Studying of SU(N) LGT in external chromomagnetic field with QCDGPU

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QCDGPU is an open-source package for Monte Carlo lattice simulations of SU(N) gluodynamics in external field and O(N) models. The package is designed to produce lattice gauge configurations as well as to perform measurements. The code is implemented in OpenCL, tested on AMD and nVidia GPUs, AMD and Intel CPUs and may run on other OpenCL-compatible devices.

init.dat file:

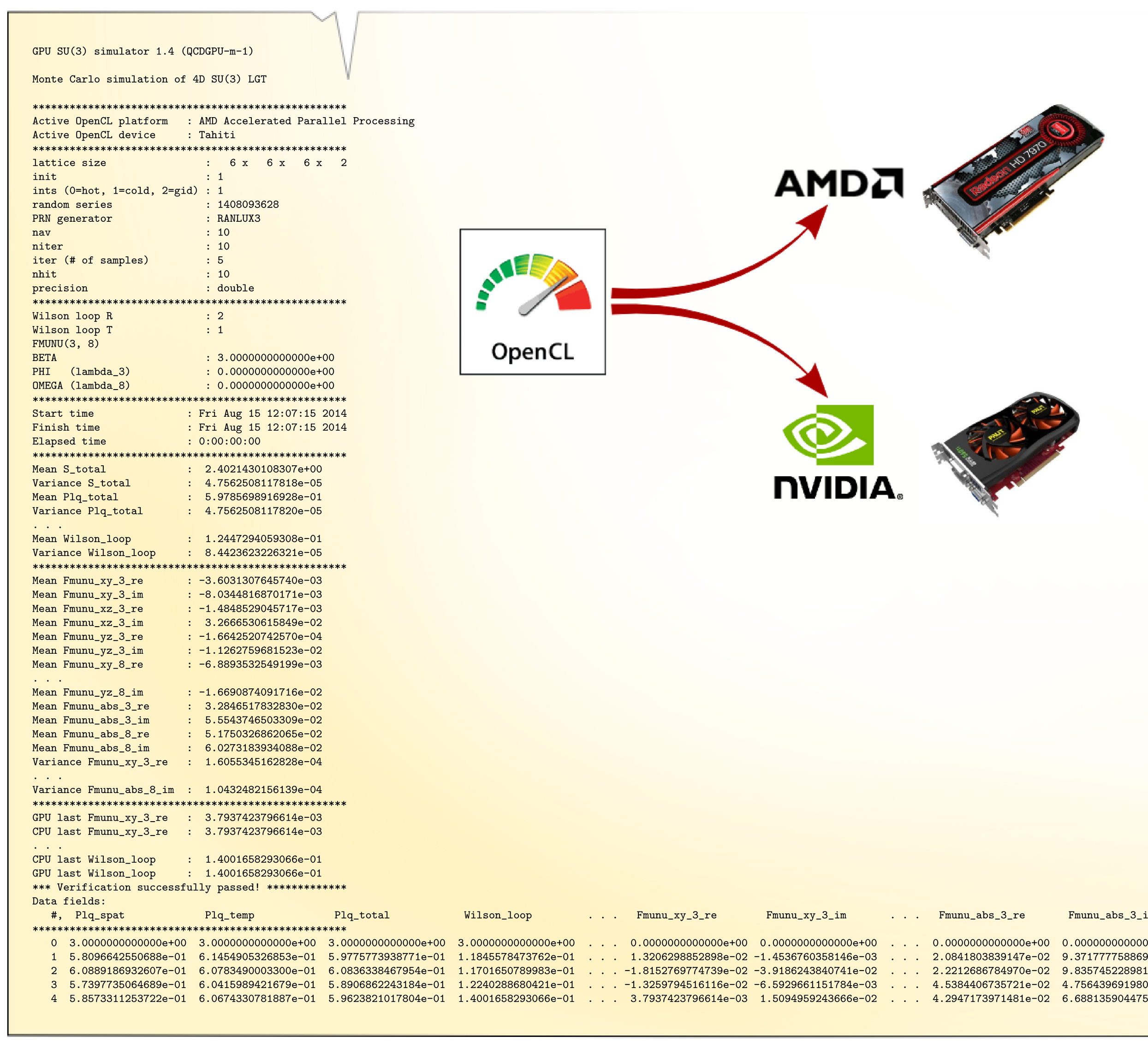
```
PLATFORM = 1              – number of desired OpenCL platform[1]
DEVICE = 0                – number of desired OpenCL device on the platform
FINISHPATH = ./
OUTPUTPATH = ./Output/
GROUP = 3                 – range of gauge group
ND = 4                    – number of space-time directions
LT = 2                    – number of sites in temporal direction (full lattice)
LS = 6                    – number of sites in spatial direction (full lattice)
NT = 2                    – number of sites in temporal direction (part of lattice)
NS = 6                    – number of sites in spatial direction (part of lattice)
ITER = 5                  – number of working iterations
NITER = 10                – number of discarded iterations (for decorrelation)
NHIT = 10                 – parameter of multihit update procedure
BETA = 3.0                – inverse coupling
NAV = 10                  – number of thermalization sweeps
INTS = 1                  – type of start (0 – hot, 1 – cold)
RANDSERIES = 0            – pseudorandom numbers series (0 – current time)
PRNG = RANLUX3            – pseudorandom numbers generator
WILSONR = 2               – size of Wilson loop in spatial direction
WILSONT = 1               – size of Wilson loop in temporal direction
PHI = 0.0                 – value of external chromomagnetic flux φ3
OMEGA = 0.0               – value of external chromomagnetic flux φ8
FMUNU = 1                 – measurement of Fμν tensor components
PL_LEVEL = 2              – Polyakov loop level (0 – do not measure,
                            1 – measue only Re L and Im L,
                            2 – measure also |L|^2 and |L|^4,
                            3 – differential Polyakov loop.
```

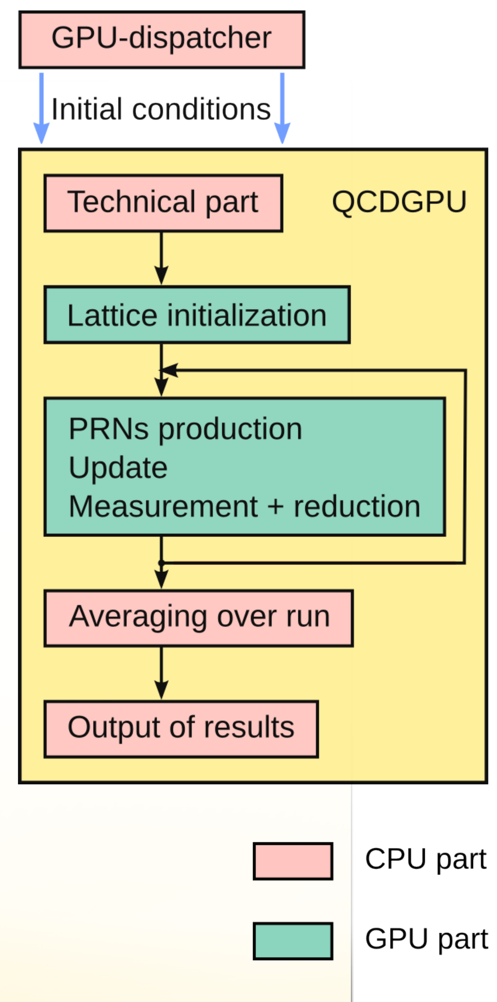
Command line

```
> rm finish.txt #if exists
> ./QCDGPU init.dat
```

[1] in this example Node 2 of [1] is used,

Output file:





Measured quantities

- mean plaquette
- mean Wilson action
- Wilson loop
- Polyakov loop L (real and imaginary parts, L^2, L^4)
- components of $F^a_{\mu\nu}$ tensor, their absolute values
- differential Polyakov loop ($L(x)$, $L(y)$, $L(z)$)
- differential Wilson action ($S(x)$, $S(y)$, $S(z)$).

Output:

- mean over configuration
- mean over run
- variance over run

Simulation details

- pseudorandom numbers are produced with PRNGCL library [2]
- multihit (pseudo) heat-bath update
- checkerboard scheme
- GID-start, GID-update for debugging
- possibility to discard a number of configuration for decorrelation
- possibility to save configuration and run conditions with user-defined frequency to allow the interruption of calculations
- checking measurement results on CPU (can be switched off)

SU(N) matrices on GPU

SU(2):

$$m = \begin{pmatrix} u1 & u2 \\ -u2^* & u1^* \end{pmatrix}$$

```
m.uv1 = (Re u1, Im u1, Re u2, Im u2);
```

SU(3):

$$m = \begin{pmatrix} u1 & u2 & u3 \\ v1 & v2 & v3 \\ w1 & w2 & w3 \end{pmatrix}, \quad \vec{w} = (\vec{u} \times \vec{v})^*$$

```
m.uv1 = (Re u1, Re u2, Re u3, Re v3);
m.uv2 = (Im u1, Im u2, Im u3, Im v3);
m.uv3 = (Re v1, Re v2, Im v1, Im v2);
```

Twisted boundary conditions (external chromomagnetic field introduction)

External chromomagnetic field is supposed to be constant and directed along z axis, $\vec{H}^e = (0, 0, H^e_z) = \overrightarrow{\mathrm{const}}$, $H^e = H^e_a T_a$, T_a are generators of the SU(N) group. It is introduced as additional flux through plaquettes.

$$U_{\mu\nu}(n) = e^{ia^2 F_{\mu\nu}(n)}.$$

In case of external flux in z direction, supposing $A'_y = A_y + xH^e_z$, it transforms into

$$U'_{xy}(n) \approx \exp\left\{ia^2(\partial_x A'_y(n) - \partial_y A_x(n) + i[A_x(n), A'_y(n)]\right\} = \exp\left\{ia^2\left(\partial_x A_y - \partial_y A_x + i[A_x, A_y] + \partial_x A^e_y + i[A_x, A^e_y]\right)\right\}$$

Suppose the external field is sufficiently weak to neglect the commutators with the external field. So, from the last equation

$$U'_{xy}(n) = U_{xy}(n) e^{ia^2 H^e_z}.$$

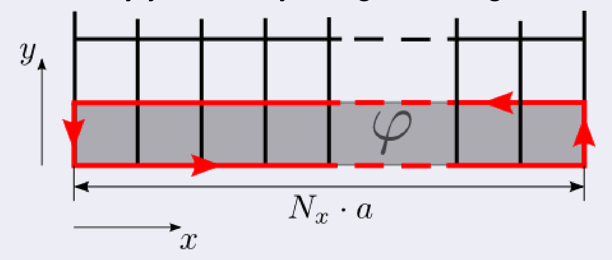
This is implemented through redefinition of the one of links forming plaquette:

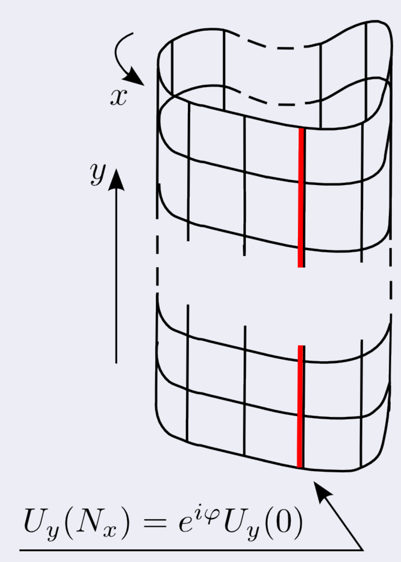
$$U'_y(n + \hat{x}) = U_y(n + \hat{x}) e^{ia^2 H^e_z(n)}.$$

It is possible to perform such "twist" only on the edge of the lattice:

$$U'_y(N_x, y, z, t) = U_y(0, y, z, t) e^{i\varphi}, \qquad \varphi = a^2 N_x H^e_z.$$

This corresponds to the flux through $1 \times N_x$ stripe of plaquettes in (xy) plane (see figure below).
This transformation is made for every y, so finally flux goes through the whole (xy) plane.





Direct measurement of $F_{\mu\nu}$ components

QCDGPU package allows direct measurement of Cartesian components of SU(N) electromagnetic tensor.
The expansion of plaquette for small lattice spacing:

$$U_{\mu\nu}(n) = 1 + ia^2 F^b_{\mu\nu} T_b - \frac{1}{2} a^4 F^b_{\mu\nu}(n) F^{b'}_{\mu\nu}(n) T_b T_{b'} + \mathcal{O}(a^5)$$

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

$$a^2 F^b_{\mu\nu}(n) = -i\, Tr\left[U_{\mu\nu}(n) T^b\right] + \mathcal{O}(a^4) \text{ [3]}$$

Direct measurement of condensate chromomagnetic field

The average of chromomagnetic components over simulated configuration can give a direct measurement of the homogeneous chromomagnetic field generated spontaneously in the deconfinement phase.
Cartesian components of total chromomagnetic field $\vec{H}$ are supposed to be Gaussian with the variance σ^2. Then, the simulated field strength $\vec{H}$ is described by the following probability distribution function (p.d.f.),

$$p(\vec{H}) = \frac{1}{(2\pi)^{3/2}\sigma^3} e^{-(\vec{H} - \vec{H}_c)^2/2\sigma^2},$$

$\vec{H}_c$ is the condensate field.
The spatial direction of $\vec{H}_c$ is unknown at each configuration generated in the Monte Carlo (MC) run. It is random, at least it has a random part. That is why the average of Cartesian components of $\vec{H}$ over the run cannot be used to extract the condensate value from the generated Boltzmann ensemble.
Actually we are interested in the absolute value of the condensate field, rather than in its direction.
Using spherical coordinates and integrating over angles the distribution of the absolute value of field strength can be obtained:

$$p(H) = \frac{4\pi}{(2\pi)^{3/2}\sigma^3} H^2 e^{-H^2/2\sigma^2} e^{-H_c^2/2\sigma^2} \frac{\sinh(HH_c/\sigma^2)}{HH_c/\sigma^2}.$$

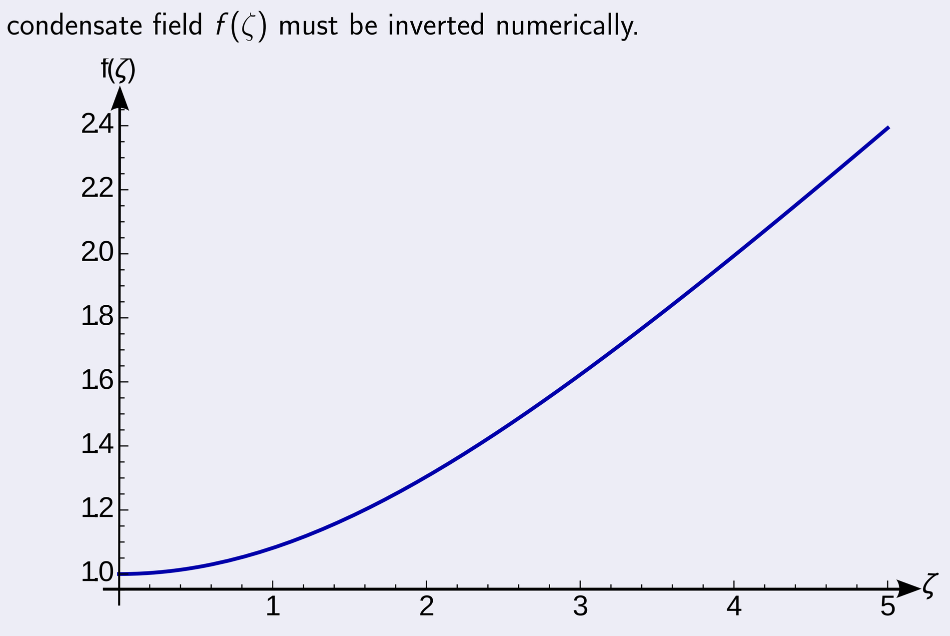
At $H_c = 0$ it becomes ordinary Maxwell distribution. The non-Maxwell shape of distribution means the presence of condensate field. The only one unknown parameter of the distribution H_c can be determined by fitting of the theoretical expectation of the mean value by the corresponding sample mean from the MC run.
For qualitative analysis it is convenient to use the p.d.f. in dimensionless terms:

$$p(\eta) = \frac{16\eta}{\zeta\pi^{3/2}} e^{-\zeta^2/4} e^{-4\eta^2/\pi} \sinh\frac{2\zeta\eta}{\sqrt{\pi}},$$

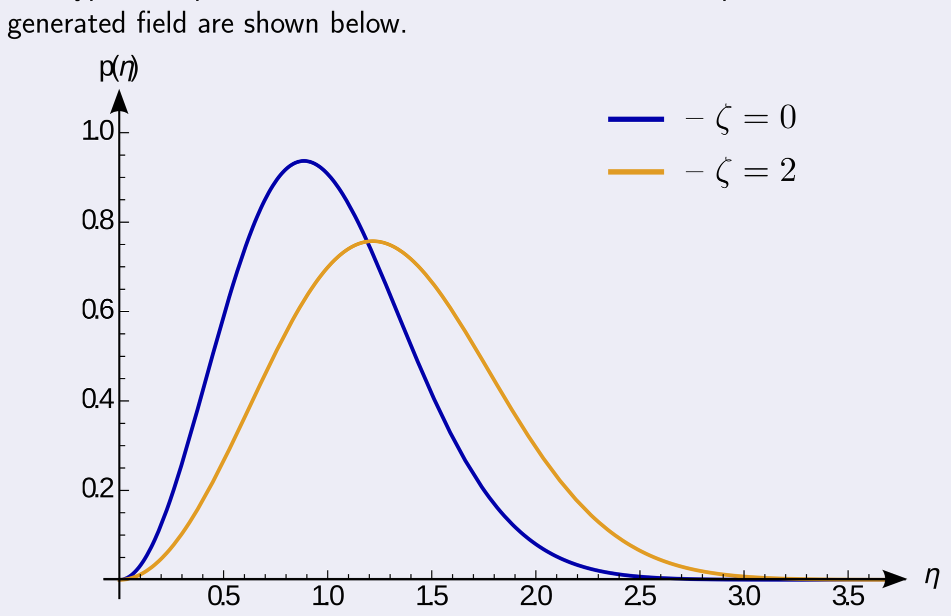
$\eta = H/H_0$, $\zeta = H_c\sqrt{2}/\sigma$, $H_0 = 4\sigma/\sqrt{2\pi}$ – the mean value of H in case of $H_c = 0$.
The mean value of η:

$$\bar{\eta} = \frac{16}{\zeta\pi^{3/2}} e^{-\zeta^2/4} \int_0^\infty \eta^2 e^{-4\eta^2/\pi} \sinh\frac{2\zeta\eta}{\sqrt{\pi}}\, d\eta = f(\zeta).$$

It can be associated with its estimator from lattice simulations. To calculate condensate field $f(\zeta)$ must be inverted numerically.



The typical shapes of the distributions of normalized H in presence and absence of generated field are shown below.



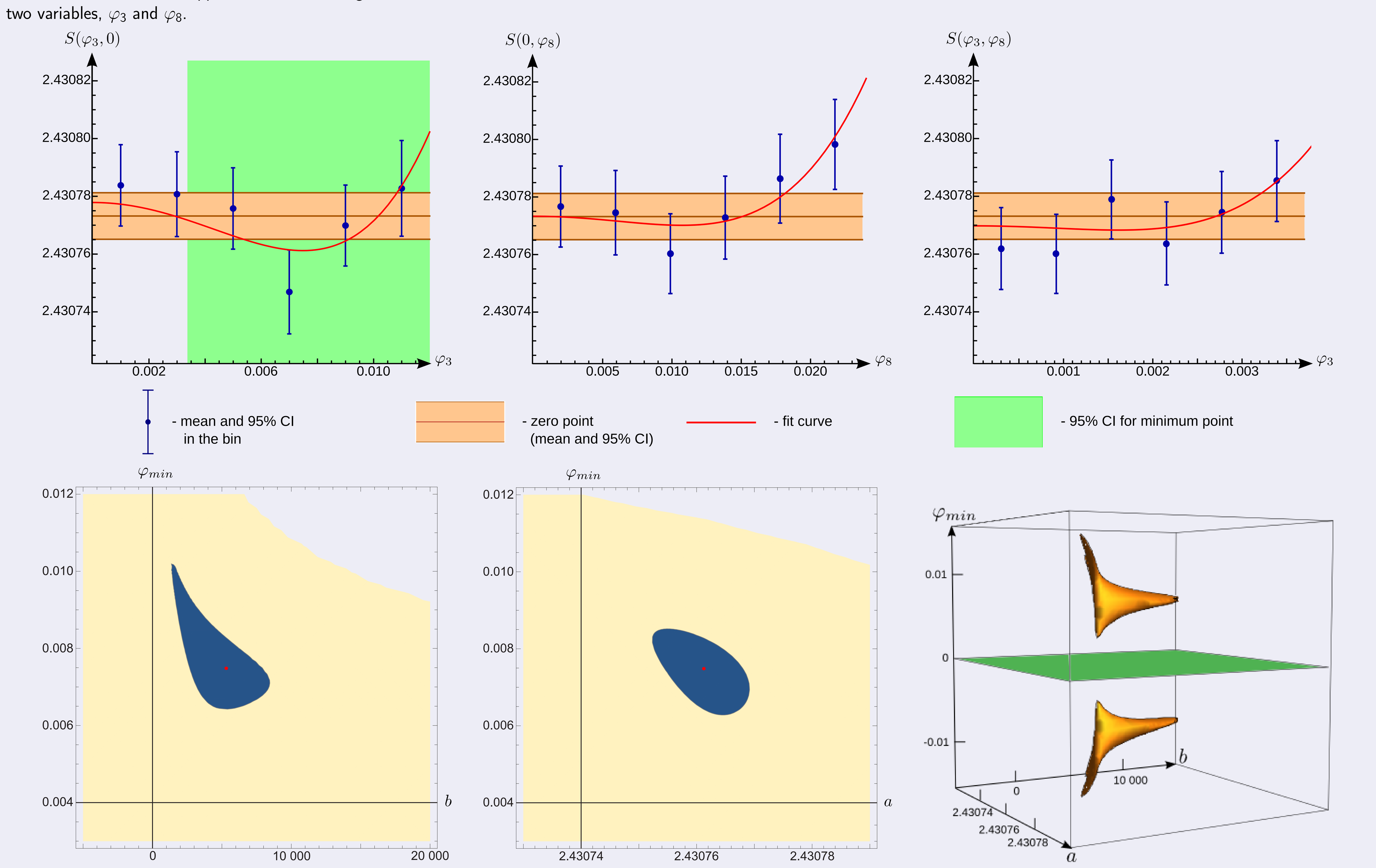
Plans for future

- implement overrelaxation algorithm
- implement fermionic fields
- implement external field by Cosmai & Cea method [5]

Spontaneous vacuum magnetization in pure SU(3) LGT

The spontaneous vacuum magnetization at high temperature is one of the possible mechanisms to produce magnetic fields in the early Universe.
The magnetized vacuum means that the state with non-zero chromomagnetic field is energetically preferable for it.
The main idea of current approach is to find the global minimum of the action as function of two variables, φ_3 and φ_8.

Because of reproducing of the whole surface $S(\varphi_3, \varphi_8)$ is very time consuming, only some sections of this surface are investigated: $\varphi_8 = 0$, $\varphi_3 = 0$, and $\varphi_8 = 6.17\varphi_3$. Such choice of the directions in (φ_3, φ_8) space is connected with investigation in [4].
The presented results are obtained on 4×16^3 lattice at $\beta = 6$. This corresponds to temperature $T \sim 370$ MeV.



It is obtained within 2σ accuracy that the spontaneous chromomagnetic field generation takes place along direction $\varphi_8 = 0$ (the left top plot). The data are collected into bins and fitted by χ^2 method with 4-th power parabola $f(\varphi) = a + b(\varphi^2 - \varphi^2_{min})^2$. The estimate of field flux in the minimum: $\varphi_{3_{min}} = \left(7.484^{+11.27}_{-4.106}\right) \times 10^{-3}$. This corresponds to field strength $H_3 \approx 487.5$ MeV^2. In the bottom plots the surface obtained from confidence intervals for fitted parameters (the right plot) and its sections by planes $a = a_{min}$ and $b = b_{min}$ are shown.

It can be seen that this surface doesn't cross the plane $\varphi_{min} = 0$, so the trivial minimum of the action is excluded at 95% CL.
In other investigated directions such minima are non-distinguishable.
So, the minimum of the S surface in point with $H_3 \approx 487.5$ and $H_8 = 0$ is obtained within 2σ accuracy and the vacuum of SU(3) gluodynamics can be spontaneously magnetized.

References

[1] http://hgpu.org/.

[2] https://github.com/vadimdi/PRNGCL.

[3] V. Demchik, A. Gulov and N. Kolomoyets, "Spontaneous generation of chromomagnetic fields at finite temperature in the SU(3) gluodynamics on a lattice," arXiv:1212.6185 [hep-lat].

[4] V. V. Skalozub and A. V. Strelchenko, "On generation of Abelian magnetic fields in SU(3) gluodynamics at high temperature," Eur. Phys. J. C 33, 105 (2004) [hep-ph/0208071].

[5] P. Cea and L. Cosmai, "Abelian chromomagnetic background field at finite temperature on the lattice," hep-lat/0101017.

https://github.com/vadimdi/QCDGPU