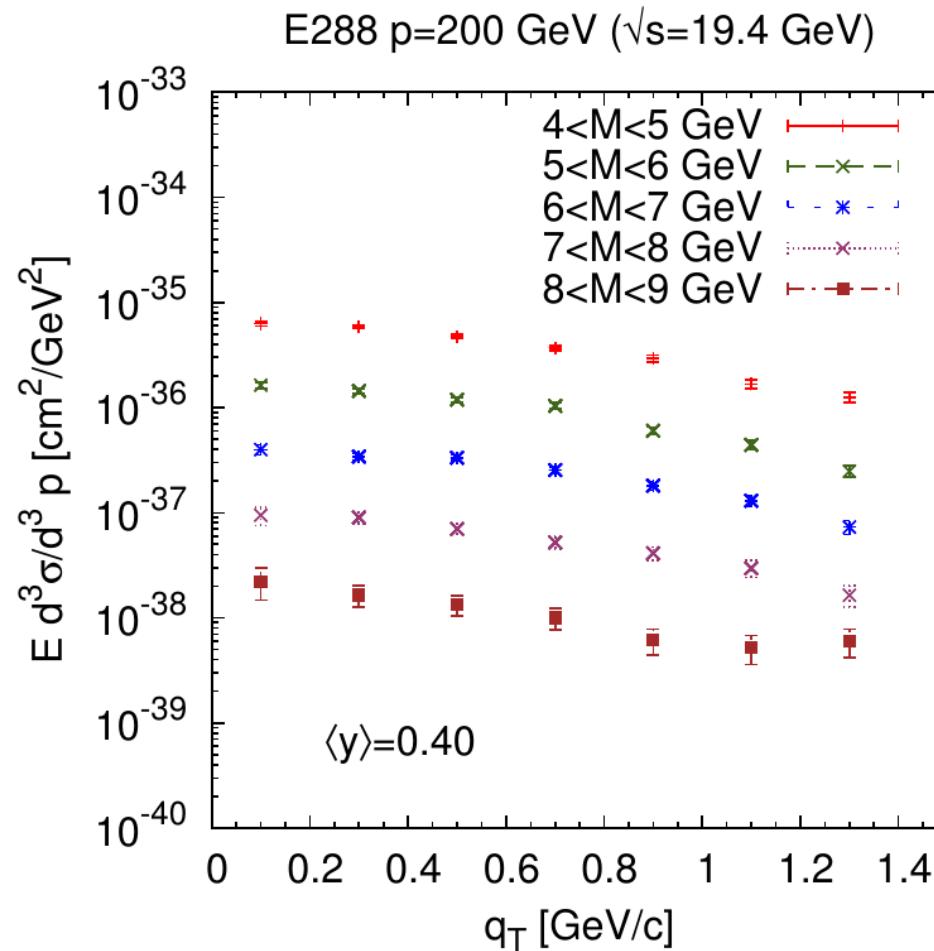
Outline

- Phenomenology with Gaussian Models
- Conclusions I
- Resummation & TMD evolution
- Conclusions II

Phenomenology with Gaussian Models

Gaussian approach

➤ Why Gaussians?



➤ Low transverse momentum distributions are Gaussian!

Gaussian approach

- Step 1: Simple phenomenological ansatz

$$f(x, k_{\perp}; Q^2) = f(x; Q^2) \frac{e^{-\frac{k_{\perp}^2}{\langle k_{\perp}^2 \rangle}}}{\pi \langle k_{\perp}^2 \rangle}$$

Factorization of longitudinal and transverse degrees of freedom;
Gaussian distribution of transverse momentum

- Step 2: The TMD cross section is given by the product of the born cross section and the convolution of the "TMD" PDFs:

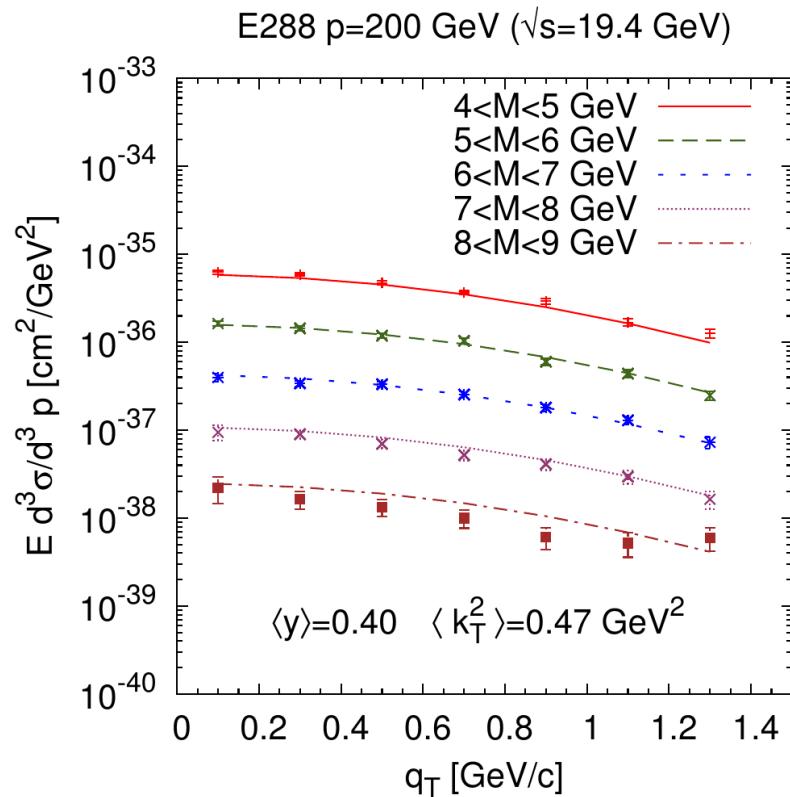
$$\frac{d\sigma}{dP_T^2} \propto \frac{4\pi\alpha_{em}}{9Q^2} \sum_q e_q^2 f_{q/h_1}(x_1; Q^2) \bar{f}_{q/h_2}(x_2; Q^2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$

In this way the distribution in P_T
is just a Gaussian!

- For pp or pN scattering we just have: $\langle P_T^2 \rangle = 2\langle k_{\perp}^2 \rangle$

Gaussian approach

$$\frac{d\sigma}{dP_T^2} \propto \frac{4\pi\alpha_{em}}{9Q^2} \sum_q e_q^2 f_{q/h_1}(x_1; Q^2) \bar{f}_{q/h_2}(x_2; Q^2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$



Nice!

Further information: the M^2 dependence is described by the model and it is given by the interplay between $1/M^2$ born cross section +DGLAP+Kinematics

Gaussian approach: SIDIS

- Simple phenomenological ansatz

$$f(x, k_\perp; Q^2) = f(x; Q^2) \frac{e^{-\frac{k_\perp^2}{\langle k_\perp^2 \rangle}}}{\pi \langle k_\perp^2 \rangle} \quad D_{h/q}(z, p_\perp; Q^2) = D_{h/q}(z; Q^2) \frac{e^{-\frac{p_\perp^2}{\langle p_\perp^2 \rangle}}}{\pi \langle p_\perp^2 \rangle}$$

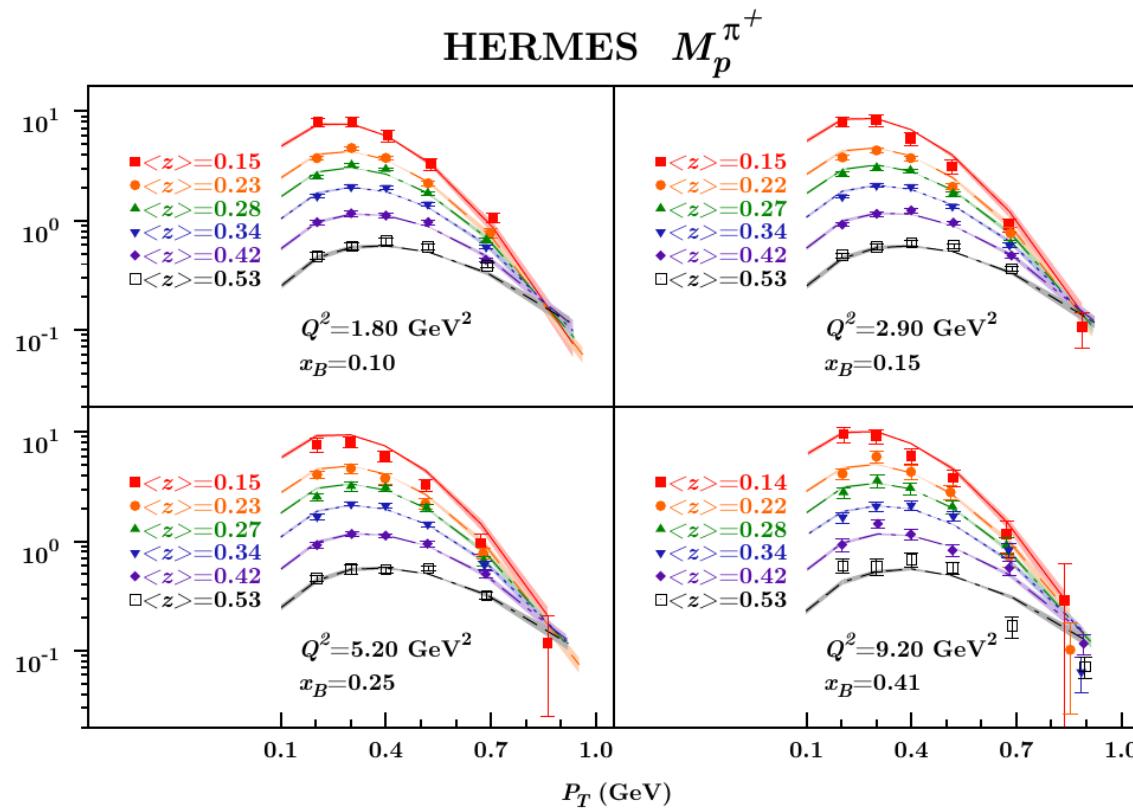
$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

Gaussian approach: SIDIS

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

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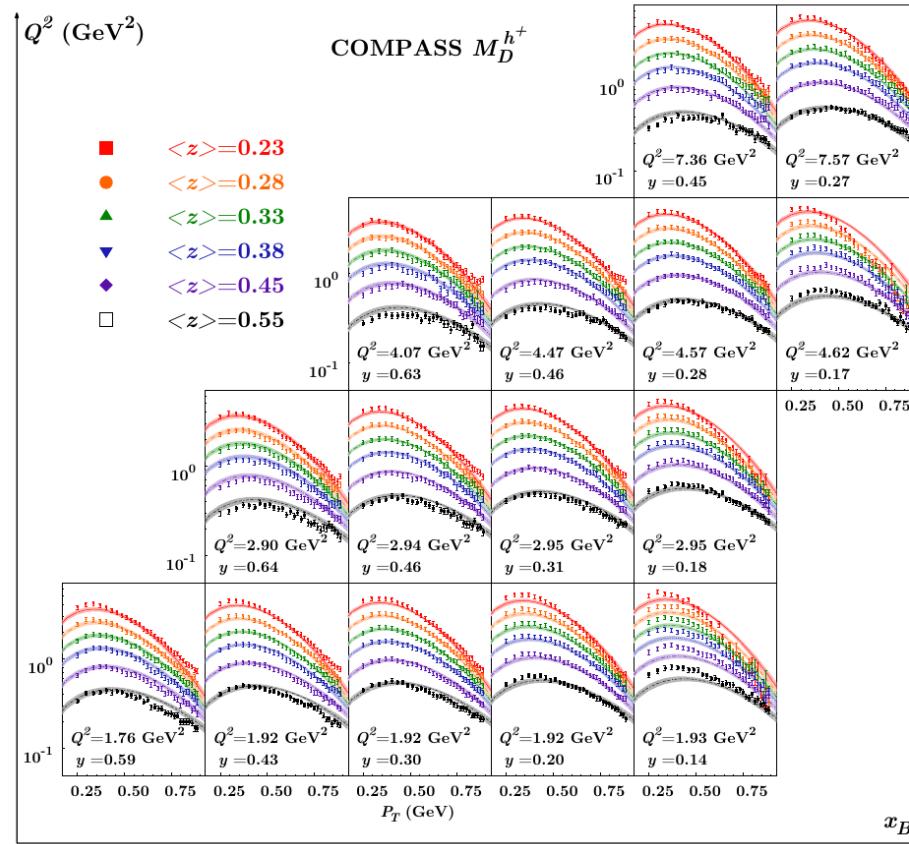


Anselmino et al. JHEP 1404 (2014) 005

Gaussian approach: SIDIS

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$



$$\langle k_\perp^2 \rangle = 0.60 \pm 0.14 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.20 \pm 0.02 \text{ GeV}^2$$

$$\chi^2_{\text{dof}} = 3.42$$

$$N_y = A + B y$$

$$A = 1.06 \pm 0.06$$

$$B = -0.43 \pm 0.14$$

Gaussian approach: SIDIS

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

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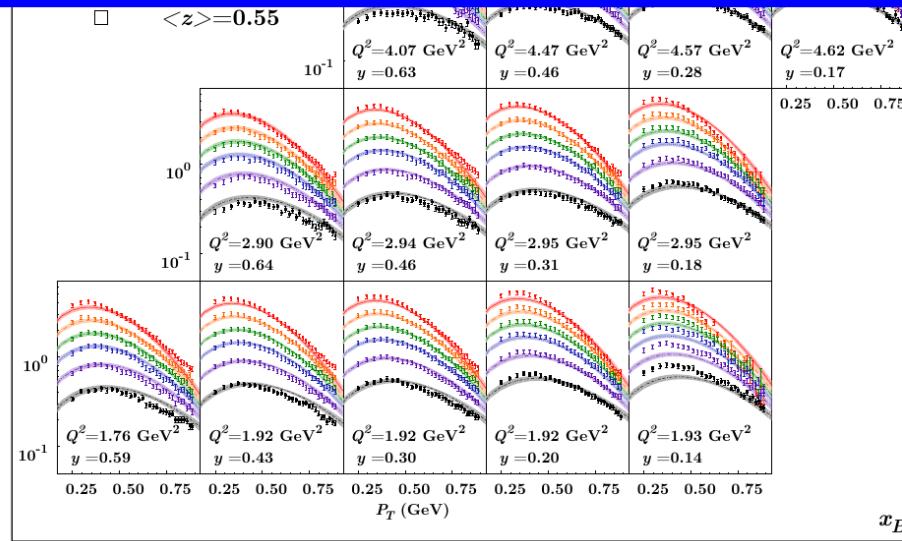
- COMPASS has already published p_T^2 dependent multiplicities from 2004 data in EPJC 73 (2013) 2531
- However, issues in this analysis were detected, which can affect the overall x, y, z normalization of multiplicities up to 40%, but the shape as a function of p_T^2 are not significantly affected. Erratum in preparation

M. Stolarski (LIP)

SPIN 2014

21-X-2014

27 / 35



$$N_y = A + B y$$

$$A = 1.06 \pm 0.06$$

$$B = -0.43 \pm 0.14$$

Sivers function

Gaussian approach and evolution (Sivers)

- The Sivers function is factorized in x and k_\perp and proportional to the unpolarized PDF.

$$\begin{aligned}\Delta^N \hat{f}_{q/p^\uparrow}(x, k_\perp; Q) &= 2\mathcal{N}_q(x) h(k_\perp) \hat{f}_{q/p}(x, k_\perp; Q) \\ &= 2\mathcal{N}_q(x) f_{q/p}(x; Q) \sqrt{2e} \frac{k_\perp}{M_1} \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle}\end{aligned}$$

Collinear PDF (DGLAP)

$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

$$\langle k_\perp^2 \rangle_S = \frac{M_1^2 \langle k_\perp^2 \rangle}{M_1^2 + \langle k_\perp^2 \rangle}$$

$$\Delta^N \hat{f}_{q/p^\uparrow}(x, k_\perp) = -\frac{2k_\perp}{m_p} f_{1T}^\perp(x, k_\perp)$$

Fit of HERMES and COMPASS SIDIS data

➤ Data sets:

- HERMES (2009) $\pi^+ \pi^- \pi^0 K^+ K^-$
- COMPASS Deuteron (2004) $\pi^+ \pi^- K^+ K^-$
- COMPASS Proton (2011) $h^+ h^-$

$$\chi^2_{\text{d.o.f}} = 1.26$$

11 free parameters, 261 points

fixed parameters

$$\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.20 \text{ GeV}^2$$

As in Anselmino et al. Phys. Rev. D71 074006 (2005)

$$N_{u_v} = 0.45^{+0.25}_{-0.17}$$

$$N_{d_v} = -1.00^{+0.85}_{-0.00}$$

$$N_{\bar{u}} = -0.03^{+0.25}_{-0.31}$$

$$N_{\bar{d}} = -0.47^{+0.43}_{-0.53}$$

$$N_s = 0.29^{+0.71}_{-0.87}$$

$$N_{\bar{s}} = 1.00^{+0.00}_{-1.07}$$

$$\alpha_{u_v} = 1.08^{+0.68}_{-0.62}$$

$$\alpha_{d_v} = 1.7^{+1.15}_{-0.91}$$

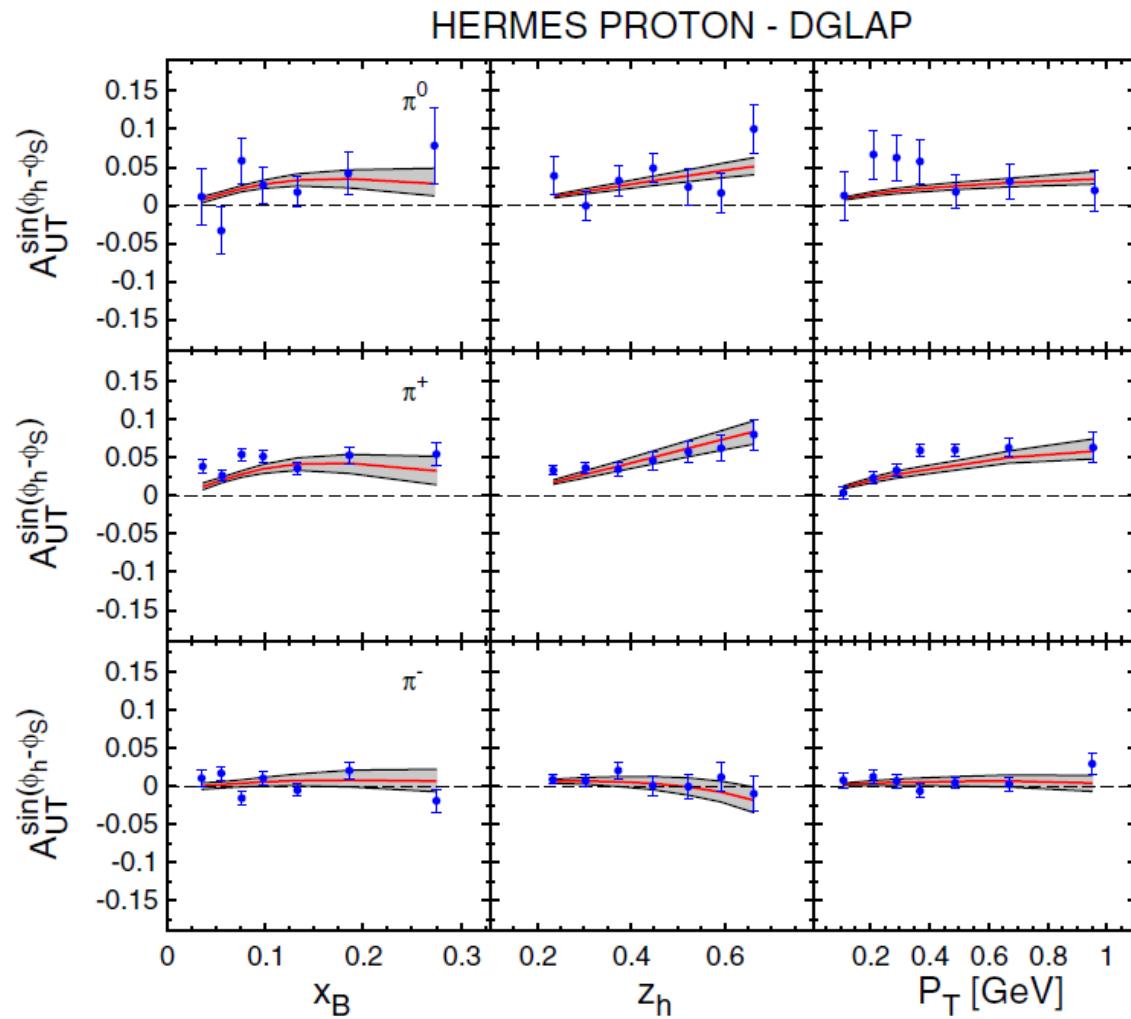
$$\alpha_{sea} = 1.21^{+1.14}_{-0.92}$$

$$\beta = 6.9^{+6.4}_{-4.1}$$

$$M_1^2 = 0.19^{+0.77}_{-0.10} (\text{GeV}/c)^2$$

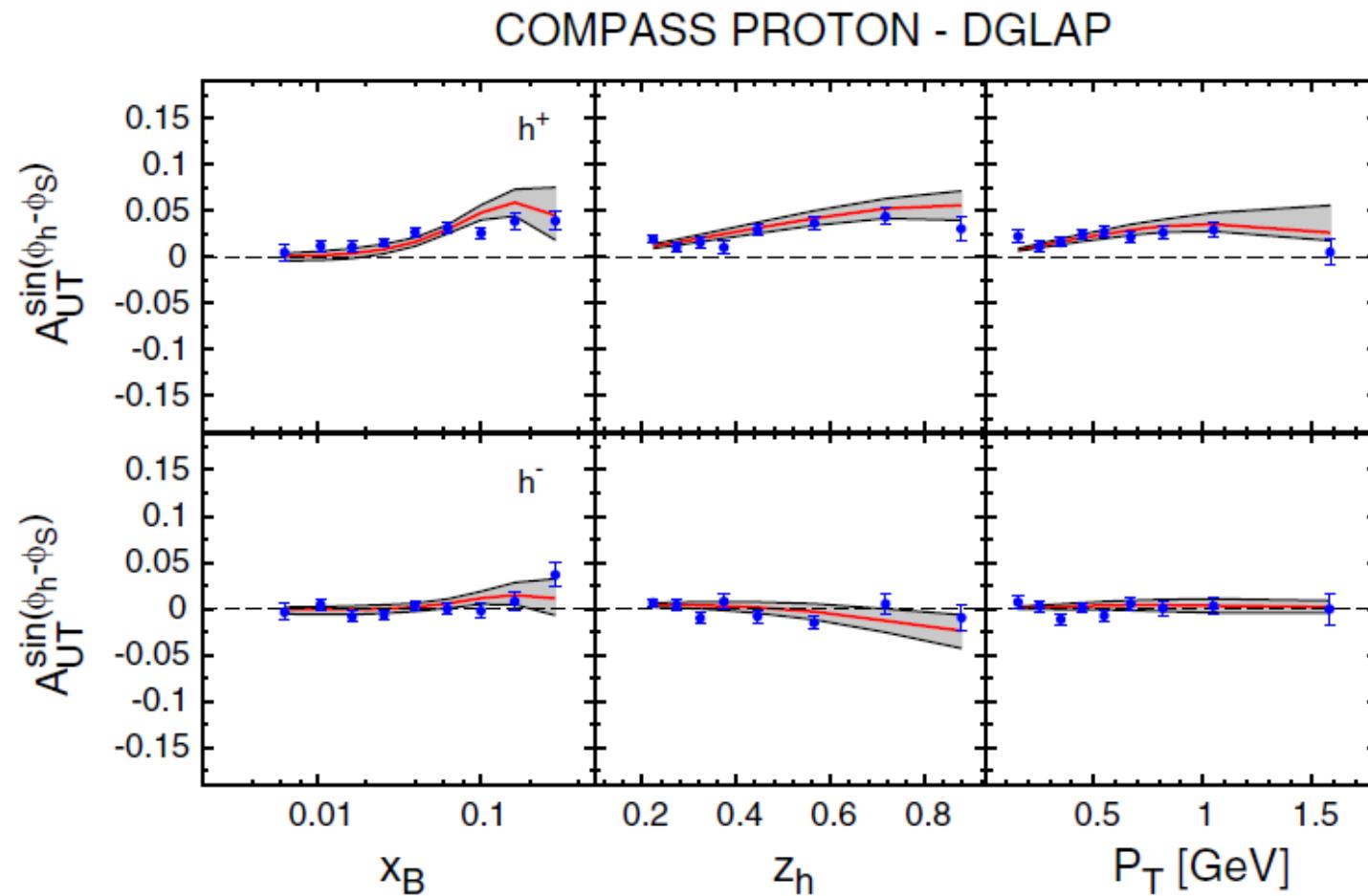
Fit of HERMES and COMPASS SIDIS data

A. Airapetian et al., Phys. Rev. Lett. 103, 152002 (2009), arXiv:0906.3918 [hep-ex]



χ^2/points HERMES: 1.20

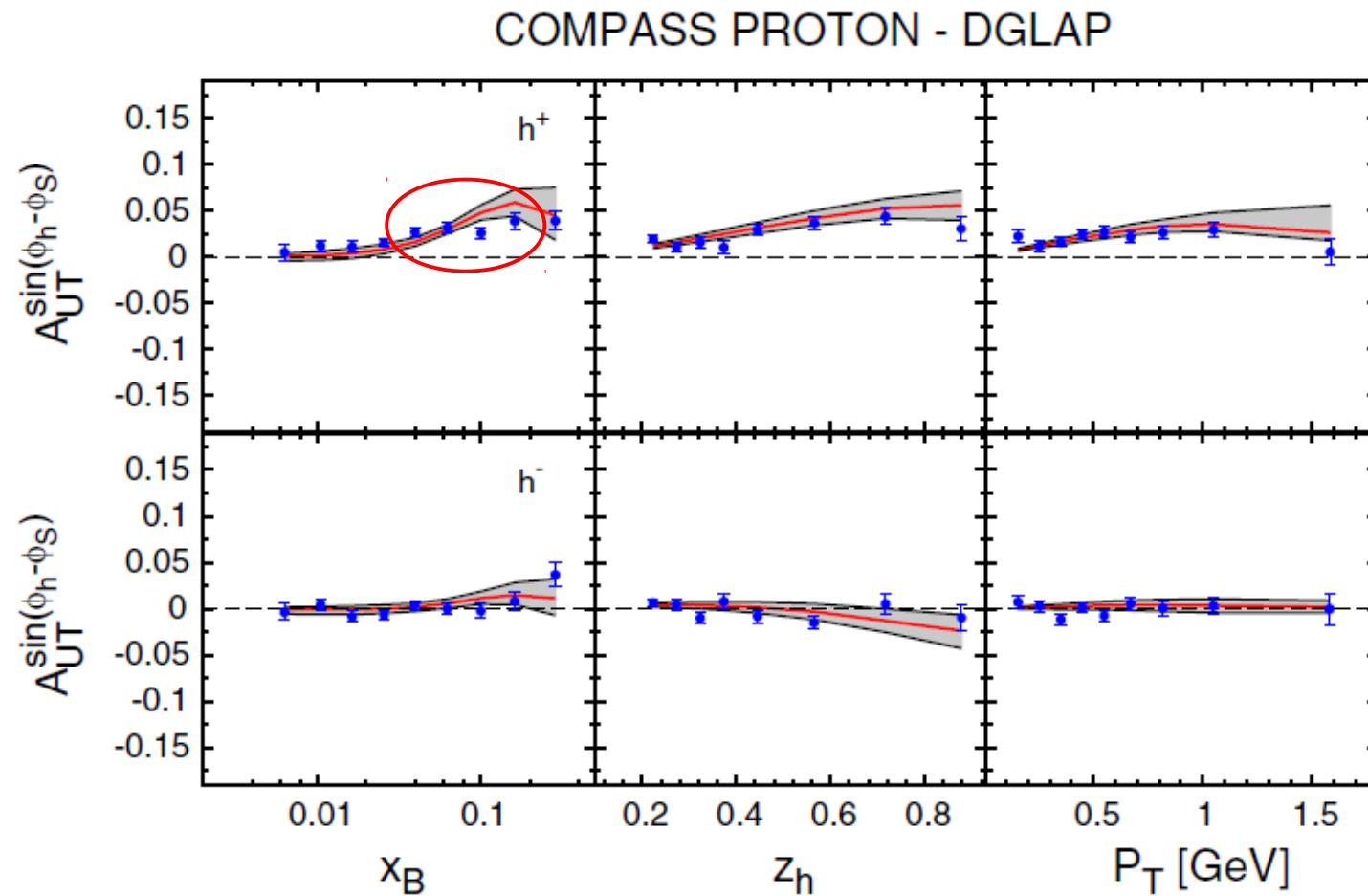
Fit of HERMES and COMPASS SIDIS data



COMPASS coll., Phys.Lett. B717 (2012) 383-389,
Nuovo Cim. C035N2 (2012) 107-114

χ^2/points COMPASS P: 1.80

Fit of HERMES and COMPASS SIDIS data



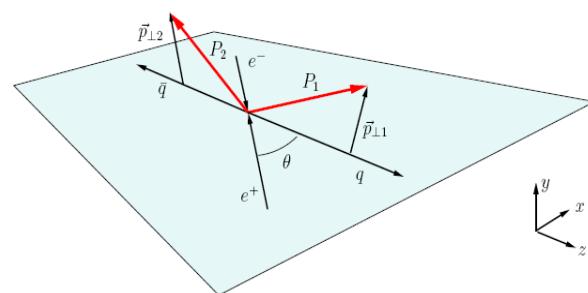
COMPASS coll., Phys.Lett. B717 (2012) 383-389,
Nuovo Cim. C035N2 (2012) 107-114

χ^2/points COMPASS P: 1.80

Transversity and Collins function

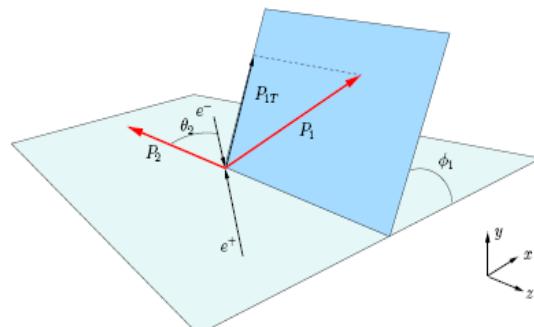
Extraction of transversity & Collins functions

- Azimuthal asymmetry in polarized SIDIS (Transversity & Collins functions)
- $e^+e^- \rightarrow h_1 h_2 X$ (Collins functions)



A_{12} asymmetry

Thrust axis method



A_0 asymmetry

Hadronic plane method

Extraction of transversity & Collins functions

- To avoid acceptance effects the BELLE Collaboration considered ratio of different combinations of hadron pairs:

Unlike-sign ($\pi^+ \pi^- + \pi^- \pi^+$)  A^{UL} asymmetry

Like-sign ($\pi^+ \pi^+ + \pi^- \pi^-$)

Unlike-sign ($\pi^+ \pi^- + \pi^- \pi^+$)  A^{UC} asymmetry

Charged ($\pi^+ \pi^+ + \pi^- \pi^- + \pi^+ \pi^- + \pi^- \pi^+$)

➤ A_{12}^{UL} A_{12}^{UC} A_0^{UL} A_0^{UC}

Gaussian parametrizations

➤ Transversity function:

$$\Delta_T q(x, k_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) [f_{q/p}(x) + \Delta q(x)] \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T}$$

➤ Collins function:

$$\begin{aligned} \Delta^N D_{\pi/q^\uparrow}(z, p_\perp) &= 2\mathcal{N}_q^C(z) h(p_\perp) D_{\pi/q}(z, k_\perp) \\ &= 2\mathcal{N}_q^C(z) \sqrt{2e} \frac{p_\perp}{M_C} e^{-p_\perp^2 / M_C^2} \frac{e^{-\frac{p_\perp^2}{\langle p_\perp^2 \rangle}}}{\pi \langle p_\perp^2 \rangle} D_{\pi/q}(z) \end{aligned}$$

$$\mathcal{N}_q^T(x) = N_q^T x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma + \delta)^{(\gamma + \delta)}}{\gamma^\gamma \delta^\delta}$$

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z, p_\perp)$$

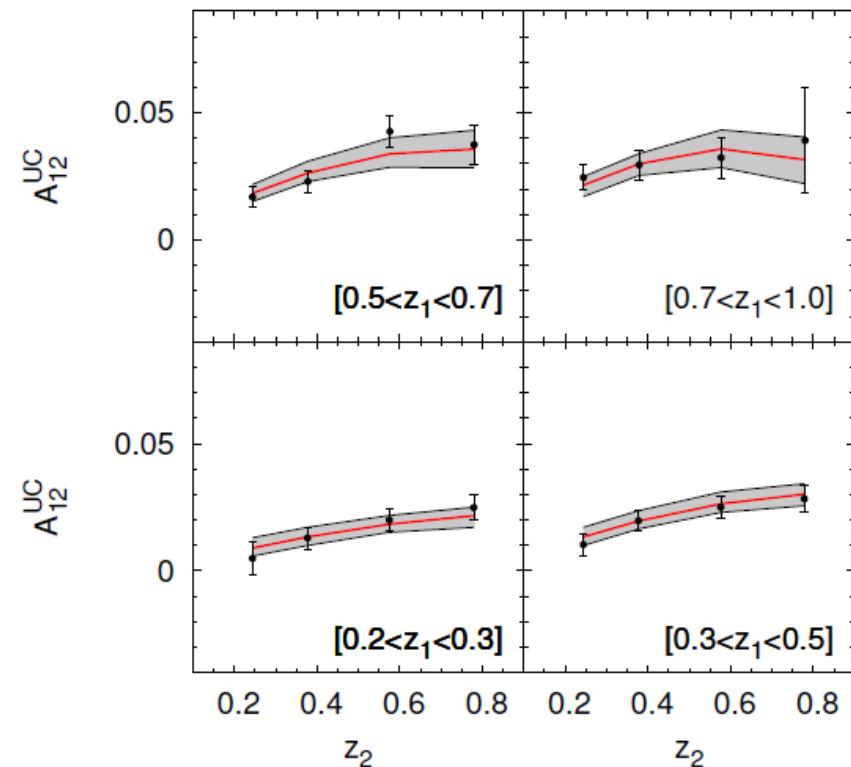
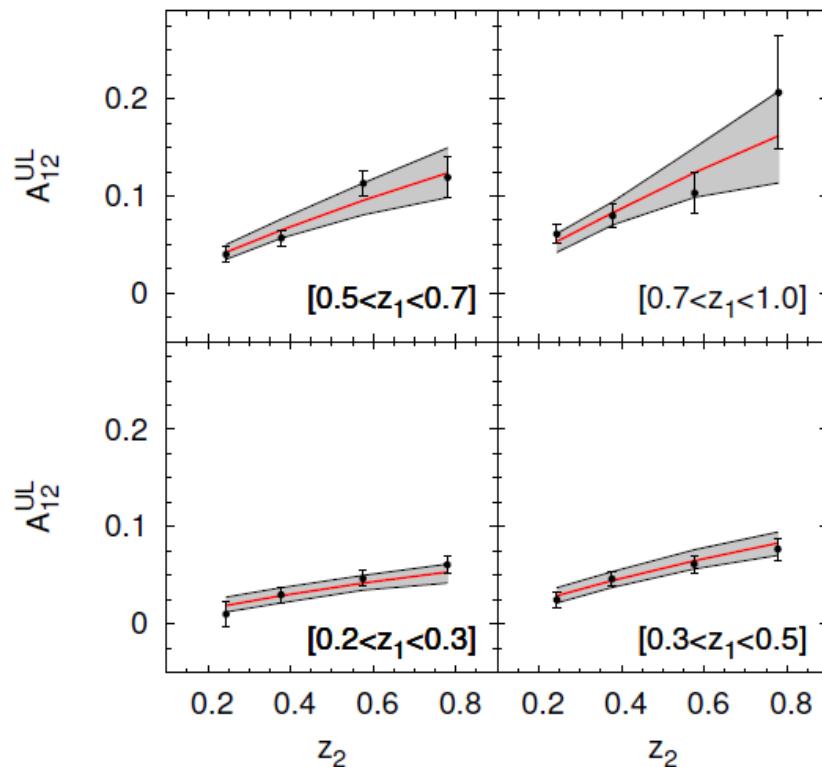
2013 Update of the extraction

- New analysis (PRD87, 2013):
 - HERMES (2009) $\pi^+ \pi^-$
 - COMPASS Deuteron (2004) $\pi^+ \pi^-$
 - COMPASS Proton (2013) $\pi^+ \pi^-$
 - BELLE A_{12} or A_0 (BELLE ERRATUM 2012, PRD86)
- U and d quarks transversity, favored and disfavored Collins functions
- Two separate fits for A_{12} and A_0 sets

Anselmino et al. Phys. Rev. D87, 094019

Extraction of transversity & Collins functions

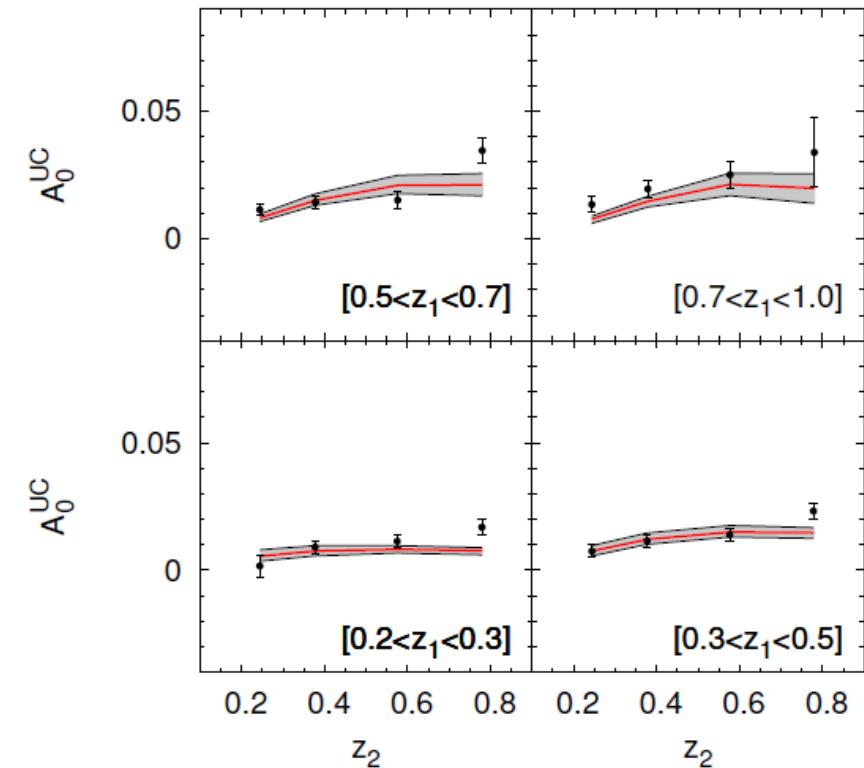
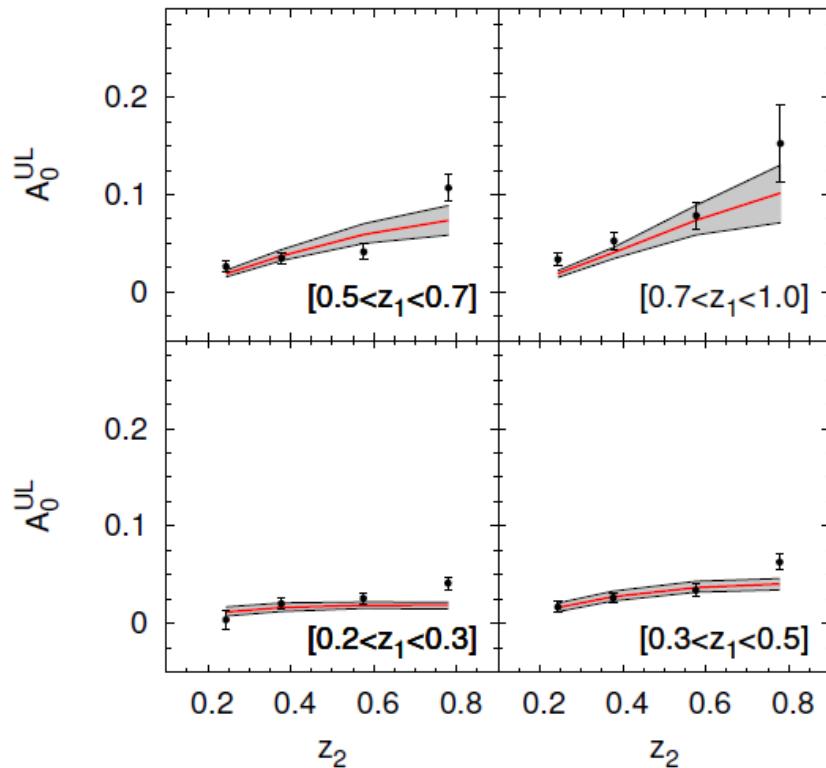
➤ FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES



➔ Full compatibility between UL and UC, contrary to 2008 BELLE data

Extraction of transversity & Collins functions

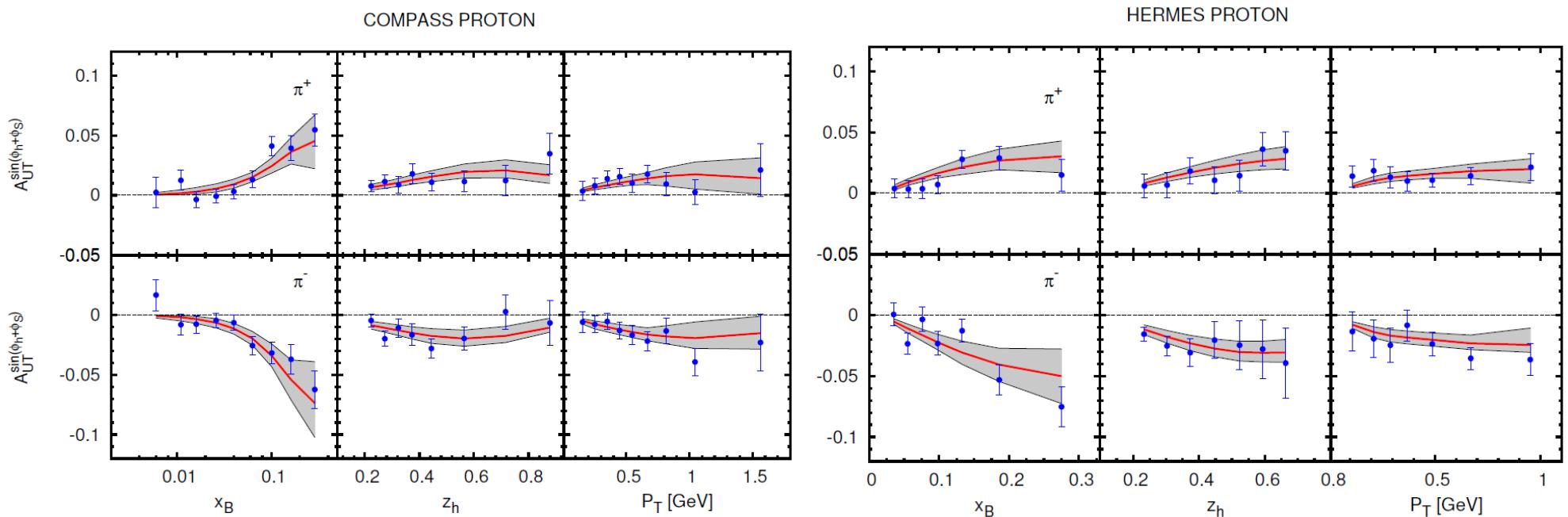
➤ FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES



➔ Still tension between the two methods A_0 and A_{12}

Extraction of transversity & Collins functions

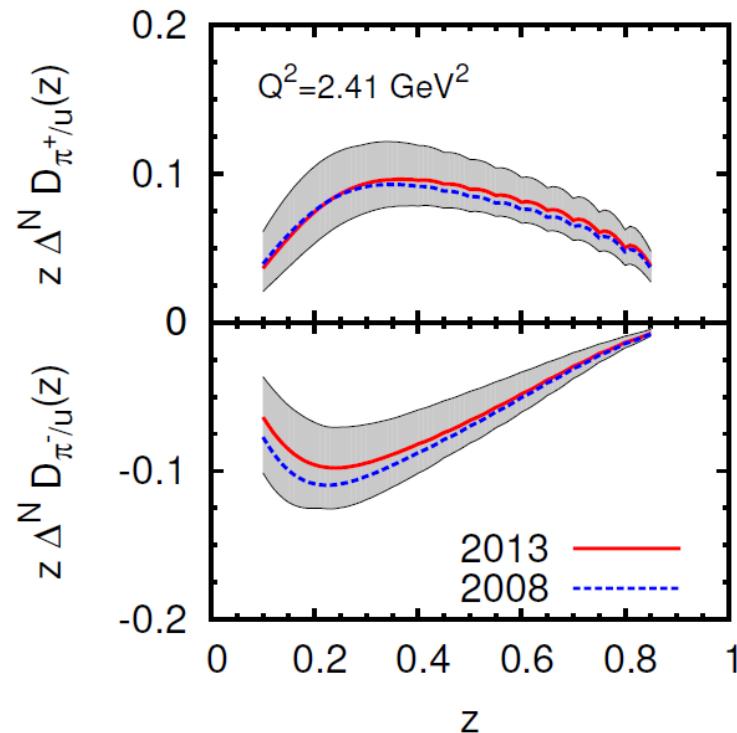
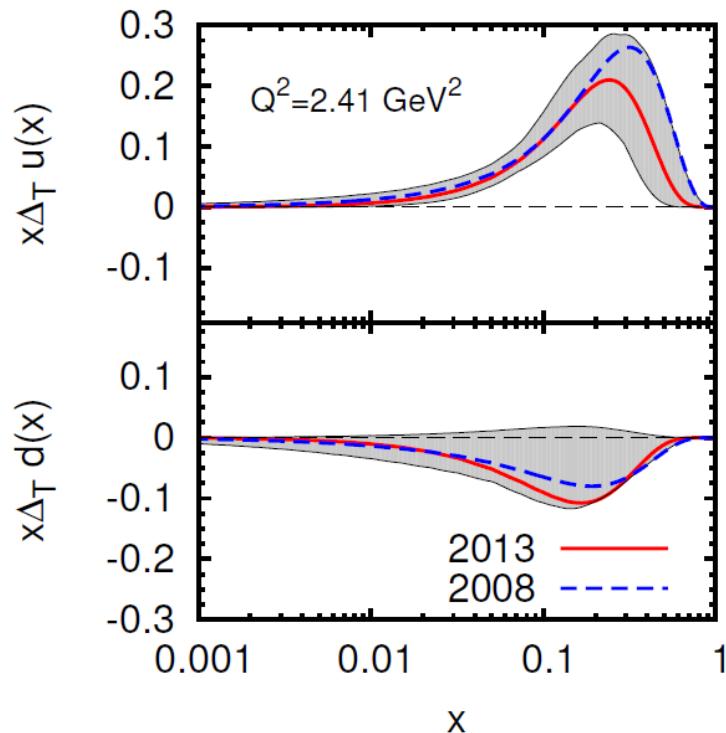
➤ FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES



➔ Similar good description of HERMES and COMPASS

Extraction of transversity & Collins functions

➤ FIT I: A_{12} BELLE data UL & UC +COMPASS+ HERMES



➔ Results similar to 2008 extraction

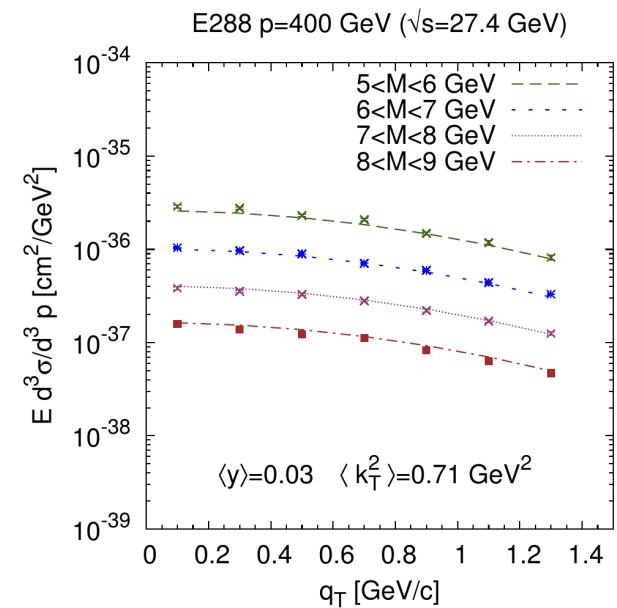
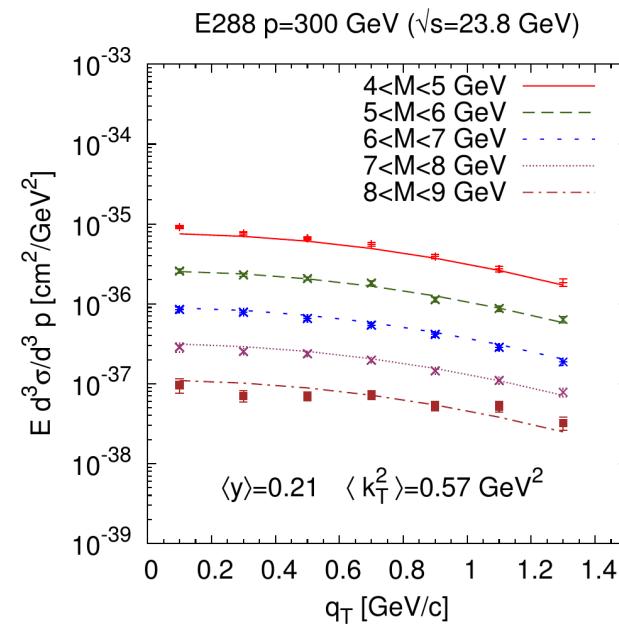
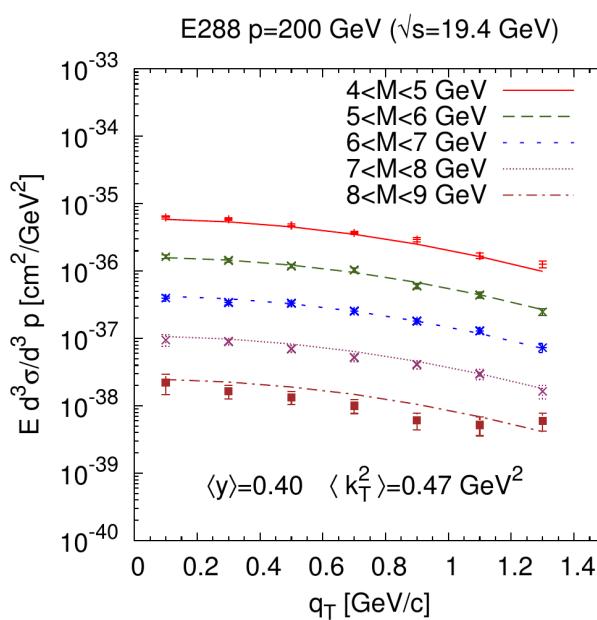
$N_u^T = 0.46^{+0.20}_{-0.14}$	$N_d^T = -1.00^{+1.17}_{-0.00}$
$\alpha = 1.11^{+0.89}_{-0.66}$	$\beta = 3.64^{+5.80}_{-3.37}$
$N_{\text{fav}}^C = 0.49^{+0.20}_{-0.18}$	$N_{\text{dis}}^C = -1.00^{+0.38}_{-0.00}$
$\gamma = 1.06^{+0.45}_{-0.32}$	$\delta = 0.07^{+0.42}_{-0.07}$
$M_h^2 = 1.50^{+2.00}_{-1.12} \text{ GeV}^2$	

Conclusions I

- Gaussian models are able to describe the main futures of the data
- They are simple to understand
- They are an amazing tool to discover problems in the data sets
- But...

Predictive power of Gaussian Models

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$

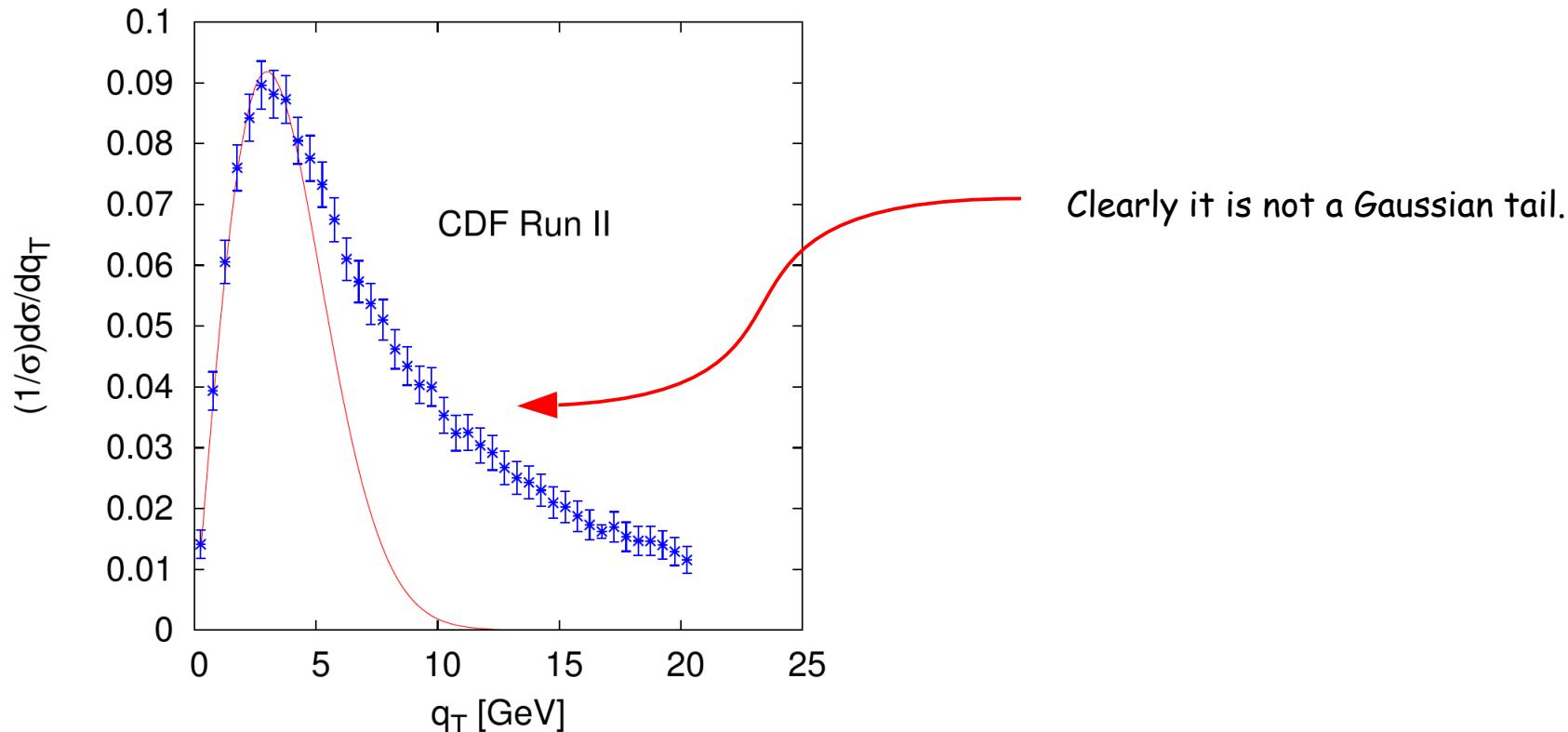


- Each data set is Gaussian but with a different width

High energy data

- Are high energy data Gaussian distributed?

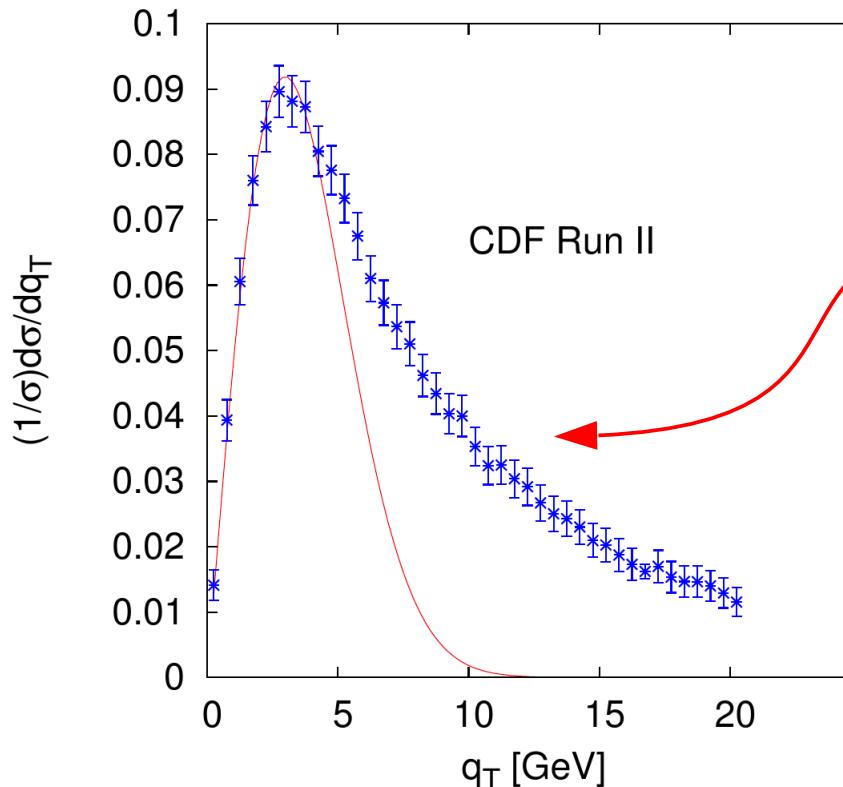
$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$



High energy data

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$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$



Clearly it is not a Gaussian tail.

➤ The tail is generated by Soft gluon emissions that can be treated using QCD

Collinear Resummation: CSS

➤ Resummation: CSS

$$\frac{1}{\sigma_0} \frac{d\sigma}{dQ^2 dy dq_T^2} = \int \frac{d^2 \mathbf{b}_T e^{i \mathbf{q}_T \cdot \mathbf{b}_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, b_T, Q) + Y(x_1, x_2, q_T, Q)$$

Soft gluon emissions resummed in b-space Regular part

$$W_j(x_1, x_2, b_T, Q) = \sum_{i,k} \exp [S_j(b_*, Q)] \left[C_{ji} \otimes f_i(x_1, \mu_b) \right] \left[C_{jk} \otimes f_k(x_2, \mu_b) \right] F_{NP}(x_1, x_2, b_T, Q)$$

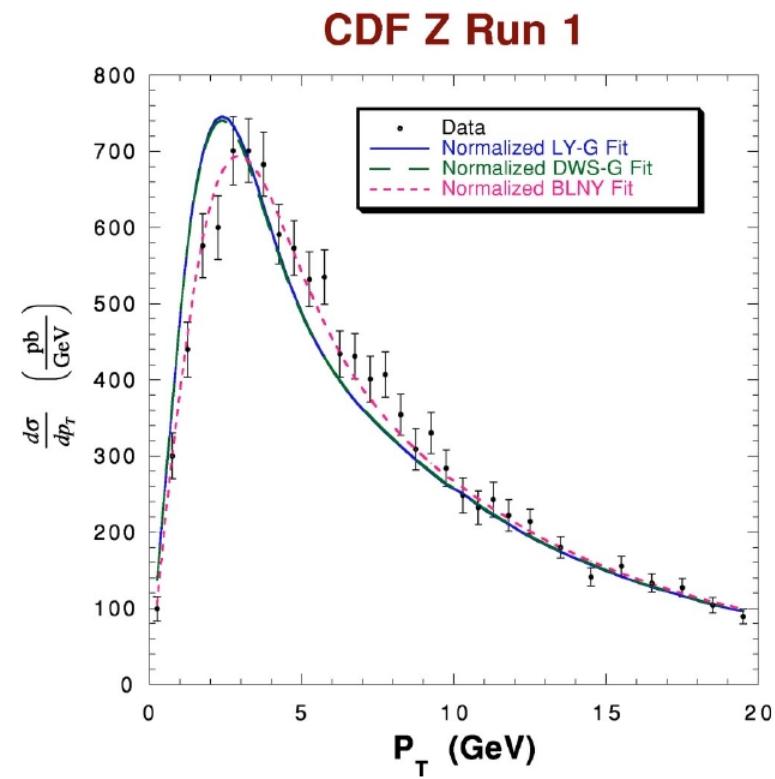
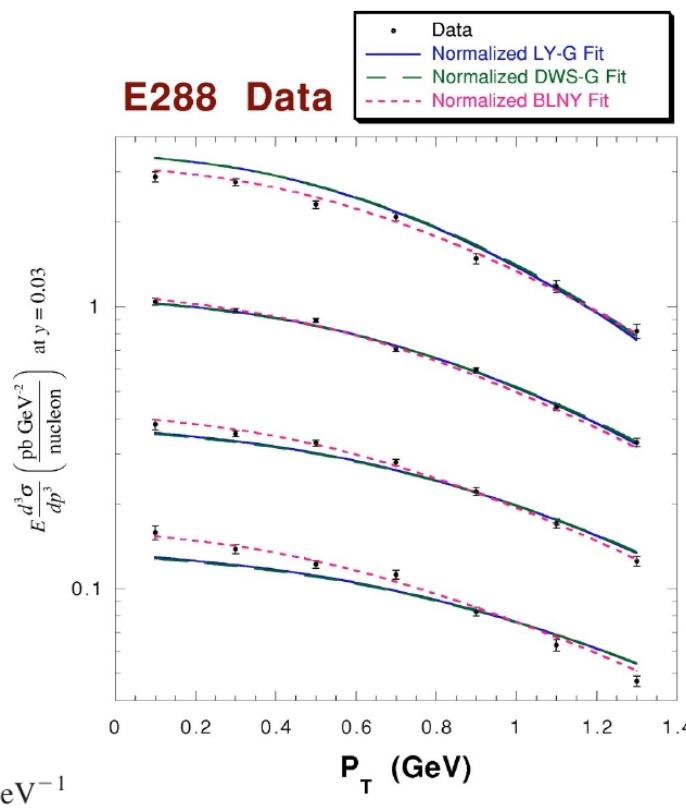
$$S_j(b_T, Q) = \int_{C_1^2/b_T^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} \left[A_j(\alpha_s(\kappa)) \ln \left(\frac{Q^2}{\kappa^2} \right) + B_j(\alpha_s(\kappa)) \right] \quad b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \quad \mu_b = C_1/b_*$$

Brock-Landry-
Nadolsky-Yuan (BLNY)

$$\exp \left[-g_1 - g_2 \ln \left(\frac{Q}{2Q_0} \right) - g_1 g_3 \ln(100x_1 x_2) \right] b^2$$

CSS Phenomenology

Nadolsky et al.* analyzed successfully low energy DY data and Z_0 production data using different parametrizations



*Nadolsky et al., Phys.Rev. D67,073016 (2003)

CSS Resummation and TMD evolution (2011)

➤ 2011 - Proper definition of a TMD (in b space):

$$\tilde{F}(x, b_T, Q, \zeta_F) = \sum_j \int_x^1 \frac{dy}{y} \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) f_j(y, \mu_b) \\ \exp \left\{ \ln \left(\frac{\sqrt{\zeta_F}}{\mu_b} \right) \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^Q \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln \left(\frac{\sqrt{\zeta_F}}{\kappa} \right) \gamma_K(\kappa) \right\} \\ \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\}$$

New scale ζ_F related to rapidity divergences

CSS Resummation and TMD evolution (2011)

➤ In phenomenological applications:

$$\zeta_F = Q^2 \quad b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \quad \mu_b = C_1/b_*$$

➤ And previous expression simplify considerably, at NLL:

$$\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) = \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b)$$

exp $\left\{ \frac{1}{2} S^{CSS}(b_*, \mu_b) \right\}$

exp $\left\{ -g_P(x, b_T) - g_K(b_T) \ln \left(\frac{Q}{Q_0} \right) \right\}$

Roughly speaking the TMD evolution
reduces to a CSS resummation.

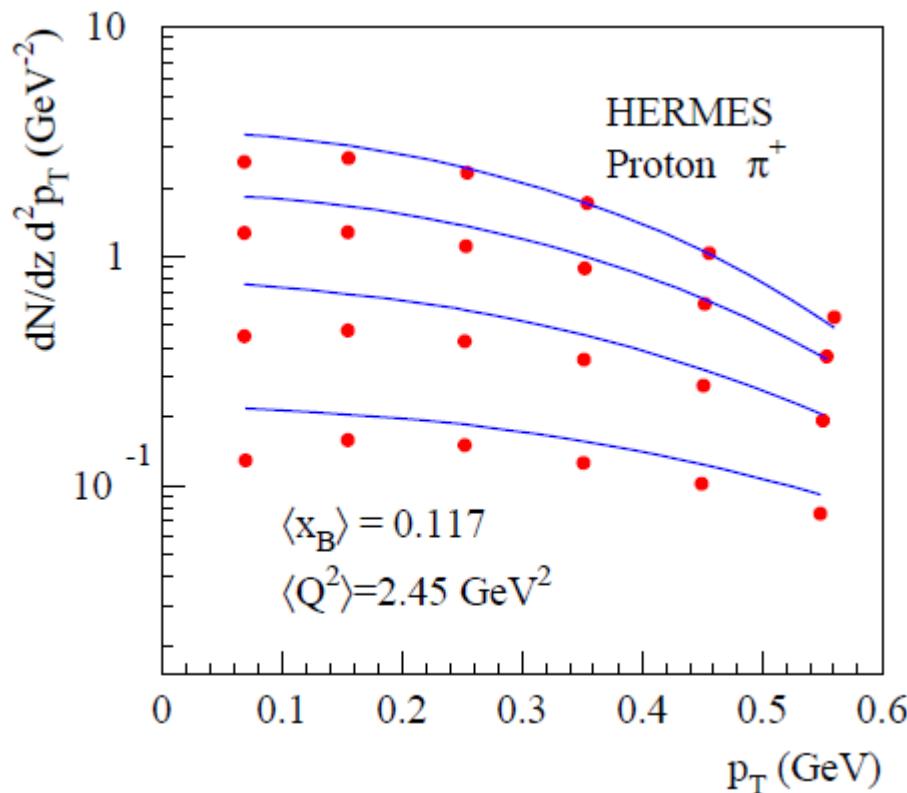
CSS results can be used to study TMDs.

CSS/TMD evolution and HERMES/COMPASS data

- CSS (and therefore TMD evolution) can describe DY data
- What about HERMES/COMPASS SIDIS data?

CSS/TMD evolution and HERMES/COMPASS data

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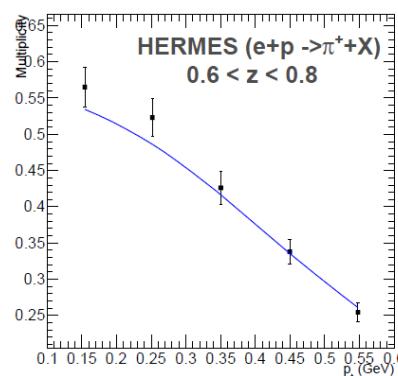
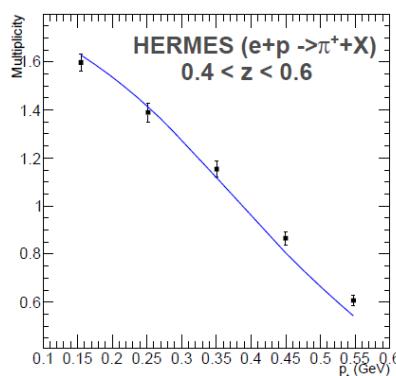
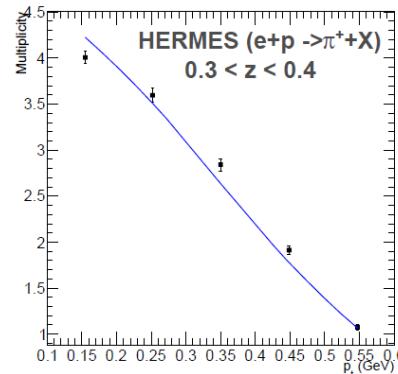
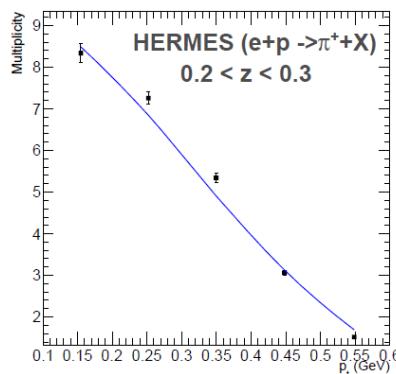


Echevarria, Idilbi, Kang, Vitev
Phys .Rev. D89 (2014) 074013

- Global Fit DY+SIDIS
- TMD evolution
- Wilson Coefficient at LO
- No full multidimensional data analysis
- No χ^2 provided

CSS/TMD evolution and HERMES/COMPASS data

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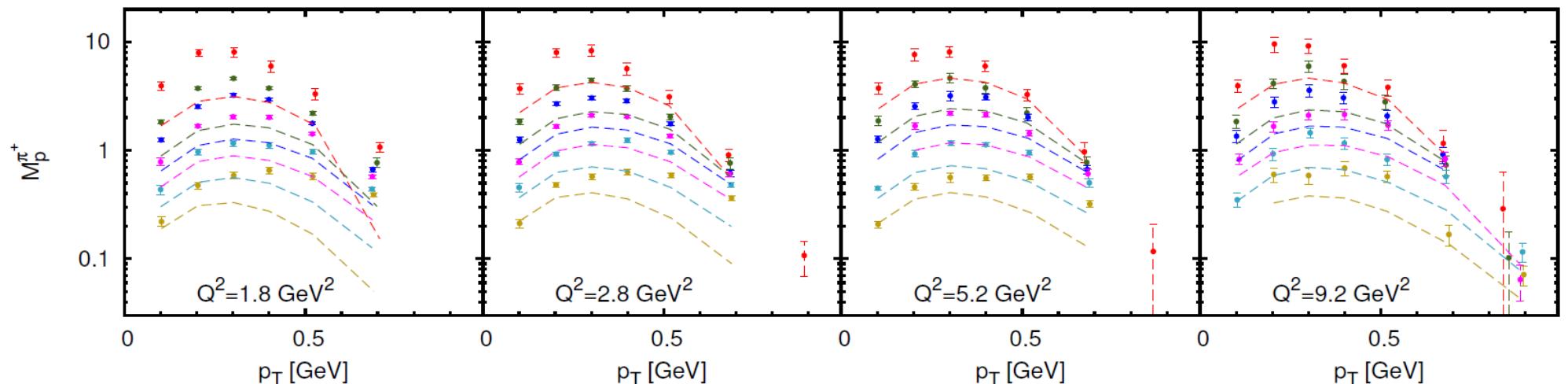
P. Sun, J. Isaacson, C.P. Yuan, F. Yuan
Arxiv: 1406.3073

- CSS evolution
- No fit??
- Normalization different for each z bin
- Only an example at fixed Q^2

$$Q^2 = 3.14 \text{ GeV}^2$$

CSS/TMD evolution and HERMES/COMPASS data

- CSS (and therefore TMD evolution) can describe DY data
- What about HERMES/COMPASS SIDIS data?



- CSS evolution
- HERMES NLL-NLO fit

S. Melis

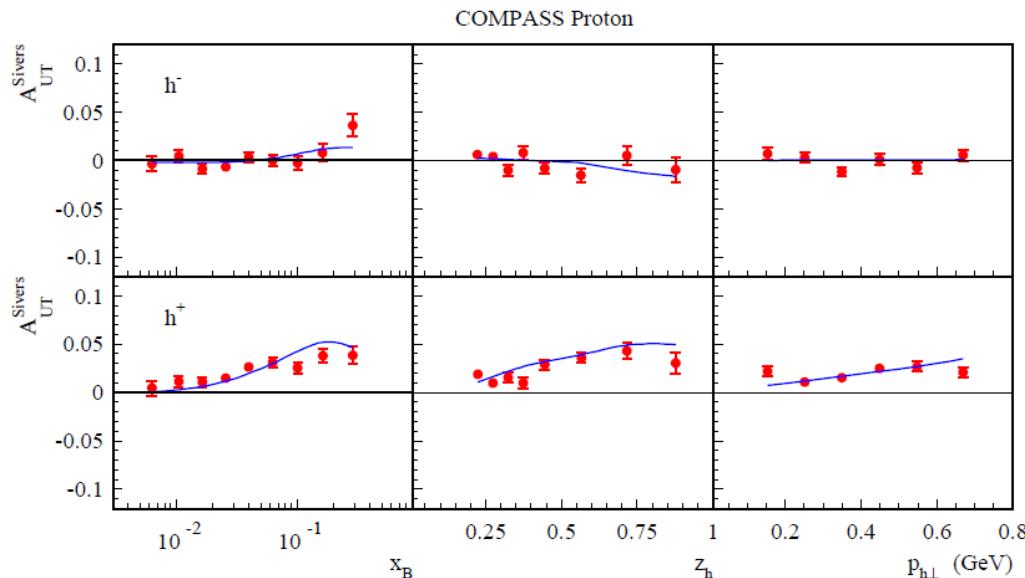
CSS/TMD evolution and HERMES/COMPASS data

- CSS (and therefore TMD evolution) can describe DY data
- What about HERMES/COMPASS SIDIS data?

... conclusion: the situation is still unclear

CSS/TMD evolution and Sivers asymmetry

➤ How TMD evolution describes the Sivers asymmetry in SIDIS?



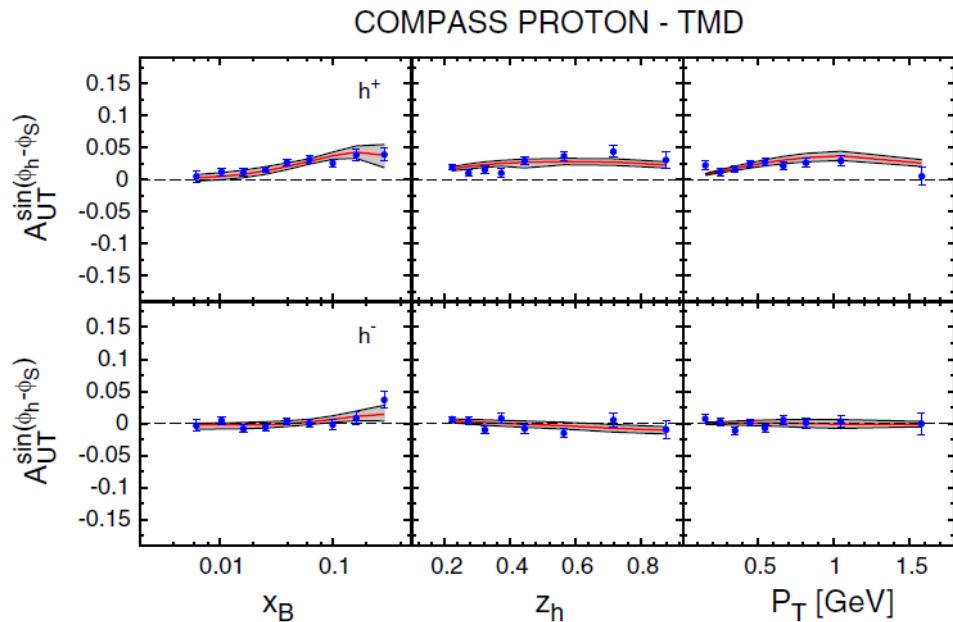
Echevarria, Idilbi, Kang, Vitev
Phys .Rev. D89 (2014) 074013

- TMD evolution
- Wilson Coefficient at LO
- $\chi^2=1.3$

$$F_{UT}^{\sin(\phi_h - \phi_s)} = \frac{1}{4\pi} \int_0^\infty db b^2 J_1(P_{h\perp} b/z_h) \sum_q e_q^2 T_{q,F}(x_B, x_B, c/b_*) D_{h/q}(z_h, c/b_*) \\ \times \exp \left\{ - \int_{c^2/b_*^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left(A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \exp \left\{ - b^2 \left(g_1^{\text{ff}} + g_1^{\text{sivers}} + g_2 \ln \frac{Q}{Q_0} \right) \right\}$$

CSS/TMD evolution and Sivers asymmetry

- How TMD evolution describes the Sivers asymmetry in SIDIS?



Anselmino, Boglione, Melis
Phys .Rev. D86 (2012) 014028

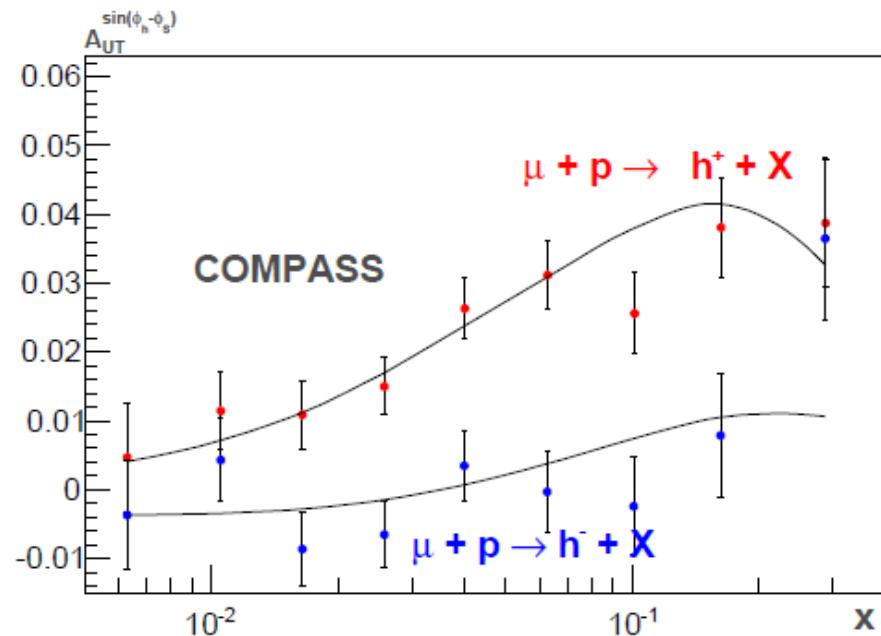
- TMD evolution
- Wilson Coefficient at LO
- No Sudakov at the initial scale
- $\chi^2 = 1.02$

$$\tilde{F}(x, \mathbf{b}_T; Q) = \tilde{F}(x, \mathbf{b}_T; Q_0) \tilde{R}(Q, Q_0, b_T) \exp \left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$$\tilde{F}(x, b_T, Q_0, Q_0^2) = f(x, Q_0) \exp \left[-\frac{\langle k_\perp^2 \rangle}{4} b_T^2 \right]$$

CSS/TMD evolution and Sivers asymmetry

➤ How TMD evolution describes the Sivers asymmetry in SIDIS?



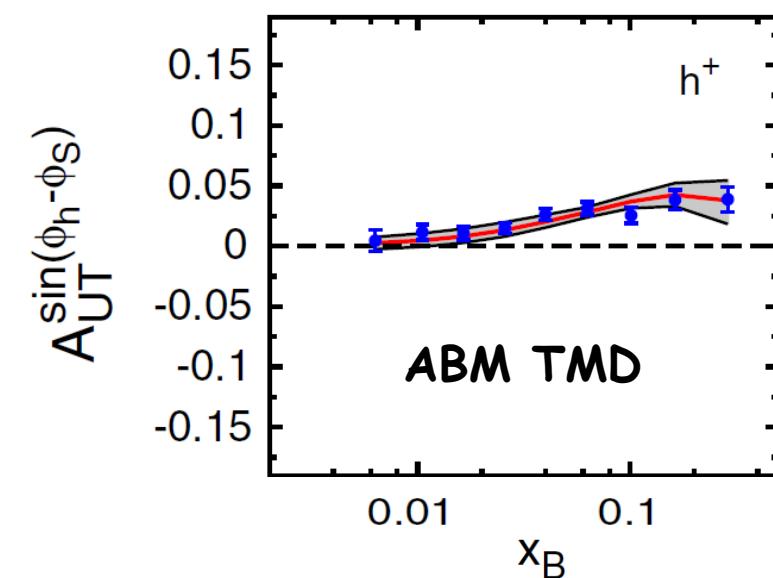
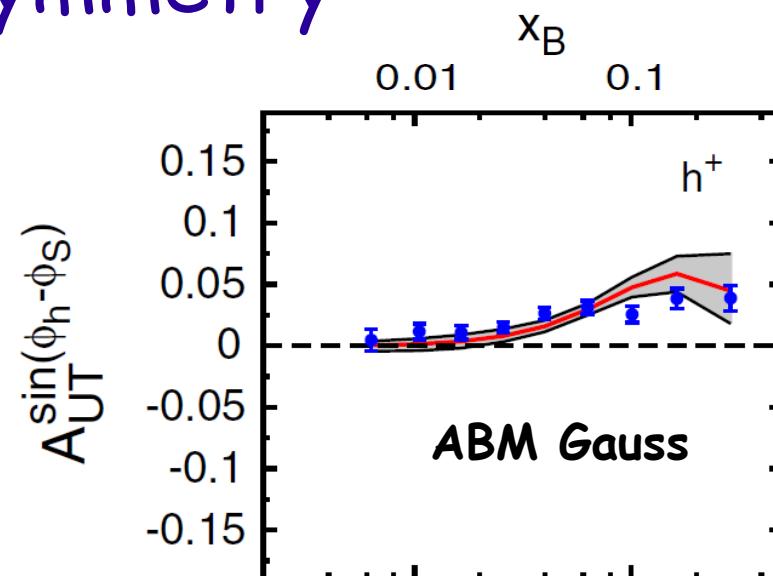
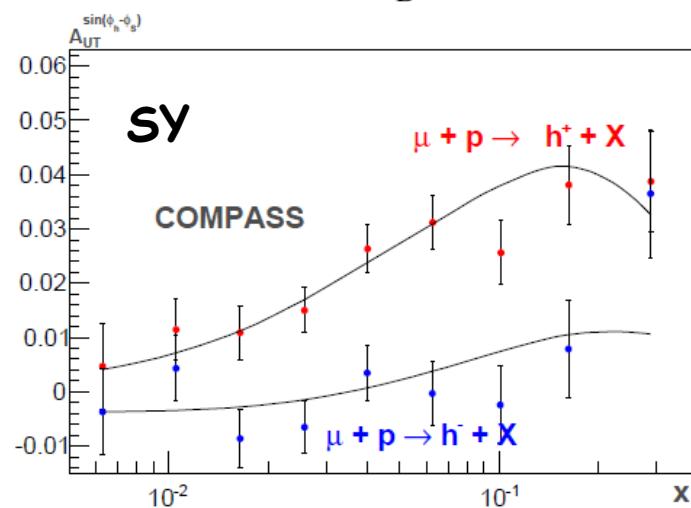
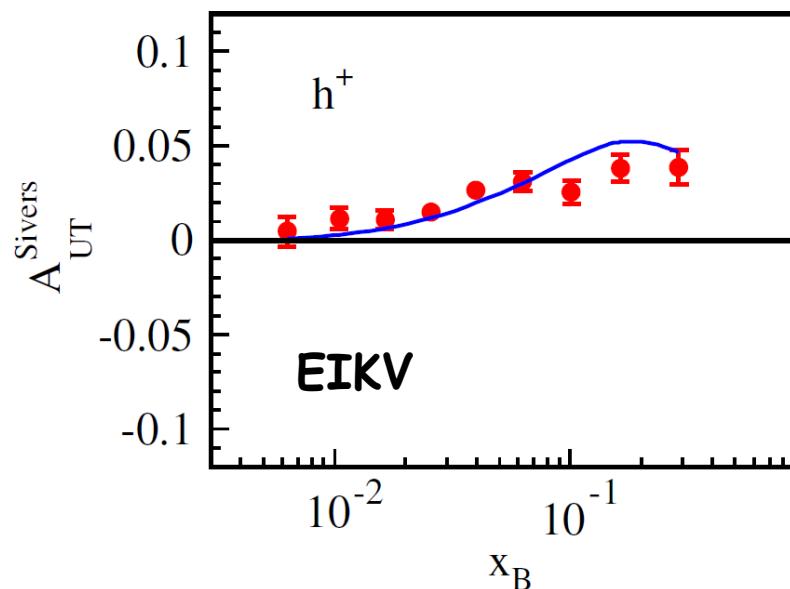
P. Sun and F. Yuan
 Phys. Rev. D88, 034016 (2013)
 Phys. Rev. D88, 114012 (2013)

➤ TMD?? ad hoc Sudakov
 ➤ $\chi^2 = 1.08$

$$\tilde{F}_{\text{sivers}}^\alpha(Q_0, b) = \frac{ib_\perp^\alpha M}{2} \sum_q e_q^2 \Delta f_q^{\text{sivers}}(x) D_q(z) e^{-(g_0 - g_s)b^2 - g_h b^2/z_h^2}$$

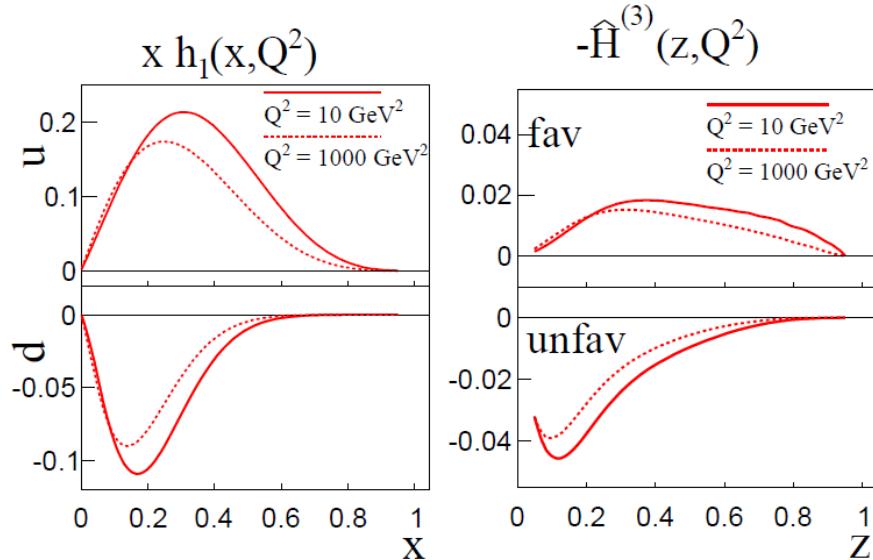
$$S_{\text{Sud}} = 2C_F \int_{Q_0}^Q \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2} \right]$$

CSS/TMD evolution and Sivers asymmetry



CSS/TMD evolution and Transversity

➤ Extraction of transversity function using TMD evolution

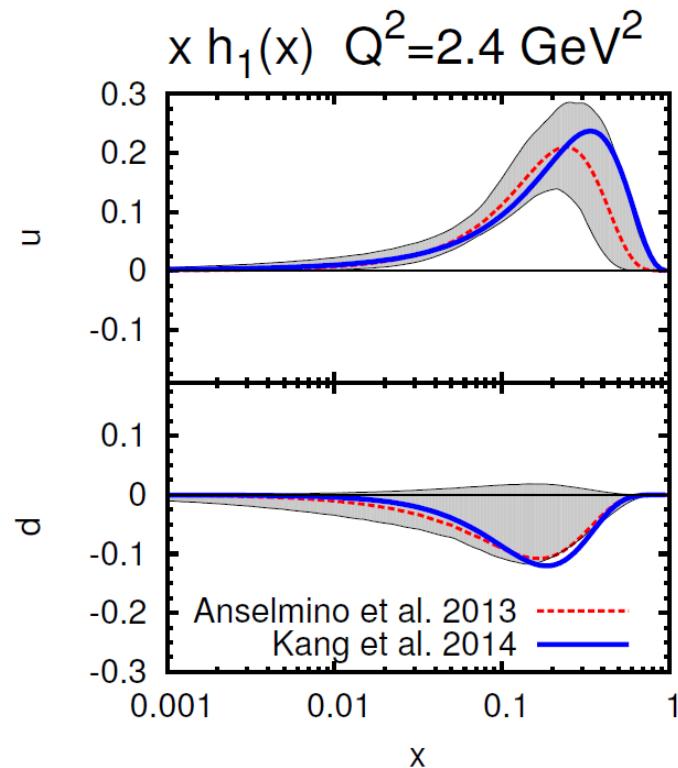


Kang, Prokudin, Sun and Yuan
Arxiv:1410.4877

$$F_{UT} = -\frac{1}{2z_h^3} \int \frac{db b^2}{(2\pi)} J_1\left(\frac{P_{h\perp} b}{z_h}\right) e^{-S_{\text{PT}}(Q, b_*) - S_{\text{NP coll}}^{(\text{SIDIS})}(Q, b)} \\ \times \delta C_{q \leftarrow i} \otimes h_1^i(x_B, \mu_b) \delta \hat{C}_{j \leftarrow q}^{(\text{SIDIS})} \otimes \hat{H}_{h/j}^{(3)}(z_h, \mu_b), (2)$$

CSS/TMD evolution and Transversity

- Extraction of transversity function using TMD evolution



Kang, Prokudin, Sun and Yuan
Arxiv:1410.4877

$$F_{UT} = -\frac{1}{2z_h^3} \int \frac{db b^2}{(2\pi)} J_1\left(\frac{P_{h\perp} b}{z_h}\right) e^{-S_{\text{PT}}(Q, b_*) - S_{\text{NP coll}}^{(\text{SIDIS})}(Q, b)} \\ \times \delta C_{q \leftarrow i} \otimes h_1^i(x_B, \mu_b) \delta \hat{C}_{j \leftarrow q}^{(\text{SIDIS})} \otimes \hat{H}_{h/j}^{(3)}(z_h, \mu_b), (2)$$

Conclusions II

- TMD/CSS evolution describes DY data
- SIDIS (low energy) data difficult to fit
- Sivers asymmetries still to be understood
- Preliminary results on transversity extracted using TMD evolution compatible with other extractions...
- ...There are also other methods/approaches/prescriptions related to TMD evolution

Alternative evolution equation

$$\begin{aligned}
 \frac{\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2)}{\tilde{F}(x, b_T, Q_0, \zeta_{F0} \equiv Q_0^2)} &= \exp \left\{ \int_Q^{Q_0} \frac{d\kappa}{\kappa} [\gamma_F(\kappa; 1) - \gamma_K(\kappa) \ln(Q/\kappa)] \right\} \\
 &\quad \exp \left[- \int_{\mu_b}^{Q_0} \frac{d\kappa}{\kappa} \gamma_K(\kappa) \ln(Q/Q_0) \right] \exp [-g_K(b_T) \ln(Q/Q_0)] \\
 &= \tilde{R}(Q, Q_0, b_T) \exp [-g_K(b_T) \ln(Q/Q_0)]
 \end{aligned}$$

$$\tilde{F}(x, b_T, Q, Q^2) = \tilde{F}(x, b_T, Q_0, Q_0^2) \tilde{R}(Q, Q_0, b_T) \exp [-g_K(b_T) \ln(Q/Q_0)]$$

Output function Input function

Notice that:

$$\frac{\tilde{f}'^\perp_{1T}(x, b_T, Q, \zeta_F)}{\tilde{f}'^\perp_{1T}(x, b_T, Q_0, \zeta_{F0})} = \frac{\tilde{f}_1(x, b_T, Q, \zeta_F)}{\tilde{f}_1(x, b_T, Q_0, \zeta_{F0})} \equiv \frac{\tilde{F}(x, b_T, Q, \zeta_F)}{\tilde{F}(x, b_T, Q_0, \zeta_{F0})}$$

Alternative evolution equation

$$\begin{aligned}
 \frac{\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2)}{\tilde{F}(x, b_T, Q_0, \zeta_{F0} \equiv Q_0^2)} &= \exp \left\{ \int_Q^{Q_0} \frac{d\kappa}{\kappa} [\gamma_F(\kappa; 1) - \gamma_K(\kappa) \ln(Q/\kappa)] \right\} \\
 &\quad \exp \left[- \int_{\mu_b}^{Q_0} \frac{d\kappa}{\kappa} \gamma_K(\kappa) \ln(Q/Q_0) \right] \exp [-g_K(b_T) \ln(Q/Q_0)] \\
 &= \tilde{R}(Q, Q_0, b_T) \exp [-g_K(b_T) \ln(Q/Q_0)]
 \end{aligned}$$

$$\tilde{F}(x, b_T, Q, Q^2) = \tilde{F}(x, b_T, Q_0, Q_0^2) \tilde{R}(Q, Q_0, b_T) \exp [-g_K(b_T) \ln(Q/Q_0)]$$

Output function Input function

Notice that:

$$\frac{\tilde{f}'^\perp_{1T}(x, b_T, Q, \zeta_F)}{\tilde{f}'^\perp_{1T}(x, b_T, Q_0, \zeta_{F0})} = \frac{\tilde{f}_1(x, b_T, Q, \zeta_F)}{\tilde{f}_1(x, b_T, Q_0, \zeta_{F0})} \equiv \frac{\tilde{F}(x, b_T, Q, \zeta_F)}{\tilde{F}(x, b_T, Q_0, \zeta_{F0})}$$

EIKV phenomenology

- TMD evolution in the CSS-like version

$$\begin{aligned}\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) &= \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \\ &\quad \exp\left\{\frac{1}{2}S^{CSS}(b_*, \mu_b)\right\} \\ &\quad \exp\left\{-g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right)\right\}\end{aligned}$$

- Some approximations to make life simpler

$$\tilde{C}_{ji}(z, \alpha(\mu)) = \delta_{ij} \delta(1-z) \quad \text{At LO; PDF at LO}$$

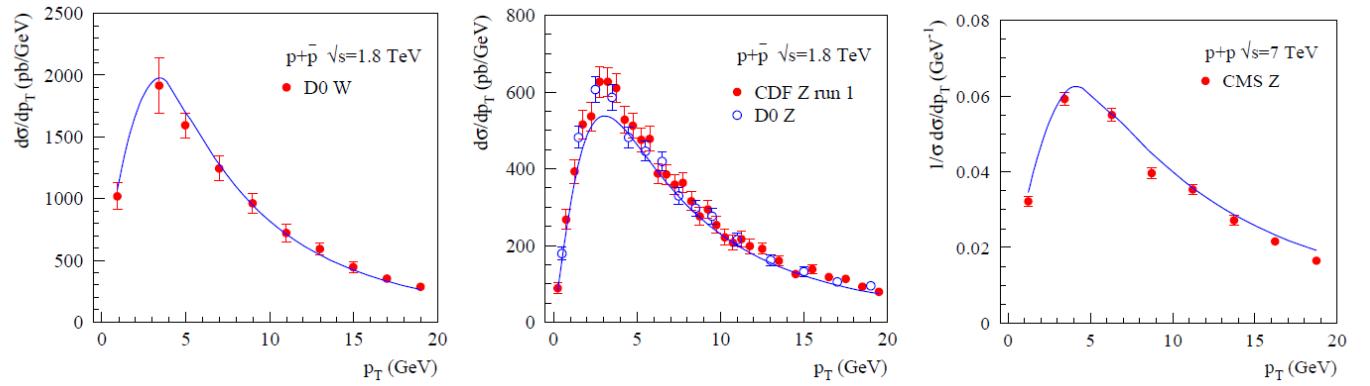
- Simple parametrizations for the non-perturbative part:

$$F_{NP}(b_T, Q)^{\text{pdf}} = \exp\left[-b_T^2 \left(g_1^{\text{pdf}} + \frac{g_2}{2} \ln(Q/Q_0)\right)\right]$$

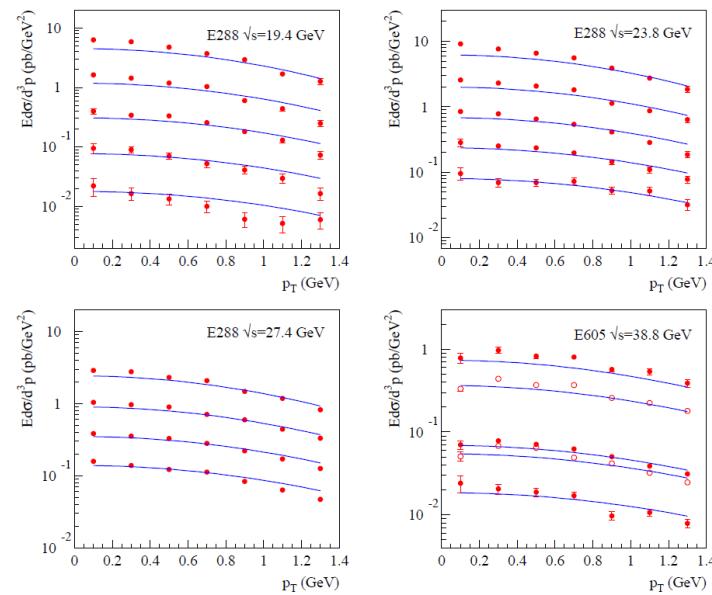
$$F_{NP}(b_T, Q)^{\text{ff}} = \exp\left[-b_T^2 \left(g_1^{\text{ff}} + \frac{g_2}{2} \ln(Q/Q_0)\right)\right]$$

EIKV phenomenology

➤ Fit DY data and SIDIS data....

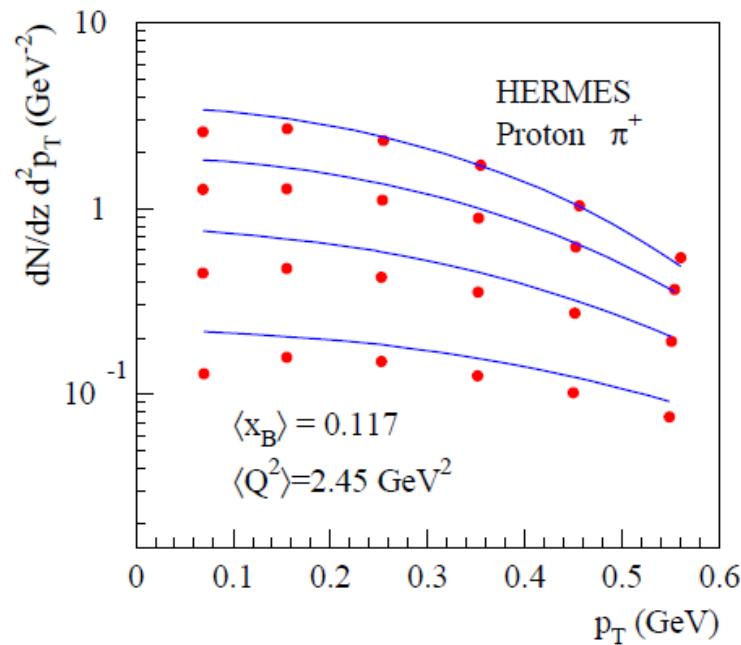
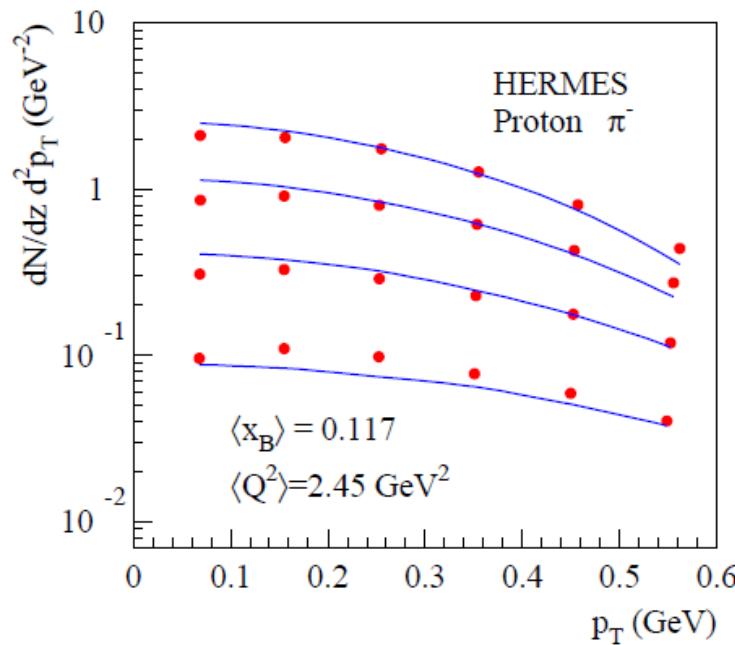


Z and W-Boson Production
Low energy DY



EIKV phenomenology

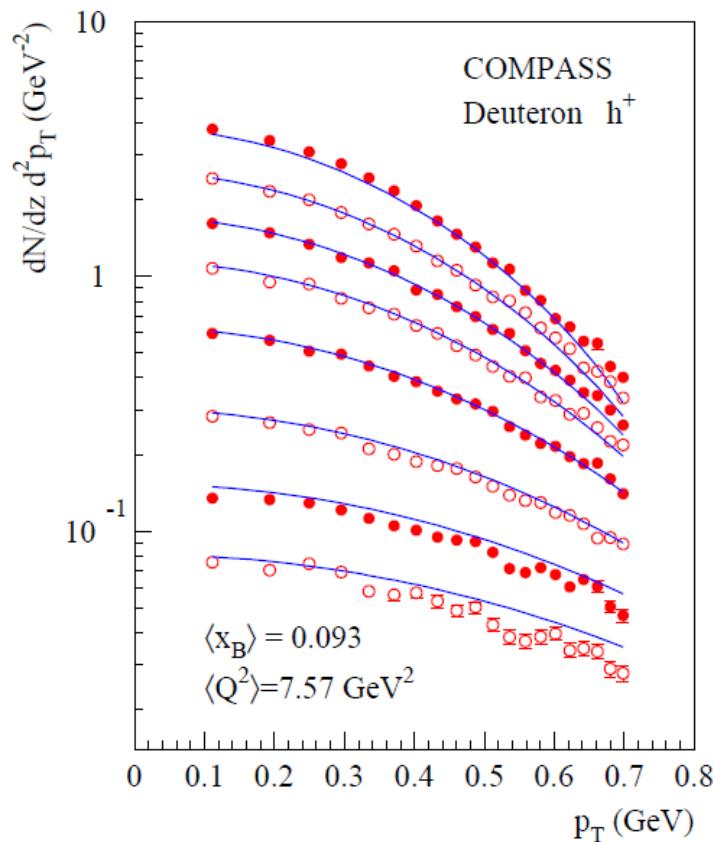
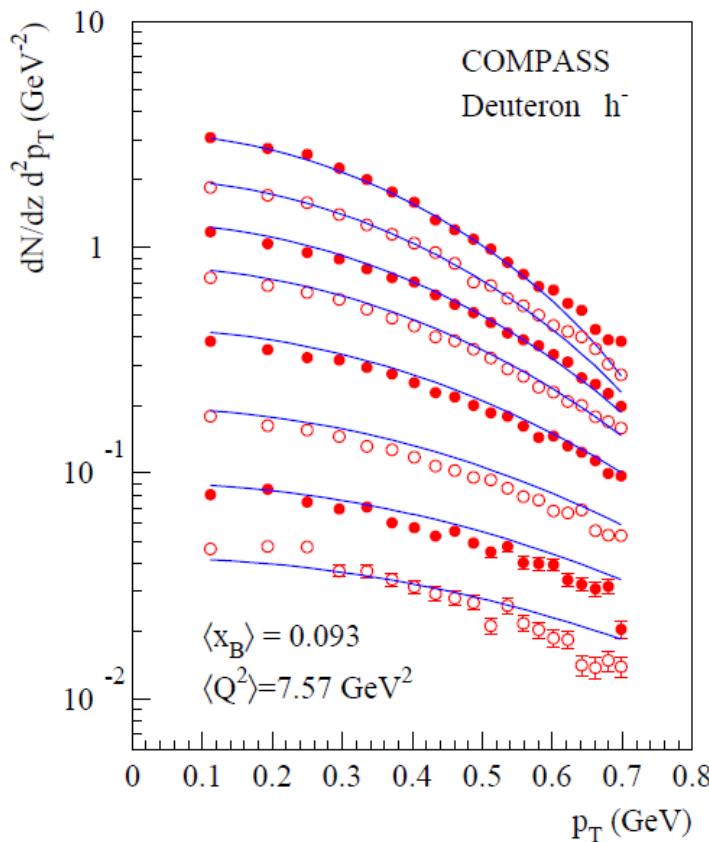
HERMES SIDIS data



MSTW2008 PDF and DSS

EIKV phenomenology

(some...only two bins?) COMPASS SIDIS data



MSTW2008 PDF and DSS

TMD evolution modelling

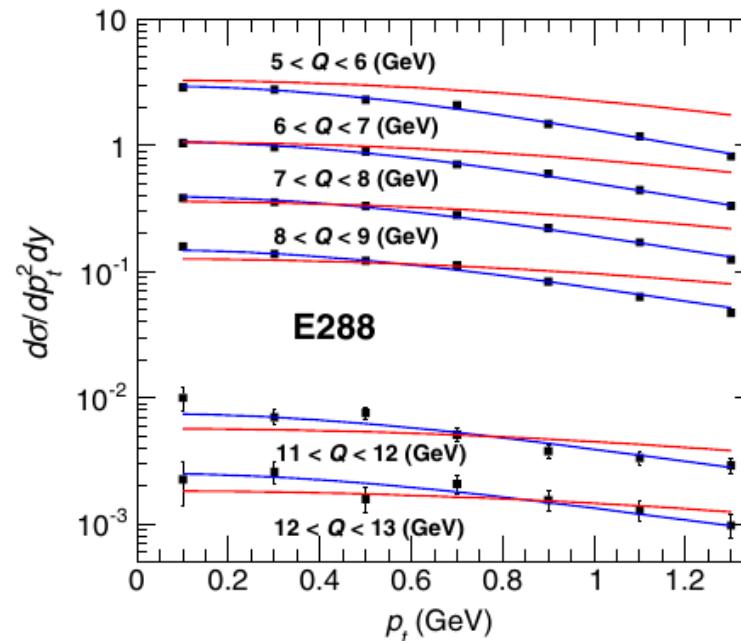
Rogers & Aybat

$$\tilde{F}(x, \mathbf{b}_T; Q) = \tilde{F}(x, \mathbf{b}_T; Q_0) \tilde{R}(Q, Q_0, b_T) \exp \left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$\tilde{F}(x, b_T, Q_0, Q_0^2) = f(x, Q_0) \exp \left[-\frac{\langle k_\perp^2 \rangle}{4} b_T^2 \right]$
 $g_K(b_T) = \frac{1}{2} g_2 b_T^2$ g_2 from DY

Average transverse momentum from SIDIS (HERMES)

Red line, prediction based
on the above formula
with the parameter as
in Rogers,Aybat 2011



Alternative TMD evolution Yuan-Sun phenomenology

- Yuan-Sun explanation: the Sudakov form factor must be modified taking into account that low energy data are almost in a non perturbative region.

$$\mathcal{S}_{\text{Sud}} = 2C_F \int_{Q_0}^Q \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2} \right]$$

$$\tilde{F}_{UU}(Q; b) = e^{-\mathcal{S}_{\text{sud}}(Q, Q_0, b)} \tilde{F}_{UU}(Q_0; b),$$

$$\tilde{F}_{UU}(Q_0, b) = \sum_q e_q^2 f_q(x_B, \mu = Q_0) D_q(z_h, \mu = Q_0) e^{-g_0 b^2 - g_h b^2 / z_h^2}$$

- Notice that there is not any b^* and therefore any b_{\max} .

See for a interesting discussion Section VII of Aidala, Field, Gamberg, Rogers, Phys.Rev. D89 (2014) 094002

Alternative TMD evolution Yuan-Sun phenomenology

- Gaussian parametrization for the PDF and the fragmentation function at the scale of HERMES.

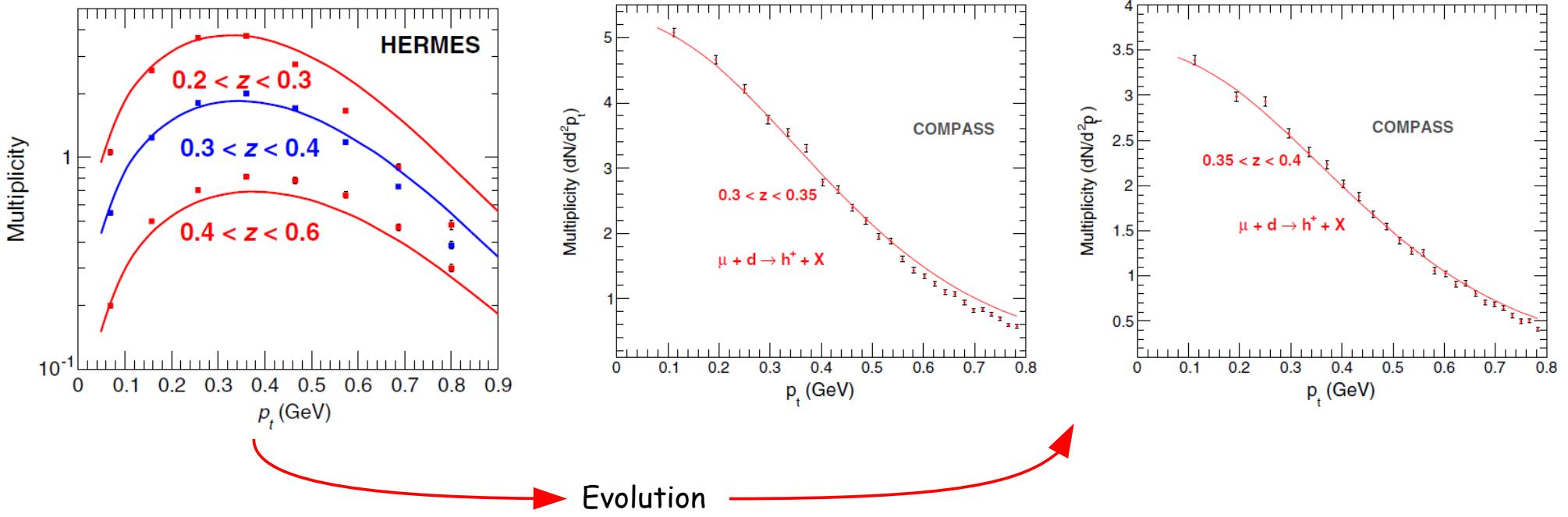
$$\tilde{F}_{UU}(Q_0, b) = \sum_q e_q^2 f_q(x_B, \mu = Q_0) D_q(z_h, \mu = Q_0) e^{-g_0 b^2 - g_h b^2 / z_h^2}$$

$$\tilde{W}_{UU}(Q_0, b) = \sum_q e_q^2 f_q(x, \mu = Q_0) f_{\bar{q}}(x', \mu = Q_0) e^{-g_0 b^2 - g_0 b^2},$$

Alternative TMD evolution

Yuan-Sun phenomenology

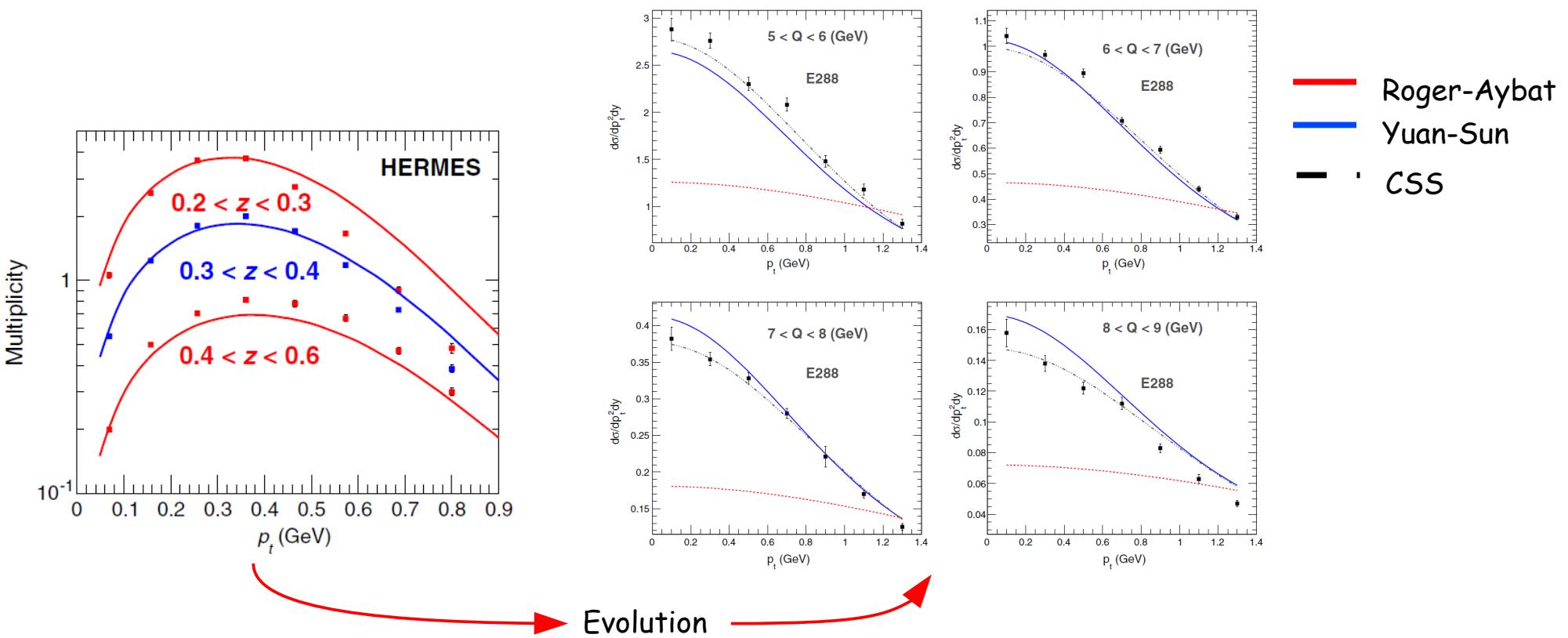
- Gaussian parametrization for the PDF and the fragmentation function at the scale of HERMES.
- Parameters g_0 and g_h as in Schweitzer et al, Phys. Rev. D81, 094019 (2010)



Alternative TMD evolution

Yuan-Sun phenomenology

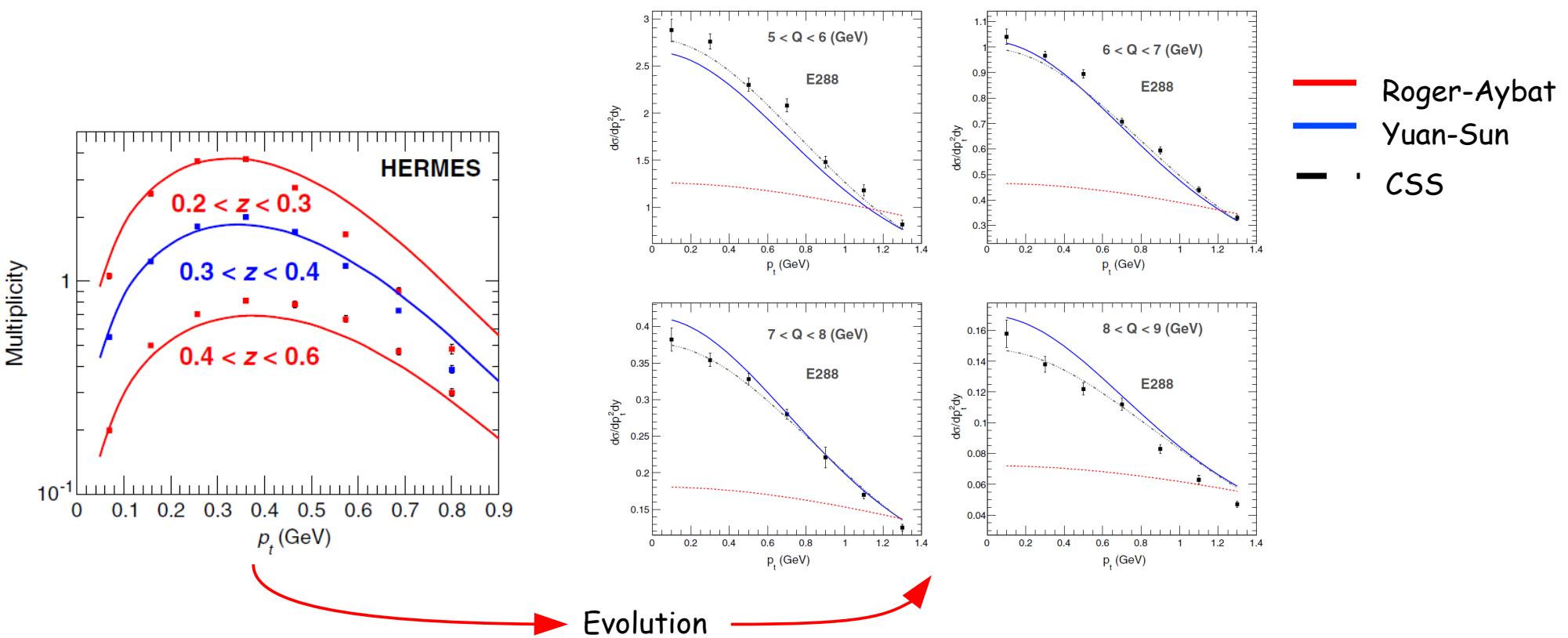
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Alternative TMD evolution

Yuan-Sun phenomenology

- Gaussian parametrization for the PDF and the fragmentation function at the scale of HERMES.
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The Sivers function from SIDIS data

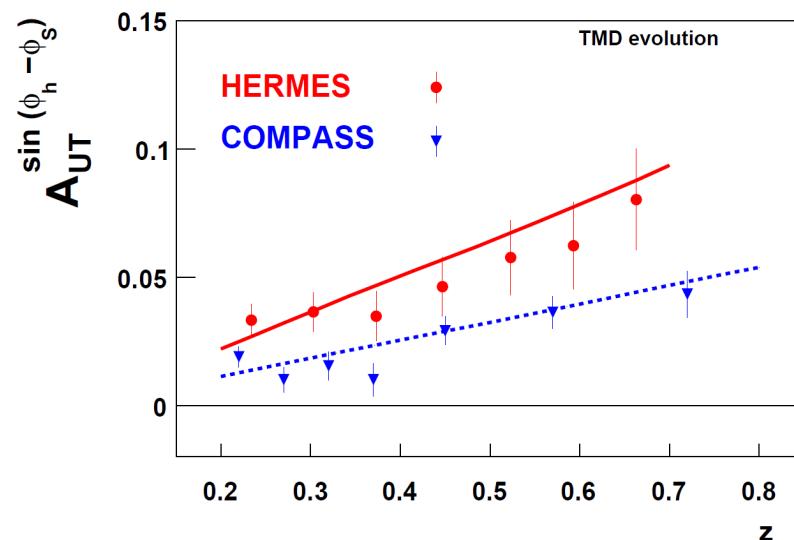
Sivers phenomenology

➤ Aybat-Roger-Prokudin: TMD EVO IO

No FIT Qual. OK

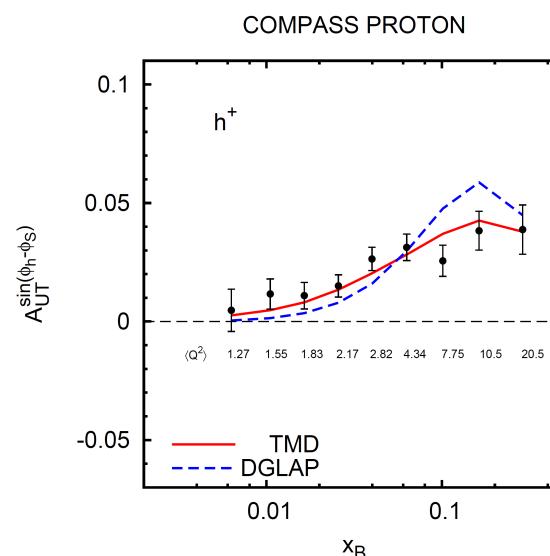
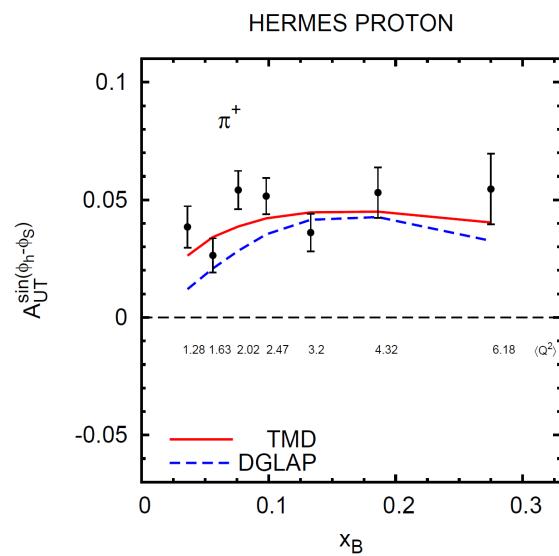
$$\tilde{F}(x, \mathbf{b}_T; Q) = \tilde{F}(x, \mathbf{b}_T; Q_0) \tilde{R}(Q, Q_0, b_T) \exp \left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$$\tilde{F}(x, b_T, Q_0, Q_0^2) = f(x, Q_0) \exp \left[-\frac{\langle k_\perp^2 \rangle}{4} b_T^2 \right] \quad g_K(b_T) = \frac{1}{2} g_2 b_T^2 \quad g_2 \text{ from DY}$$



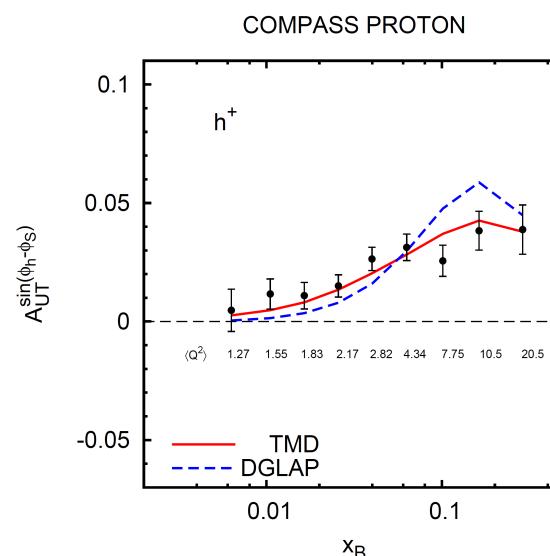
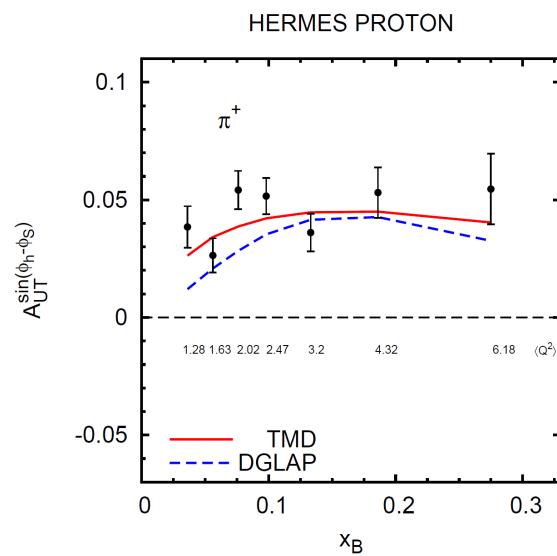
Sivers phenomenology

- Aybat-Roger-Prokudin: TMD EVO IO No FIT Qual. OK
- Anselmino-Boglione-Melis: Gaussian FIT $\chi^2=1.26$



Sivers phenomenology

- | | | |
|--|--------|---------------|
| ➤ Aybat-Roger-Prokudin: TMD EVO IO | No FIT | Qual. OK |
| ➤ Anselmino-Boglione-Melis: Gaussian | FIT | $\chi^2=1.26$ |
| ➤ Anselmino-Boglione-Melis: TMD EVO IO | FIT | $\chi^2=1.02$ |

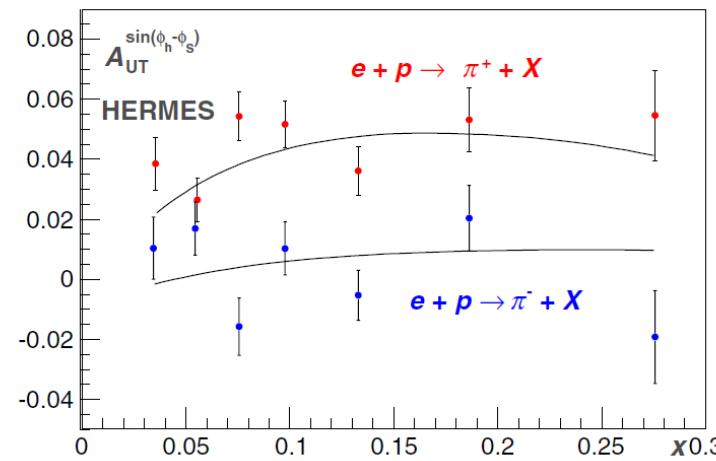
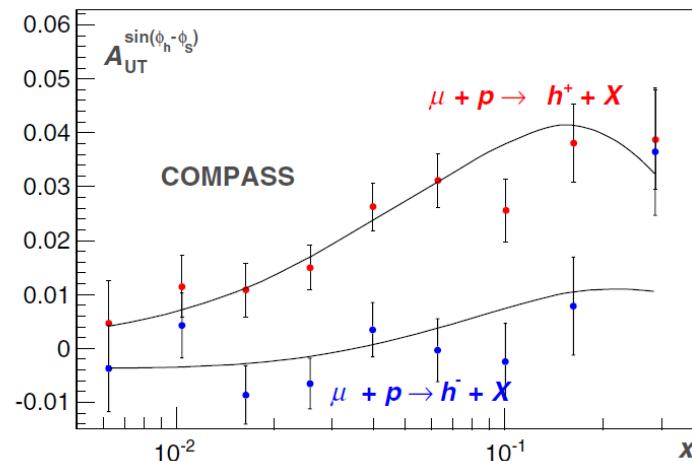


Sivers phenomenology

- | | | |
|--|--------|---------------|
| ➤ Aybat-Roger-Prokudin: TMD EVO IO | No FIT | Qual. OK |
| ➤ Anselmino-Boglione-Melis: Gaussian | FIT | $\chi^2=1.26$ |
| ➤ Anselmino-Boglione-Melis: TMD EVO IO | FIT | $\chi^2=1.02$ |
| ➤ Sun-Yuan: TMD EVO IO+ Modified Sudakov | FIT | $\chi^2=1.08$ |

$$\tilde{F}_{\text{sivers}}^\alpha(Q_0, b) = \frac{ib_\perp^\alpha M}{2} \sum_q e_q^2 \Delta f_q^{\text{sivers}}(x) D_q(z) e^{-(g_0 - g_s)b^2 - g_h b^2/z_h^2}$$

$$\mathcal{S}_{\text{Sud}} = 2C_F \int_{Q_0}^Q \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2} \right]$$



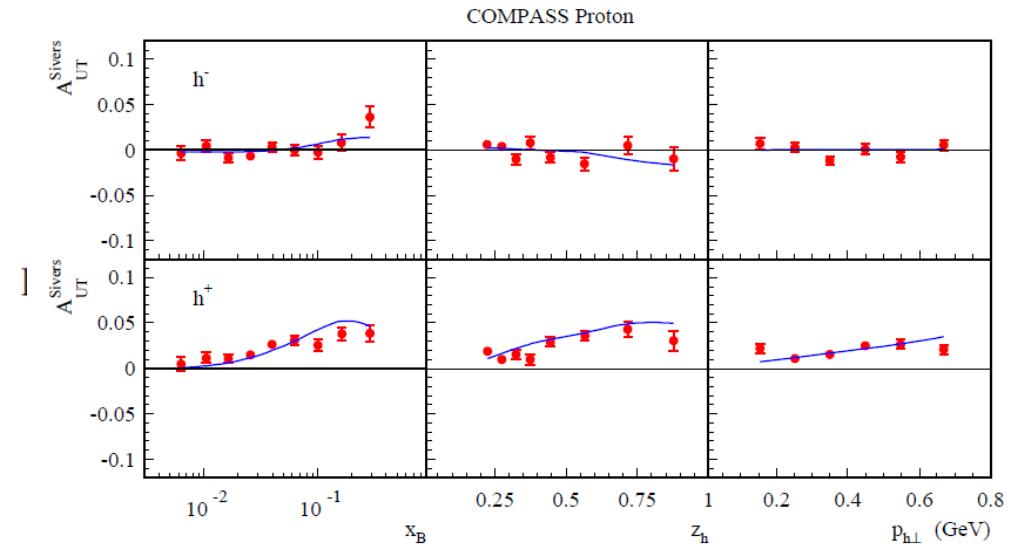
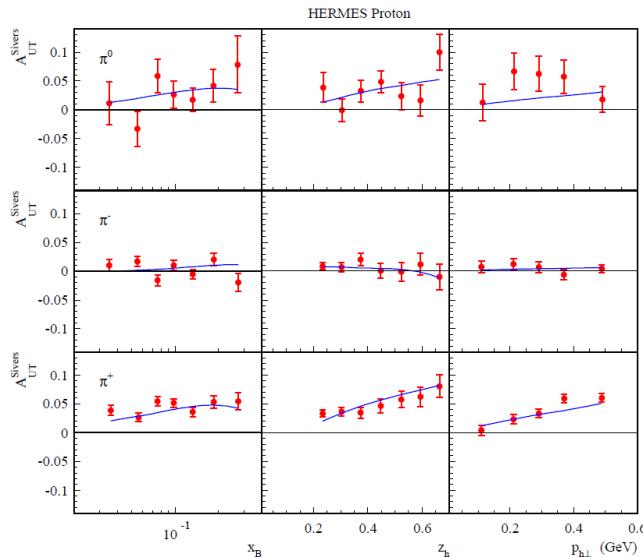
Sivers phenomenology

➤ Aybat-Roger-Prokudin: TMD EVO IO	No FIT	Qual. OK
➤ Anselmino-Boglione-Melis: Gaussian	FIT	$\chi^2=1.26$
➤ Anselmino-Boglione-Melis: TMD EVO IO	FIT	$\chi^2=1.02$
➤ Sun-Yuan: TMD EVO IO+ Modified Sudakov	FIT	$\chi^2=1.08$
➤ EIKV: TMD Evo a la CSS+ C at LO	FIT	$\chi^2=1.3$

$$\begin{aligned}
 F_{UT}^{\sin(\phi_h - \phi_s)} = & \frac{1}{4\pi} \int_0^\infty db b^2 J_1(P_{h\perp} b/z_h) \sum_q e_q^2 T_{q,F}(x_B, x_B, c/b_*) D_{h/q}(z_h, c/b_*) \\
 & \times \exp \left\{ - \int_{c^2/b_*^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left(A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \exp \left\{ - b^2 \left(g_1^{\text{ff}} + g_1^{\text{sivers}} + g_2 \ln \frac{Q}{Q_0} \right) \right\}
 \end{aligned}$$

Sivers phenomenology

- | | | |
|--|--------|---------------|
| ➤ Aybat-Roger-Prokudin: TMD EVO IO | No FIT | Qual. OK |
| ➤ Anselmino-Boglione-Melis: Gaussian | FIT | $\chi^2=1.26$ |
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Sivers phenomenology

➤ Aybat-Roger-Prokudin: TMD EVO IO	No FIT	Qual. OK
➤ Anselmino-Boglione-Melis: Gaussian	FIT	$\chi^2=1.26$
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➤ EIKV: TMD Evo a la CSS+ C at LO	FIT	$\chi^2=1.3$

Unpolarized phenomenology

Can these methods
describe unpolarized processes?
SIDIS DY

➤ Aybat-Roger-Prokudin: TMD EVO IO	No	No
➤ Anselmino-Boglione-Melis: Gaussian	Maybe (separately)	Maybe low energy No High energy
➤ Anselmino-Boglione-Melis: TMD EVO IO	No	No
➤ Sun-Yuan: TMD EVO IO+ Modified Sudakov	No Hermes YES/Maybe COMPASS	Yes low energy No High energy
➤ EIKV: TMD Evo a la CSS+ C at LO	No Hermes YES/Maybe COMPASS	YES

Unpolarized phenomenology

Can these methods
describe unpolarized processes?
SIDIS DY

➤ Aybat-Roger-Prokudin: TMD EVO IO	No	No
➤ Anselmino-Boglione-Melis: Gaussian	Maybe (separately)	Maybe low energy No High energy
➤ Anselmino-Boglione-Melis: TMD EVO IO	No	No
➤ Sun-Yuan: TMD EVO IO+ Modified Sudakov	No Hermes YES/Maybe COMPASS	Yes low energy No High energy
➤ EIKV: TMD Evo a la CSS+ C at LO	No Hermes YES/Maybe COMPASS	YES
➤ This is a comparison list! There other works related to the unpolarized processes!		

Yuan-Sun phenomenology

- Then Anselmino et al like parametrization for the Sivers function at the scale of HERMES

$$\tilde{F}_{\text{sivers}}^{\alpha}(Q_0, b) = \frac{ib_1^{\alpha}M}{2} \sum_q e_q^2 \Delta f_q^{\text{sivers}}(x) D_q(z) e^{-(g_0 - g_s)b^2 - g_h b^2/z_h^2}$$

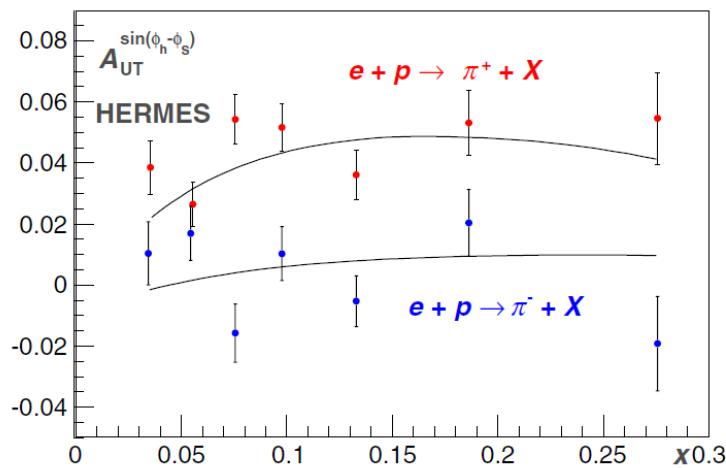
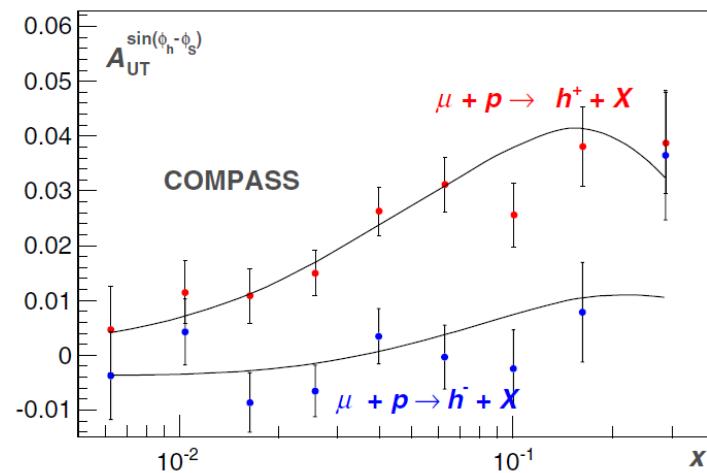
$$\Delta f_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{\alpha_q + \beta_q}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} f_q(x)$$

Yuan-Sun phenomenology

TABLE I. Parameters $\{a_i^0\}$ describing our optimum Δf_i in Eq. (5) at the input scale $Q^2 = 2.4$ GeV.

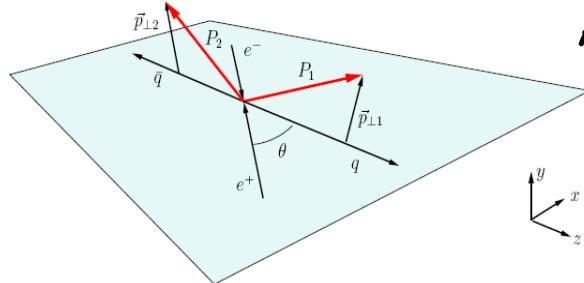
flavor i	N_i	α_i	β_i	g_s (GeV 2)
u	0.13 ± 0.023	0.81 ± 0.16	4.0 ± 1.2	0.062 ± 0.005
d	-0.27 ± 0.12	1.41 ± 0.28	4.0 ± 1.2	0.062 ± 0.005
s	0.07 ± 0.06	0.58 ± 0.39	4.0 ± 1.2	0.062 ± 0.005
\bar{u}	-0.07 ± 0.05	0.58 ± 0.39	4.0 ± 1.2	0.062 ± 0.005
\bar{d}	-0.19 ± 0.12	0.58 ± 0.39	4.0 ± 1.2	0.062 ± 0.005

$$\chi^2/\text{d.o.f} = 1.08$$



Extraction of transversity & Collins functions

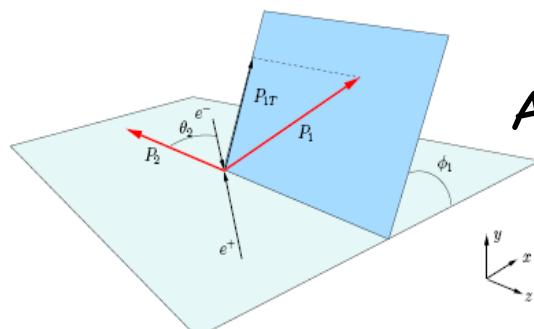
➤ $e^+e^- \rightarrow h_1 h_2 X$ BELLE Data



A_{12} asymmetry

Thrust axis method

$$A(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d\cos\theta d(\varphi_1 + \varphi_2)} \\ = 1 + \frac{1}{8} \frac{\sin^2 \theta}{1 + \cos^2 \theta} \cos(\varphi_1 + \varphi_2) \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\dagger}(z_1) \Delta^N D_{h_2/\bar{q}^\dagger}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$



A_0 asymmetry

Hadronic plane method

$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi} \frac{z_1 z_2}{z_1^2 + z_2^2} \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos(2\phi_1) \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\dagger}(z_1) \Delta^N D_{h_2/\bar{q}^\dagger}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$

Parametrizations

➤ Gaussian parametrization of the unpolarized PDF & FF:

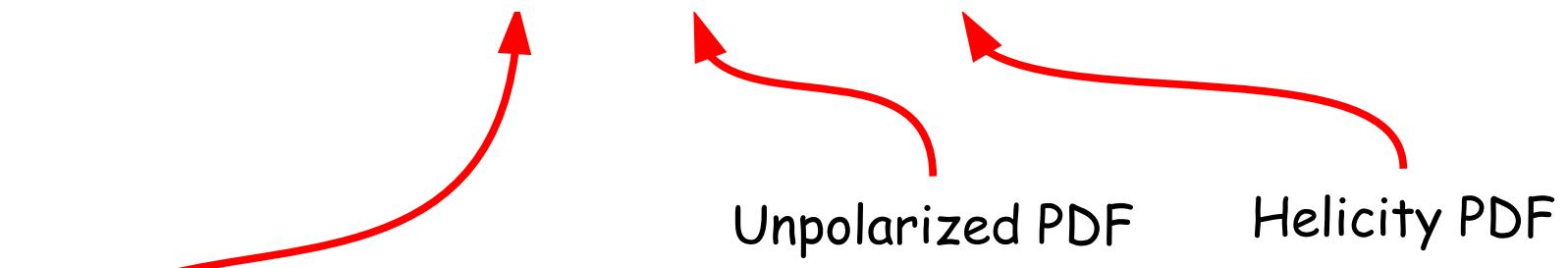
- $f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$
- $D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$

[*] $\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2$ $\langle p_\perp^2 \rangle = 0.20 \text{ GeV}^2$

Parametrizations

➤ Parametrization of Transversity function:

☞ $\Delta_T q(x, k_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) [f_{q/p}(x) + \Delta q(x)] \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T}$



$$\mathcal{N}_q^T(x) = N_q^T x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

N_q^T , α , β free parameters

Parametrizations

➤ Parametrization of the Collins function:

 $\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = 2 \mathcal{N}_q^C(z) h(p_\perp) D_{\pi/q}(z, p_\perp)$

- $\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma + \delta)^{(\gamma + \delta)}}{\gamma^\gamma \delta^\delta}$

- $h(p_\perp) = \sqrt{2e} \frac{p_\perp}{M_h} e^{-p_\perp^2/M_h^2}$

$N_q^C, \gamma, \delta, M_h$ free parameters

Unpolarized FF

✓ Bound:

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) \leq 2 D_{\pi/q}(z, k_\perp)$$

✓ Torino vs Amsterdam notation

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z, p_\perp)$$

Extraction of transversity & Collins functions

➤ FIT II: A_0 BELLE data UL & UC +COMPASS+ HERMES ???

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{d.o.f} = 0.80$	$\chi^2_{tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
Standard Parameterization $\chi^2_{d.o.f} = 1.12$	$\chi^2_{tot} = 190$	$\chi^2 = 125$	$\chi^2 = 20$ NO FIT	$\chi^2 = 12$ NO FIT	$\chi^2 = 35$	$\chi^2 = 30$

➔ A_0 data cannot be nicely described even if fitted...

Standard parametrization of the Collins function

- Parametrization of the z-dependent part of the Collins function:

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = 2 \mathcal{N}_q^C(z) h(p_\perp) D_{\pi/q}(z, p_\perp)$$

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma + \delta)^{(\gamma + \delta)}}{\gamma^\gamma \delta^\delta}$$

Our standard parametrization



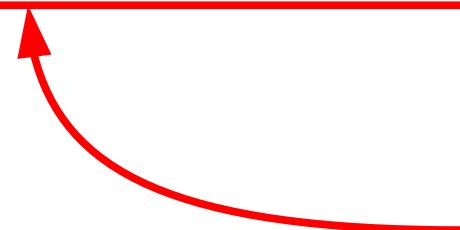
- It is equal to 0 at $z=0$ and $z=1$

New parametrization of the Collins function

- Let us try to change the parametrization of the z-dependent part of the Collins function:

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z) h(p_\perp) D_{\pi/q}(z, p_\perp)$$

$$\mathcal{N}_q^C(z) = N_q^C z [(1 - a - b) + a z + b z^2]$$



NEW Polynomial parametrization

- It is equal to 0 at $z=0$ and equal to N_q at $z=1$

Extraction of transversity & Collins functions

➤ FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{d.o.f} = 0.80$	$\chi^2_{tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
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Polynomial Parameterization $\chi^2_{d.o.f} = 0.81$	$\chi^2_{tot} = 136$	$\chi^2 = 123$	$\chi^2 = 8$	$\chi^2 = 5$	$\chi^2 = 45$ NO FIT	$\chi^2 = 39$ NO FIT
Polynomial Parameterization $\chi^2_{d.o.f} = 1.01$	$\chi^2_{tot} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

Extraction of transversity & Collins functions

➤ FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{d.o.f} = 0.80$	$\chi^2_{tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
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- ➔ If we fit A_{12} data we get the same description obtained with the std par.
- ➔ Almost identical Collins function, again the description of A_0 is not so good

Extraction of transversity & Collins functions

➤ FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{d.o.f} = 0.80$	$\chi^2_{tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
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	$\chi^2_{tot} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

- ➔ If we fit A_0 data we can improve their description
- ➔ Still tension with A_{12}

Extraction of transversity & Collins functions

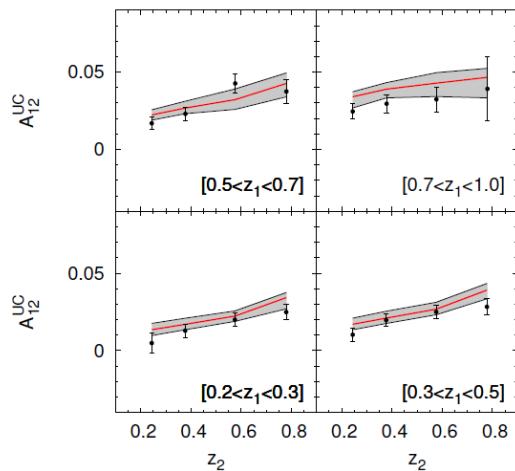
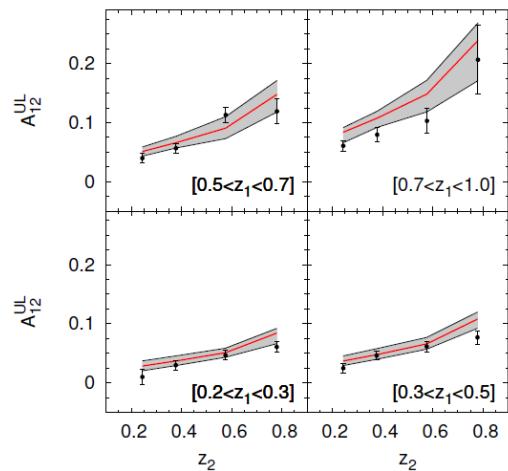
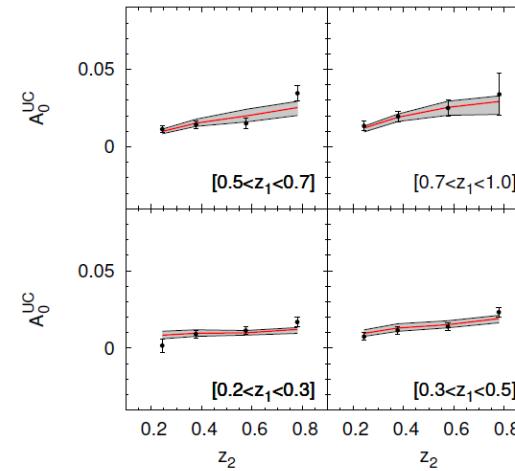
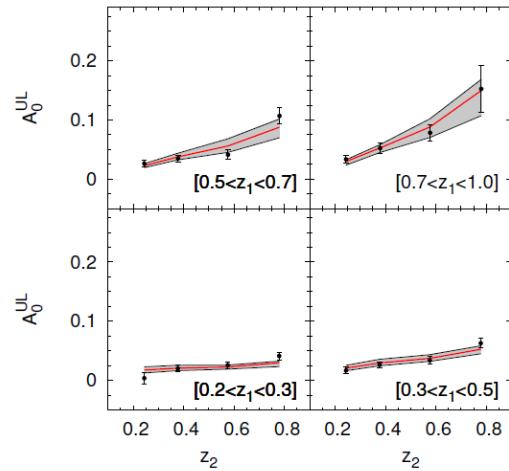
➤ FIT III and IV: Polynomial Parametrization

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
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- ➔ If we fit A_0 data we can improve their description
- ➔ Still tension with A_{12}

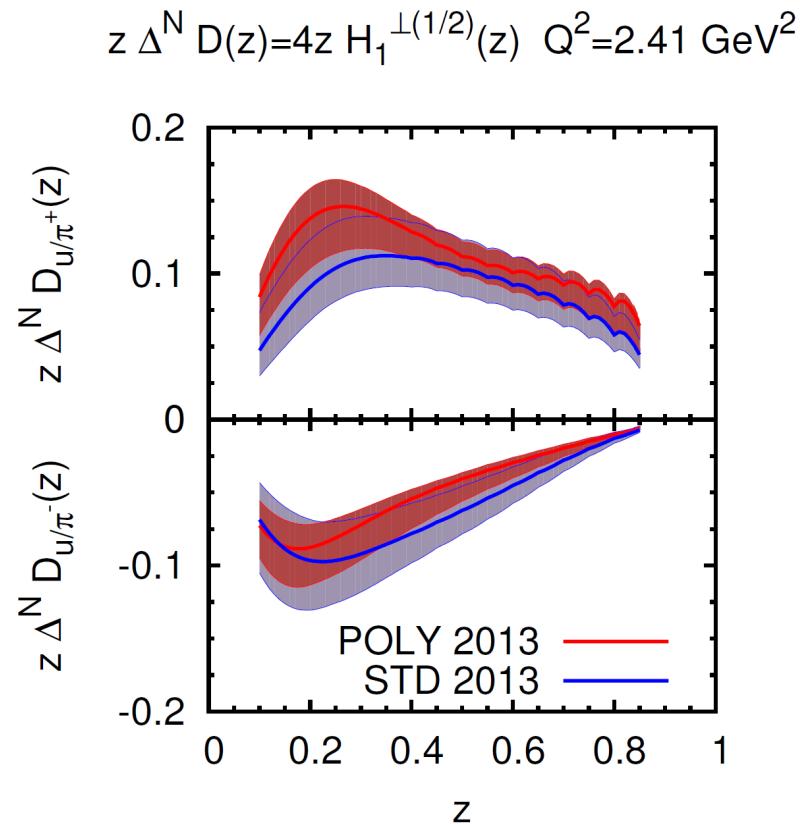
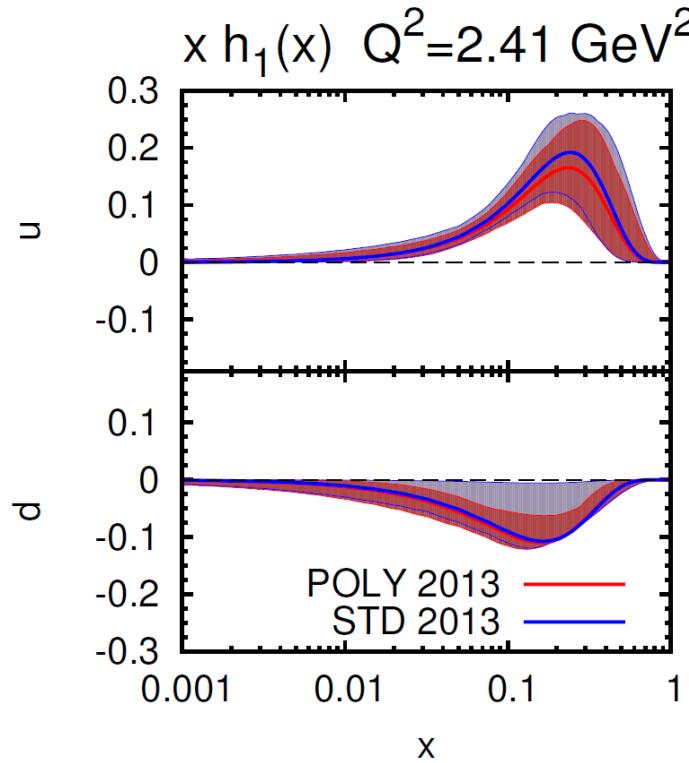
Extraction of transversity & Collins functions

➤ FIT IV: A_0 BELLE data UL & UC +COMPASS+ HERMES-POLYNOMIAL



Extraction of transversity & Collins functions

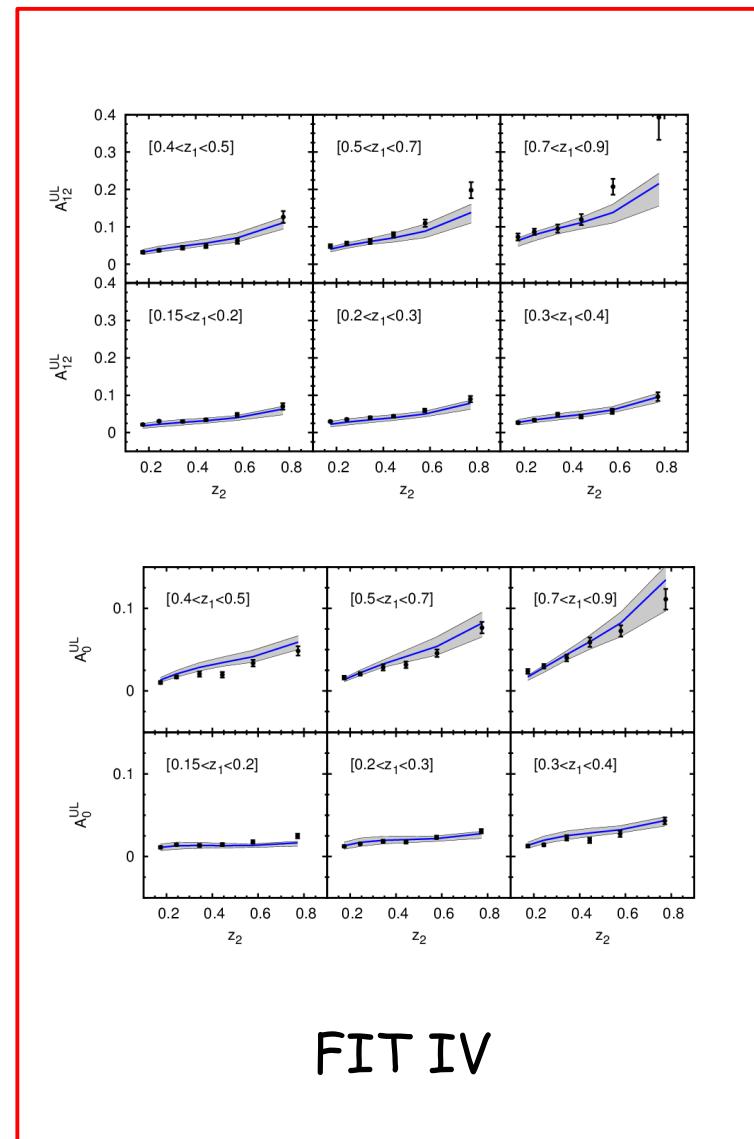
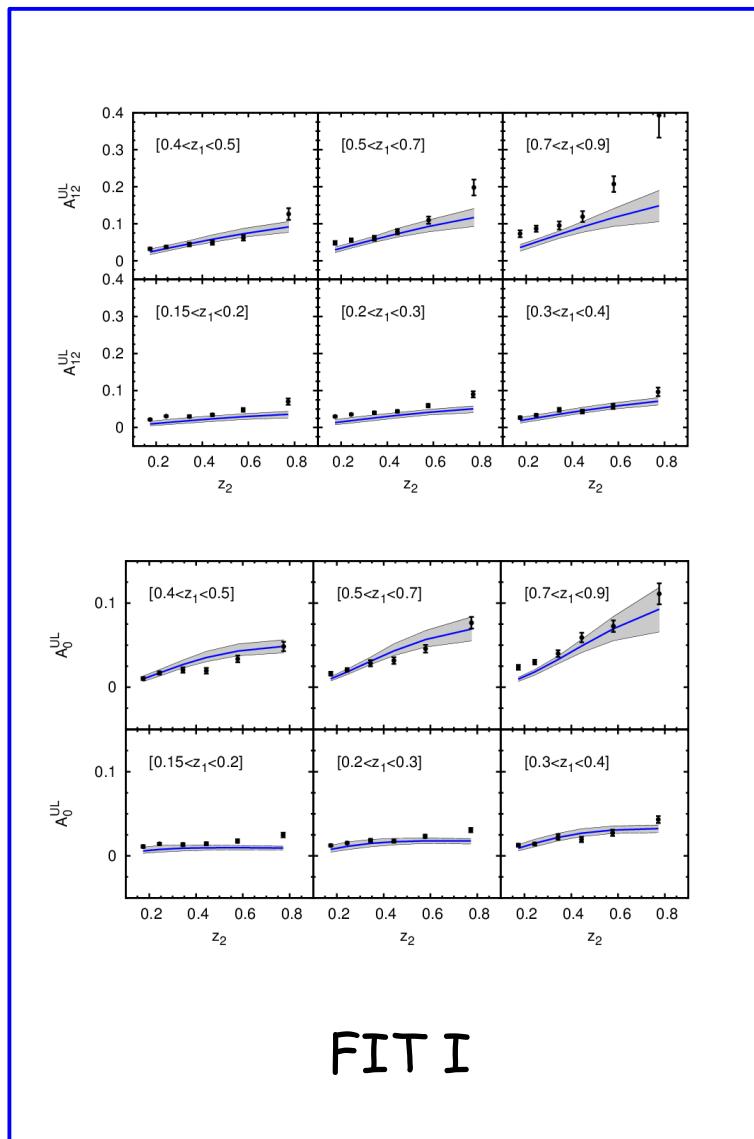
➤ FIT II vs FIT IV (POLYNOMIAL vs STD; FITTED A_0)



→ Same transversity

→ Different Collins functions (but not dramatically different)

BaBar Predictions



Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

TMD evolution (exact)

$$\chi_{\text{tot}}^2 = 255.8$$

$$\chi_{\text{d.o.f}}^2 = 1.02$$

DGLAP evolution

$$\chi_{\text{tot}}^2 = 315.6$$

$$\chi_{\text{d.o.f}}^2 = 1.26$$

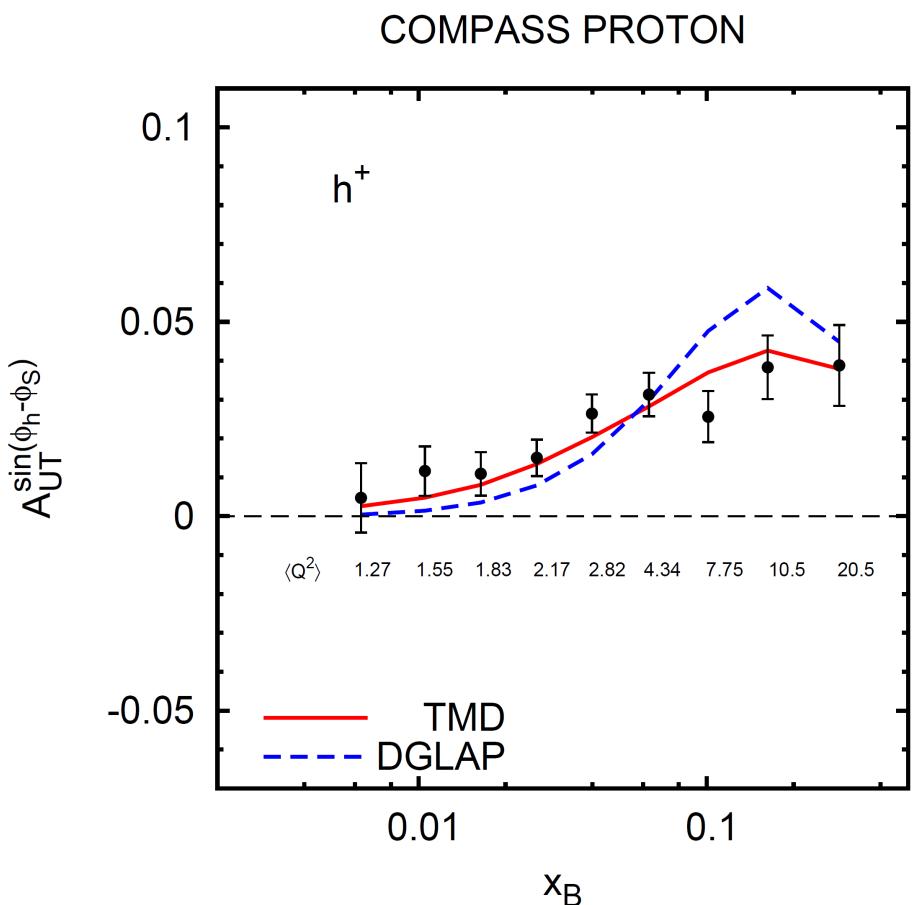
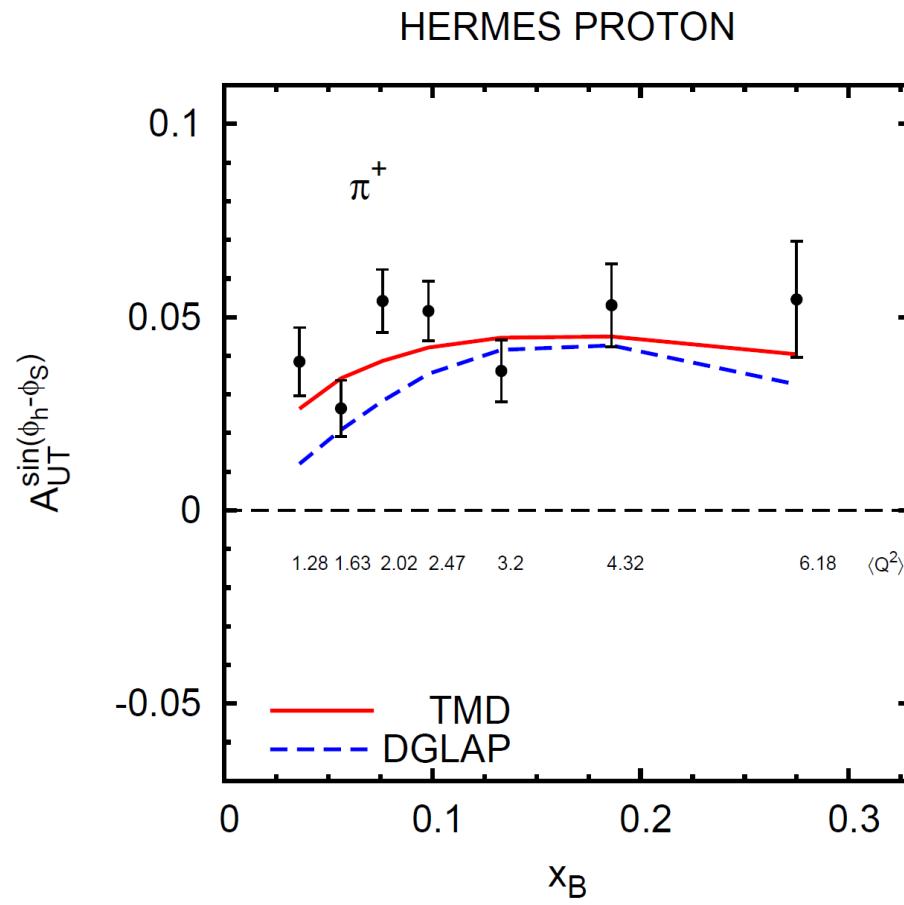
Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

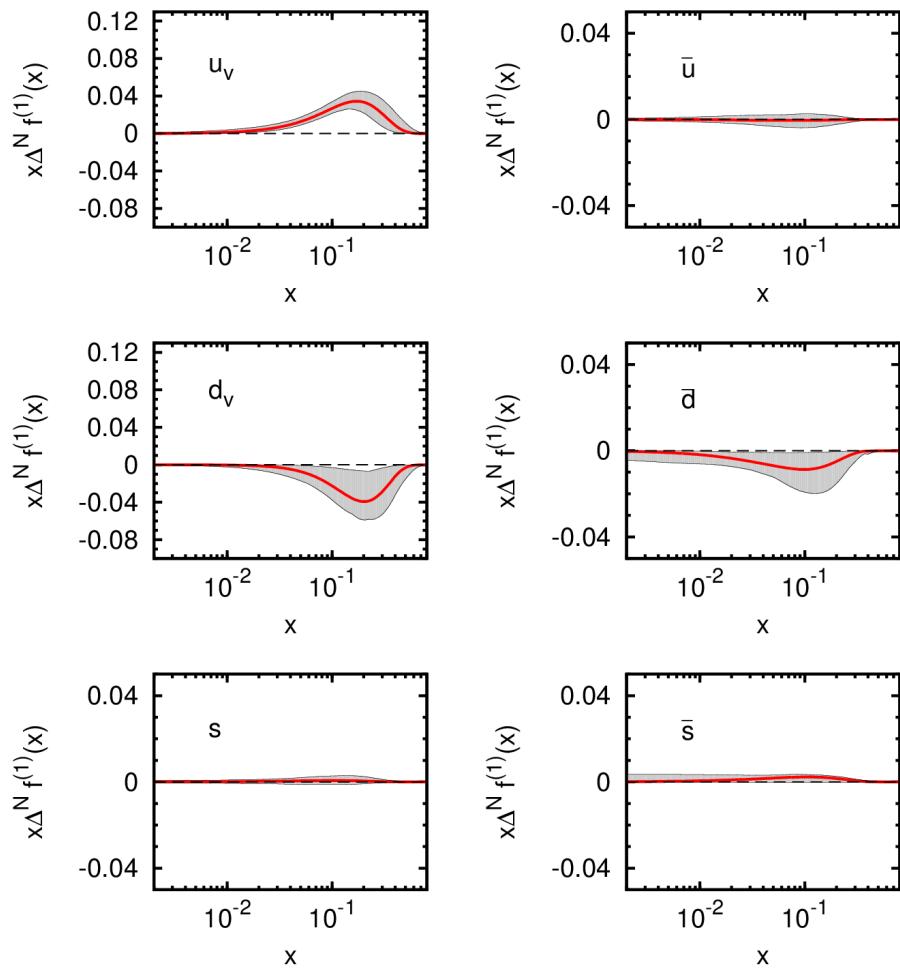
TMD Evolution (Exact)		DGLAP Evolution
$\chi_{tot}^2 = 255.8$		$\chi_{tot}^2 = 315.6$
$\chi_{d.o.f}^2 = 1.02$		$\chi_{d.o.f}^2 = 1.26$
HERMES	$\chi_x^2 = 10.7$	$\chi_x^2 = 27.5$
π^+	$\chi_z^2 = 4.3$	$\chi_z^2 = 8.6$
	$\chi_{P_T}^2 = 9.1$	$\chi_{P_T}^2 = 22.5$
COMPASS	$\chi_x^2 = 6.7$	$\chi_x^2 = 29.2$
h^+	$\chi_z^2 = 17.8$	$\chi_z^2 = 16.6$
	$\chi_{P_T}^2 = 12.4$	$\chi_{P_T}^2 = 11.8$

Fit of HERMES and COMPASS SIDIS data

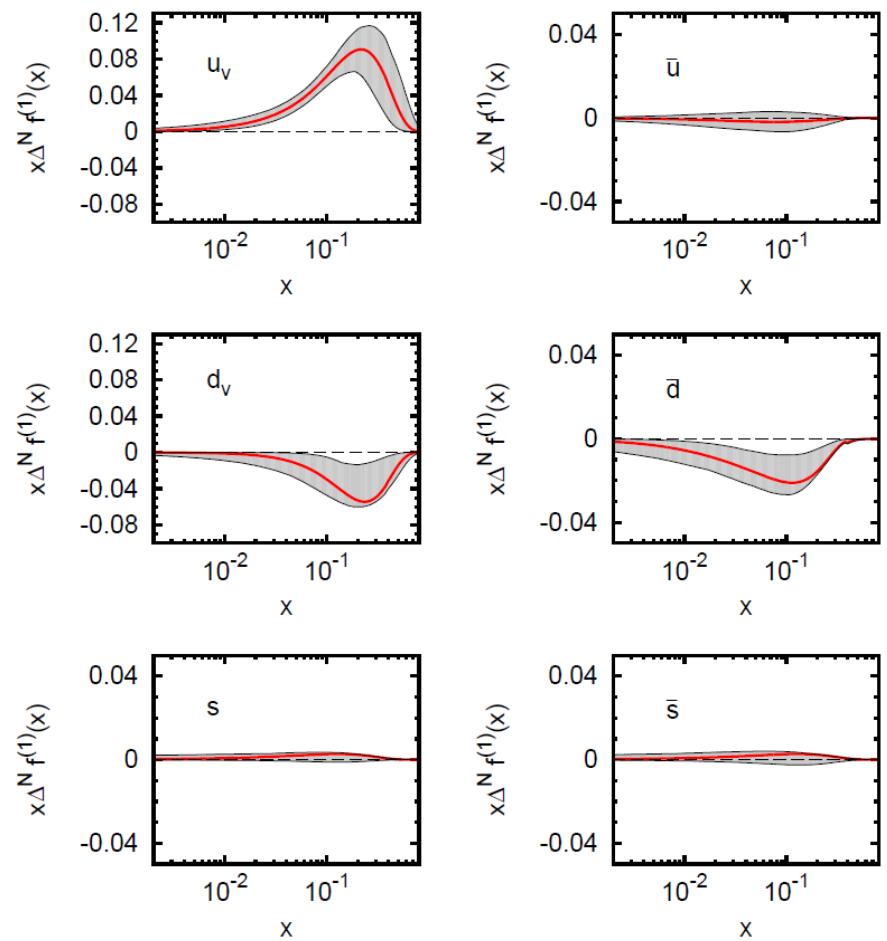


Sivers functions

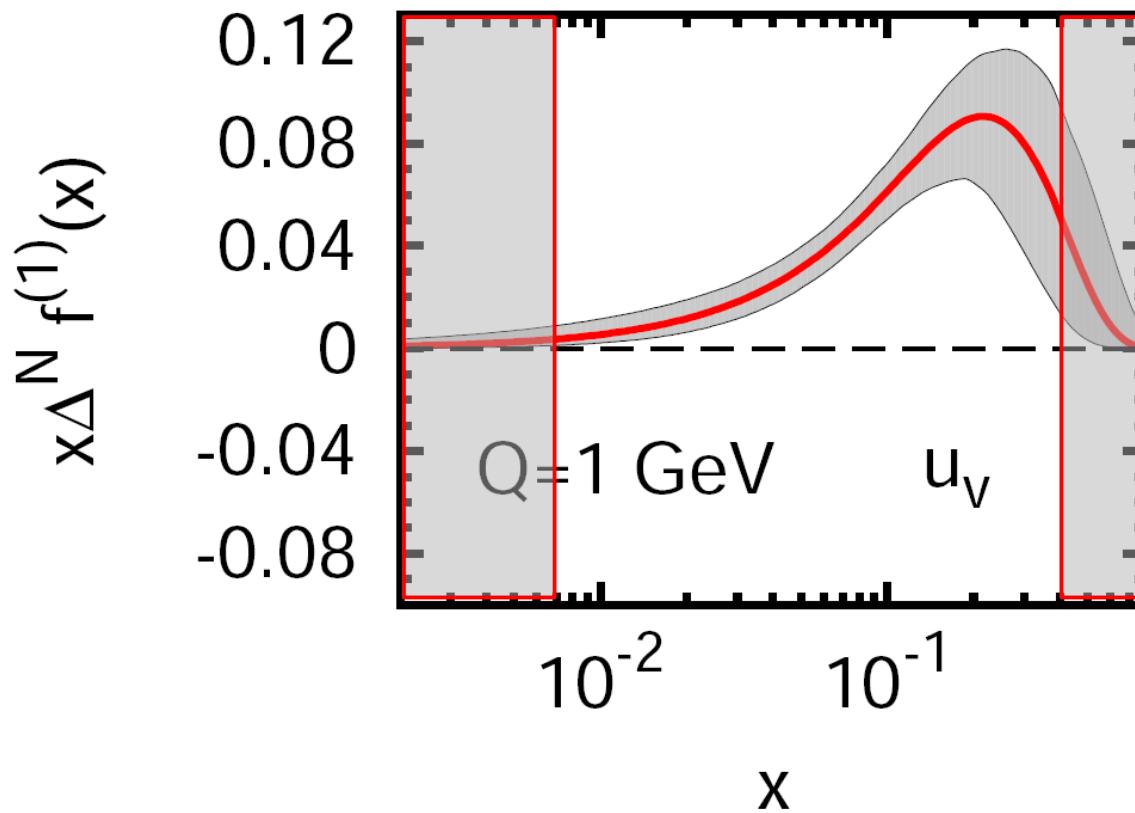
SIVERS FUNCTION - DGLAP



SIVERS FUNCTION - TMD



Sivers functions



Turin standard approach (DGLAP)

- Unpolarized TMDs are factorized in x and k_{\perp} . Only the collinear part evolves with DGLAP evolution equation. No evolution in the transverse momenta:

$$\hat{f}_{q/p}(x, k_{\perp}; Q) = f_{q/p}(x; Q) \frac{e^{-k_{\perp}^2/\langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}$$

Collinear PDF (DGLAP evolution)

Normalized Gaussian: no evolution

Collins TMD evolution of the Sivers function (PRD85,2012)

$$\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) = \tilde{F}'^{\perp f}_{1T}(x, b_T; \mu_0, Q_0^2) \exp \left\{ \ln \frac{\sqrt{\zeta_F}}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right. \\ \left. + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{\sqrt{\zeta_F}}{Q_0} \gamma_K(g(\mu')) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}. \quad (44)$$

$$\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) = \sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1 d\hat{x}_2}{\hat{x}_1 \hat{x}_2} \tilde{C}_{f/j}^{\text{Sivers}}(\hat{x}_1, \hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b)) T_{Fj/P}(\hat{x}_1, \hat{x}_2, \mu_b) \\ \times \exp \left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp \left\{ -g_{f/P}^{\text{Sivers}}(x, b_T) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}. \quad (47)$$

Drell-Yan phenomenology

- Are data distributed as a Gaussian? Do data scale as $1/M^2 + \text{DGLAP} + \text{KIN}$

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$

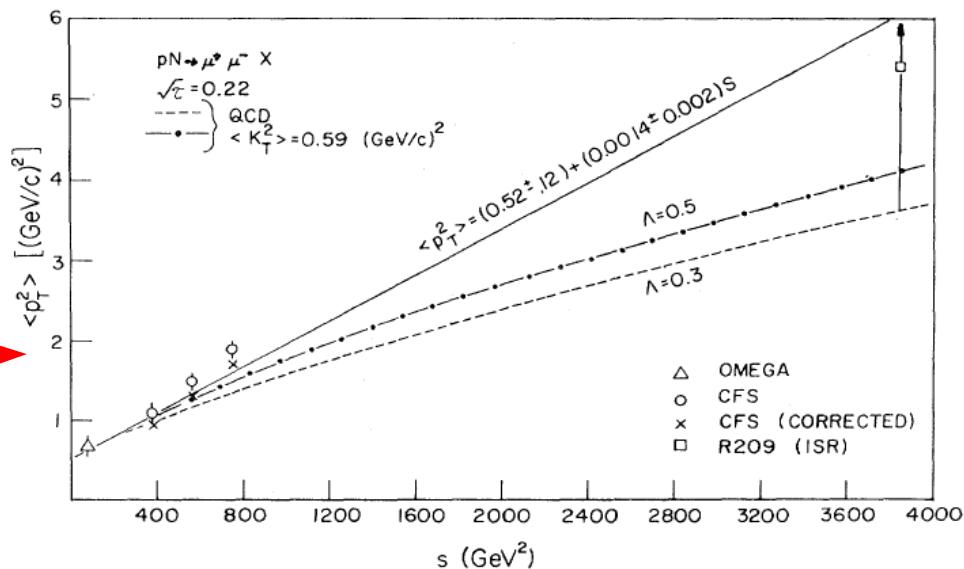


FIG. 3. $\langle p_T^2 \rangle$ vs s for dimuons produced in p -nucleon interactions. The solid curve is the linear fit to the data. The dashed and dot-dash curves are the predictions of first-order QCD using the Altarelli *et al.* prescription for different values of Λ .

$$\langle K_\perp^2 \rangle = \alpha_s(Q^2) \sum f(\tau, \alpha_s(Q^2)) + \dots$$

Cox and Malhotra, Phys. Rev. D29(1984)

Drell-Yan phenomenology

- Are data distributed as a Gaussian? Do data scale as $1/M^2 + \text{DGLAP} + \text{KIN}$

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$

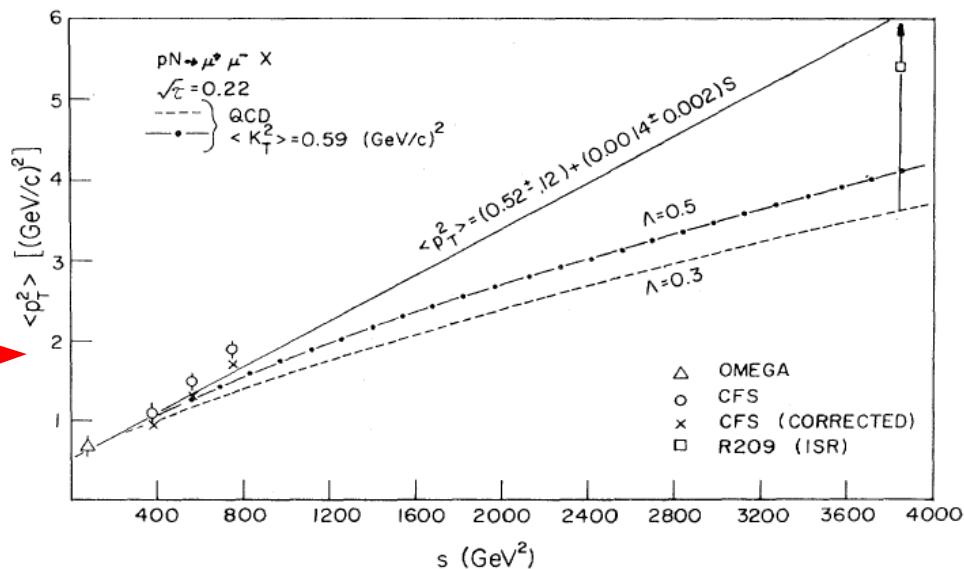


FIG. 3. $\langle p_T^2 \rangle$ vs s for dimuons produced in p -nucleon interactions. The solid curve is the linear fit to the data. The dashed and dot-dash curves are the predictions of first-order QCD using the Altarelli *et al.* prescription for different values of Λ .

$$\langle K_T^2 \rangle = \alpha_s(Q^2) \sum f(\tau, \alpha_s(Q^2)) + \dots$$

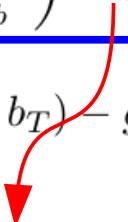
Cox and Malhotra, Phys. Rev. D29(1984)

TMD evolution

TMD in the b space:

$$\tilde{F}(x, b_T, Q, \zeta_F) = \sum_j \int_x^1 \frac{dy}{y} \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) f_j(y, \mu_b)$$

$$\boxed{\exp \left\{ \ln \left(\frac{\sqrt{\zeta_F}}{\mu_b} \right) \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^Q \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln \left(\frac{\sqrt{\zeta_F}}{\kappa} \right) \gamma_K(\kappa) \right\}}$$

$$\exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\}$$


Related to the evolution in the cut off parameter of the TMD:

$$\frac{\partial \ln \tilde{F}(x, \mathbf{b}_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(\mathbf{b}_T; \mu)$$

However.... at first order in the strong coupling constant:

$$\tilde{K}(\mu, b_T) = -\frac{\alpha_s(\mu)}{\pi} \ln(\mu^2 b_T^2 / C_1^2) \quad \text{if } \mu_b = C_1/b_* \quad \tilde{K}(b_*, \mu_b) = 0$$

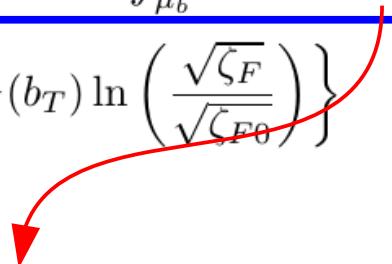
TMD evolution

TMD in the b space:

$$\tilde{F}(x, b_T, Q, \zeta_F) = \sum_j \int_x^1 \frac{dy}{y} \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) f_j(y, \mu_b)$$

$$\exp \left\{ \ln \left(\frac{\sqrt{\zeta_F}}{\mu_b} \right) \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^Q \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln \left(\frac{\sqrt{\zeta_F}}{\kappa} \right) \gamma_K(\kappa) \right\}$$

$$\exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\}$$



Second part of the part of the Sudakov form factor, notice that depends on ζ_F

$$\gamma_F(\mu; \zeta_F/\mu^2) = \alpha_s(\mu) \frac{C_F}{\pi} \left(\frac{3}{2} - \ln \left(\frac{\zeta_F}{\mu^2} \right) \right)$$

at order α_s :

$$\gamma_K(\mu) = 2C_F \frac{\alpha_s(\mu)}{\pi}$$

Unpolarized data phenomenology

- Tmd factorization has been proved for two kinds of processes:

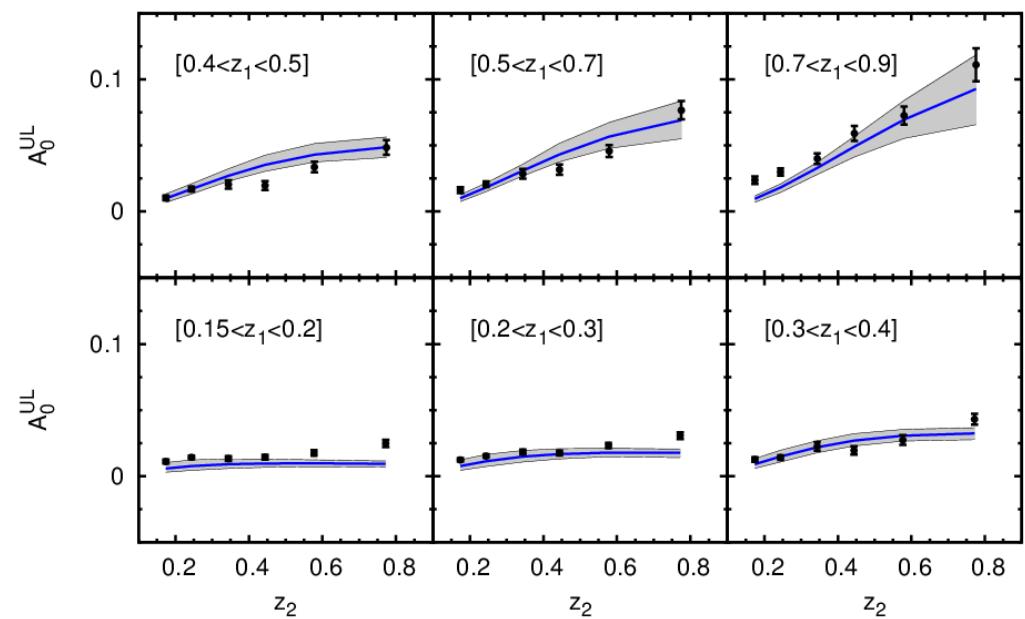
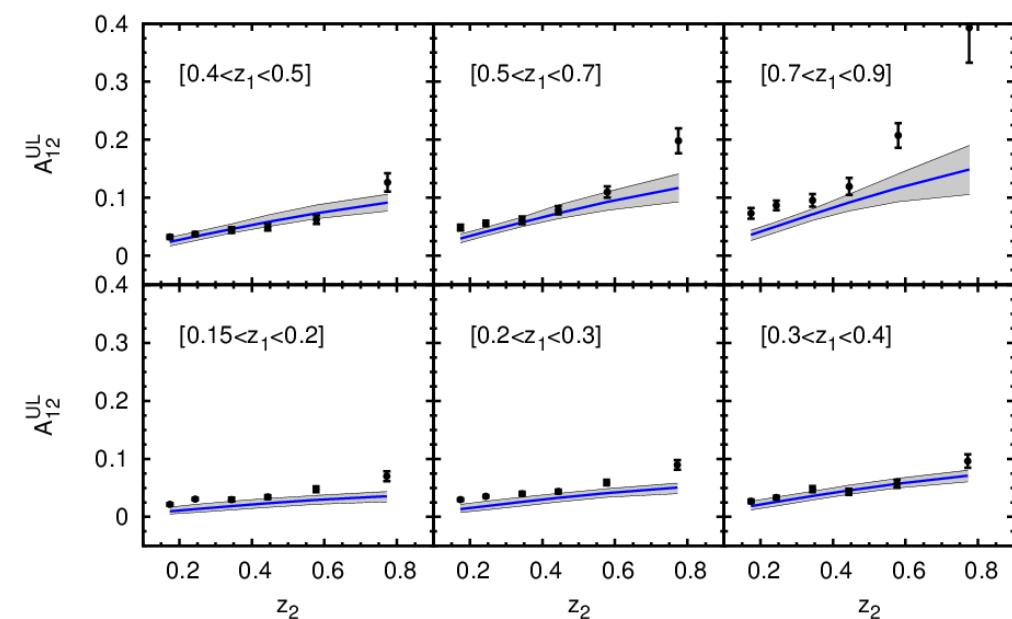
DRELL-YAN

- $\sqrt{s} \sim 20-69 \text{ GeV}; 1-7 \text{ TeV}$
- $4 < Q < 9; 10.5 < Q < 25 \text{ GeV}; M_{Z_0}$
- $0.1 < P_T < \text{tens GeV}; 1-\text{hundreds GeV}$
- (Absolute) Cross sections
- $\langle P_T^2 \rangle$
- Azimuthal asymmetries

SIDIS (JLAB, HERMES, COMPASS)

- $\sqrt{s} \sim 3.6-7-18 \text{ GeV}$
- $1 < Q < 3.2 \text{ GeV}$
- $0.1 < P_T < \text{few GeV}$
- Multiplicity
- $\langle P_T^2 \rangle$
- Azimuthal asymmetries

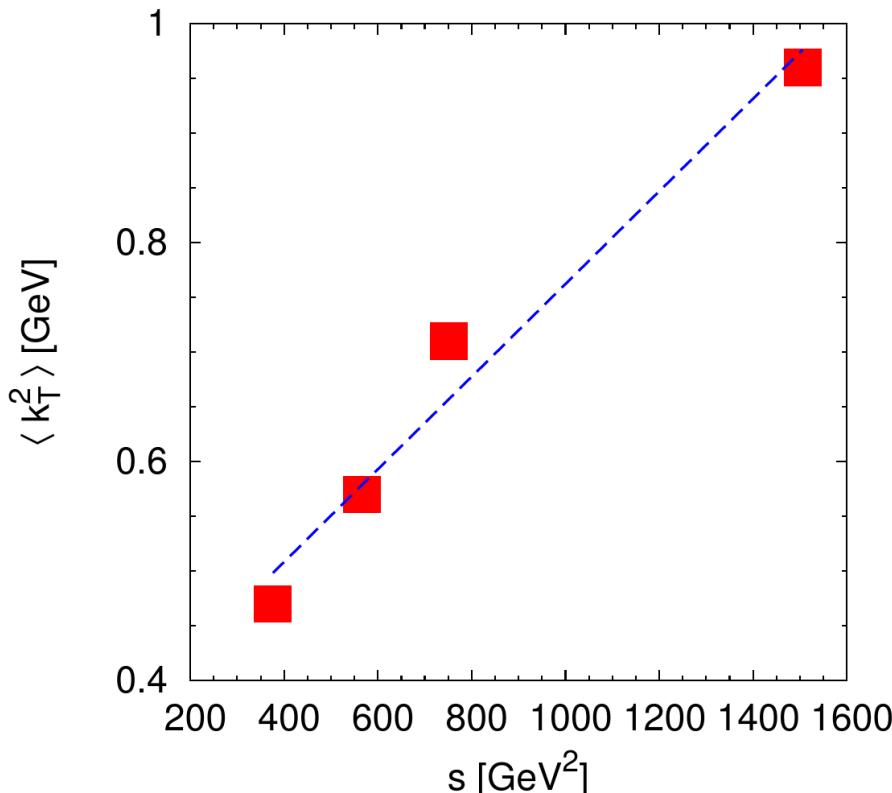
BaBar Predictions



Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$



- QCD prediction?

$$\langle K_{\perp}^{2 \text{ pert.}} \rangle = \alpha_s(Q^2) \sum f(\tau, \alpha_s(Q^2)) + \dots$$

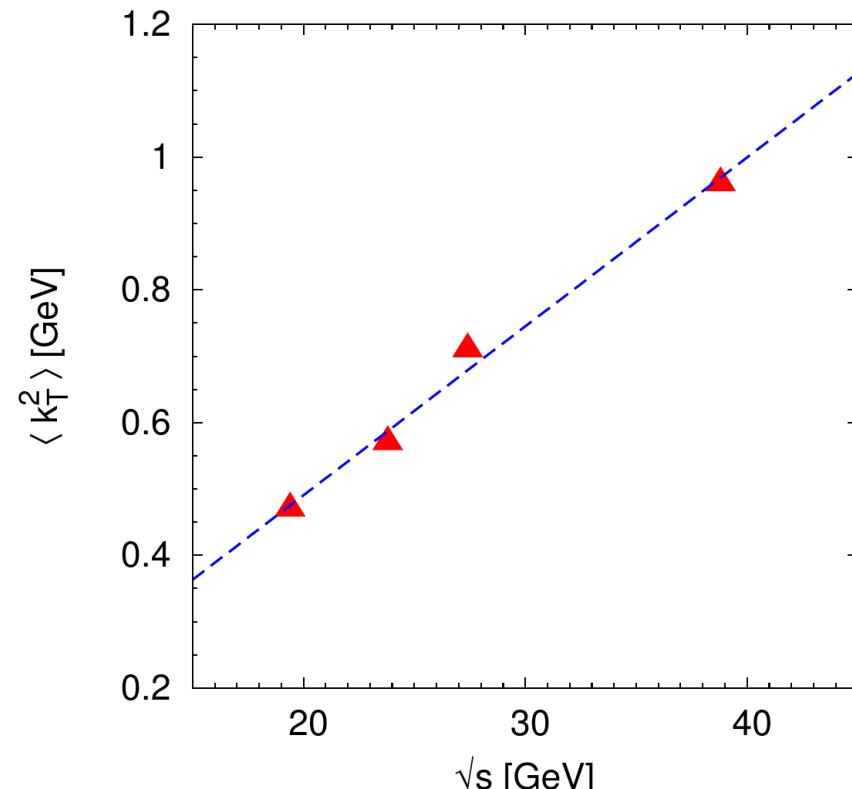
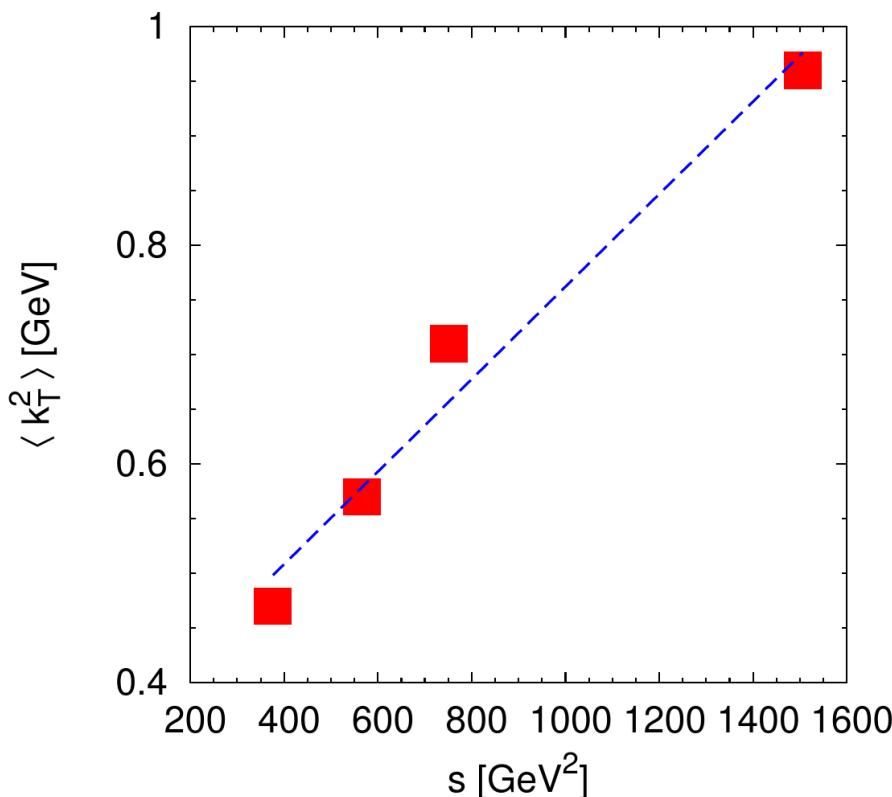
- Altarelli, Parisi and Petronzio
Phys.Lett. B76 (1978) 351

See, for SIDIS, also
Schweitzer, Metz, Teckentrup
Phys.Rev. D81 (2010) 094019

Drell-Yan phenomenology

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CSS Resummation and TMD evolution (2011)

➤ In phenomenological applications:

$$\zeta_F = Q^2 \quad b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \quad \mu_b = C_1/b_*$$

➤ And previous expression simplify considerably:

$$\begin{aligned} \tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) &= \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \\ &\quad \exp \left\{ \int_{\mu_b}^Q \frac{d\kappa}{\kappa} \gamma_F(\kappa; Q^2/\kappa^2) \right\} \\ &\quad \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left(\frac{Q}{Q_0} \right) \right\} \end{aligned}$$

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$$\exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left(\frac{Q}{Q_0} \right) \right\}$$

Convolution of the collinear PDFs
with the Wilson coefficient

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exp $\left\{ \int_{\mu_b}^Q \frac{d\kappa}{\kappa} \gamma_F(\kappa; Q^2/\kappa^2) \right\}$

Sudakov factor 

$$\exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left(\frac{Q}{Q_0} \right) \right\}$$

EIKV phenomenology

- TMD evolution in the CSS-like version

$$\begin{aligned}\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) &= \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \\ &\quad \exp\left\{\frac{1}{2}S^{CSS}(b_*, \mu_b)\right\} \\ &\quad \exp\left\{-g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right)\right\}\end{aligned}$$

- Some approximations to make life simpler

$$\tilde{C}_{ji}(z, \alpha(\mu)) = \delta_{ij} \delta(1-z) \quad \text{At LO; PDF at LO}$$

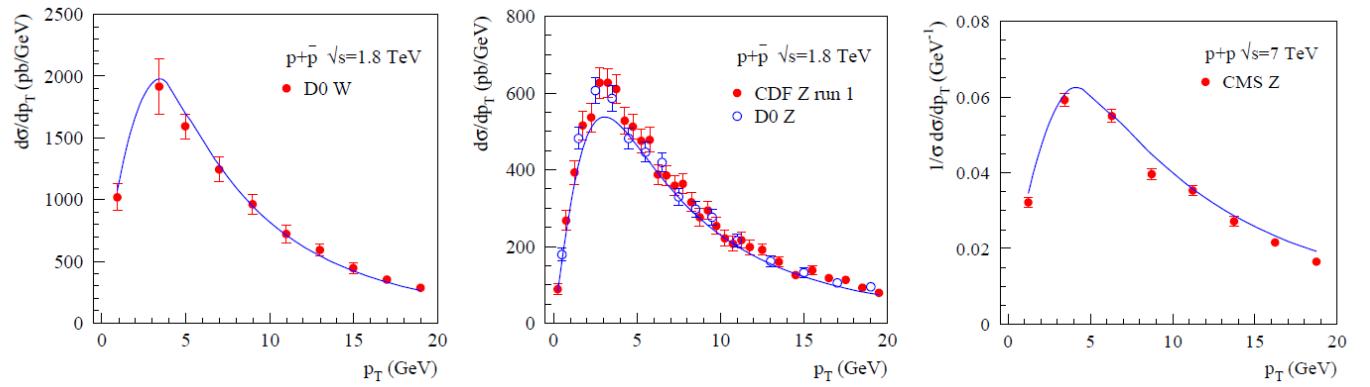
- Simple parametrizations for the non-perturbative part:

$$F_{NP}(b_T, Q)^{\text{pdf}} = \exp\left[-b_T^2 \left(g_1^{\text{pdf}} + \frac{g_2}{2} \ln(Q/Q_0)\right)\right]$$

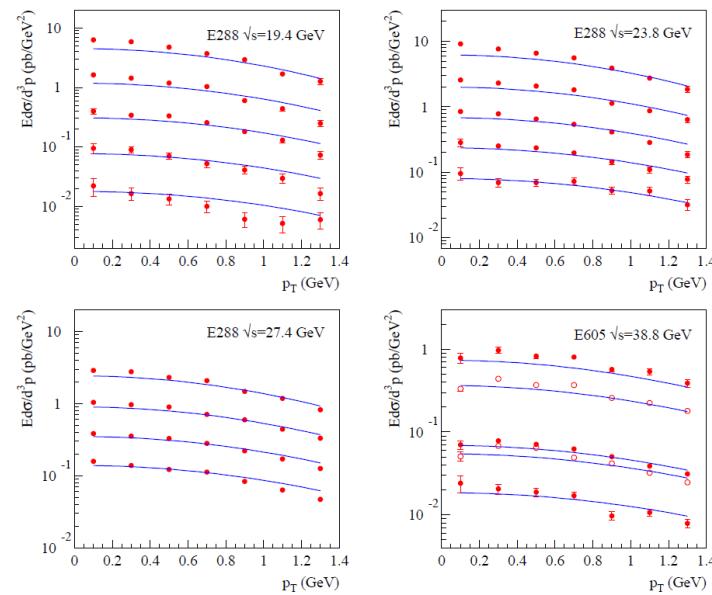
$$F_{NP}(b_T, Q)^{\text{ff}} = \exp\left[-b_T^2 \left(g_1^{\text{ff}} + \frac{g_2}{2} \ln(Q/Q_0)\right)\right]$$

EIKV phenomenology

➤ Fit DY data and SIDIS data....

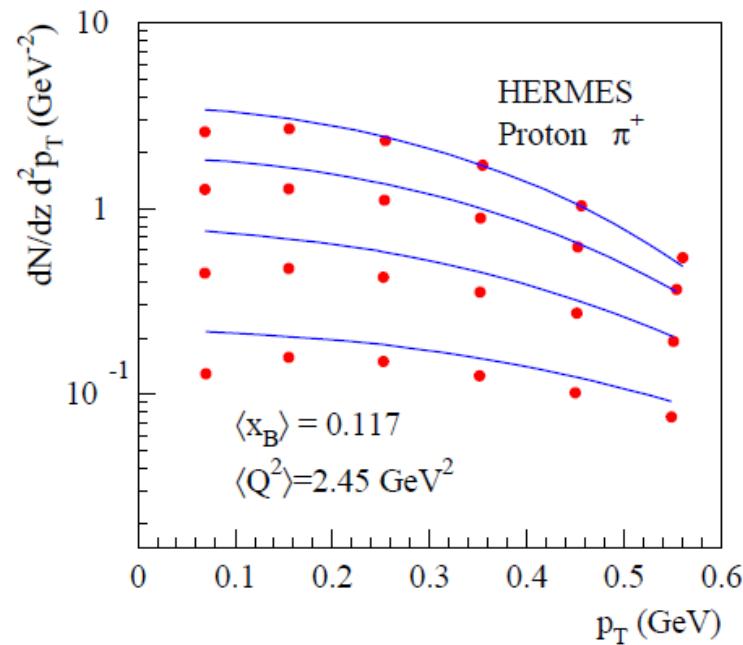
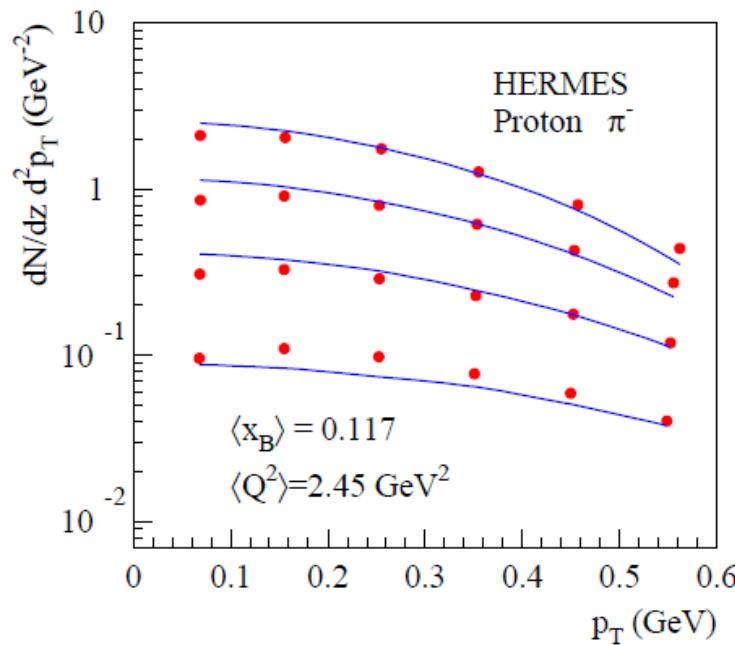


Z and W-Boson Production
Low energy DY



EIKV phenomenology

HERMES SIDIS data

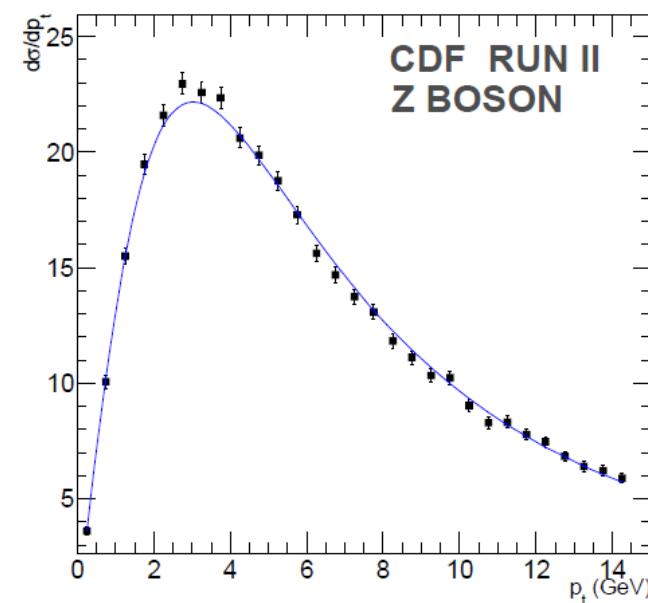
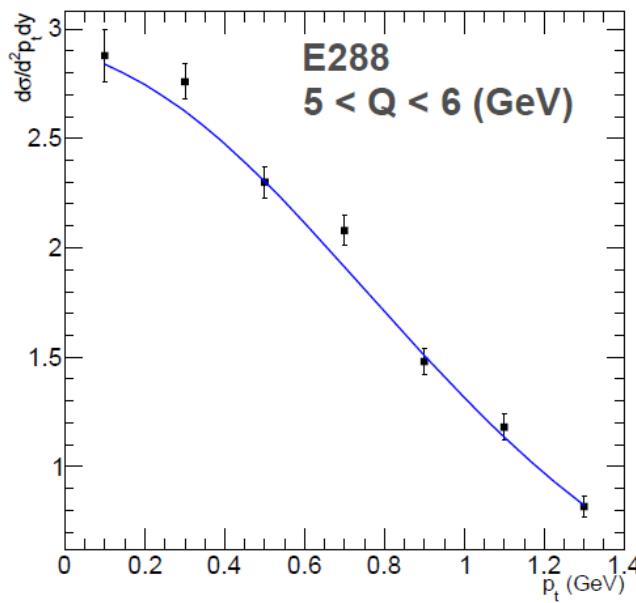


MSTW2008 PDF and DSS

SJYY phenomenology (CSS)

- Two step fit. First DY data:

$$S_{NP} = g_1 b^2 + g_2 \ln(b/b_*) \ln(Q/Q_0) + g_3 b^2 \left((x_0/x_1)^\lambda + (x_0/x_2)^\lambda \right)$$

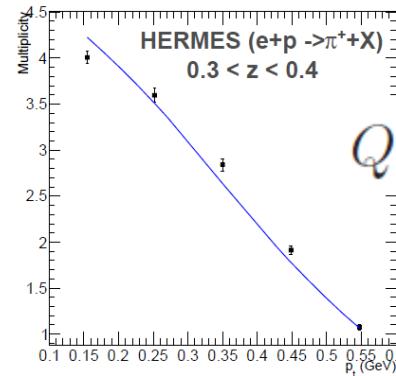
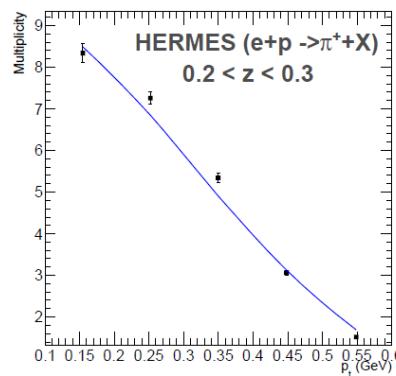


$$\chi^2/D.O.F \simeq 197/(140 - 10) = 1.5$$

SJYY phenomenology (CSS)

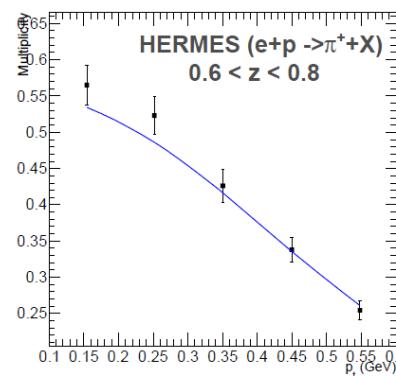
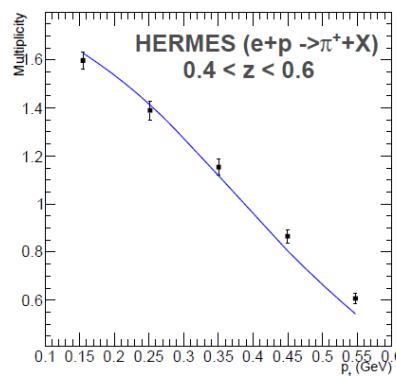
➤ Then SIDIS data:

$$S_{NP}^{(DIS)} = g_2 \ln(b/b_*) \ln(Q/Q_0) + g_1 b^2/2 + g_3 (x_0/x_B)^\lambda + g_h b^2/z_h^2$$



$$Q^2 = 3.14 \text{ GeV}^2$$

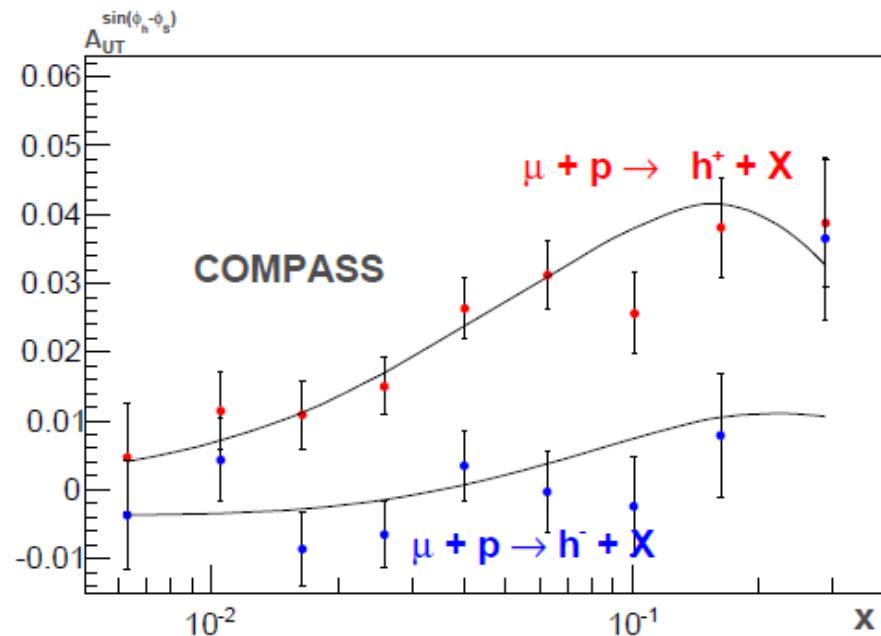
$$g_h = 0.042$$



- No fit?
- Normalization different for each z bin
- Only an example at fixed Q²

CSS/TMD evolution and Sivers asymmetry

➤ How TMD evolution describes the Sivers asymmetry in SIDIS?



P. Sun and F. Yuan
 Phys. Rev. D88, 034016 (2013)
 Phys. Rev. D88, 114012 (2013)

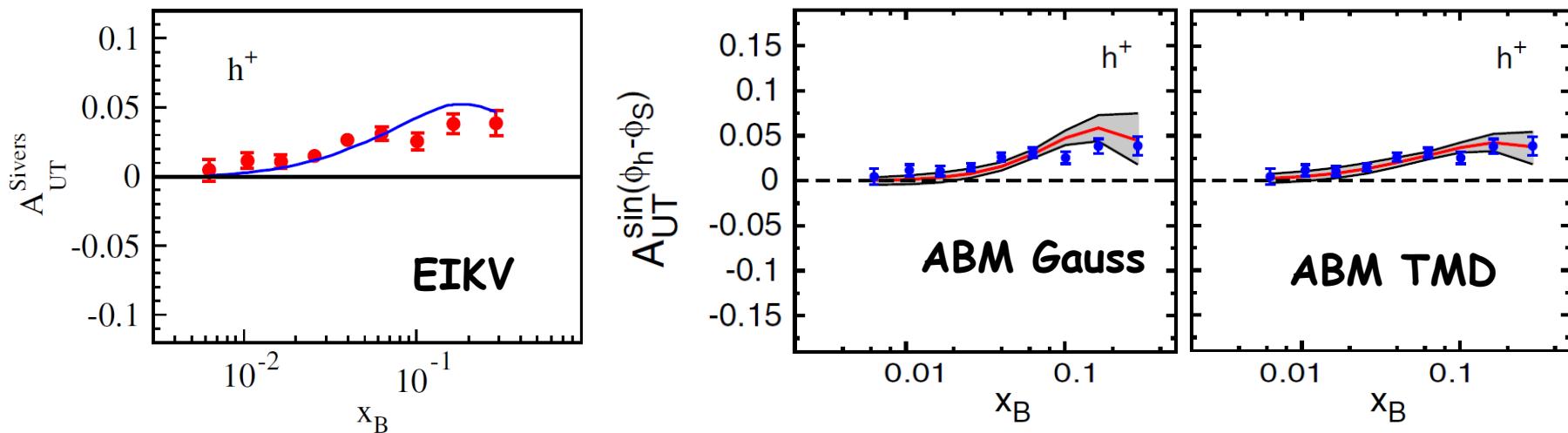
➤ TMD Modified ad hoc Sudakov
 ➤ $\chi^2 = 1.08$

$$\tilde{F}_{\text{sivers}}^\alpha(Q_0, b) = \frac{ib_\perp^\alpha M}{2} \sum_q e_q^2 \Delta f_q^{\text{sivers}}(x) D_q(z) e^{-(g_0 - g_s)b^2 - g_h b^2/z_h^2}$$

$$S_{\text{Sud}} = 2C_F \int_{Q_0}^Q \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2} \right]$$

CSS/TMD evolution and Sivers asymmetry

- How TMD evolution describes the Sivers asymmetry in SIDIS?



CSS/TMD evolution and Sivers asymmetry

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