The Phenomenological Holographic Approach to QCD

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Introduction

Quantum ChormoDynamics (QCD) is the best undestood piece of the Standard Model, from the point of view of the fundamental degrees of freedom:

- It is $SU(3)$ Yang-Mills theory with six quark flavors in the fundamental representation, weakly coupled at high energies.
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- At low energies it becomes non-pertrubative: confinement, chiral symmetry breaking, a rich phase diagram...

  Many approaches have been tried to describe these phenomena (perturbation theory, lattice, Dyson-Schwinger, chiral Lagrangians...). Holography, like all these techniques, has advantages and limits. It offers a simple and powerful tool to compute non-perturbative observables and it is especially useful in out-of-equilibrium situations.
- In this talk I will describe a class of phenomenological holographic models that provide a good descriptions of many aspects of the non-perturbative physics

$SU(N)$ Yang-Mills

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- Free at high energy: $\lambda(E) \simeq (\beta_0 \log E/\Lambda_{IR})^{-1}$; strongly coupled at low energy.
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- The spectrum is discrete (glueballs) and organized in towers associated to gauge-invariant operators, e.g:

$$TrF^2 \Leftrightarrow J^{PC} = 0^{++}, \quad TrF\tilde{F} \Leftrightarrow 0^{-+} \quad T^{tt}_{\mu\nu} \Leftrightarrow 2^{++} \quad \ldots$$

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- 't Hooft: N large, λ fixed $\Rightarrow$ the theory simplifies (e.g. glueballs become weakly interacting)

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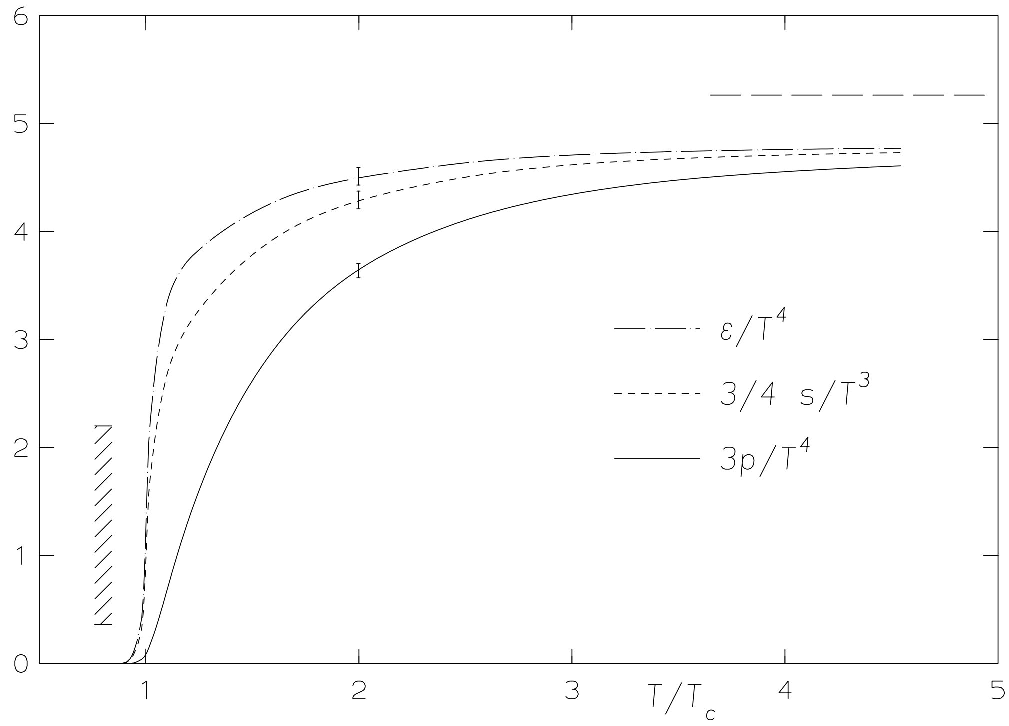

Equation of state $p(T)$ from lattice

Karsch, hep-lat/0106019

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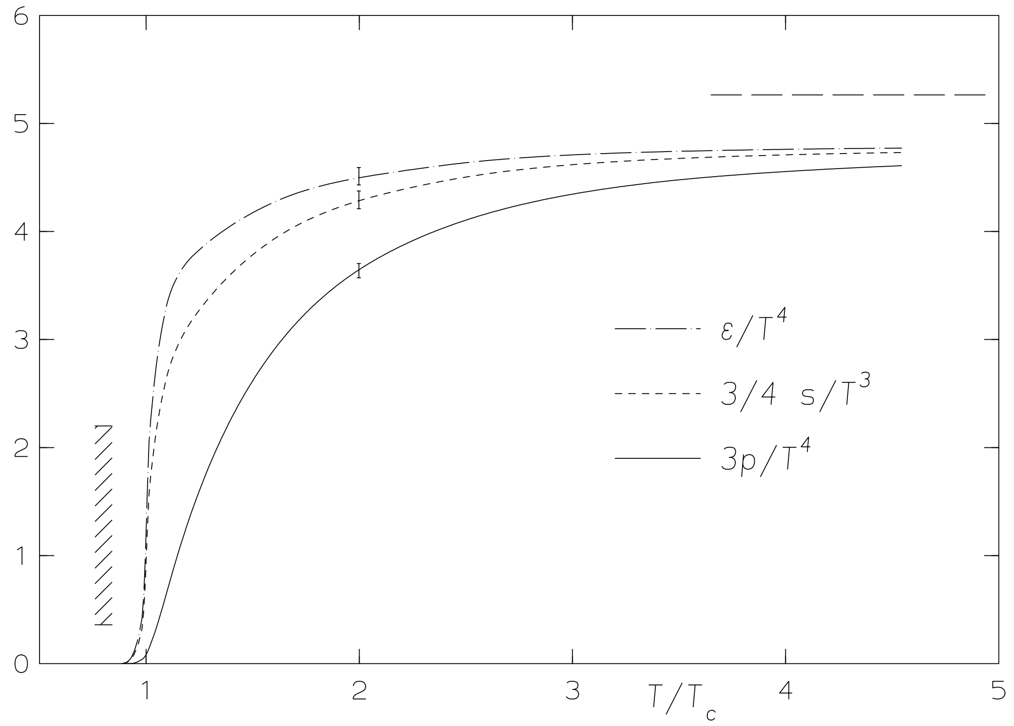

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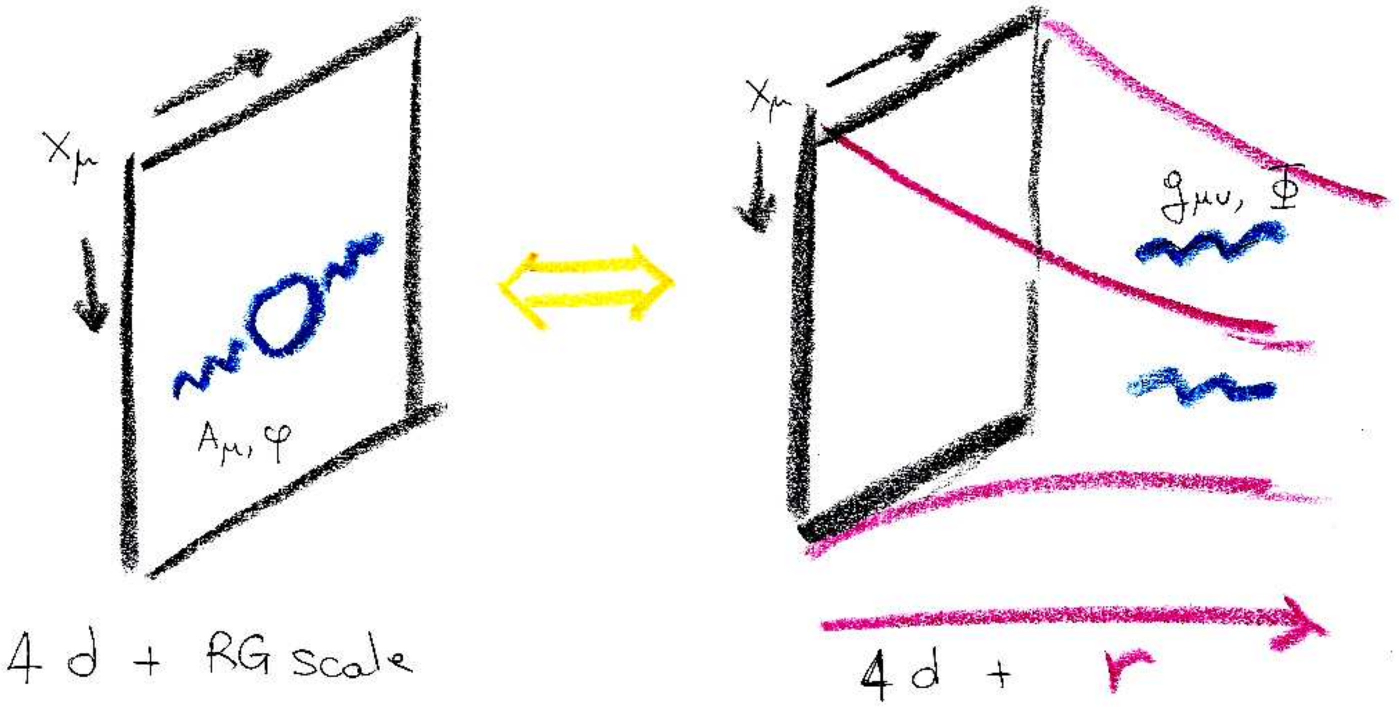
- In the deconfined phase the spectrum is continuous and the long-wavelenght physics is expected to be described by (viscous) hydrodynamics.

AdS/CFT

The AdS/CFT duality: conjecture that certain quantum field theories are equivalent to theories of gravity in higher dimensions.



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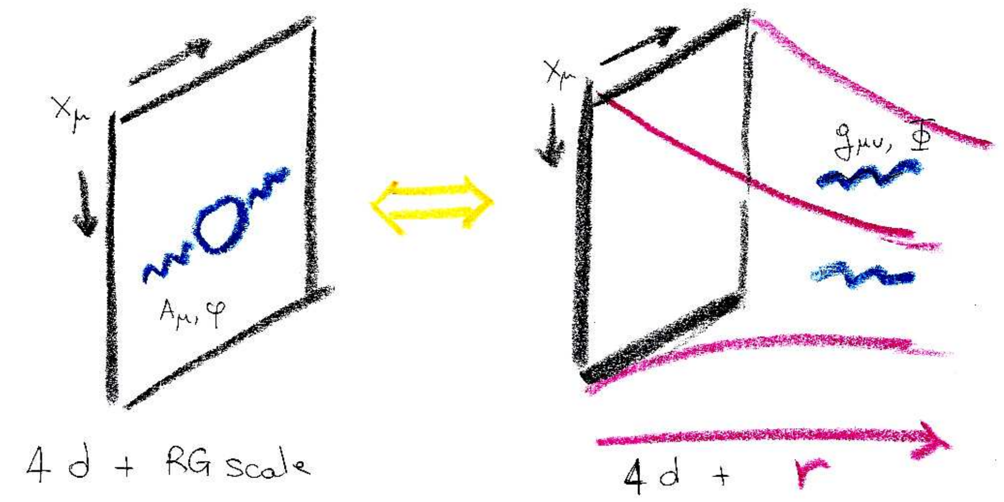

- Equivalent means that the two theories contain the same degrees of freedom, but arranged in differnt ways.
- Depending on the situation, one side or the other may be easier to handle.

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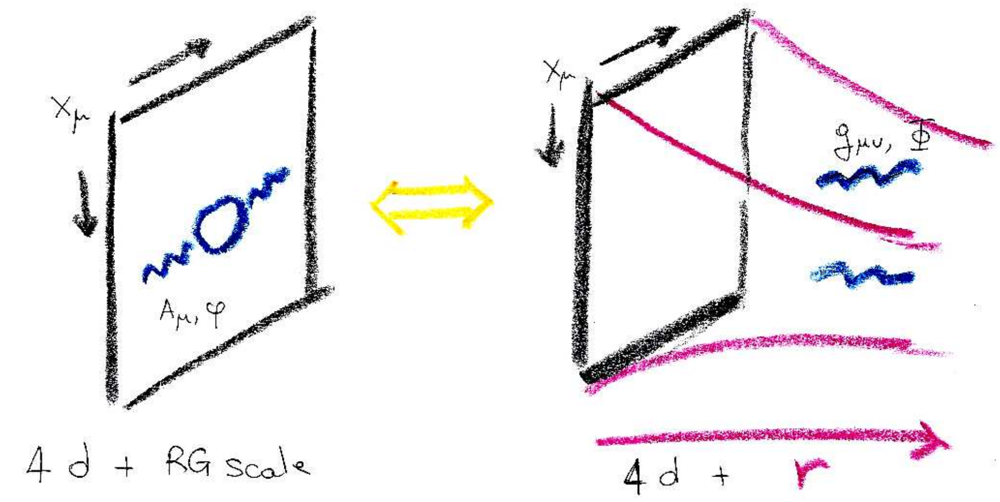
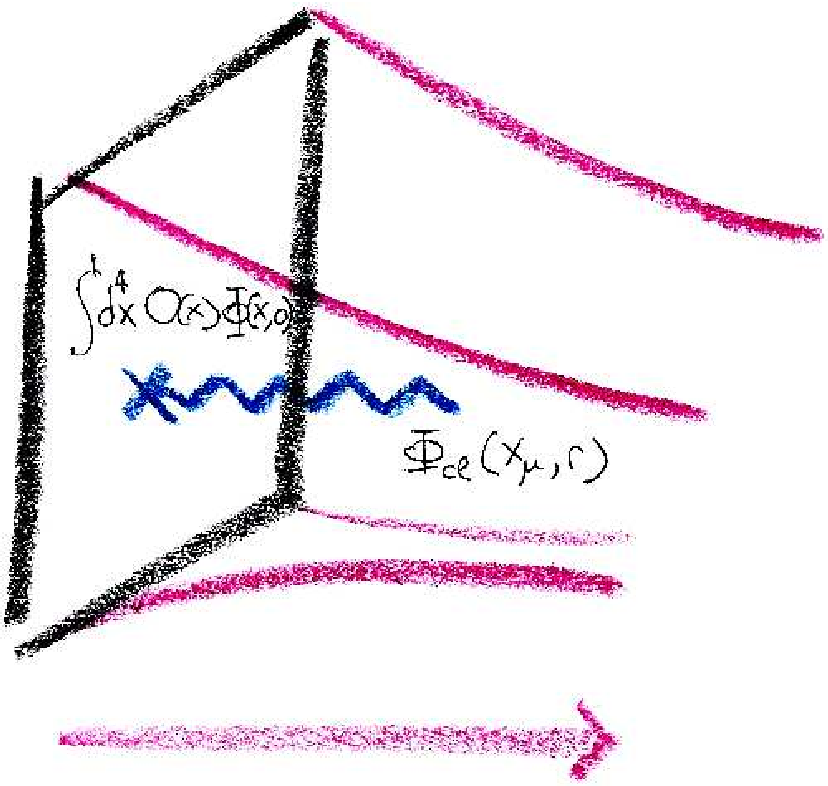


- $\mathcal{N} = 4$ SYM theory in 4D $\Leftrightarrow$ IIB String theory on $AdS_5 \times S^5$
- large N, large λ: Gravity side becomes classical and non-stringy.
- Conformal invariance $\Leftrightarrow$ AdS spacetime $ds^2 = r^{-2}(dr^2 + dx_\mu^2)$, Scaling isometry $r \to \lambda r$, $x_\mu \to \lambda x_\mu$.
- RG scale $\Leftrightarrow$ radial coordinate r; UV $\Leftrightarrow$ AdS boundary $r = 0$.

Field/Operator correspondence

- QFT operator $O(x) \Leftrightarrow$ Bulk field $\Phi(x, r)$.
- $\Phi_0(x) = \Phi(x, 0)$ is a source for $O(x)$ in the QFT:



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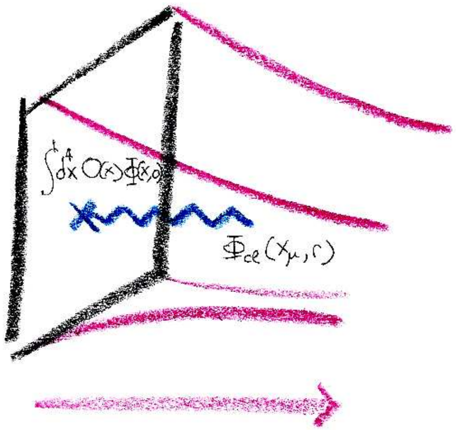

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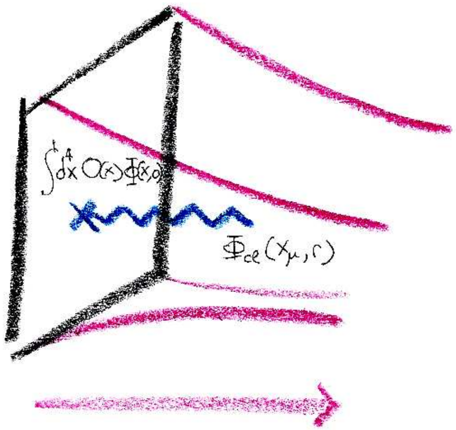

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$$\mathcal{Z}_{QFT}[\Phi_0(x)] = \exp iS_{cl}[\Phi_0(x)]$$

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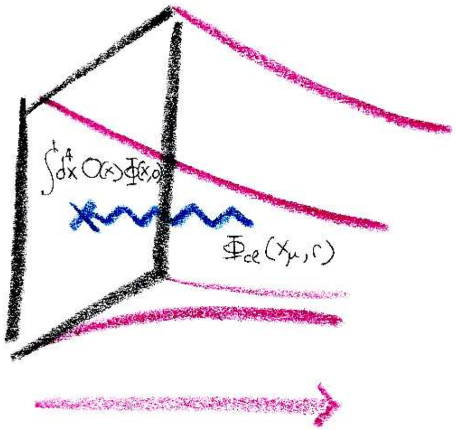

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$$\langle O(x_1) \ldots O(x_n) \rangle = \frac{\delta}{\delta\Phi_0(x_1)} \cdots \frac{\delta}{\delta\Phi_0(x_n)} S_{cl}[\Phi_0]$$

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Top-down Construct string theory backgrounds which break susy/conformal invarariance.

Bottom-up: Construct phenomenological gravity models such that, using holography rules, we can match the properties of the QFT.

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- Not a controlled approximation of a more fundamental theory;
- Free parameters can be used to fit data from other techniques and have a quantitative match.

Minimal phenomenological setup

- The bulk theory is five-dimensional (x^μ + RG coordinate r)
- Include only lowest dimension YM operators ($\Delta = 4$)

4D Operator		Bulk field	Coupling
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- Breaking of conformal symmetry, mass gap, confinement, and all non-perturbative dynamics driven by the dilaton dynamics (aka the Yang-Mills coupling).
- Treat Φ as a 5D string theory dilaton: $g^{(s)}_{\mu\nu} = e^{4/3\Phi} g^{(E)}_{\mu\nu}$ (the true dual, if it exists, should be a non-critical 5d string theory)

5-D Eistein-Dilaton Theory

Bulk dynamics described by a 2-derivative action:

$$S_E = -M_p^3 N_c^2 \int d^5x \sqrt{-g} \left[R - \frac{4}{3}(\partial\Phi)^2 - V(\Phi) \right]$$

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- Effective Planck scale $\sim N_c^2$ is large.
- Features: asymptotic freedom, confinement, discrete linear glueball spectrum, correct thermodynamics and phase diagram
- Drawbacks: it is not a controlled approximation of a more complete theory: unknown size of possible corrections (e.g. from higher dimension operators)

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- UV: $\lambda \to 0, \qquad V(\lambda) \sim \frac{12}{\ell^2}\left(1 + v_0\lambda + v_1\lambda^2 \ldots\right)$
  $\Rightarrow$ asymptotically AdS solutions ($r \to 0$) $+ \log$-corrections:

$$e^A \simeq \frac{\ell}{r}, \quad \lambda(r) \simeq \frac{1}{\beta_0 \log r\Lambda}, \quad \beta_0 = \frac{8}{9}v_0$$

- IR: Generically, $e^A \to 0$;. Detailed behavior depends on $V(\lambda)$ at large λ. Assume no IR AdS fixed point, eventually $\lambda \to \infty$ at large r.

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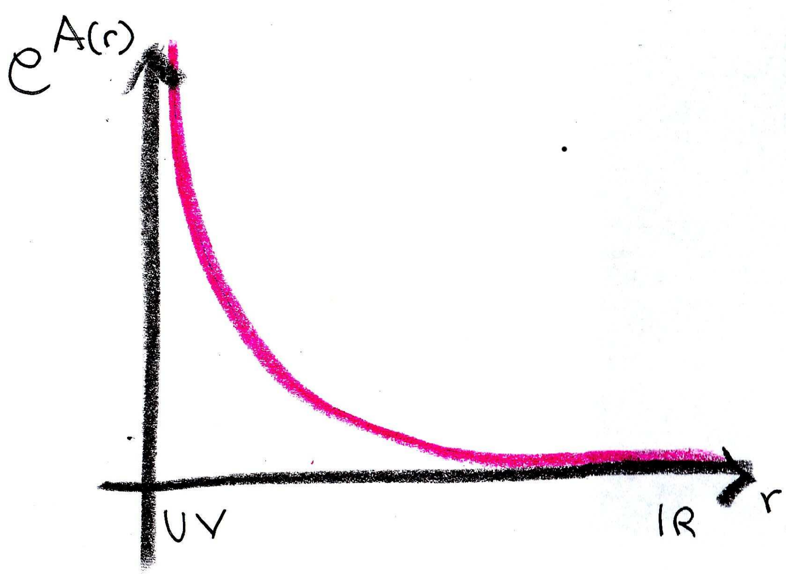

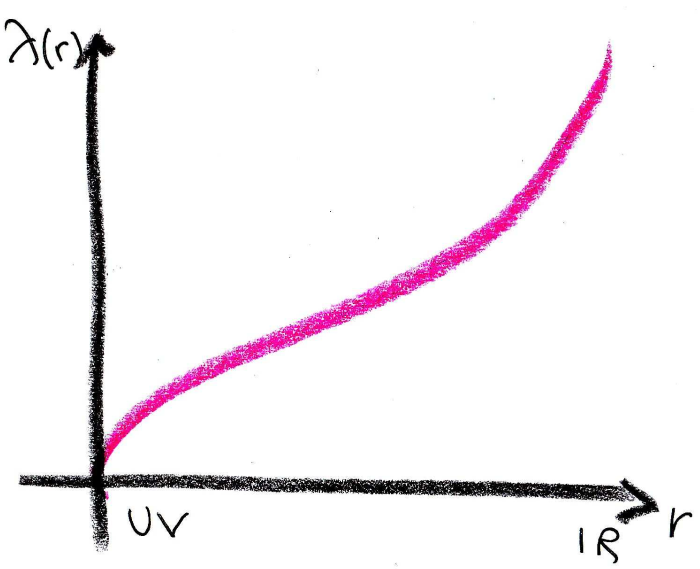

AdS/CFT and Confinement

Confinement in AdS/CFT is decided via the dual of the Wilson loop test. QFT Wilson Loop operator:

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Confinement $\Leftrightarrow$ Area Law for large size of γ: $W \sim \exp\{i\sigma_c Area_\gamma\}$

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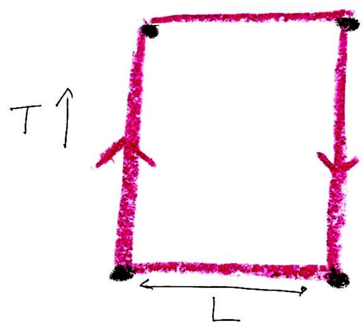
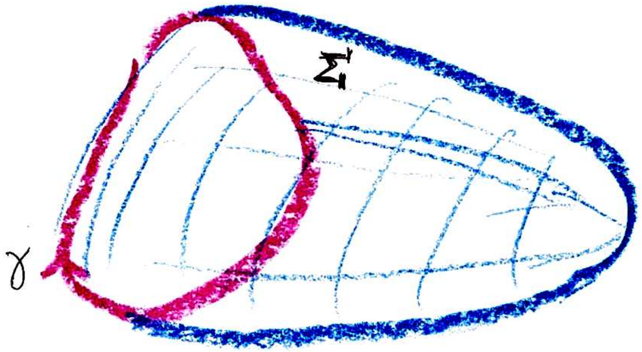


$\Rightarrow$ linear potential between two quarks at large separation:

$$W = e^{iS_\gamma}, \quad S_\gamma = TV(L) \sim \sigma_c TL \Rightarrow V(L) = \sigma_c L$$

AdS/CFT and Confinement

The holographic dual of the Wilson Loop is the action of a string attaching to the contour on the boundary, and closing into the interior.



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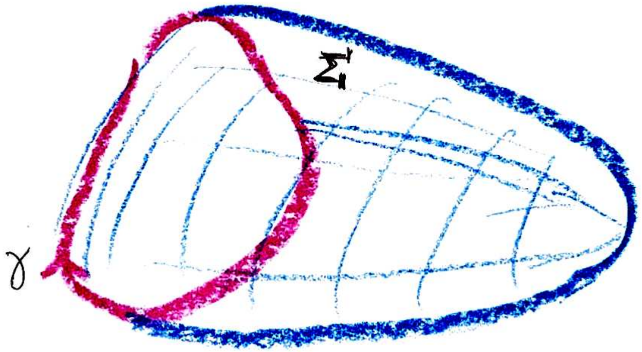

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Σ is chosen as the minimal (i.e. geodesic) surface ending on γ.

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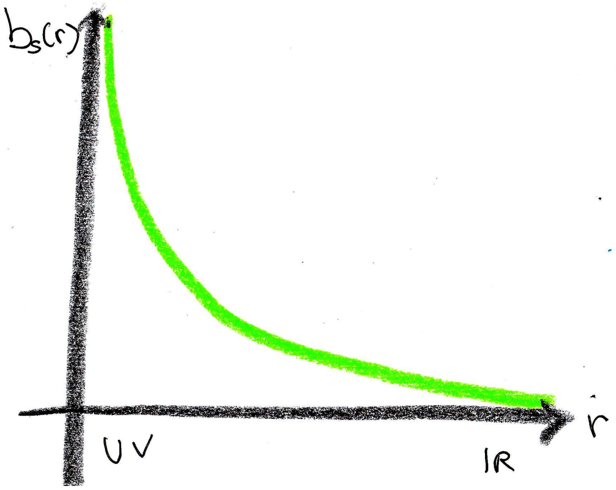

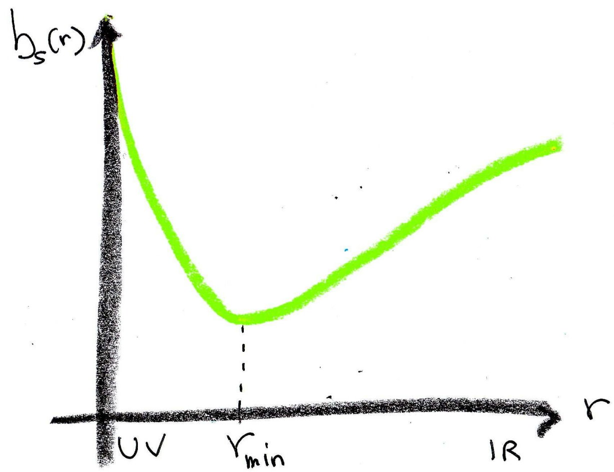

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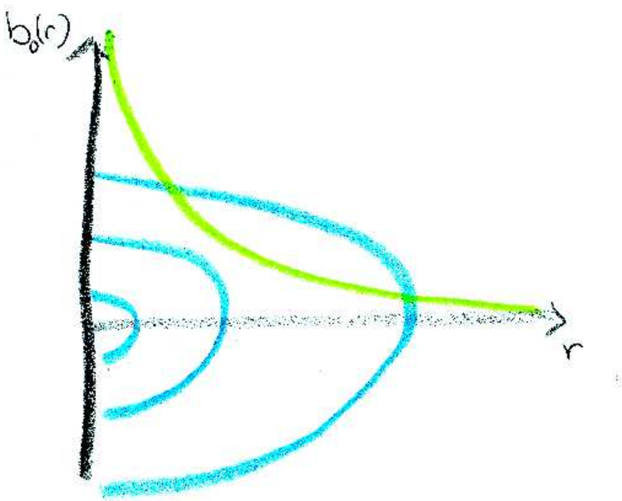

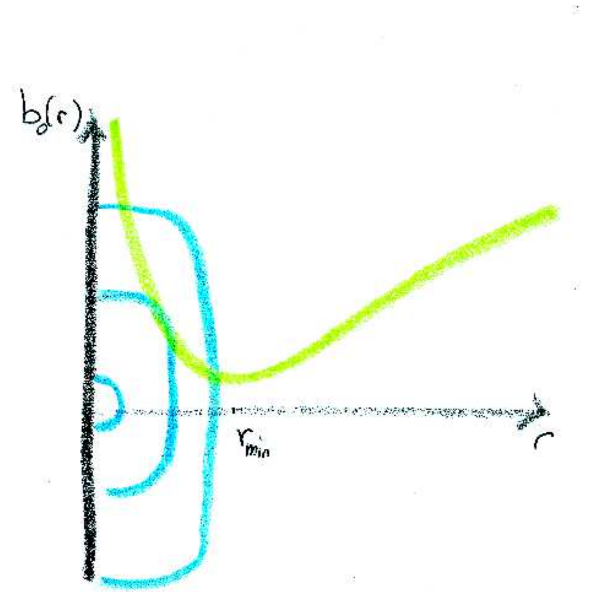

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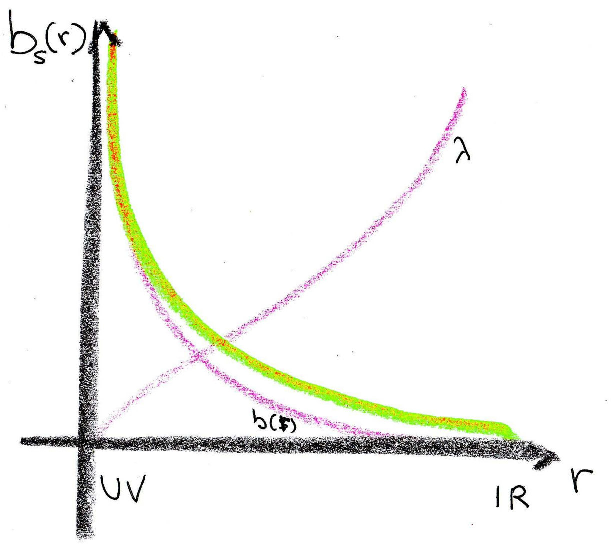

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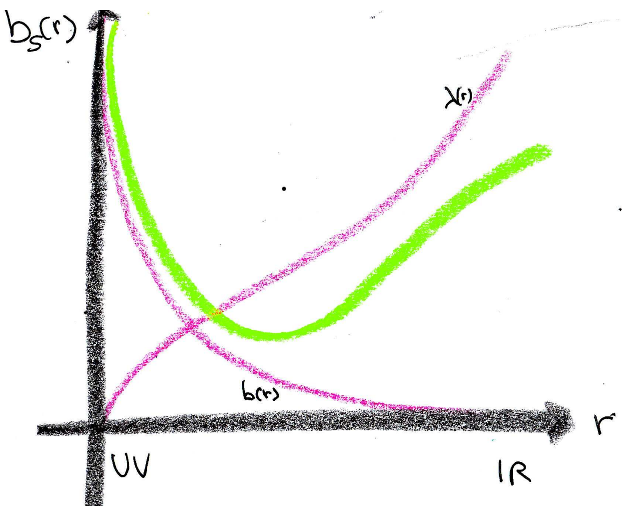

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Spectrum of states associated to O $\Leftrightarrow$ spectrum of normalizable fluctuations of the corresponding bulk field Φ

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Fluctuation equations equivalent to a Scrhödinger problem:

$$-\ddot{\Psi} + V_s(r)\Psi = -k^2\Psi \qquad k^2 = -m^2$$

$V_s(r)$ determined by the background solution $(A(r), \lambda(r))$

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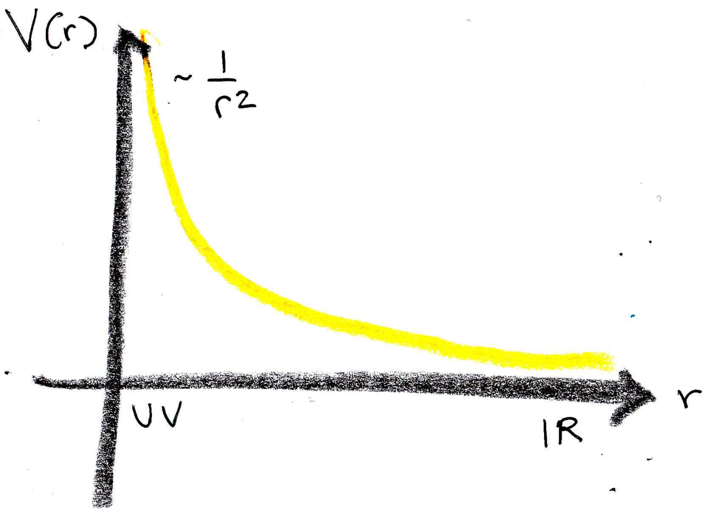
The Schroedinger potentials behave as:



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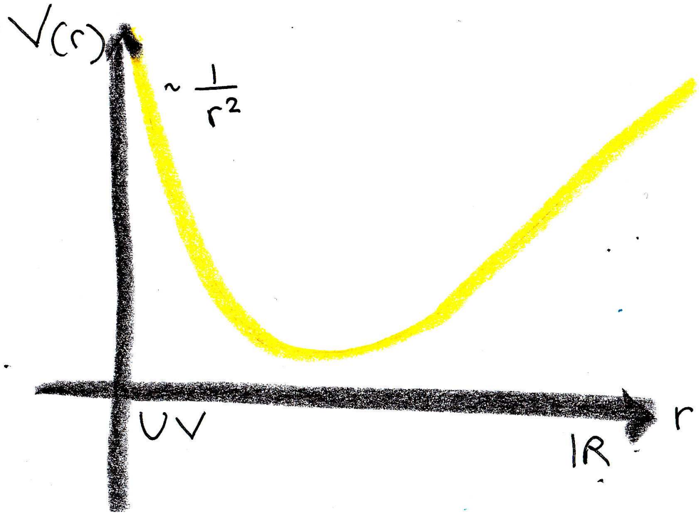

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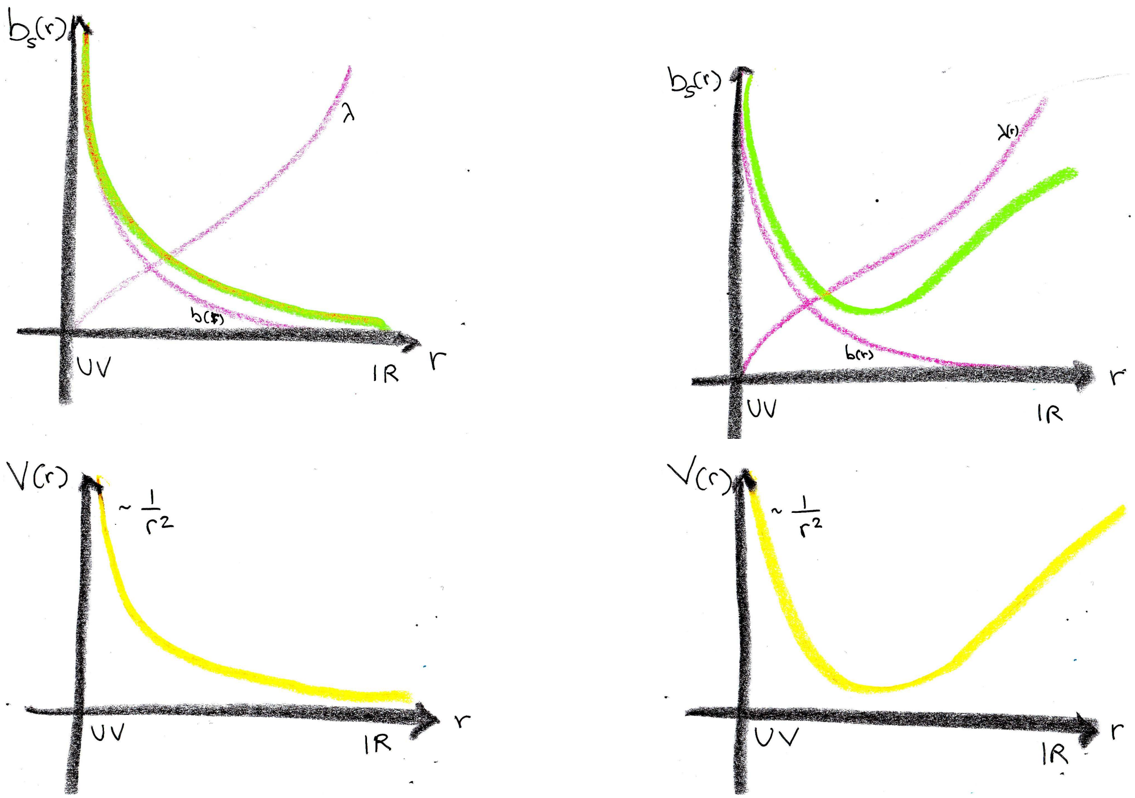

non-confining confining

Area law $\Leftrightarrow$ Gaped discrete tower of spin 2 and spin 0 states

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- Phase transition happens at T_c where $\mathcal{F}_1(T_c) = \mathcal{F}_2(T_c)$

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Thermally excited vacuum:

$$ds^2_{TG} = e^{2A(r)} \left[dr^2 + d\tau^2 + d\vec{x}^2\right]$$

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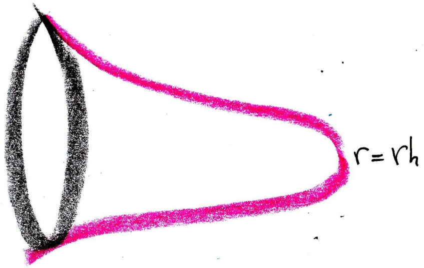

Thermally excited vacuum:

$$ds^2_{TG} = e^{2A(r)} \left[dr^2 + d\tau^2 + d\vec{x}^2\right]$$

Black hole:

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Finite Temperature

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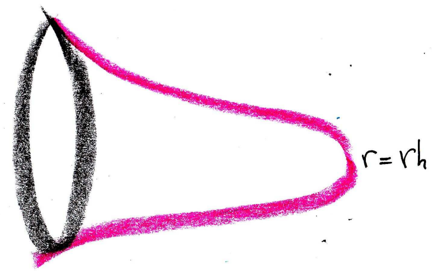

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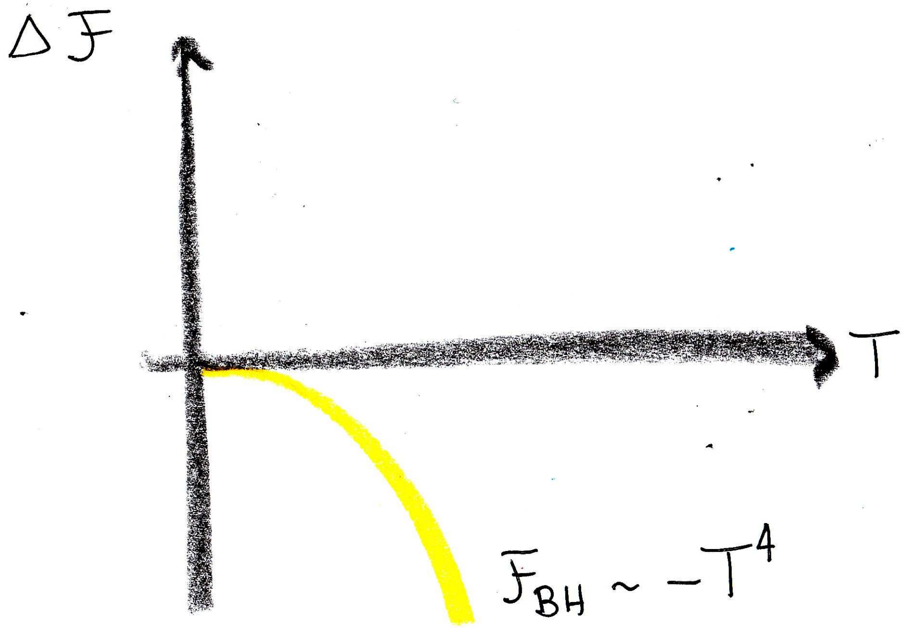
- the black hole always corresponds to a deconfined phase ($g_{tt} = 0$ at the horizon, and the string tension vanishes)

Phase diagram

- $\mathcal{N} = 4$ SYM has no scale $\Rightarrow$ no phase transition
  $\mathcal{F}_{BH} = -cT^4$, at any finite T BH phase dominates.



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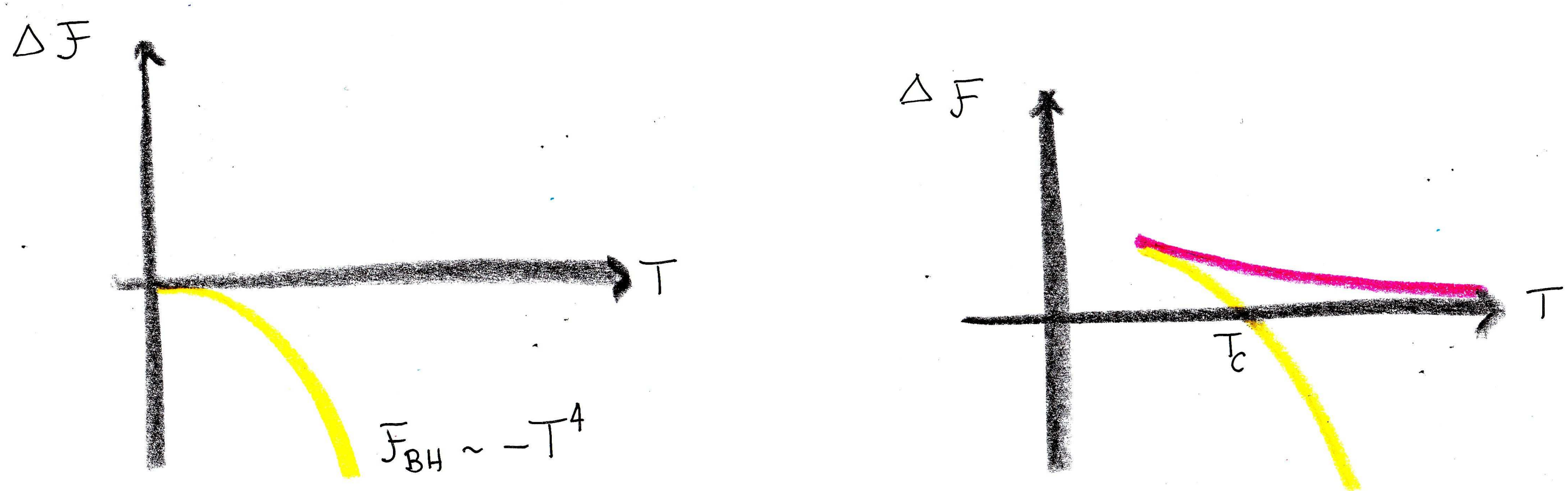

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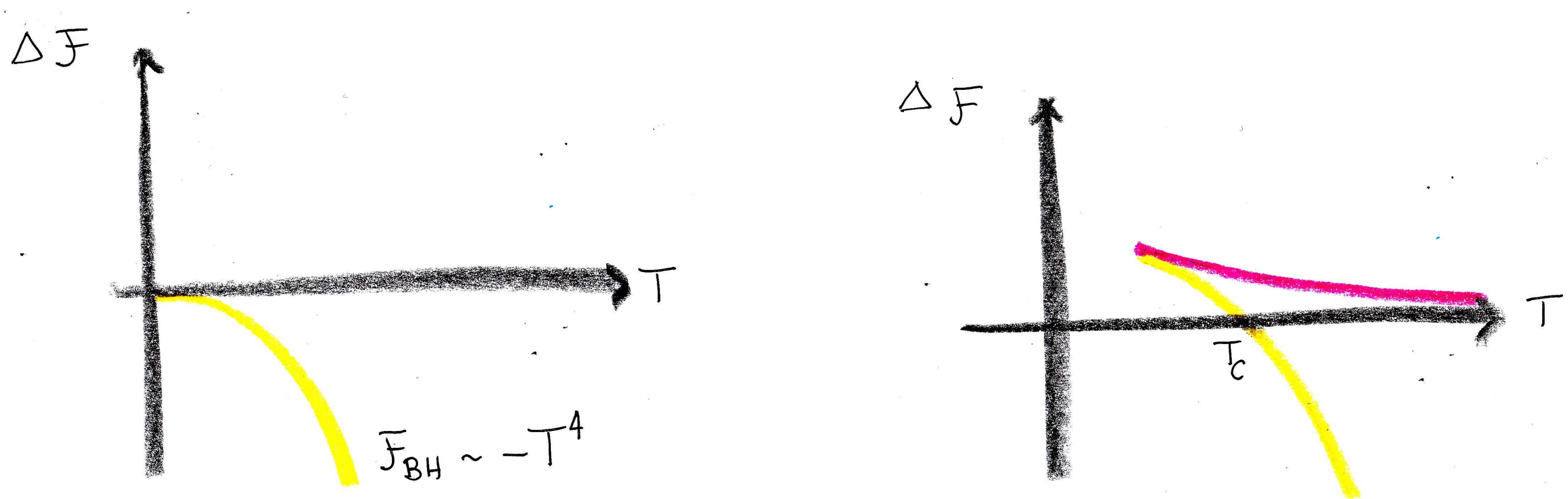
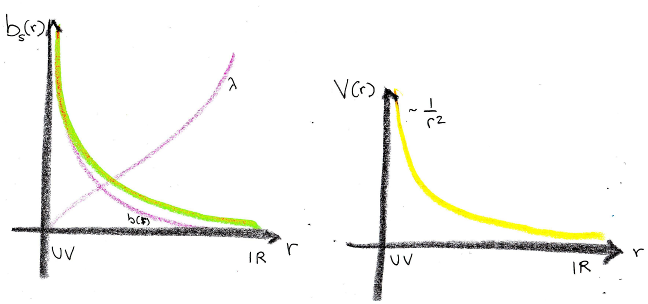
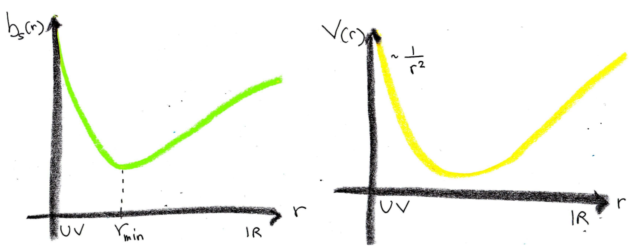




$V(\lambda) \lesssim O(\lambda^{4/3})$



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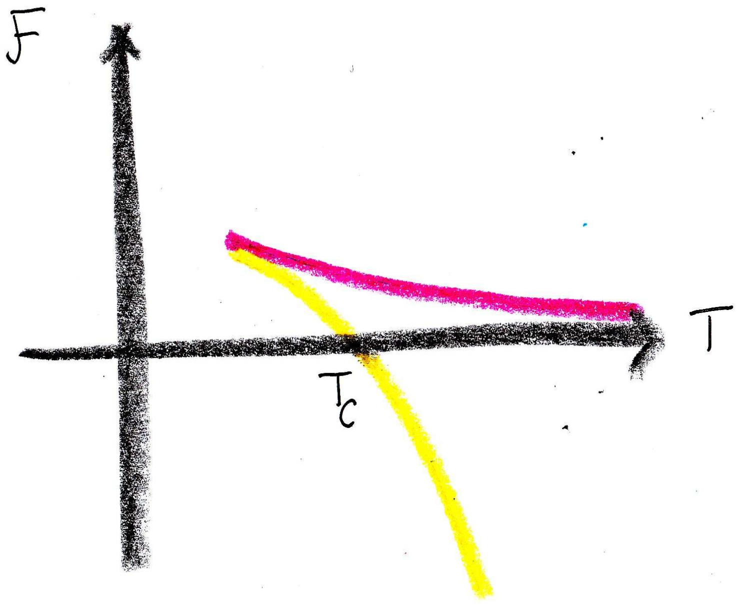

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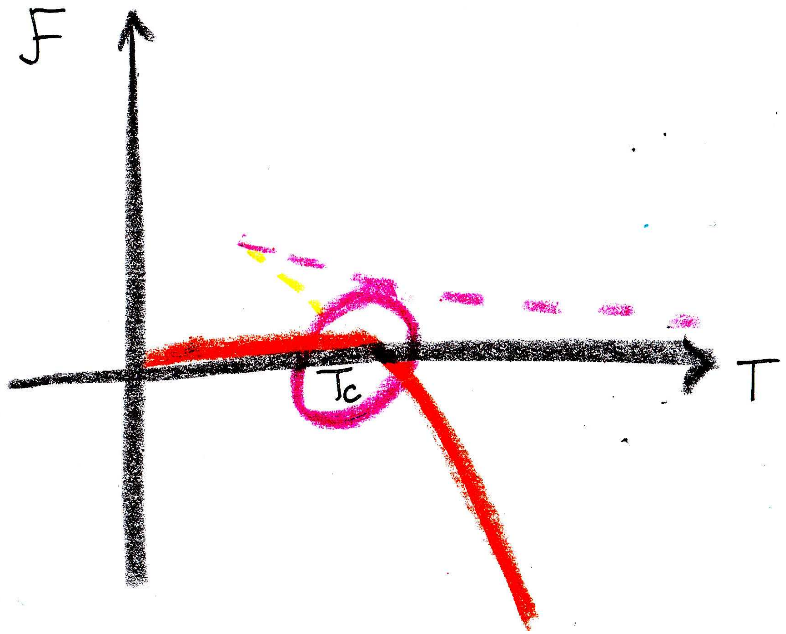

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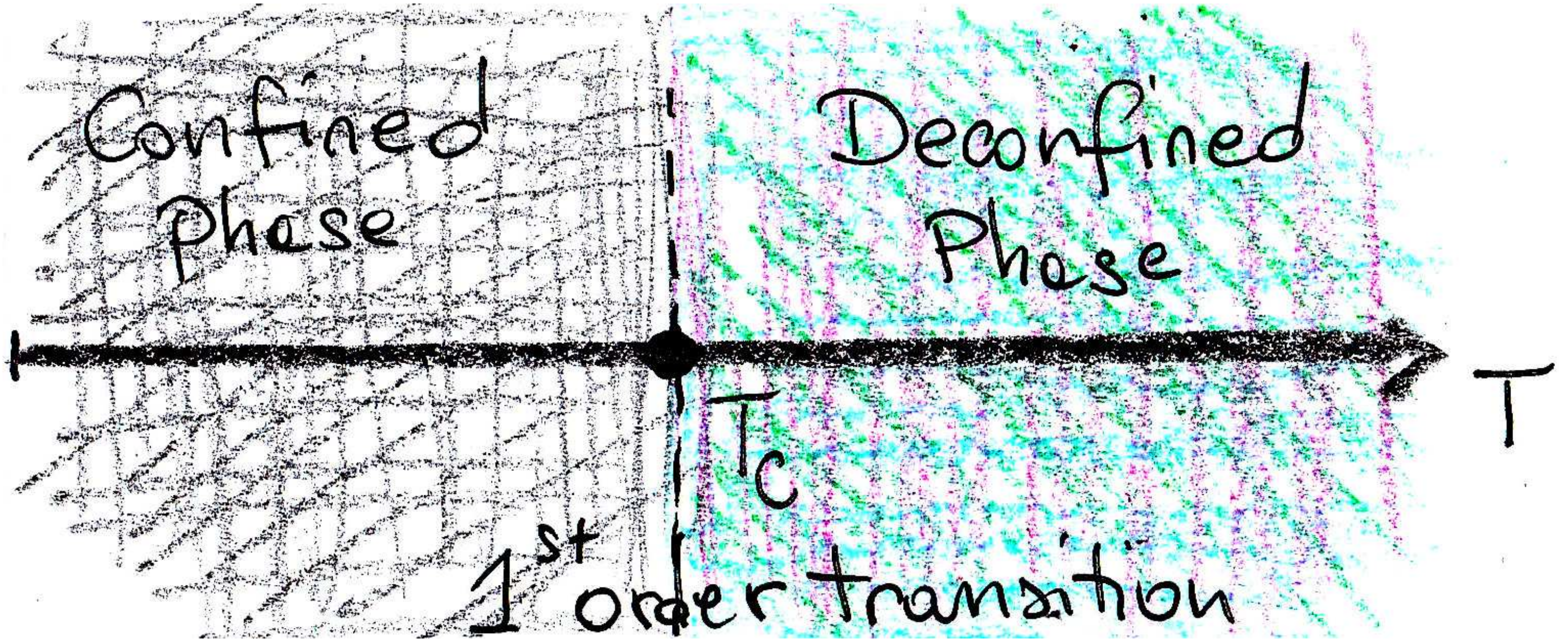

Confining geometries display a first order transition to a black hole phase for $T > T_c$: Exact correlation between Wilson loop area law, mass gap, and thermal phase transitions.

Matching Pure YM Thermodynamics

Appropriate dilaton potential (fixed asymptotics plus 2 fit parameters) $\Rightarrow$ Good agreement with lattice YM thermodynamics.

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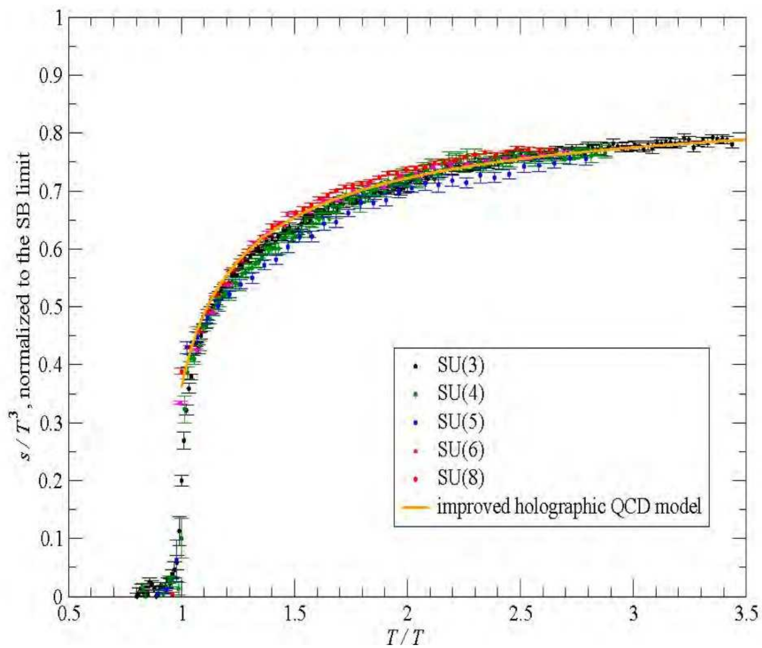

$s(T)/T^3$ lattice data: Panero, hep-lat/0106019

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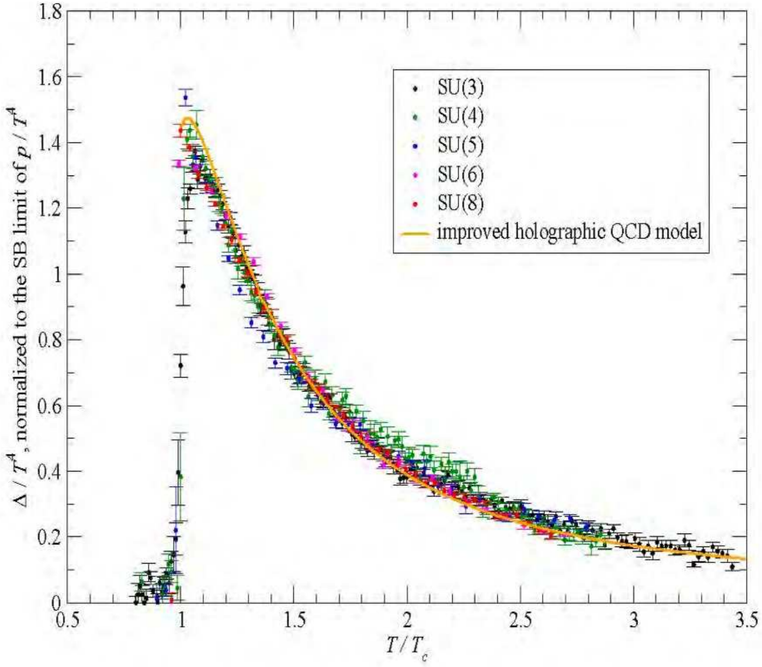

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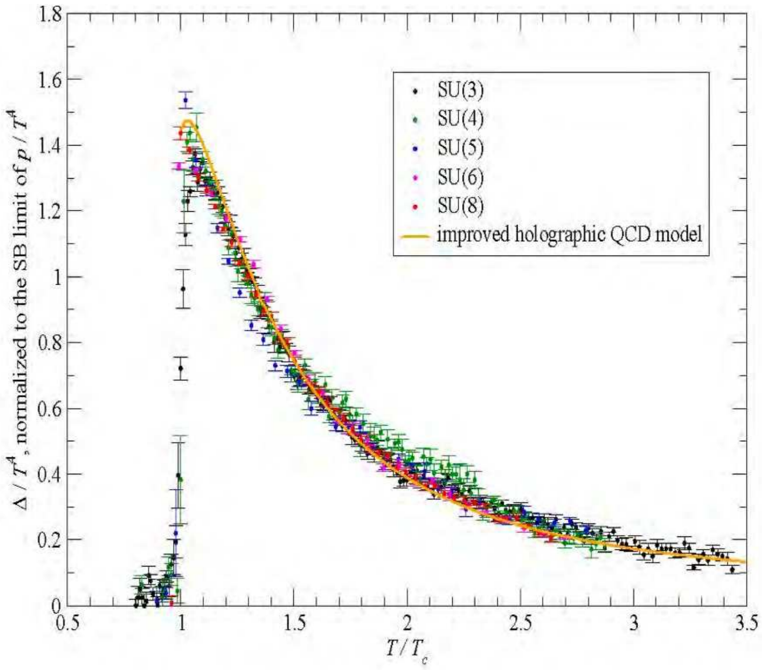

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AdS black hole may provide a good description of the deconfined phase at high T, but close to T_c non-conformality becomes important.

Beyond Equilibrium

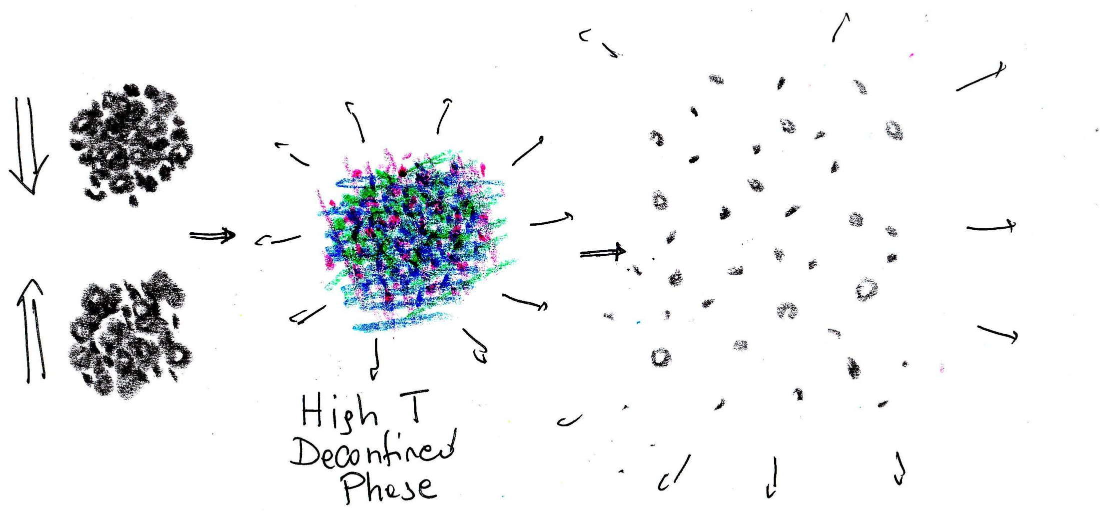
Everything discussed so far can be computed on the lattice. Up to this point, we modeled a holographic setup to reproduce lattice results.

Where AdS/CFT techniques can really lead to advancement is the

- Real-time dynamics of the deconfined phase.
- Finite baryon density

Quark-Gluon plasma

The deconfined phase of of QCD is studied in Relativistic Heavy Ion collision experiments (RHIC and LHC).



After a thermalization phase, the dynamics is indeed well described by a hadrodynamic limit in terms of a few quantities (transport coefficients):

- Flow parameters
- Bulk and Shear Viscosity
- Transport related to heavy probes (energy loss, jet quenching)

Hydrodynamic Transport

In the long-wavelength limit the dynamics is described by energy transport with a hydrodynamic stress-tensor:

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + p)\, u^\mu u^\nu + p\, g^{\mu\nu} \\ &\quad - P^{\mu i} P^{\nu j} \left[\eta \left(\partial_i u_j + \partial_j u_i - \frac{2}{3} g_{ij} \partial \cdot u \right) + \zeta\, g_{ij}\, \partial \cdot u \right] \end{aligned}$$

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\end{aligned}
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η : Shear viscosity.

- RHIC data consistent with very small η/s, $\sim 0.08 \,-\, 0.2$
- Closest match: Strong coupling holographic computation in $\mathcal{N} = 4$ SYM: $\eta/s = (4\pi)^{-1} \approx 0.08$
- IHQCD setup: same result as in $\mathcal{N} = 4$ SYM (universal in 2-derivative models)

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ζ : Bulk viscosity

- Vanishes in a conformal fluid (like $\mathcal{N} = 4$ plasma), but one expects a non-zero answer away from conformality.
- How signigicant is ζ close to T_c?

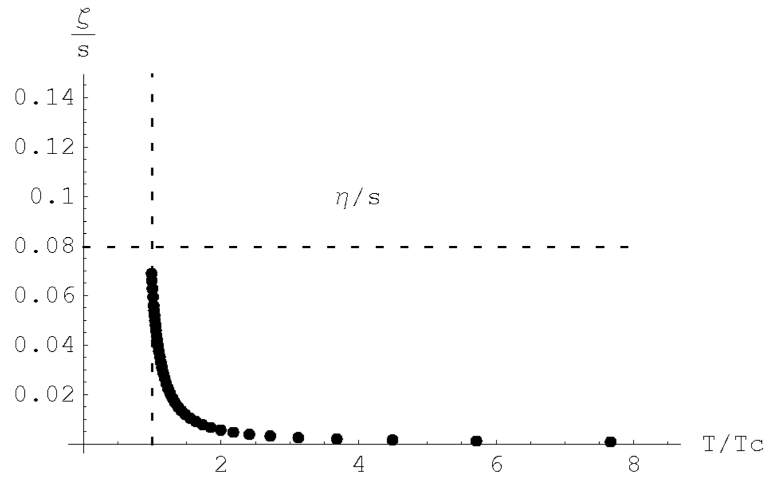
  Important for fit to experiment: if ζ too large, linear hydrodinamic models break down

Bulk Viscosity

The viscosity is computed by Kubo formula

$$\eta \sim \int dt d^3x \, \langle T_{ii}(x) T_{ii}(0) \rangle_{ret}$$

Correlator obtained holographically from the low-frequncy isotropic scalar mode fluctuations around BH with infalling boundary conditions at the horizon.



Matches indication from lattice (Meyer '08) and result from lattice thermodynamics + QCD sum rules (karsch 08)

Axial sector

In YM We can consider the pseudoscalar operator:

$\tilde{O} = TrF\tilde{F} \Leftrightarrow a(x,r) \qquad \Delta = 4 \Rightarrow m_a^2 = 0$

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Background geometry

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$\Downarrow$ Background geometry $\qquad$ $\Downarrow$ probe pseudoscalar $a(x,r)$

- Shift symmetry in the large-N limit $\Rightarrow$ No potential for a.
- $Z(\lambda)$ to be fixed phenomenologically.

Chern-Simons diffusion

The low frequency limit of the correlator gives a *diffusion constant*

$$\Gamma_{CS} = \propto \int d^4x \langle TrF\tilde{F}(x)\, TrF\tilde{F}(0)\rangle_R$$

This quantity plays an important role in the chiral magnetic effect:

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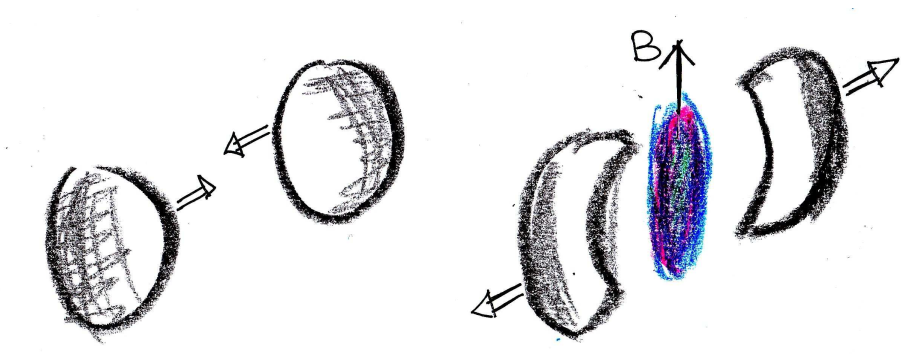

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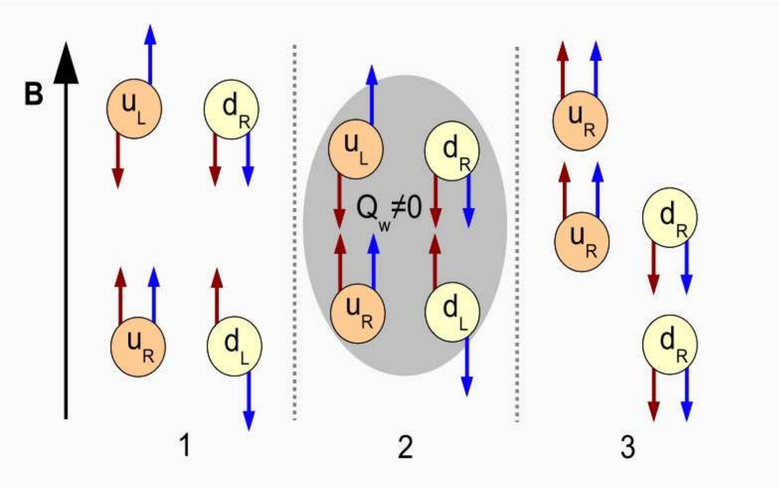

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$$J^5 = \mu_5 B, \qquad \mu_5 \propto \Gamma_{CS}$$

- Holographic computation in the deconfined phase:

$$\Gamma_{CS} = \frac{sT}{N^2}\frac{Z(r_h)}{2\pi}$$

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$$\downarrow \qquad\qquad \downarrow$$

Finite χ_{top} $\quad$ 0^{-+} glueball asymptotics

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Free parameters to fix by matching lattice/experiment

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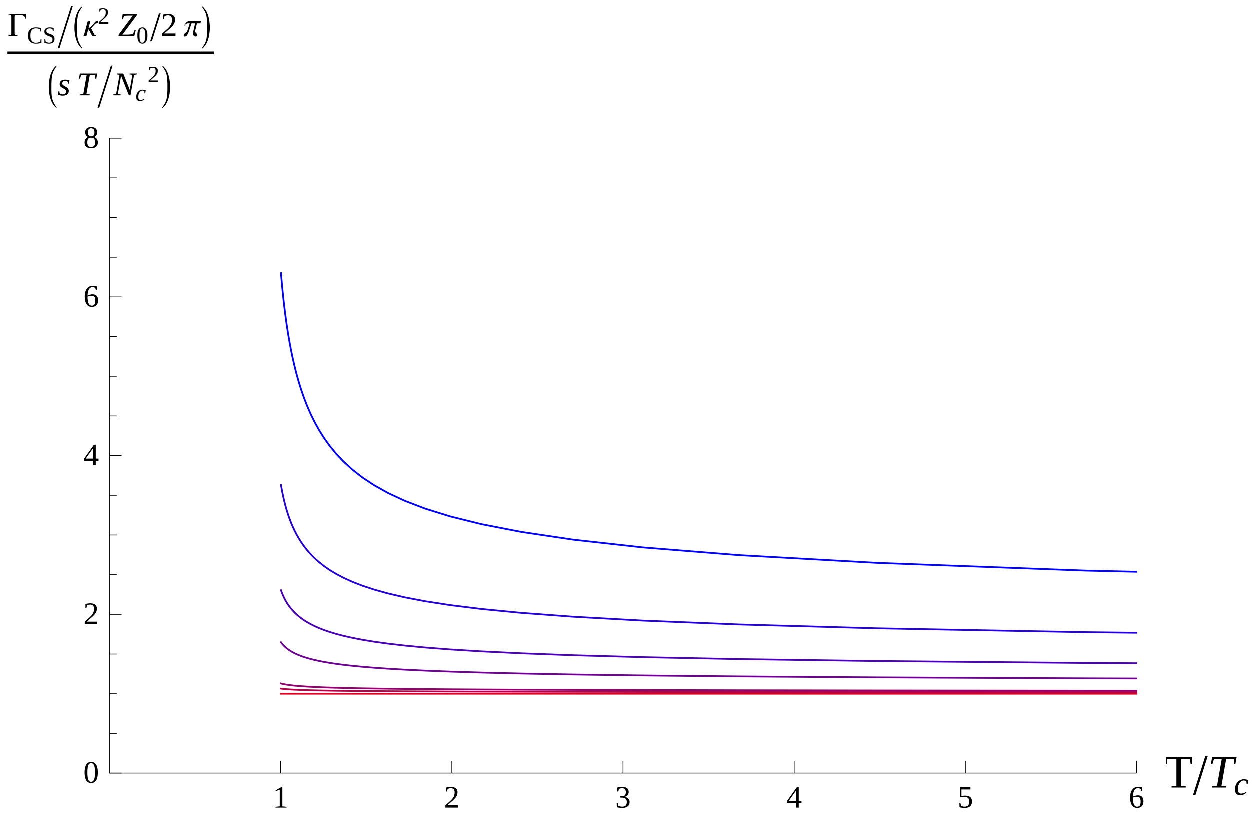

Adding Flavor

N_f quark flavors: spacetime-filling $D4 - \bar{D}4$ branes. E. Kiritsis and collaborators

- Worldvolume fields:

$$\begin{array}{ccc} T^i_j & \Leftrightarrow & \bar{q}^i q_j \\ A^L_\mu, A^R_\mu & \Leftrightarrow & J^{R,L}_\mu \end{array}$$

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$$S = \quad S_{glue}[g_{ab}, \lambda] + M^3 N_f\, N \int d^5x, V_0(\lambda) e^{-a(\lambda)T^2} \sqrt{-\det(g_{ab} + h(\lambda)\partial_a T \partial_b T)}$$

- χSB : $T \to \infty$ in the IR.

’t Hooft vs. Veneziano limit

- ’t Hooft limit: $N \to \infty$, N_f fixed $\Rightarrow$ neglect backreaction of flavors on colors.
  - Confined phase $\Rightarrow$ $T \to +\infty$ in the IR
  - Deconfined phase (BH) $\Rightarrow$ no regular solutions at r_h with non-trivial T $\Rightarrow$ chiral symmetry restored

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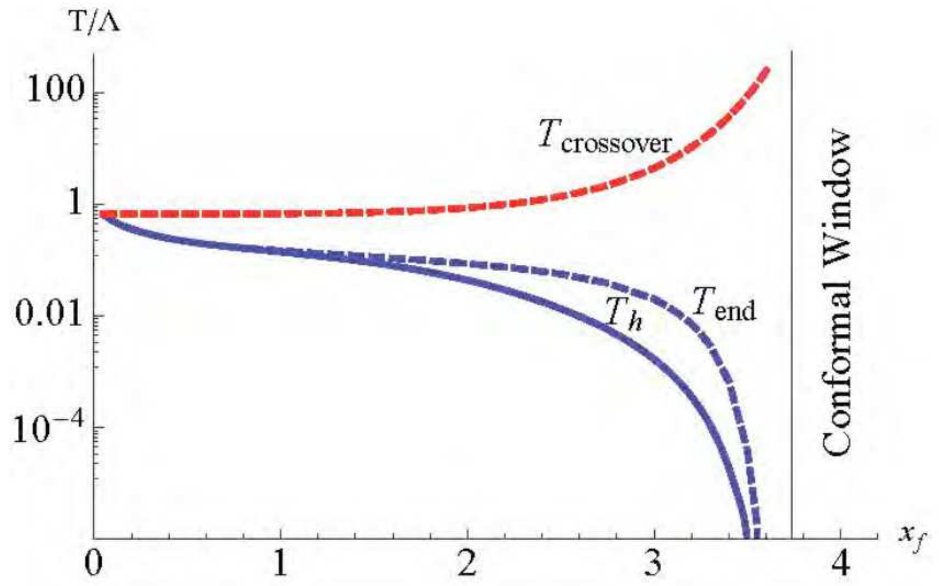
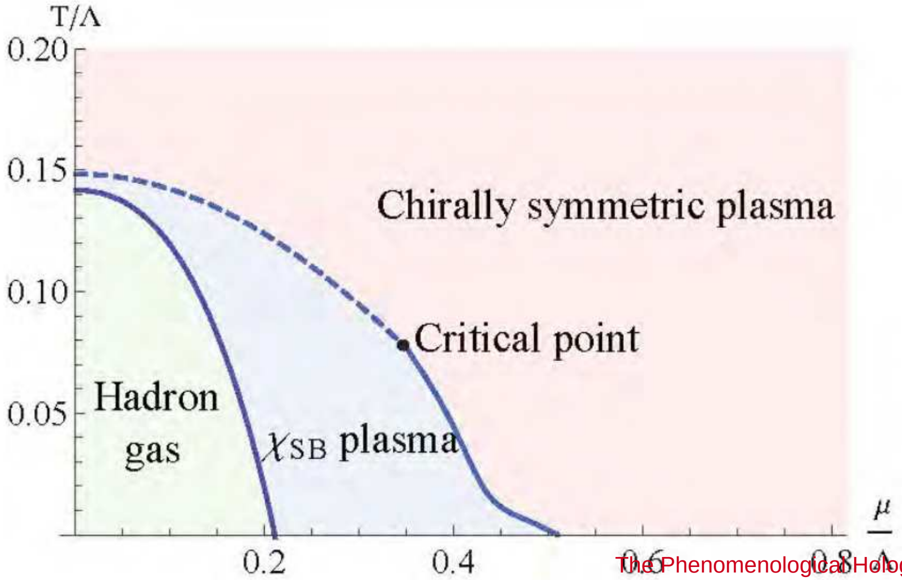
  - Deconfined phase (BH) $\Rightarrow$ no regular solutions at r_h with non-trivial T $\Rightarrow$ chiral symmetry restored
- Veneziano limit: $N, N_f \to \infty$, $x = N_f/N$ fixed $\Rightarrow$ backreaction of T is included
  - IR fixed point with unbroken χ_s in a conformal window for $x_c < x < 5.5$, $x_c \simeq 4$.
  - $x < x_c \Rightarrow$ the would-be conformal fixed point is not reached (T tachyonic at the fixed point) $\Rightarrow$ confined phase with broken χ_s

Phase diagrams

Finite Temperature



Finite Temperature and Density



Conclusion

Phenomenological holographic models provide a calculational framework for quantities that cannot be tackled with other techniques. In situations where other techniques can be applied these models give qualitatively correct and even quantitatively accurate results.

Contrary to top-down string theory models, some approximations and assumptions not under control. A more precise contact with string theory would be desired.

Parametrizing the axion Lagrangian

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Discrete 0^{-+} spectrum with asymptotics (from WKB method)

$$m_n^2 \sim n, \qquad f_n \sim n$$

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For $c_1 = 0$, $c_4 = 0.26$ one finds a good match with Lattice result for the lowest lying 0^{-+} states.

	5d model	lattice hep-lat/9901004
$m_{0^{-+}}/m_{0^{++}}$	1.50	1.50(4)
$m_{0^{*-+}}/m_{0^{++}}$	2.10	2.11(6)