Nuclear Structure Functions in the Resonance Region

Sergey Kulagin

Institute for Nuclear Research, Moscow

Talk at the HiX-2014 Workshop
November 17–21, 2014
INFN at Frascati

Motivations and outline

- A large fraction of nuclear structure function data falls into a resonance region of $W^2 \lesssim 4$ GeV^2 and we need to understand how to compute nuclear effects in this region.
- An approach to calculate nuclear structure functions requires both (i) the model of the proton and neutron structure functions and (ii) a theoretical framework to relate proton and neutron structure functions and the nuclear structure functions. The latter should consistently account for finite Q^2 effects. We certainly have to avoid the Bjorken limit.
- Discuss first the light nuclei such as ^{2}H and ^{3}H for which the resonace structure is more pronounced and less affected by in-medium interactions.

Nucleon resonance region

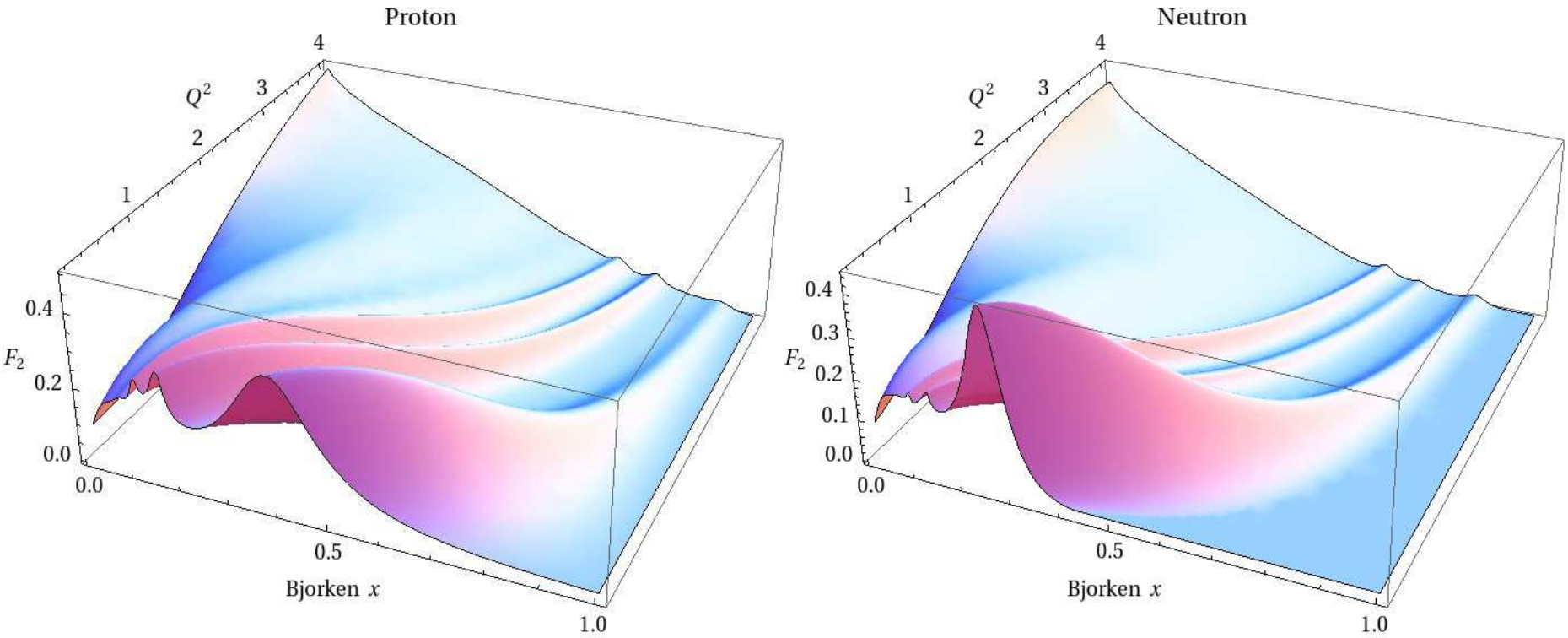
At low $Q^2 \lesssim 1$ GeV^2 and $W^2 < 4$ GeV^2 the inelastic cross sections/structure functions are dominated by the resonance production.

- 3 distinctive resonance enhancements in cross section. There are almost 20 established nucleon resonances which contribute to these enhancements.
- The 1st resonance region is due to a single $\Delta_{33}(1232)$ state
- The 2nd resonace region involves contributions from $S_{11}(1535)$ and $D_{13}(1520)$ states. Note that the data contain the tails of the higher-mass resonce structures as well as non-resonace contributions.
- The 3d resonance region is a composition of many resonance states.

An empirical fit to resonance cross section data by *Christy & Bosted*:

- SLAC and JLab proton and deuteron data included for $Q^2 < 10$ GeV^2 and $M + m_\pi < W < 3$ GeV
- The fit includes both Breit-Wiegner resonance contributions and non-resonance background
- Separation of L and T cross sections for the proton and the neutron. A smooth transition to $Q^2 \to 0$.
- The fit describes data at the 3% or better level

Christy & Bosted proton and neutron structure functions



- Resonances move with Q^2 (recall the relation $x = Q^2/(W^2 - M^2 + Q^2)$)
- Strength of resonances falls off quickly with Q^2

High W DIS region

Theory approach is based on the operator product (twist) expansion in QCD:

$$F_2(x,Q^2) = F_2^{LT,TMC}(x,Q^2) + \frac{H_2(x,Q)}{Q^2} + \cdots$$

The leading term is given in terms of PDFs convoluted with coefficient functions:

$$F_2^{\mu,\,LT} = C_q(\alpha_S) \otimes x \sum_q e_q^2 (q + \bar{q}) + \cdots$$

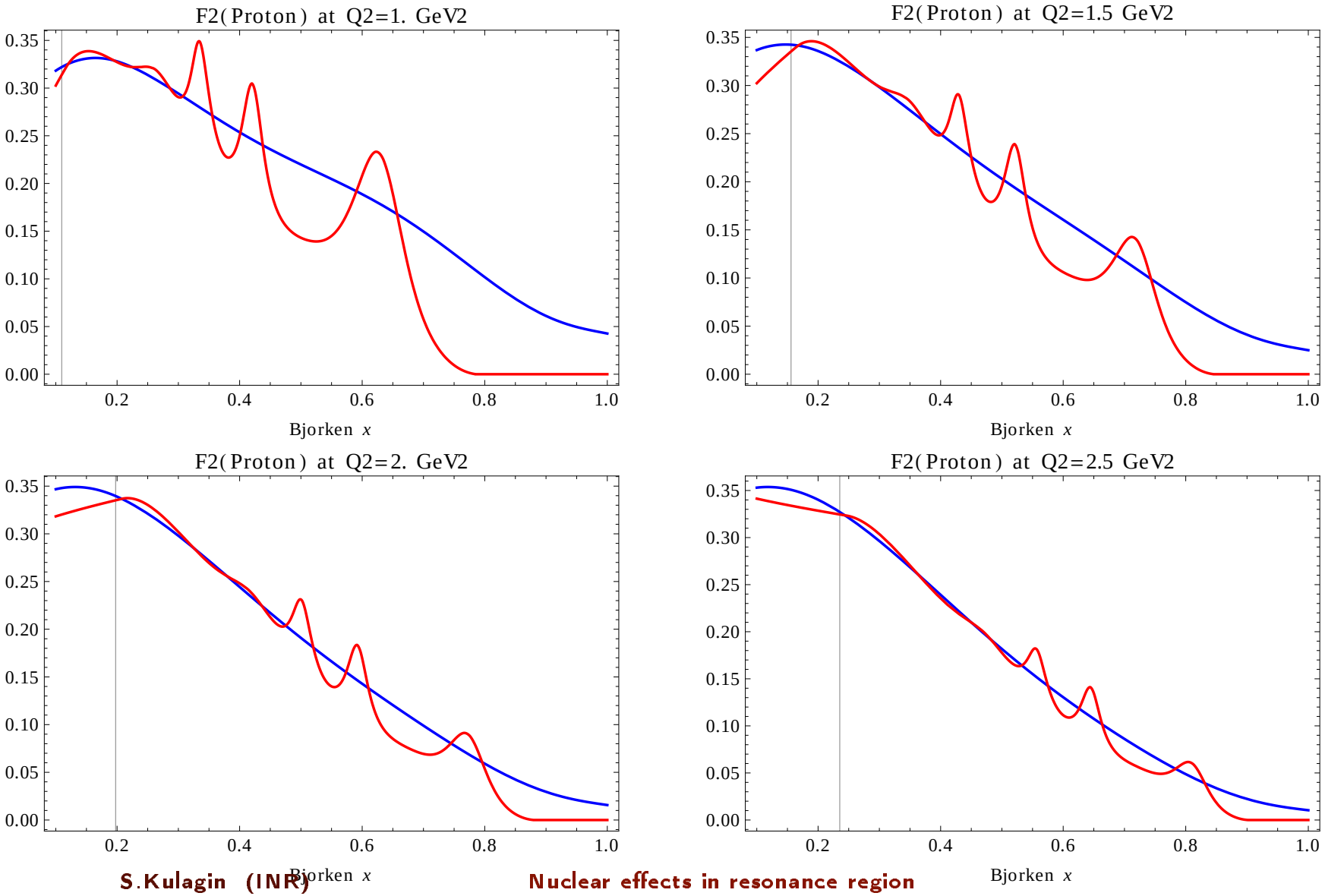
The HT terms involve interaction between quarks and gluons and lack simple probabilistic interpretation.
In the region of high Bjorken x and/or low Q^2 one has to worry about target mass correction *Georgi & Politzer, 1976*

$$F_2^{LT,TMC}(x,Q^2) = \frac{x^2}{\xi^2\gamma^2} F_2^{LT}(\xi,Q^2) + \frac{6x^3M^2}{Q^2\gamma^4} \int_\xi^1 \frac{\mathrm{d}z}{z^2} F_2^{LT}(z,Q^2) + \mathcal{O}(Q^{-4})$$

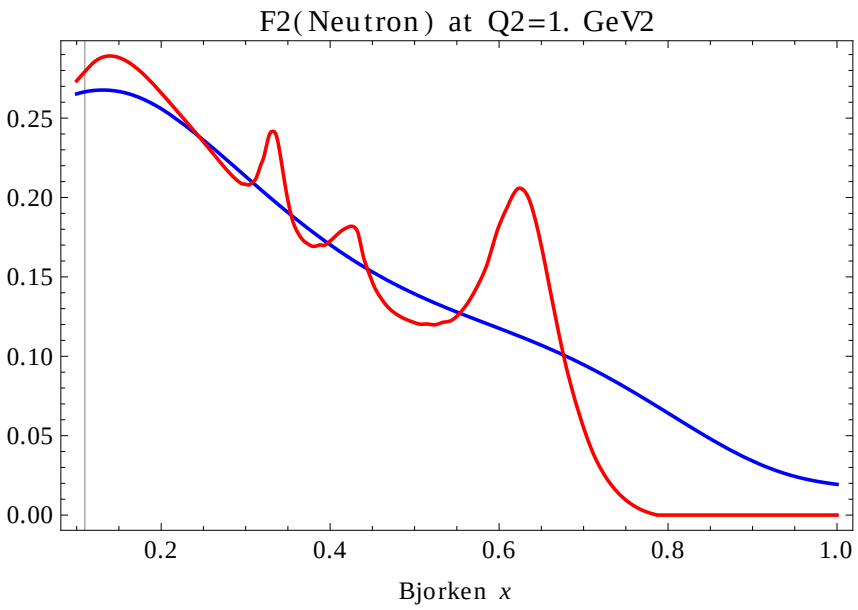
$\xi = 2x/(1+\gamma)$ is Nachtmann variable and $\gamma^2 = 1 + 4x^2M^2/Q^2$

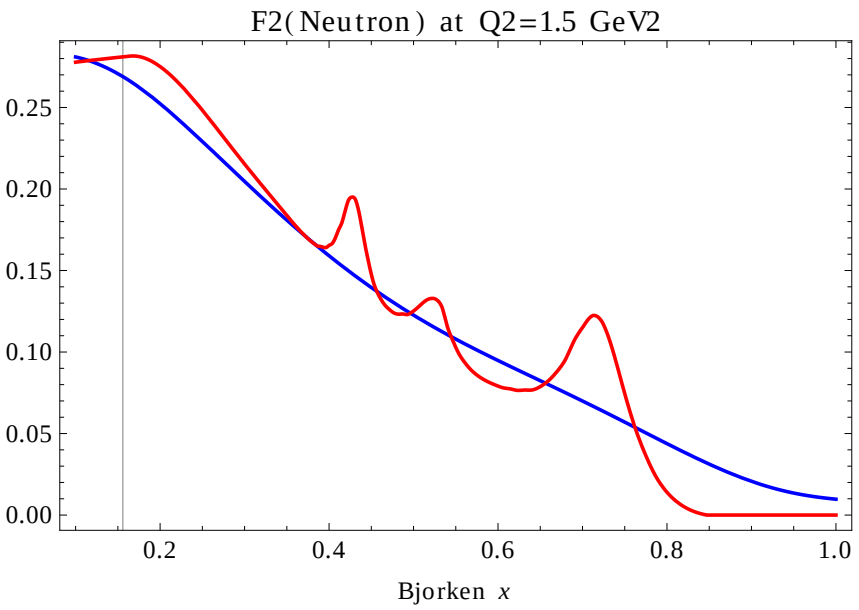
Scaling–Resonance correspondance (Proton)

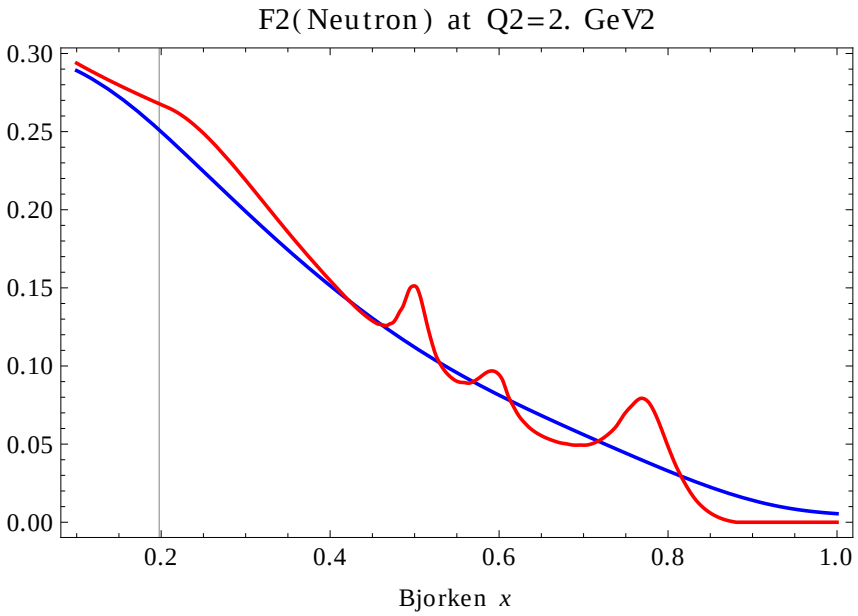
BC parametrization of resonance data and NNLO DIS structure functions in terms of Alekhin PDFs (2008 analysis, HT and TMC included).

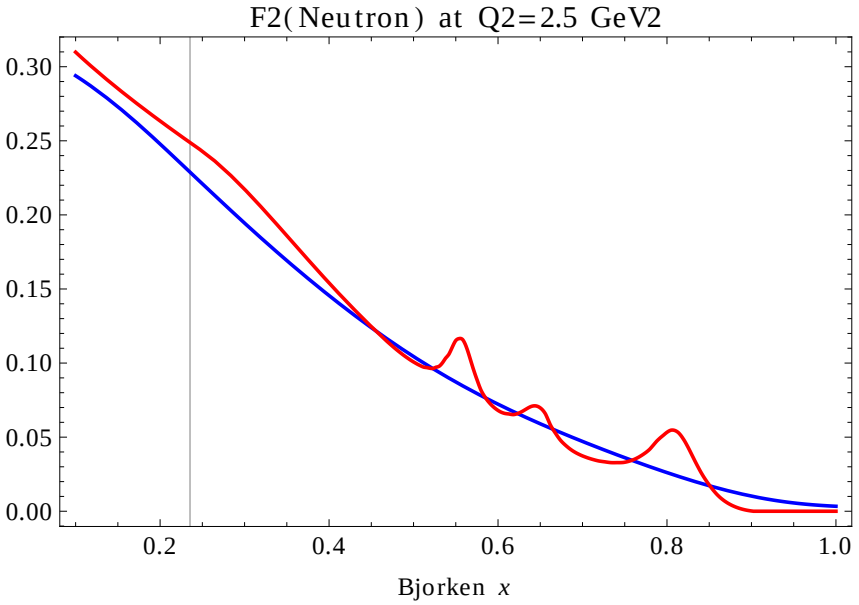


Neutron

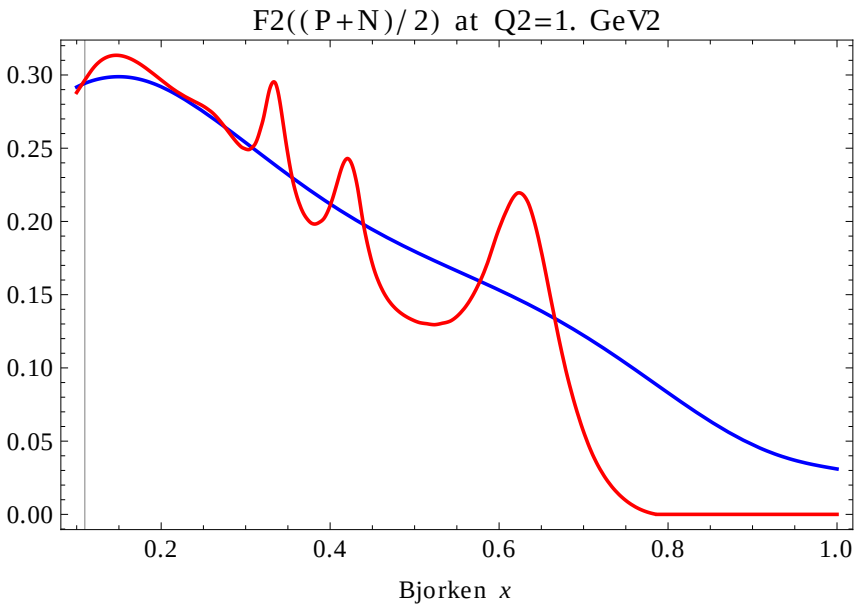

F2(Neutron) at Q2=1. GeV2
0.25
0.20
0.15
0.10
0.05
0.00
0.2
0.4
0.6
0.8
1.0
Bjorken x

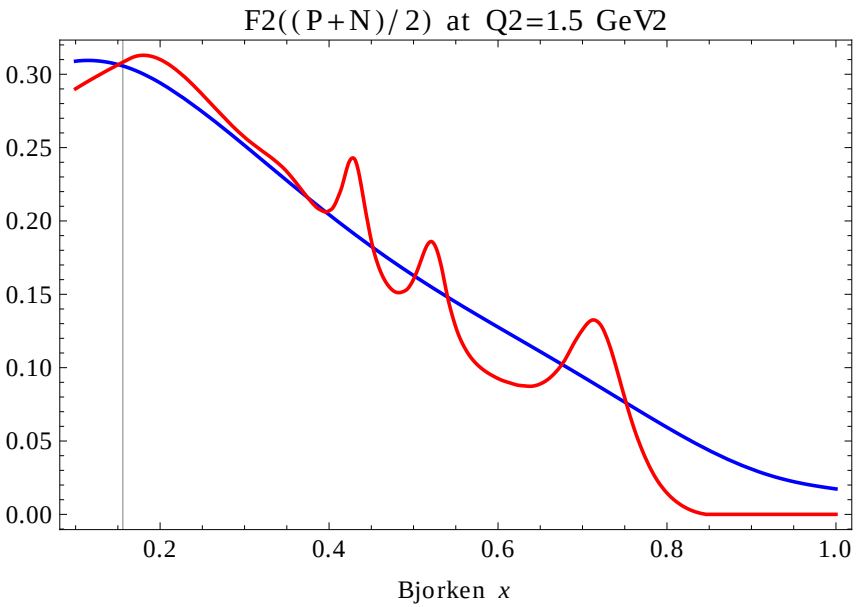

F2(Neutron) at Q2=1.5 GeV2
0.25
0.20
0.15
0.10
0.05
0.00
0.2
0.4
0.6
0.8
1.0
Bjorken x

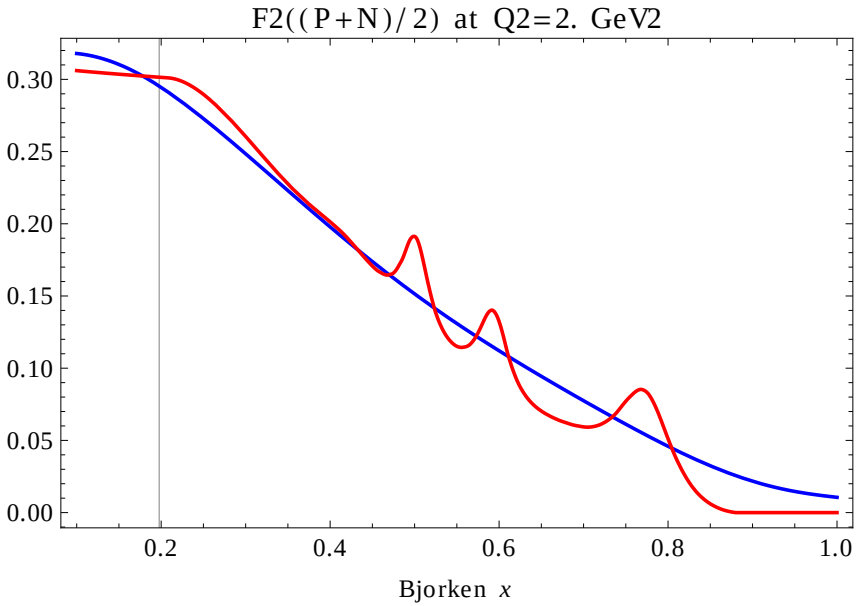

F2(Neutron) at Q2=2. GeV2
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.2
0.4
0.6
0.8
1.0
Bjorken x

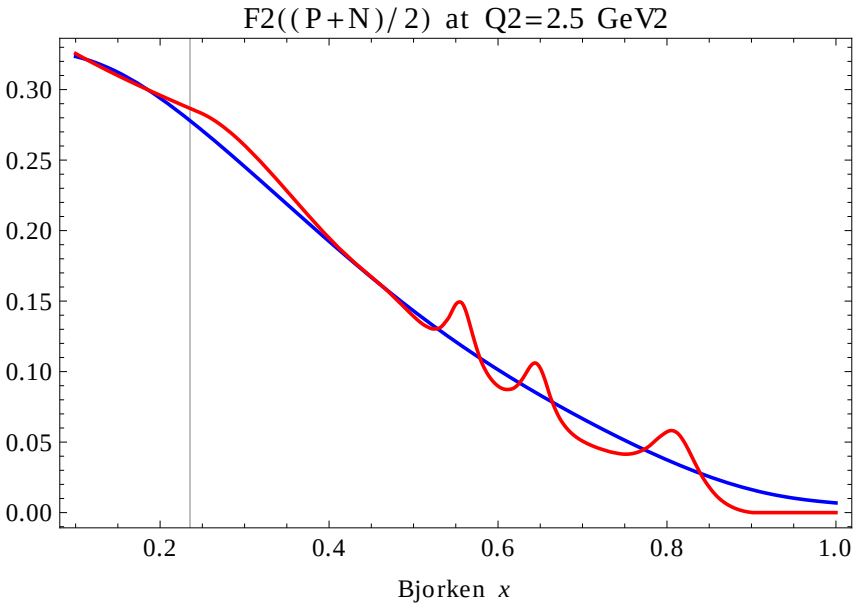

F2(Neutron) at Q2=2.5 GeV2
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.2
0.4
0.6
0.8
1.0
Bjorken x

Isoscalar nucleon $(p+n)/2$

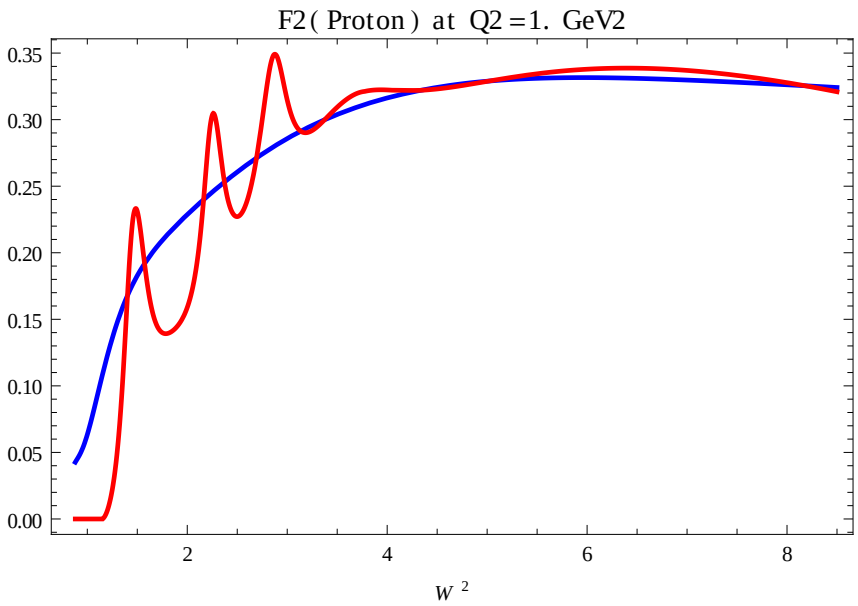


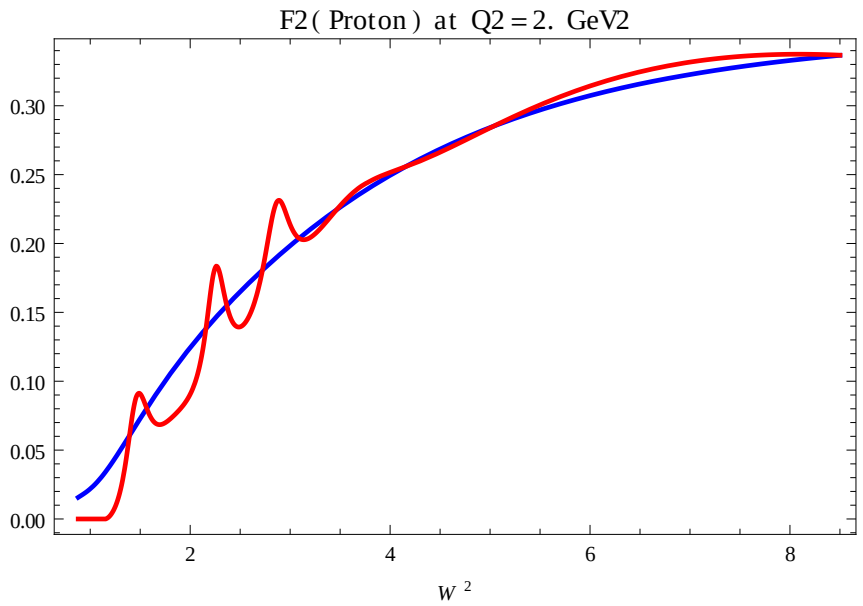






Integral scaling–resonance duality





Bloom & Gilman (1970s): Scaling and resonance descriptions of electro-production are dual in an integral sense:

$$\int_{(M+m_\pi)^2}^{W_R^2} \mathrm{d}W^2 \; F_2^{\text{Scaling}} = \int_{(M+m_\pi)^2}^{W_R^2} \mathrm{d}W^2 \; F_2^{\text{Resonance}}$$

- The integral duality between scaling SF (Alekhin08 PDFs) and resonance SF (BC09 parametrization) holds with $W_R^2 = 4.5$ GeV^2 at the level of 3% and better for all $1 < Q^2 < 10$ GeV^2 for the proton and the isoscalar combination $(p+n)/2$.
- Somewhat worse agreement for the neutron. This is likely because of different treatment of nuclear corrections for the deuteron in Alekhin's and BC's fits.
- The BG duality allows to constrain the low-W (i.e. large Bjorken x) behaviour of SF in DIS region from the studies of the resonance region at much lower Q^2. This also should be true in the reverse direction.

- The studies of the resonance region in electroproduction could be a useful tool to constrain the neutrino interaction in a similar kinematical region.
- IF the duality holds for both the electron and neutrino scattering THEN relations between resonance electron and neutrino production Example:

IF in Scaling region $$F_2^{\mu N} = \frac{5}{18} F_2^{\nu N}$$

THEN in Resonance region $$\int \mathrm{d}W^2 F_2^{\mu N} = \frac{5}{18} \int \mathrm{d}W^2 F_2^{\nu N}$$

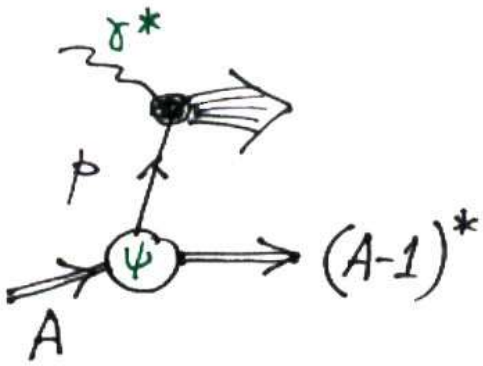
Note this relation may be modified by higher-twist terms as they may be different for electromagnetic and weak currents.

Calculating nuclear effects at large Bjorken x

A reasonable starting point is approximation of nuclear scattering amplitude as a sum of incoherent scattering off bound protons and neutrons.

$$W^A_{\mu\nu}(P_A, q) = \sum_{\tau=p,n} \int \mathrm{d}^4 p \; \mathrm{Tr}\left[\widehat{\mathcal{W}}^\tau_{\mu\nu}(p,q)\mathcal{A}^\tau(p;A)\right]$$

$$\mathcal{A}^\tau_{\alpha\beta}(p;A) = \int \mathrm{d}t \mathrm{d}^3\boldsymbol{r} e^{ip_0 t - i\boldsymbol{p}\cdot\boldsymbol{r}} \langle A|\overline{\Psi}^\tau_\beta(t,\boldsymbol{r})\,\Psi^\tau_\alpha(0)|A\rangle$$



- $\Psi^\tau_\alpha(t,\boldsymbol{r})$ the nucleon Dirac field operator.
- The off-shell nucleon tensor $\widehat{\mathcal{W}}_{\mu\nu}(p,q)$ is the matrix in the Dirac space.

On the mass shell $p^2 = M^2$, averaging $\widehat{\mathcal{W}}_{\mu\nu}(p,q)$ over the nucleon polarizations we have 2 structure functions:

$$W^\tau_{\mu\nu}(p,q) = \frac{1}{2}\mathrm{Tr}\left[(\not{p}+M)\widehat{\mathcal{W}}^\tau_{\mu\nu}(p,q)\right] = F_1\,\widetilde{g}_{\mu\nu} + F_2\,\widetilde{p}_\mu\widetilde{p}_\nu/p\cdot q,$$

$$\widetilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \qquad \widetilde{p}_\mu = p_\mu - \frac{pq}{q^2}q_\mu$$

How many structure functions do we have off-mass-shell $p^2 \neq M^2$?

- Expand in the Dirac basis:

$$\widehat{W}_{\mu\nu} = \sum_n W^n_{\mu\nu} \Gamma_n$$

$$\Gamma_n = I, \gamma^\alpha, \sigma^{\alpha\beta}, \gamma^\alpha \gamma_5, \gamma_5$$

- Require the symmetry under P and T transformations AND keeping ONLY current-conserving terms ($q_\mu W_{\mu\nu} = 0$) we have 7 independent structure functions

Melnitchouk, Schreiber & Thomas, 1994; *SK, Piller & Weise, 1994*

$$2\,\widehat{\mathcal{W}}^{\rm sym}_{\mu\nu}(p,q) = -\,\widetilde{g}_{\mu\nu}\left(\frac{f_1^{(0)}}{M} + \frac{f_1^{(1)}\not{p}}{M^2} + \frac{f_1^{(2)}\not{q}}{p\cdot q}\right)$$
$$+\,\frac{\widetilde{p}_\mu \widetilde{p}_\nu}{p\cdot q}\left(\frac{f_2^{(0)}}{M} + \frac{f_2^{(1)}\not{p}}{M^2} + \frac{f_2^{(2)}\not{q}}{p\cdot q}\right) + \frac{f_2^{(3)}}{p\cdot q}\,\widetilde{p}_{\{\mu}\widetilde{g}_{\nu\}\alpha}\gamma^\alpha,$$

- Contribution of each of these structure functions is goverened by corresponding matrix element $\langle \overline{\Psi}\Gamma_n \Psi\rangle$.
- On the mass shell $p^2 = M^2$ we only have 2 independent structure functions

$$F_1 = f_1^{(0)} + f_1^{(1)} + f_1^{(2)},$$
$$F_2 = f_2^{(0)} + f_2^{(1)} + f_2^{(2)} + f_2^{(3)}$$

Weak binding approximation (WBA)

SK, 1984

- Assume the nuclear ground state to be nonrelativistic:
  - $|\boldsymbol{p}| \ll M,\ |p_0 - M| \ll M$
  - No strong scalar and vector fields in nuclei
- Reduce the four-component relativistic field Ψ to a two-component nonrelativistic operator ψ *Landau & Lifshitz, Field theory*

$$\Psi(\boldsymbol{p},t) = e^{-iMt} Z \begin{pmatrix} \psi(\boldsymbol{p},t) \\ (\boldsymbol{\sigma}\cdot\boldsymbol{p}/2M)\,\psi(\boldsymbol{p},t) \end{pmatrix}$$

- The renormalization operator $Z = 1 - \boldsymbol{p}^2/8M^2$ provides a correct normalization of the nonrelativistic two-component nucleon field ψ: $\int \mathrm{d}^3p\Psi^\dagger\Psi = \int \mathrm{d}^3p\psi^\dagger\psi$ to order $\boldsymbol{p}^2/M^2$.
- Separate the nucleon mass M from the energy p_0, $p = (M+\varepsilon, \boldsymbol{p})$. Examine and reduce all the Lorentz–Dirac structures of $\widehat{W}_{\mu\nu}$. The result to order ε/M and $\boldsymbol{p}^2/M^2$ can be summarized as

Nuclear hadronic tensor: $\dfrac{W^A_{\mu\nu}(P_A,q)}{M_A} = \sum_{\tau=p,n} \int \dfrac{\mathrm{d}^4 p}{M+\varepsilon} \mathcal{P}^\tau(\varepsilon,\boldsymbol{p}) W^\tau_{\mu\nu}(p,q),$

Nuclear spectral function: $\mathcal{P}(\varepsilon,\boldsymbol{p}) = \int \mathrm{d}t\, e^{-i\varepsilon t} \langle \psi^\dagger(\boldsymbol{p},t)\psi(\boldsymbol{p},0)\rangle$

In the WBA regime (a "weak"off-shellness of bound nucleons) we do not have to consider all possible $f_i^{(j)}$ inelastic structure functions independently. Their contribution combine into 2 bound nucleon structure functions *SK, Piller & Weise, 1994*; *SK & Petti, 2004*:

$$W^\tau_{\mu\nu}(p,q) = F_1\, \widetilde{g}_{\mu\nu} + F_2\, \widetilde{p}_\mu \widetilde{p}_\nu / p\cdot q,$$

$$F_1(x,Q^2,p^2) = f_1^{(0)}\left(1+\frac{p^2-M^2}{2M^2}\right) + f_1^{(1)}\frac{p^2}{M^2} + f_1^{(2)},$$

$$F_2(x,Q^2,p^2) = f_2^{(0)}\left(1+\frac{p^2-M^2}{2M^2}\right) + f_2^{(1)}\frac{p^2}{M^2} + f_2^{(2)} + f_2^{(3)},$$

Relations beween A & bound p, n structure functions

$$F_T^A(x,Q^2) = \sum_{\tau=p,n} \int \frac{\mathrm{d}\varepsilon \mathrm{d}^3\boldsymbol{p}}{(2\pi)^4}\, \mathcal{P}^\tau(\varepsilon,\boldsymbol{p})\, \left(1+\frac{\gamma p_z}{M}\right) \left(F_T^\tau + \frac{2x'^2\boldsymbol{p}_\perp^2}{Q^2} F_2^\tau\right),$$

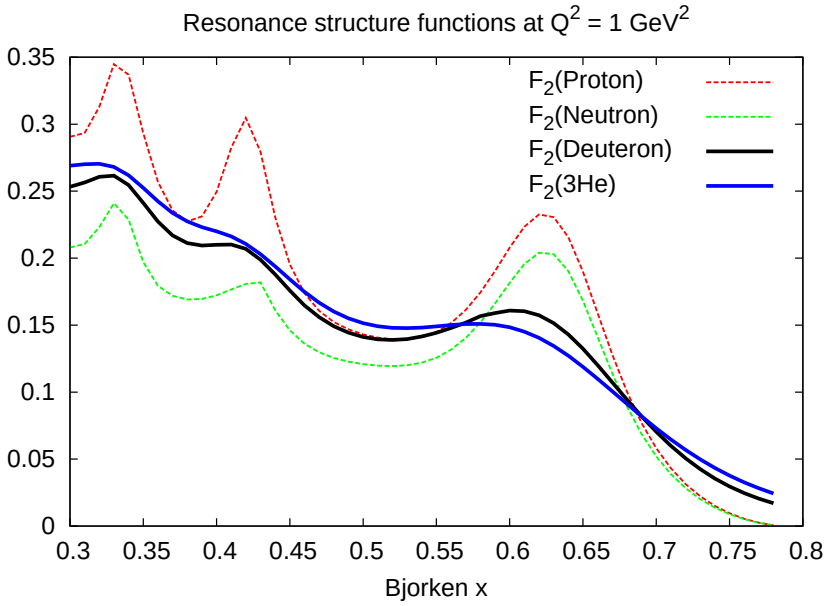
$$F_L^A(x,Q^2) = \sum_{\tau=p,n} \int \frac{\mathrm{d}\varepsilon \mathrm{d}^3\boldsymbol{p}}{(2\pi)^4}\, \mathcal{P}^\tau(\varepsilon,\boldsymbol{p})\, \left(1+\frac{\gamma p_z}{M}\right) \left(F_L^\tau + \frac{4x'^2\boldsymbol{p}_\perp^2}{Q^2} F_2^\tau\right),$$

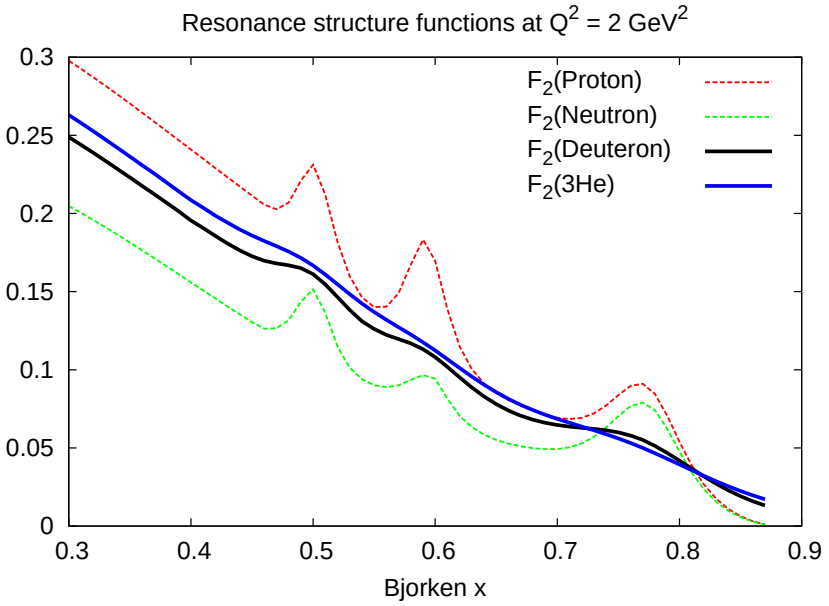
$$\gamma^2 F_2^A(x,Q^2) = \sum_{\tau=p,n} \int \frac{\mathrm{d}\varepsilon \mathrm{d}^3\boldsymbol{p}}{(2\pi)^4}\, \mathcal{P}^\tau(\varepsilon,\boldsymbol{p})\, \left(1+\frac{\gamma p_z}{M}\right) \left(\gamma'^2 + \frac{6x'^2\boldsymbol{p}_\perp^2}{Q^2}\right) F_2^\tau.$$

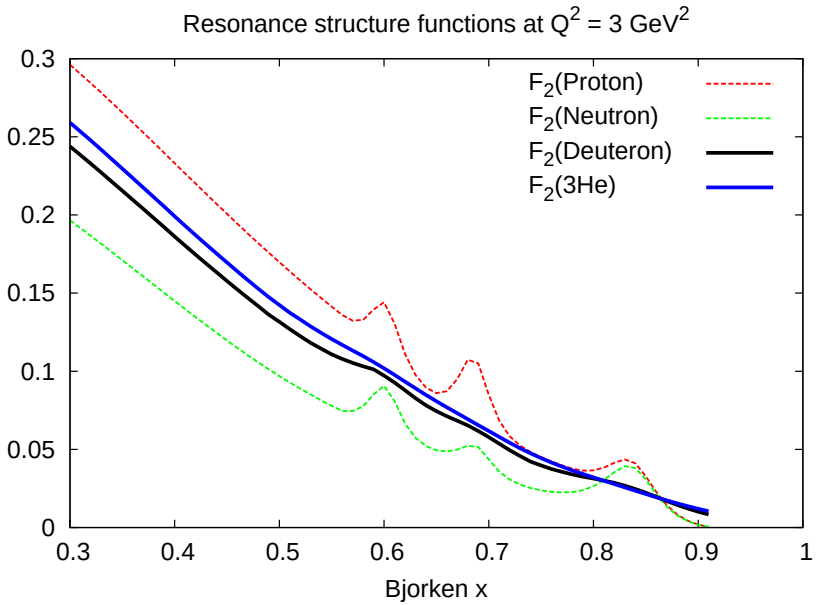
$$xF_3^A(x,Q^2) = \sum_{\tau=p,n} \int \frac{\mathrm{d}\varepsilon \mathrm{d}^3\boldsymbol{p}}{(2\pi)^4}\, \mathcal{P}^\tau(\varepsilon,\boldsymbol{p})\, \left(1+\frac{p_z}{\gamma M}\right) x' F_3^\tau.$$

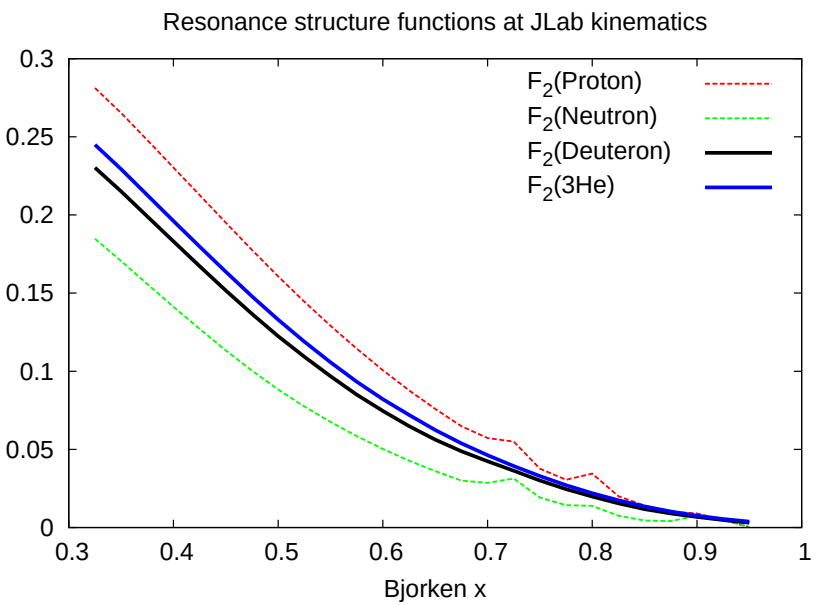
The off-shell nucleon structure functions $F_i(x',Q^2,p^2)$ depend on $x' = Q^2/(2p{\cdot}q) = x/[1+(\varepsilon+\gamma p_z)/M]$, momentum transfer square Q^2 and the virtuality $p^2 = (M+\varepsilon)^2 - \boldsymbol{p}^2$ as additional variable, $\gamma = |\boldsymbol{q}|/q_0 = 1 + 4M^2x^2/Q^2$.

Structure functions in the resonance region

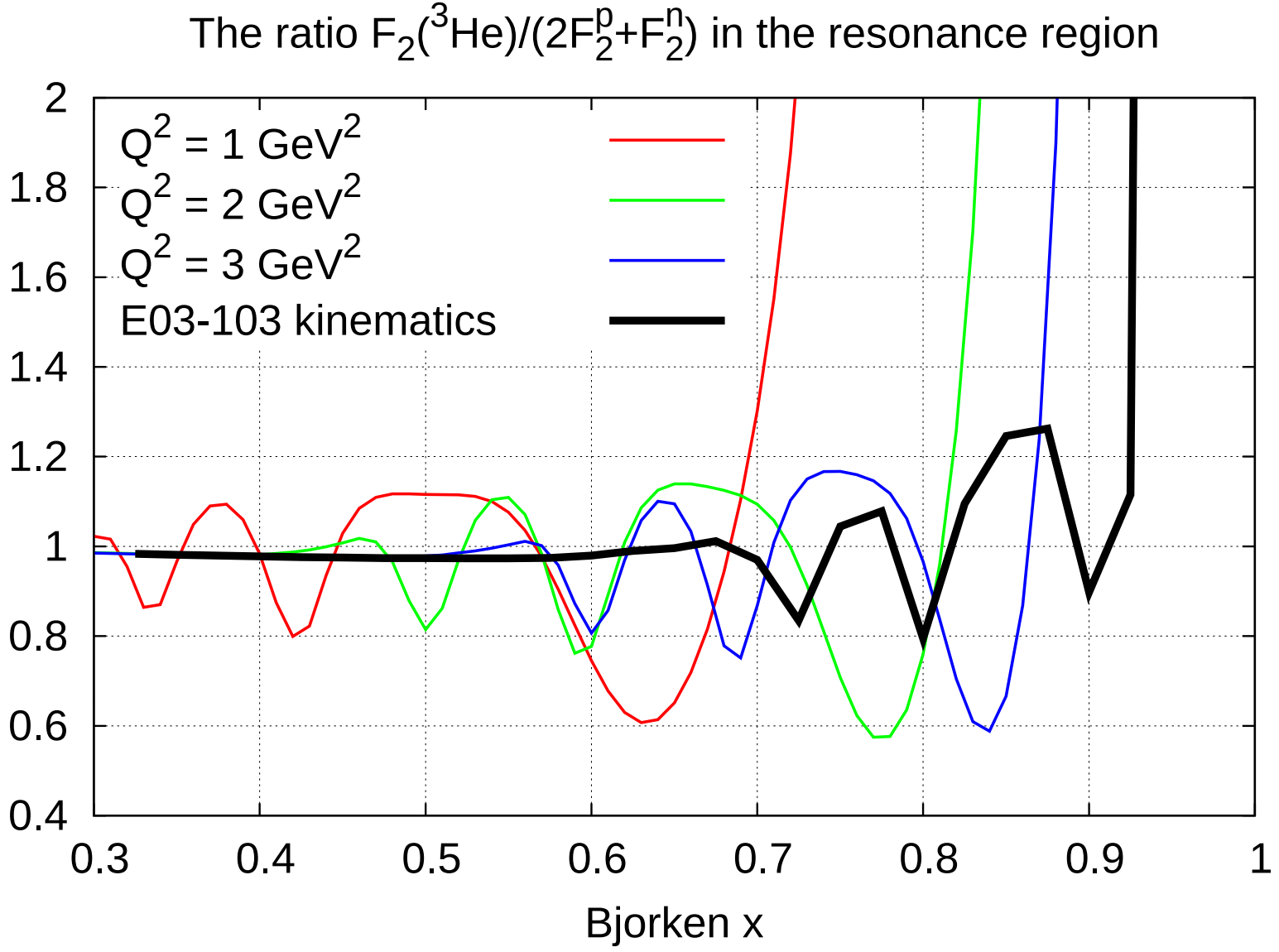




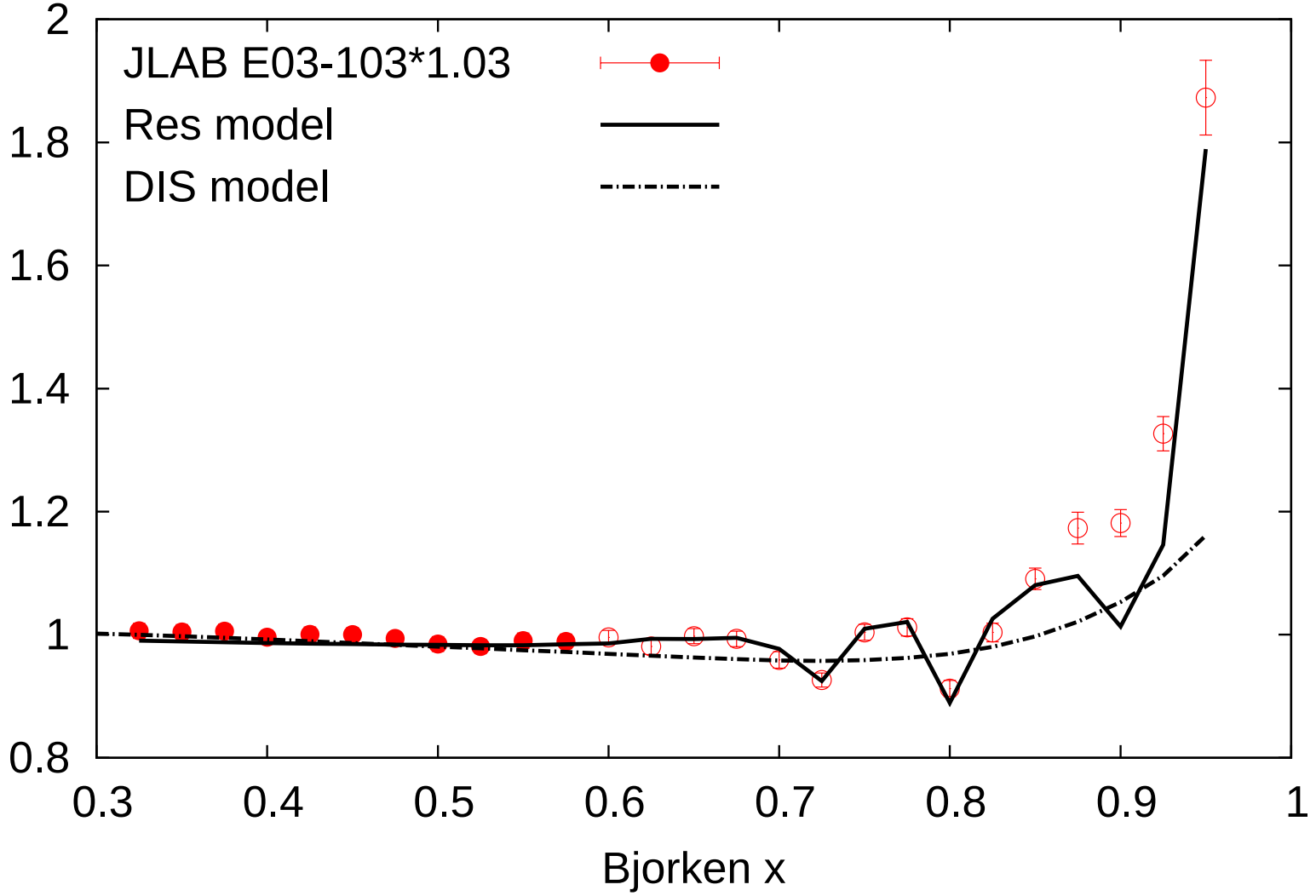


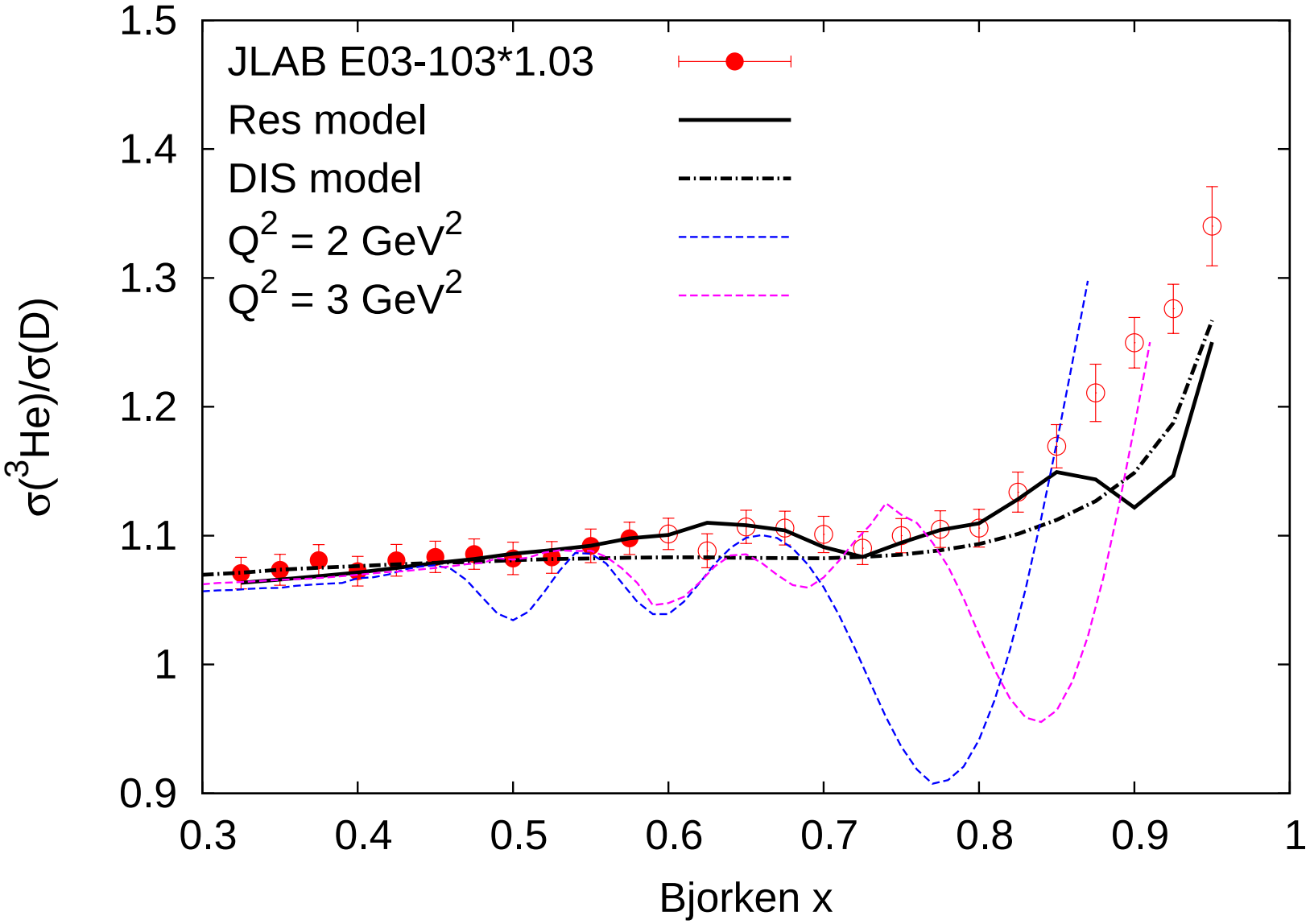


Nuclear ratios in resonance region



The ratio $F_2(^3He)/(F_2^D+F_2^p)$ in the resonance region




JLAB E03-103*1.03
Res model
DIS model
Q2 = 2 GeV2
Q2 = 3 GeV2
σ(3He)/σ(D)
Bjorken x
1.5
1.4
1.3
1.2
1.1
1
0.9
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Summary

- The resonance structure in nuclear ratios is largely smeared out. However the resonanse structure is present at large x for JLab E03-103 kinematics.
- The results on the nuclear ratios with Res and DIS models in our approach are consistent for $W > 2$ GeV and describe data very well. Also we have a good agreement with data in the 3d and the 2nd resonance regions.
- The nuclear ratios calculated with Res and DIS structure functions are not identical for $W < 2$ GeV with noticable difference at large Bjorken x (low W). The resonance structure in nuclear ratios appears mainly because of the deuterium in the denominator.
- While the higher resonance region is described well, the $\Delta(1232)$ region is not yet fully understood. We anticipate a noticable correction from QE contribution in the $\Delta(1232)$ region.