



# Transversity PDF

$$h_1 = \text{circle with up arrow} - \text{circle with down arrow}$$

$$h_1^q(x; Q^2)$$

FIT !

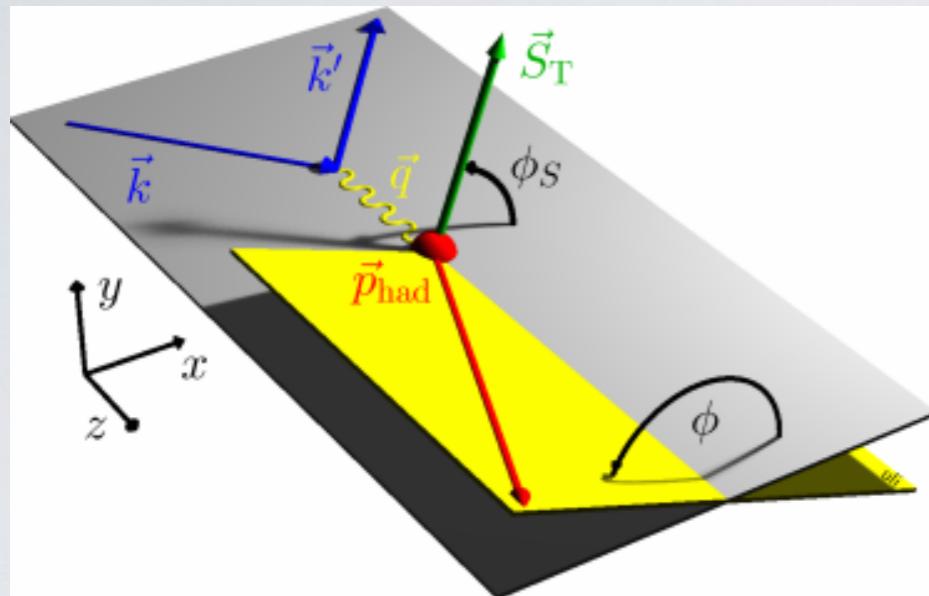


**Marco Radici**  
INFN - Pavia

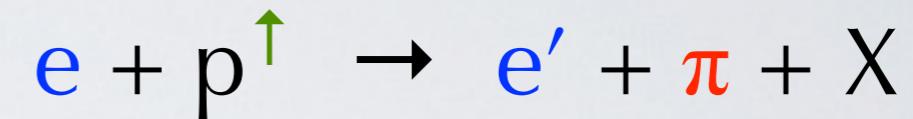


in collaboration with:  
A. Bacchetta (Univ. Pavia)  
A. Courtoy (Univ. Liege &  
U.N.A.M.)

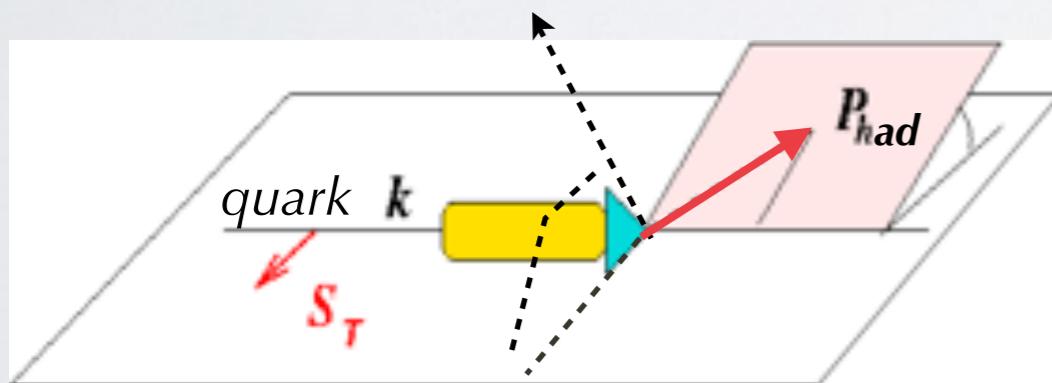
# transversity from Collins effect



single-hadron Semi-Inclusive DIS



Collins effect

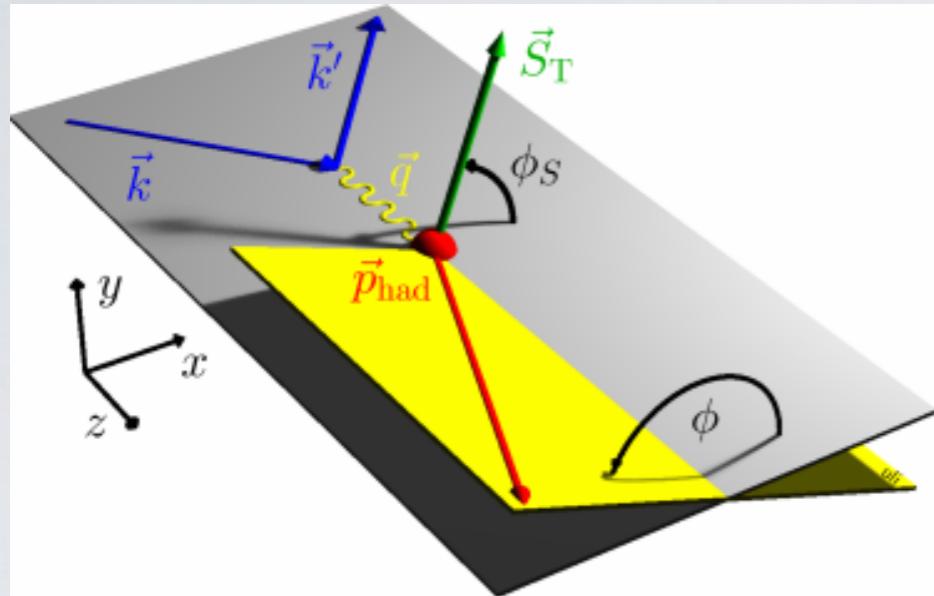


J. Collins, NPB396 (93)

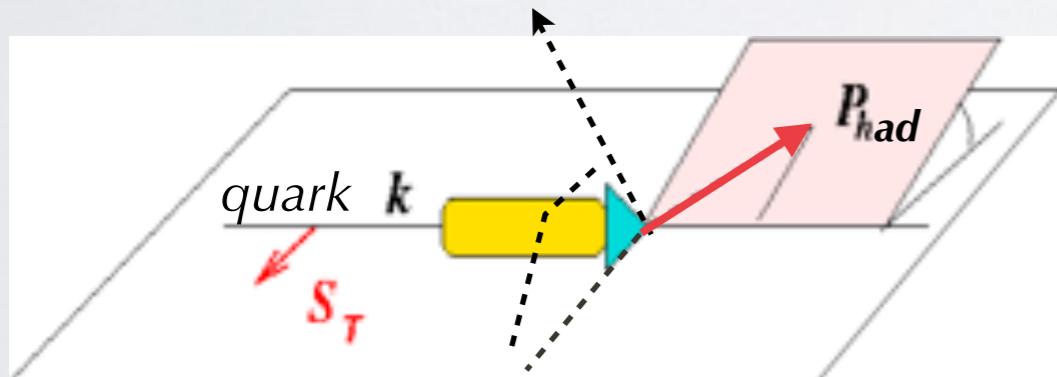
$$\mathbf{k} \times \mathbf{P}_h \cdot \mathbf{S}_T \propto \sin(\phi + \phi_S)$$

Collins angle

# transversity from Collins effect



Collins effect



*J. Collins, NPB396 (93)*

$$\mathbf{k} \times \mathbf{P}_h \cdot \mathbf{S}_T \propto \sin(\phi + \phi_S)$$

Collins angle

single-hadron Semi-Inclusive DIS



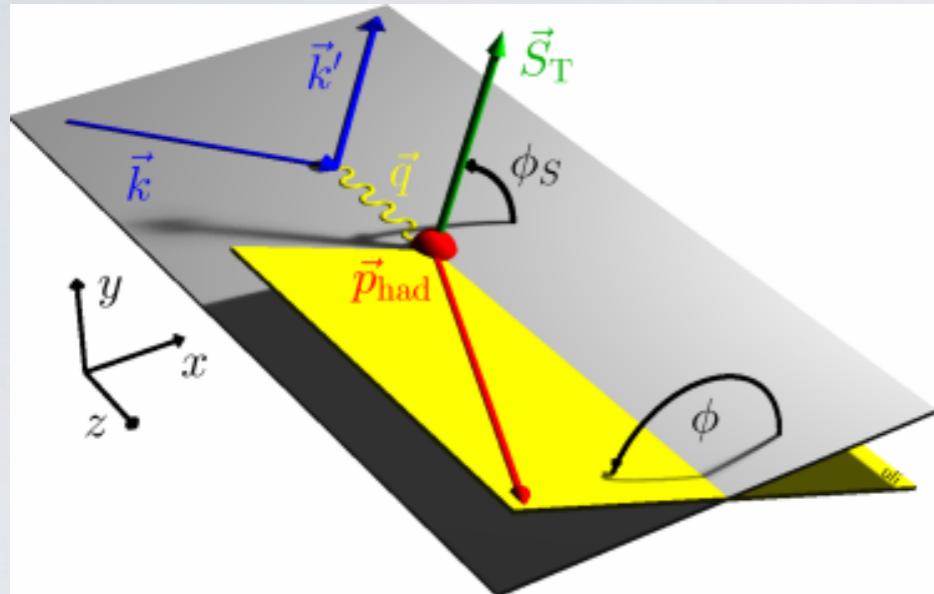
single-spin Asymmetry

$$A_{UT}^{\sin(\phi+\phi_S)}(x, z, \phi, \mathbf{P}_{hT}^2) = \frac{1}{\sin(\phi + \phi_S)} \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

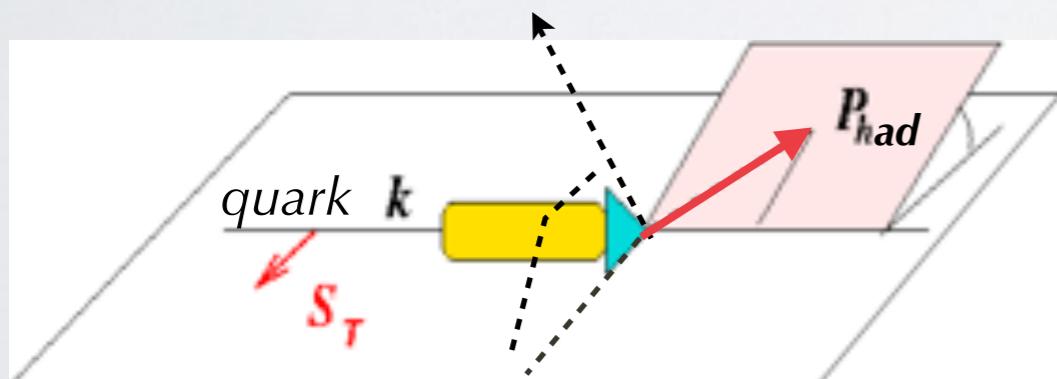
$$\propto \frac{\sum_q e_q^2 \left[ h_1^q \otimes_w H_1^{\perp q} \right] (x, z, \mathbf{P}_{hT})}{\sum_q e_q^2 [f_1^q \otimes D_1^q] (x, z, \mathbf{P}_{hT})}$$

$\mathbf{P}_{hT} \neq 0$  transverse momentum  
of hadron required  
 $\Rightarrow$  TMD factorization required

# transversity from Collins effect



Collins effect



*J. Collins, NPB396 (93)*

$$\mathbf{k} \times \mathbf{P}_h \cdot \mathbf{S}_T \propto \sin(\phi + \phi_S)$$

Collins angle

single-hadron Semi-Inclusive DIS



single-spin Asymmetry

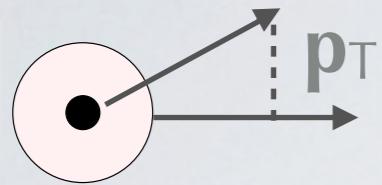
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$\mathbf{P}_{hT} \neq 0$  transverse momentum  
of hadron required  
 $\Rightarrow$  TMD factorization required

but transversity is  
a collinear PDF

# transversity is a PDF



transverse momentum dependent  
parton distributions TMD ( $x, p_T$ )

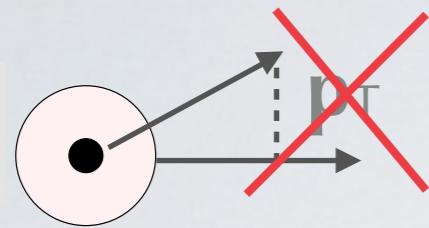
(see next talk and  
Thursday afternoon session)

leading-twist TMD map  
quark polarization

	U	L	T
U	$f_1$		$h_1^\perp$
L		$g_L$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	$h_1 \ h_{1T}^\perp$

nuclon polarization

# transversity is a PDF



transverse momentum dependent  
parton distributions TMD ( $x, p_T$ )

(see next talk and  
Thursday afternoon session)

leading-twist TMD map       PDF map  
quark polarization

	U	L	T
U	$f_1$		$h_1^\perp$
L		$g_L$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	$h_1 \ h_{1T}^\perp$

nucleon polarization

$$f_1 = \text{circle with dot}$$

$$g_1 = \text{circle with dot and green arrow} - \text{circle with dot and red arrow}$$

(tomorrow  
session)

$$h_1 = \text{circle with dot and green arrow up} - \text{circle with dot and red arrow down}$$

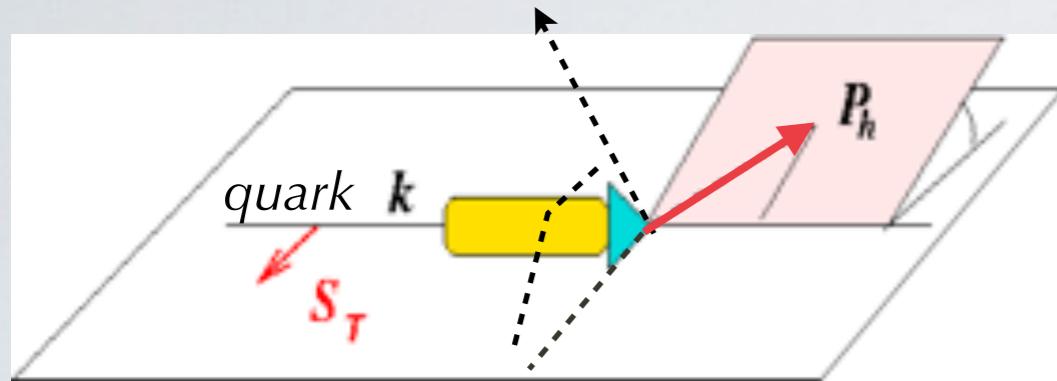
this talk

# Outline

- how to extract transversity in a collinear framework
- review of existing results about extraction of transversity
- new fit : what's new ?
- conclusions and outlooks

# from Collins effect to Di-hadron Fragmentation

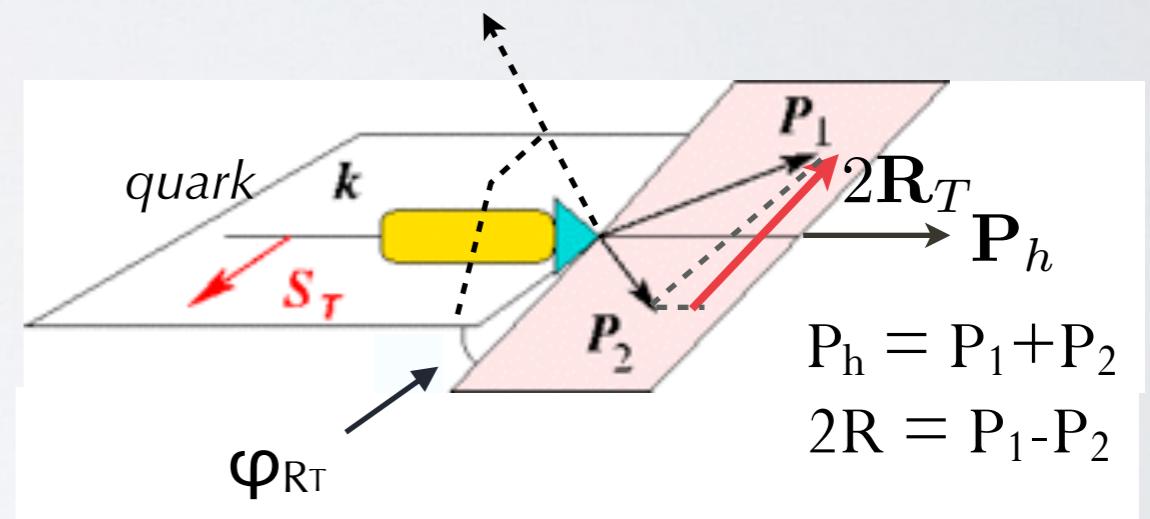
Collins effect



*J. Collins, NPB396 (93)*

$$\mathbf{k} \times \mathbf{P}_h \cdot \mathbf{S}_T \propto \sin(\phi + \phi_S)$$

DiFF effect

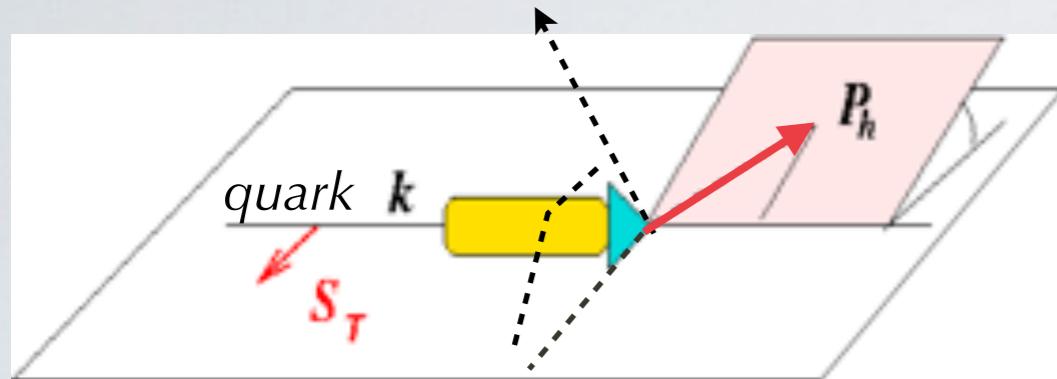


*Collins, Heppelman, Ladinsky, NP B420 (94)*

$$\mathbf{P}_h \times \mathbf{R}_T \cdot \mathbf{S}_T \propto \sin(\phi_{RT} + \phi_S)$$

# from Collins effect to Di-hadron Fragmentation

## Collins effect



*J. Collins, NPB396 (93)*

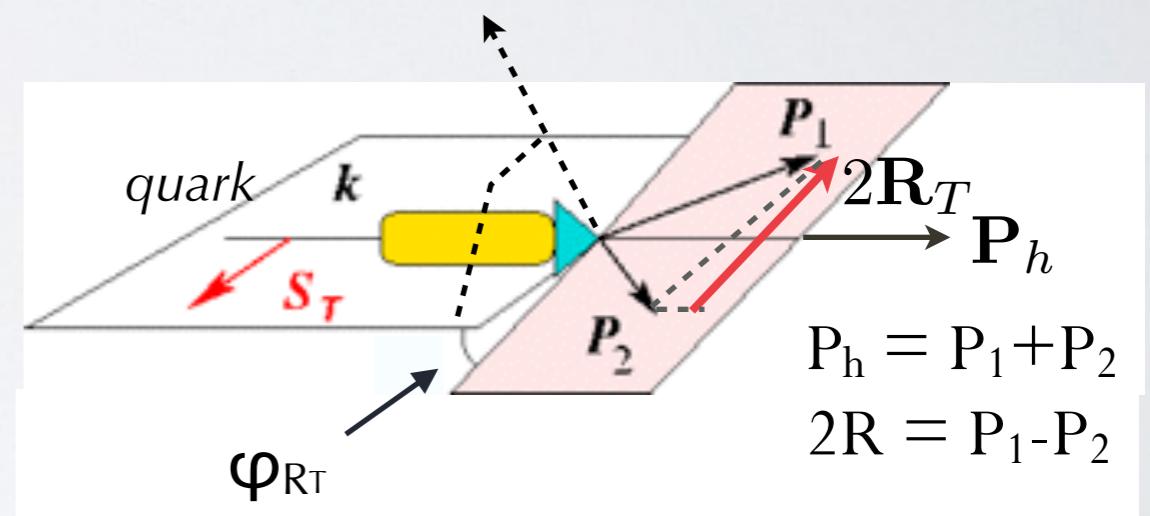
$$\mathbf{k} \times \mathbf{P}_h \cdot \mathbf{S}_T \propto \sin(\phi + \phi_S)$$

effect relies on  $\mathbf{R}_T \neq 0$

$\mathbf{P}_{hT} = 0$  the pair is collinear

framework of  
collinear factorization

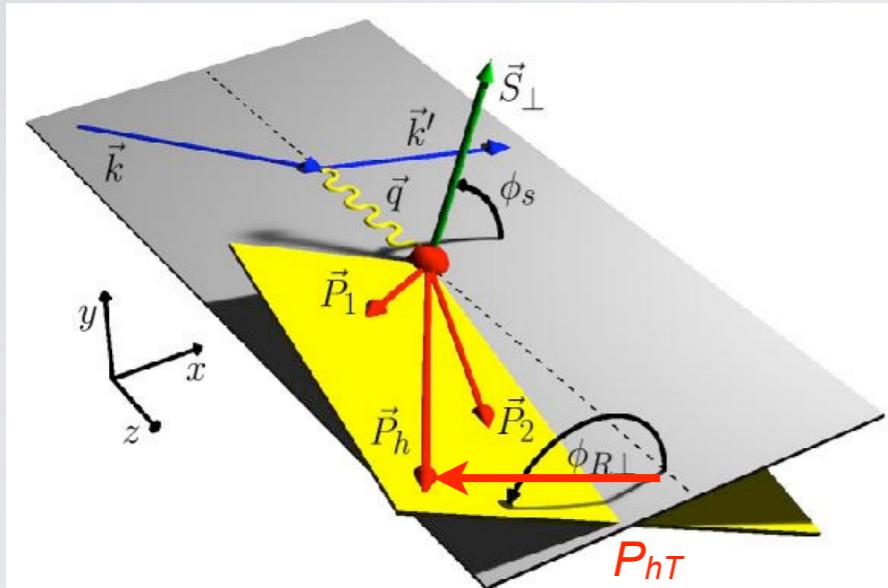
## DiFF effect



*Collins, Heppelman, Ladinsky, NPB420 (94)*

$$\mathbf{P}_h \times \mathbf{R}_T \cdot \mathbf{S}_T \propto \sin(\phi_{R_T} + \phi_S)$$

# transversity from DiFF



di-hadron Semi-Inclusive DIS

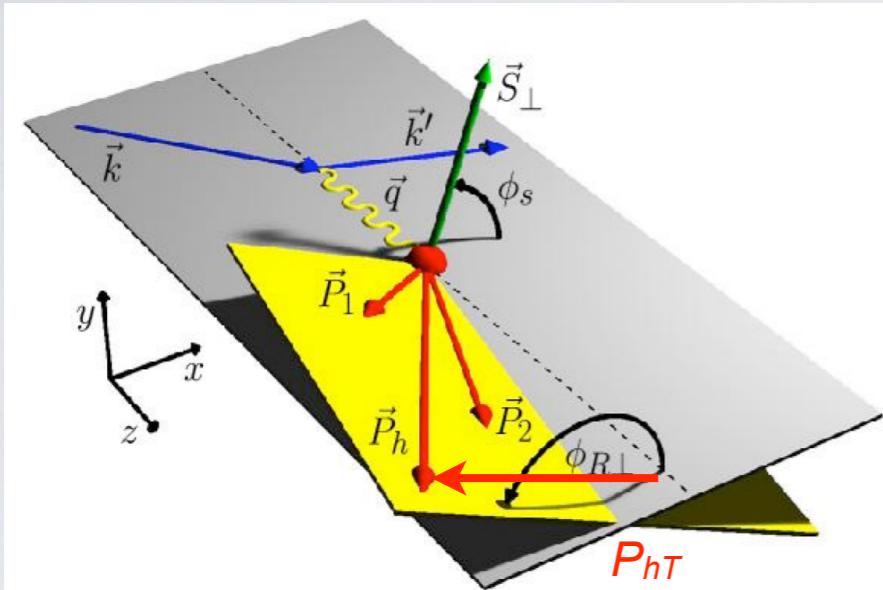


single-spin Asymmetry

$$A_{UT}^{\sin(\phi_R + \phi_S)}(x, z, M_h^2) \propto -\frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^{q*}(x) H_1^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$

$$\frac{|\mathbf{R}|}{M_h} = \sqrt{1 - \frac{4m_\pi^2}{M_h^2}}$$

# transversity from DiFF



di-hadron Semi-Inclusive DIS



single-spin Asymmetry

$$A_{UT}^{\sin(\phi_R + \phi_S)}(x, z, M_h^2) \propto -\frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^q(x) H_1^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$

$$\frac{|\mathbf{R}|}{M_h} = \sqrt{1 - \frac{4m_\pi^2}{M_h^2}}$$

$P_{hT} = 0$

collinear factorization



simple product  
of  $h_1$  and  $H_1^*$

*Radici, Jakob, Bianconi  
PR D65 (02)*  
*Bacchetta and Radici,  
PR D67 (03)*

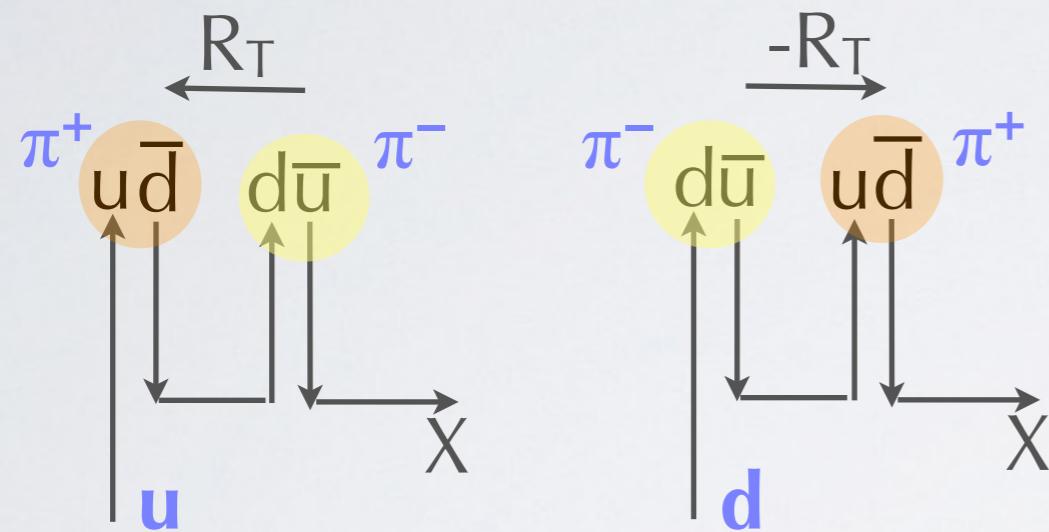


DGLAP evolution eq.  
(well known)

*Ceccopieri, Radici, Bacchetta,  
PL B650 (07)*

# first extraction of valence $h_1^{q_v}$

$$A_{UT}^{\sin(\phi_R + \phi_S)}(x, z, M_h^2) \propto -\frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^q(x) H_1^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$



isospin symmetry + charge conjugation

$$H_1^{\triangleleft u} = -H_1^{\triangleleft d}$$

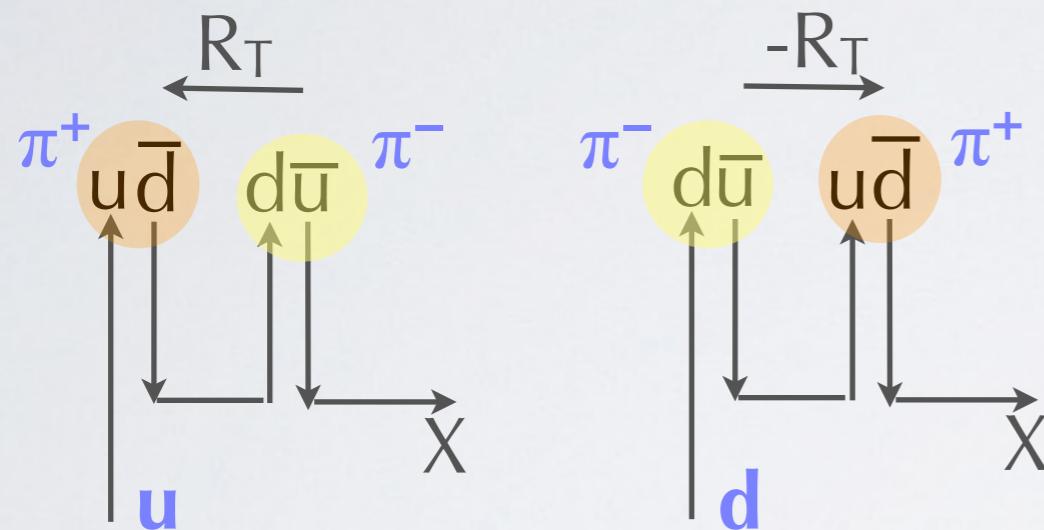
$$H_1^{\triangleleft q} = -H_1^{\triangleleft \bar{q}}$$

$$D_1^q = D_1^{\bar{q}}$$

consistent with  
sign of exp.  $A_{UT}$

# first extraction of valence $h_1^{q_v}$

$$A_{UT}^{\sin(\phi_R + \phi_S)}(x, z, M_h^2) \propto -\frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^q(x) H_1^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$



isospin symmetry + charge conjugation

$$H_1^{\triangleleft u} = -H_1^{\triangleleft d}$$

$$H_1^{\triangleleft q} = -H_1^{\triangleleft \bar{q}}$$

$$D_1^q = D_1^{\bar{q}}$$

consistent with  
sign of exp.  $A_{UT}$

proton target

$$xh_1^p(x) \equiv xh_1^{u_v}(x) - \frac{1}{4}xh_1^{d_v}(x)$$

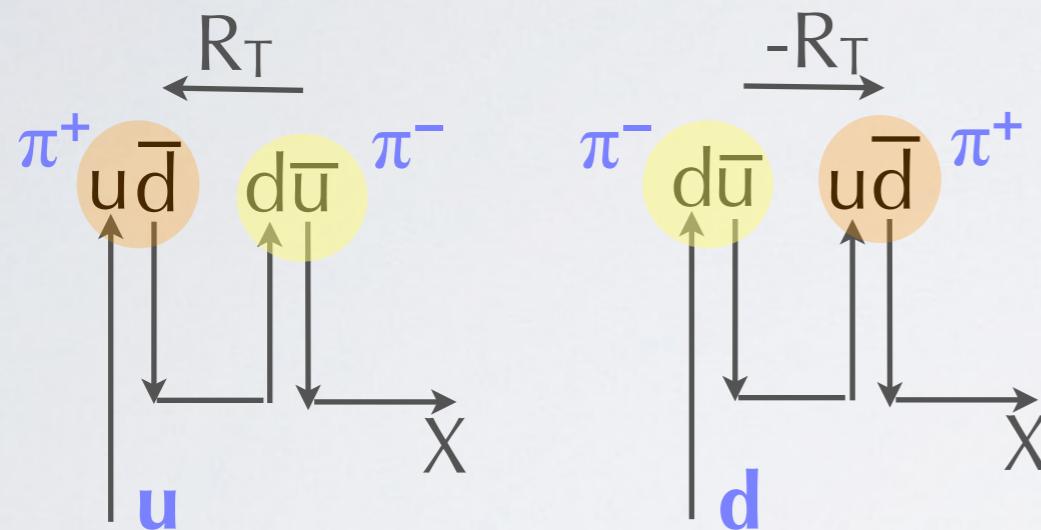
2h-SIDIS data

$$\propto -\frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 H_1^{\triangleleft u}} \left[ \sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_1^{q+\bar{q}}(x) \int dz dM_h^2 D_1^q \right]$$

point-by-point extraction

# first extraction of valence $h_1^q$

$$A_{UT}^{\sin(\phi_R + \phi_S)}(x, z, M_h^2) \propto -\frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^q(x) H_1^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$



isospin symmetry + charge conjugation

$$H_1^{\triangleleft u} = -H_1^{\triangleleft d}$$

$$H_1^{\triangleleft q} = -H_1^{\triangleleft \bar{q}}$$

$$D_1^q = D_1^{\bar{q}}$$

consistent with  
sign of exp.  $A_{UT}$

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$$xh_1^p(x) \equiv xh_1^{u_v}(x) - \frac{1}{4}xh_1^{d_v}(x)$$

2h-SIDIS data

$$\propto -\frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 H_1^{\triangleleft u}}$$

$f_1^q$  parametrization

$$\left[ \sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_1^{q+\bar{q}}(x) \int dz dM_h^2 D_1^q \right]$$

extract **DiFF** from  
 $e^+e^- \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)$

point-by-point extraction



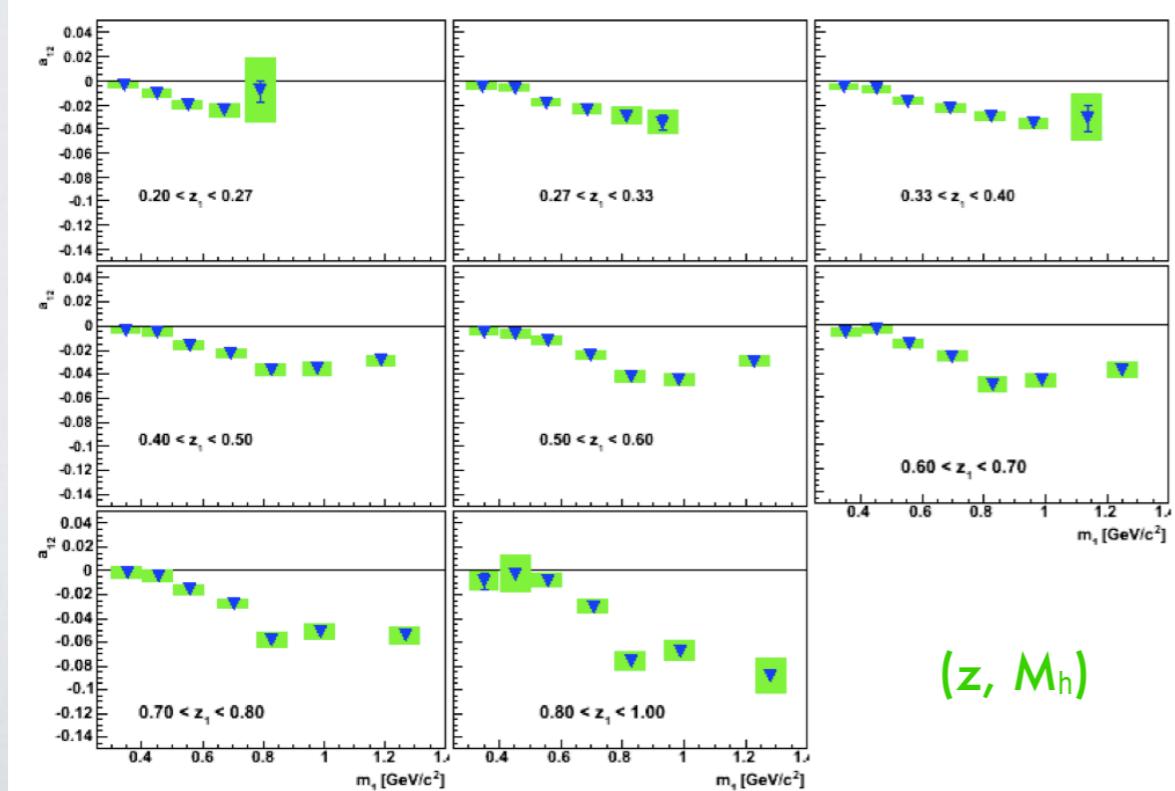
fit  data  $\rightarrow$  extract DiFF

$$A^{\cos(\phi_R + \bar{\phi}_R)} = \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \frac{|\mathbf{R}_T|}{M_h} \frac{|\overline{\mathbf{R}}_T|}{\overline{M}_h}$$

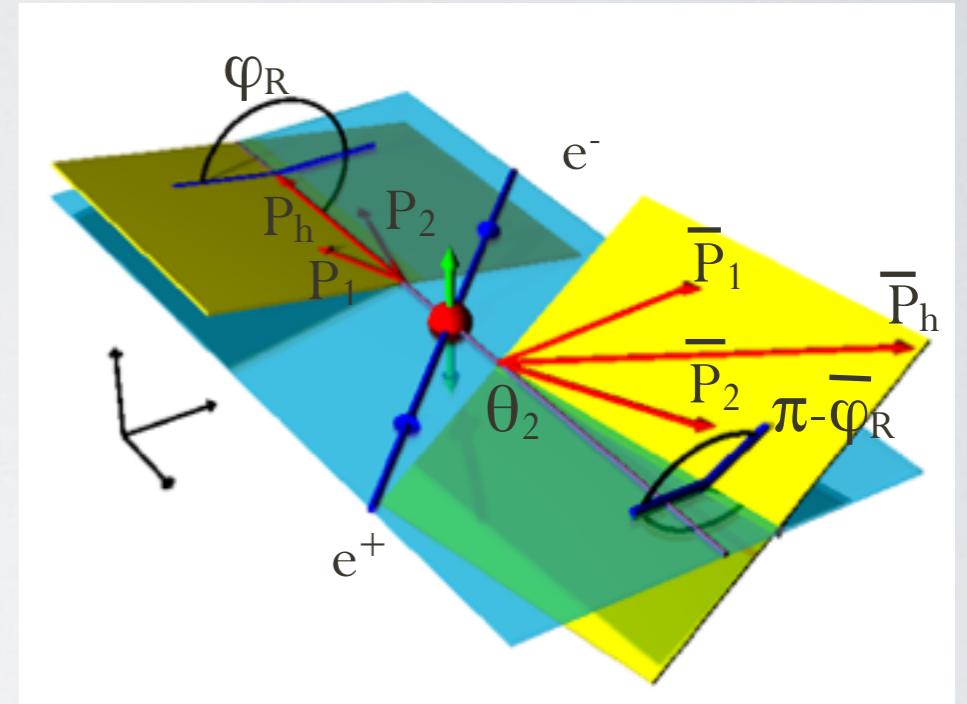
$$\times \frac{\sum_q e_q^2 H_{1,sp}^{< q}(z, M_h^2) \overline{H}_{1,sp}^{< \bar{q}}(\bar{z}, \overline{M}_h^2)}{\sum_q e_q^2 D_1^q(z, M_h^2) \overline{D}_1^{\bar{q}}(\bar{z}, \overline{M}_h^2)}$$

*Boer, Jakob, Radici, P.R. D67 (03) 094003  
 Artru & Collins, Z.Ph. C69 (96) 277*

(integrating on one hemisphere)



$e^+e^- \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)+X$



*Vossen et al., P.R.L. 107 (11) 072004*

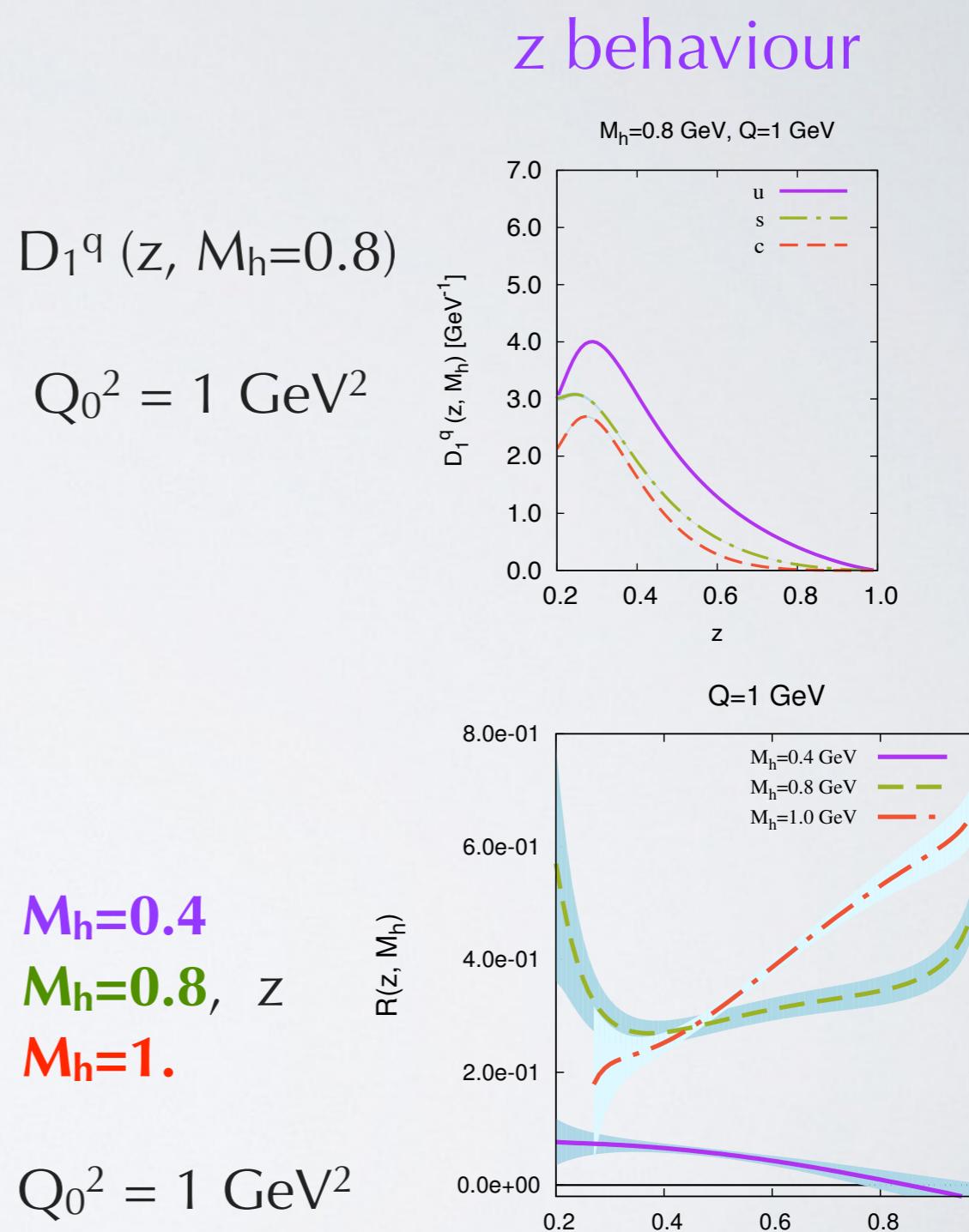
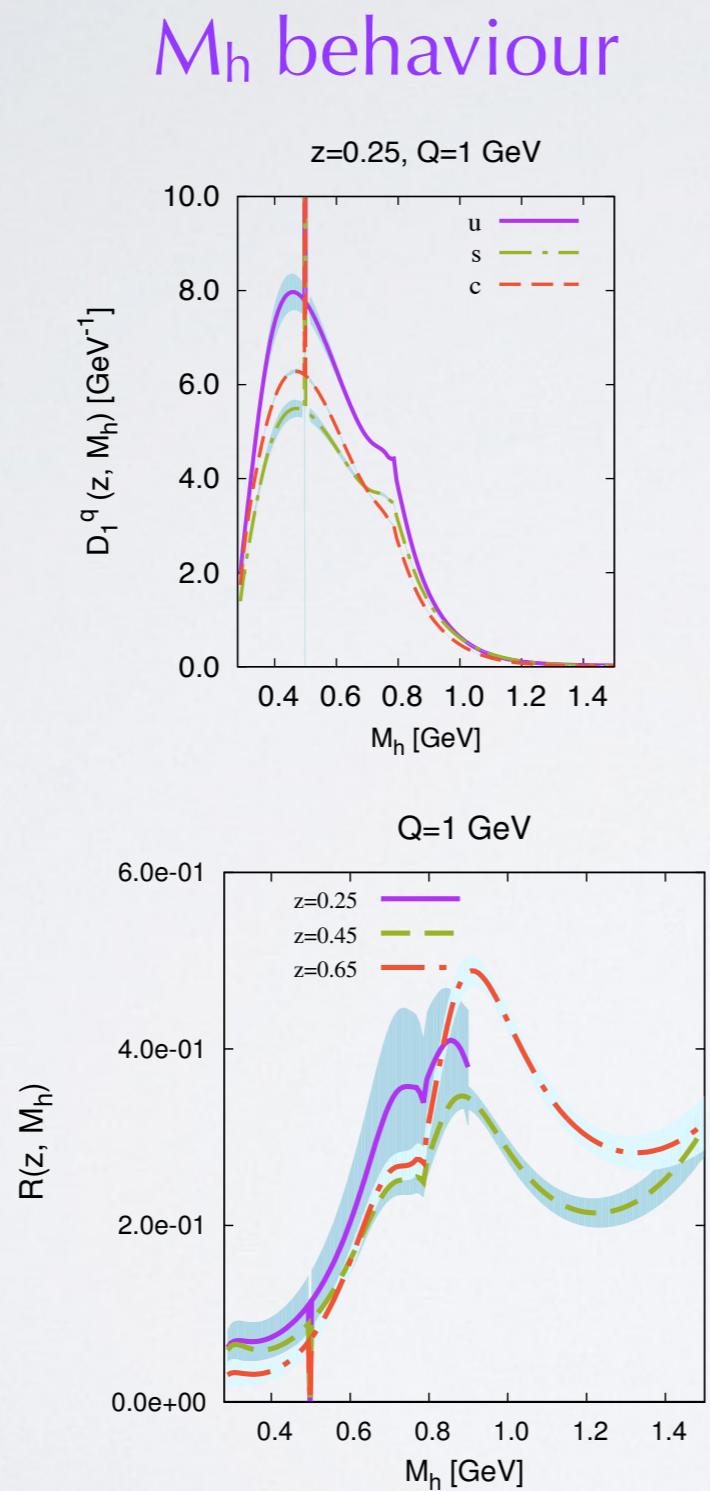
fitting  $A^{\cos(\Phi_R + \bar{\Phi}_R)} \rightarrow$  extract  $H_1^{<}$

no data for unpolarized  $d\sigma$   
 $\rightarrow D_1$  extracted from PYTHIA  
 adapted to Belle

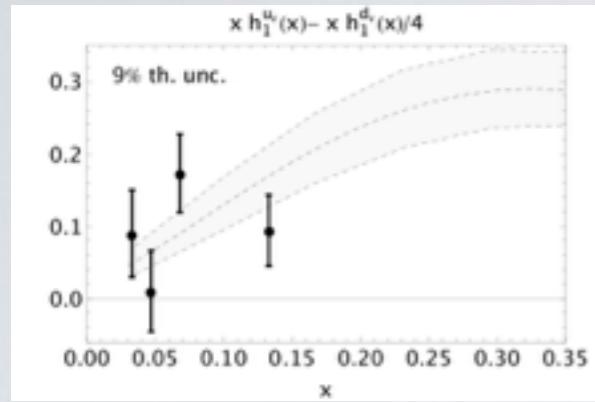
# first ever extraction of DiFF

Courtoy, Bacchetta, Radici, Bianconi, P.R. D85 (12) 114023

$$\begin{aligned}
 & D_1^q \\
 & D_1^q(z=0.25, M_h) \\
 & Q_0^2 = 1 \text{ GeV}^2 \\
 & \frac{|R|}{M_h} \frac{H_1^{< u}}{D_1^u} \\
 & z=0.25 \\
 & z=0.45, M_h \\
 & z=0.65 \\
 & Q_0^2 = 1 \text{ GeV}^2
 \end{aligned}$$



# first extraction of valence $h_1^{q_v}$



Bacchetta, Courtoy, Radici,  
P.R.L. **107** (11) 012001

proton target

$$xh_1^p(x) \equiv xh_1^{u_v}(x) - \frac{1}{4}xh_1^{d_v}(x)$$

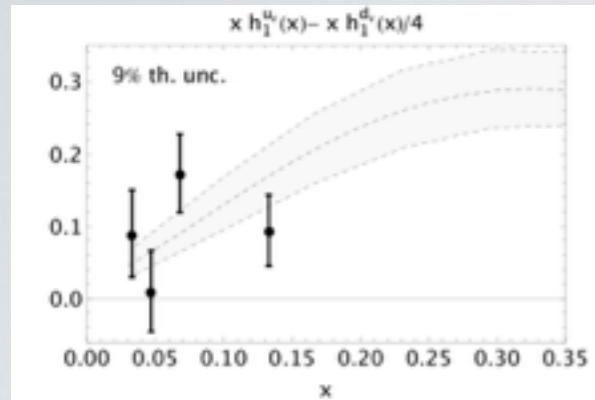


Airapetian et al.,  
JHEP **0806** (08) 017



$$A_{UT}^{\sin(\phi_R + \phi_S)}$$

# first extraction of valence $h_1^{q_v}$



Bacchetta, Courtoy, Radici,  
*P.R.L. **107** (11) 012001*

proton target

$$x h_1^p(x) \equiv x h_1^{u_v}(x) - \frac{1}{4} x h_1^{d_v}(x)$$

adding  
deuteron target

$$x h_1^D(x) \equiv x h_1^{u_v}(x) + x h_1^{d_v}(x)$$

point-by-point access  
to  $h_1^{u_v}, h_1^{d_v}$  separately

$$A_{UT}^{\sin(\phi_R + \phi_S)}$$

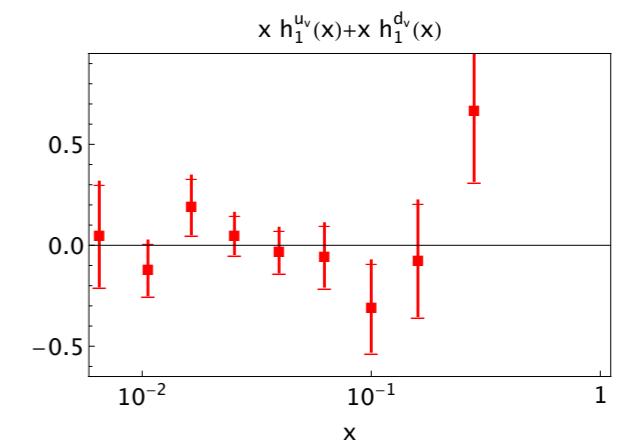
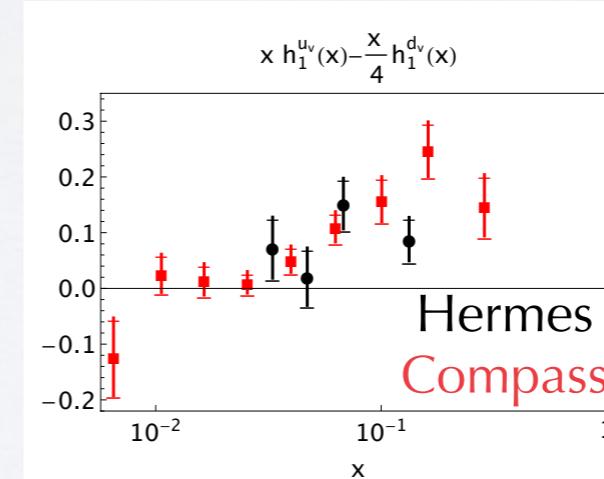
Airapetian et al.,  
*JHEP **0806** (08) 017*

C.Adolph et al.,  
*PL **B713** (12)*



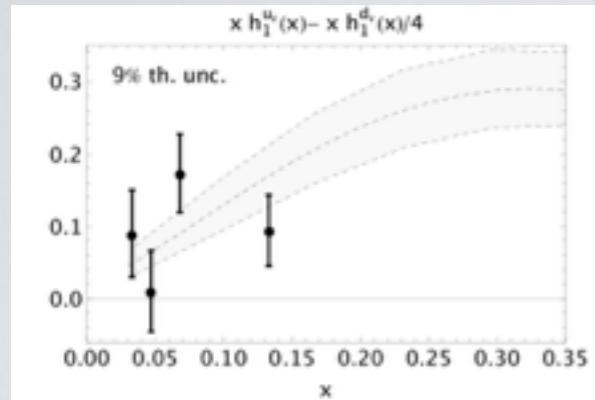
COMPASS run  
2007

COMPASS run  
2002-4



Bacchetta, Courtoy, Radici, *JHEP **1303** (13) 119*

# first extraction of valence $h_1^{q_v}$



*Bacchetta, Courtoy, Radici,  
P.R.L. **107** (11) 012001*

proton target

$$xh_1^p(x) \equiv xh_1^{u_v}(x) - \frac{1}{4}xh_1^{d_v}(x)$$

adding  
deuteron target

$$xh_1^D(x) \equiv xh_1^{u_v}(x) + xh_1^{d_v}(x)$$

point-by-point access  
to  $h_1^{u_v}, h_1^{d_v}$  separately

$$A_{UT}^{\sin(\phi_R + \phi_S)}$$

*Airapetian et al.,  
JHEP **0806** (08) 017*

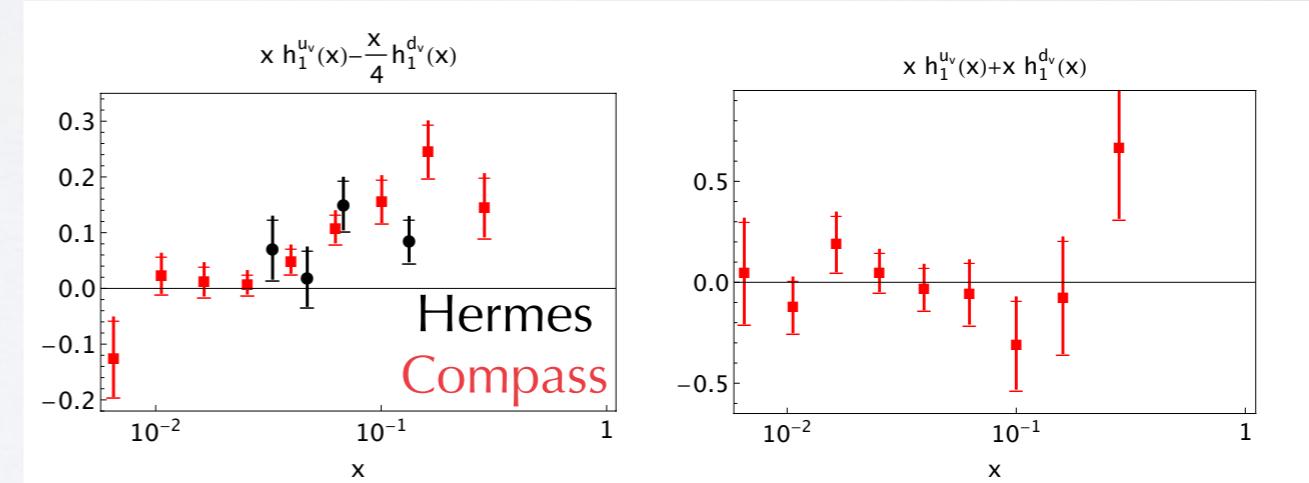
*C.Adolph et al.,  
PL **B713** (12)*



run  
2007



run  
2002-4



*Bacchetta, Courtoy, Radici, JHEP **1303** (13) 119*

we have also performed a true fit of  $x h_1^p(x)$  and  $x h_1^D(x)$

## Fit : **functional form** and **method**

**functional form** at starting scale  $Q_0^2 = 1 \text{ GeV}^2$

$$x h_1^{q_v}(x) = \tanh [\sqrt{x} (A_q + B_q x + C_q x^2 + D_q x^3)] [x \text{SB}_q(x) + x \overline{\text{SB}}_{\bar{q}}(x)]$$

satisfies **Soffer Bound** at any  $Q^2$

$$2|h_1^q(x, Q^2)| \leq 2 \text{SB}_q(x) = |f_1^q(x) + g_1^q(x)|$$

# Fit : **functional form** and **method**

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rigid



satisfies **Soffer Bound** at any  $Q^2$

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# Fit : **functional form** and **method**

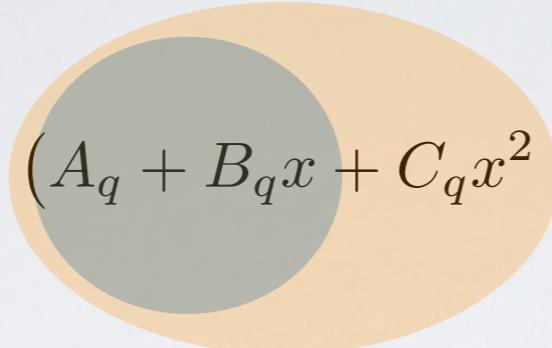
**functional form** at starting scale  $Q_0^2 = 1 \text{ GeV}^2$

$$x h_1^{q_v}(x) = \tanh [\sqrt{x} (A_q + B_q x + C_q x^2 + D_q x^3)] [x \text{SB}_q(x) + x \overline{\text{SB}}_{\bar{q}}(x)]$$

rigid



flexible



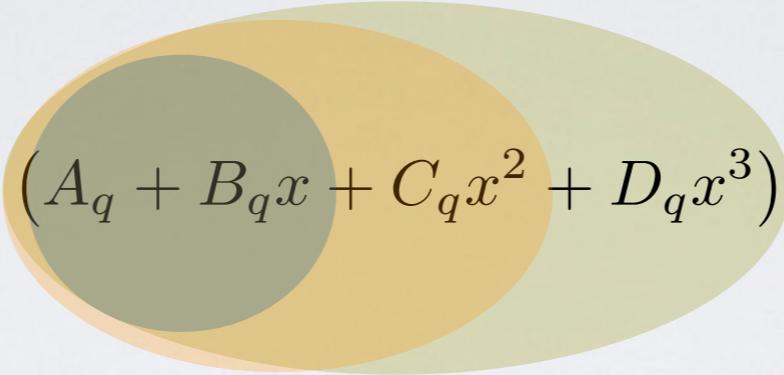
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$$2|h_1^q(x, Q^2)| \leq 2 \text{SB}_q(x) = |f_1^q(x) + g_1^q(x)|$$

# Fit : **functional form** and **method**

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rigid



flexible



extra-flexible



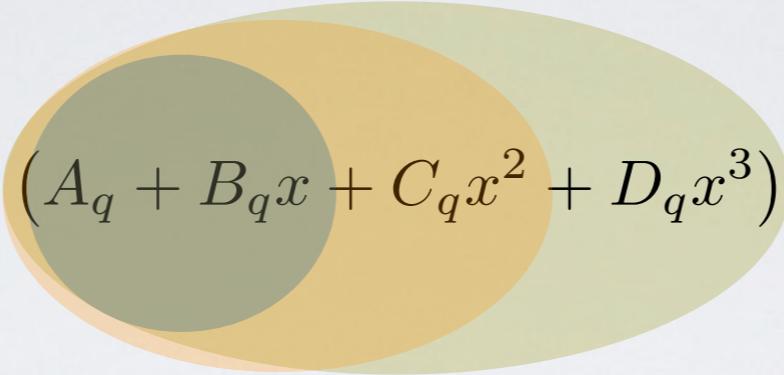
satisfies **Soffer Bound** at any  $Q^2$

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# Fit : **functional form** and **method**

**functional form** at starting scale  $Q_0^2 = 1 \text{ GeV}^2$

$$x h_1^{q_v}(x) = \tanh [\sqrt{x} (A_q + B_q x + C_q x^2 + D_q x^3)] [x \text{SB}_q(x) + x \overline{\text{SB}}_{\bar{q}}(x)]$$



rigid



flexible



extra-flexible



satisfies **Soffer Bound** at any  $Q^2$

$$2|h_1^q(x, Q^2)| \leq 2 \text{SB}_q(x) = |f_1^q(x) + g_1^q(x)|$$

**two methods**

for statistical analysis:

- standard Hessian method
- replica method

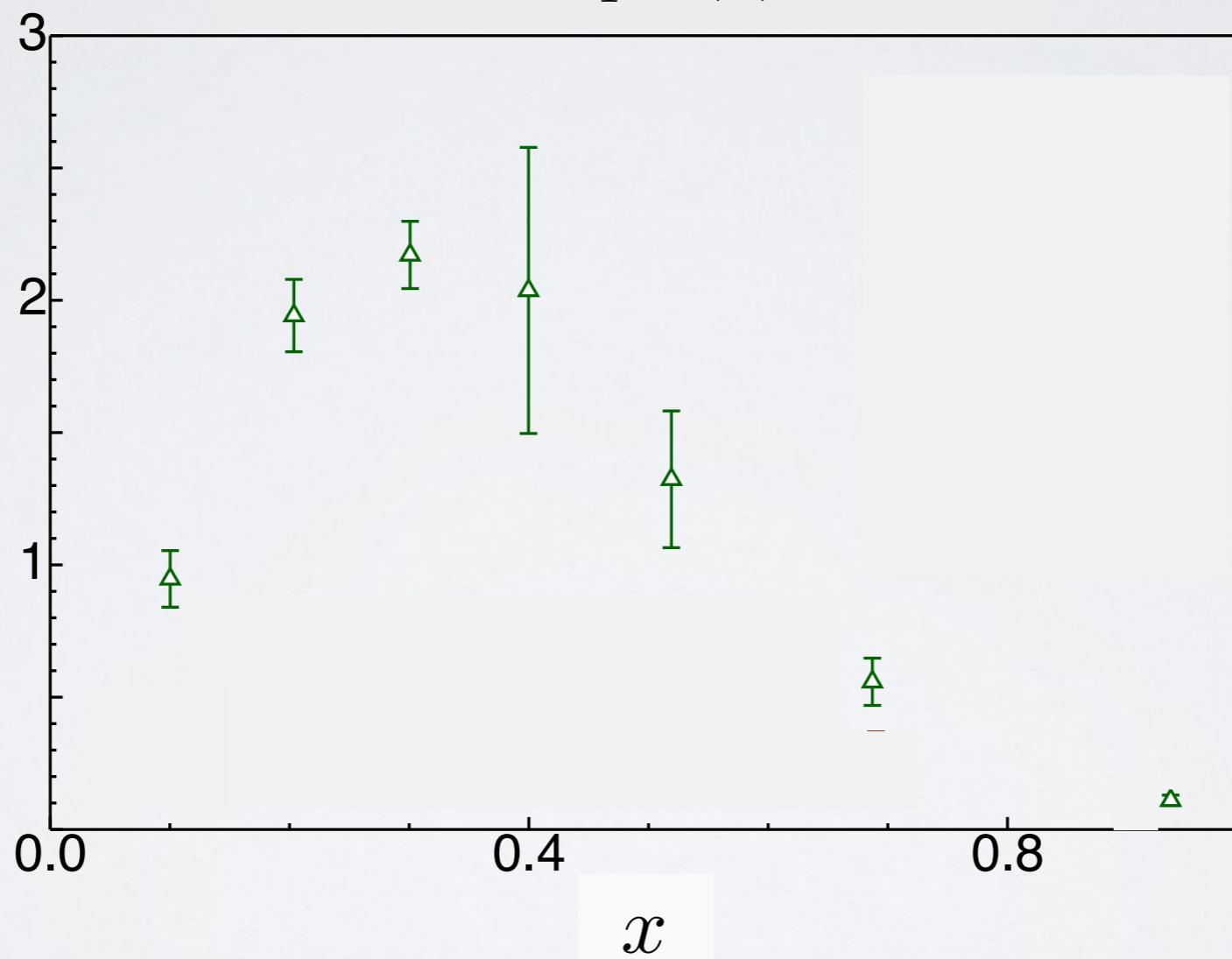
# the replica method

inspired by NNPDF

$$x h_1^{u_v} - \frac{1}{4} x h_1^{d_v} \propto -\frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 H_1^{\triangleleft u}} \cdots$$

$$x h_1^{u-\bar{u}}(x)$$

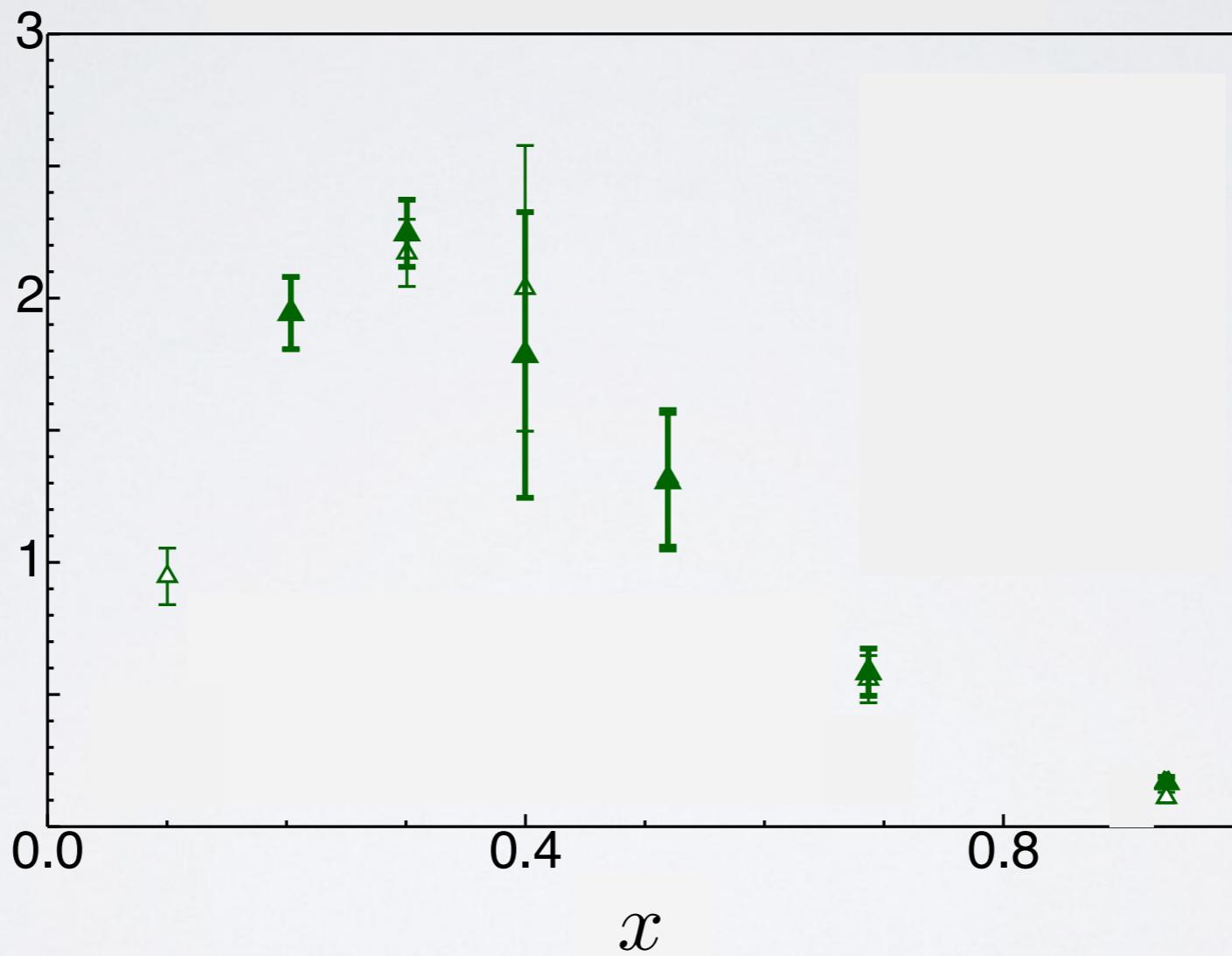
$$x h_1^{u_v} + x h_1^{d_v} \propto -\frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 H_1^{\triangleleft u}} \frac{4}{3} \cdots$$



sample of original data

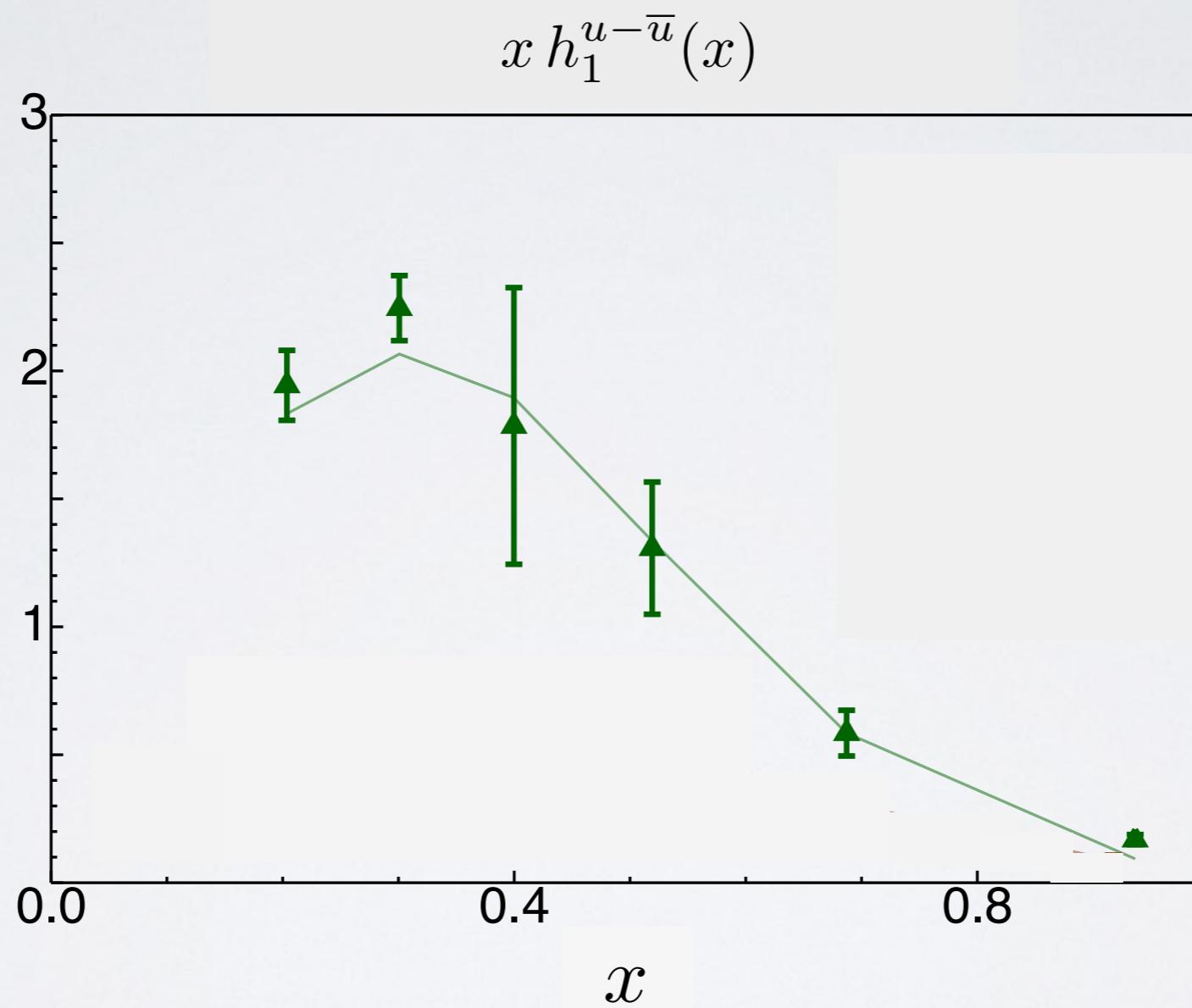
# the replica method

$$x h_1^{u_v} - \frac{1}{4} x h_1^{d_v} \propto -\frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 H_1^{\triangleleft u}} \dots$$
$$x h_1^{u_v} + x h_1^{d_v} \propto -\frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 H_1^{\triangleleft u}} \frac{4}{3} \dots$$



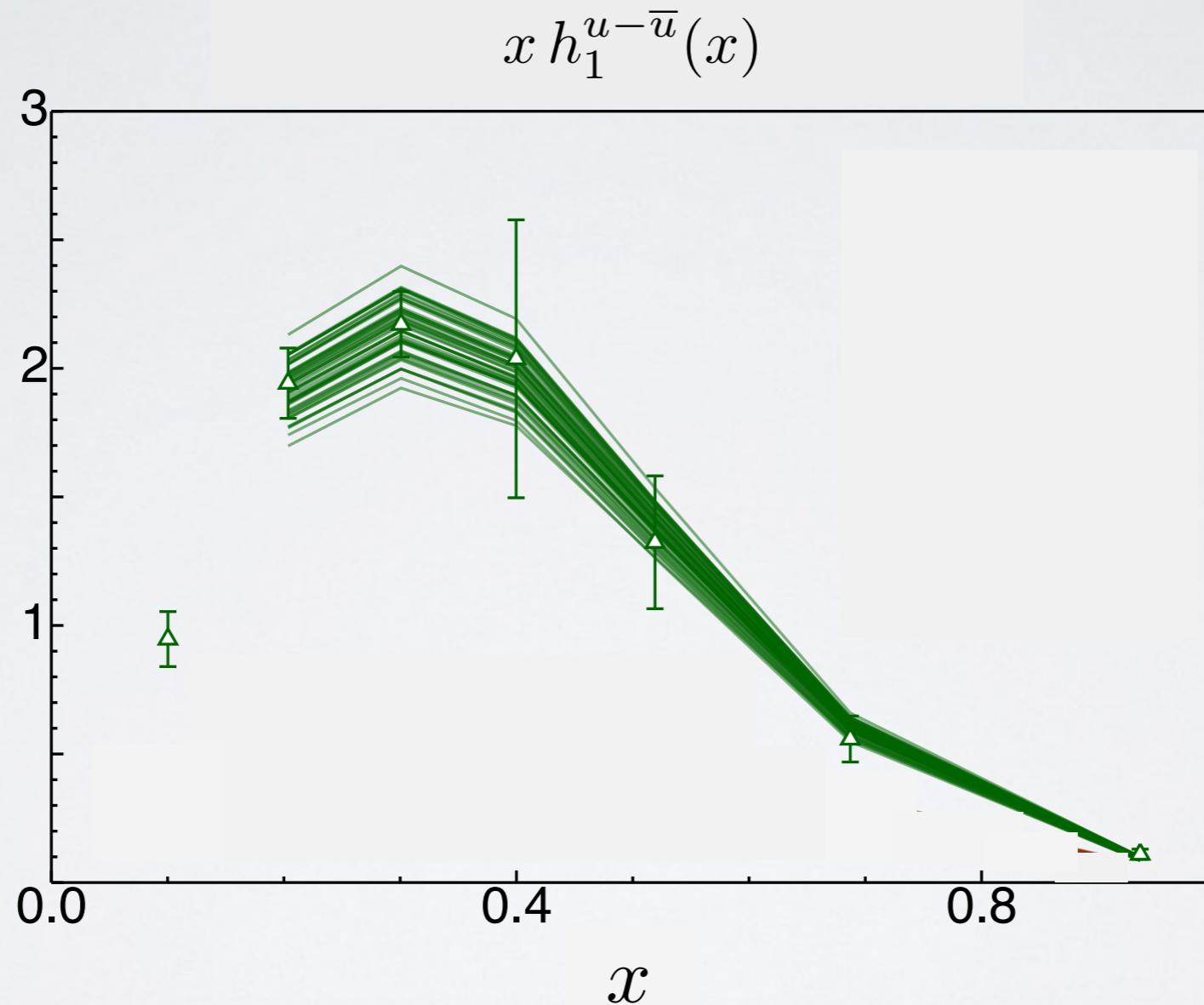
data are replicated with Gaussian noise  
within exp. variance

# the replica method

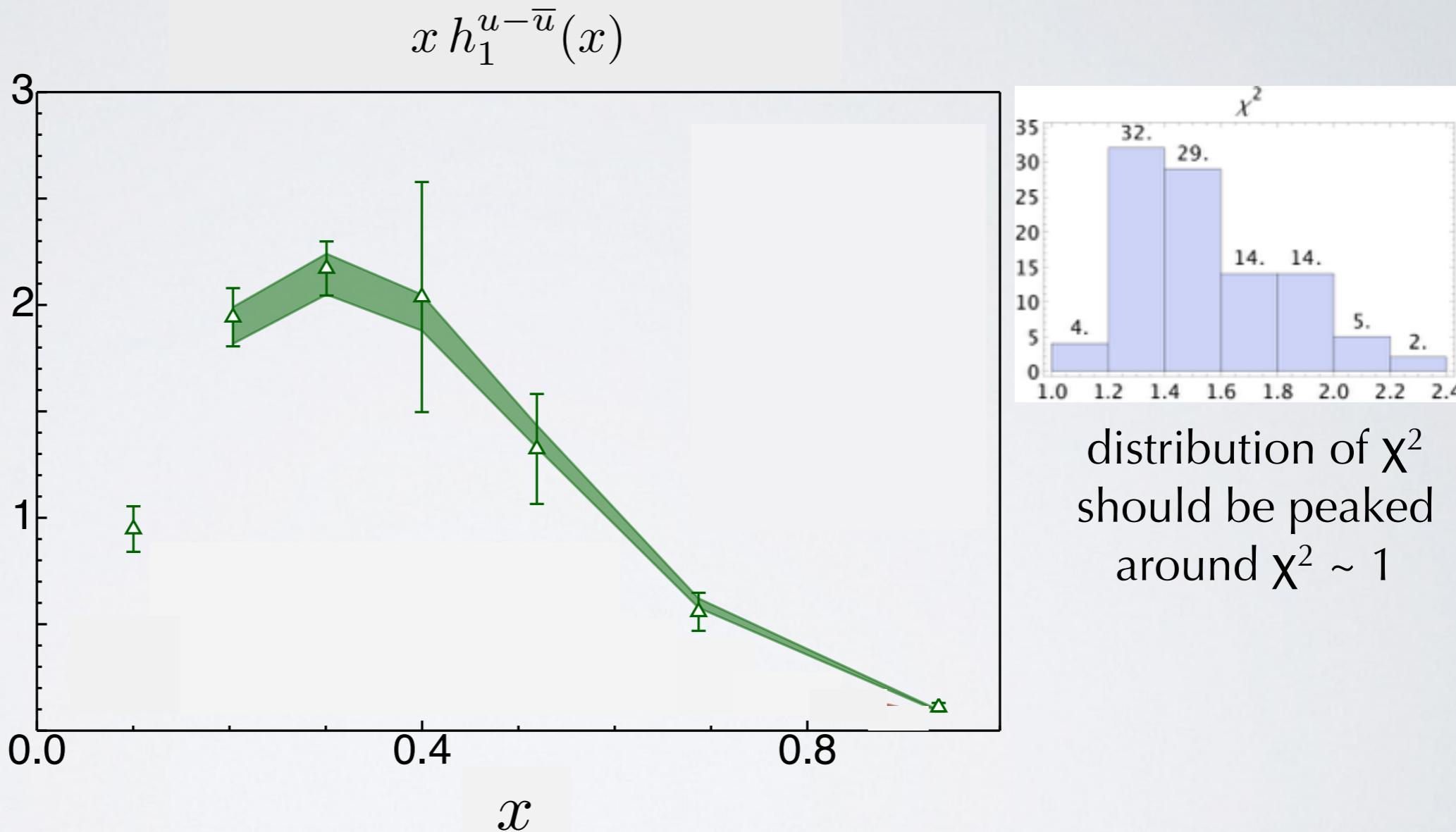


fit the replicated data

# the replica method



# the replica method

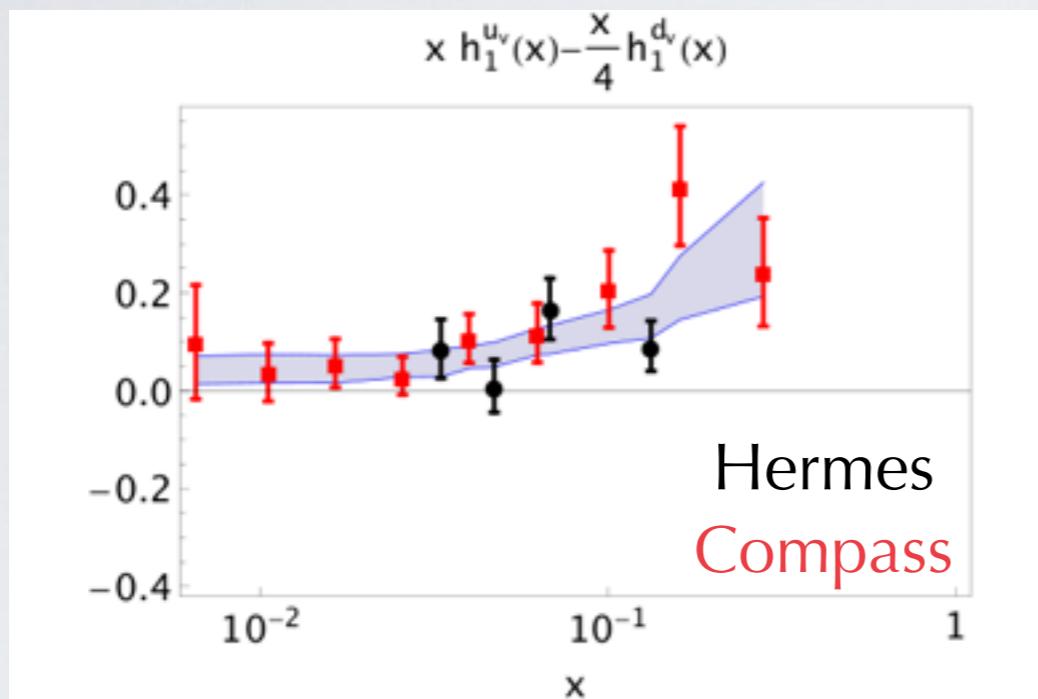


for each point, a central 68% confidence interval is identified  
(distribution is not necessarily Gaussian)

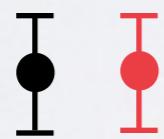
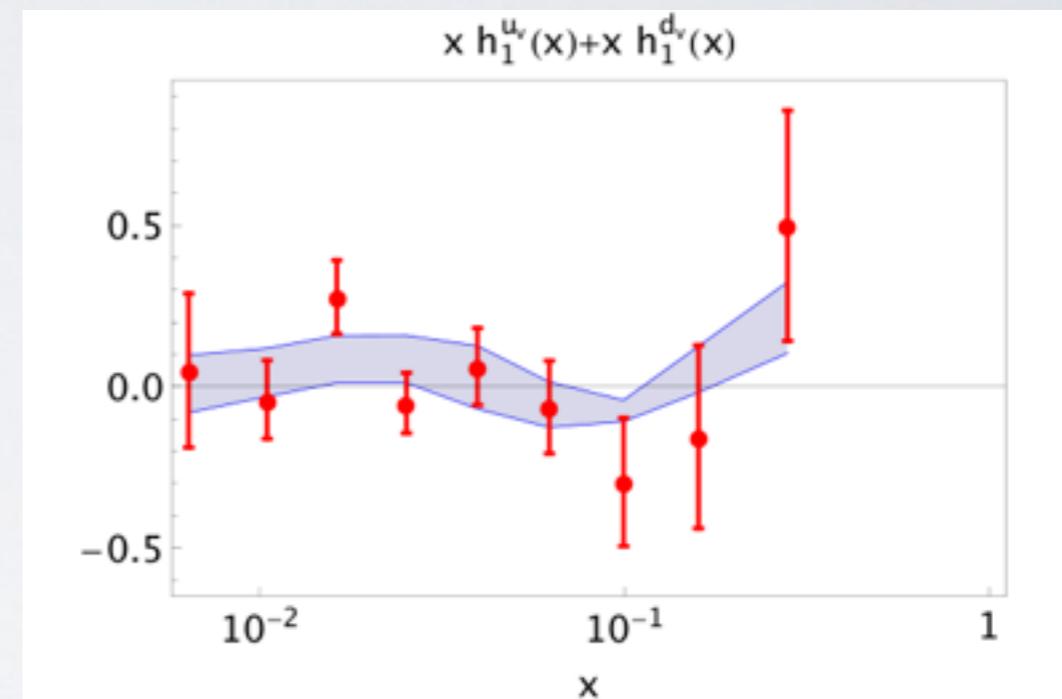
# point-by-point extraction and fit

Bacchetta, Courtoy, Radici, JHEP **1303** (13) 119

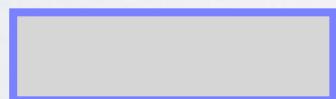
proton data  $\times h_1^p(x)$



deuteron data  $\times h_1^D(x)$



extraction point by point



fit with the replica method  
(and flexible fitting form)



globally  
 $\chi^2/\text{d.o.f.} \sim 1.2$

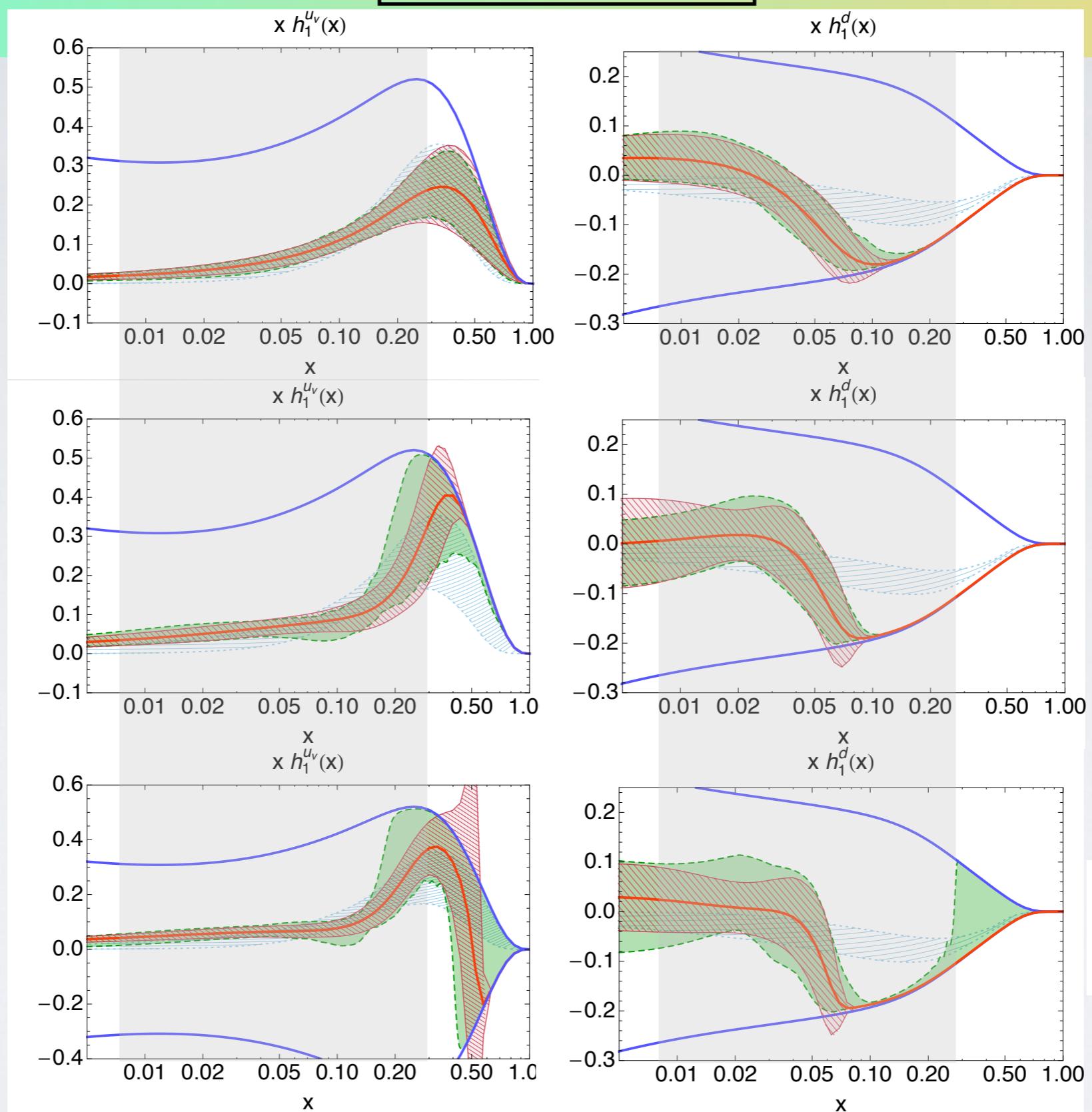
$Q^2 = 2.4 \text{ GeV}^2$

$u - \bar{u}$

$\times h_1^{q-\bar{q}}(x)$

$d - \bar{d}$

Bacchetta, Courtoy, Radici,  
JHEP 1303 (13) 119



rigid



flexible



extra  
flexible

$Q^2 = 2.4 \text{ GeV}^2$

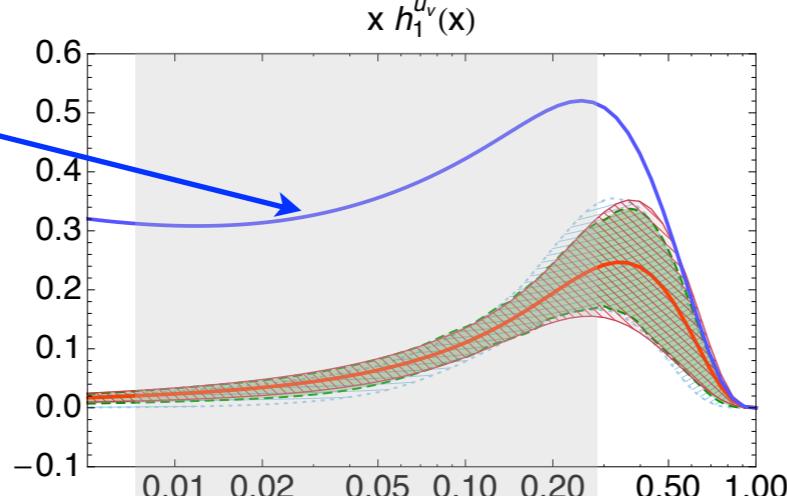
$u - \bar{u}$

$\times h_1^{q-\bar{q}}(x)$

$d - \bar{d}$

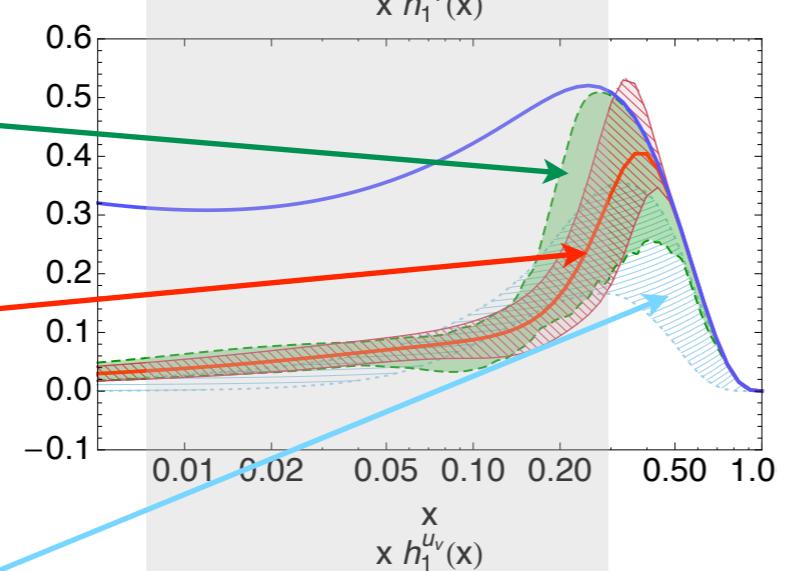
Bacchetta, Courtoy, Radici,  
JHEP 1303 (13) 119

Soffer bound

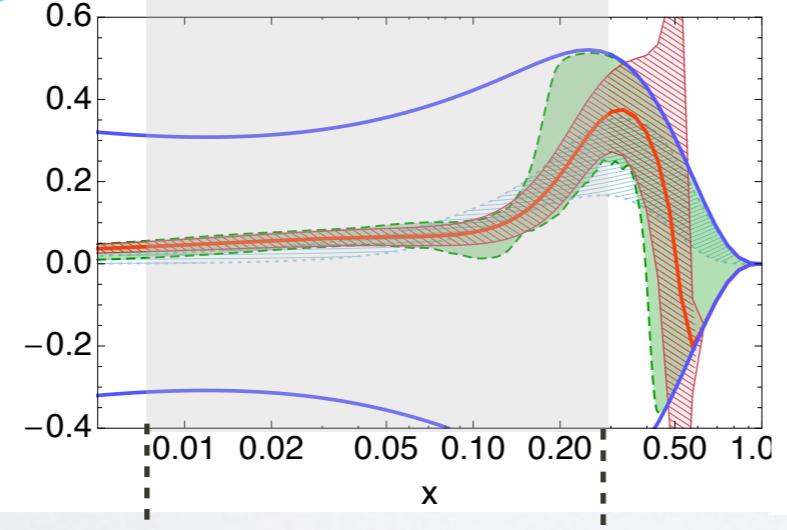


68% band of  
replicas

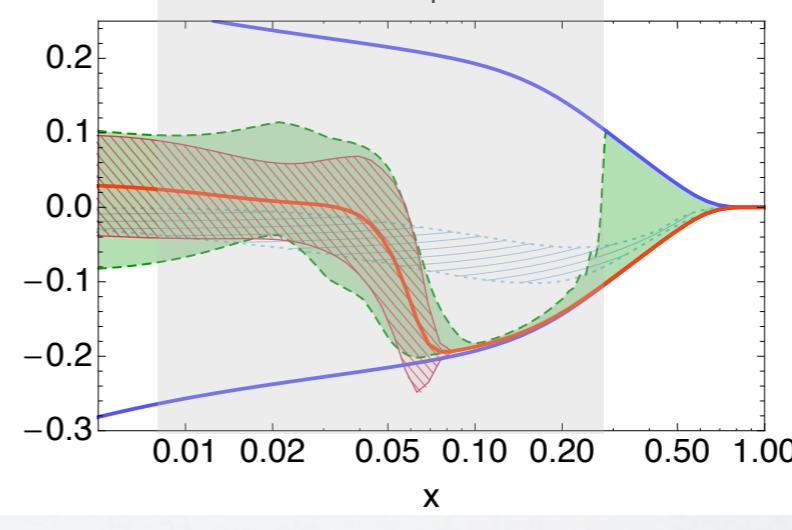
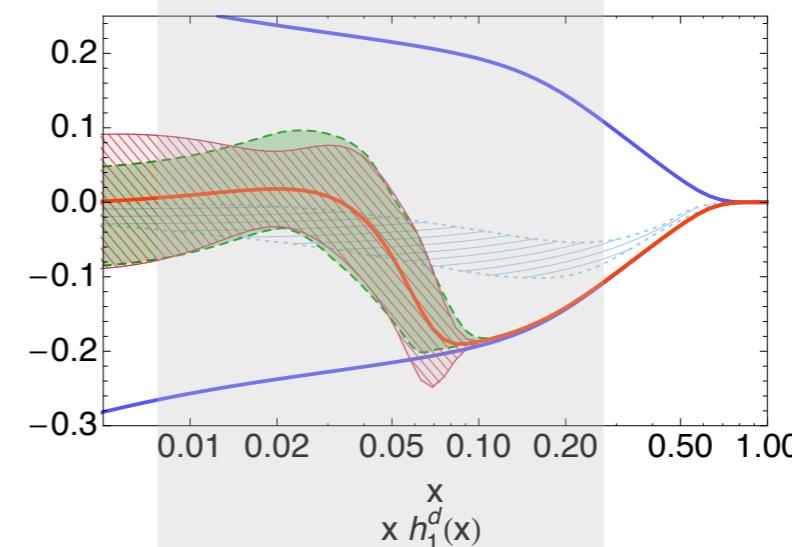
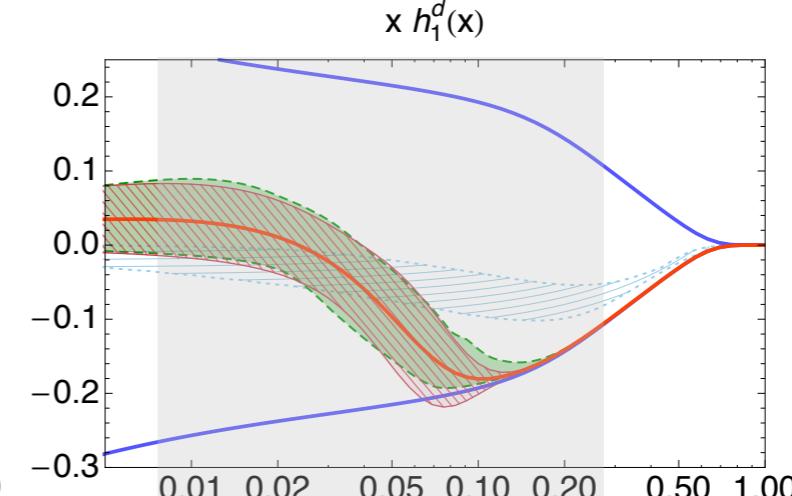
central value  
for standard fit  
with 1 $\sigma$  band



Torino 2009



← data →



rigid



flexible



extra  
flexible

$Q^2 = 2.4 \text{ GeV}^2$

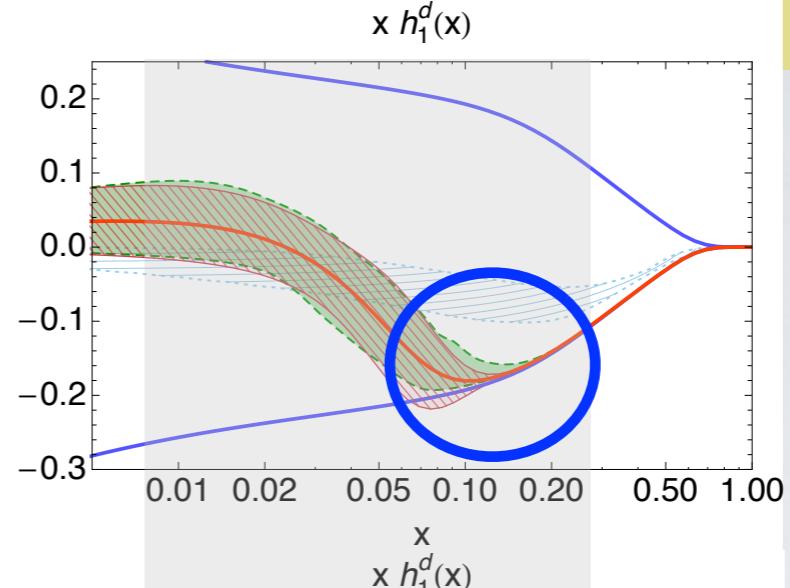
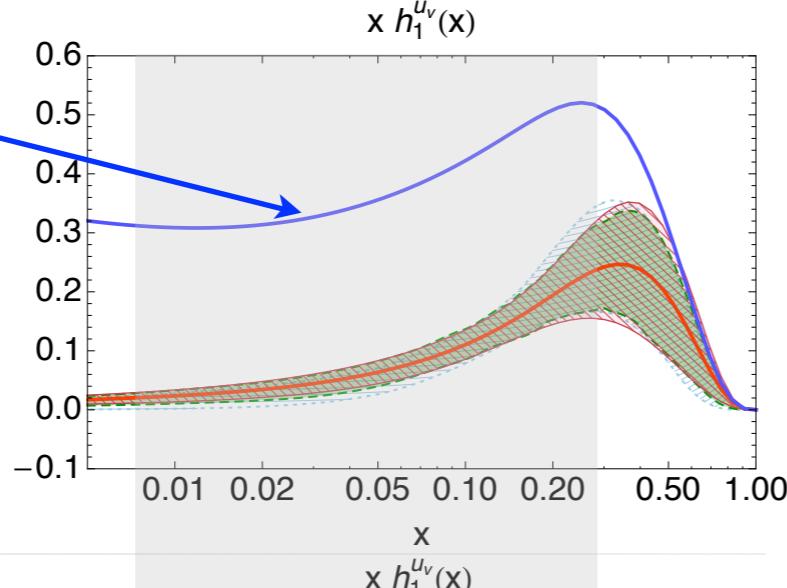
$u - \bar{u}$

$\times h_1^{q-\bar{q}}(x)$

$d - \bar{d}$

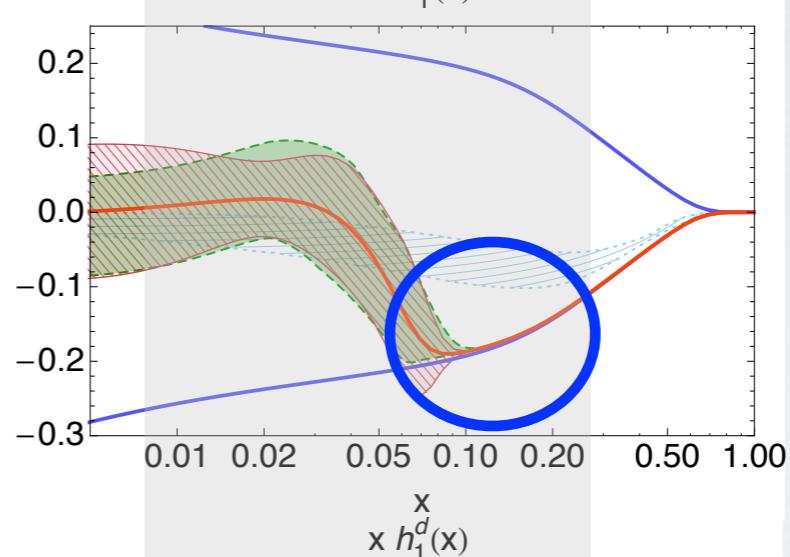
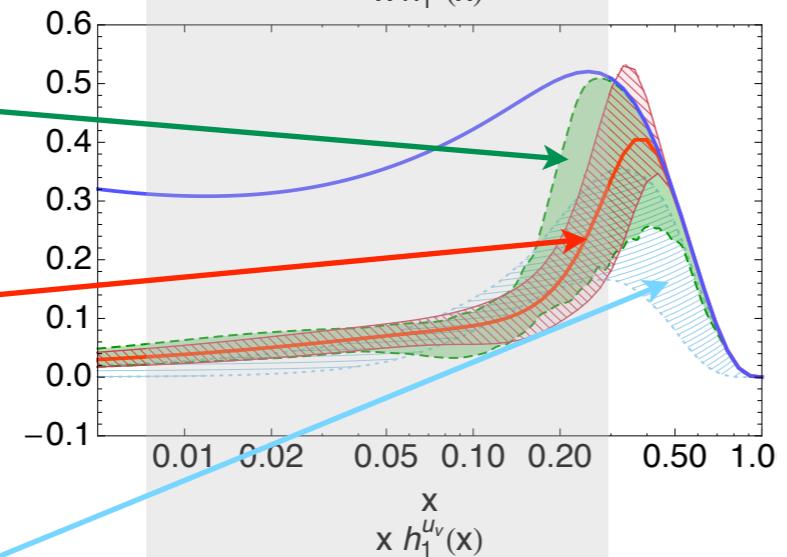
Bacchetta, Courtoy, Radici,  
JHEP 1303 (13) 119

Soffer bound

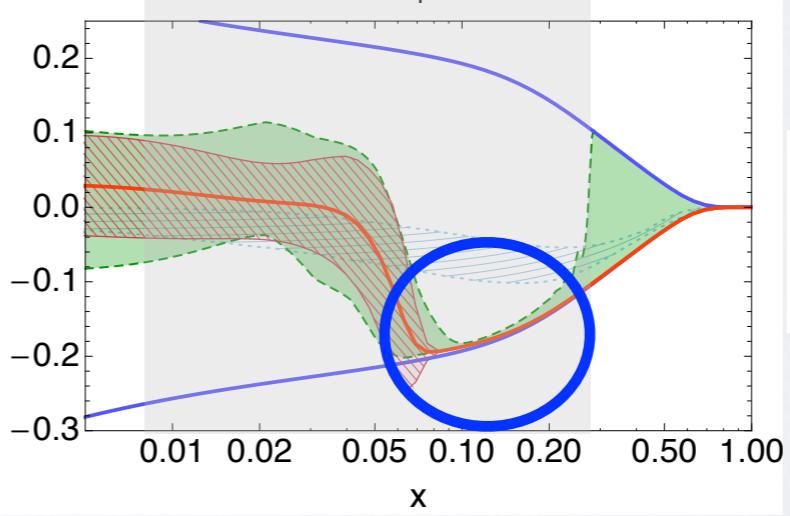
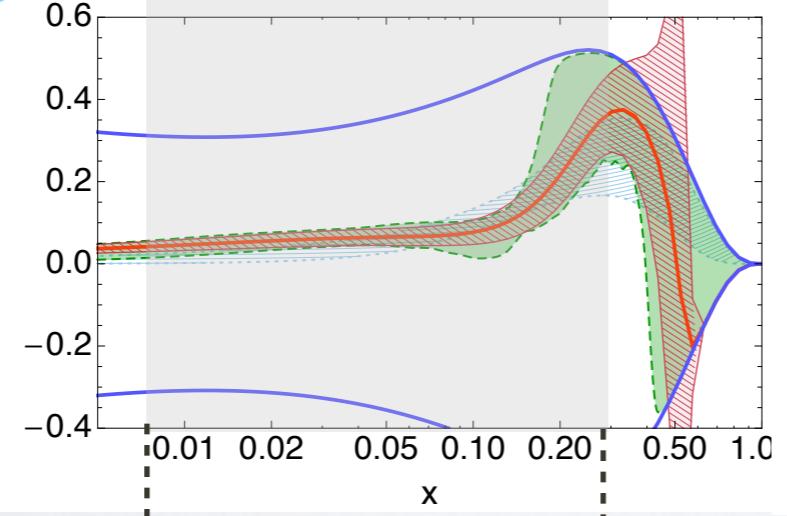


68% band of  
replicas

central value  
for standard fit  
with 1 $\sigma$  band



Torino 2009



← data →

driven by COMPASS deuteron data



rigid



flexible



extra  
flexible

new fit

1. add new  2010 proton data for  $\pi^+\pi^-$

*C.Adolph et al., P.L. **B736** (14) 124  
arXiv:1401.7873  
and  
C. Braun (Compass), PoS (DIS2014) 203*

2. use replica method to re-extract **DiFF** from  data

3. use 2 different values of  $\alpha_S(M_{Z^0}^2)$  in evolution eq.'s  
from  scale to   scales

new fit

1. add new  2010 proton data for  $\pi^+\pi^-$

*C.Adolph et al., P.L. **B736** (14) 124  
arXiv:1401.7873*

*and*

*C. Braun (Compass), PoS (DIS2014) 203*

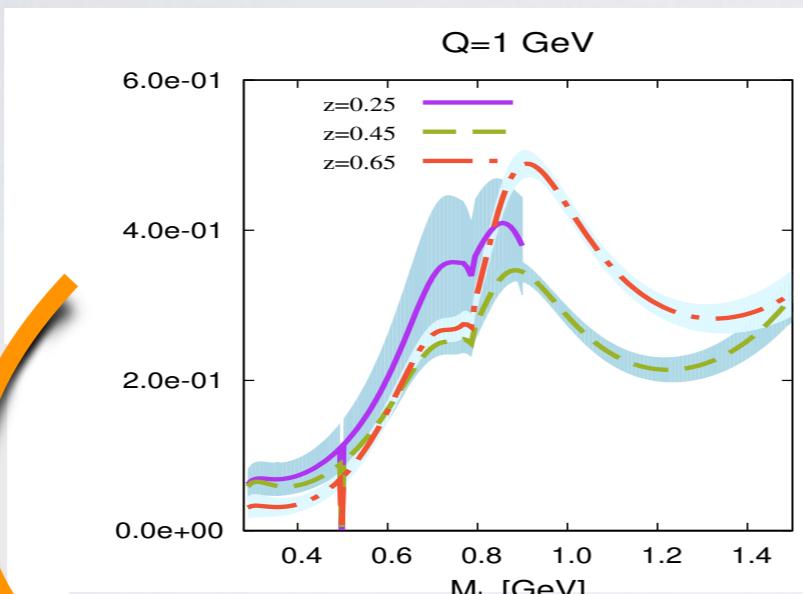
2. use replica method to re-extract **DiFF** from  data

3. use 2 different values of  $\alpha_S(M_{Z^0}^2)$  in evolution eq.'s  
from  scale to   scales

current most realistic estimate of  
uncertainty on transversity

# re-fit $H_1 \not\rightarrow q \rightarrow \pi^+\pi^-$ using replica method

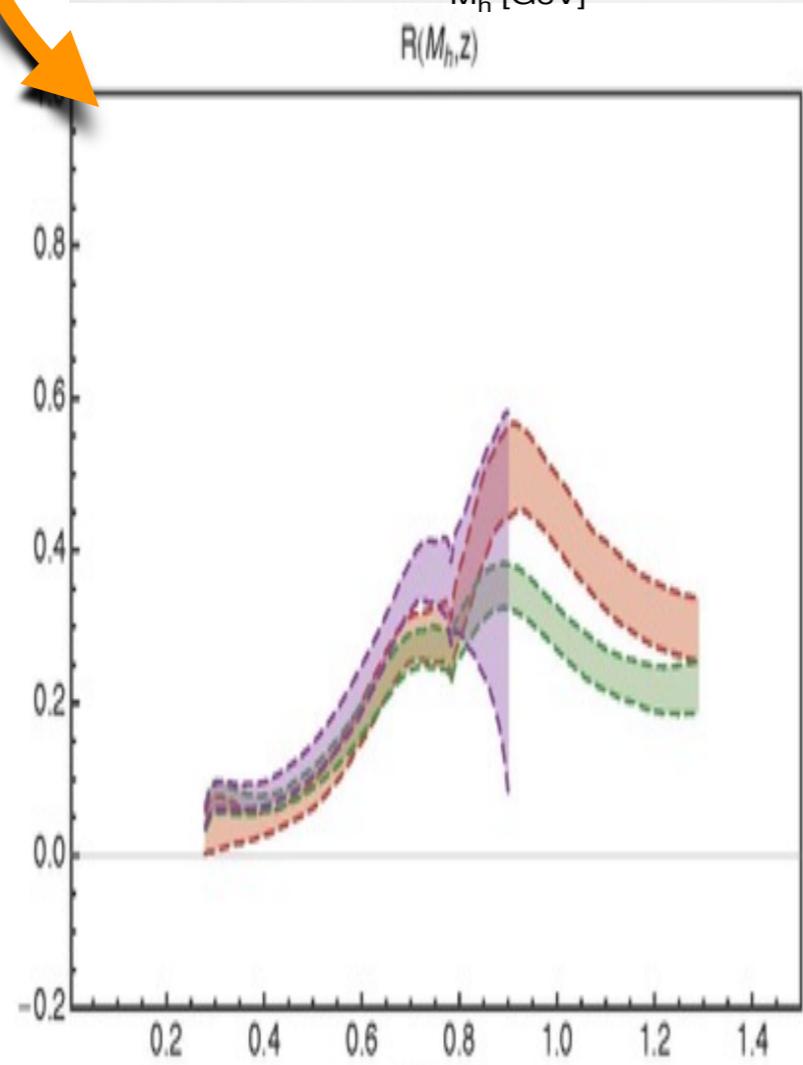
$M_h$  behaviour



$z=0.25$

$z=0.45, M_h$

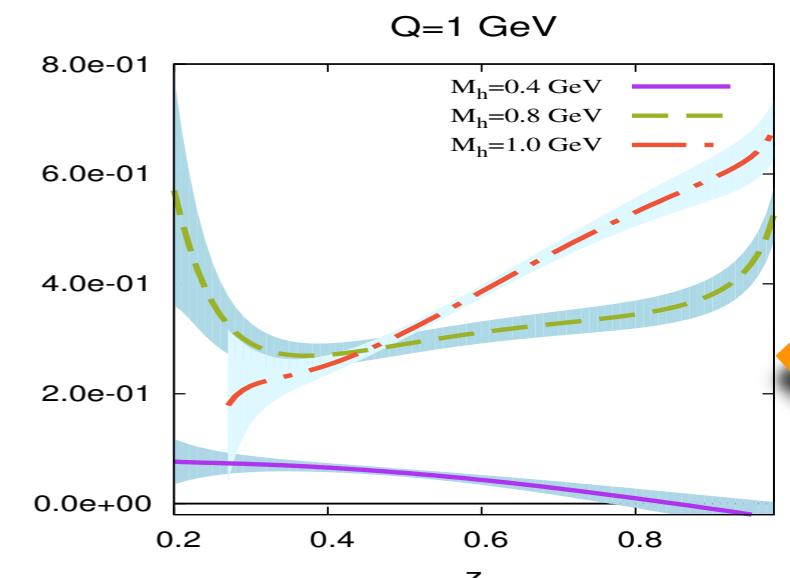
$z=0.65$



$u \rightarrow \pi^+ \pi^-$

$$\frac{|\mathbf{R}|}{M_h} \frac{H_1^{\not\rightarrow u}}{D_1^u}$$

$z$  behaviour



$M_h = 0.4$

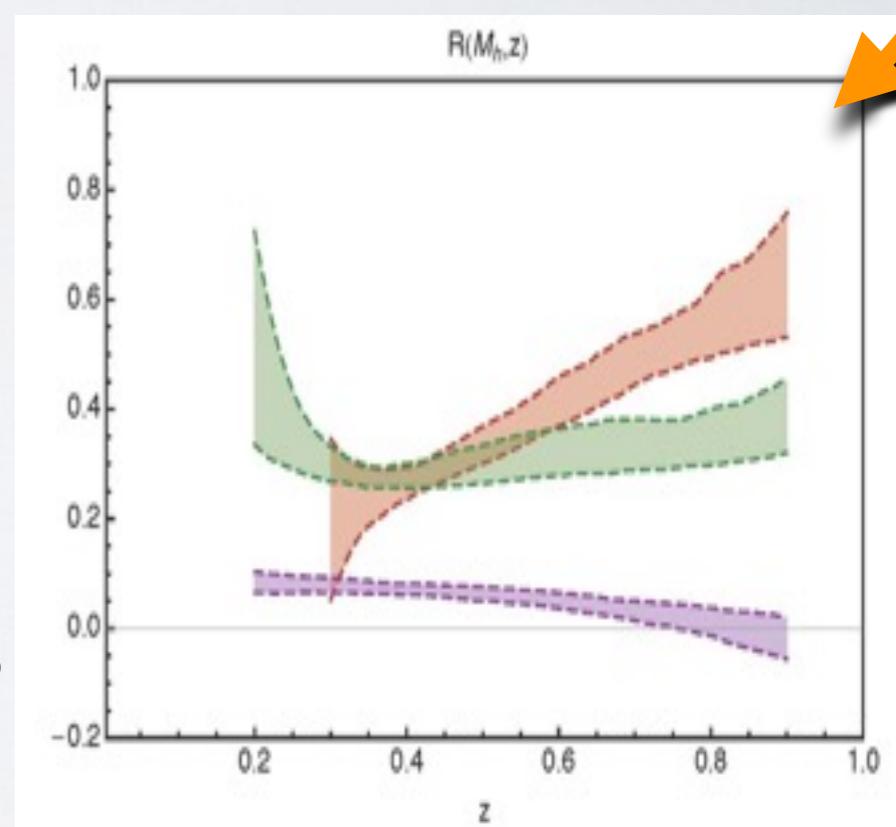
$M_h = 0.8, z$

$M_h = 1.$

$$Q_0^2 = 1 \text{ GeV}^2$$

$$\alpha_S(M_Z^2) = 0.125$$

(GRV98)



# impact on transversity extraction

Ex: proton data

$$\begin{aligned} x h_1^p(x) &\equiv x h_1^{u_v}(x) - \frac{1}{4} x h_1^{d_v}(x) \\ &\propto -\frac{A_{UT}^{\sin(\phi_R + \phi_S)}}{\int dz dM_h^2 \textcolor{red}{H}_1^{\leftarrow u}} \left[ \sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_1^{q+\bar{q}}(x) \int dz dM_h^2 D_1^q \right] \end{aligned}$$

# impact on transversity extraction

Ex: proton data

$$xh_1^p(x) \equiv xh_1^{u_v}(x) - \frac{1}{4}xh_1^{d_v}(x)$$

★ more precise  
data points

★ more realistic error  
on fit of  $H_1^{*u}$

$$\propto -\frac{A_{UT}^{\sin(\phi_R+\phi_S)}}{\int dz dM_h^2 H_1^{*u}} \left[ \sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_1^{q+\bar{q}}(x) \int dz dM_h^2 D_1^q \right]$$

★ change  $\alpha_s$   
→ uncertainty on  
evolved  $D_1$ ,  $H_1^{*u}$

replica method: alter  $A_{UT}$  data with Gaussian noise and randomly pick up corresponding  $H_1^{*u}$

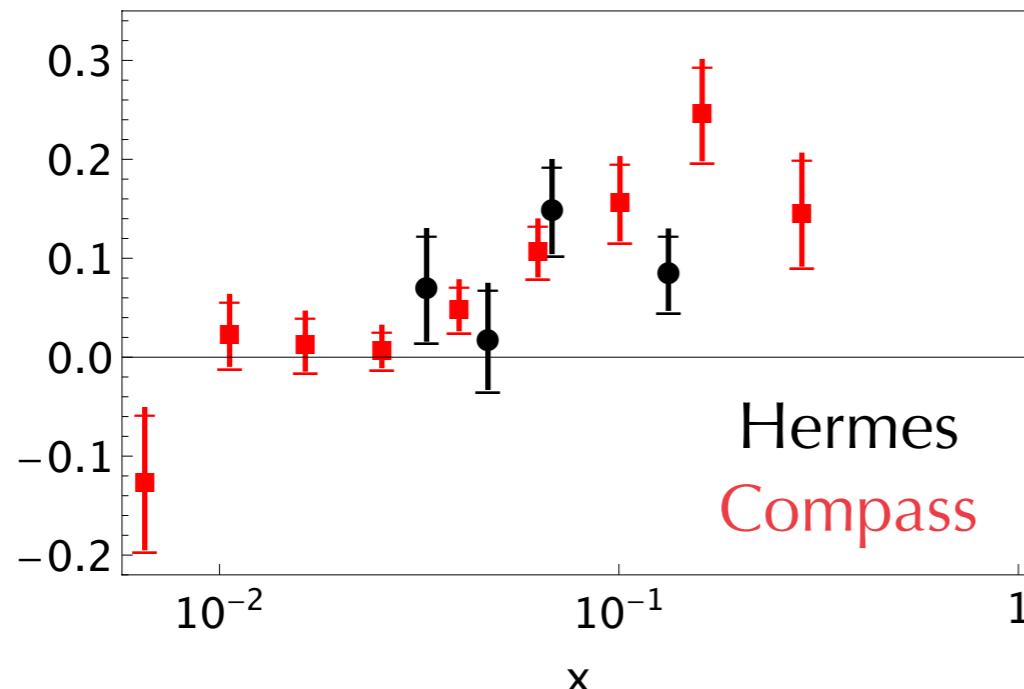
results of new fit

$$\alpha_s(M_{Z^0}^2) = 0.125 \text{ (GRV98)}$$

point-by-point extraction

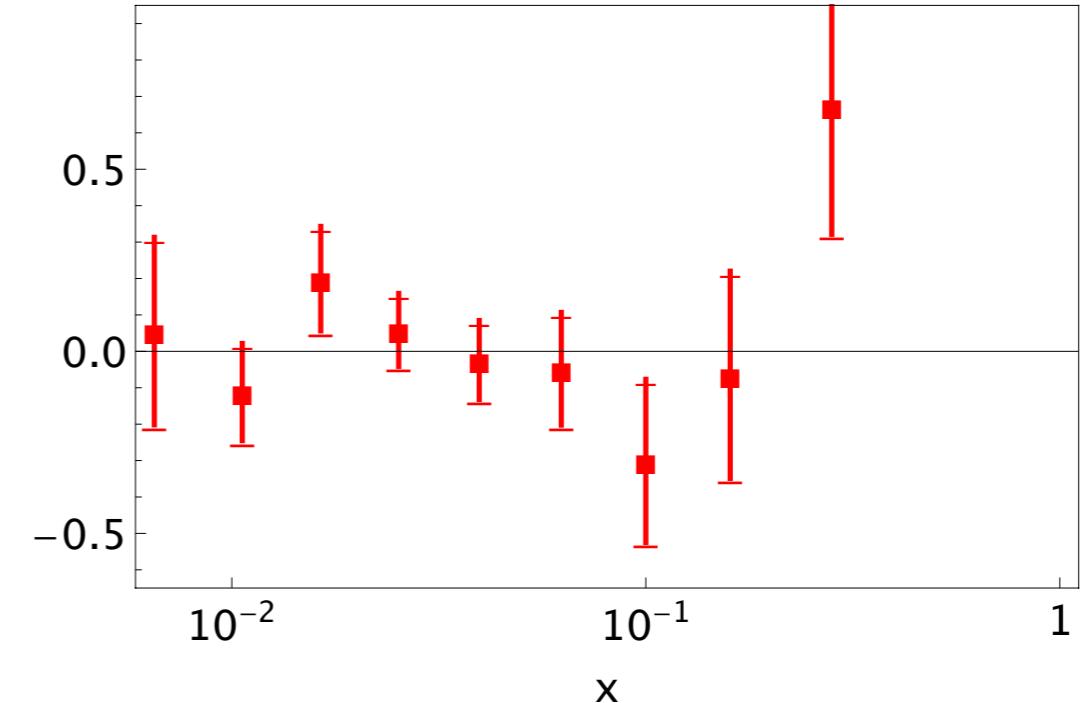
proton

$$x h_1^{u_v}(x) - \frac{x}{4} h_1^{d_v}(x)$$



deuteron

$$x h_1^{u_v}(x) + x h_1^{d_v}(x)$$

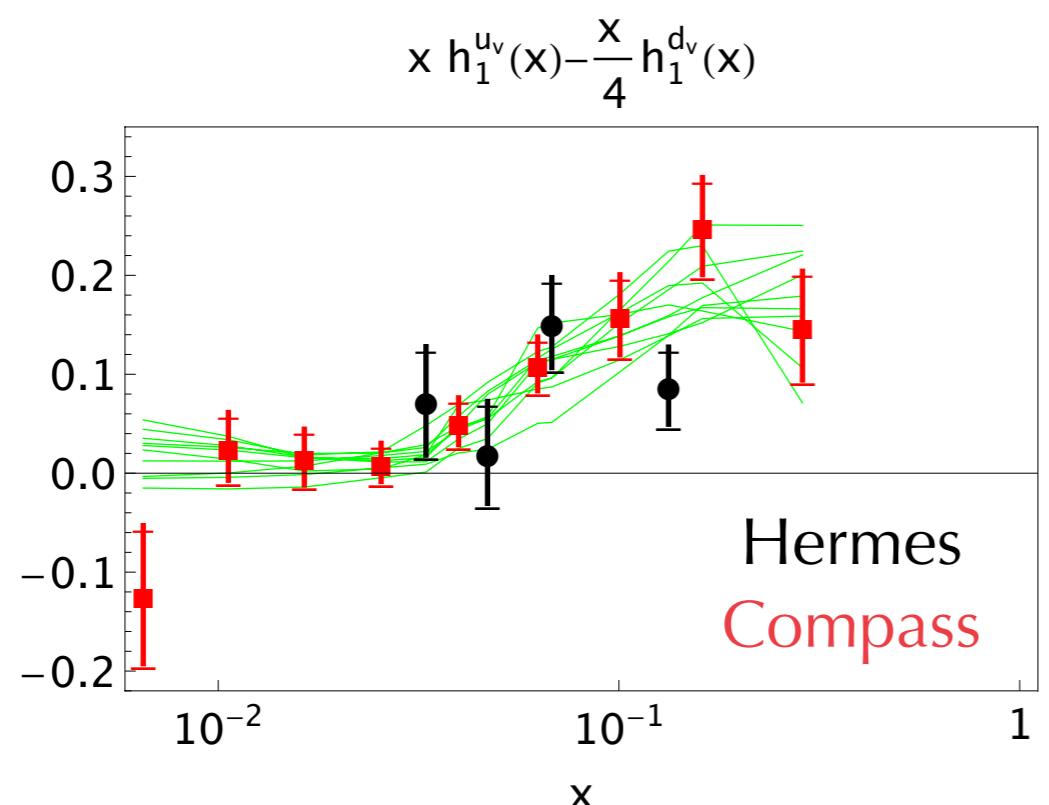


results of new fit

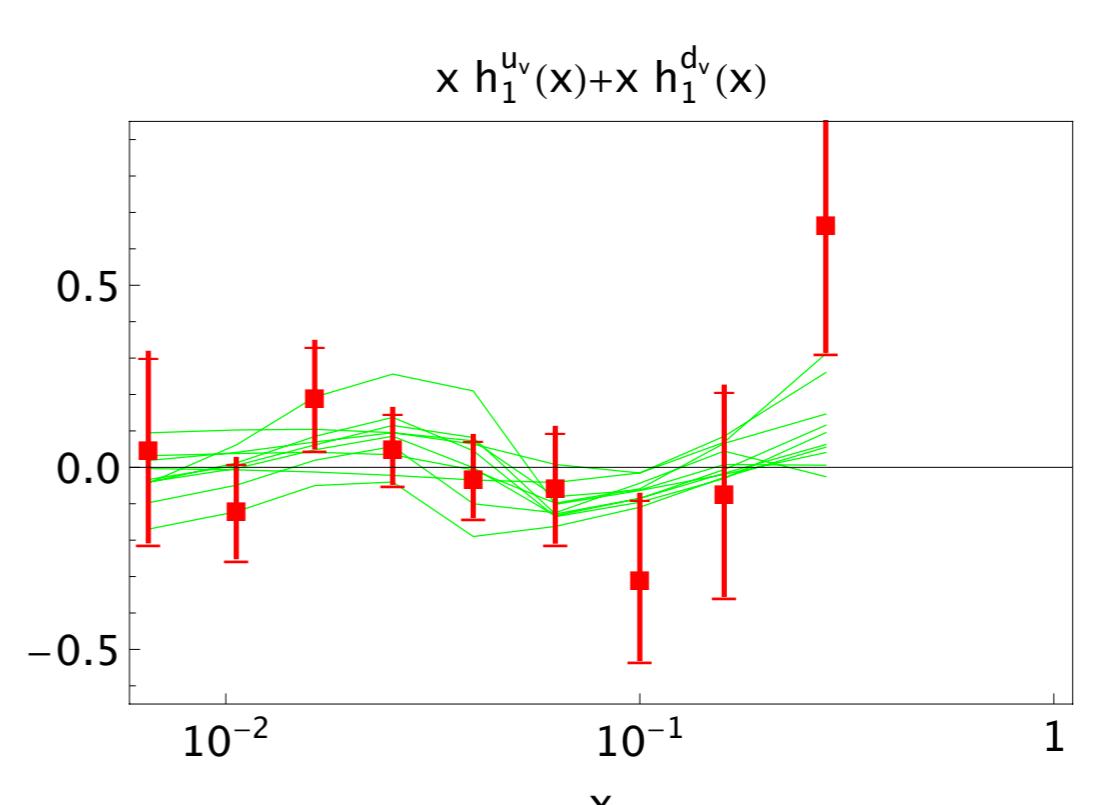
$$\alpha_s(M_{Z^0}^2) = 0.125 \text{ (GRV98)}$$

fit with 10 replica

proton



deuteron



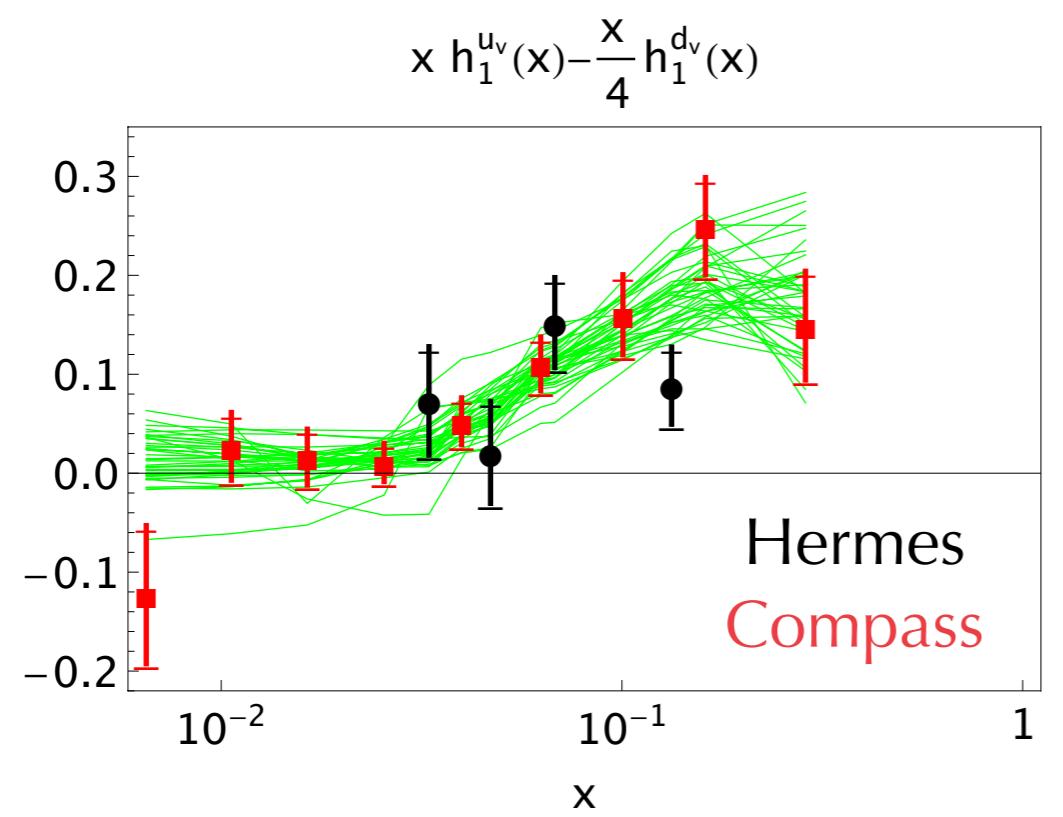
flexible

results of new fit

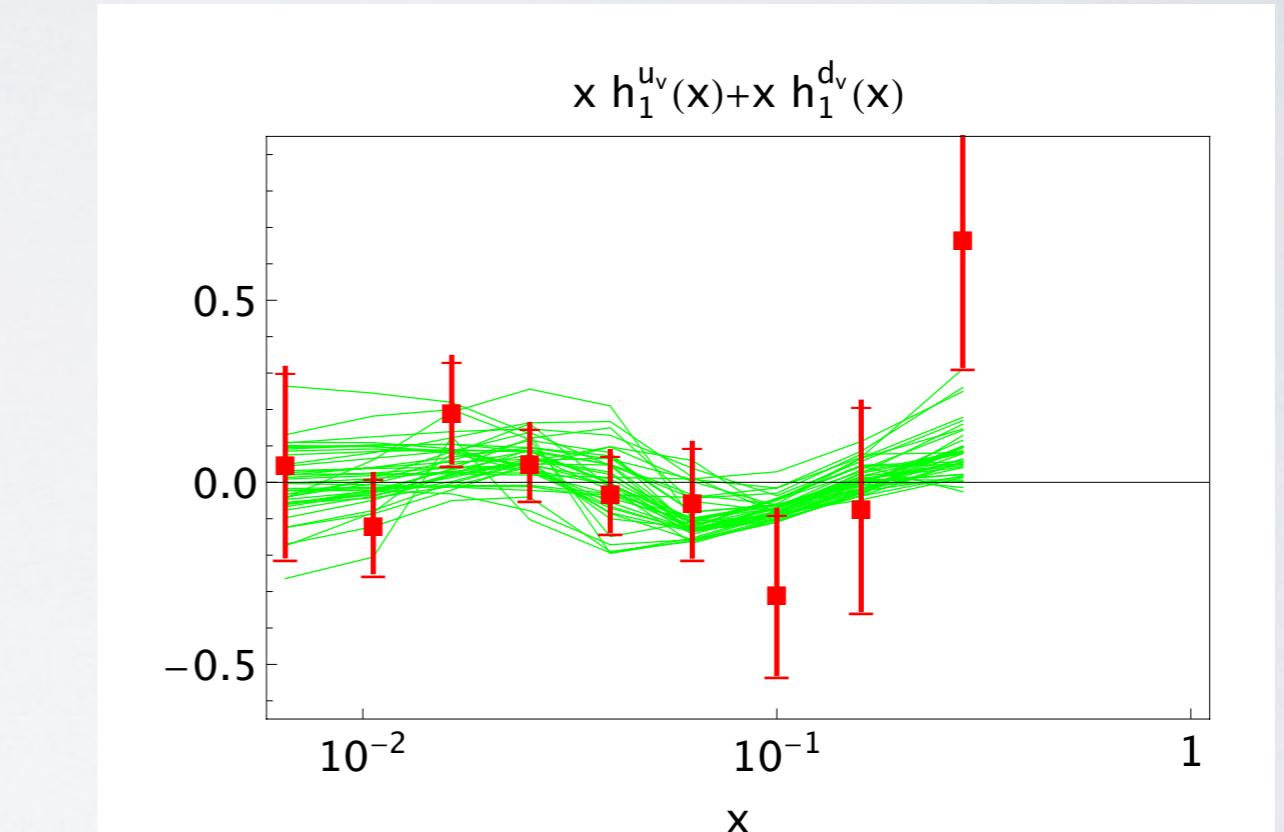
$$\alpha_s(M_{Z^0}^2) = 0.125 \text{ (GRV98)}$$

fit with **40** replica

proton



deuteron



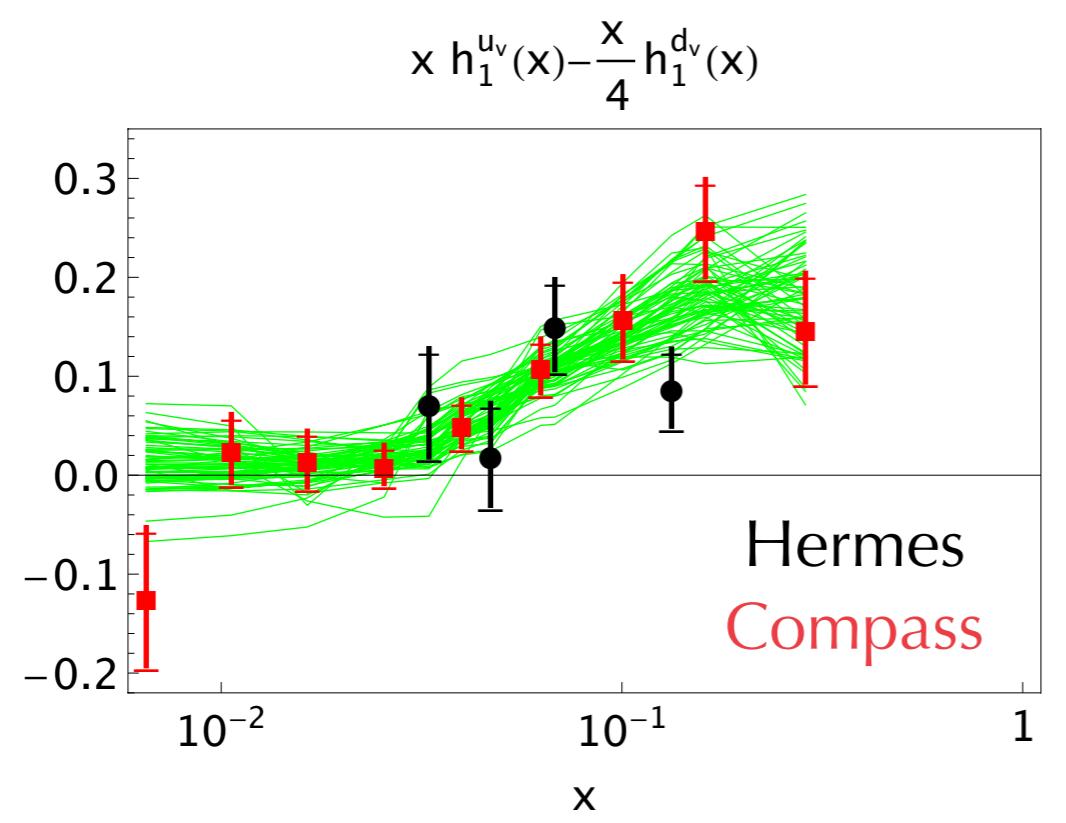
flexible

results of new fit

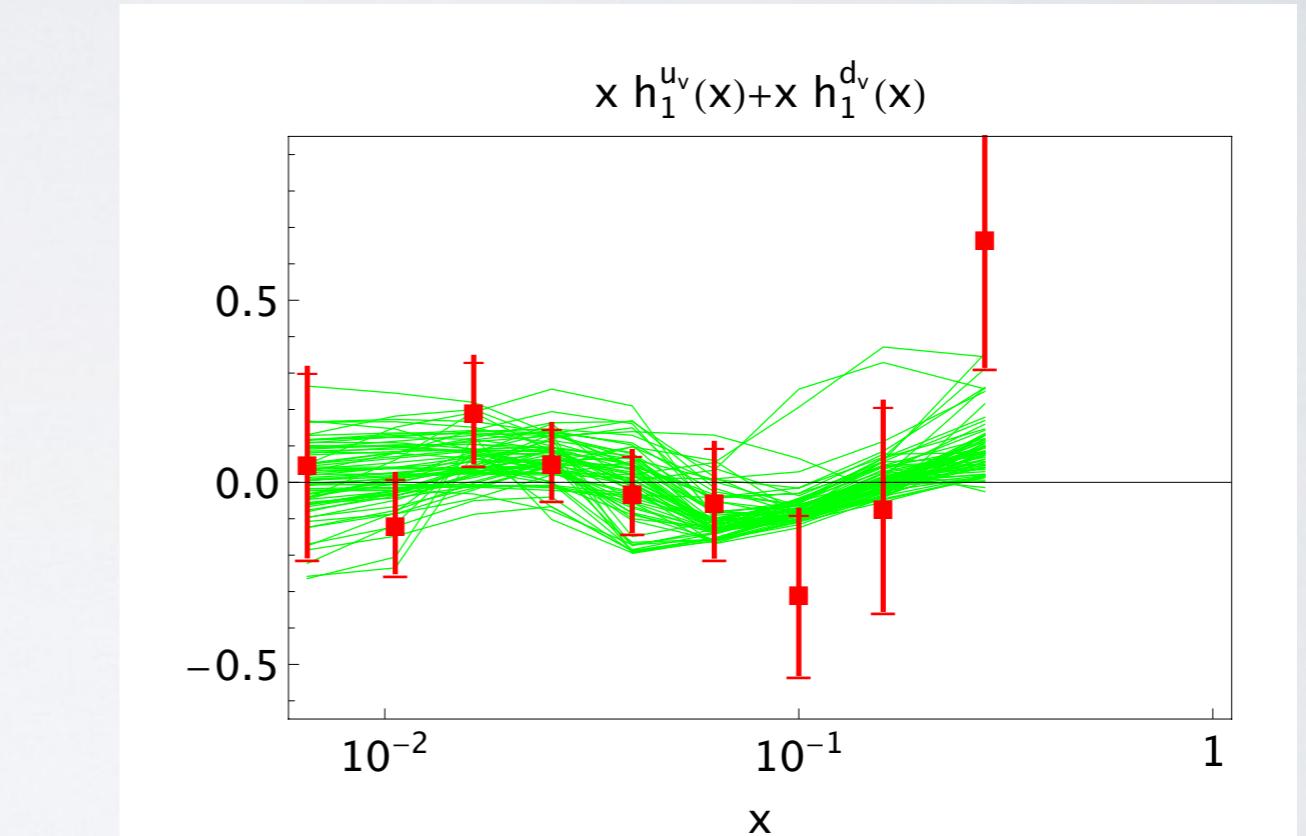
$$\alpha_s(M_{Z^0}^2) = 0.125 \text{ (GRV98)}$$

fit with 70 replica

proton



deuteron



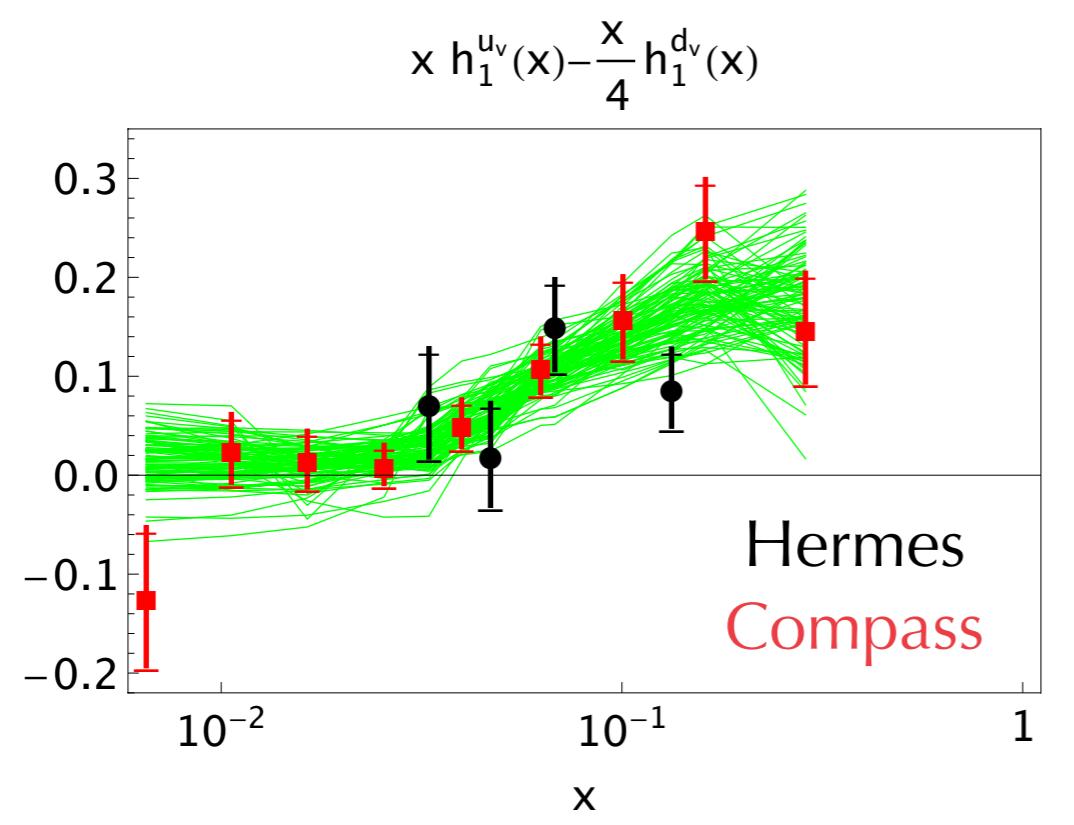
flexible

results of new fit

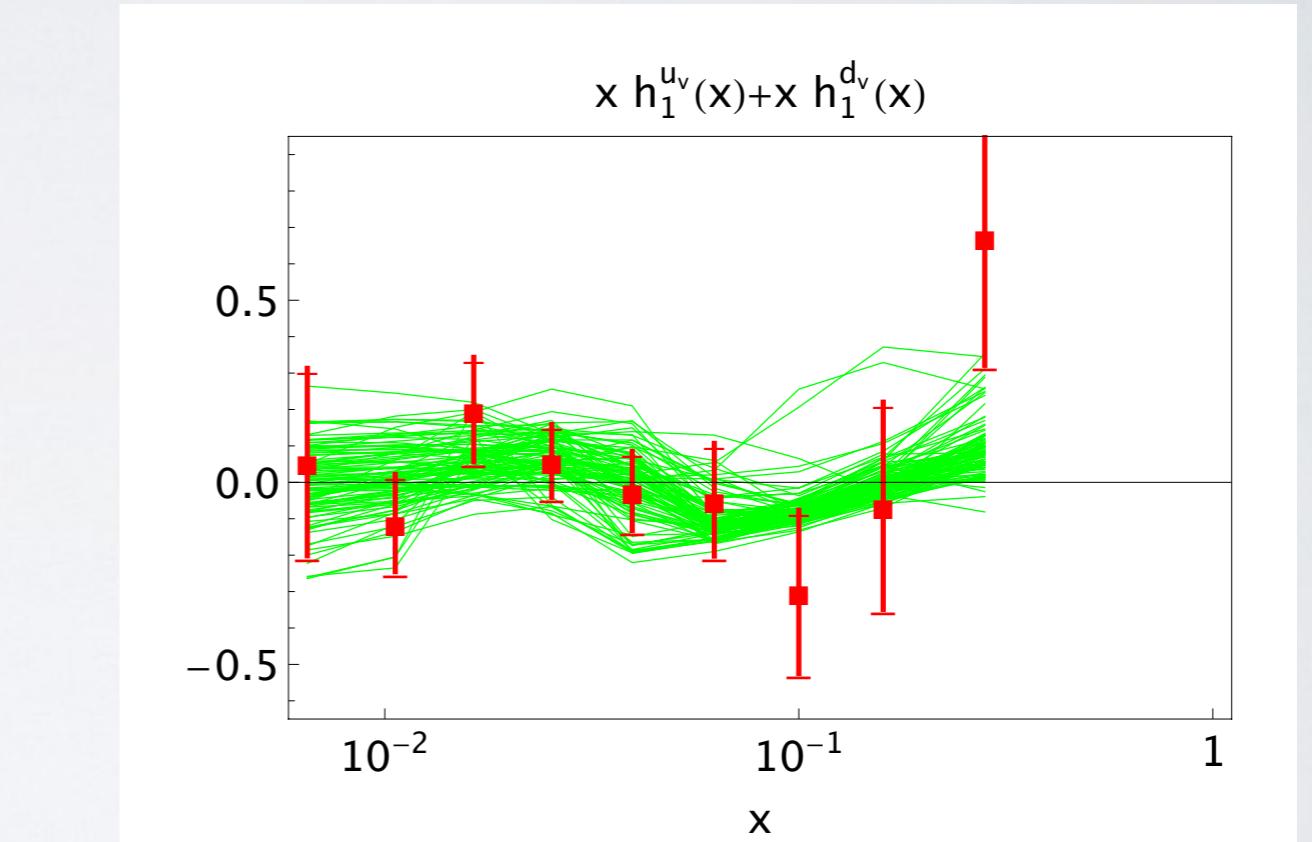
$$\alpha_s(M_{Z^0}^2) = 0.125 \text{ (GRV98)}$$

fit with **100** replica

proton



deuteron



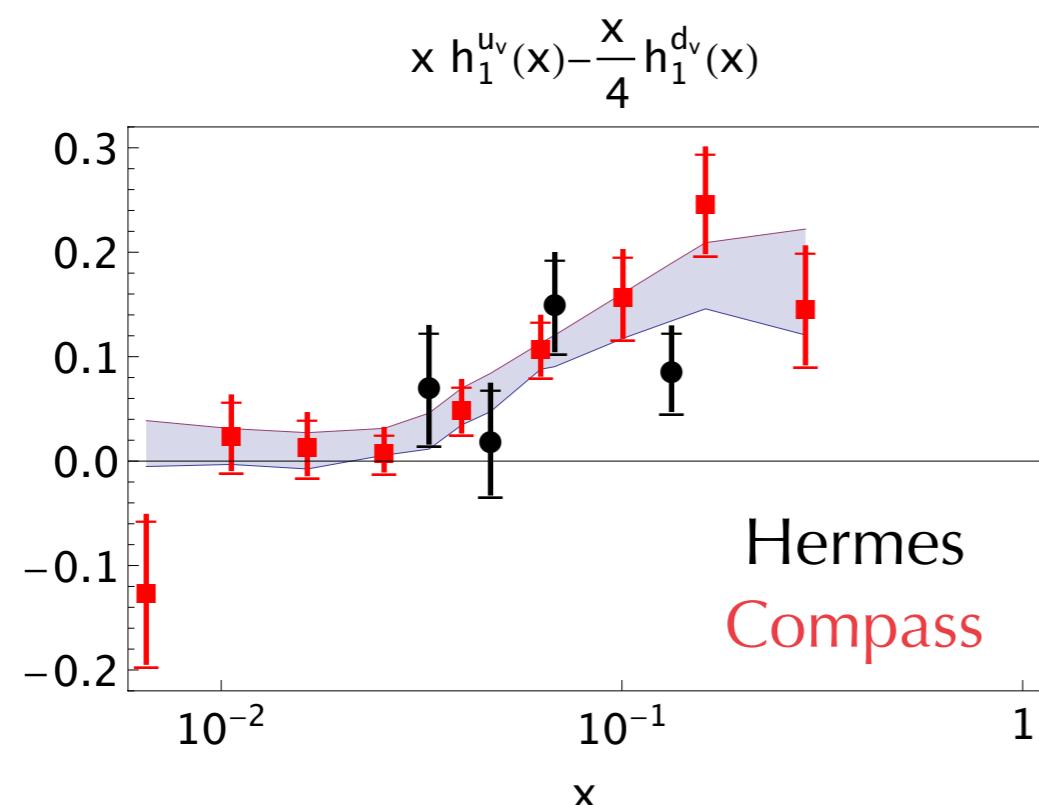
flexible

results of new fit

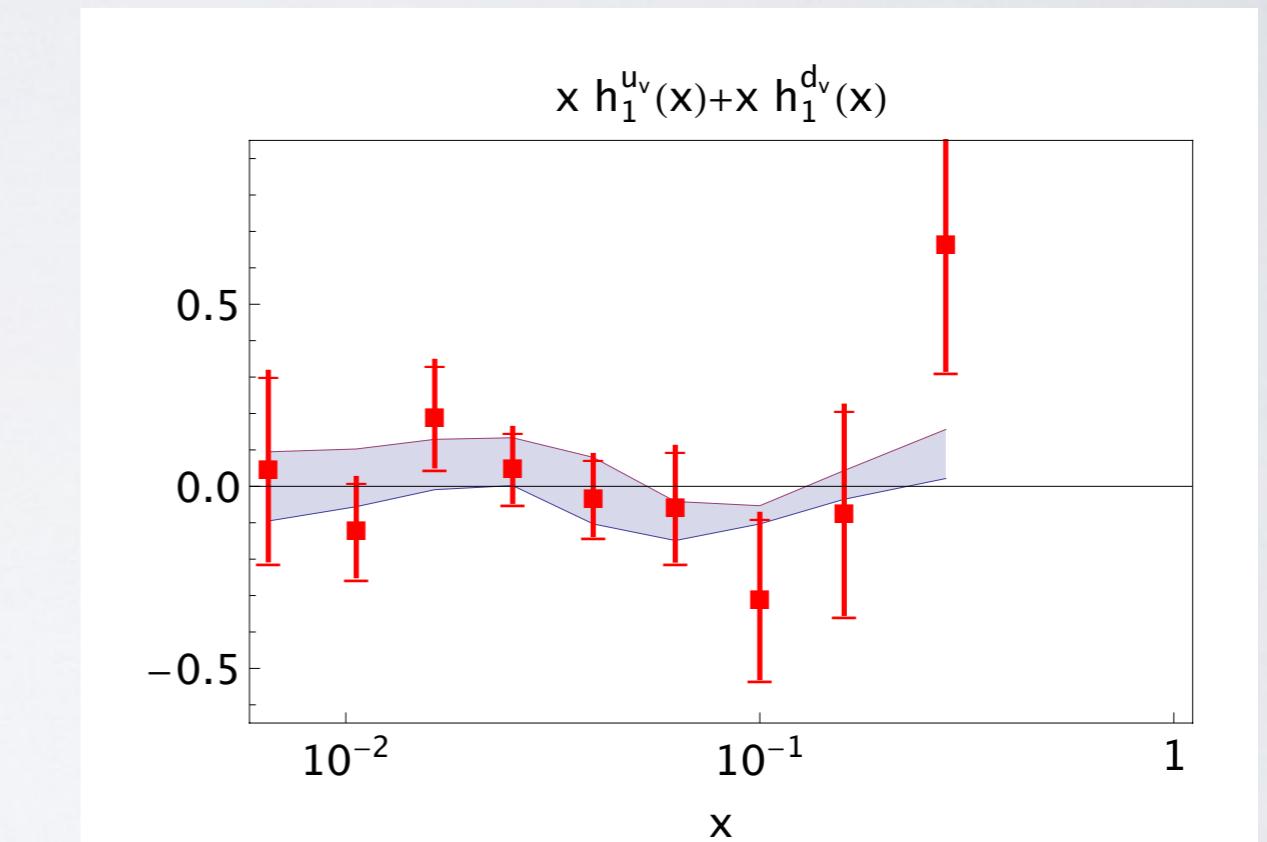
$$\alpha_s(M_{Z^0}^2) = 0.125 \text{ (GRV98)}$$

taking the **68%** band

proton



deuteron



flexible

# new fit vs. previous fit



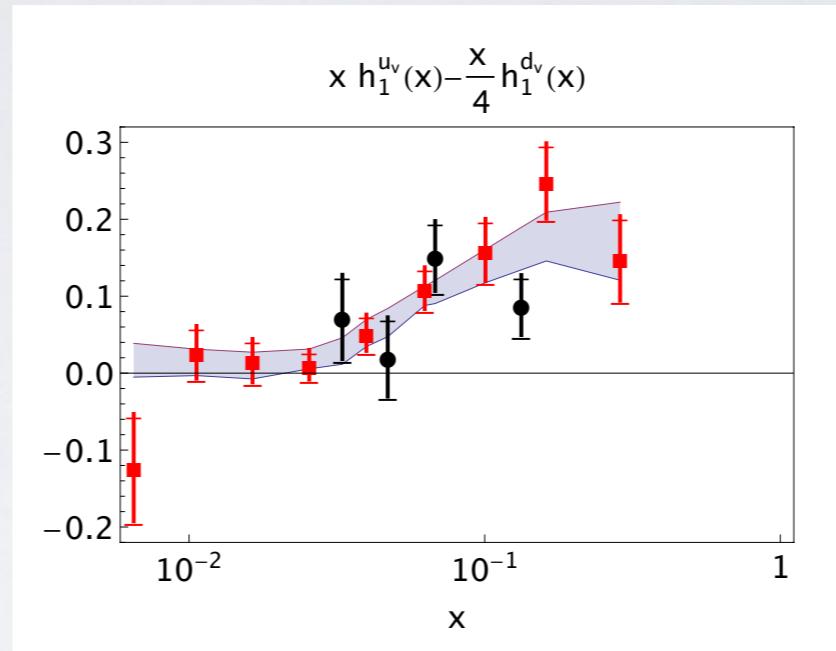
flexible

$$\alpha_s(M_Z^2) = 0.125 \quad (\text{GRV98})$$

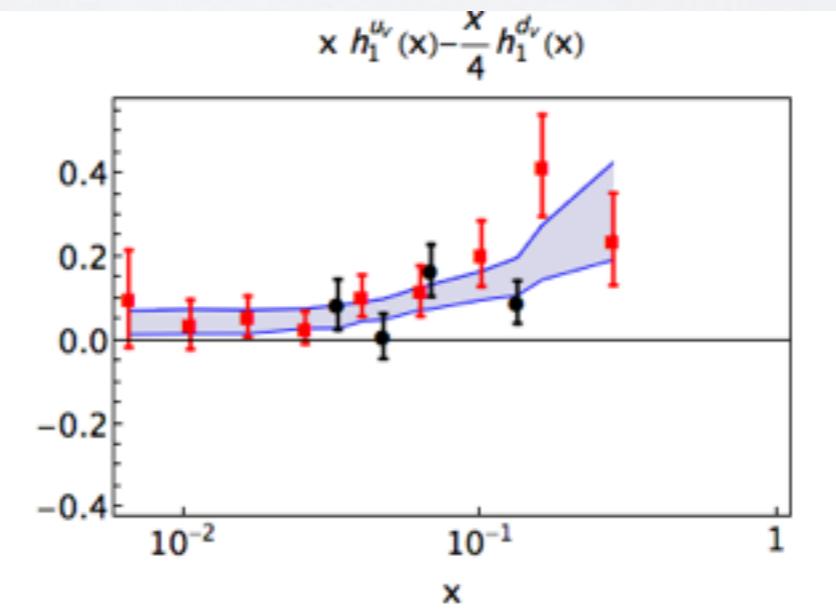
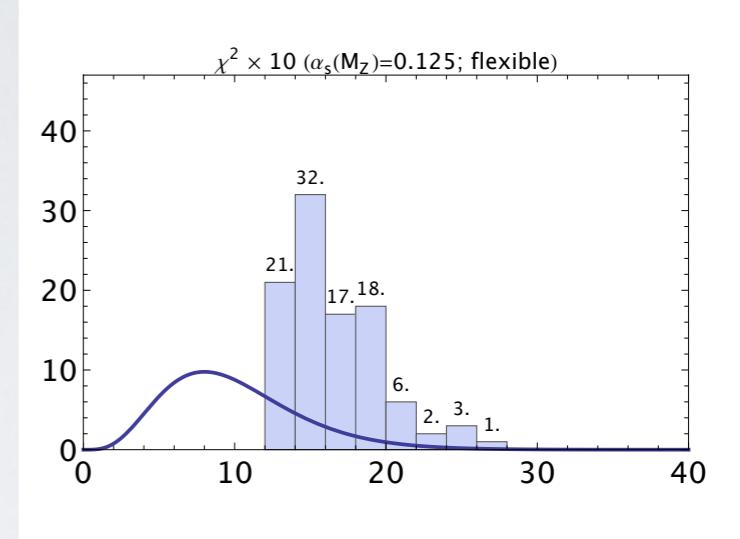
previous fit

*Bacchetta, Courtoy, Radici,  
JHEP 1303 (13) 119*

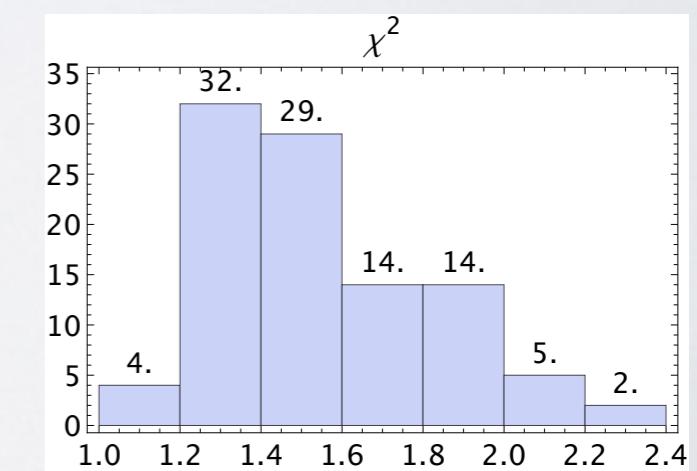
new fit



$\chi^2/\text{d.o.f.} \times 10$



$\chi^2/\text{d.o.f.}$



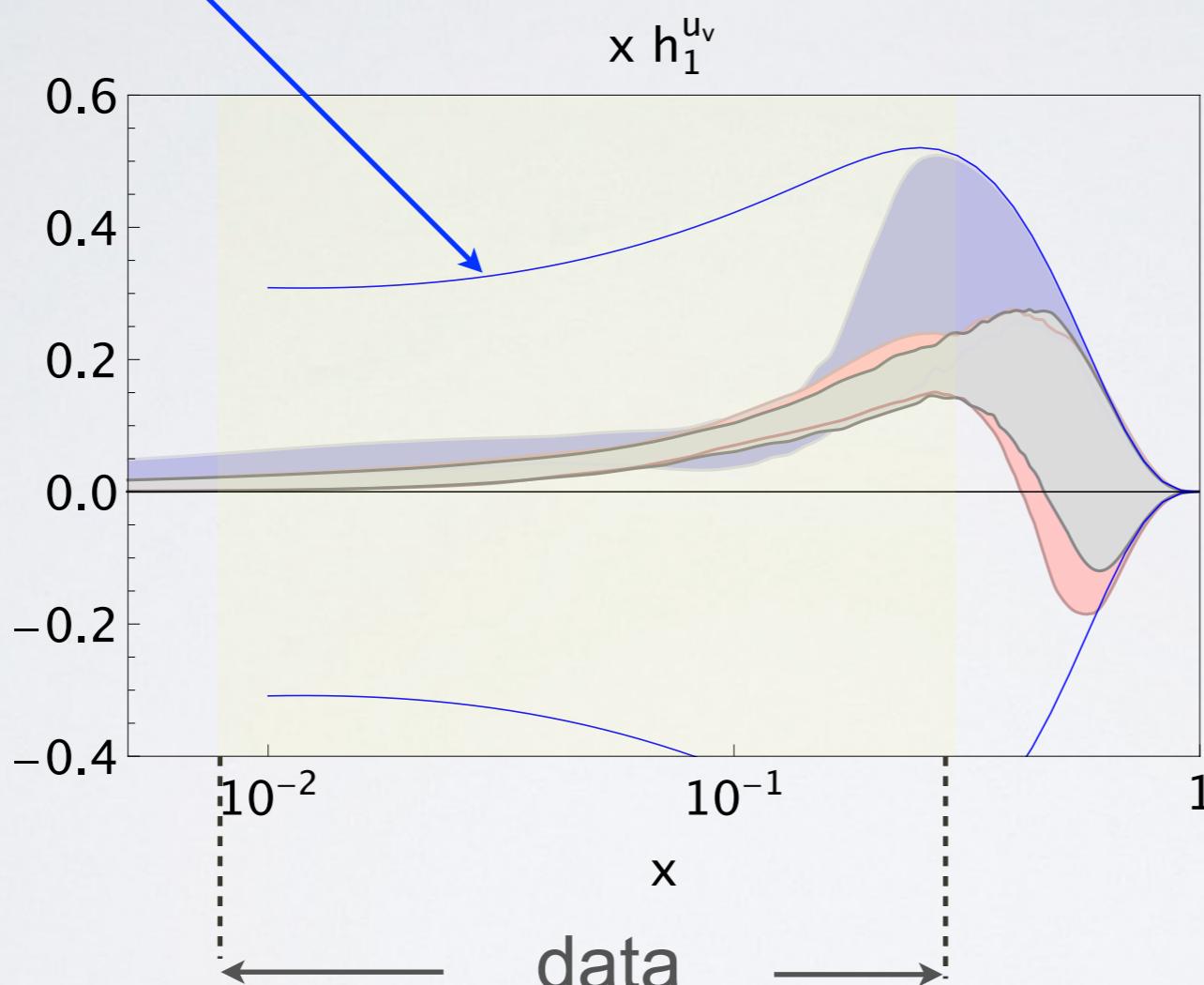
# new fit vs. previous fit

$Q^2 = 2.4 \text{ GeV}^2$

Soffer bound



flexible



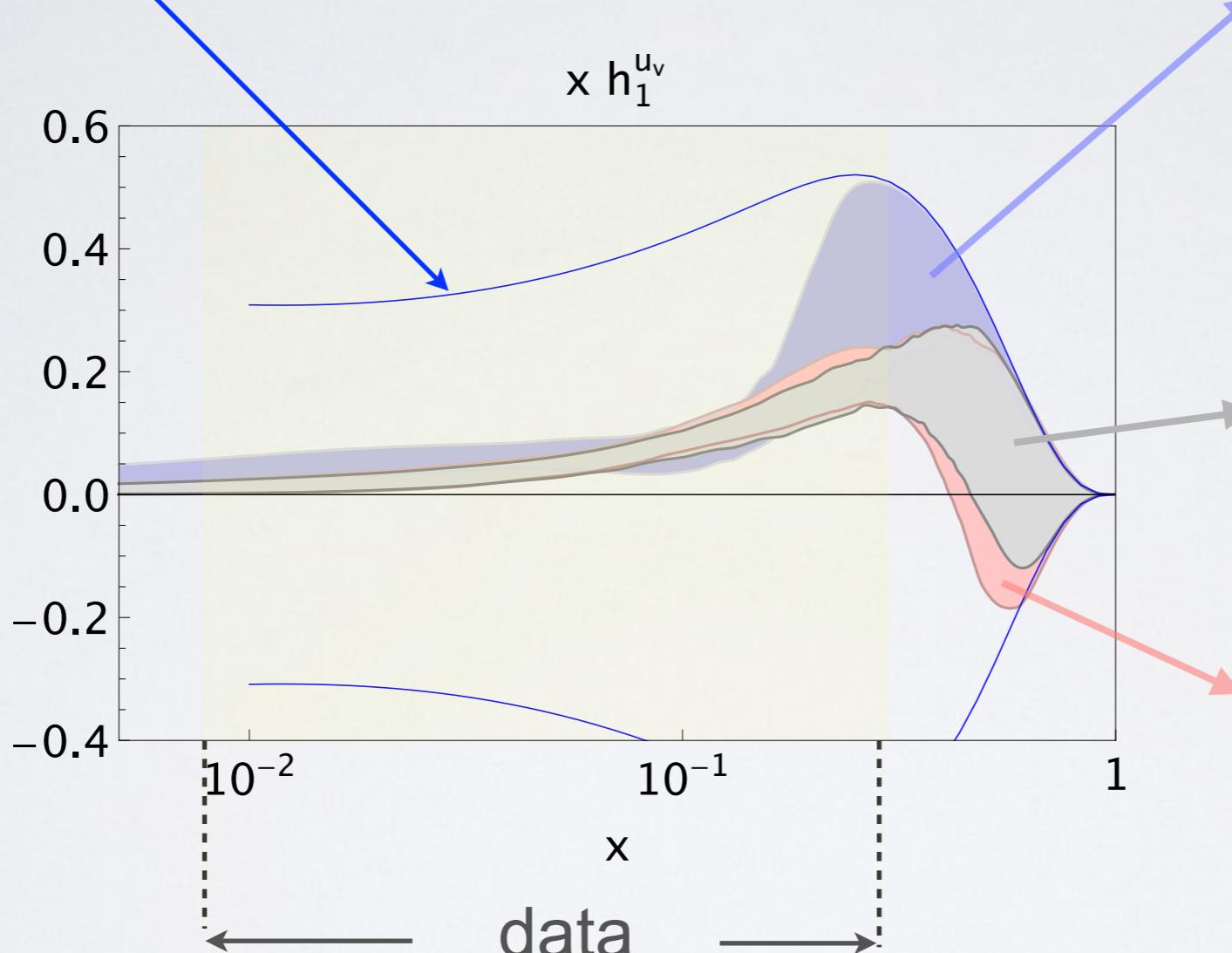
# new fit vs. previous fit

$Q^2 = 2.4 \text{ GeV}^2$

Soffer bound



flexible



previous fit

Bacchetta, Courtoy, Radici,  
*JHEP 1303* (13) 119

new fit  
 $\alpha_s(M_{Z0}^2) = 0.139$   
(MSTW08)

new fit  
 $\alpha_s(M_{Z0}^2) = 0.125$   
(GRV98)

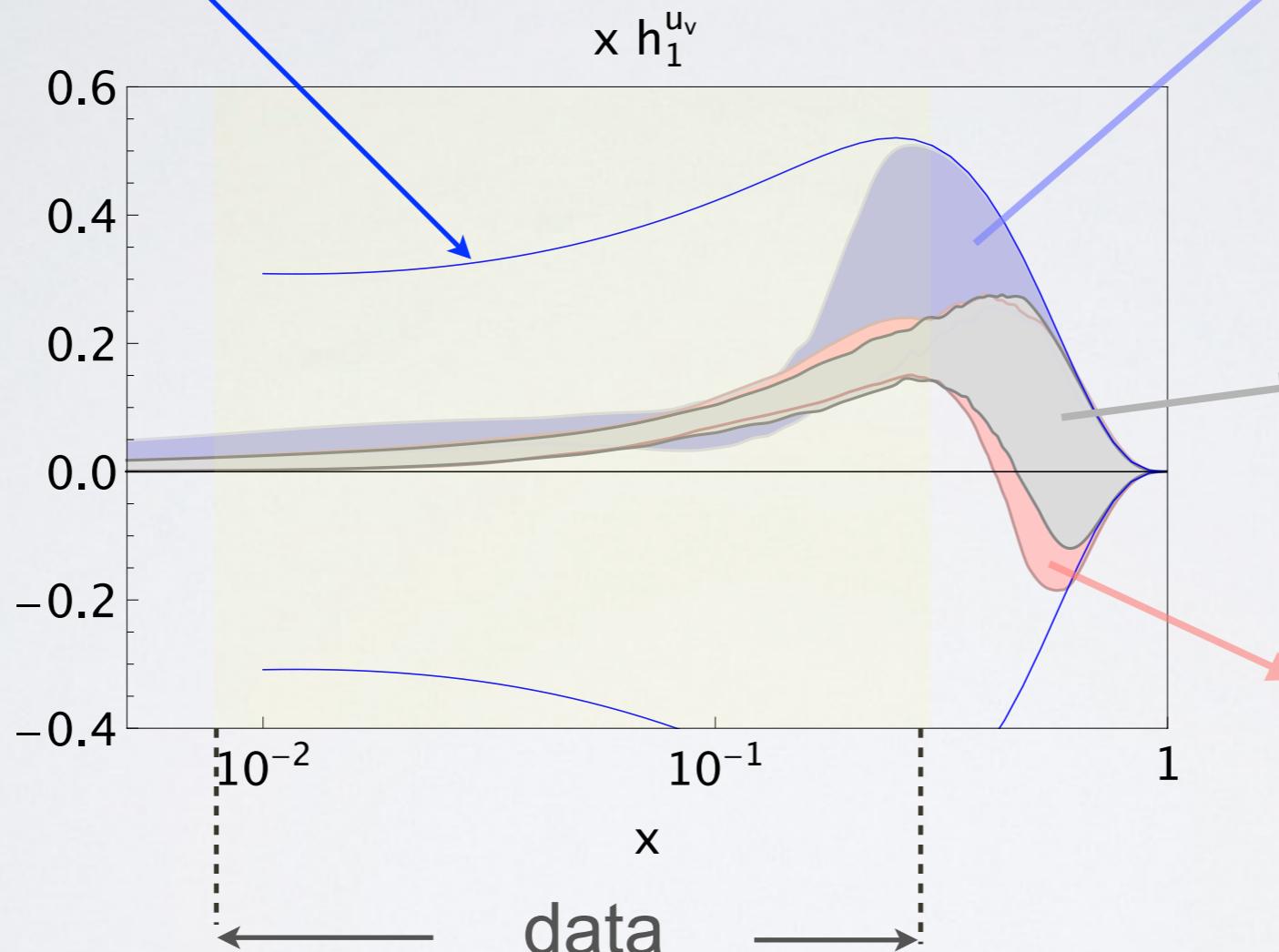
# new fit vs. previous fit

$Q^2 = 2.4 \text{ GeV}^2$



flexible

Soffer bound



previous fit

Bacchetta, Courtoy, Radici,  
JHEP 1303 (13) 119

new fit  
 $\alpha_s(M_Z^0)^2 = 0.139$   
 (MSTW08)

new fit  
 $\alpha_s(M_Z^0)^2 = 0.125$   
 (GRV98)

new 68% band for  $h_1^u$  is narrower (where there are data) and “smaller”  
 large uncertainties → need data at high  $x$  (JLab) and low  $x$  (EIC)

# comparison with Collins effect

Soffer bound

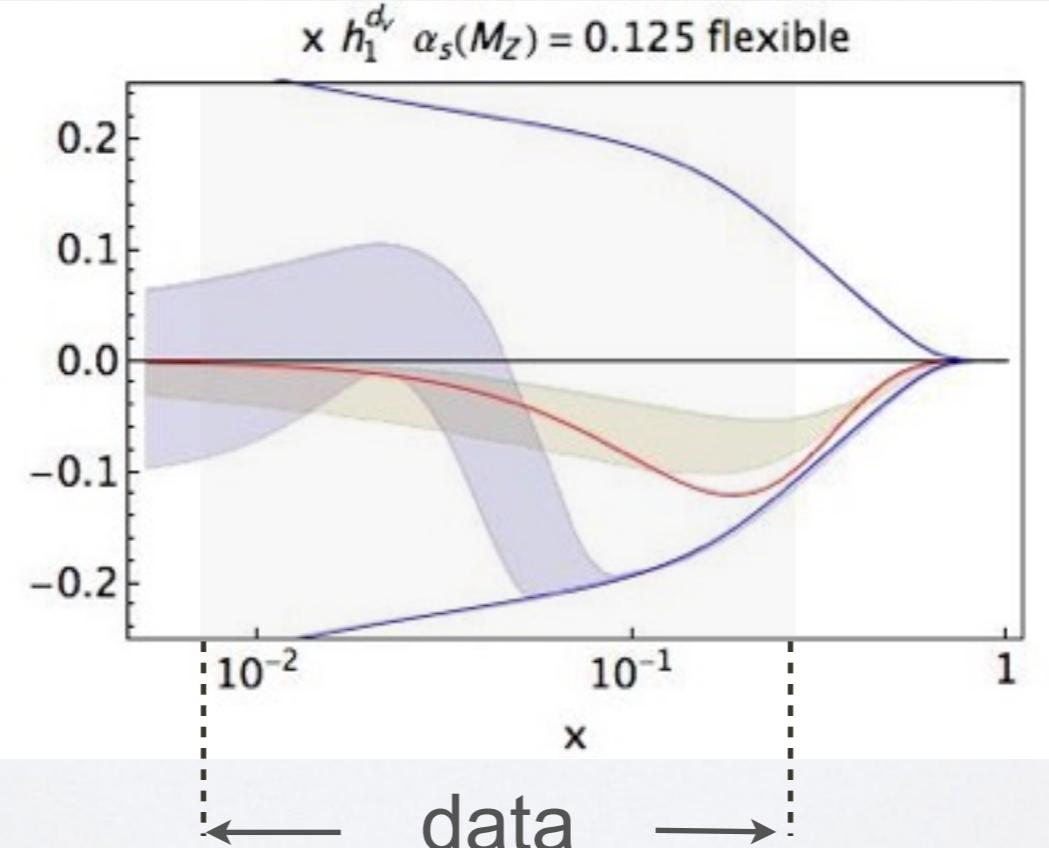
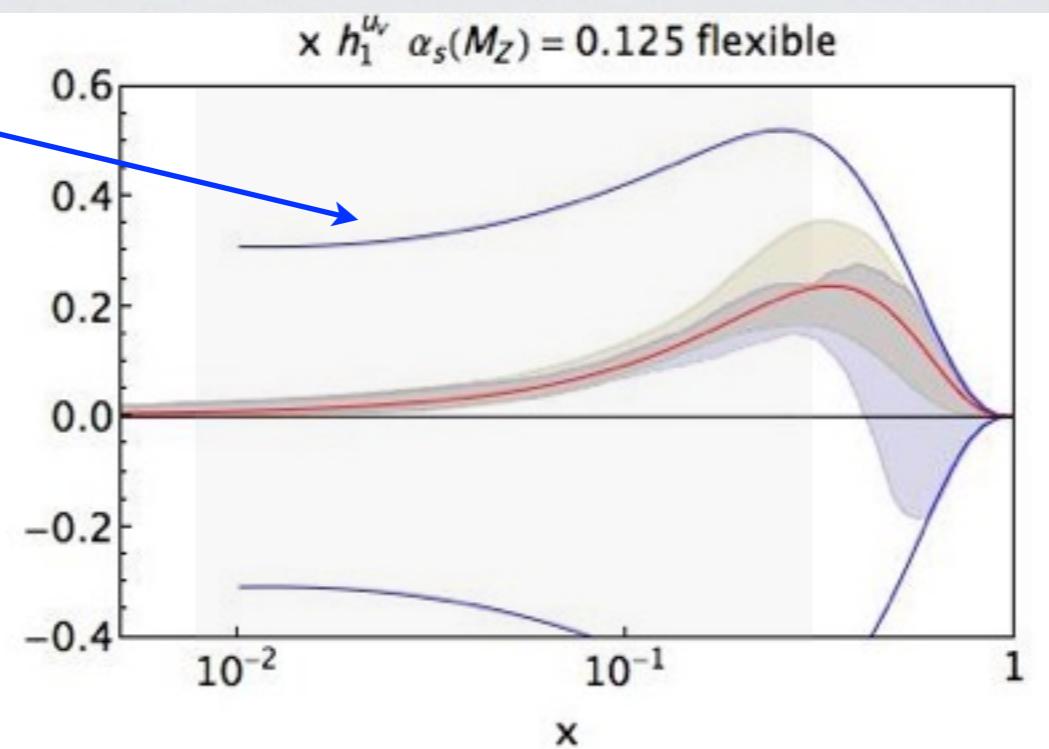
up

$x h_1^{q-\bar{q}}(x)$   
at  $Q^2 = 2.4 \text{ GeV}^2$

down



flexible



# comparison with Collins effect

Soffer bound

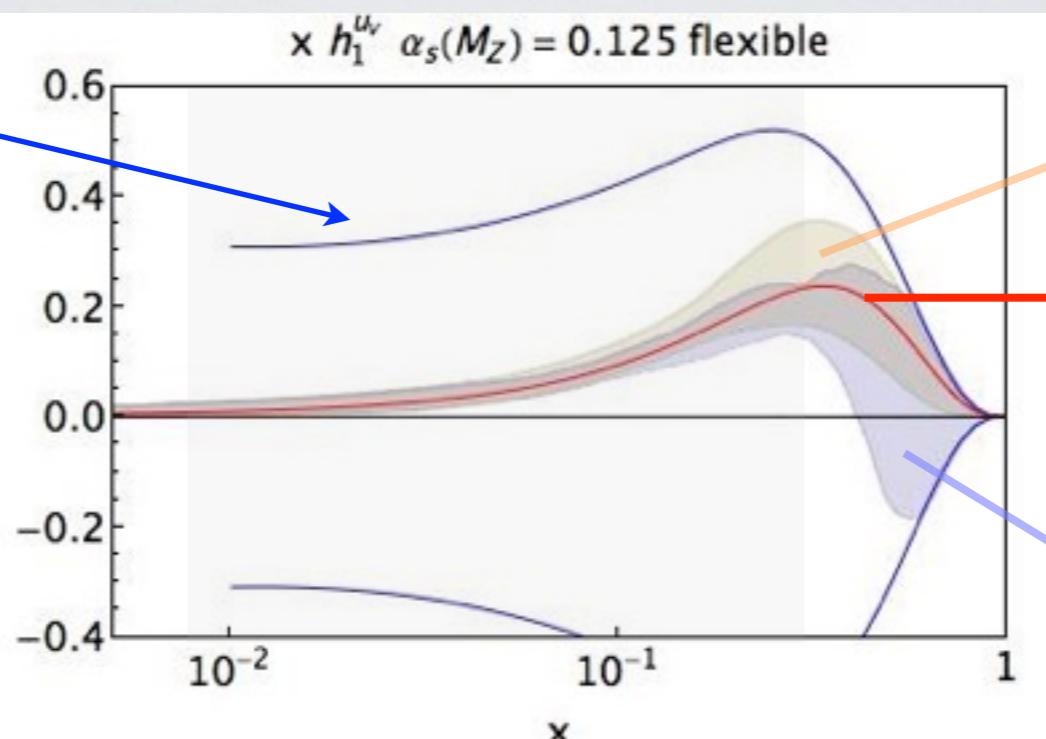
up

$x h_1^{q-\bar{q}}(x)$   
at  $Q^2 = 2.4 \text{ GeV}^2$

down



flexible



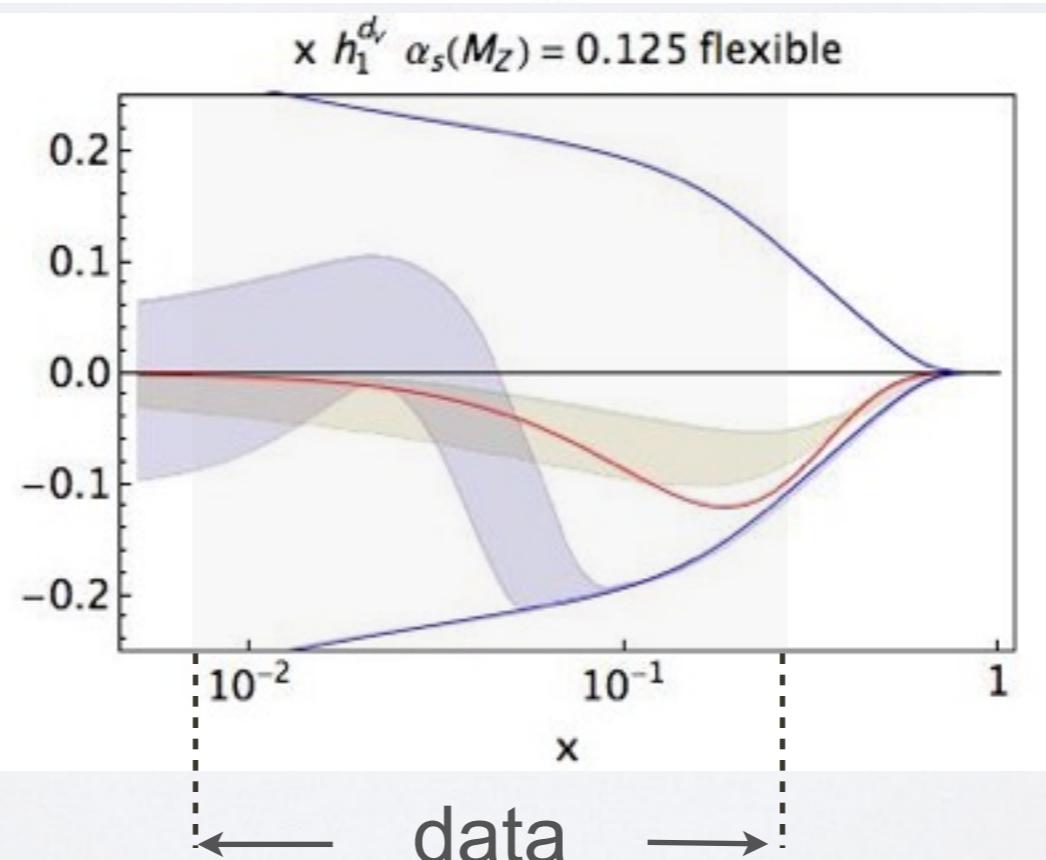
Torino 2013

Anselmino et al., P.R. D87 (13)

Collins effect  
with TMD evolution

Kang, Prokudin, Sun, Yuan,  
arXiv:1410.4877

new fit  
 $\alpha_s(M_{Z0}^2) = 0.125$   
(GRV98)



data

# comparison with Collins effect

Soffer bound

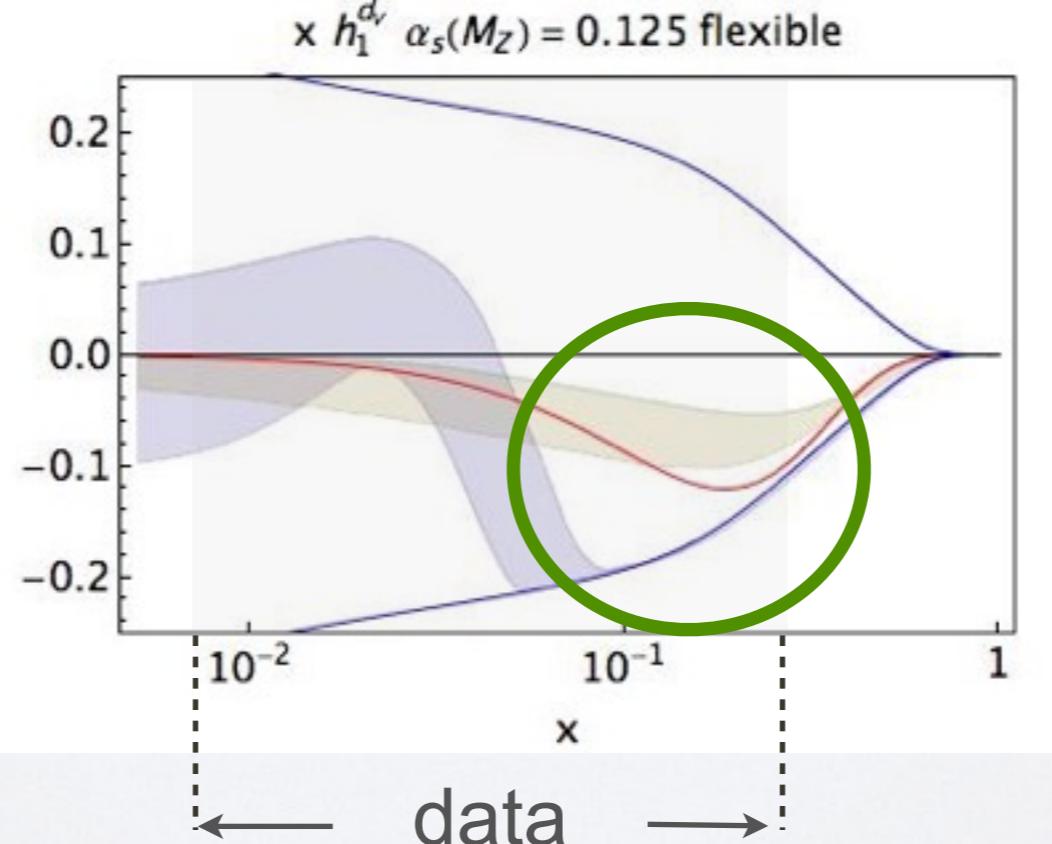
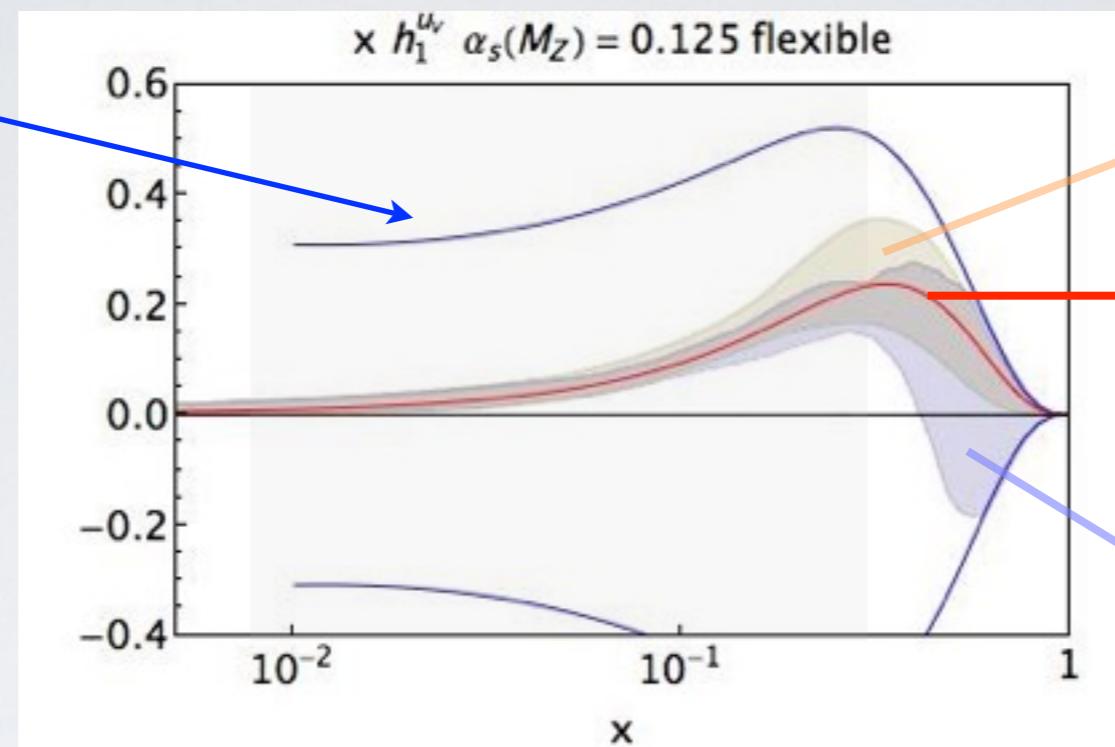
up

$x h_1^{q-\bar{q}}(x)$   
at  $Q^2 = 2.4 \text{ GeV}^2$

down



flexible



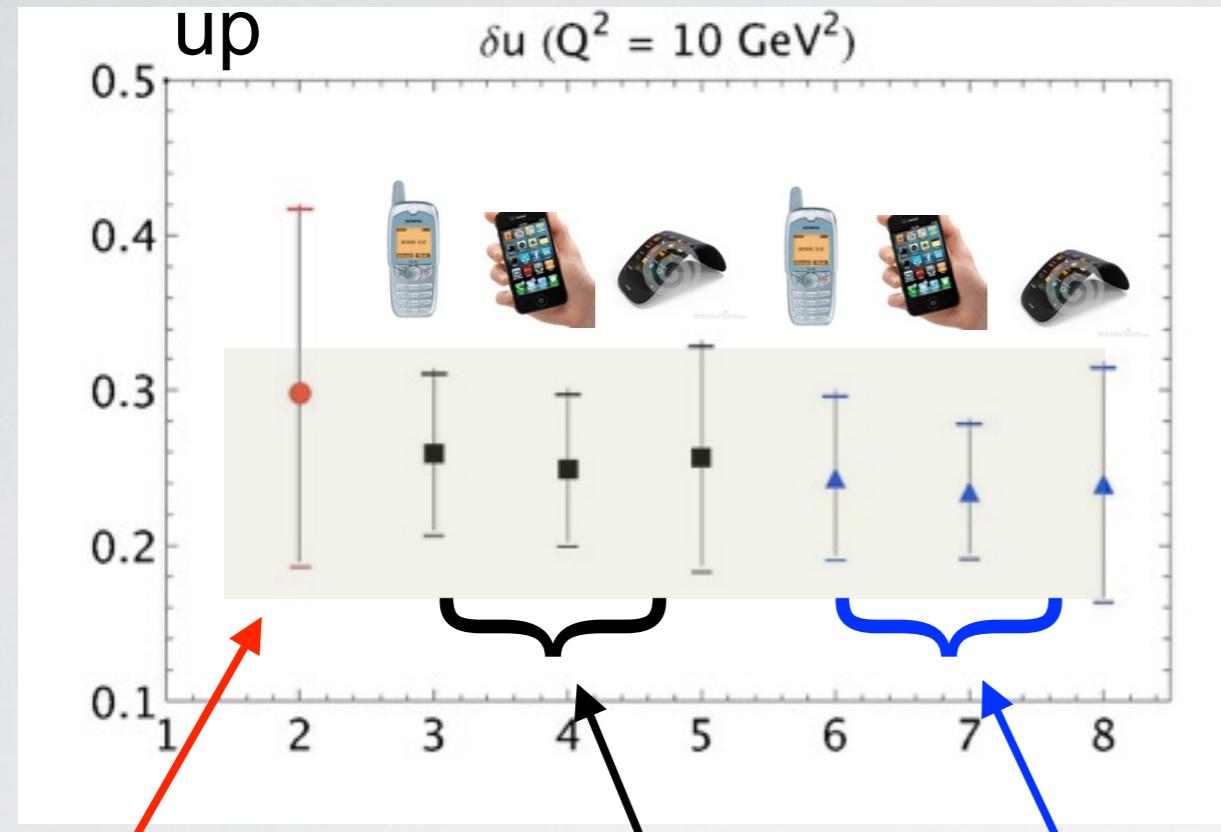
Torino 2013  
Anselmino et al., P.R. D87 (13)

Collins effect  
with TMD evolution  
Kang, Prokudin, Sun, Yuan,  
arXiv:1410.4877

new fit  
 $\alpha_s(M_Z0^2) = 0.125$   
(GRV98)

tension driven  
by COMPASS  
deuteron data

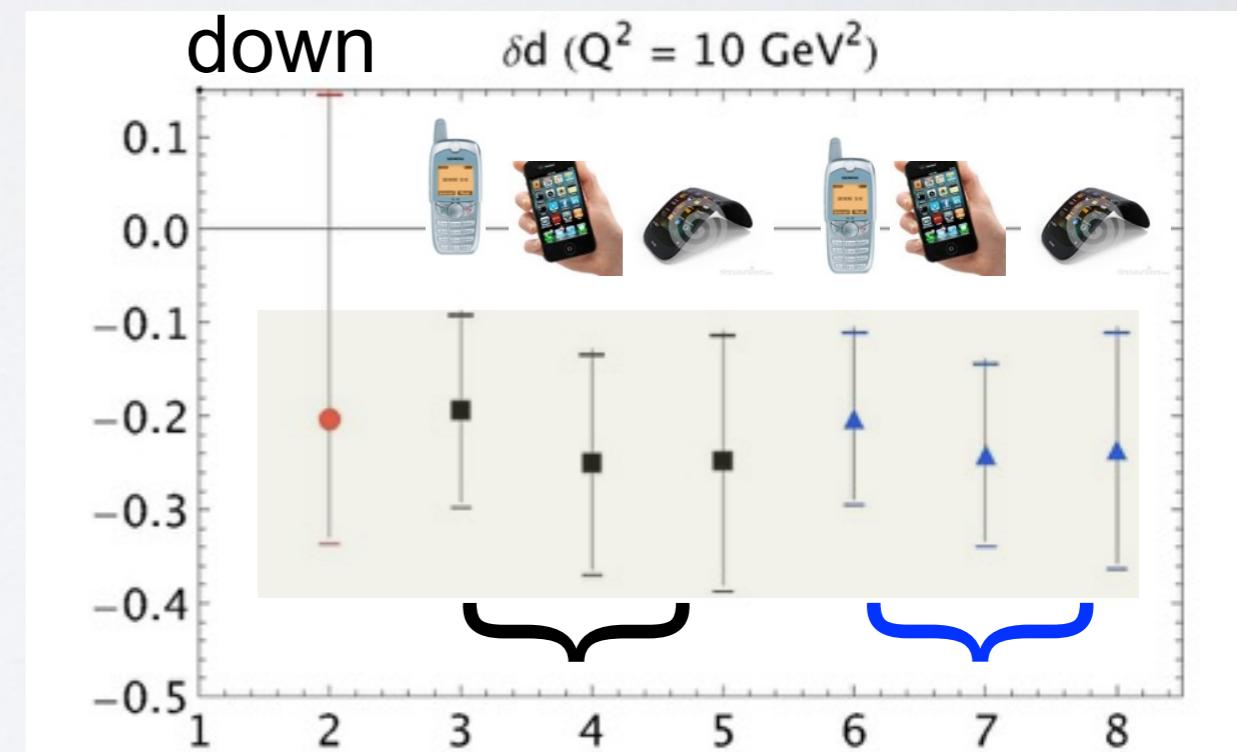
# tensor charges



Collins effect  
with TMD evol.

Kang, Prokudin, Sun, Yuan,  
arXiv:1410.4877

$$\delta q = \int_{x_{\min}}^{x_{\max}} dx h_1^{q_v}(x) \quad Q^2 = 10 \text{ GeV}^2$$



Collins  
effect

$\alpha_s = 0.125$

$\alpha_s = 0.139$

# Conclusions and outlook

- new  proton data for 2h-SIDIS induce **narrower uncertainty band for  $h_1^u$**
- new fit based also on more realistic errors on extraction of DiFF and on a first (crude) estimate of th. uncertainty in evolution  
⇒ **current most realistic estimate of errors on  $h_1$**
- $h_1^d$  basically unchanged  
 $h_1^u$  seems smaller but still compatible with Collins effect  
but still **large uncertainties, particularly at high  $x$**

# Conclusions and outlook

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  - $h_1^d$  basically unchanged  
 $h_1^u$  seems smaller but still compatible with Collins effect  
but still **large uncertainties, particularly at high  $x$**
- 

- **need 2h-SIDIS data at high  $x$  (JLab12) and low  $x$  (EIC)**
- need also  $D_1$  from  $e^+e^-$  data, not from PYTHIA..
- NLO evolution
- improve the replica method..