Dark Matter: Galaxy Formation, Small Scale Crisis, and WDM

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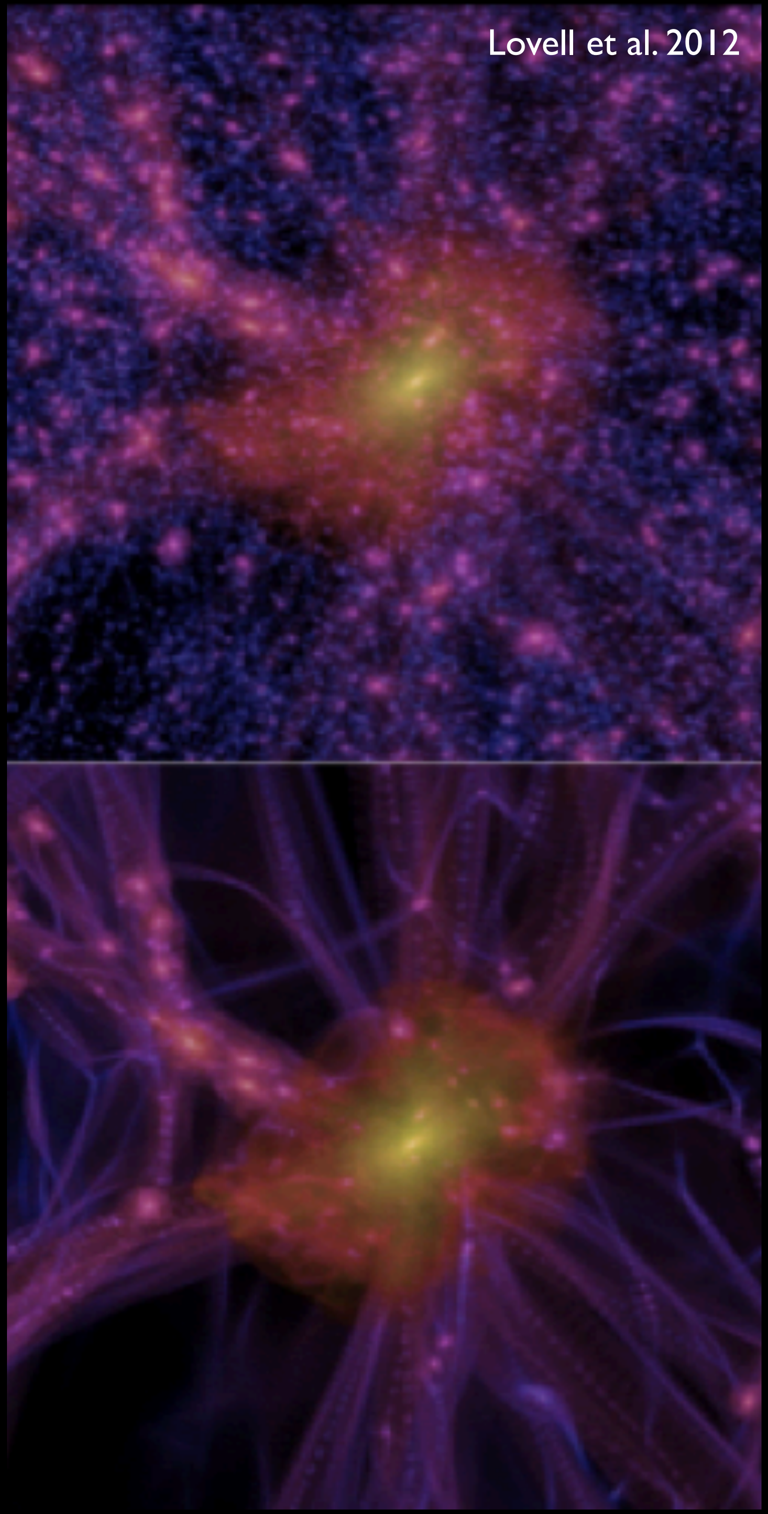
Outline



Evidences for DM
- rotation curves
- galaxy clusters
- growth of perturbatios from CMB
- concordance cosmology

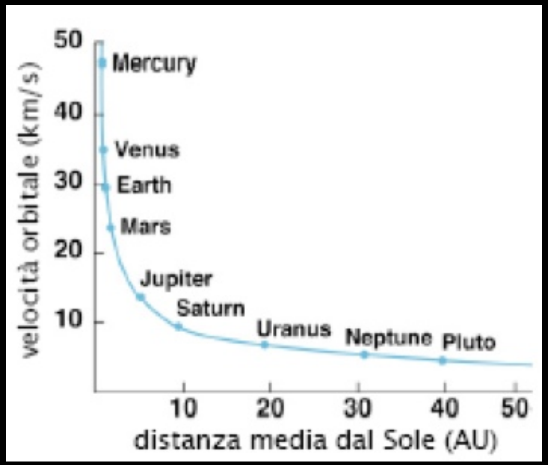
The Impact of the mass DM particles on the formation of cosmic structues

Galaxy Formation in Cold Dark Matter:
The small-scale crisis

Galaxy Formation in Warm Dark Matter scenarios

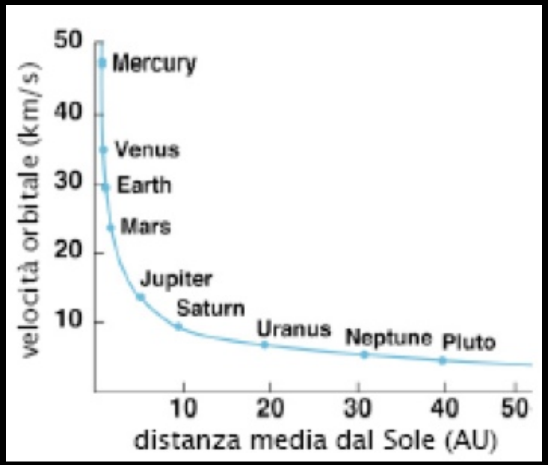
Comparing WDM constraints from galaxy formation with other bounds

Galaxy rotation curves



$$v_c = \frac{\sqrt{GM}}{\sqrt{r}}$$

Galaxy rotation curves



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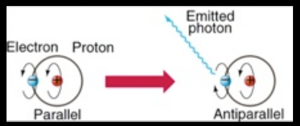
density =10-50 atoms/cm^3



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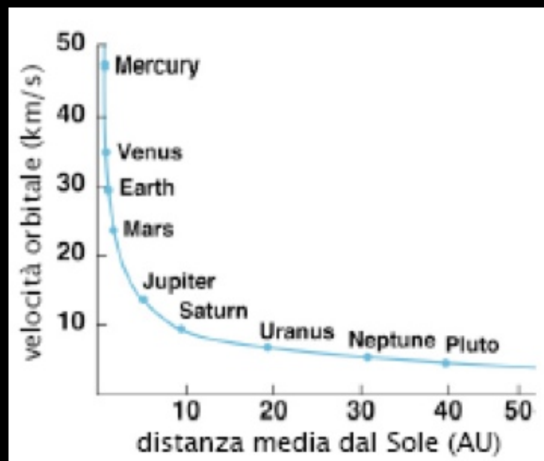

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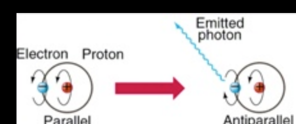
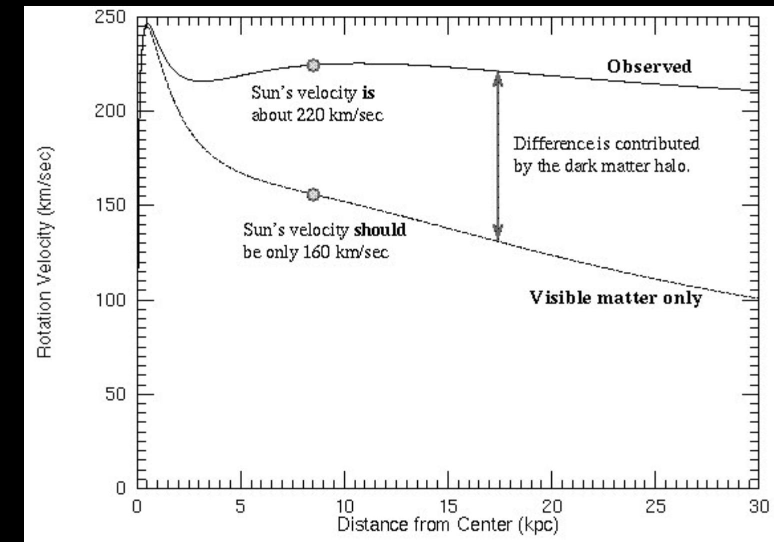
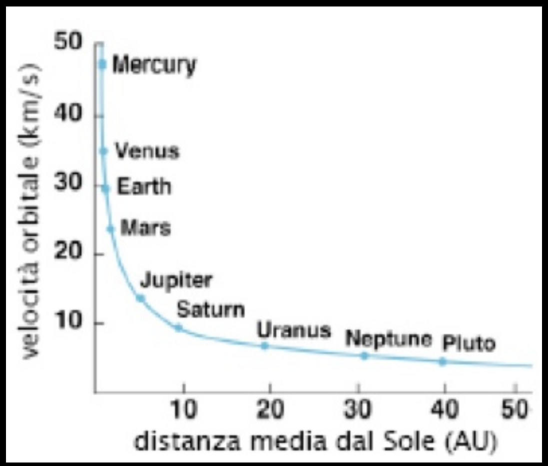




The gravity of the visible matter in the Galaxy is not enough to explain the high orbital speeds of stars in the Galaxy. For example, the Sun is moving about 60 km/sec too fast. The part of the rotation curve contributed by the visible matter only is the bottom curve. The discrepancy between the two curves is evidence for a **dark matter halo.**

Galaxy rotation curves



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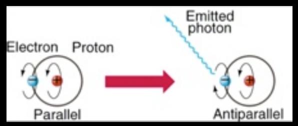
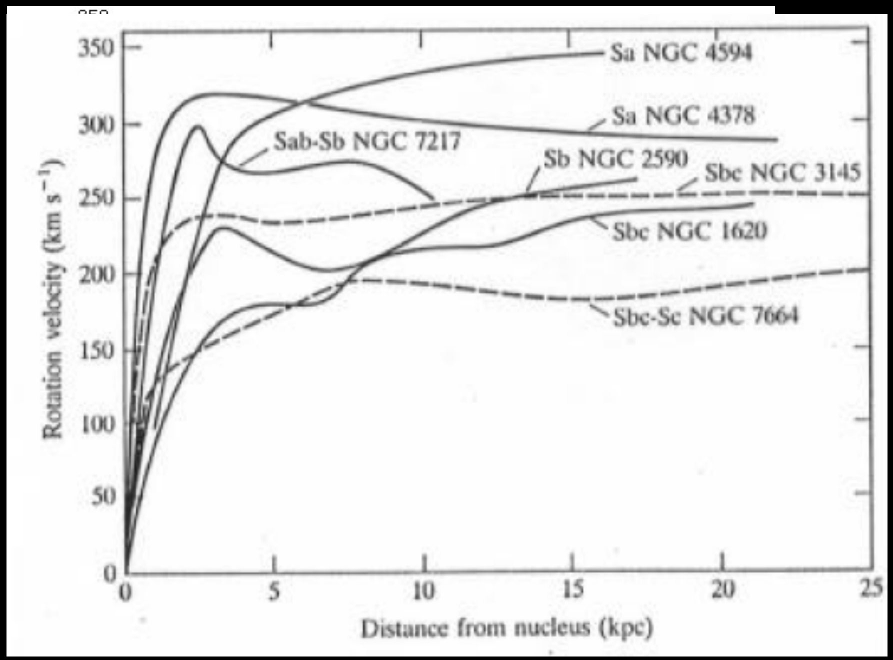
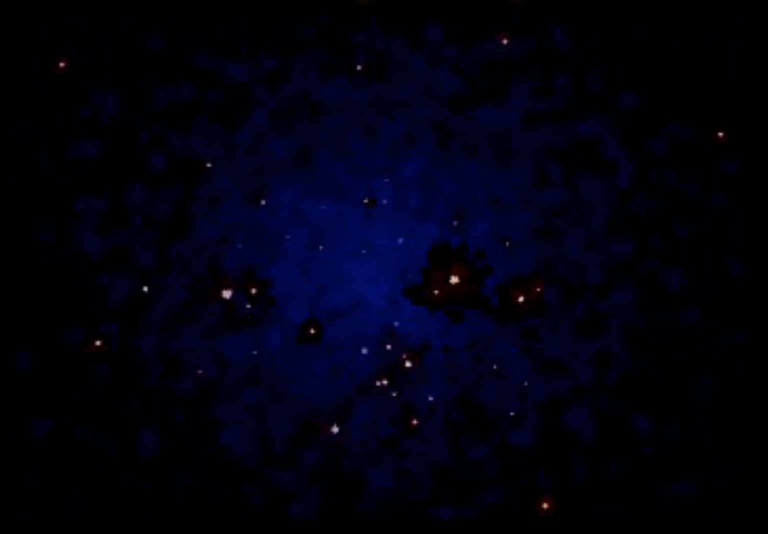




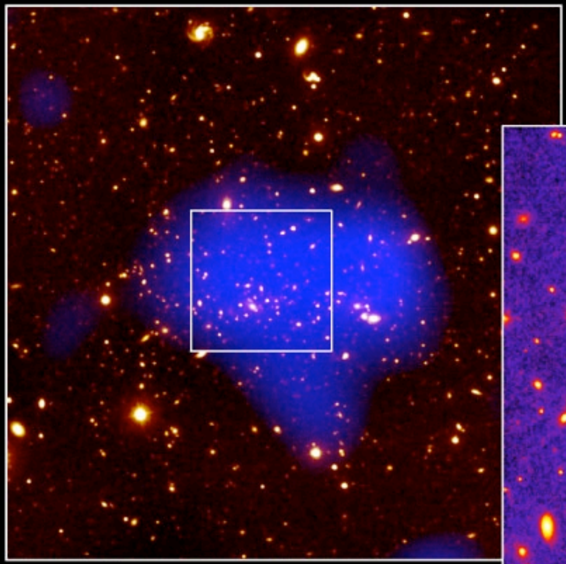
M/L≈10-50

Galaxy clusters: velocity dispersions + X-ray temperature



$$\langle v^2 \rangle = \frac{GM}{R}$$

$$KT = \frac{GM}{R} / \mu m_p$$

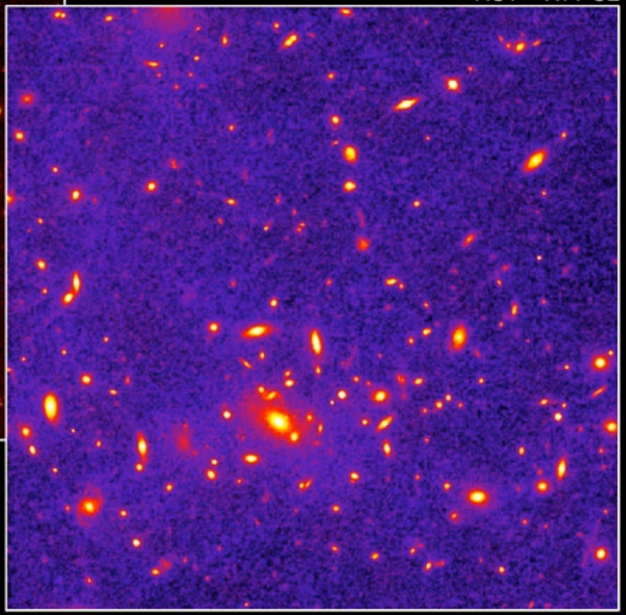


Ground + X-ray

PRC98-26 • August 19, 1998
STScI • OPO
M. Donahue (ST ScI) and NASA

Distant Galaxy Cluster
MS1054-0321

HST • WFPC2

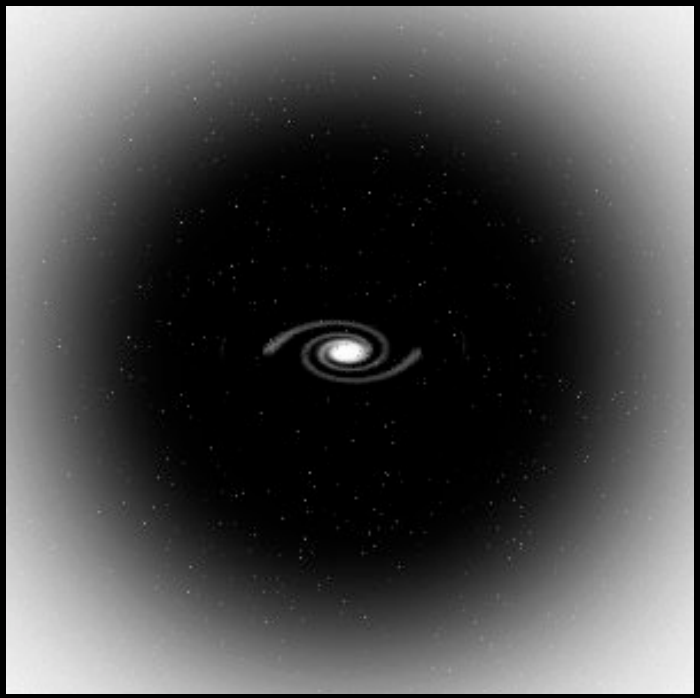


$$L_X(E) = A \frac{n^2}{\sqrt{KT}} e^{-\frac{E}{kT}}$$

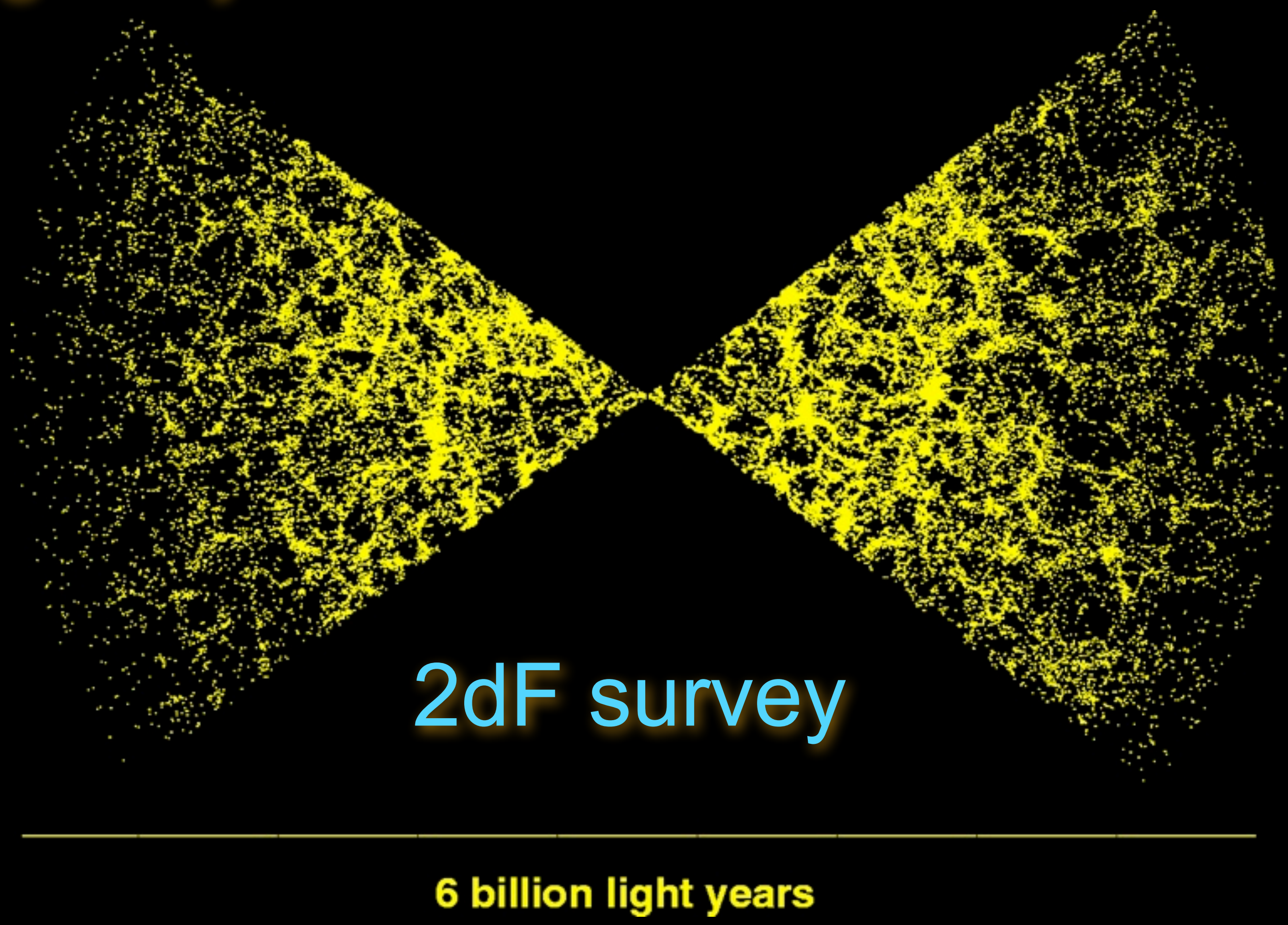
M/L~100

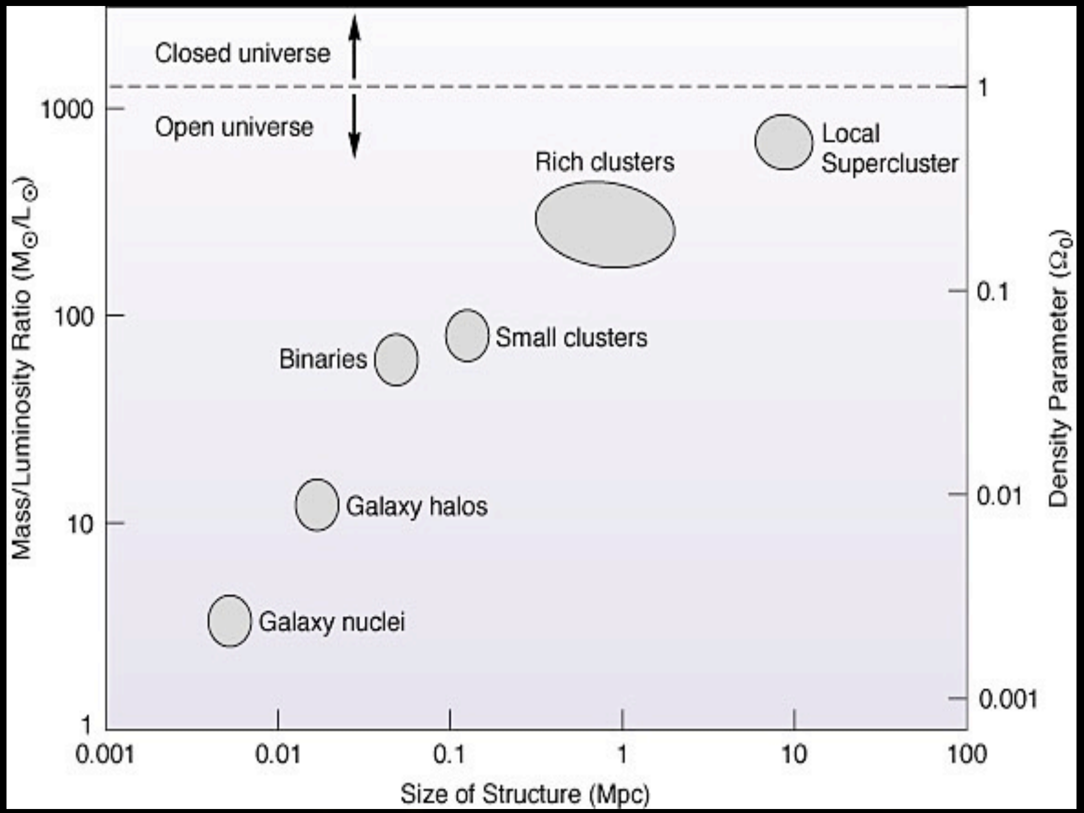
Dark Matter

M/L~100

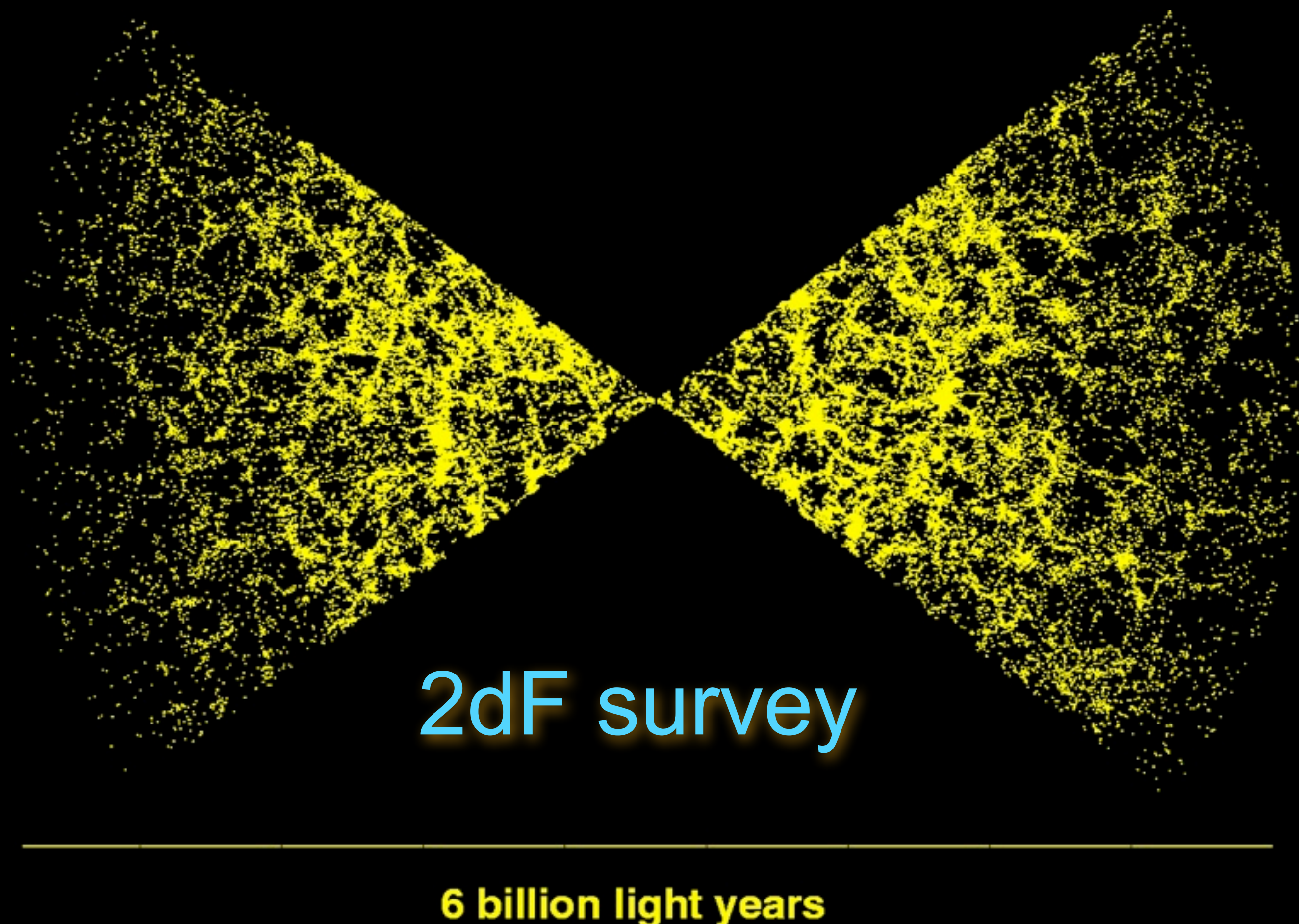


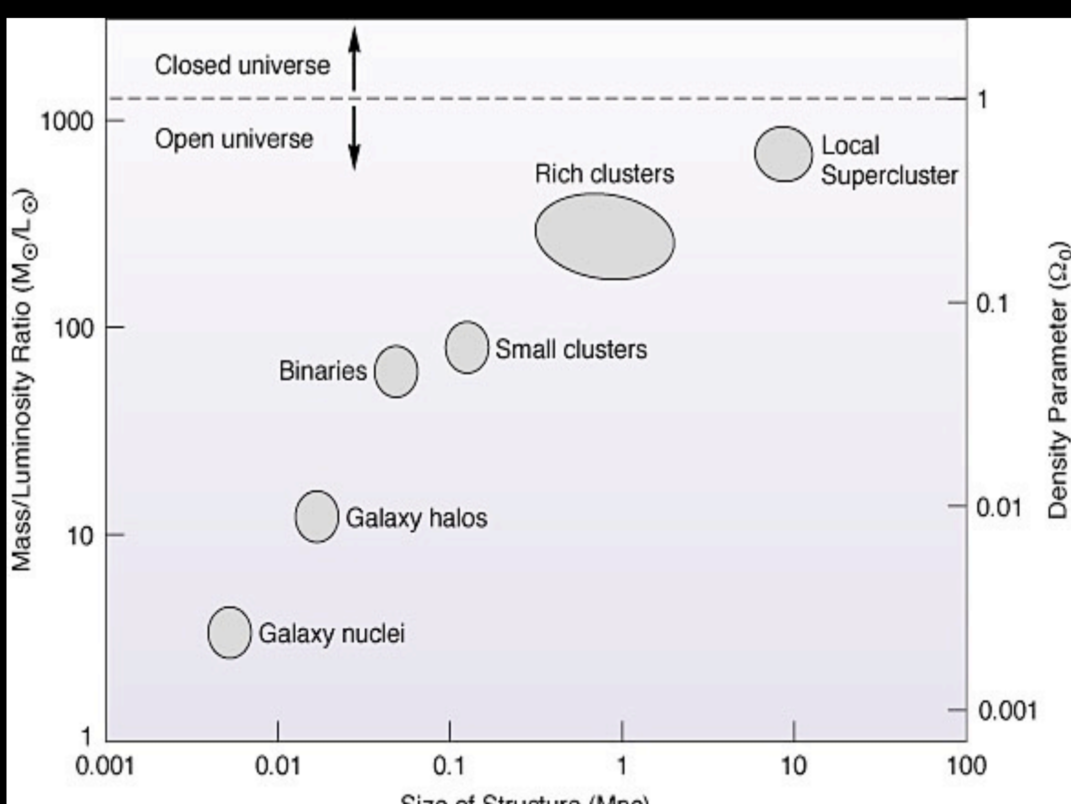
Large Scale Structures
galaxy distribution over scales ~100 Mpc





Current DM
Budget: $\Omega \approx 0.24$





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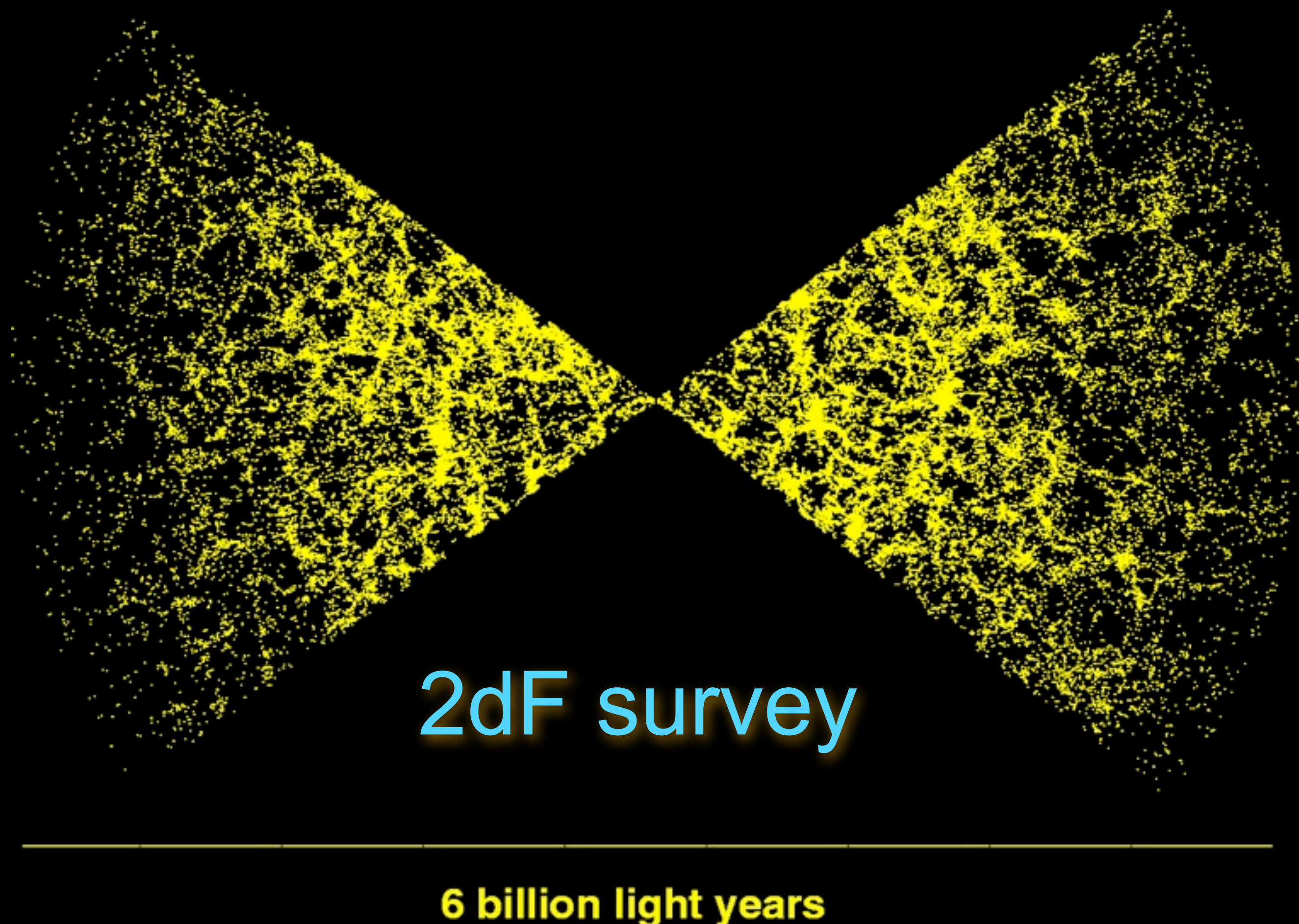

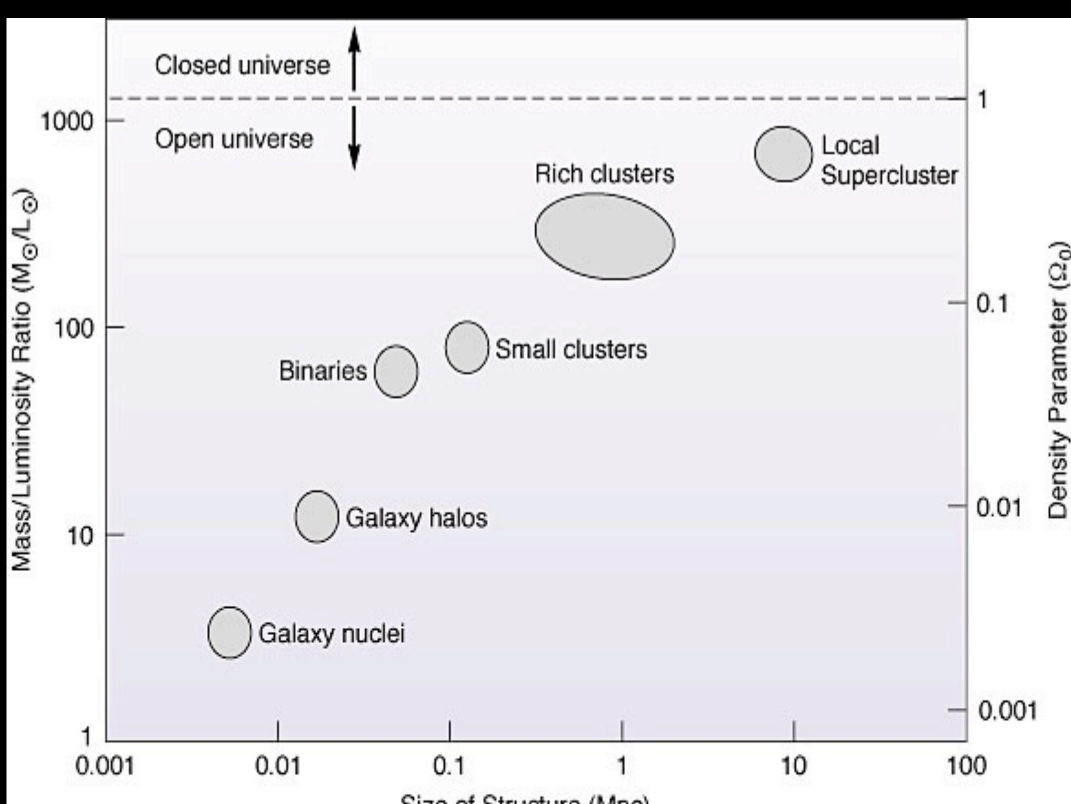

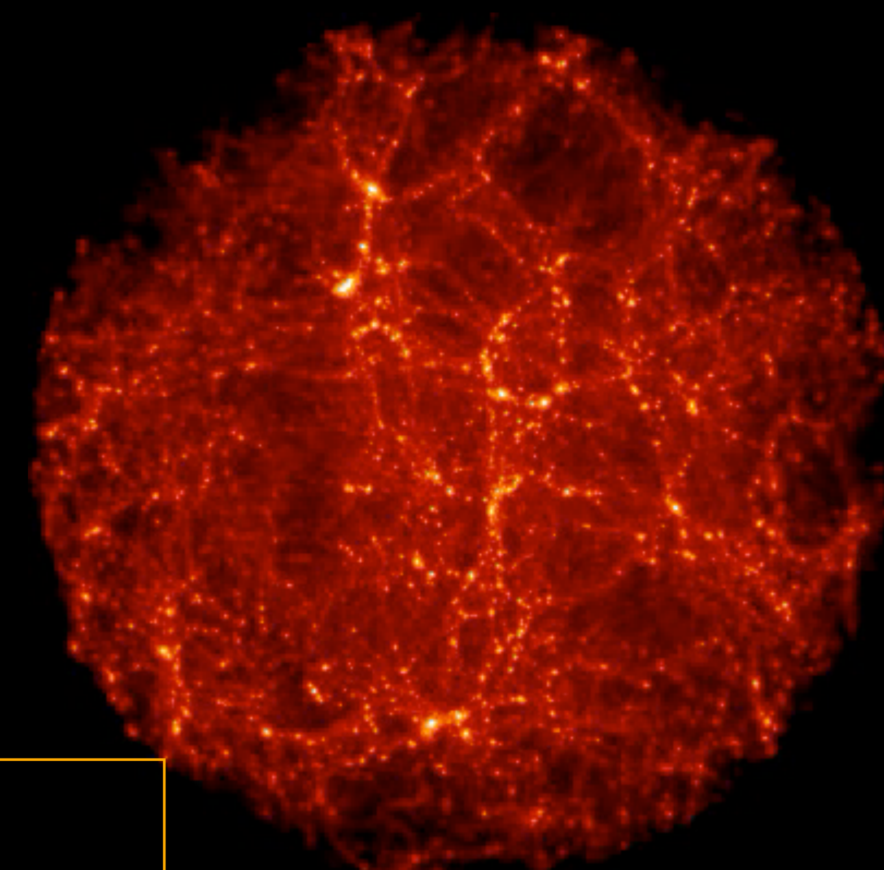

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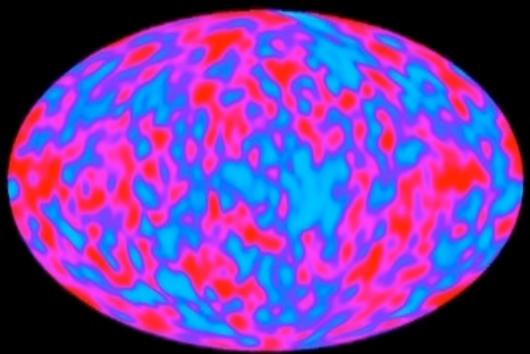
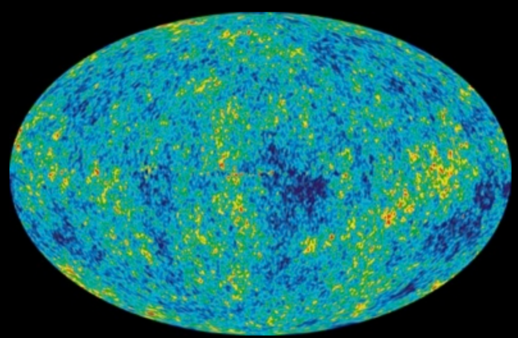
Concordance Cosmology

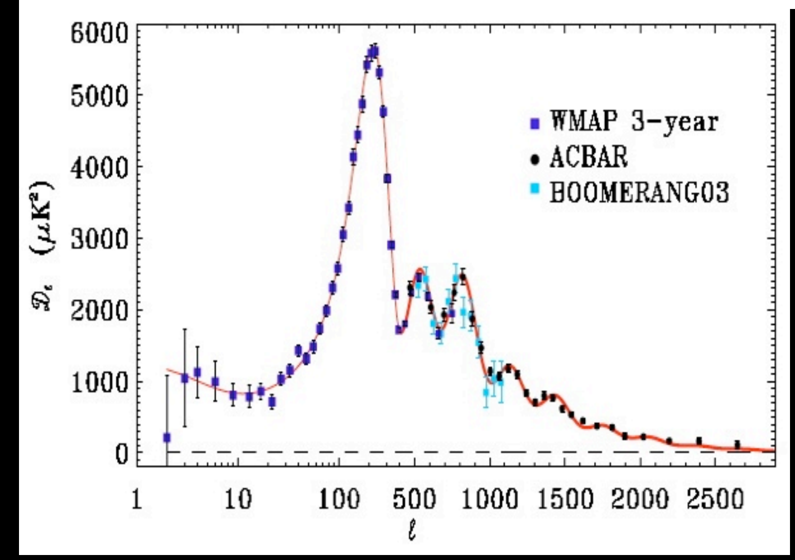




$$C(\theta)=\left\langle \frac{\delta T}{T}(\alpha)\frac{\delta T}{T}(\alpha+\theta)\right\rangle$$

Express this as sum of Legendre polynomials

$$C(\theta)=\frac{1}{4\pi}\sum_{l=2}^{\infty}a_l^2(2l+1)P_l(\cos\theta)$$

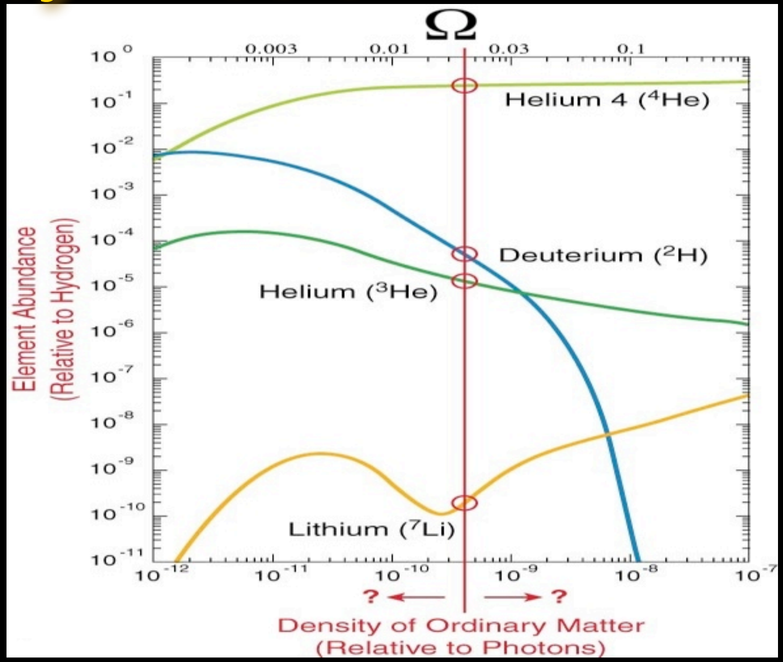


Amplitude of CMB fluctuations

$\Omega_b=0.04$

Cosmic Nucleosynthesis

$$\begin{aligned}
p+\mathrm{D} &\rightleftharpoons \mathrm{He}^3+\gamma\\
n+\mathrm{D} &\rightleftharpoons \mathrm{T}+\gamma\\
n+\mathrm{He}^3 &\rightleftharpoons p+\mathrm{T}\\
p+\mathrm{T} &\rightleftharpoons \mathrm{He}^4+\gamma\\
n+\mathrm{He}^3 &\rightleftharpoons \mathrm{He}^4+\gamma\\
p+\mathrm{Li}^7 &\rightleftharpoons \mathrm{He}^4+\mathrm{He}^4\\
n+\mathrm{Be}^7 &\rightleftharpoons \mathrm{He}^4+\mathrm{He}^4
\end{aligned}$$



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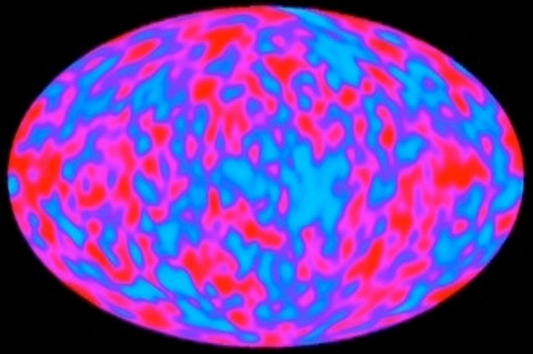

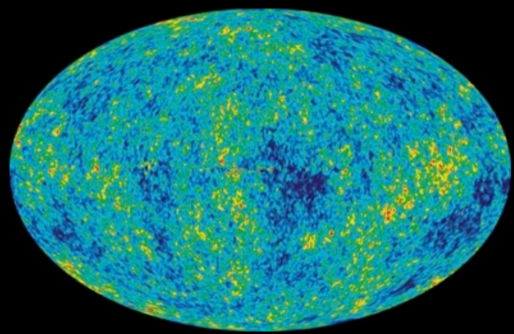

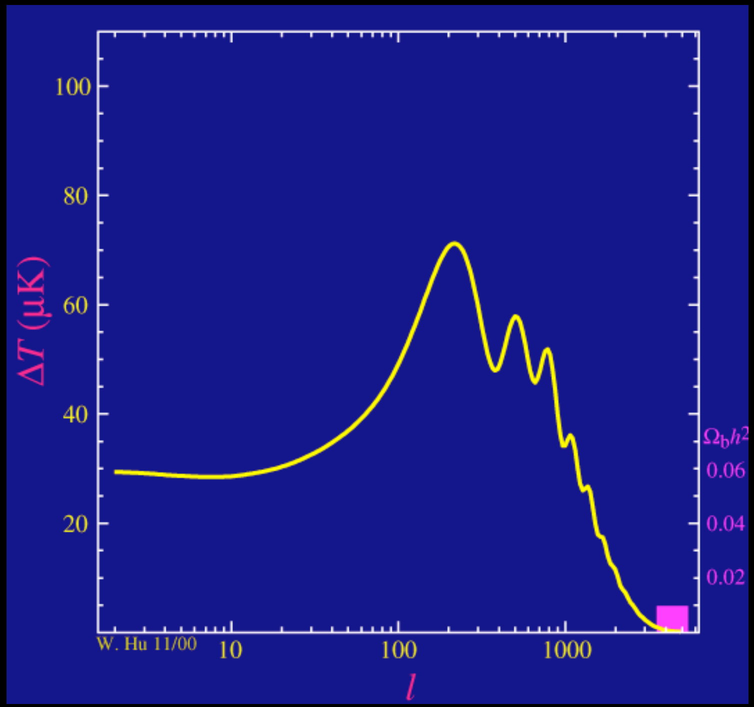

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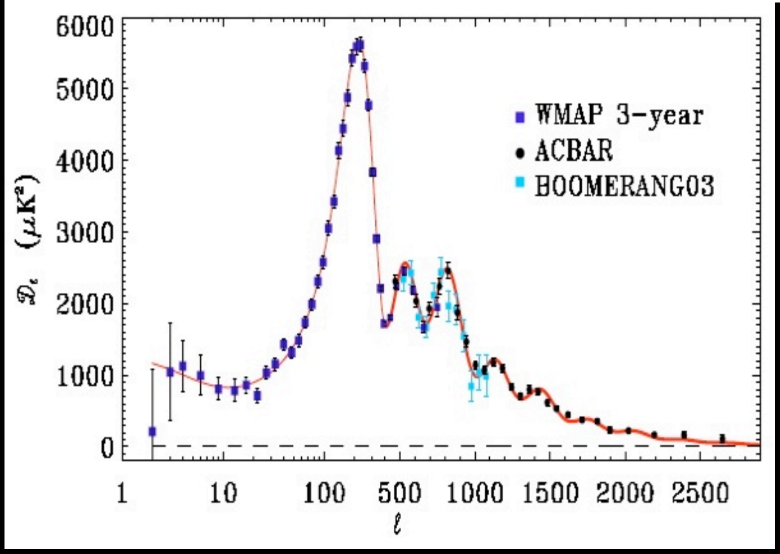

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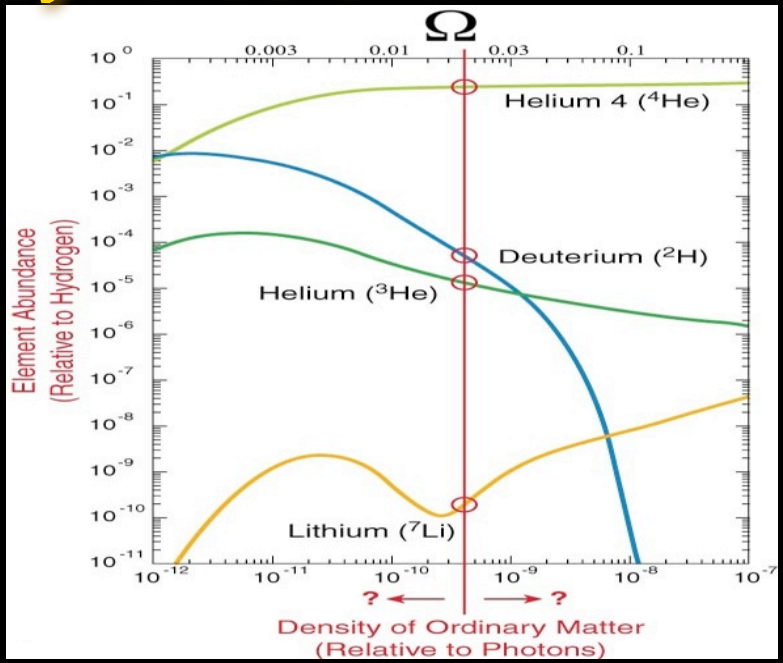
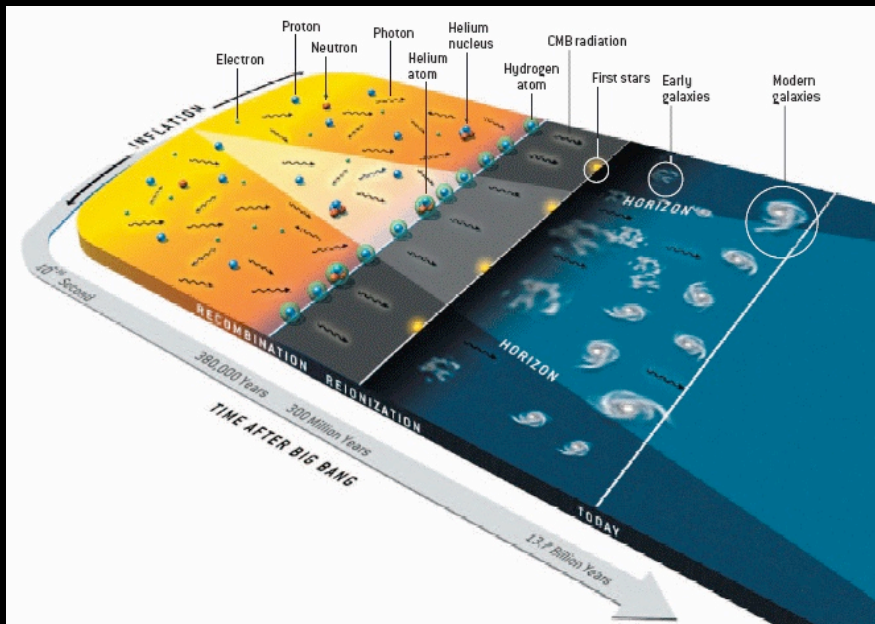


CMB and Growth of perturbations



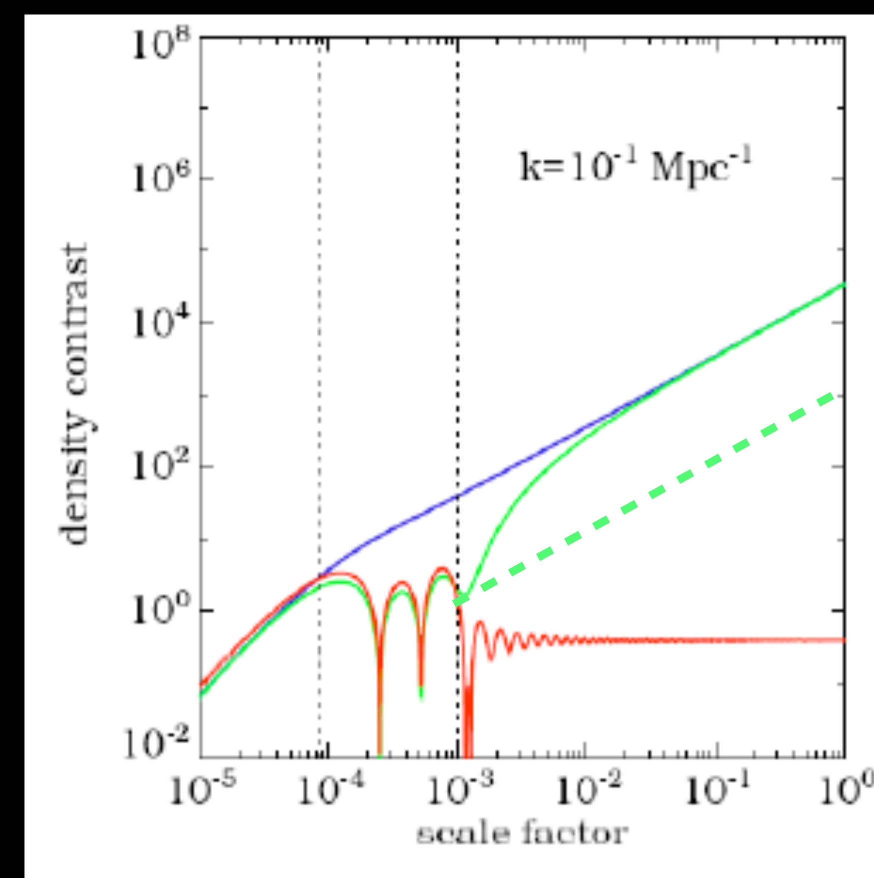
Royal Society Summer 2009 Exhibition, 'Cosmic Origins':

$$\delta(t_{rec}) \sim 10^{-5}$$

$$\delta(t) \sim t^{2/3} \sim (1+z)$$

ordinary Matter (baryons)can only grow after ricombination (z~1000).
They can grow at most a factor 1000. Observed fluctuations at recombination (CMB) $\delta \simeq 10^{-5}$
They cannot grow non-linear ($\delta \simeq 1$)

Dark Matter: starts to grow earlier. At recombination baryons fall into potential wells which are already in place



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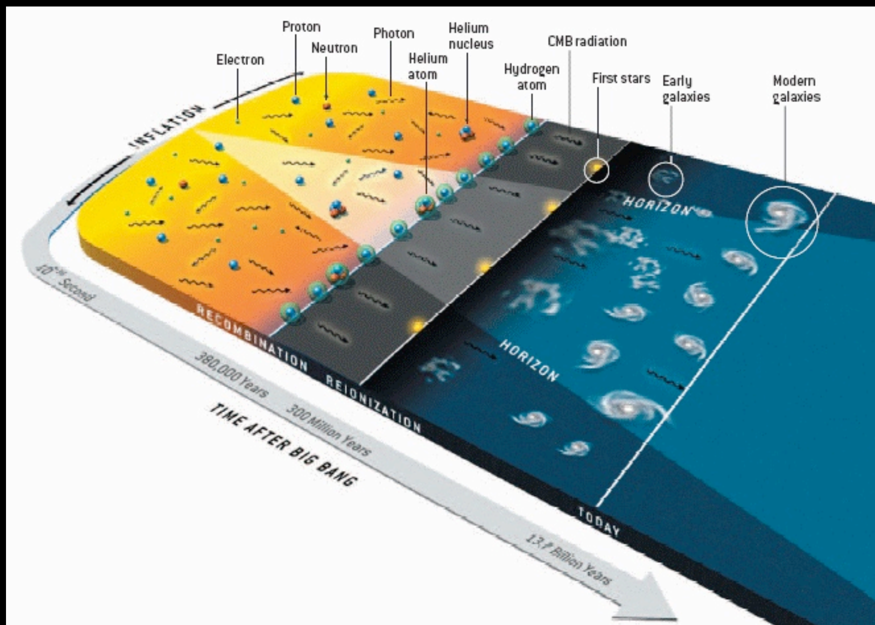

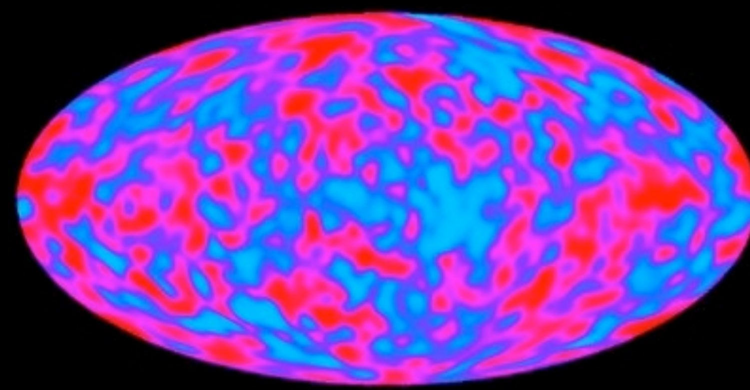

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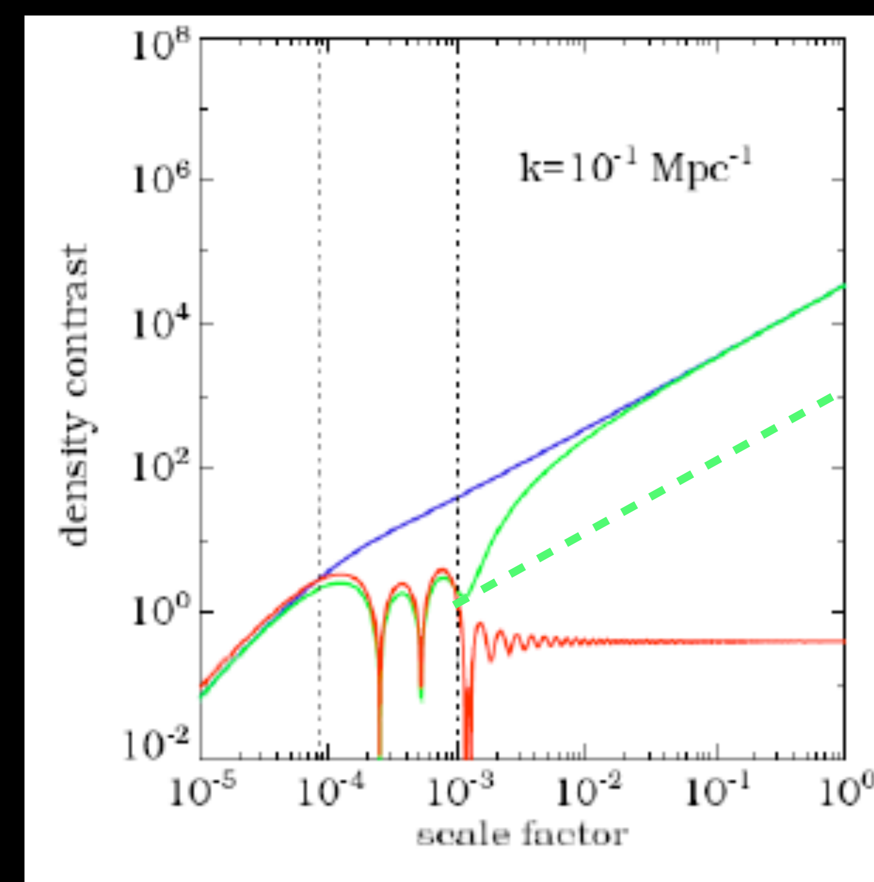
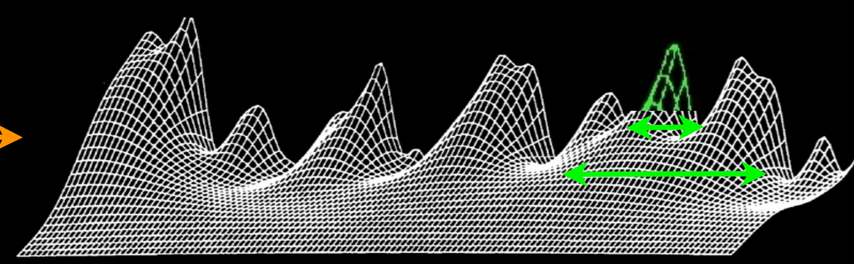
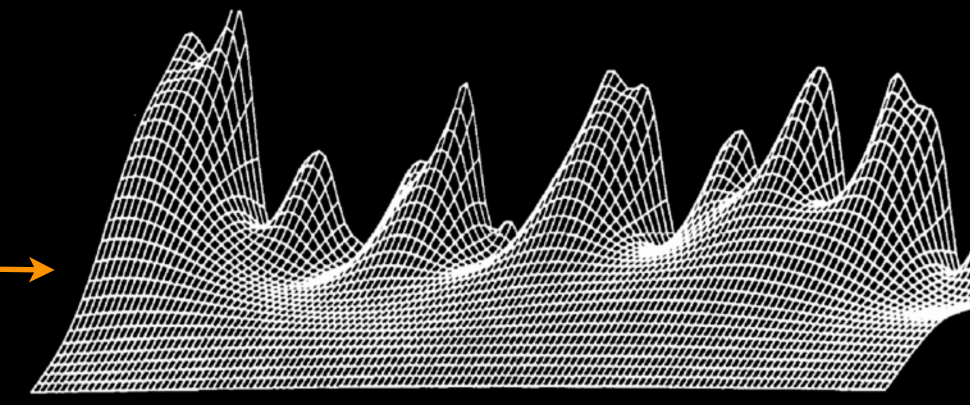


Cosmic Structures form from the collapse of overdense regions in the primordial density field, and grow by gravitational instability





R=2π/k

$$M = \frac{4\,\pi}{3}\,\rho\,R^3$$

$$\langle \delta_M^2 \rangle = \sigma^2(M)\,g(t)$$

Gaussian Random field

$$\delta = \frac{\delta\rho}{\rho}$$

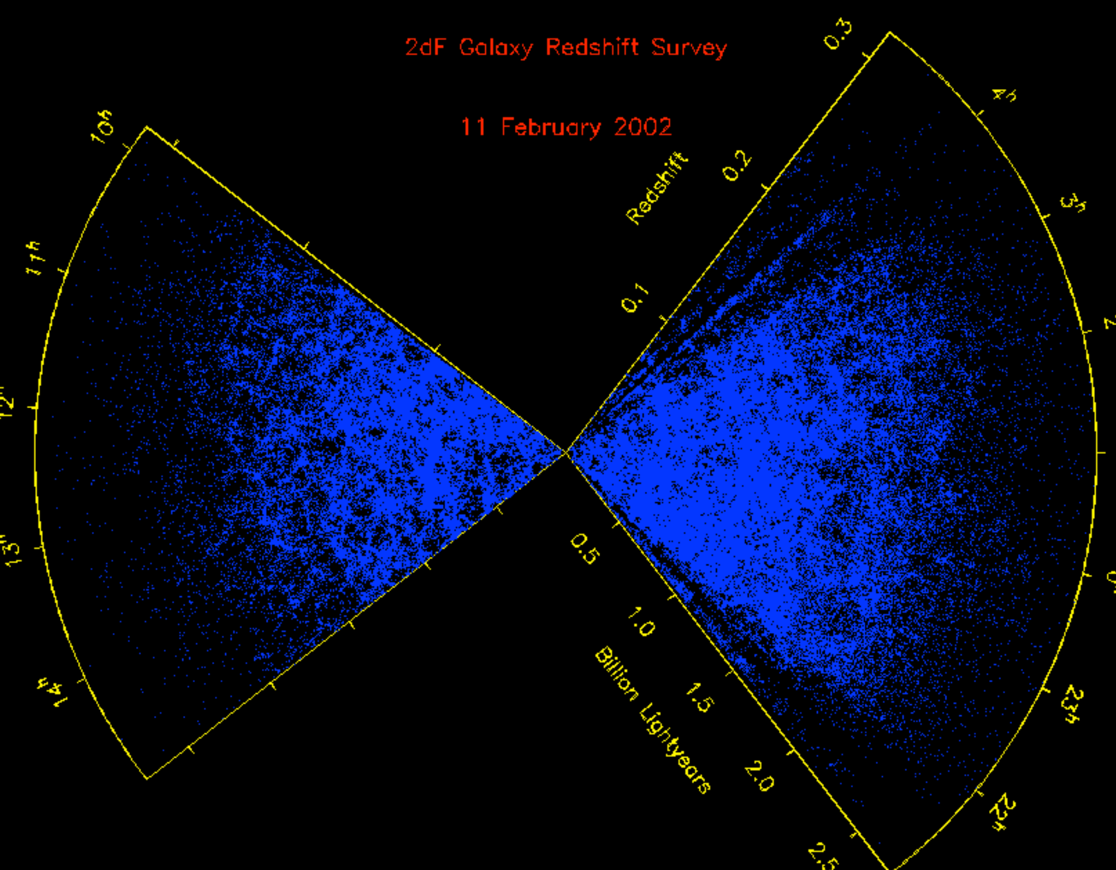
$$p(\delta_k) = \frac{1}{\sqrt{2\pi}\,\sigma_k}\,e^{\frac{\delta_k^2}{2\,\sigma_k^2}}$$

Mean (square) value of perturbations of size R(~1/k) enclosing a mass M

$$P(k) = \frac{1}{V}\,\langle |\delta_k|^2 \rangle$$

$$\sigma_M^2 = \frac{1}{(2\pi)^3\,V}\int^{M\leftrightarrow k} dk\,k^2\,P(k)$$

$$\sigma_M^2 \leftrightarrow P(k)$$



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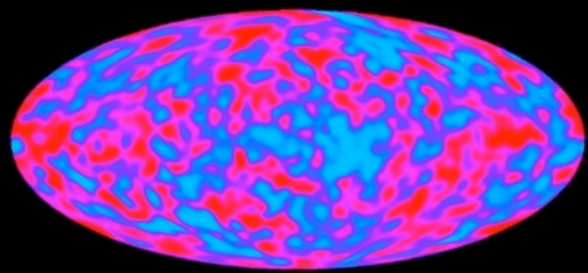

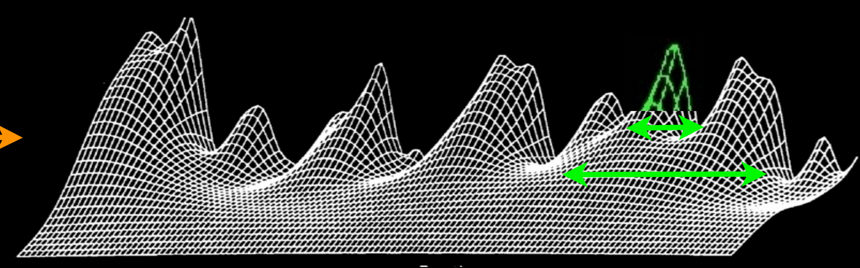

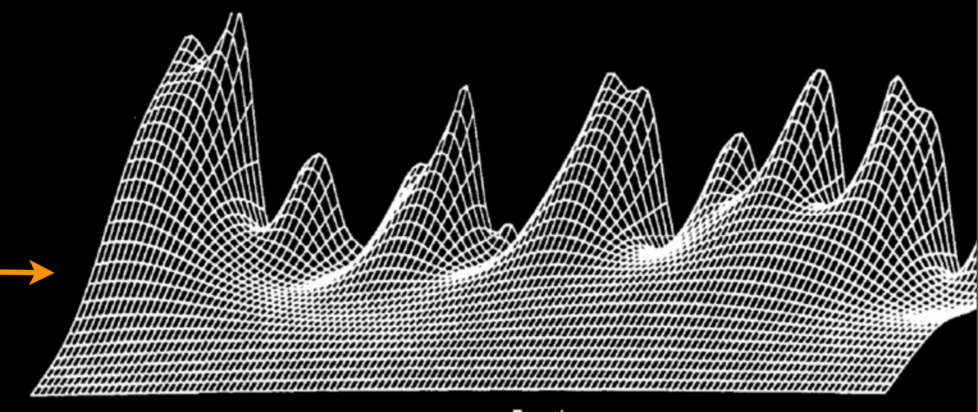

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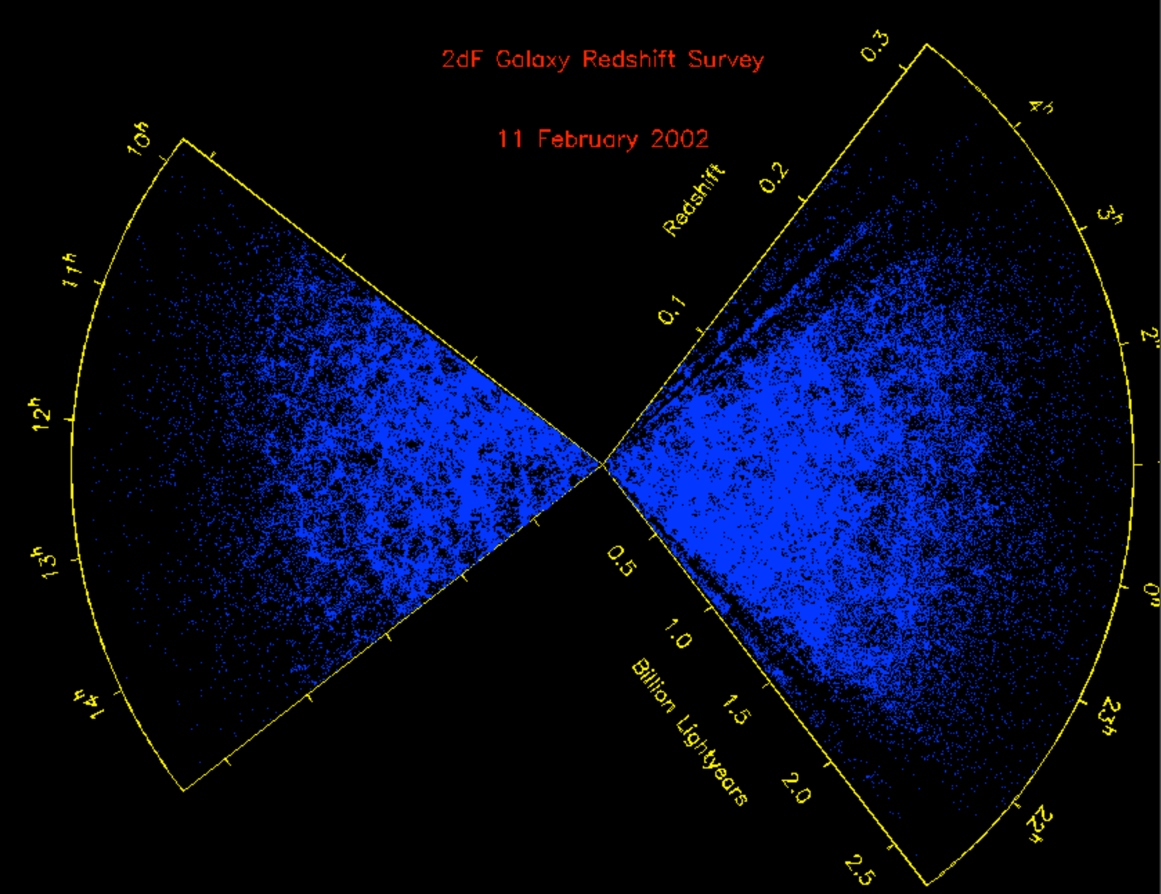
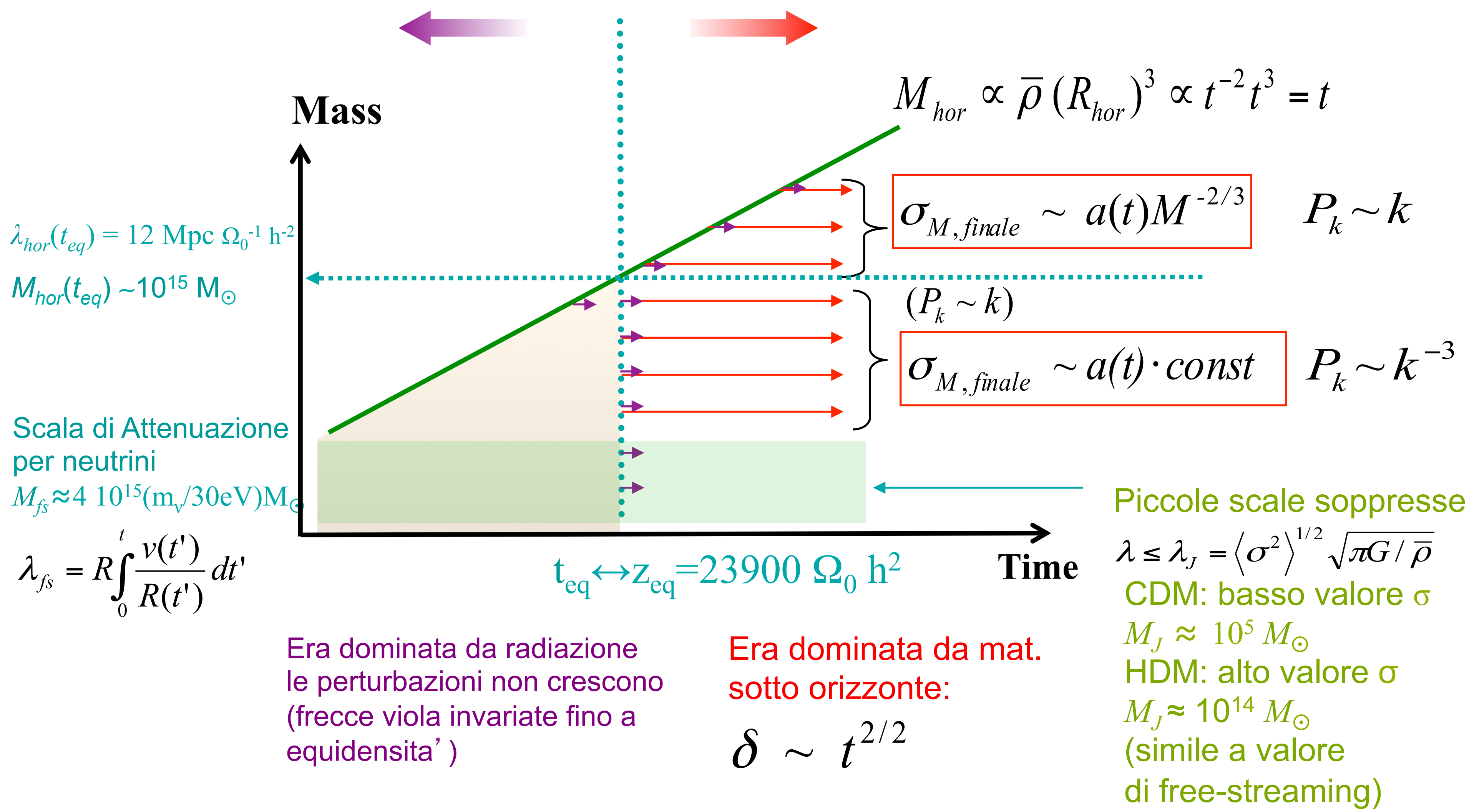
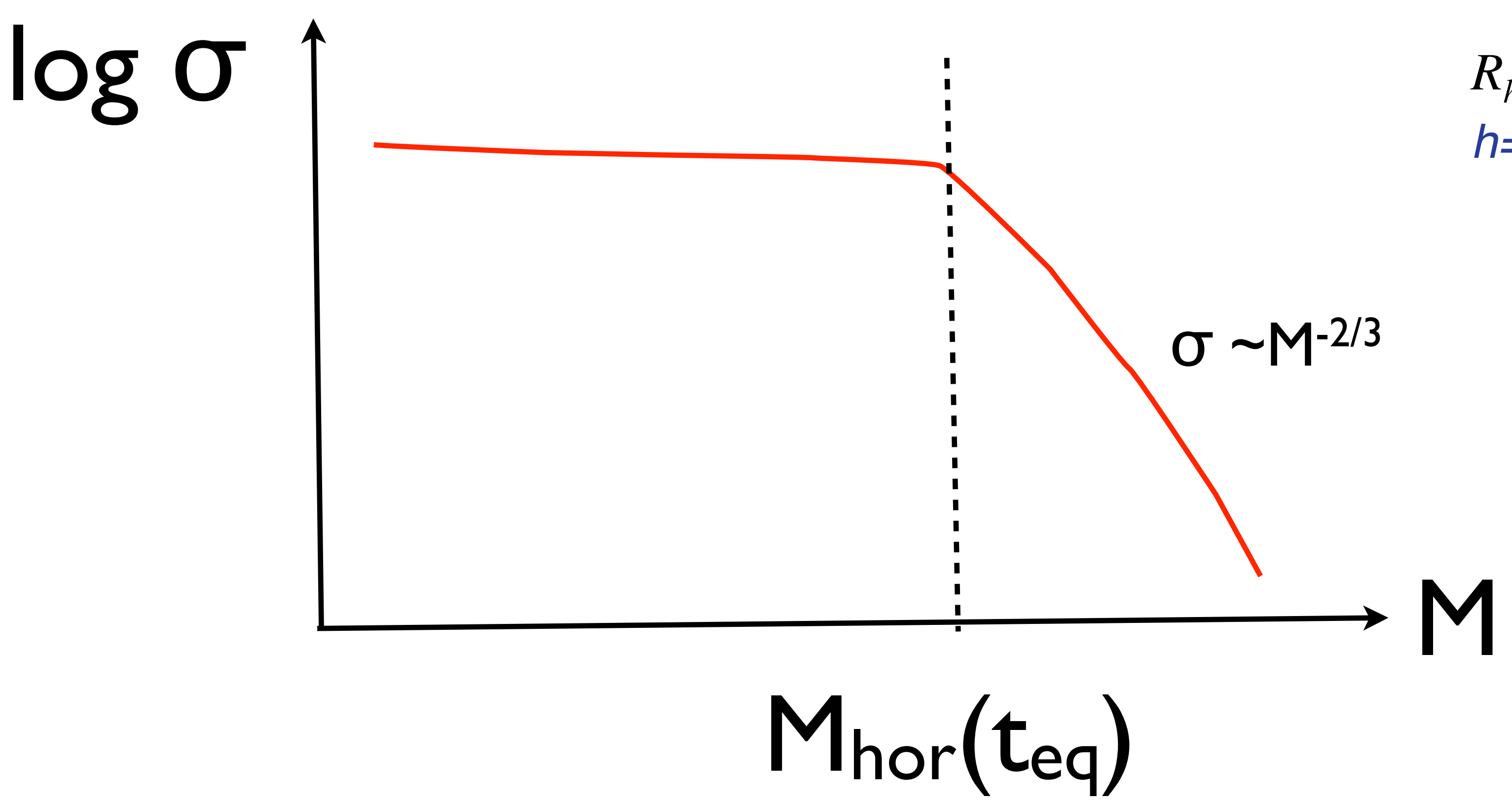




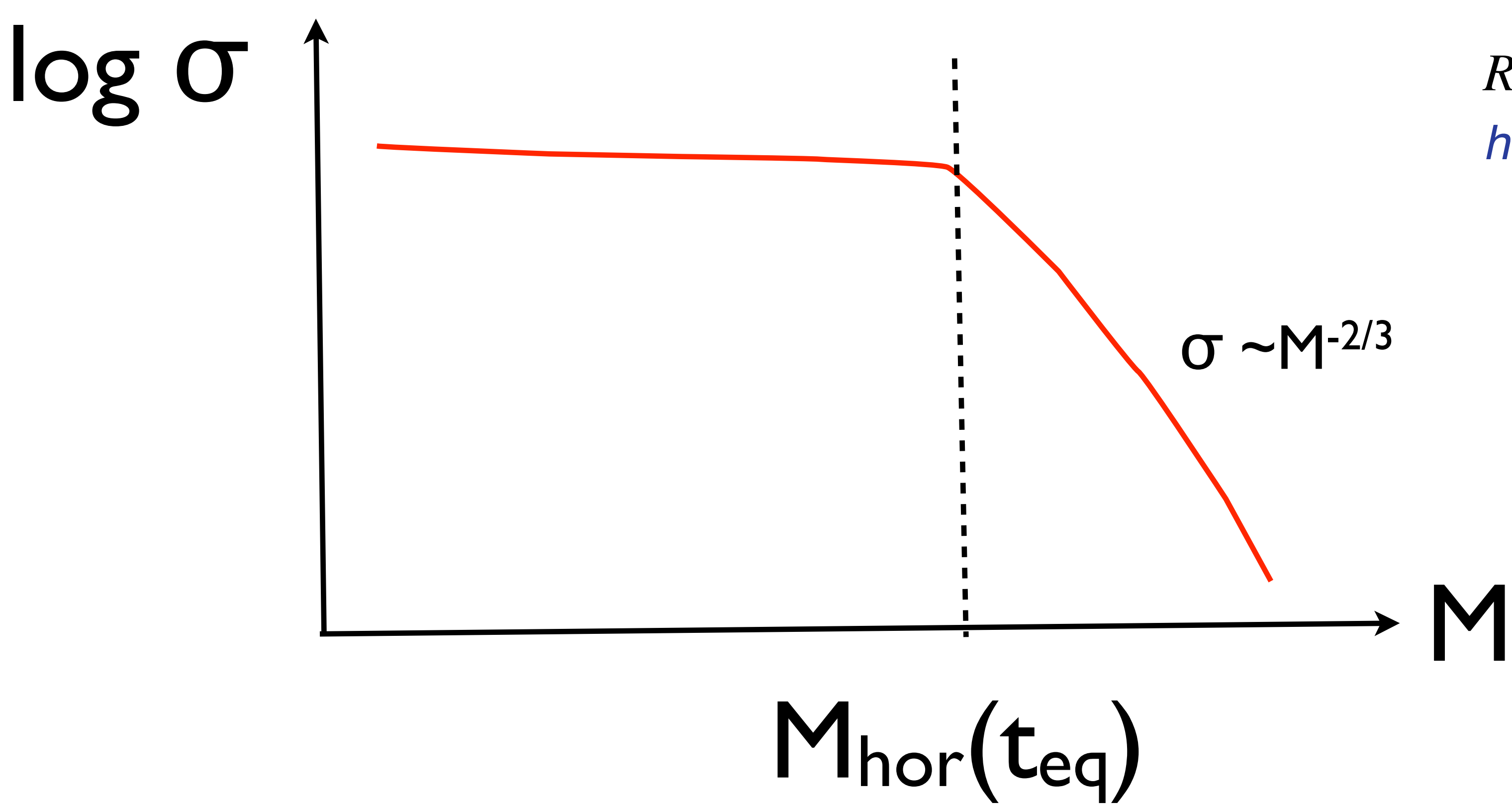
$R_{hor} = 2c\, t_{hor} = 13\ _0^{-1} h^{-2}$ Mpc = 110 Mpc for $_0$=0.3 h=0.7



Perturbations involving scales larger than that of the horizon at the equivalence start to grow later

On average, perturbations on large scales (large masses) have a lower amplitude

σ variance of the density field



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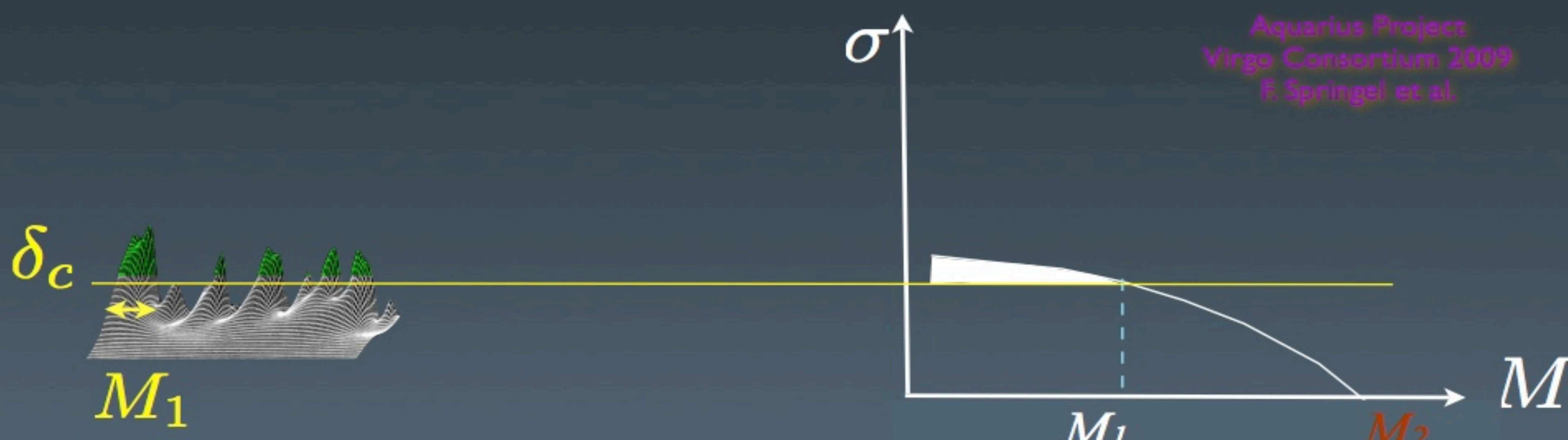
The evolution of DM perturbation

Initial density perturbations constitute a random Gaussian field.

Measurements of the CMB show that its variance is inversely related to their mass scale.

This implies that small scales collapse - on average - at earlier times



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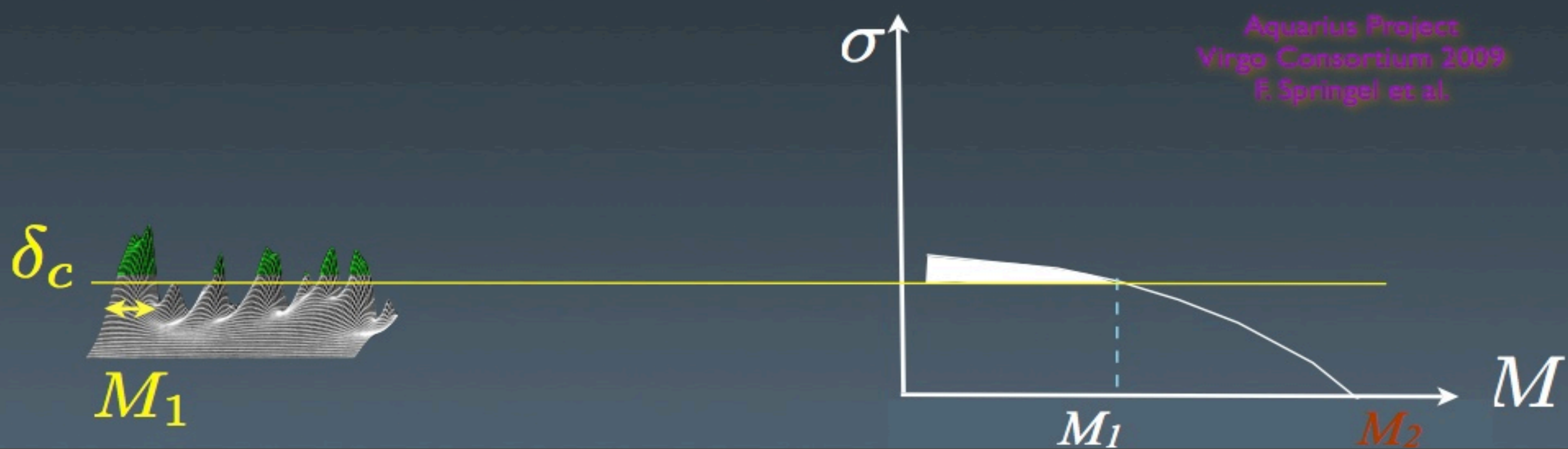

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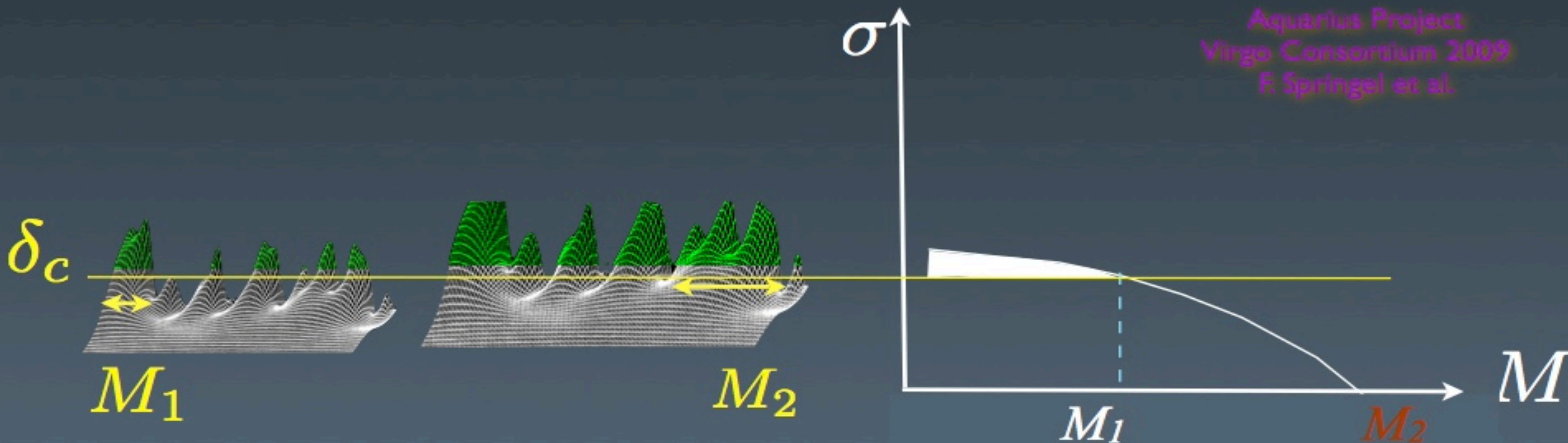
Aquarius Project
Virgo Consortium 2009
F. Springel et al.



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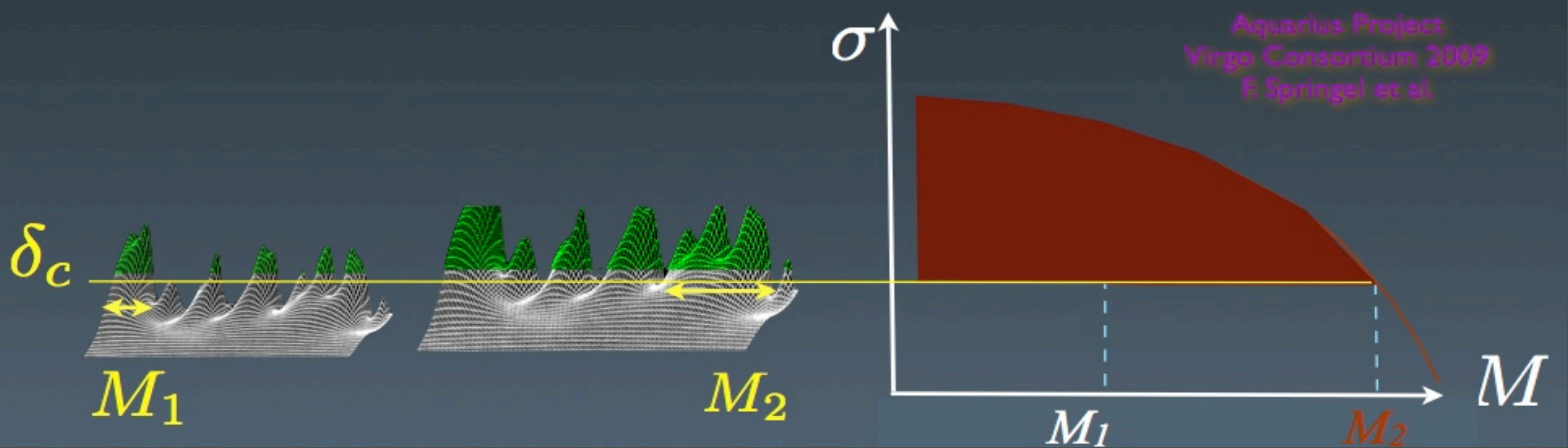


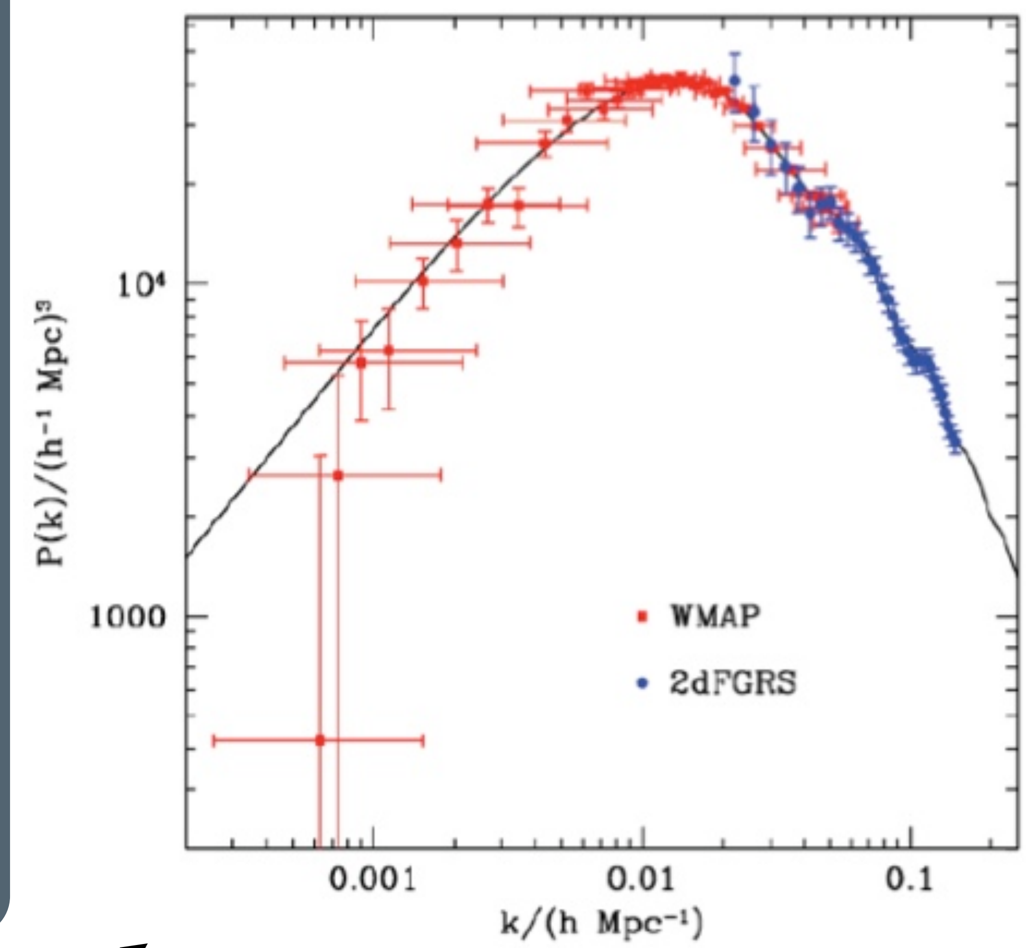
In terms of wavenumber k→Power Spectrum

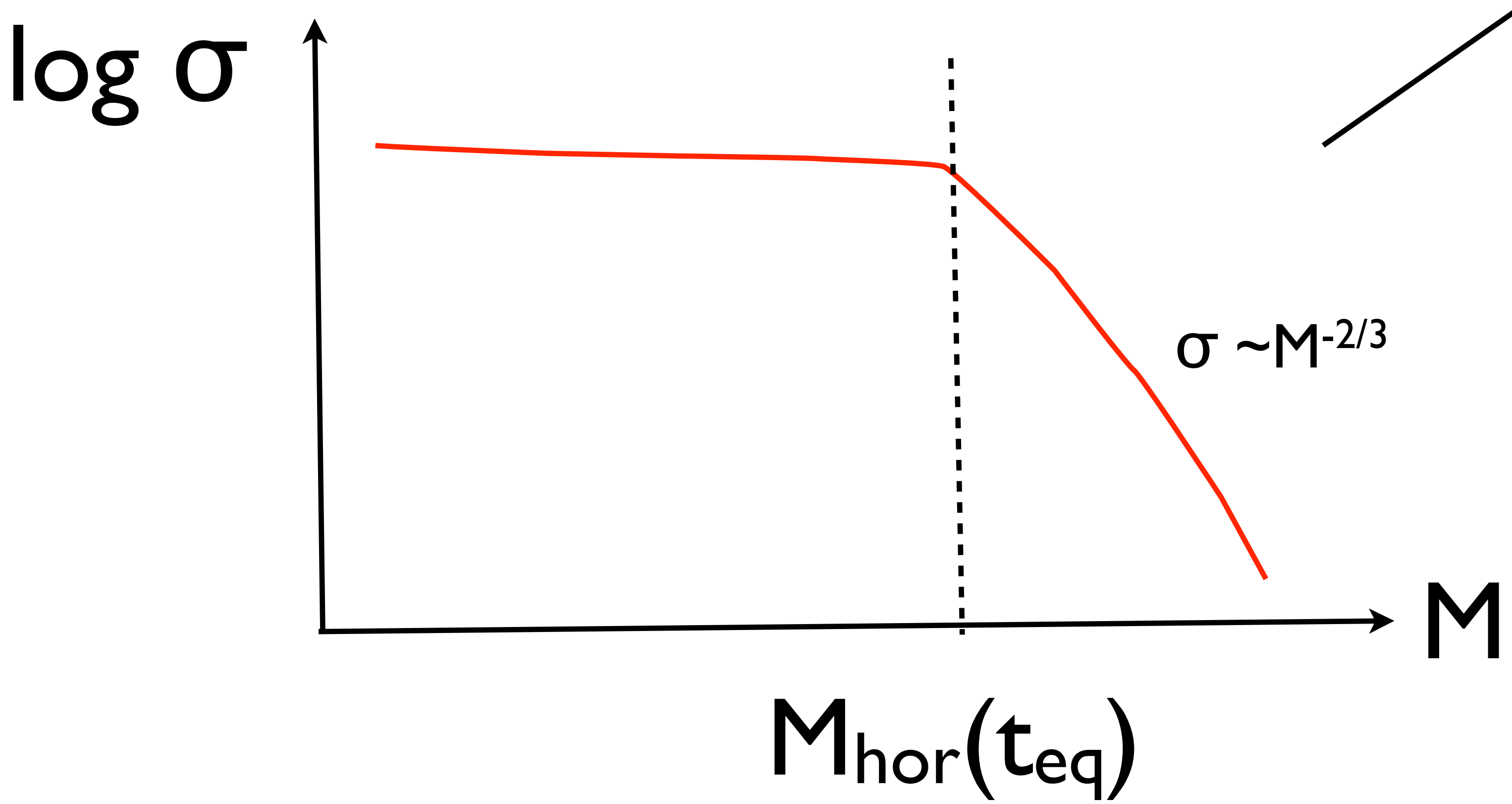
$$\sigma_M^2 = \frac{1}{(2\pi)^3 V} \int^{M \leftrightarrow k} dk\, k^2\, P(k)$$

$$\langle |\delta_k^2 \rangle = P(k) \propto k^2$$

$$\sigma_M \propto M^{-2/3} \qquad P(k) \propto k$$

$$\sigma_M = const \qquad P(k) \propto k^{-}3$$





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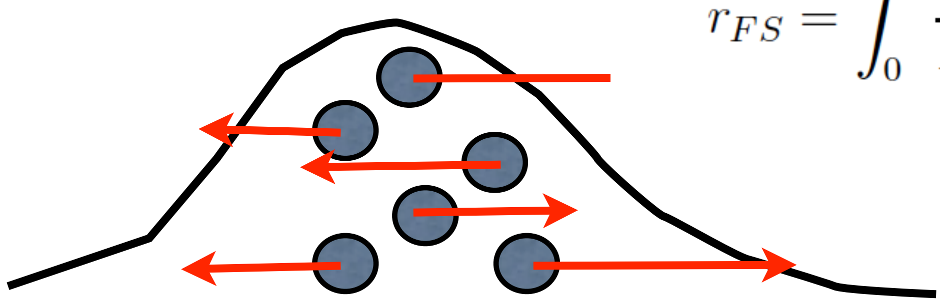
Dissipation at small scales free-streaming

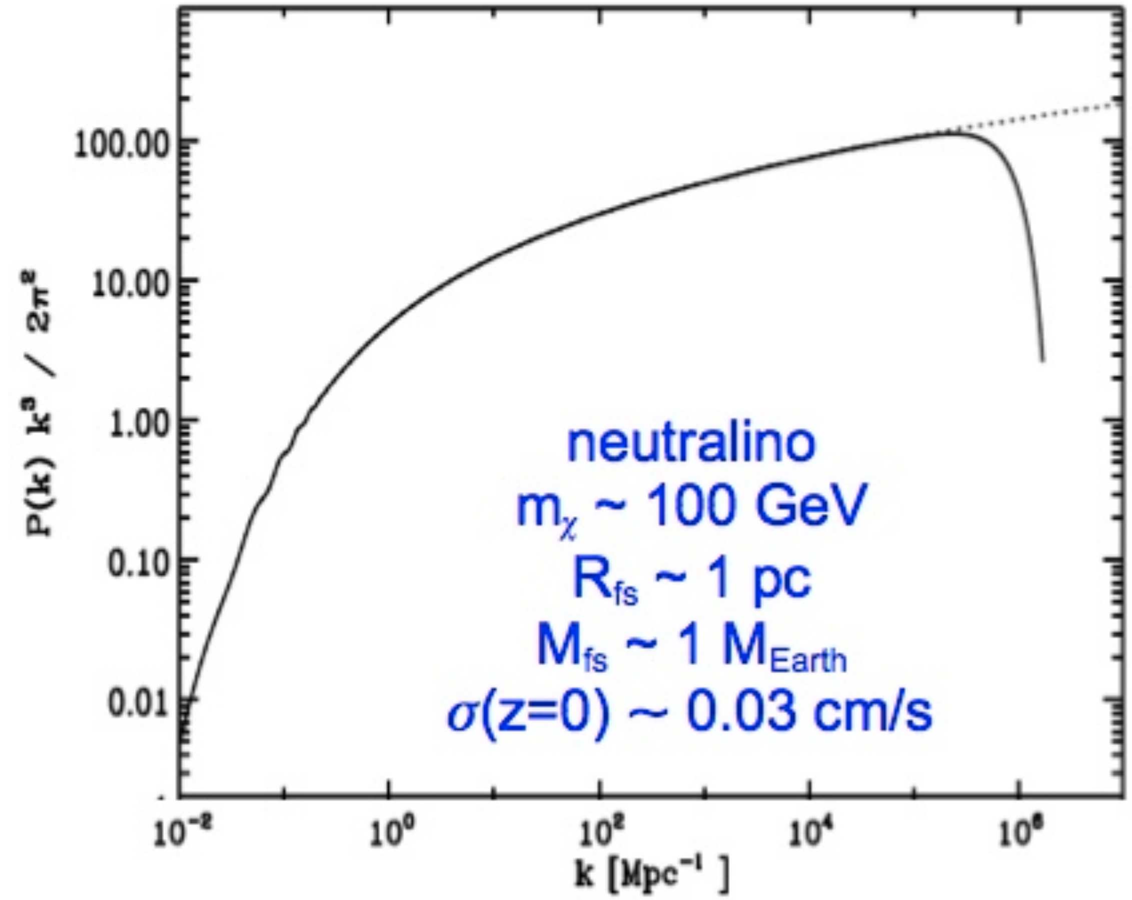


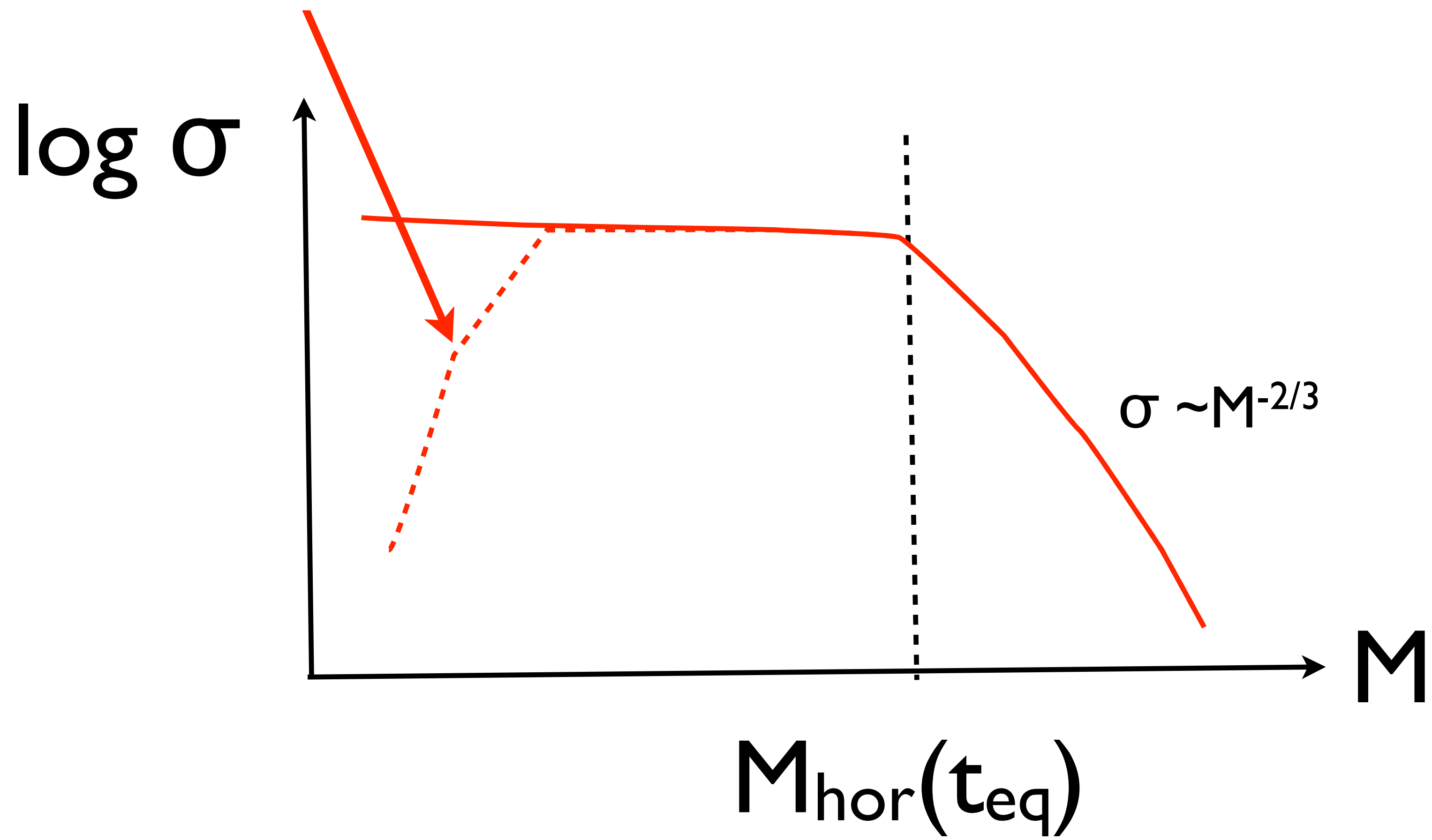
$$r_{FS} = \int_0^t \frac{v(t)}{R(t)}$$

$$\sigma_\chi \propto a^{-1} m_\chi^{-1/2}$$

$$M_{FS} = 4 \times 10^{15} \left(\frac{m_\nu}{30\ \mathrm{eV}}\right)^{-2} M_\odot$$

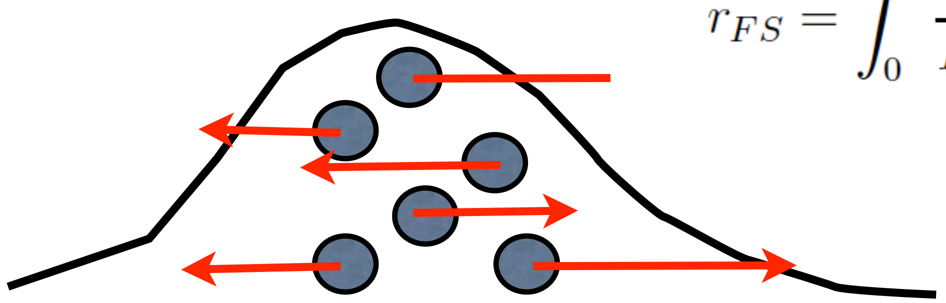
Angulo & White, 2010





Perturbations involving scales larger than that of the horizon at the equivalence start to grow later

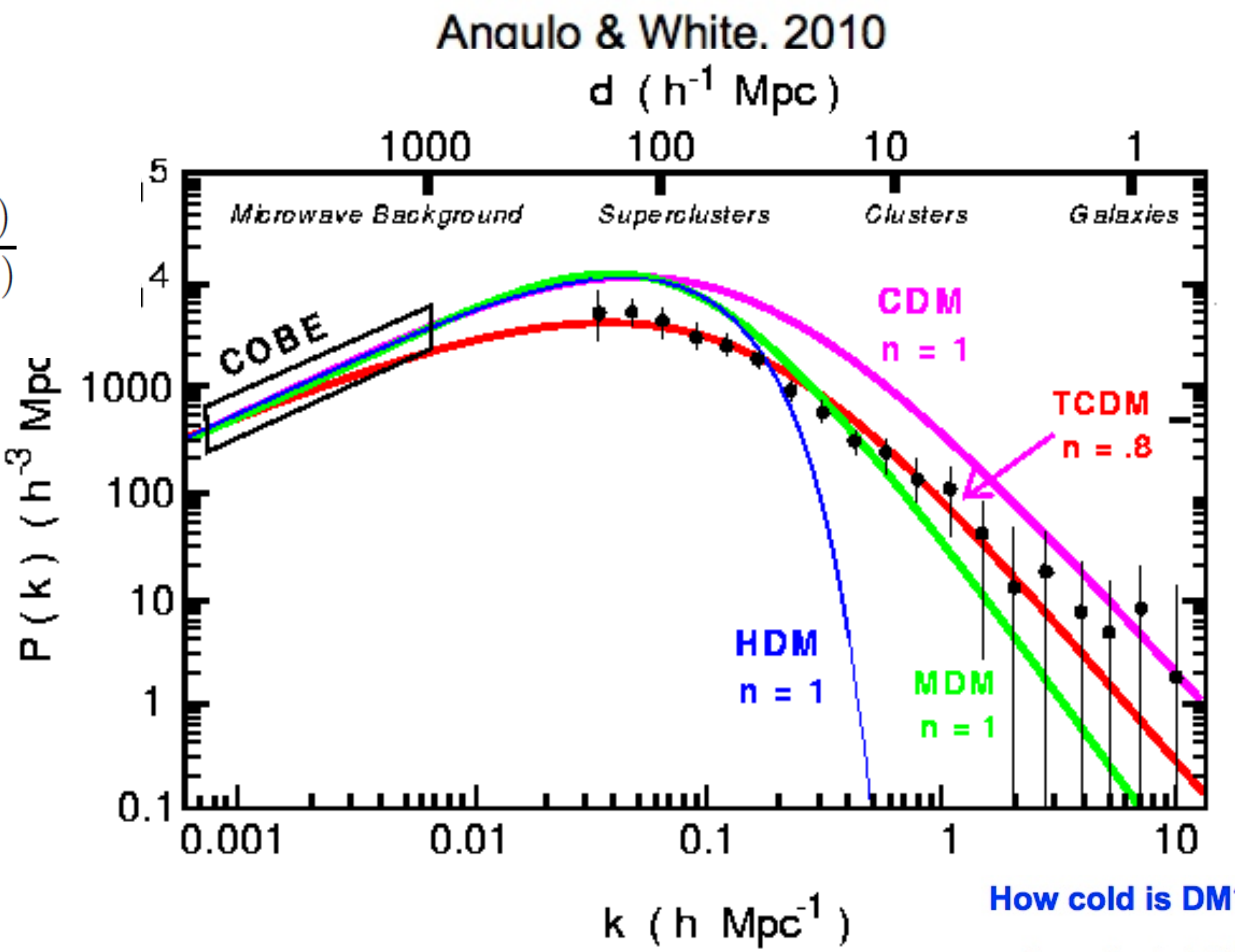
Dissipation at small scales free-streaming



$$r_{FS} = \int_0^t \frac{v(t)}{R(t)}$$

$$\sigma_\chi \propto a^{-1} m_\chi^{-1/2}$$

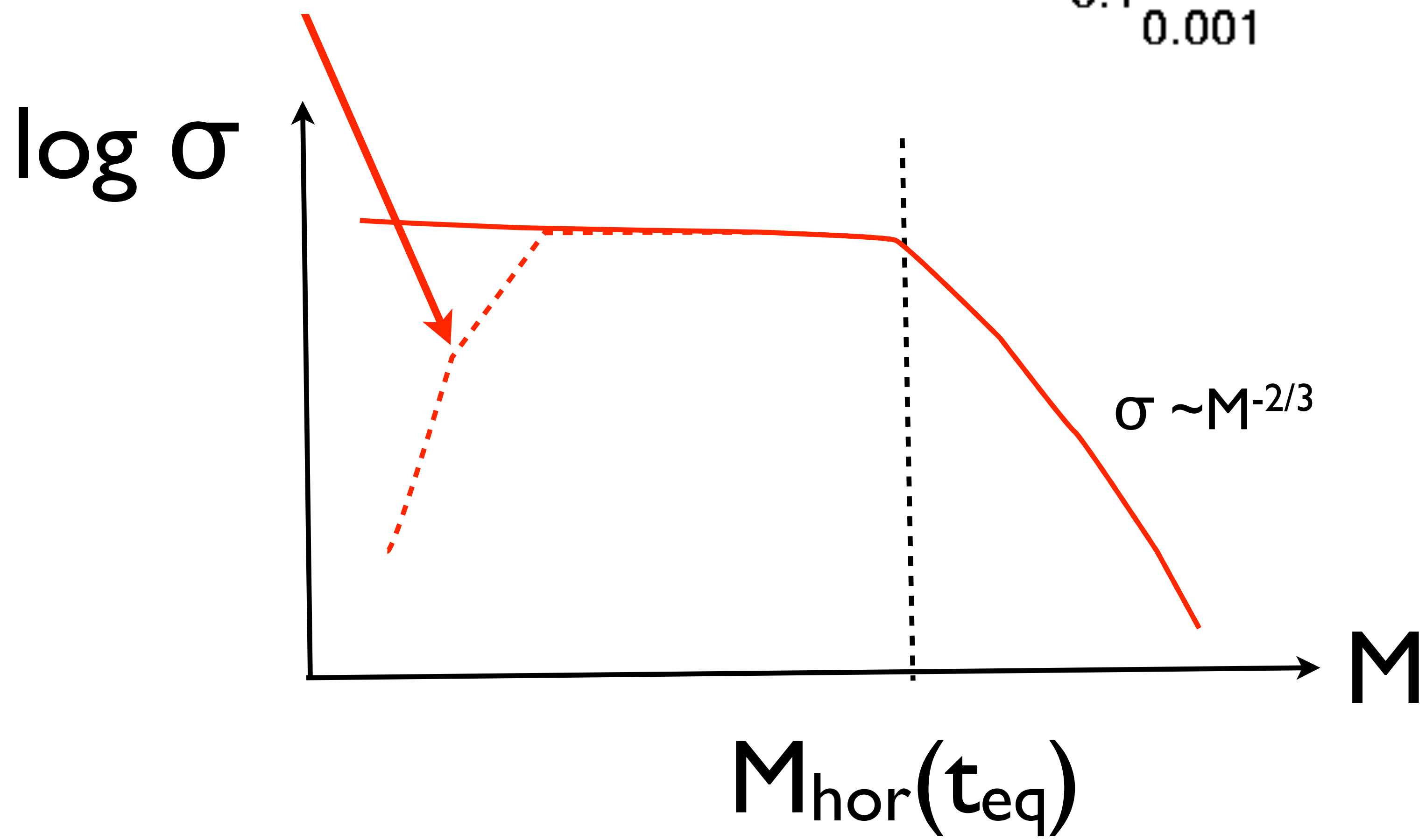
$$M_{FS} = 4 \times 10^{15} \left(\frac{m_\nu}{30\text{ eV}}\right)^{-2} M_\odot$$



How cold is DM?

e.g. Boyarsky et al. 2009

$m_\chi > 1$ keV
$R_{fs} < 500$ kpc
$M_{fs} < 10^{10}\ M_{Sun}$
$\sigma(z=0) < 0.04$ km/s



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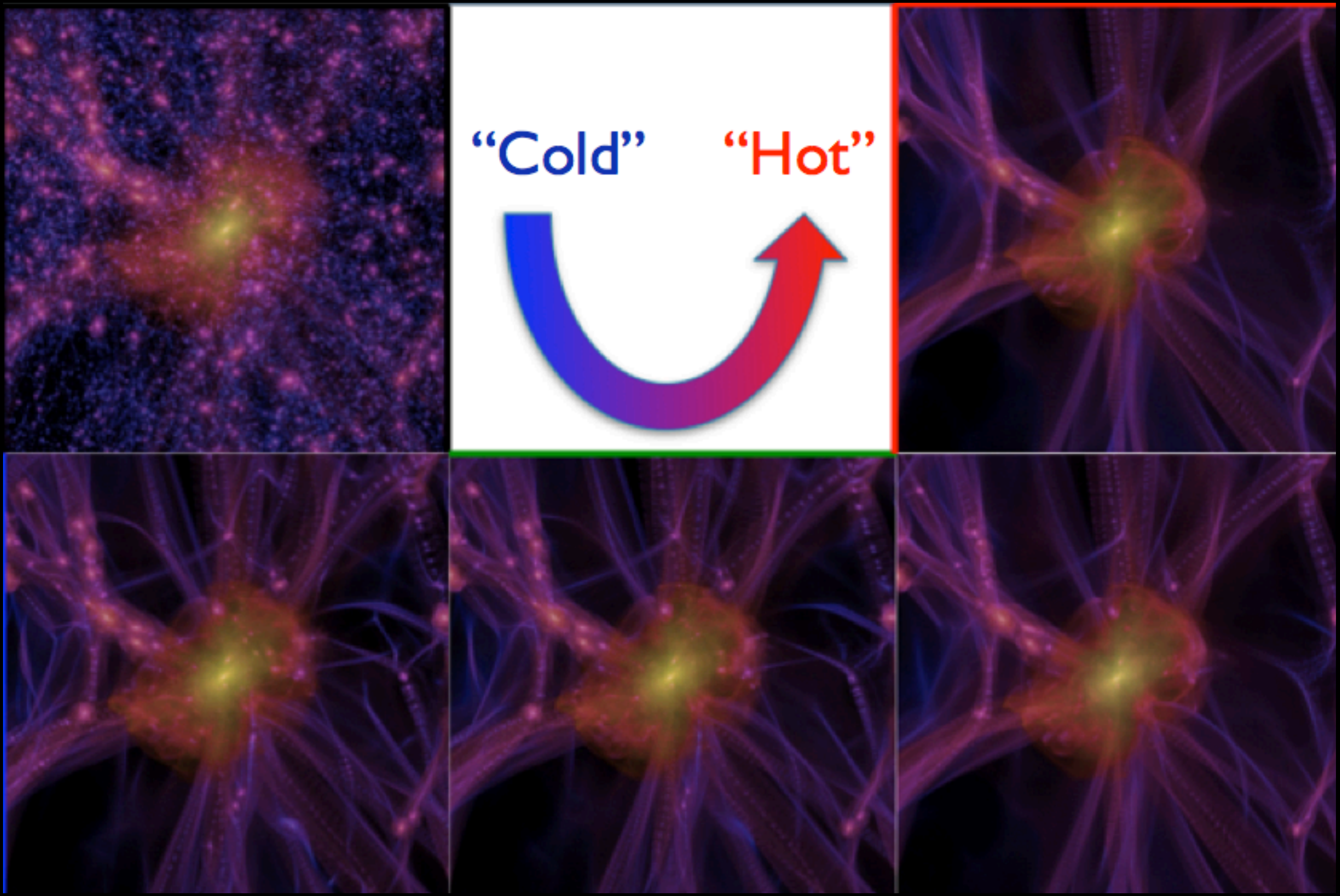


Lovell et al. 2012

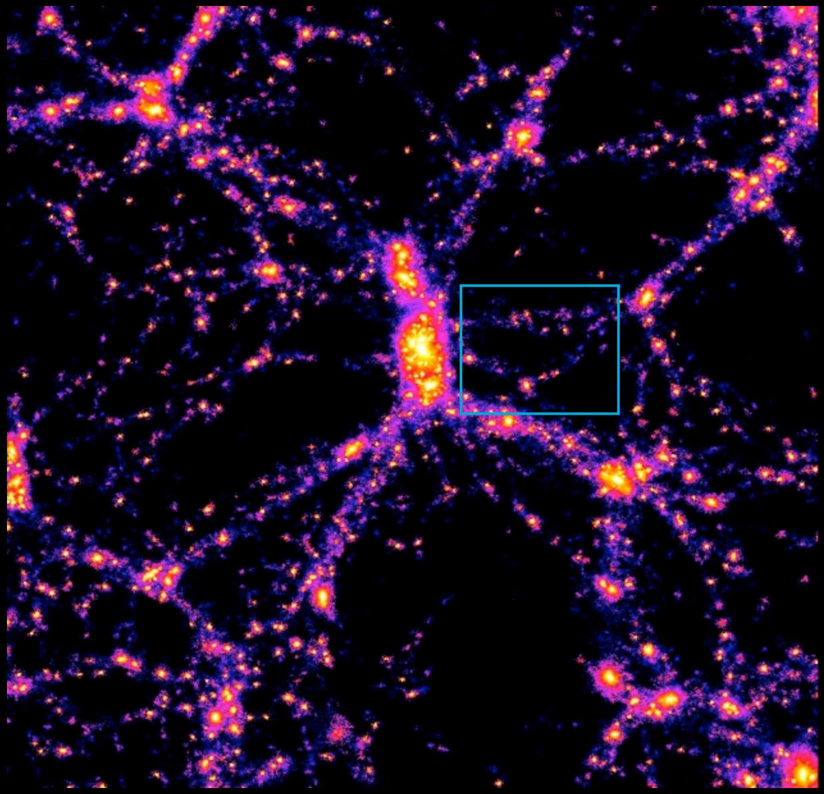
What’ so cold about CDM

For “thermal relics” such as neutrinos relatively straightforward to compute their present day abundance. Neutrinos relativistic at decoupling, velocity dispersion large.

Candidates for “Hot Dark Matter” -- ruled out by observation.

Velocity dispersion assumed to be vanishingly small

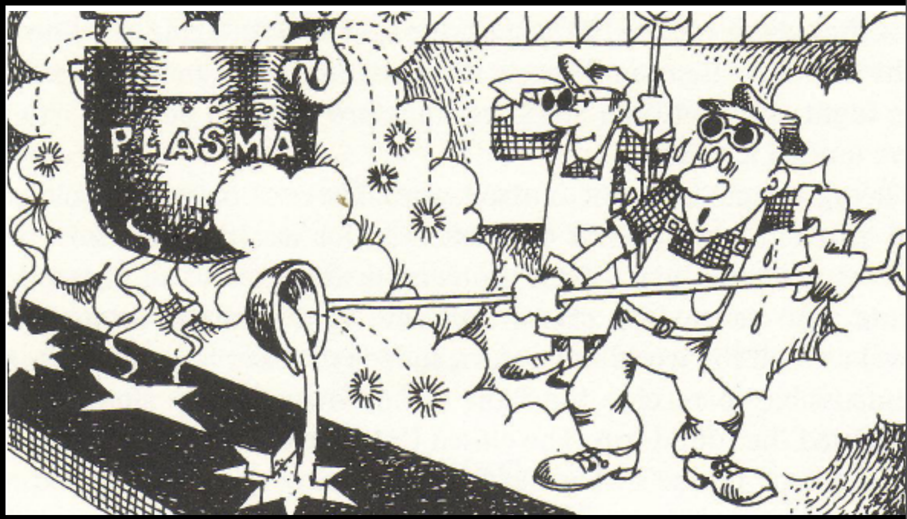
limit M_{fs} << Masses of Cosmological Relevance

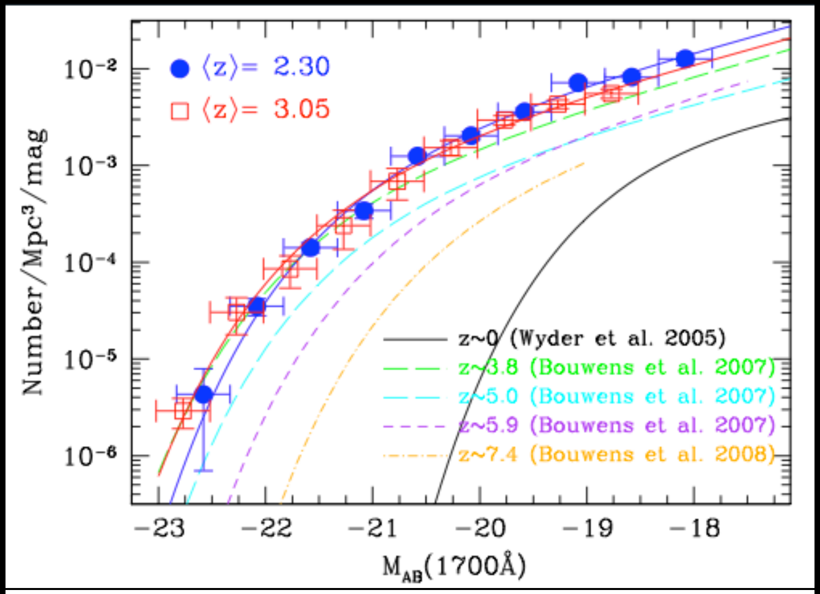


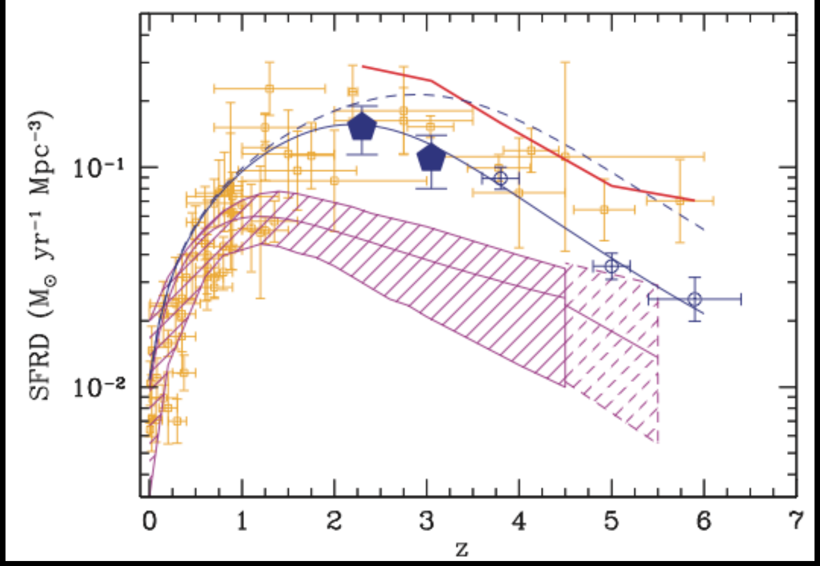
Testing the COLD DARK MATTER scenario against observations: the evolution of galaxies

Requires modeling of baryon physics inside evolving DM potential wells

- gas physic s (cooling, heating)
- disk formation
- star formation
- evolution of the stellar population
- injection of energy into the gas
  (SNae, UV background, AGN)







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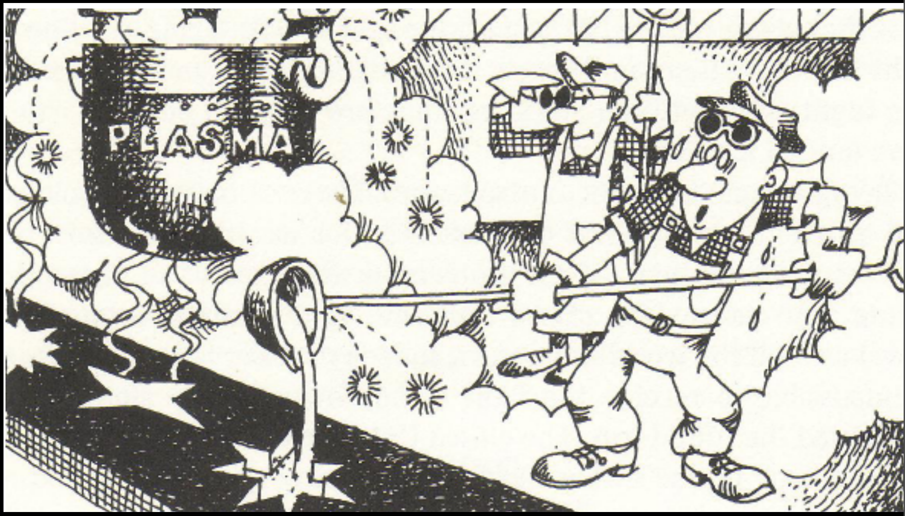
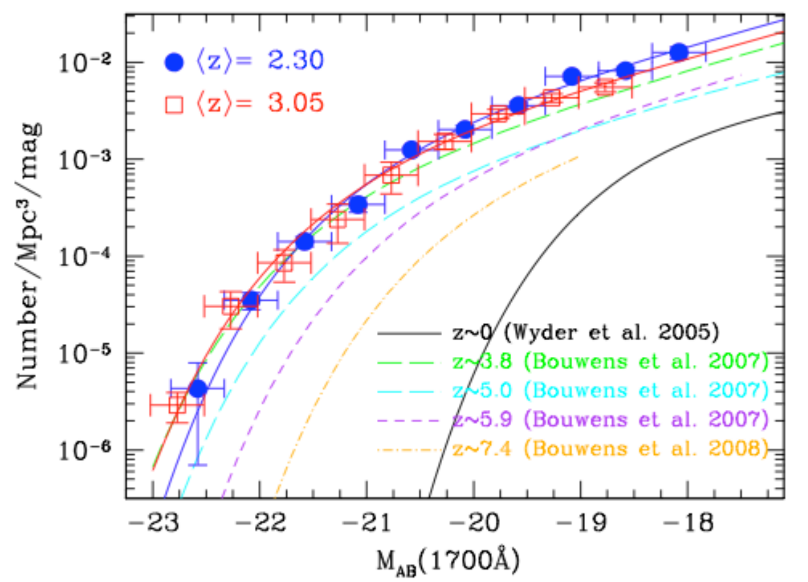
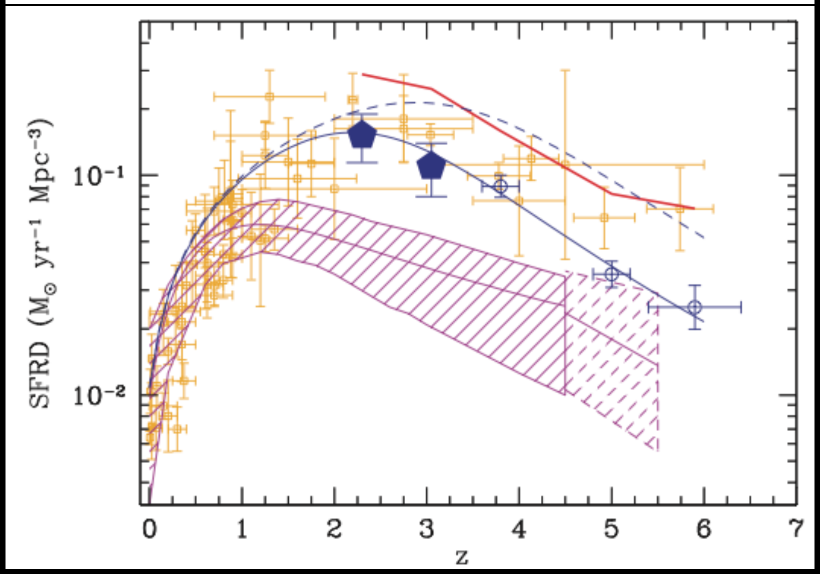
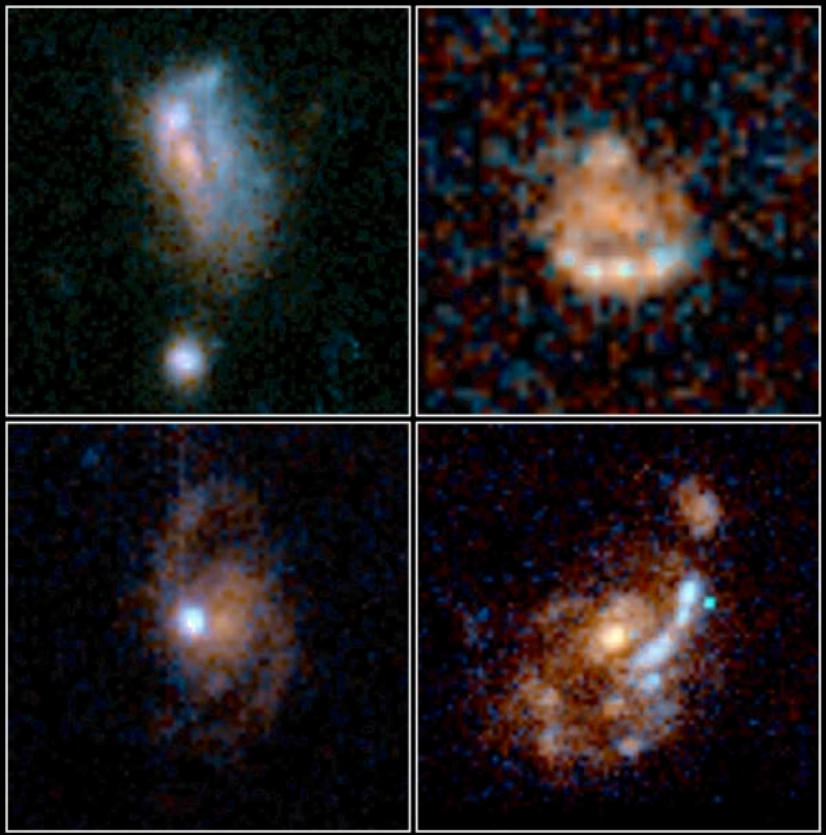








Medium Deep Survey HST • WFPC2

PRC94-39b · ST ScI OPO · R. Griffiths (JHU), NASA

Galaxy Formation in a Cosmological Context

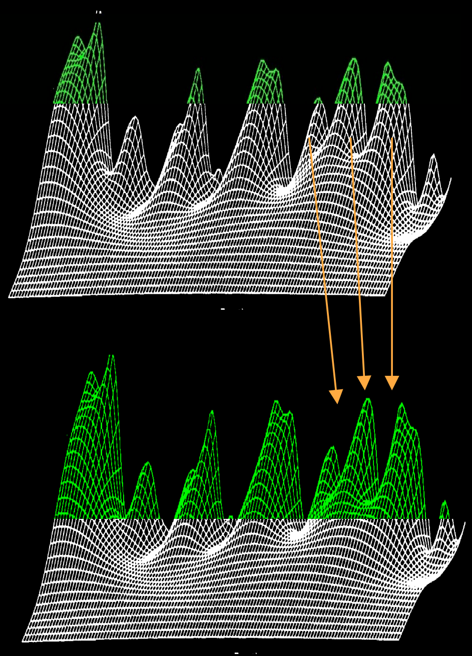
Hydrodynamcal N-body simulations

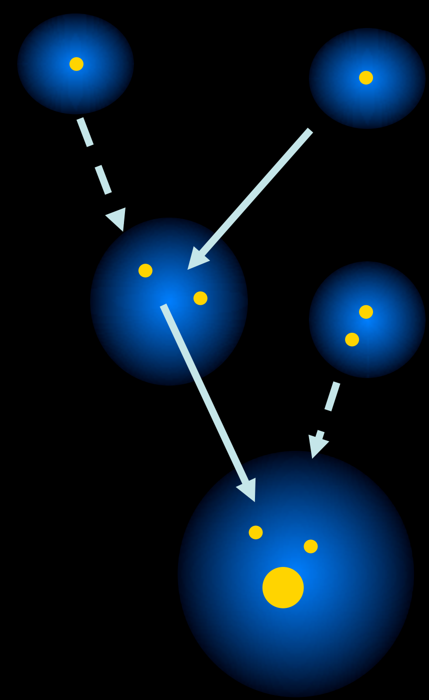
<u>Pros</u>
include hydrodynamics of gas
contain spatial information
<u>Cons</u>
numerically expensive
(limited exploration of parameter space)
requires sub-grid physics

Semi-Analytic Models Monte-Carlo realization of collapse and merging histories

<u>Pros</u>
Physics of baryons linked to DM halos through scaling laws, allows a fast spanning of parameter space
<u>Cons</u>
Simplified description of gas physics
Do not contain spatial informations

Galaxy Formation in a Cosmological Context





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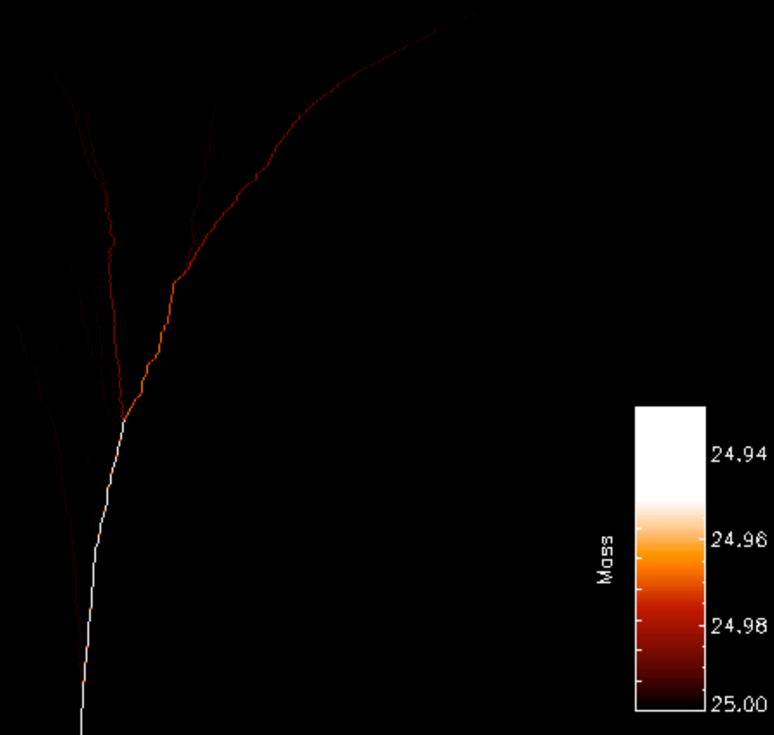


Halo Properties
Density Profiles
Virial Temperature
Virial Radius

Gas Properties
Profiles
Cooling - Heating Processes
Collapse, disk formation

Star Formation Rate

Gas Heating (feedback)
SNae
UV background

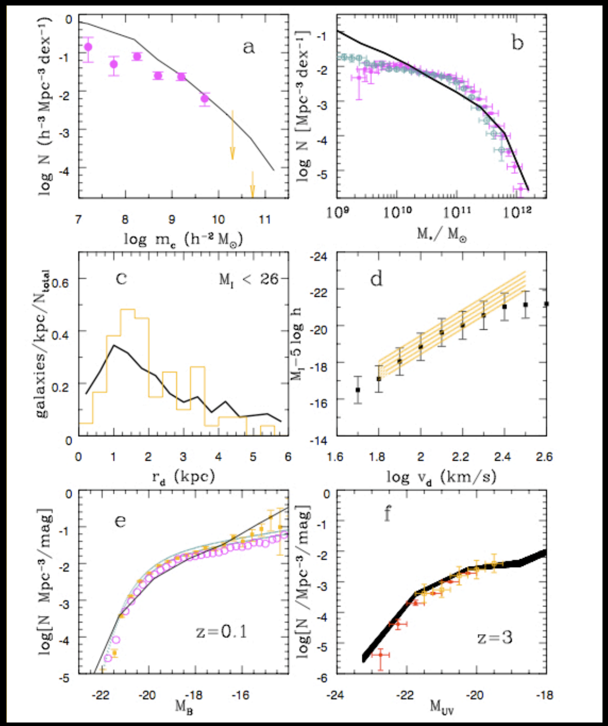
Evolution of stellar populations

Growth of Supermassive BHs
Evolution of AGNs

Galaxy Formation models in CDM scenario

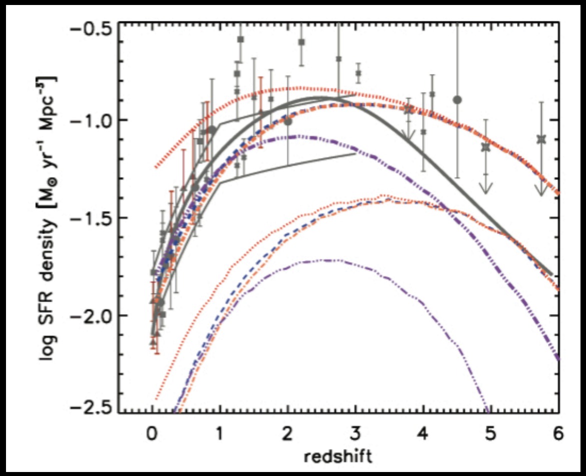
Local properties:

gas content
luminosity distribution
disk sizes
distribution of the stellar mass content

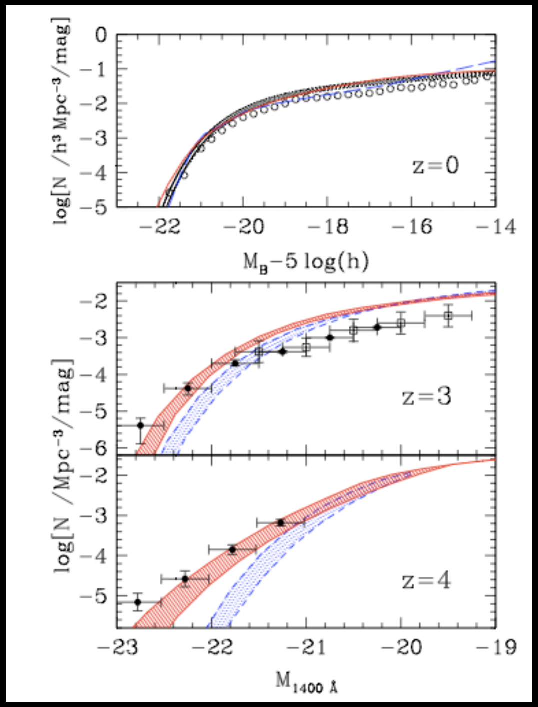


properties of distant galaxies:

luminosity distribution
evolution of the star formation rate



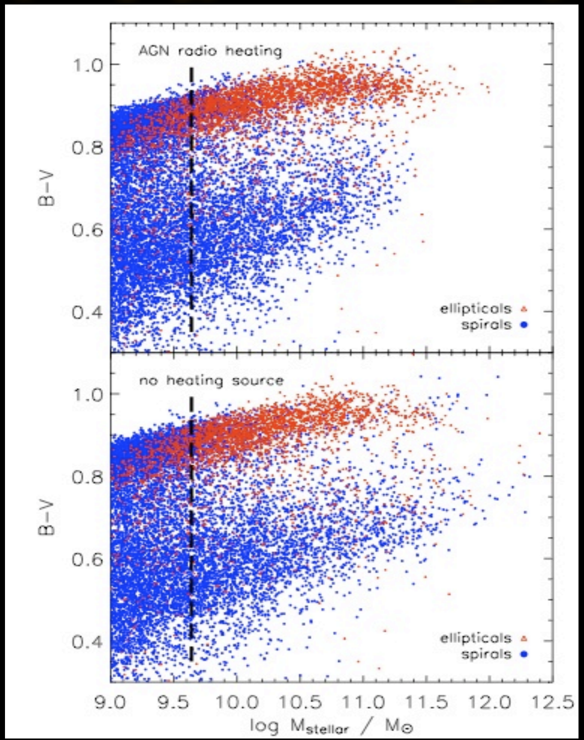
Somerville et al. 2010



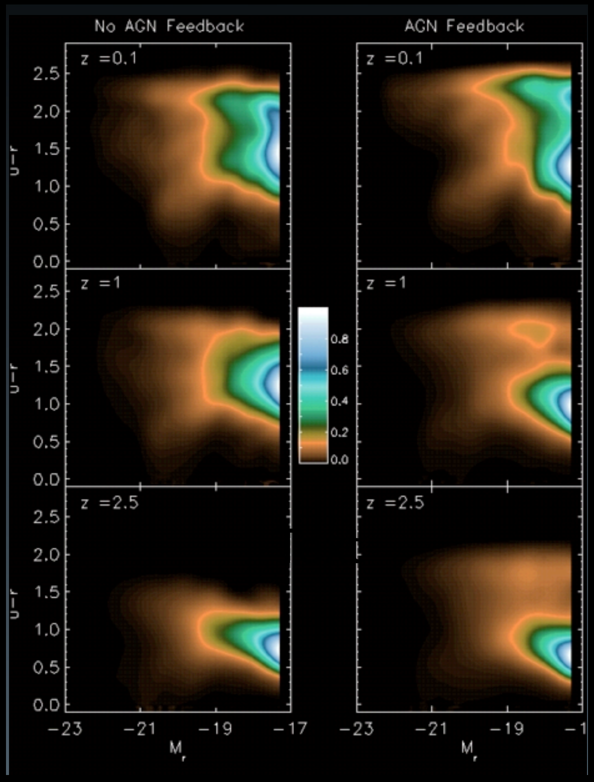
NM et al. 2006

Color Distributions:

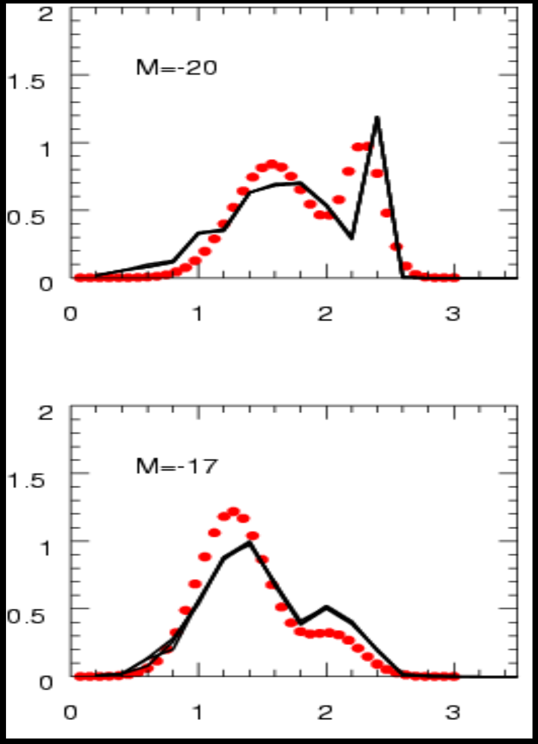
bimodal properties
(early type vs late type)



Croton et al. 2006



NM et al. 2008



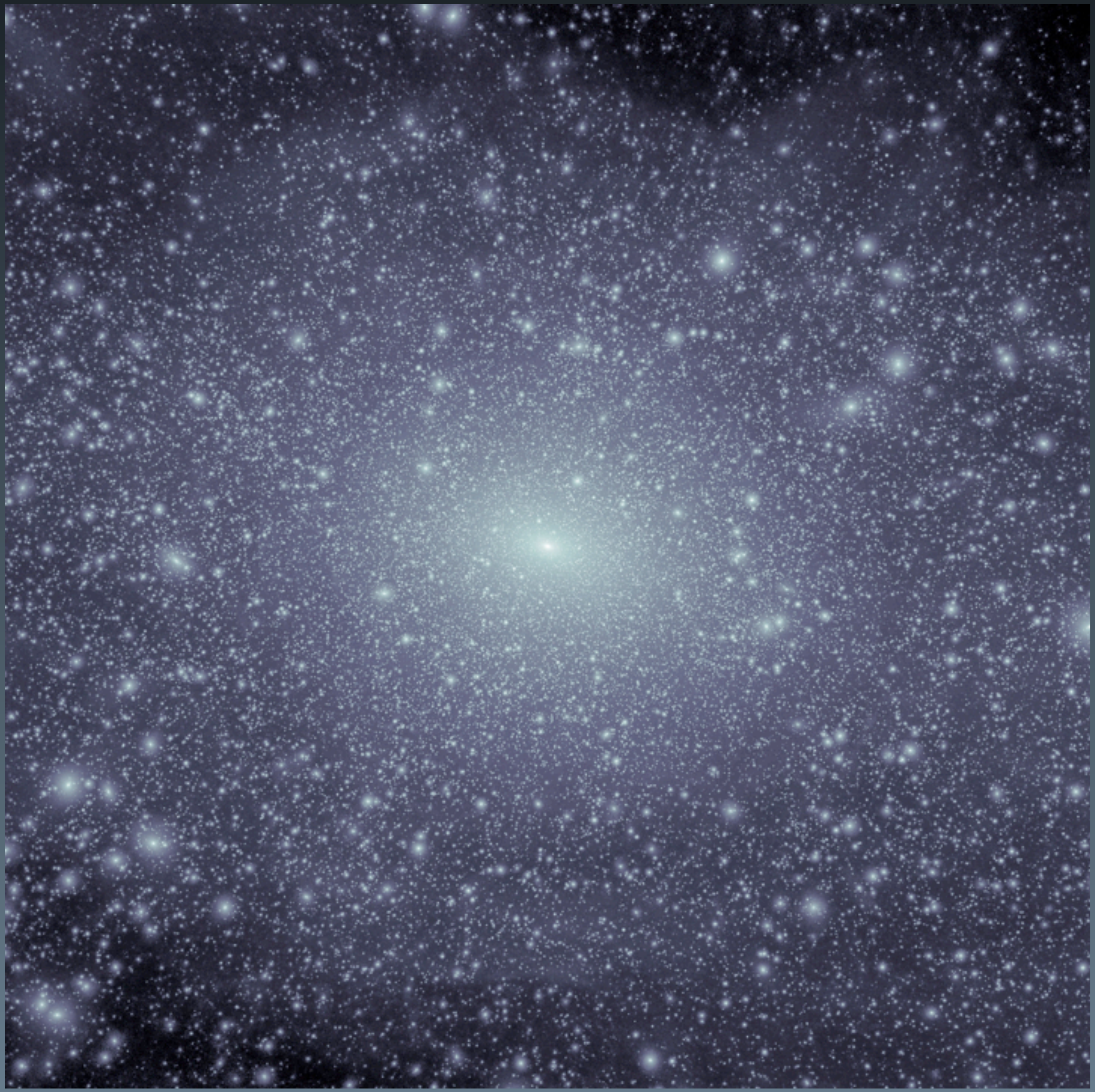
3. Critical Issues

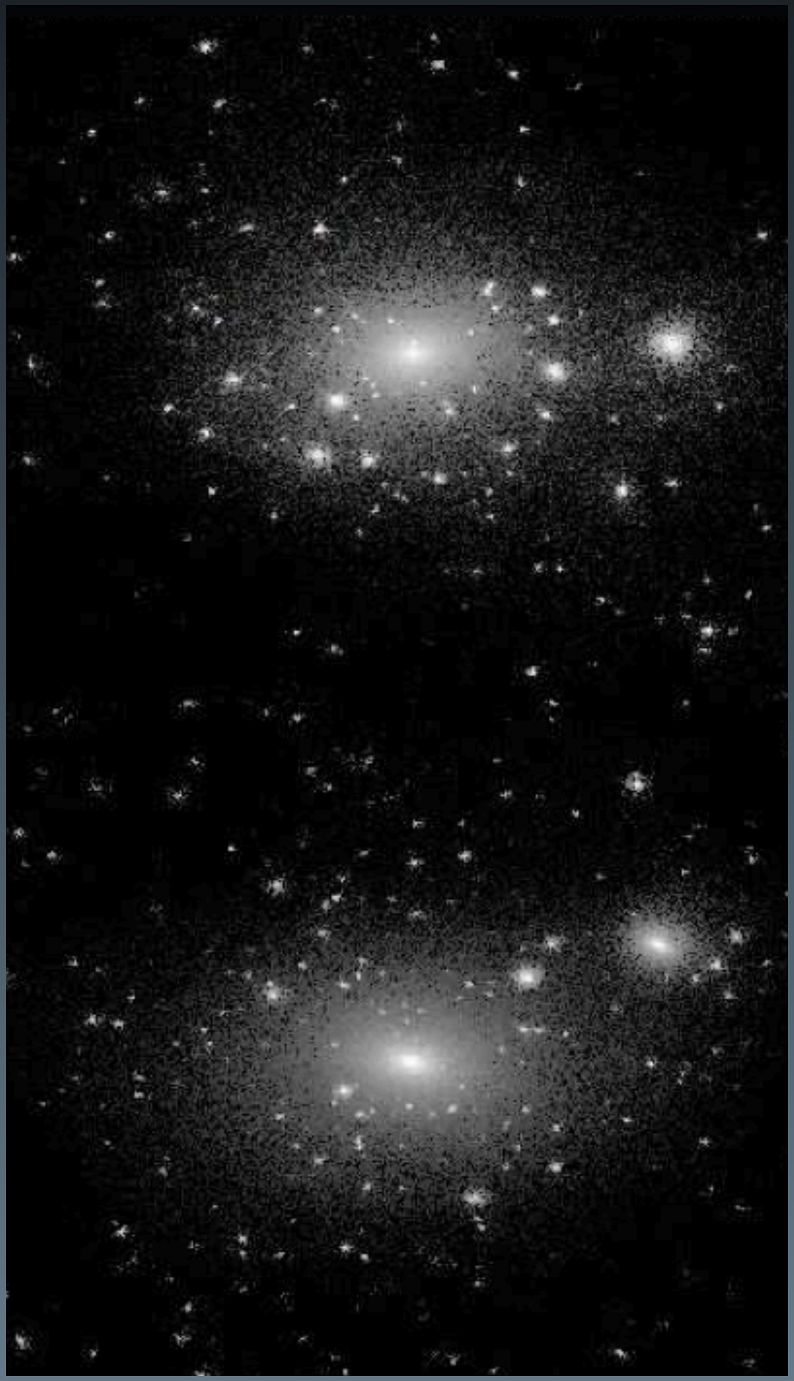
Overabundance of Low Mass Objects

i) satellite DM haloes
ii) density profiles, dwarf galaxies
iii) faint galaxies
iv) faint AGNs

Abundance of satellites

Via Lactea simulation of a Milky Way - like galaxy Diemand et al. 2008





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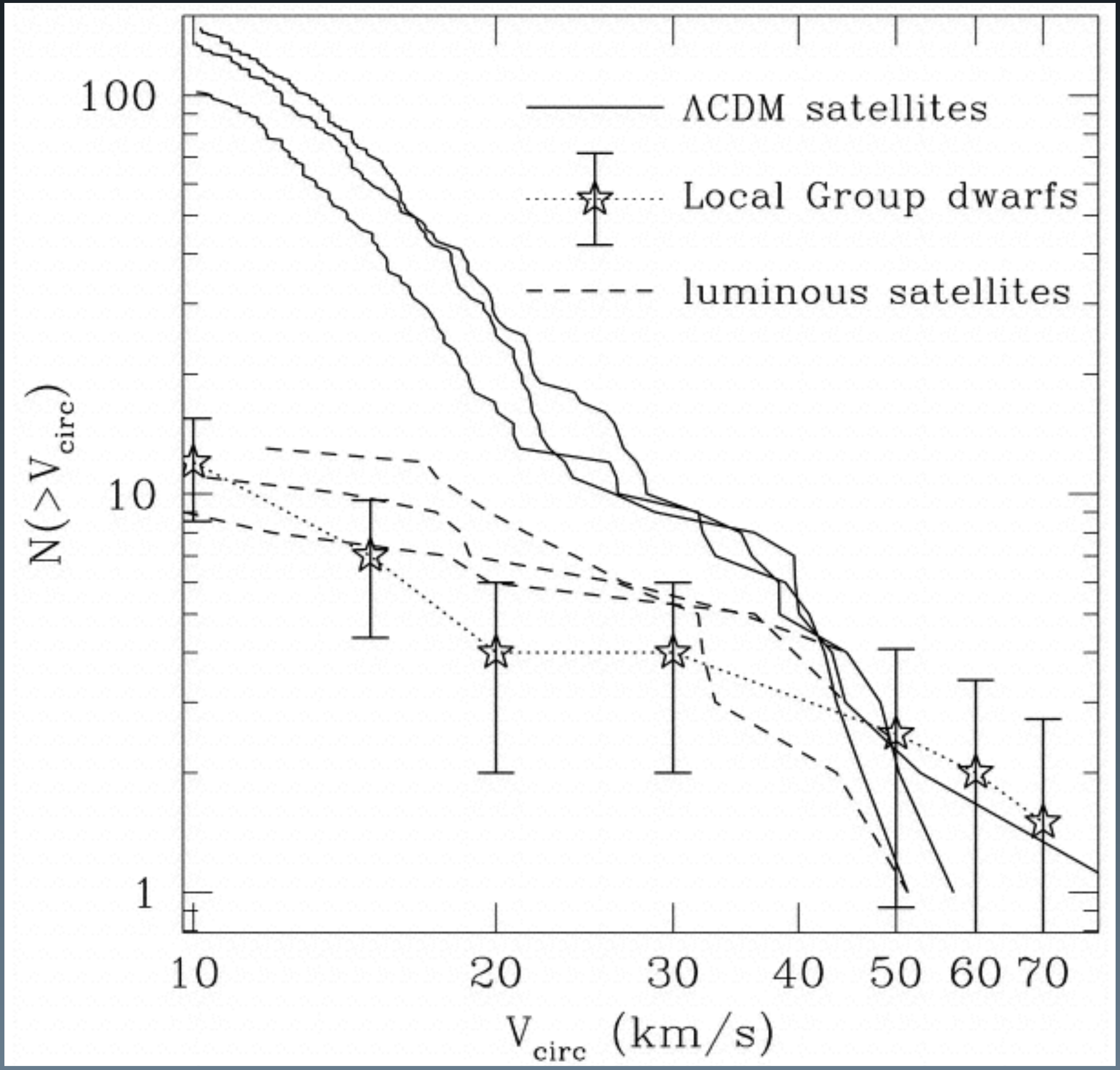
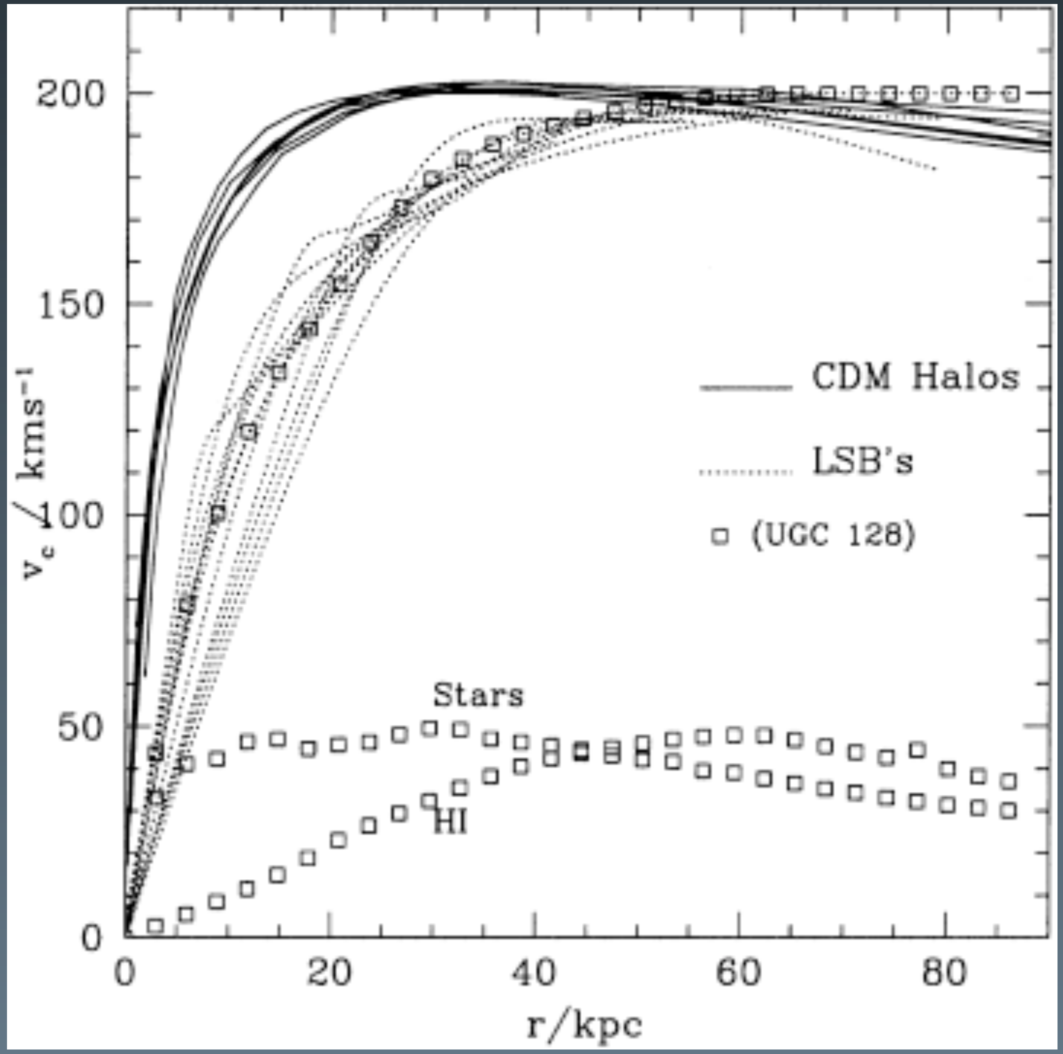


Kravtsov, Klypin, Gnedin 2004

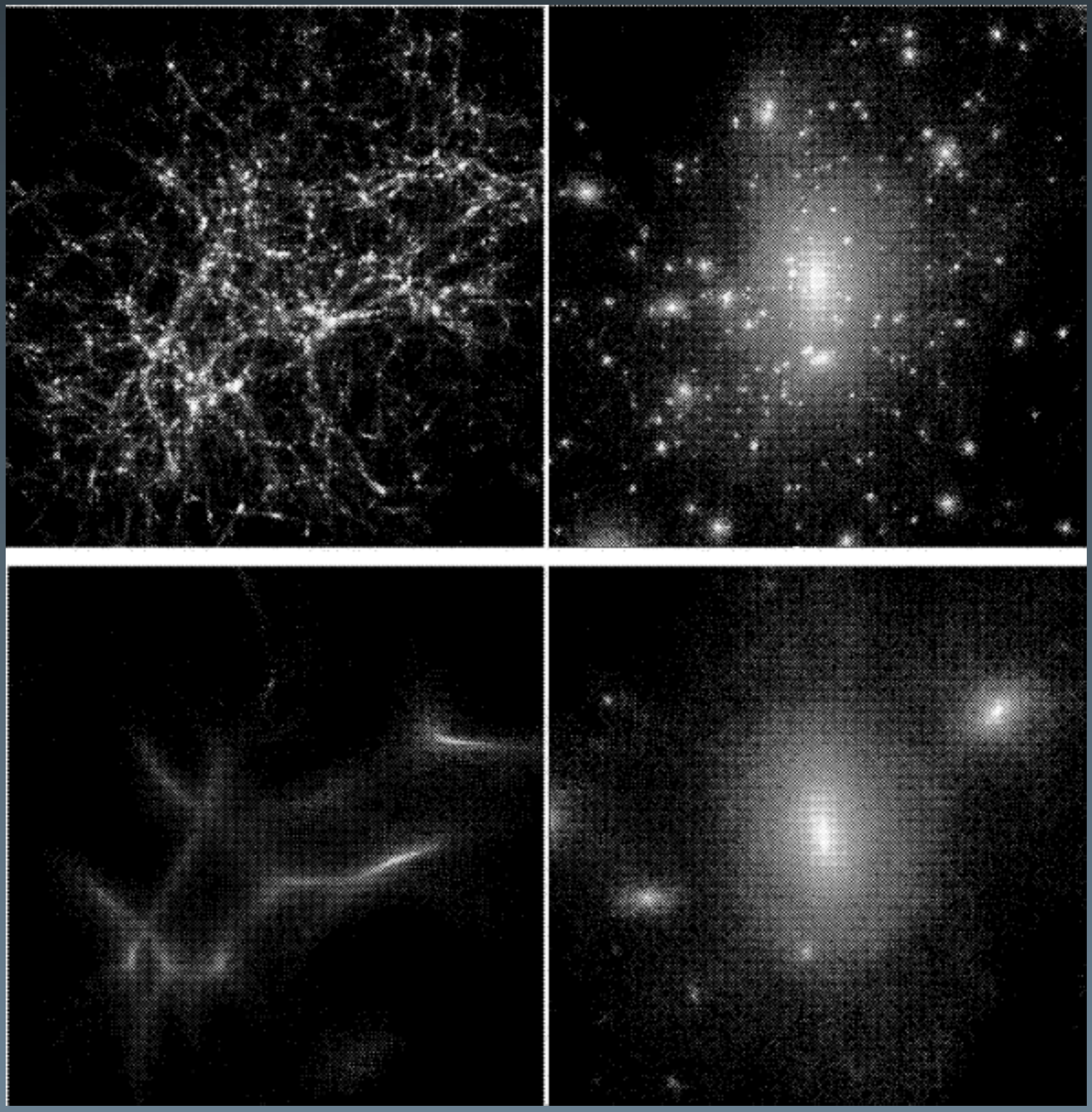
Bulgeless dwarf galaxies and dark matter cores

Most observed dwarf galaxies consist of a rotating stellar disk2 embedded in a massive dark-matter halo with a near-constant-density core. Models based on the dominance of CDM, however, invariably form galaxies with dense spheroidal stellar bulges and steep central dark-matter profiles, because low-angular-momentum baryons and dark matter sink to the centres of galaxies through accretion and repeated mergers.

Moore et al. 2002

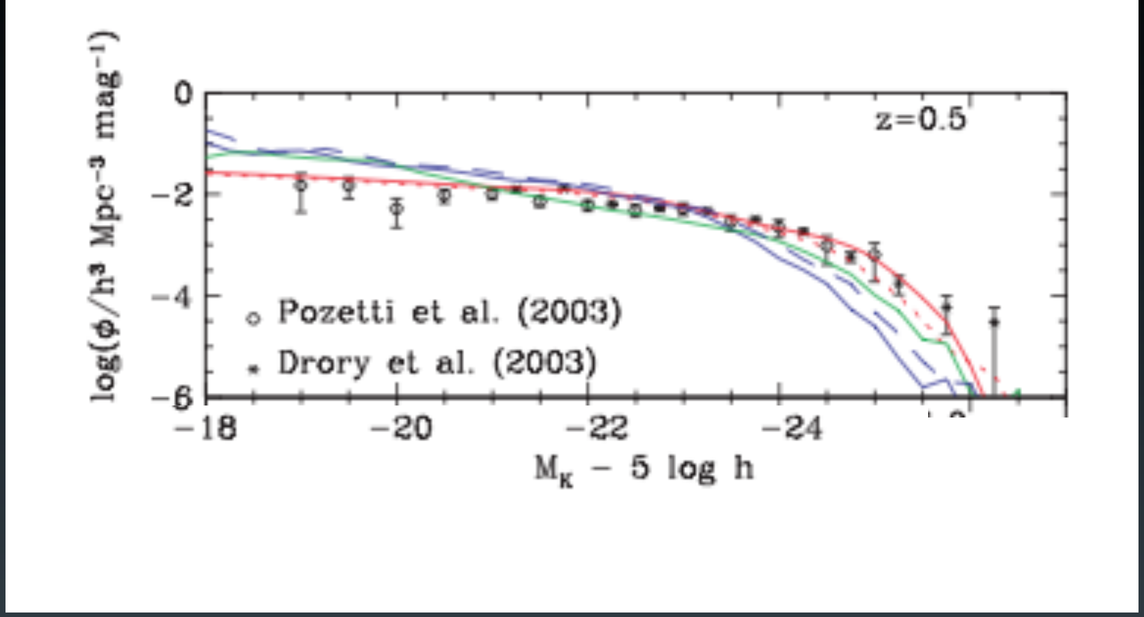


Effect of a Cutoff on Power Spectrum for r<8 Mpc



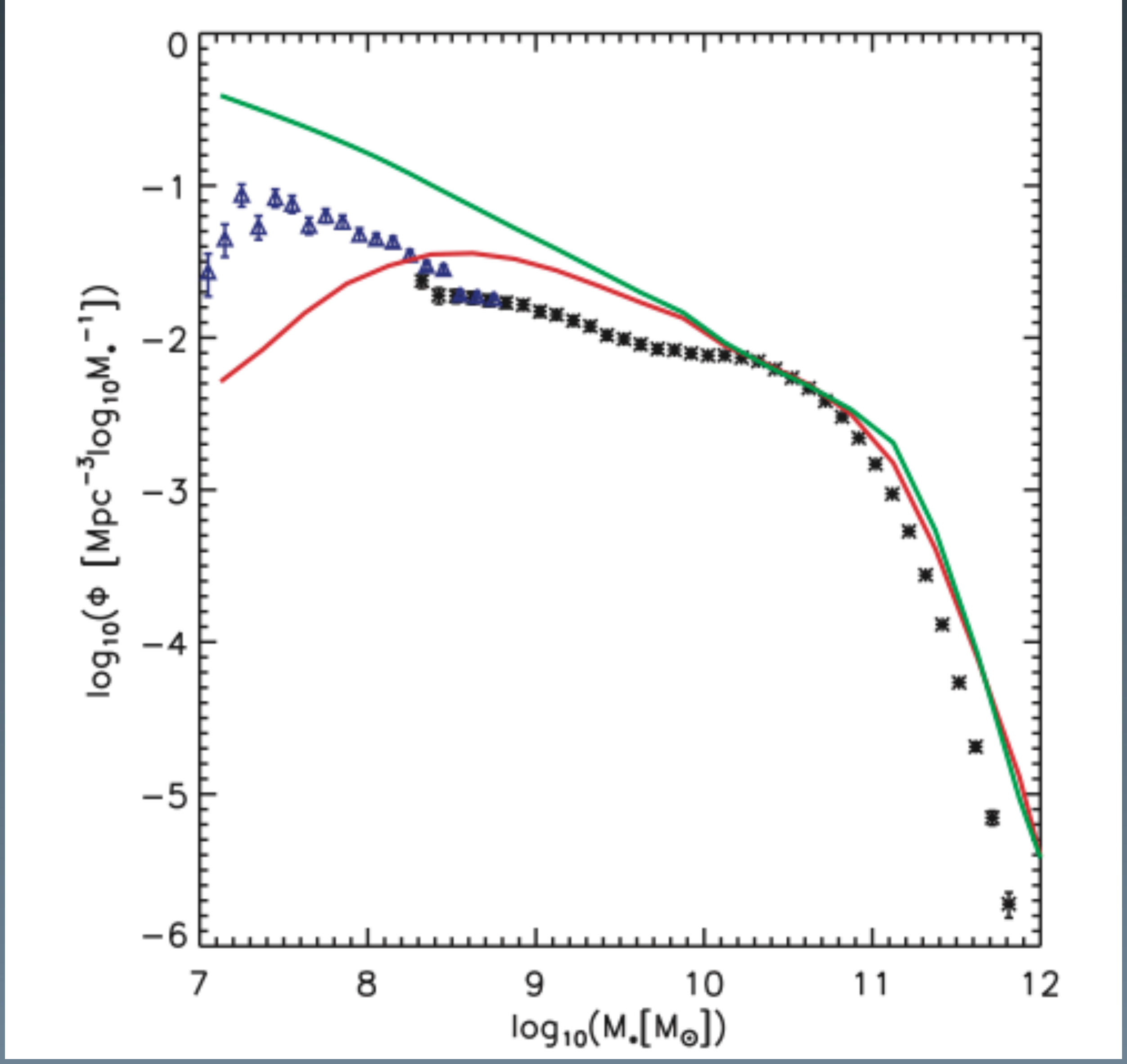
Over-prediction of Faint Galaxies

In all first-generation SAM
the number density of faint (low-mass)
galaxies was over-predicted

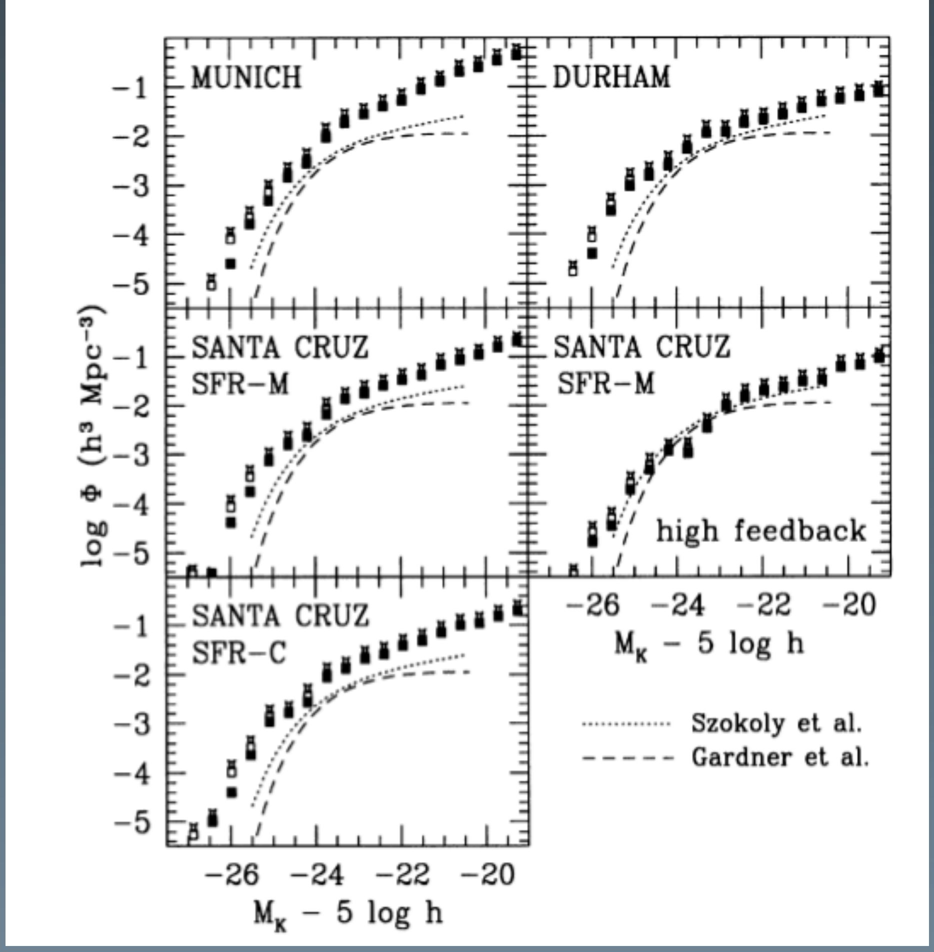


Bower et al. 2006

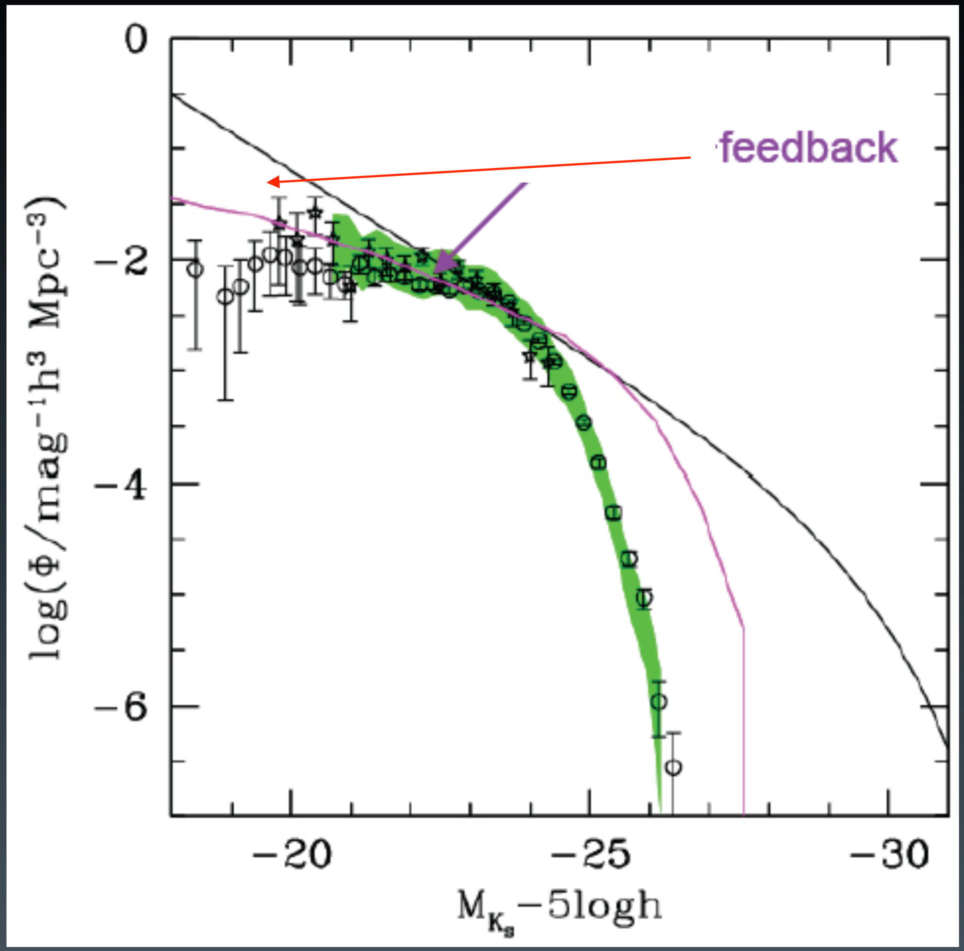
The Stellar Mass Function in the
De Lucia et al. SAM based on Millenium
merger trees



The K-Band Luminosity Function in the
Somerville et al. SAM



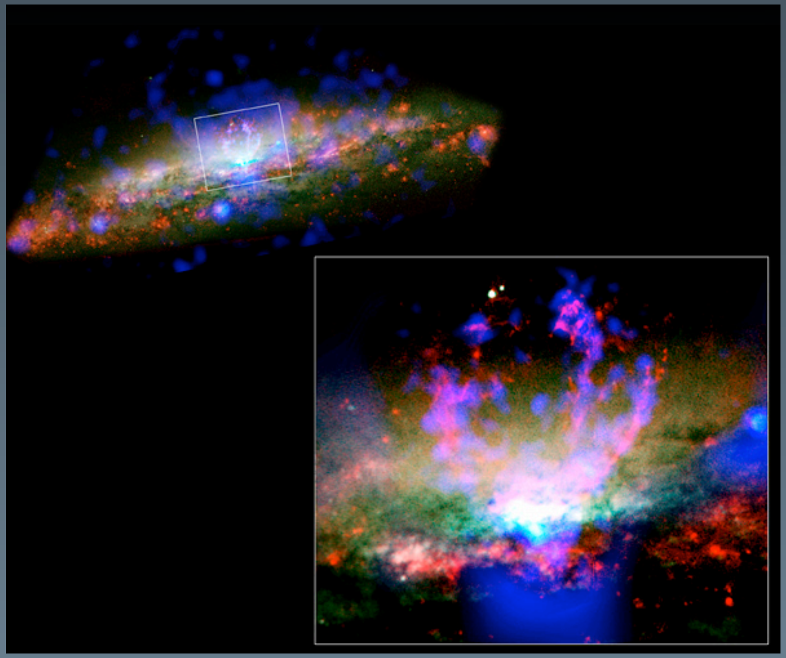
The Origin of the problem and a 1st order solution

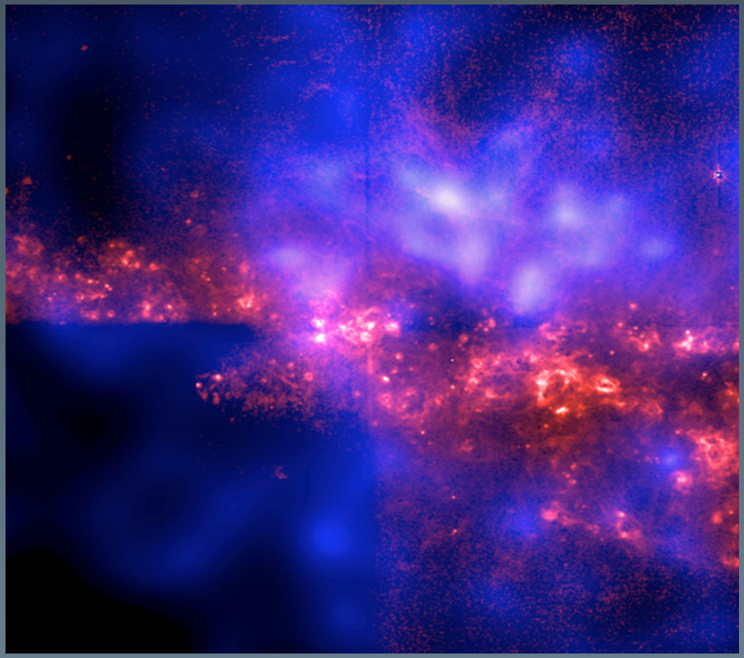


The DM halo Mass function has a steep log slope
$N \sim M^{-1.8}$
While the Observed Galaxy Luminosity Function
has a much flattter slope $N \sim L^{-1.2}$

Possible Solution
Suppress luminosity (star formation) in low-mass haloes
Heat - Expell Gas from shallow potential wells
- Enhanced SN feedback
- UV background

$$E_{SN} \approx 10^{51}\eta_{IMF}\Delta M_* \qquad v_{esc} = \sqrt{E_{SN}/M_{gas}} \approx 100 \quad km/s$$





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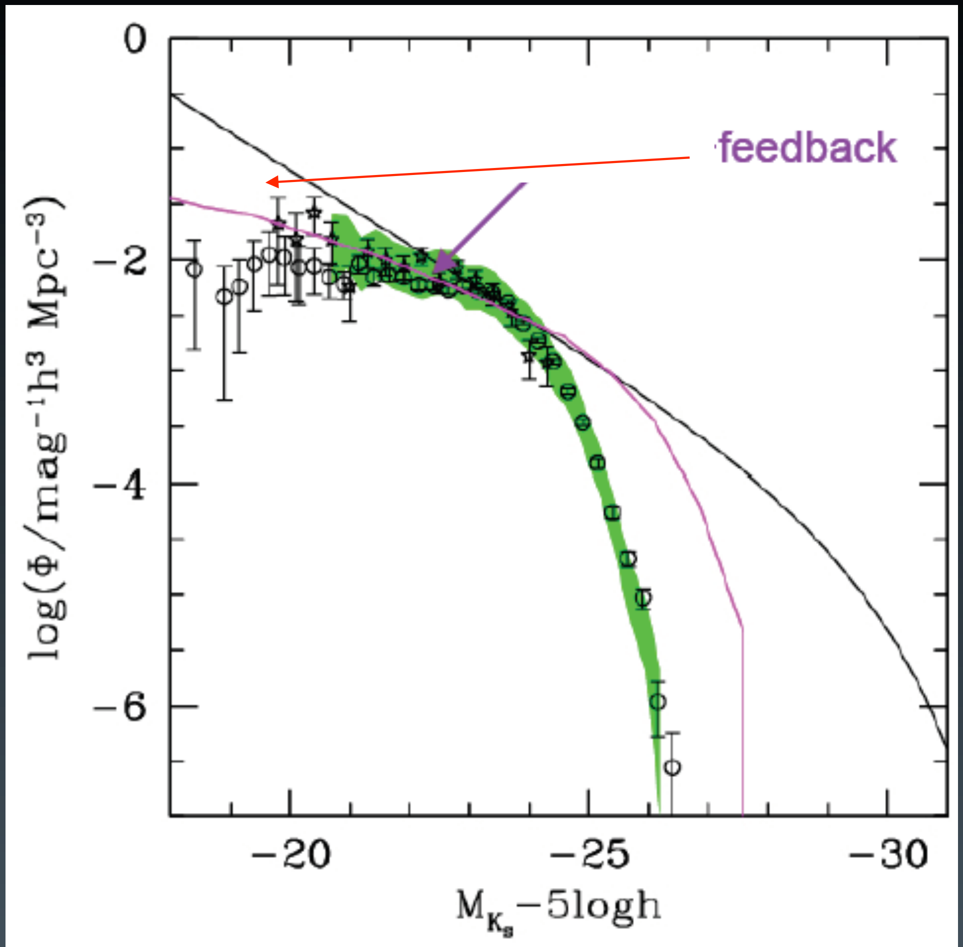

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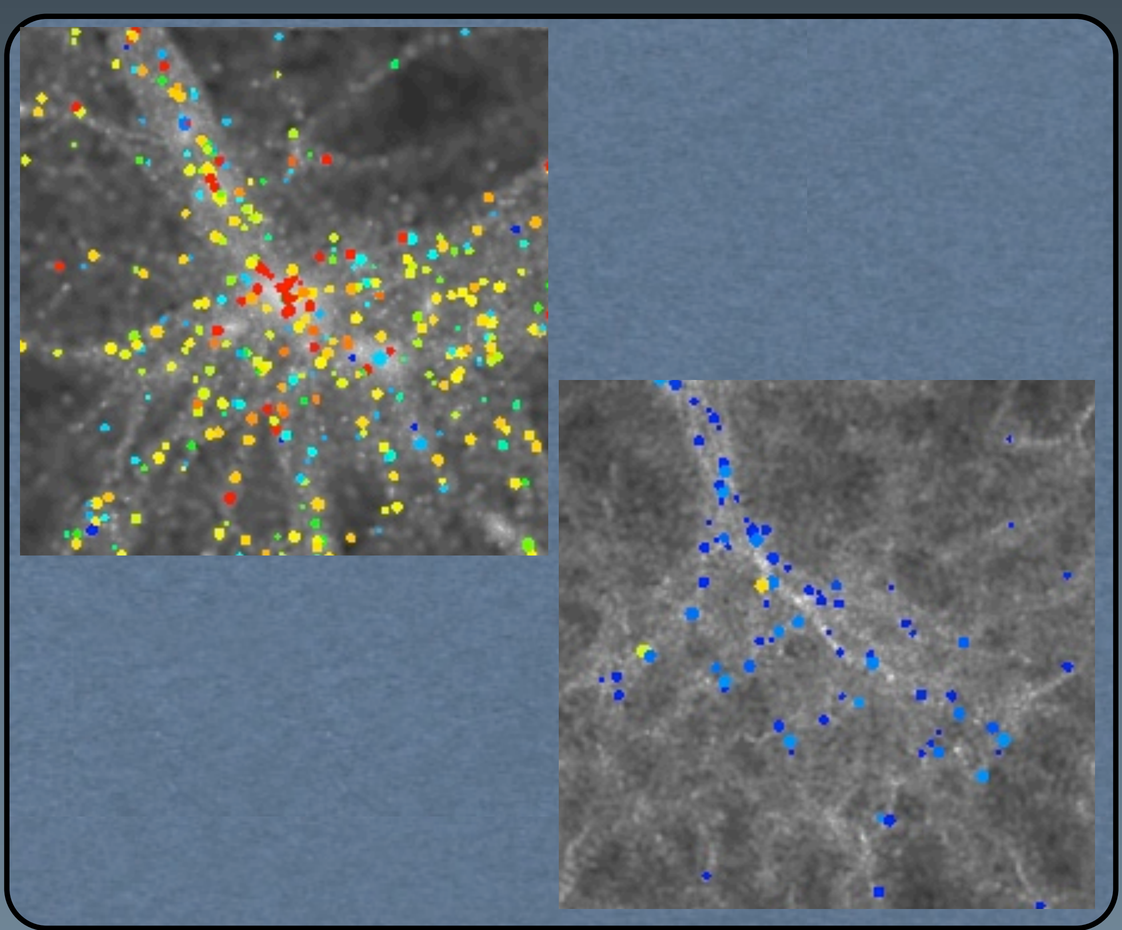

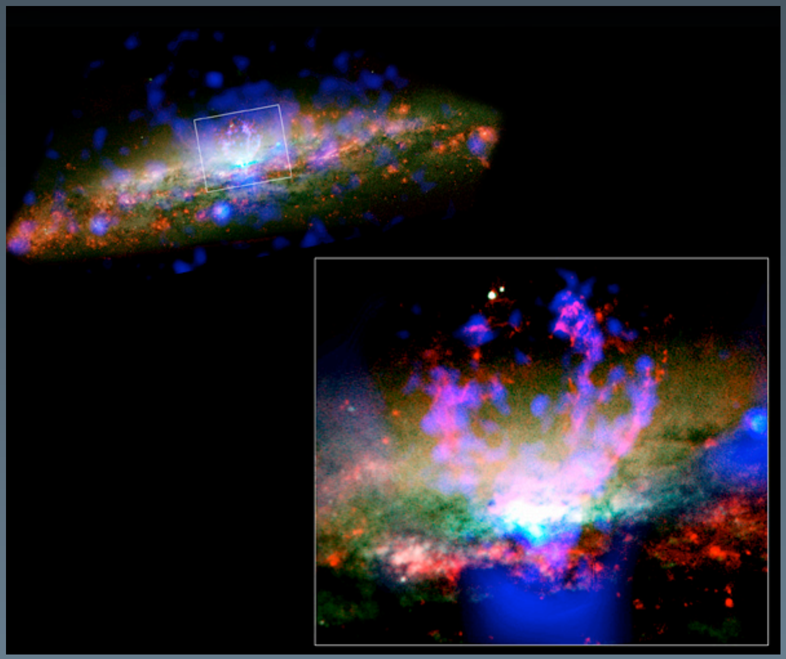

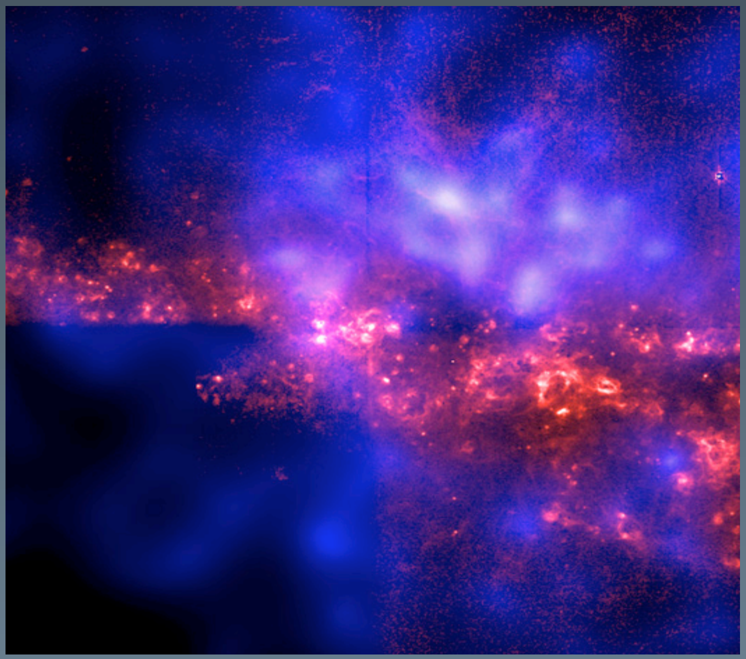

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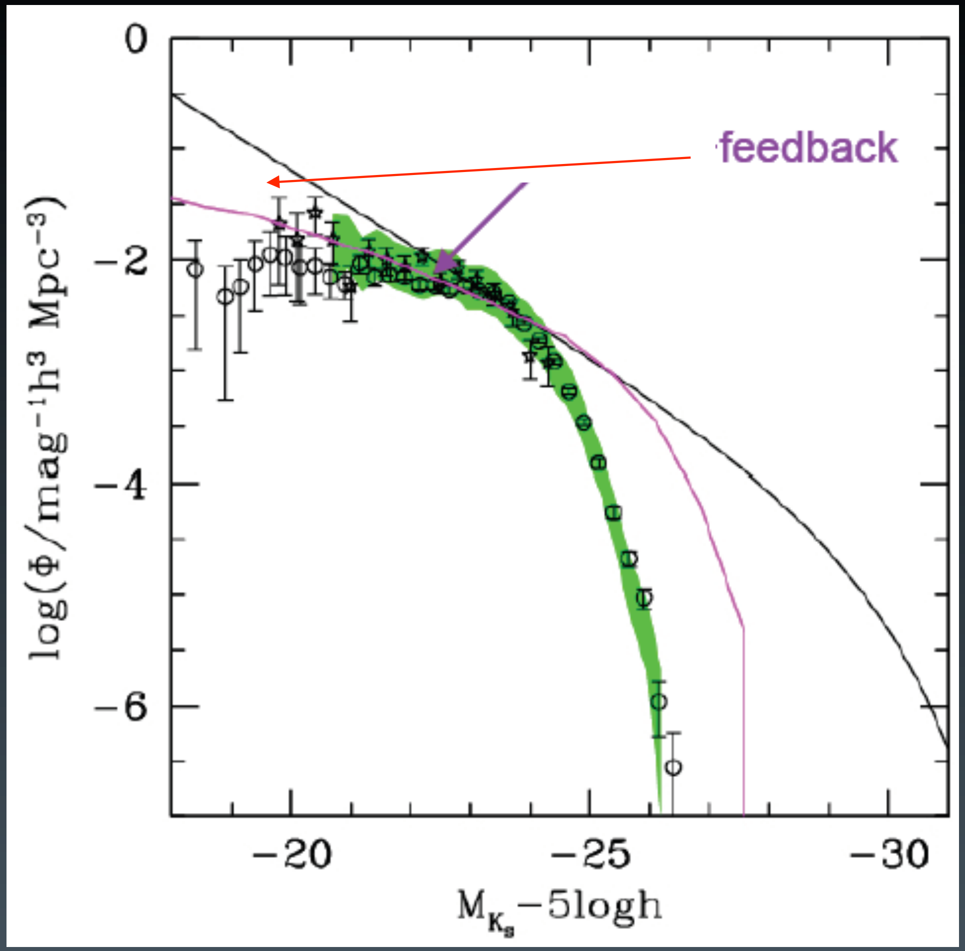

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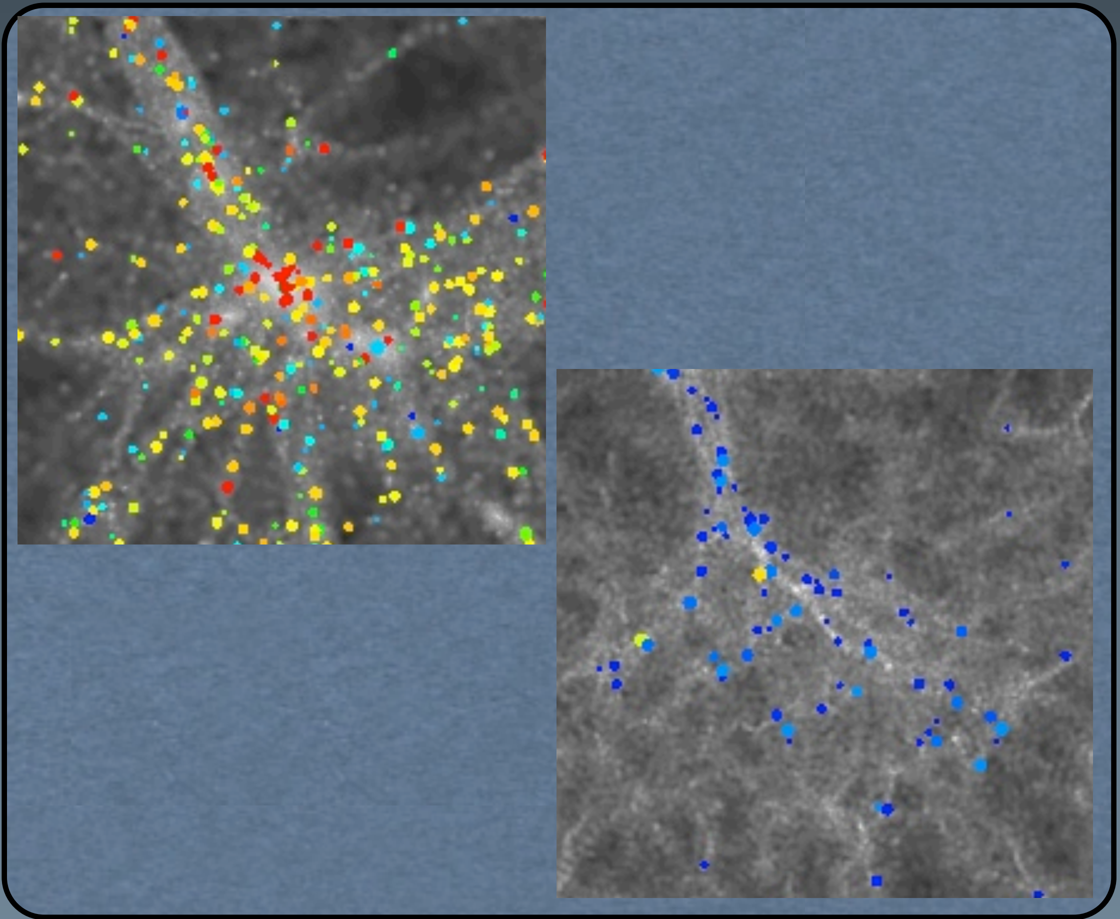
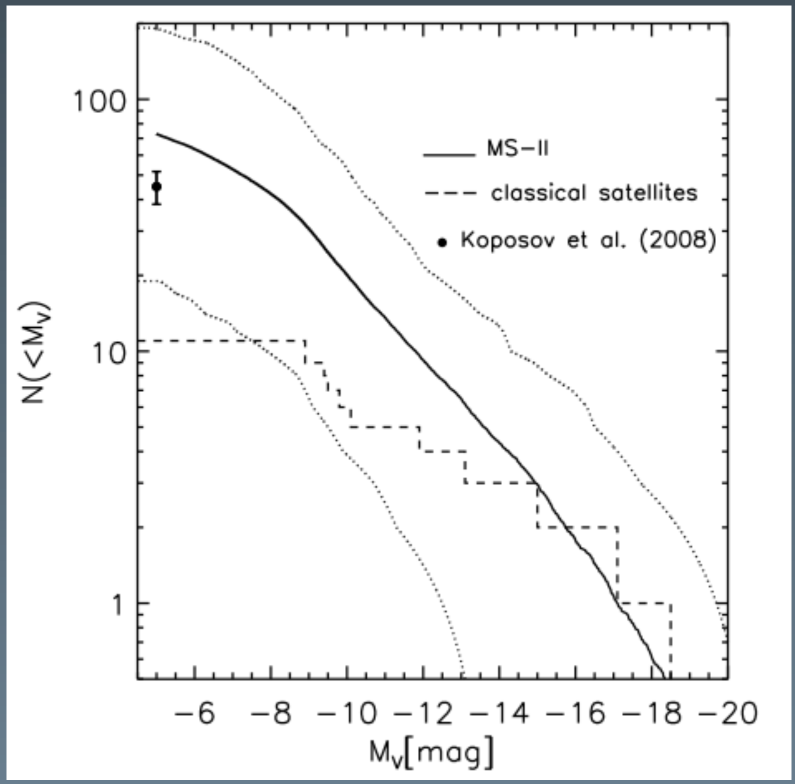
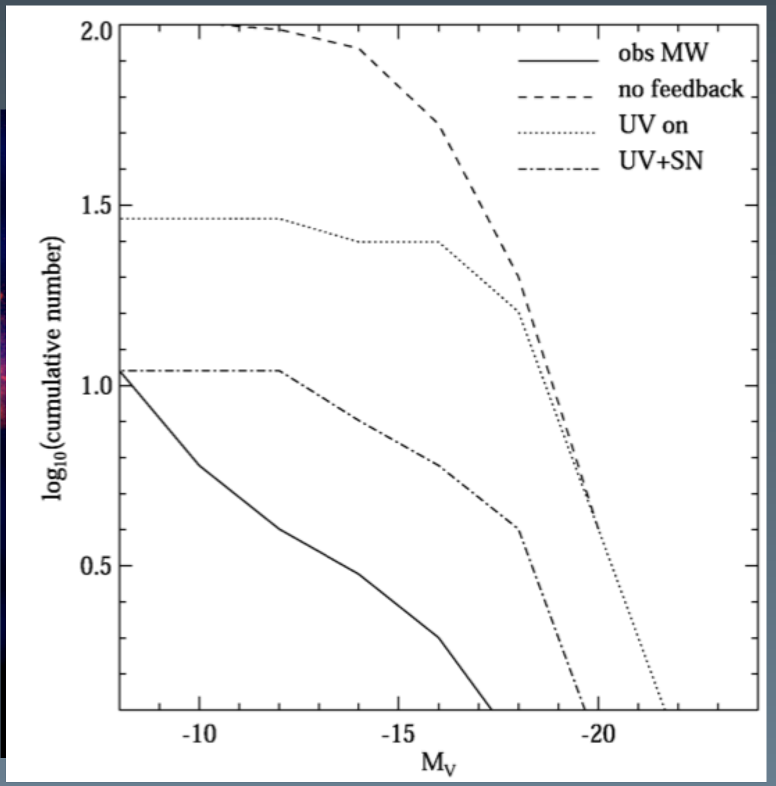
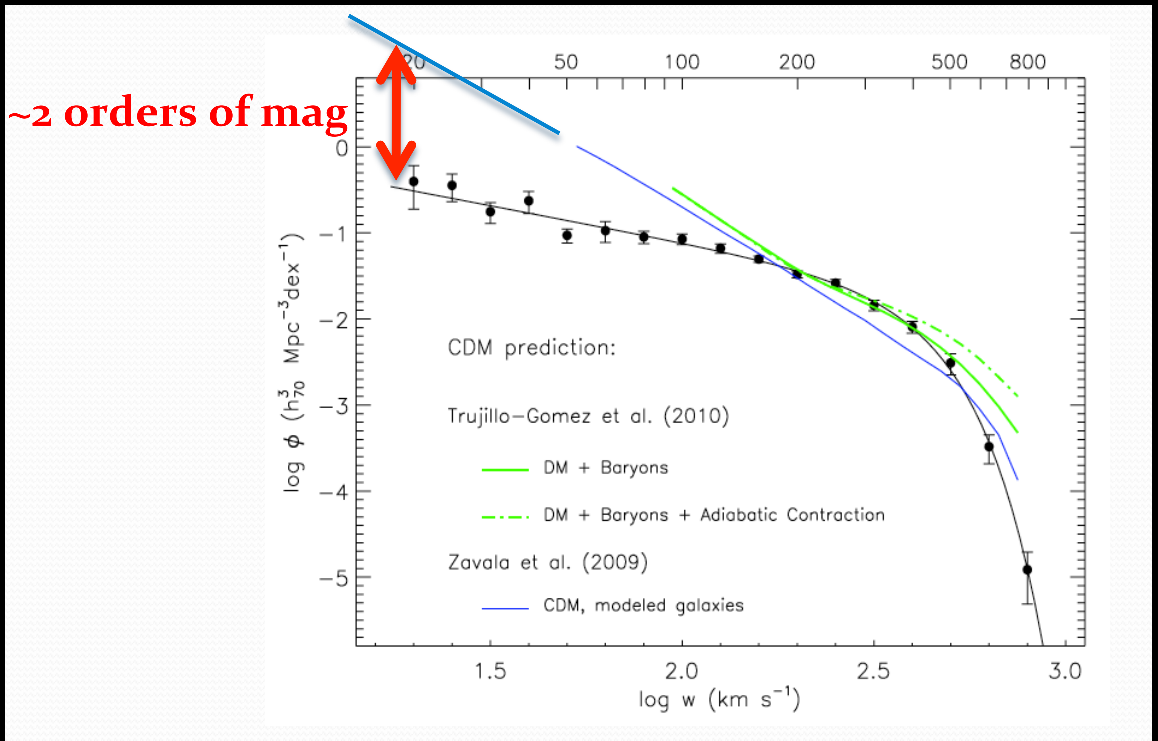






Guo et al. 2009 Governato et al. 2009

Abundance of galaxies as a function of their velocity width (gas rotation velocity)



Papastergis+ (2011)

21-cm survey done with Arecibo Telescope: 3000 deg^2; 11000 detections

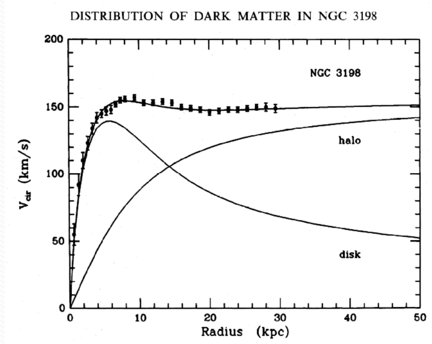
measures: redshift, velocity width, integrated flux

No spatial resolution (size, inclination, shape)

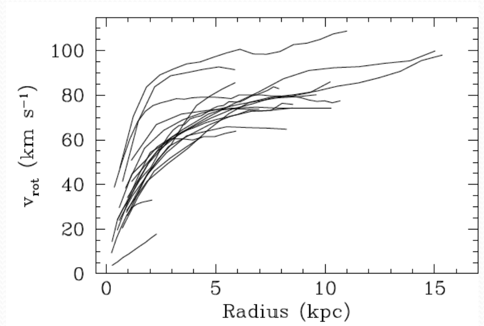
Directly measures depth of the potential well: less prone to physics of gas (feedback)

Solutions within CDM scenario ?

- large fraction of galaxies with low gas content (below the sensitivity)
- large fraction of galaxies with rising rotation curve



'flat' rotation curve



'rising' rotation curves

Swaters+ (2009)

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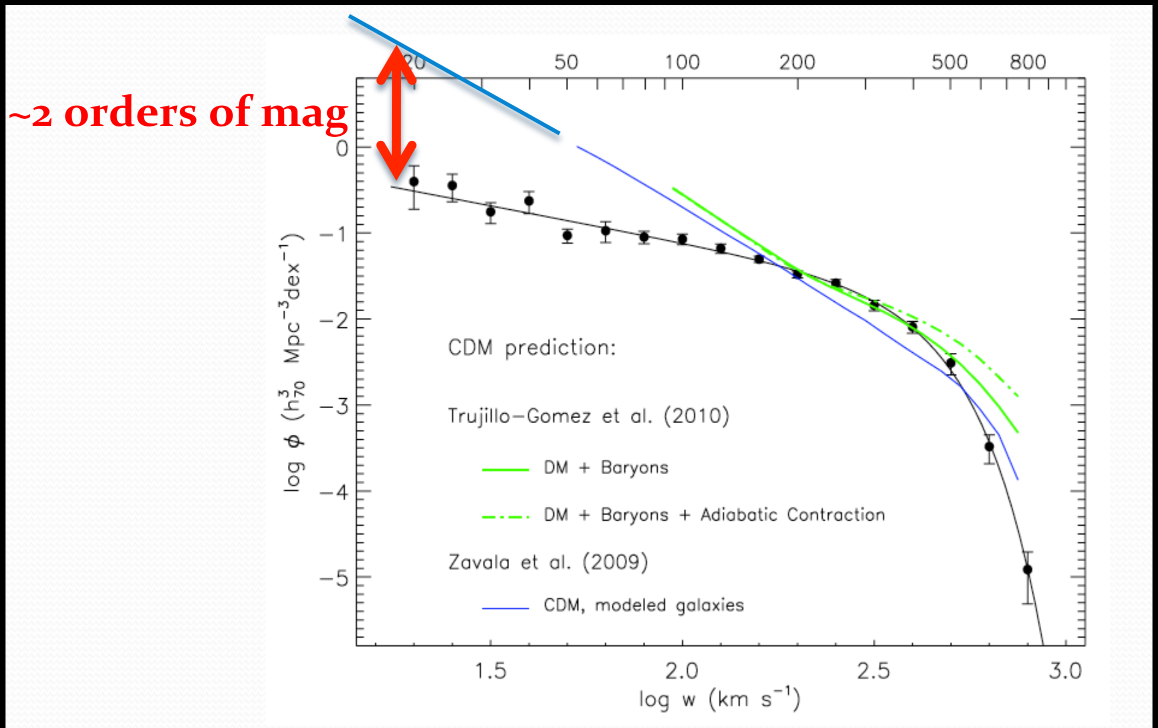

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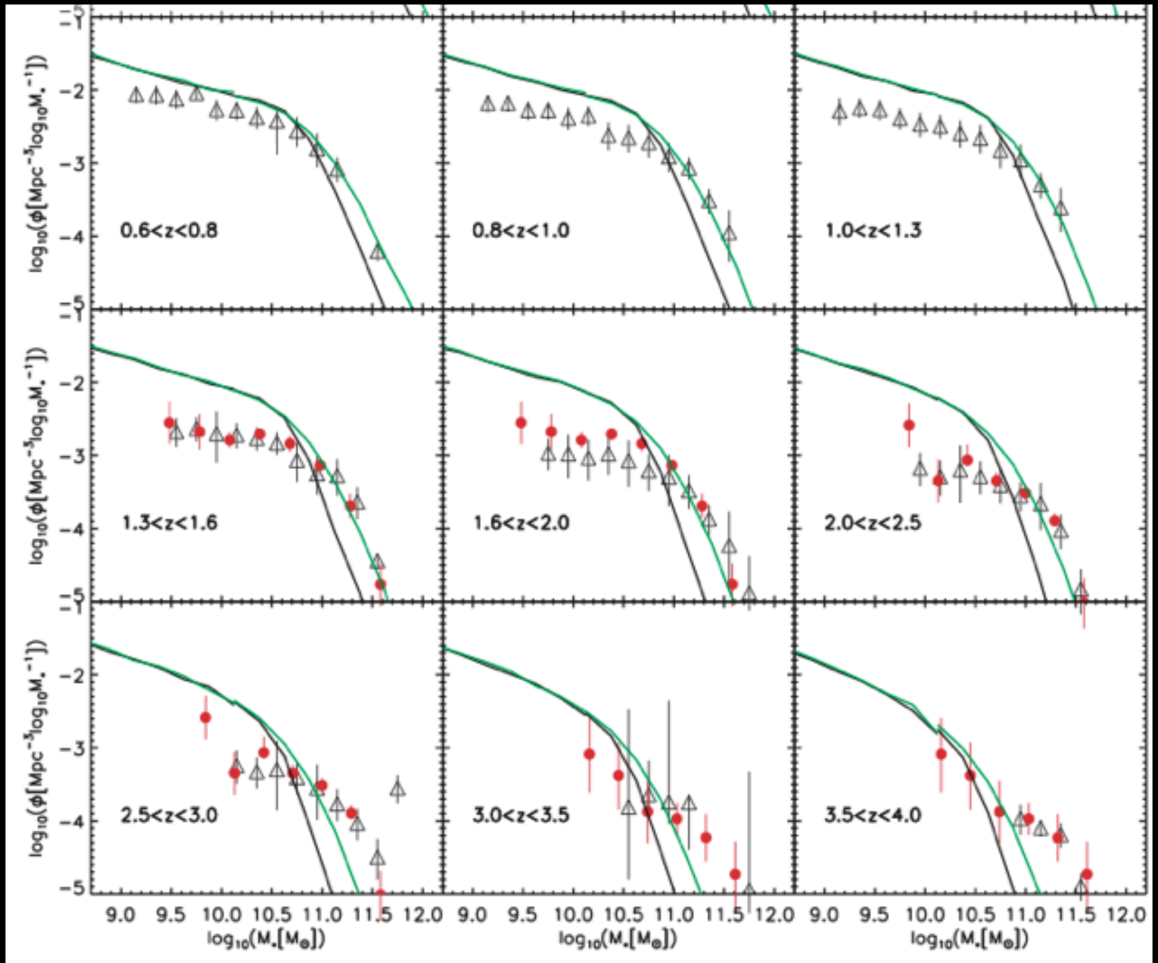


At high redshift, galaxies are denser

Difficult to expell gas from such compact objects

Even with maximized feedback, current models still over estimate the number of small mass galaxies

Galaxy formation in WDM Cosmology

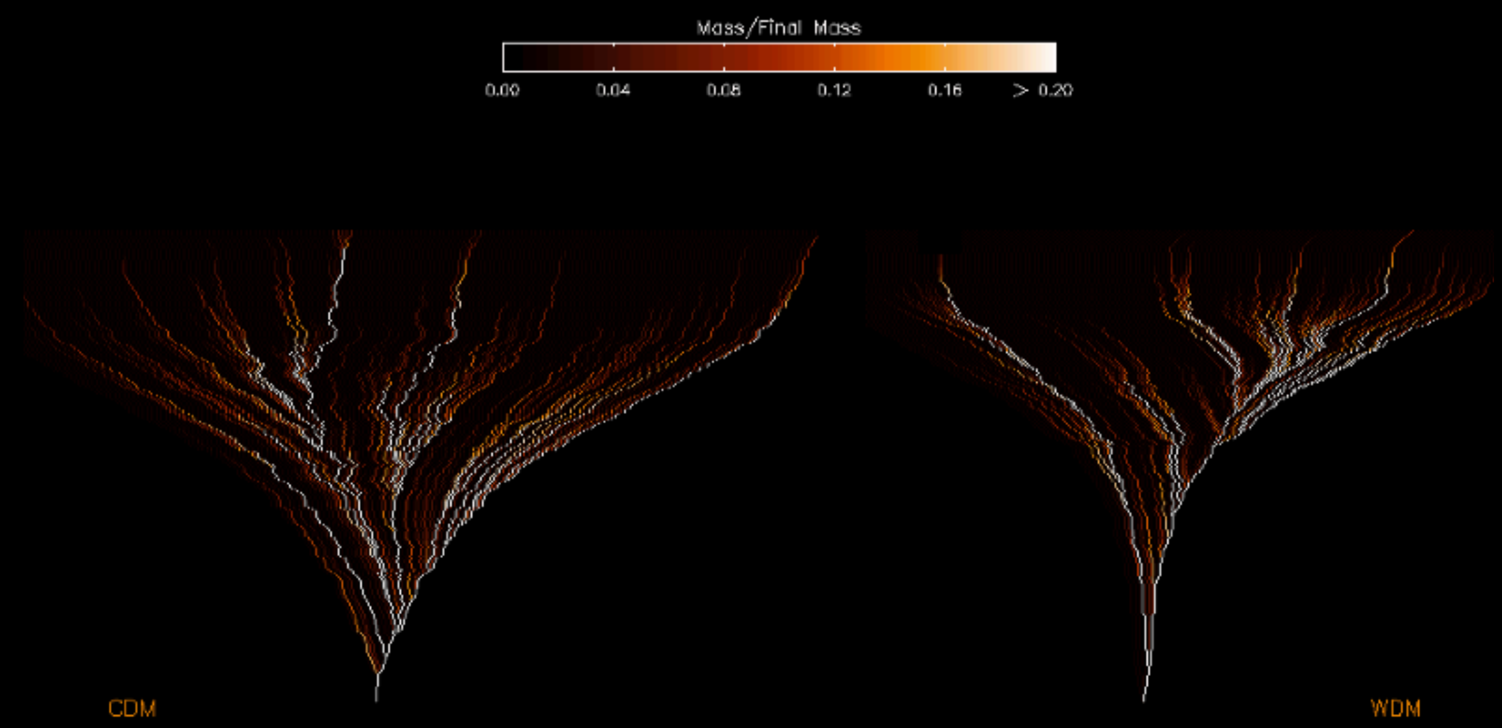
Problem Persists at high redshifts

Too many low-mass structures

Need to suppress Power Spectrum at small scales ?

can WDM solve all problems simultaneously ?

Merger Trees in CDM and WDM Cosmologies



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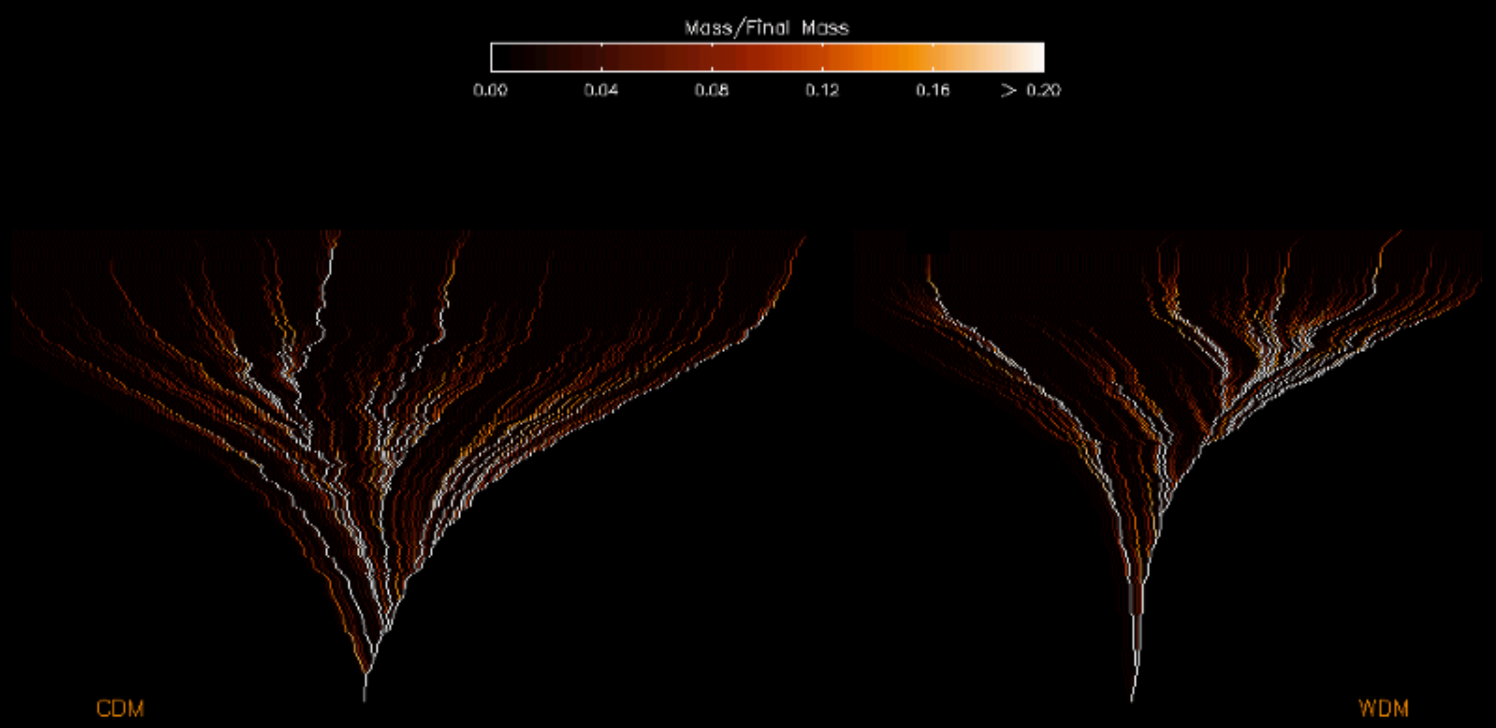
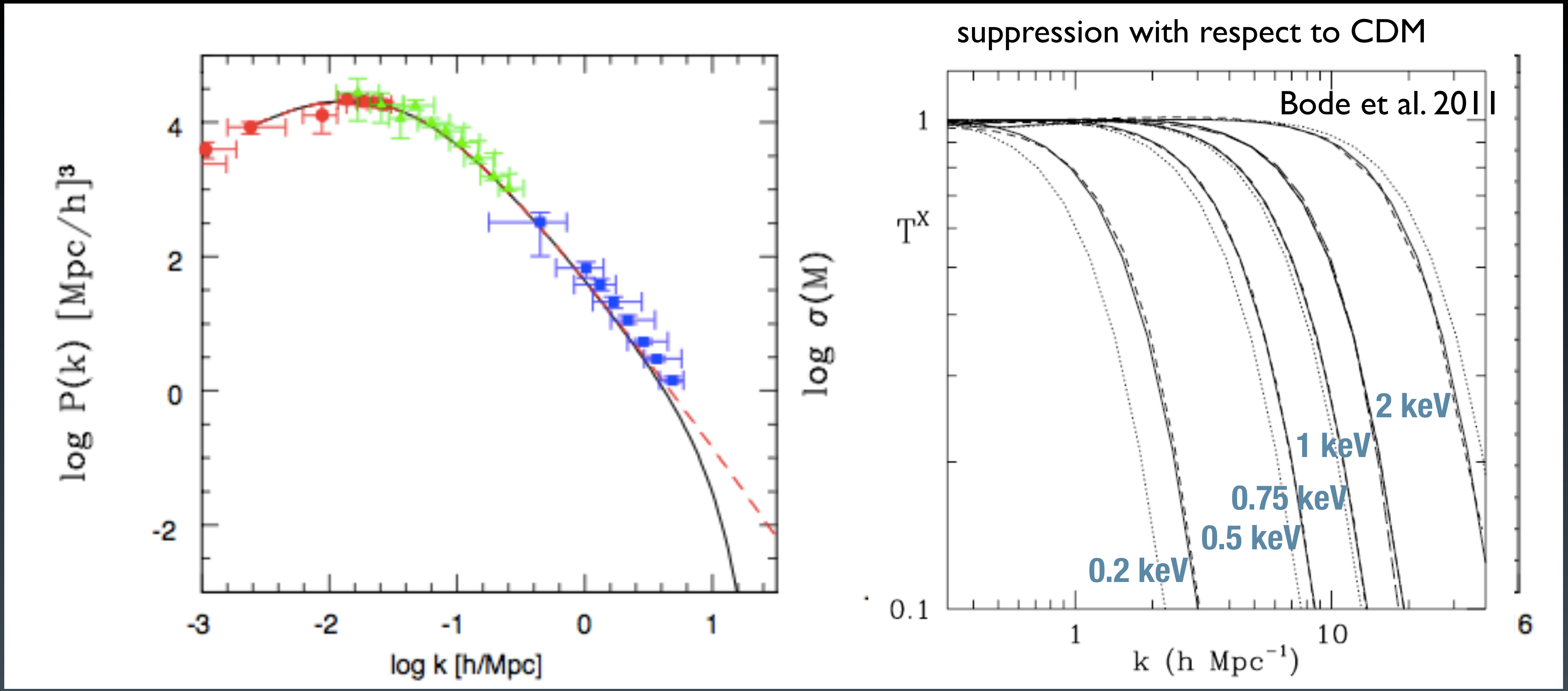


Rome PANDA model

NM, A. Lamstra



Compilation of data from Tegmark, Zaldarriga 2002

To explore the maximal effect of a power-spectrum cutoff on galaxy formation we shall consider a cutoff at scales just below 0.2 Mpc, where data from Lyman-α systems (compared to N-body simulations) yields stringer upper limits on power suppression. This corresponds to mass scales $M_{fs} \sim 5\ 10^8\ M_\odot$

$$r_{fs} \approx 0.2 \left[\frac{\Omega_X h^2}{0.15}\right]^{1/3} \left[\frac{m_X}{\mathrm{keV}}\right]^{-4/3} \mathrm{Mpc} \longrightarrow$$

WDM particle mass ~ 1 keV

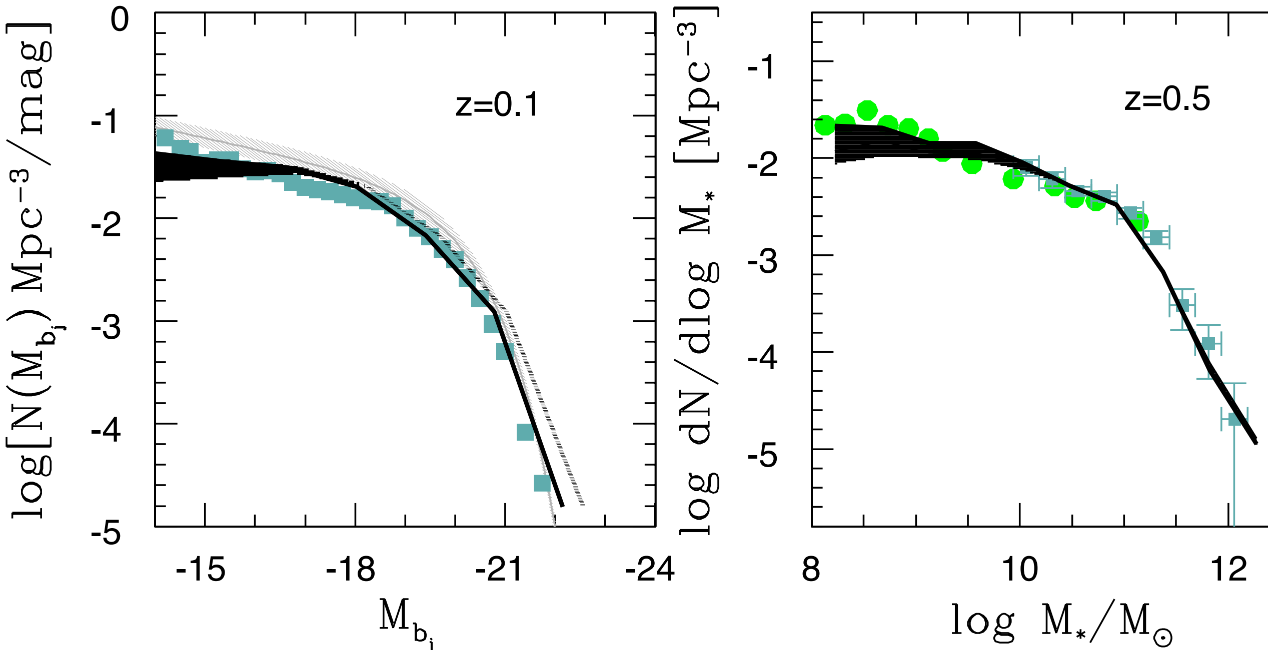
$$\frac{P_{WDM}(k)}{P_{CDM}(k)} = \left[1 + (\alpha\, k)^{2\,\mu}\right]^{-5\,\mu}$$

$$\alpha = 0.049 \left[\frac{\Omega_X}{0.25}\right]^{0.11} \left[\frac{m_X}{\mathrm{keV}}\right]^{-1.11} \left[\frac{h}{0.7}\right]^{1.22} h^{-1}\,\mathrm{Mpc}$$

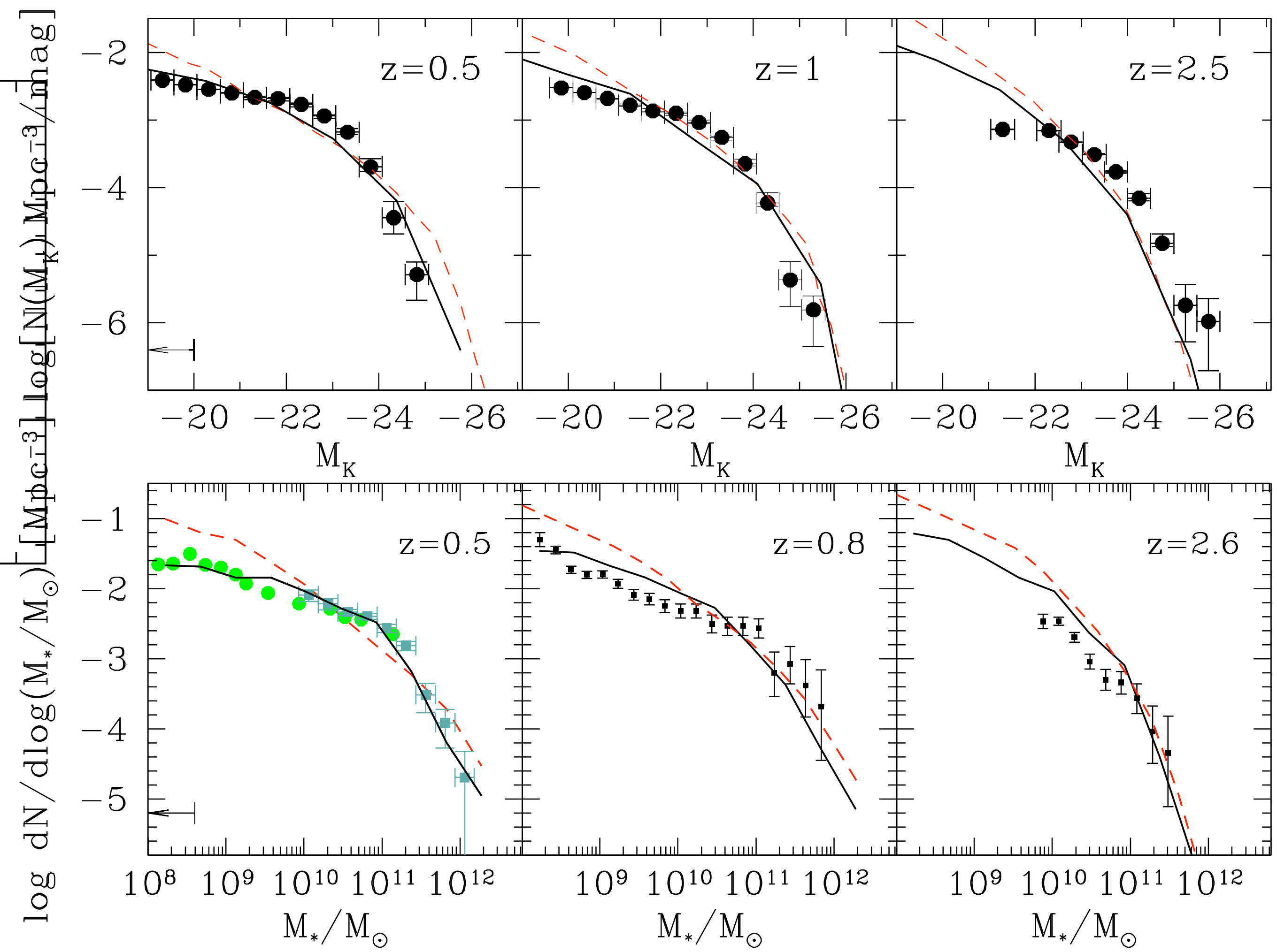
Galaxy Formation in WDM cosmology (m_{WDM}=1 keV)

NM et al. 2012-2013

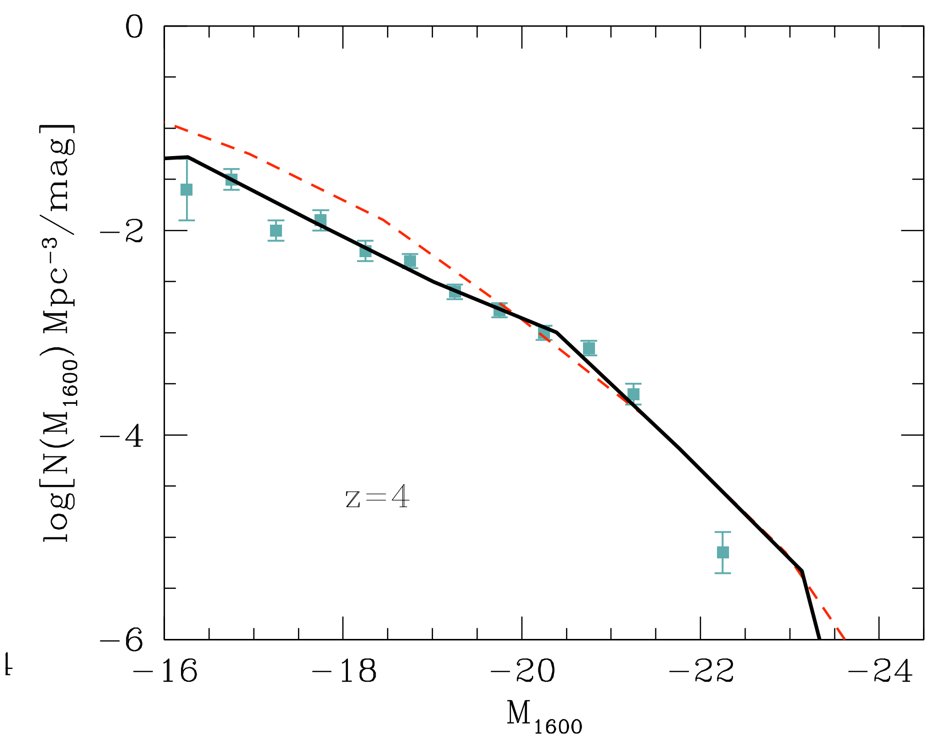
local lum. and stell. mass distributions



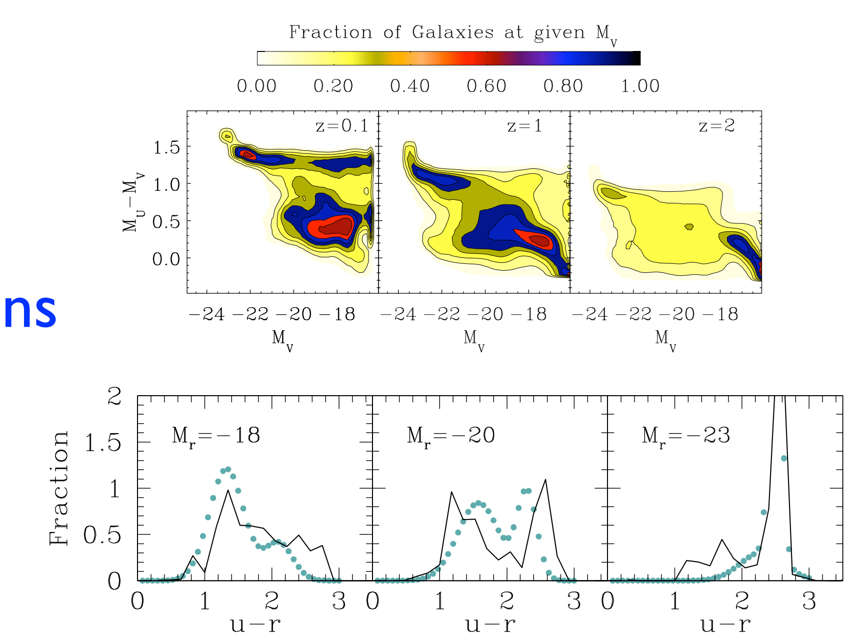
Evolution



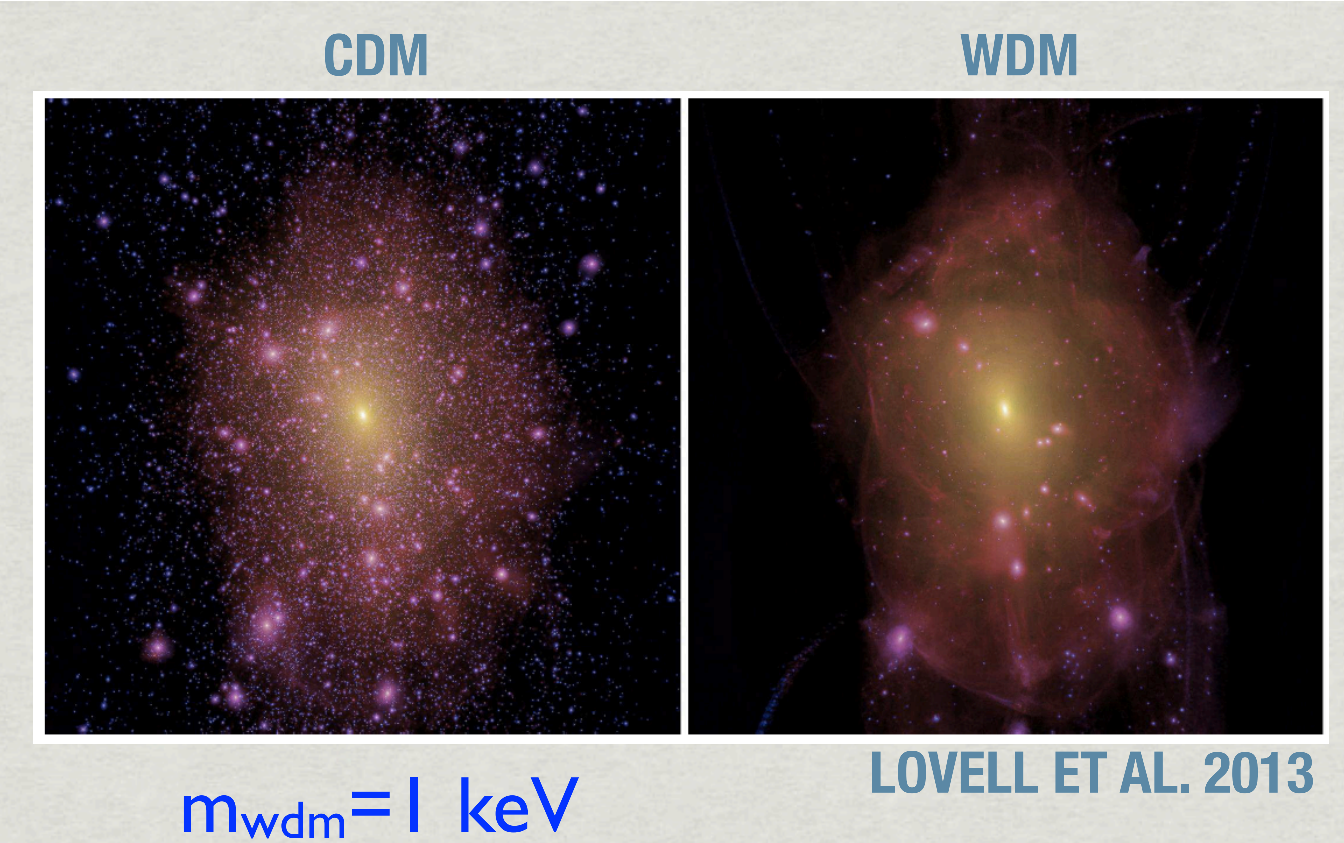
High-z (z=4) lum. distributions



color distributions

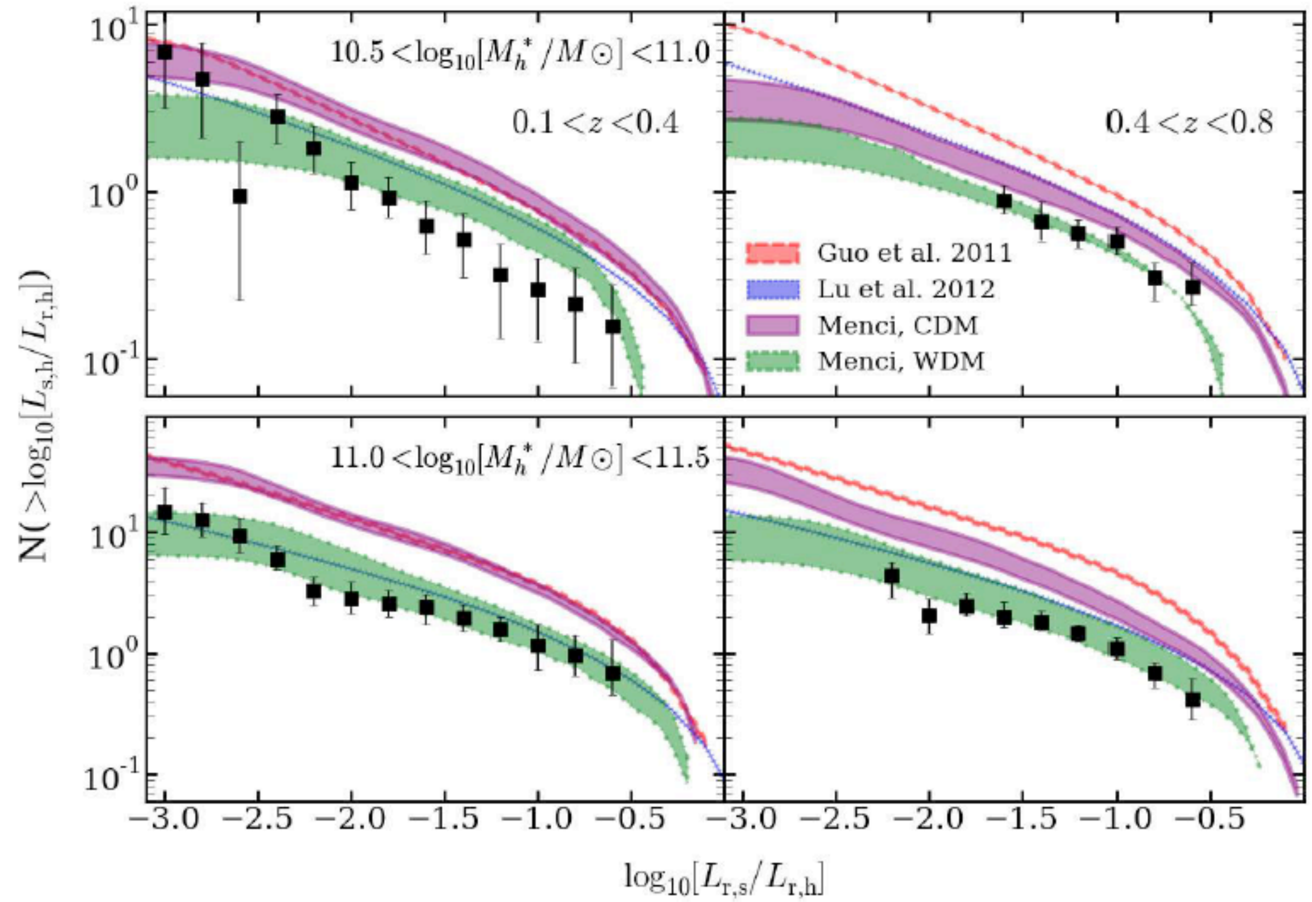


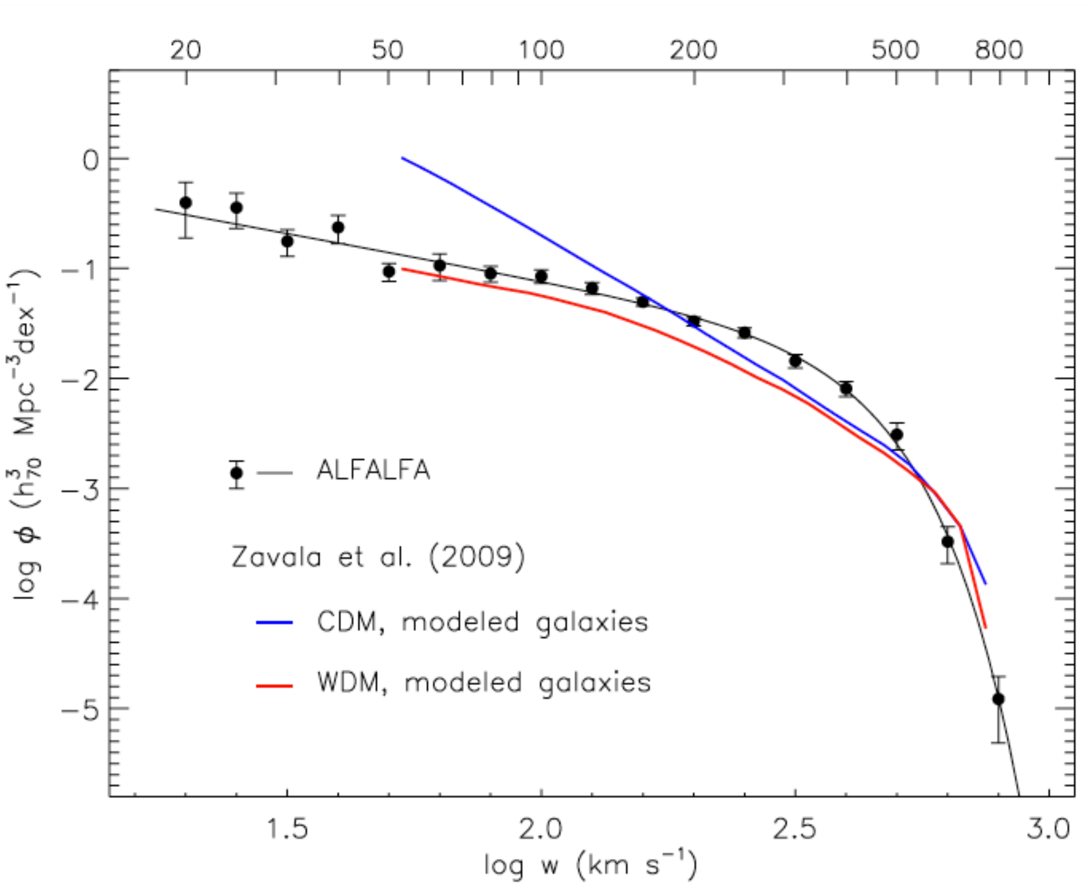
Substructure Predictions



The luminosity distribution of Satellite Galaxies
Nierenberg, Treu, NM 2013

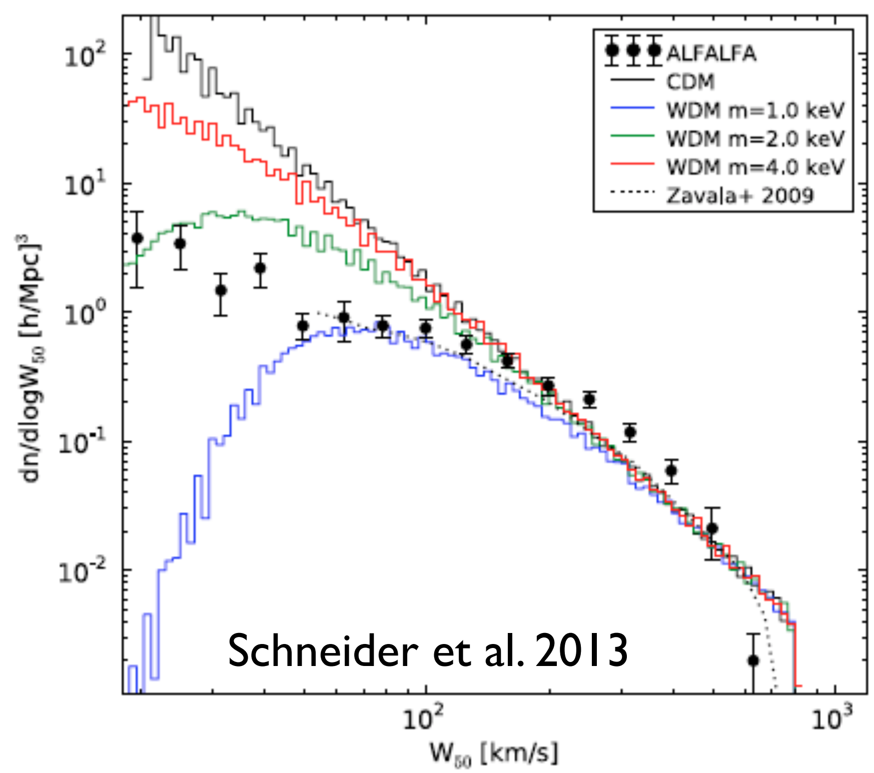
observations of satellites between 0.1 < z < 0.8 based on Hubble Space Telescope images





Papastergis+ (2011)

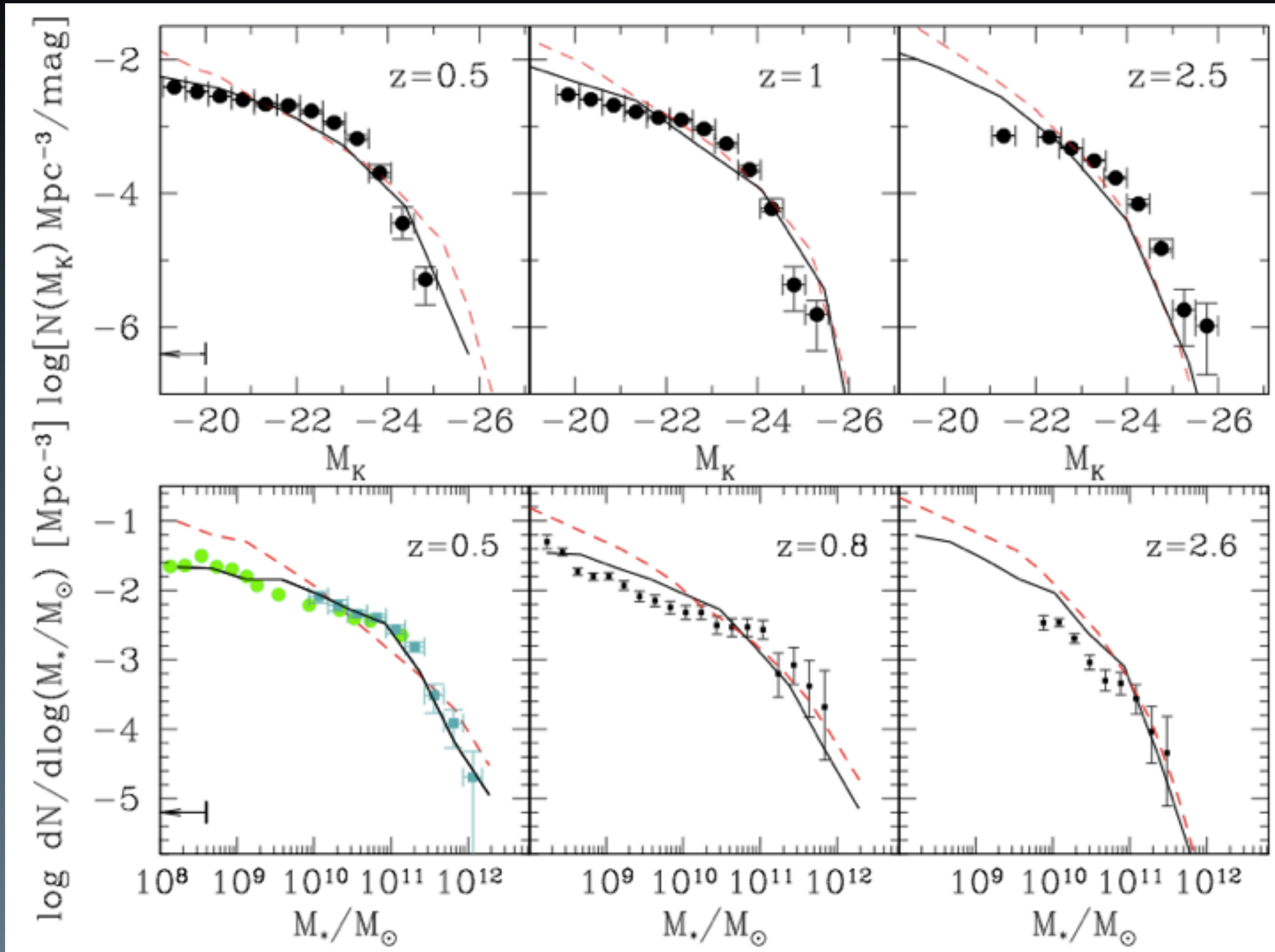
modeling: Zavala+ (2009)



The evolution of the stellar mass function

Dashed CDM
Solid WDM

NM et al. 2012



WDM solution to small-scale problems
in Galaxy Formation $\rightarrow m_{wdm}$=1-2 keV

WDM particle mass: limits from the Ly-α forest

Viel et al. 2005-2013

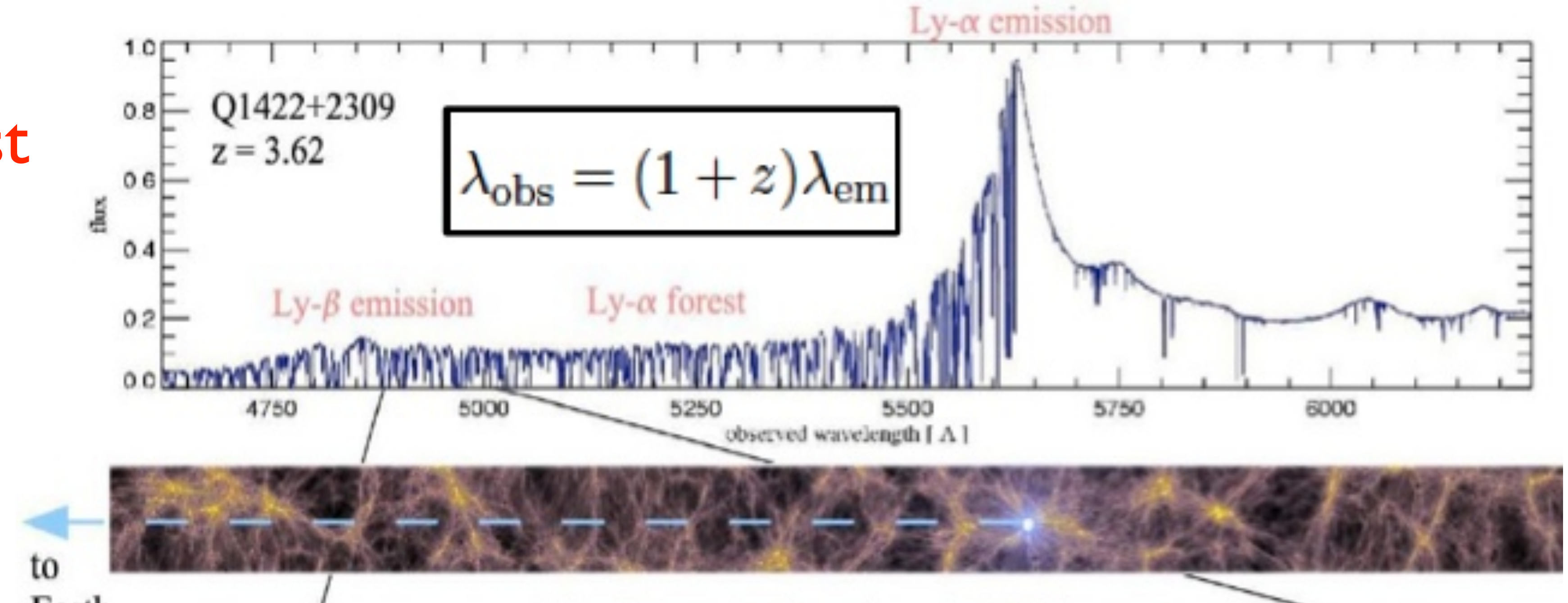
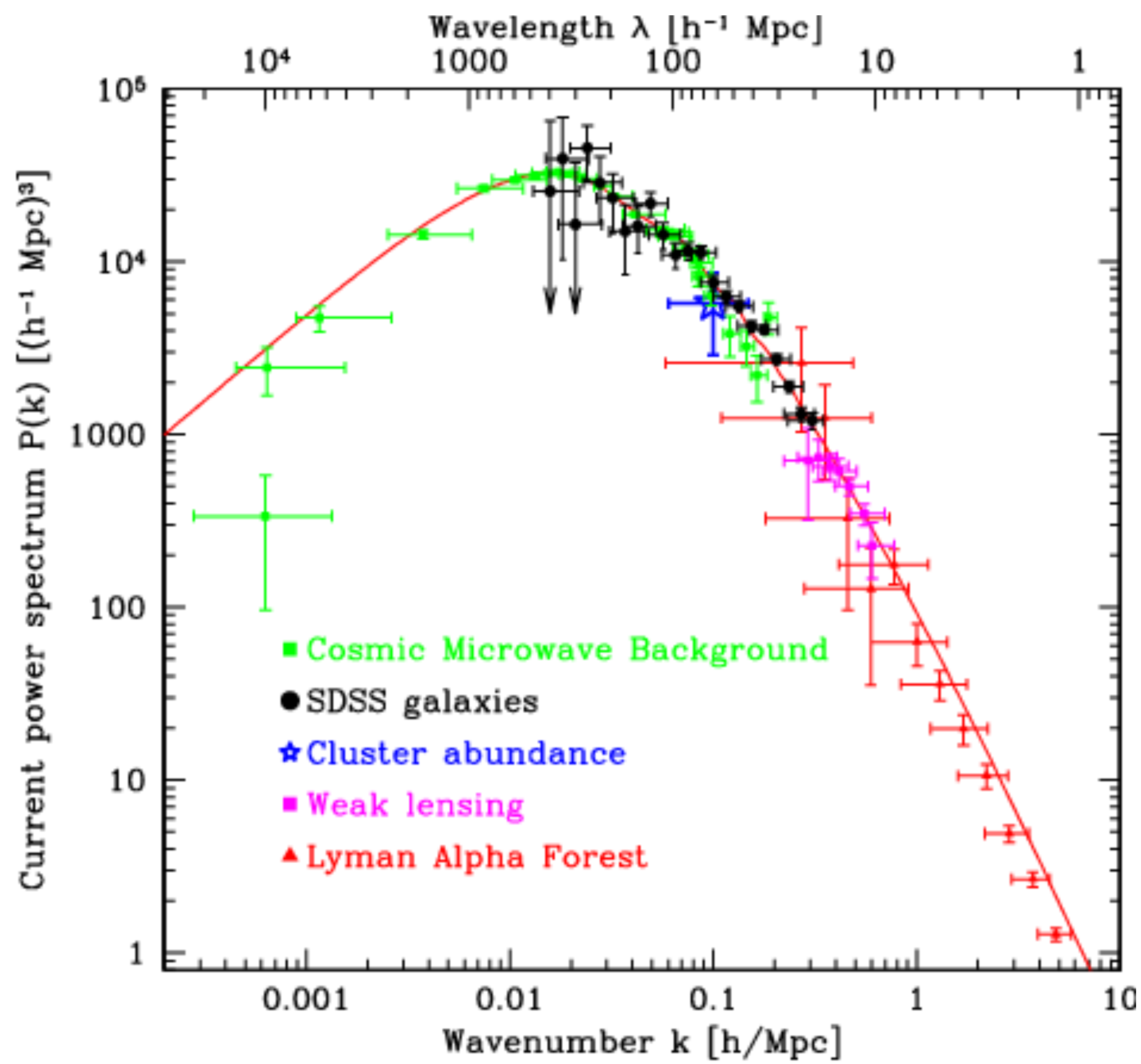


Fig. From Springel et al. 2006

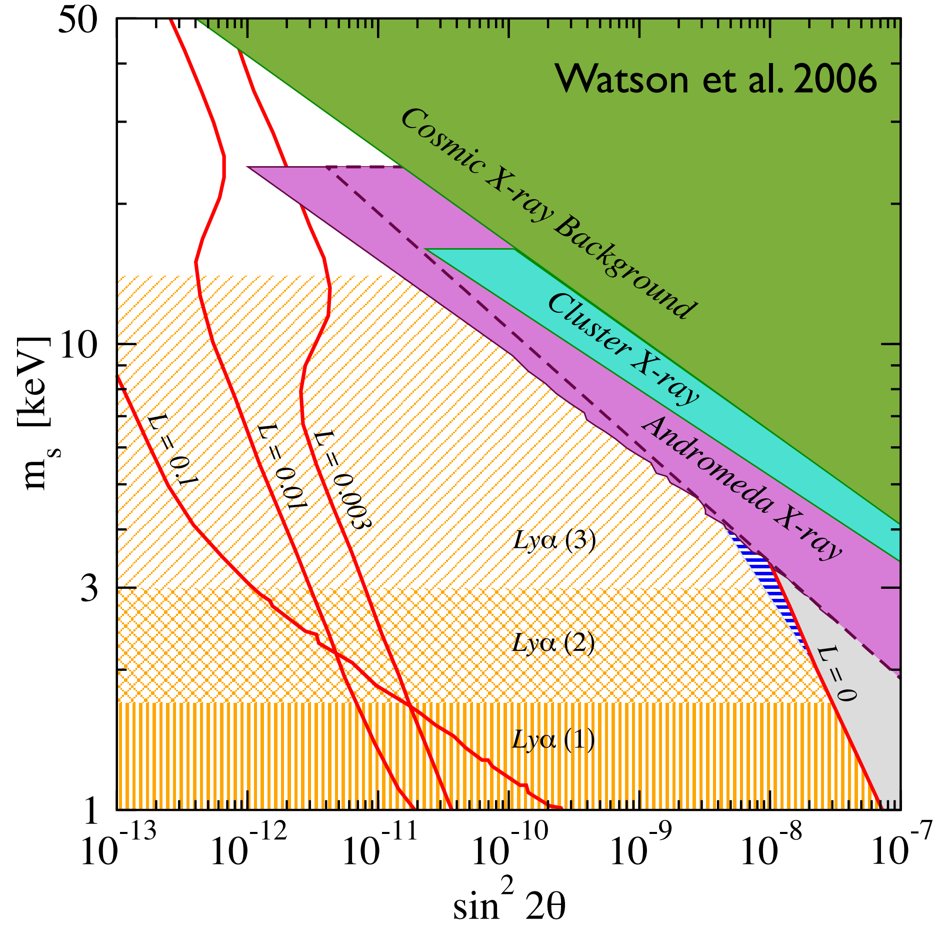
m_{WDM}>1 keV — Thermal relics WDM

m_ν>4 keV — Sterile ν WDM (DW)

m_{WDM}>4 keV — Thermal relics WDM



Constraints from X-ray emission from clusters and galaxies



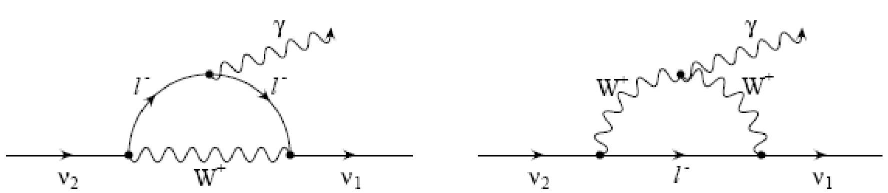
if $m_s > m_\alpha$ the radiative decay $\nu_s \rightarrow \nu_\alpha + \gamma$ becomes allowed

$$E_\gamma = \frac{1}{2} m_s \left(1 - \frac{m_\alpha^2}{m_s^2}\right) .$$

Emission lines in X-rays from DM concentrations:
- clusters (large signal but also large background)
- galaxies

The observed flux, F_{det}, gives an upper limit for the flux from decaying dark matter so Eqn. 19 can be rewritten as:

$$\begin{aligned} \Gamma_{\gamma,max} &\leq \frac{8\pi F_{det} D_L^2}{M_{tot}} \\ &= 1.34 \cdot 10^{-4}\,\mathrm{sec}^{-1} \left(\frac{F_{det}}{\mathrm{erg/cm^2/sec}}\right) \left(\frac{D_L}{\mathrm{Mpc}}\right)^2 \left(\frac{M_{tot}}{\mathrm{M_\odot}}\right)^{-1} \end{aligned}$$



Abazajian et al. 2001-2005

Summary

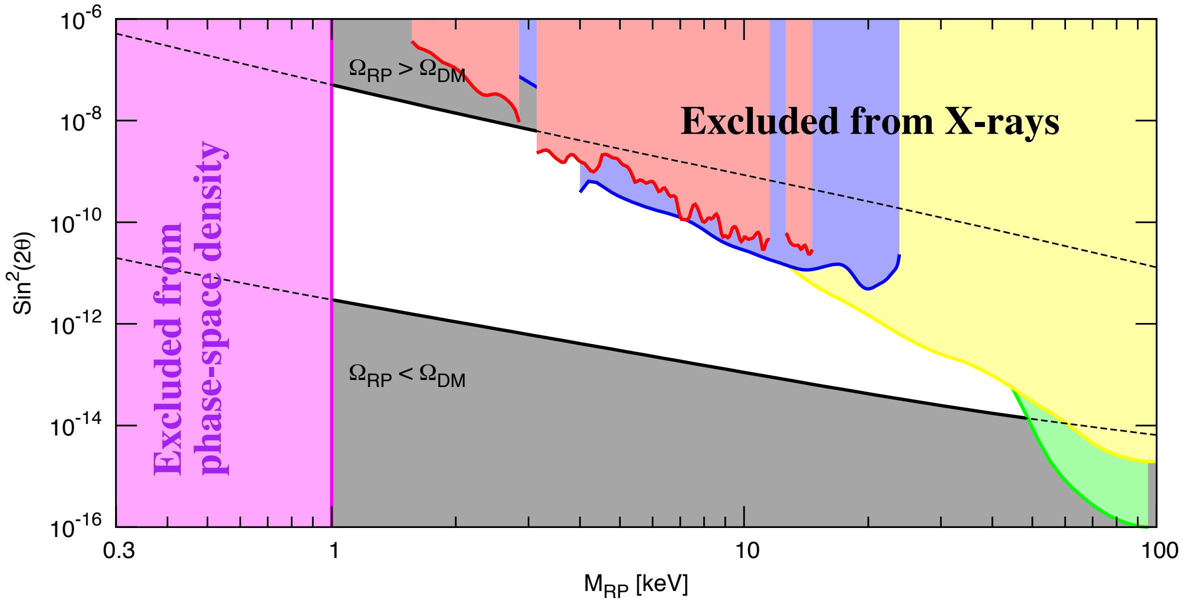
The mass of DM particles has a major impact on structure formation
(suppression of small-scale perturbations due to free-streaming)
CDM is the limit of M_{fs}<< masses of cosmological interest

CDM problems on small scales:
- cusps
- number of satellite galaxies
- abundance of low-mass (faint) galaxies at low and high redhsifts

Baryonic physics can hardly solve the problems

Galaxy formation in WDM cosmology is a viable solution

There is a tension:
current limits from high-z structure (Lyman-a forest)
suggest $m>10$ keV, but to solve the galactic small-scale crisis $m<2$ keV is needed



Window corresponds to resonant production

Upper boundary - zero lepton asymmetry

Lower boundary - maximal lepton asymmetry

Boyarsky et al 2009

6 – Sterile neutrino resonant production

In presence of a large lepton asymmetry, $\mathcal{L} \equiv (n_\nu - n_{\bar{\nu}})/n_\gamma$, matter effects become important and the mixing angle can be resonantly enhanced. [Shi, Fuller, 1998; Abazajian et al., 2001

$$\boxed{\sin^2 2\theta_m = \frac{\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\sin^2 2\theta + D^2 + (\Delta(p)\cos 2\theta - \frac{2\sqrt{2}\zeta(3)}{\pi^2} G_F T^3 \mathcal{L} + |V_T|)^2}}$$

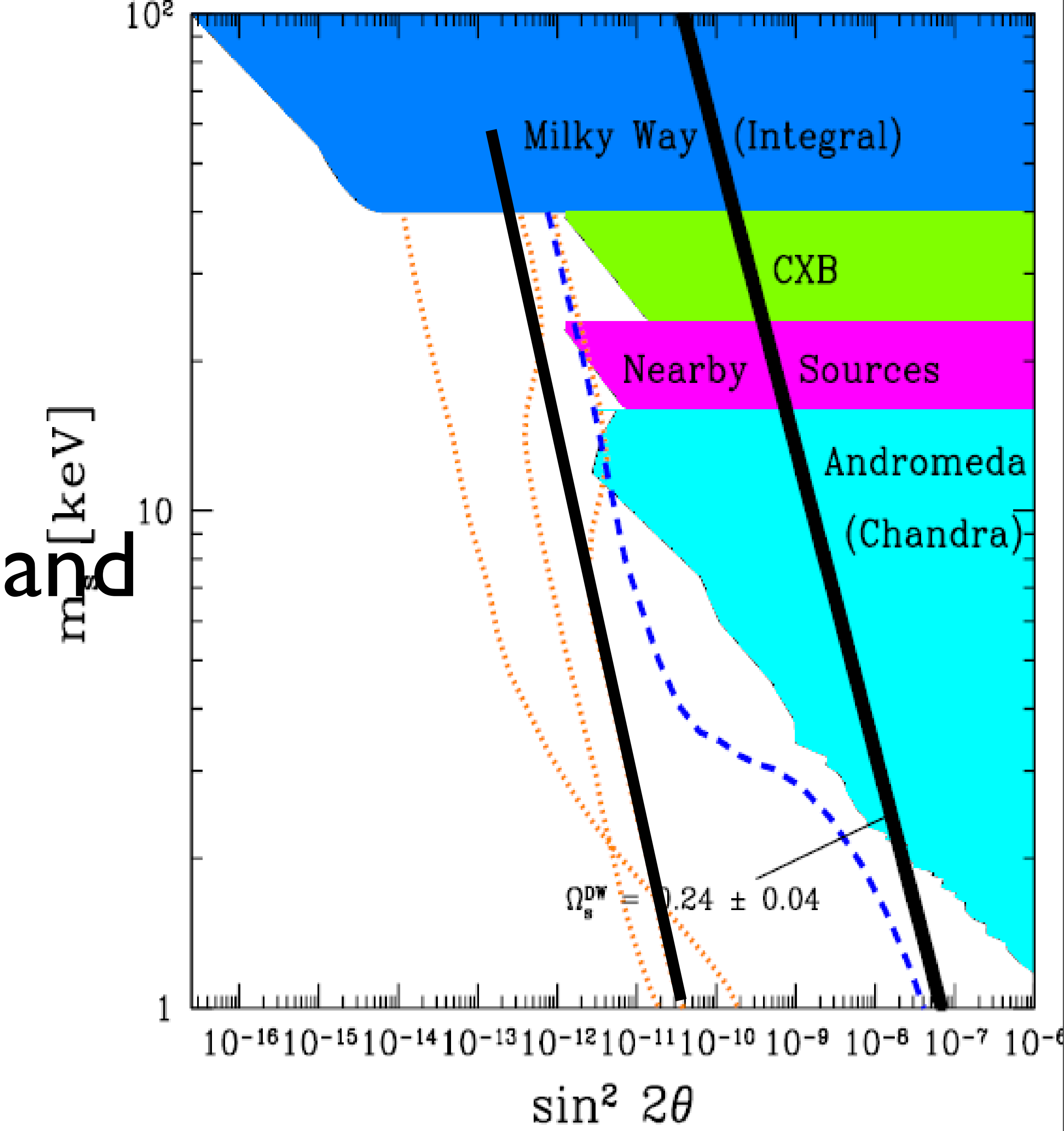
The mixing angle is maximal $\sin^2 2\theta_m = 1$ when the resonant condition is satisfied (with $\Delta(p) \equiv m_4^2/(2p)$)

$$\boxed{\Delta(p)\cos 2\theta - \frac{2\sqrt{2}\zeta(3)}{\pi^2} G_F T^3 \mathcal{L} + |V_T| = 0}$$

$$\boxed{\left(\frac{m_4}{1\mathrm{keV}}\right)^2 \simeq 0.08 \frac{p}{T} \frac{\mathcal{L}}{10^{-4}} \left(\frac{T}{100\,\mathrm{MeV}}\right)^4 + 2\left(\frac{p}{T}\right)^2 \frac{B}{\mathrm{keV}} \left(\frac{T}{100\,\mathrm{MeV}}\right)^6}$$

Watson et al. 2012

Limits from the
X-ray emission from clusters and
galaxies



Very small mixing ($\sin^2 2\theta \lesssim 10^{-7}$) between

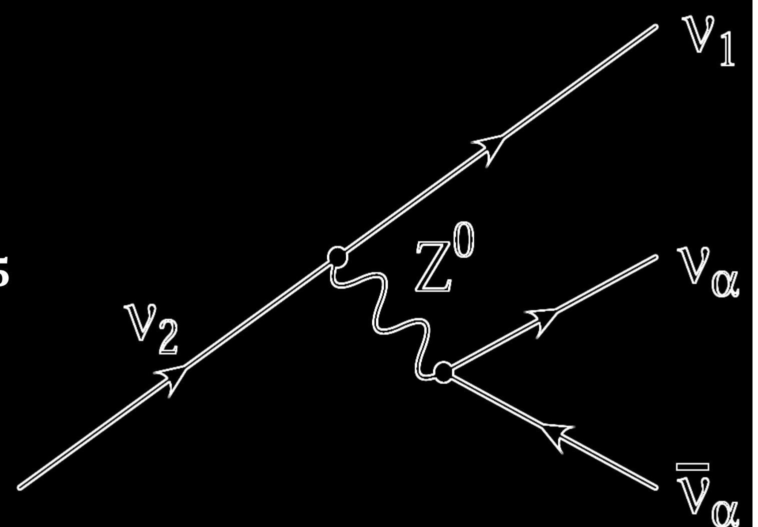
mass $|\nu_{1,2}>$ & flavor $|\nu_{\alpha,s}>$ states:

$$|\nu_\alpha\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$
$$|\nu_s\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

For $m_s < m_e$,

3ν Decay Mode Dominates:

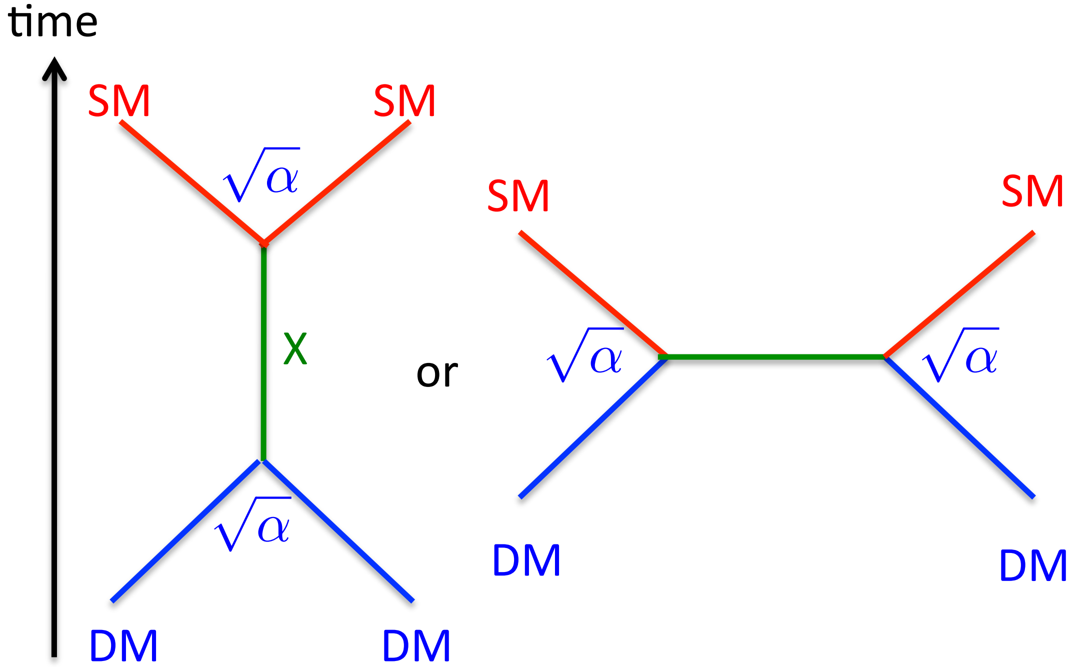
$$\Gamma_{3\nu} \backsimeq 1.74 \times 10^{-30} s^{-1} \left(\frac{\sin^2 2\theta}{10^{-10}}\right)\left(\frac{m_s}{\text{keV}}\right)^5$$



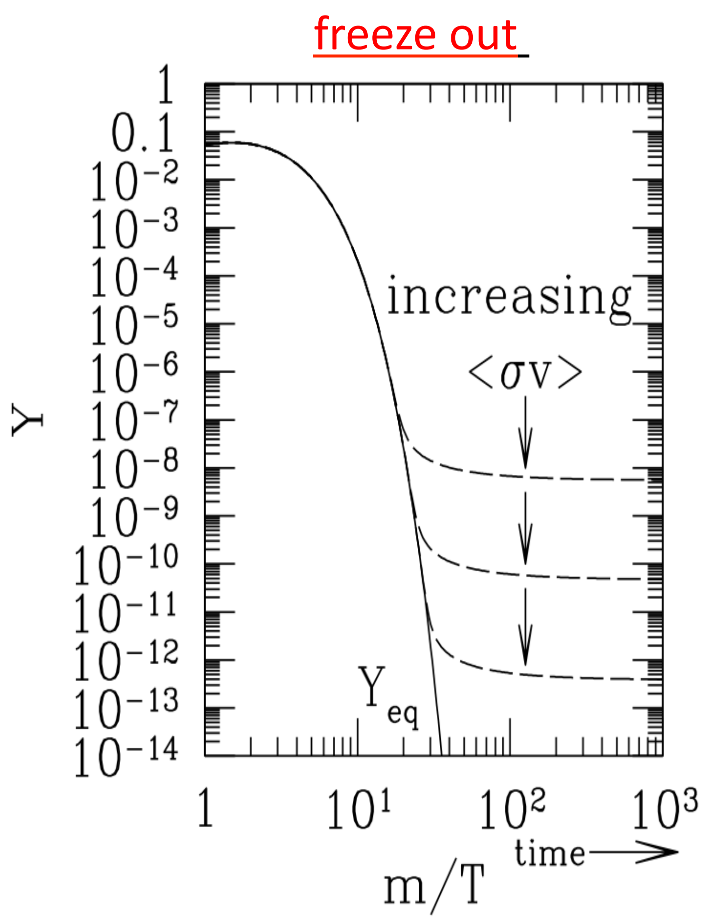
Radiative Decay Rate is:

$$\Gamma_s \backsimeq 1.36 \times 10^{-32} s^{-1} \left(\frac{\sin^2 2\theta}{10^{-10}}\right)\left(\frac{m_s}{\text{keV}}\right)^5 \quad \nu_s \to \nu_\alpha + \gamma$$

WIMP



self-annihilation



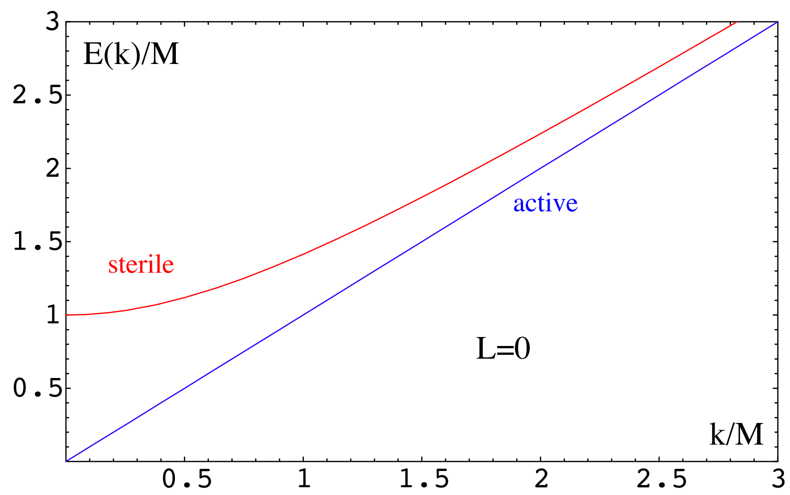
$$\Omega h^2 \approx \frac{3 \times 10^{-27}\ \mathrm{cm}^3/\mathrm{s}}{\langle \sigma_{\mathrm{ann}} v \rangle}.$$

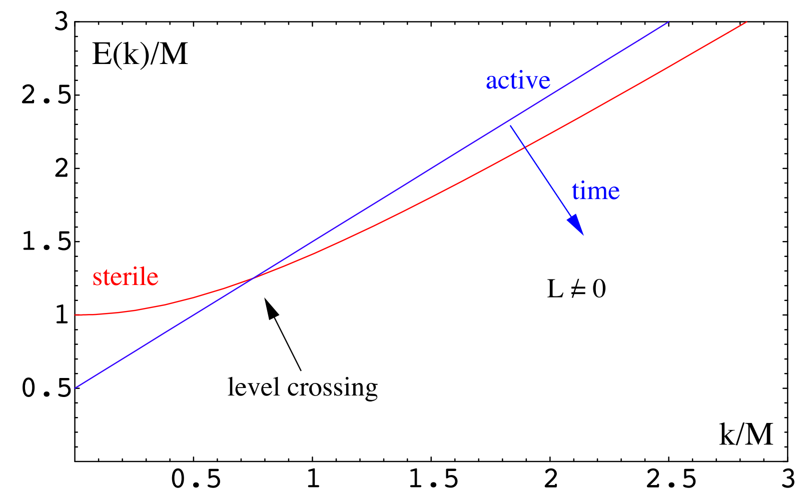
$$\langle \sigma_{\mathbf{ann}} \mathbf{v} \rangle \sim \mathbf{10^{-25}} \mathrm{cm}^3 \mathrm{s}^{-1} \left(\frac{\alpha}{\mathbf{10^{-2}}} \right)^{\mathbf{2}} \left(\frac{\mathbf{100GeV}}{\mathbf{m_X}} \right)^{\mathbf{2}}$$

Electro Weak Scale(～100GeV) WIMP naturally explains the relic abundance.

TeV scale SUSY & neutralino dark matter

Dispersional relations for active and sterile neutrinos (from real part)





Transitions $\nu \to N_1$

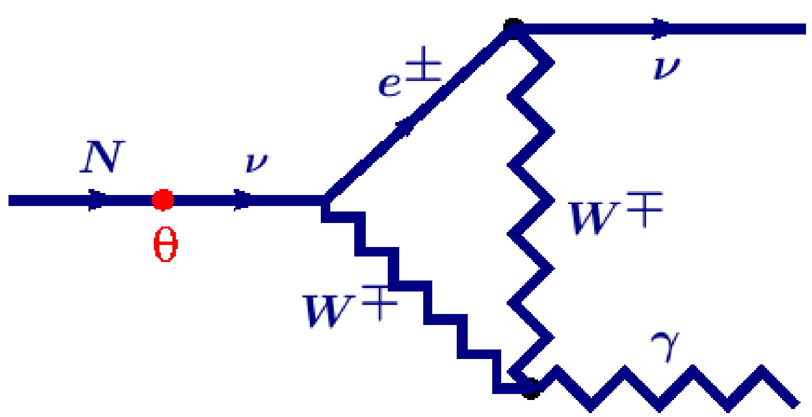
Dodelson-Widrow

Resonant transitions

Shi-Fuller

- Photon energy:

$$E_\gamma = \frac{M_1}{2}$$



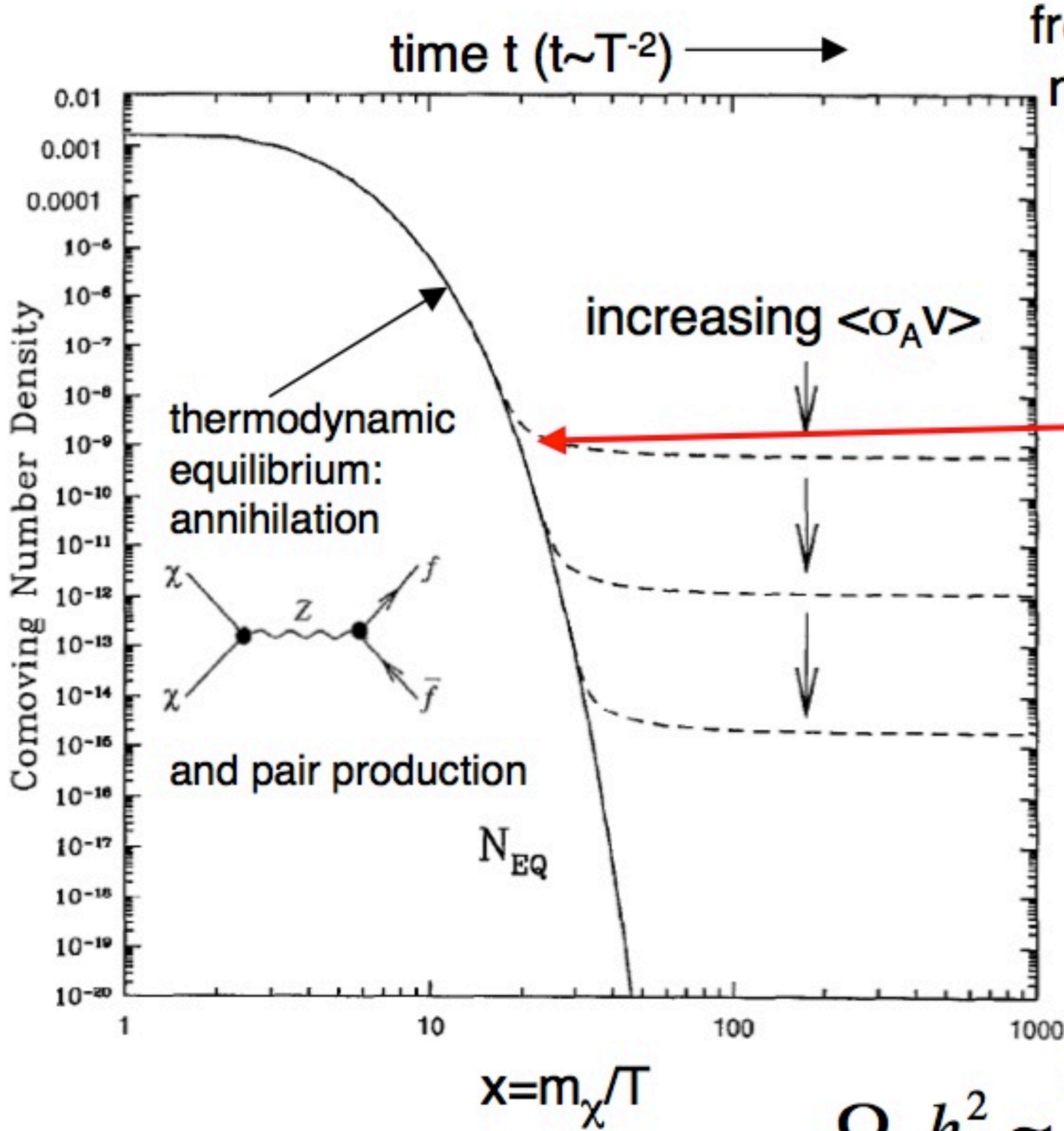
- Radiative decay width

$$\Gamma = \frac{9\alpha_{\rm EM} G_F^2}{256\pi^4}\,\theta^2\, M_1^5$$

Dark matter made of sterile neutrino is not completely dark

Dolgov & Hansen (2000)

CDM as particle Dark Matter



freeze-out of a weakly interacting massive particle (WIMP χ) when reaction rate drops below expansion rate

$T_{\text{freeze-out}} \sim 1/20 \times m(\text{WIMP})$

Cold Dark Matter:
- non-relativistic

"survival of the weakest"

At or below the weak scale

$$\Omega_\chi h^2 \approx \frac{m_\chi n_\chi}{\rho_c} \approx \frac{3\times10^{-27}\,\text{cm}^3/\text{sec}}{\langle \sigma_A v\rangle}$$