Gran Sasso Summer Institute 2014

Lecture #1: Thermonuclear Reactions and Stellar Burning

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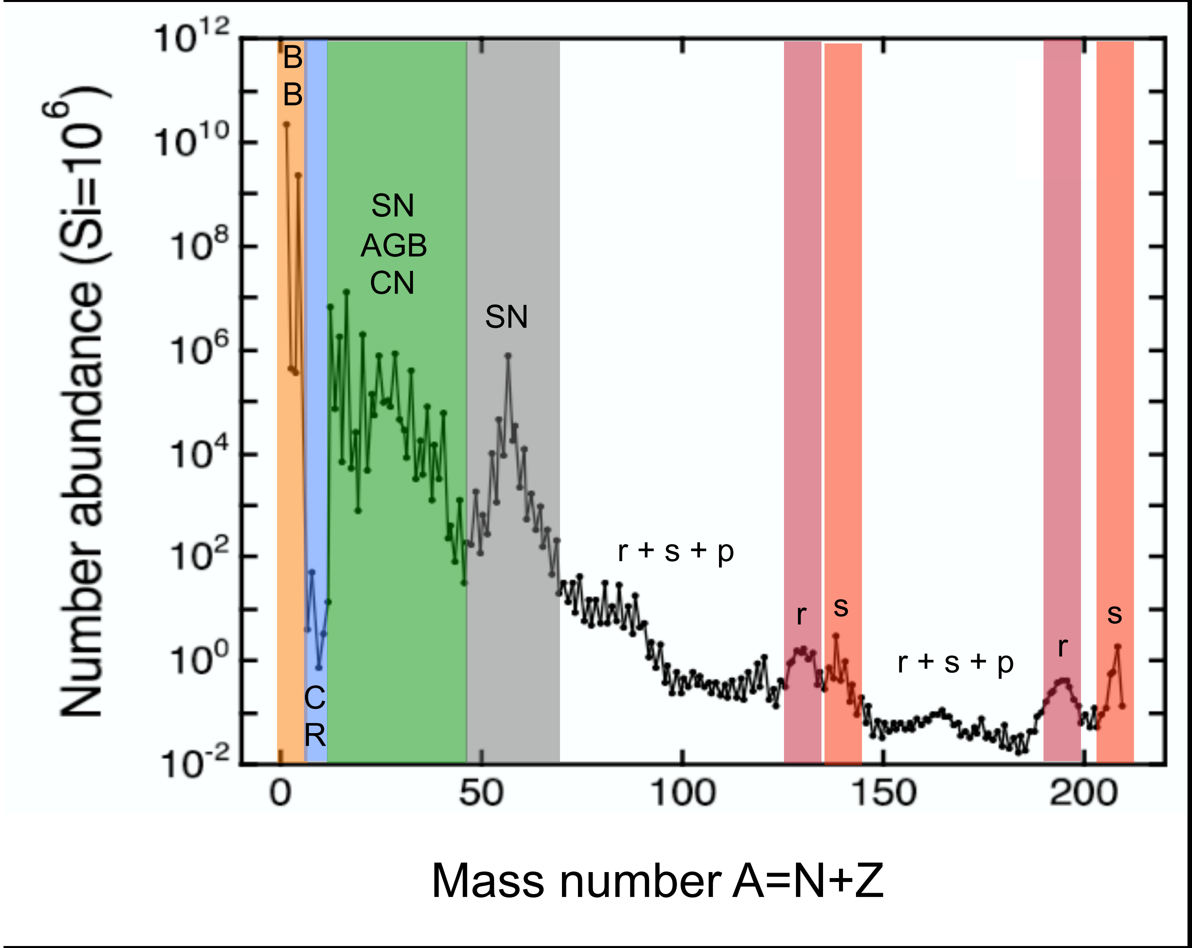
THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

TUNL

History: Solar System Abundances



Willy Fowler
(1911-95)

- Suess & Urey, Rev. Mod. Phys. 28, 53 (1956)
- Lodders, Palme & Gail, Landolt-Boernstein New Series VI/4B (Springer 2009)

Foundation of modern theory of nuclear astrophysics:

- Burbidge, Burbidge, Fowler and Hoyle, Rev. Mod. Phys. 29, 547 (1957)
- Cameron, Pub. Astron. Soc. Pac. 69, 201 (1957)

Nobel prize to Willy Fowler (1983)

Nuclear Reactions

Definition of cross section:

$$\sigma \equiv \frac{(\text{number of interactions per time})}{(\text{number of incident particles per area per time})(\text{number of target nuclei within the beam})} = \frac{N_r}{N_0 N_t}$$

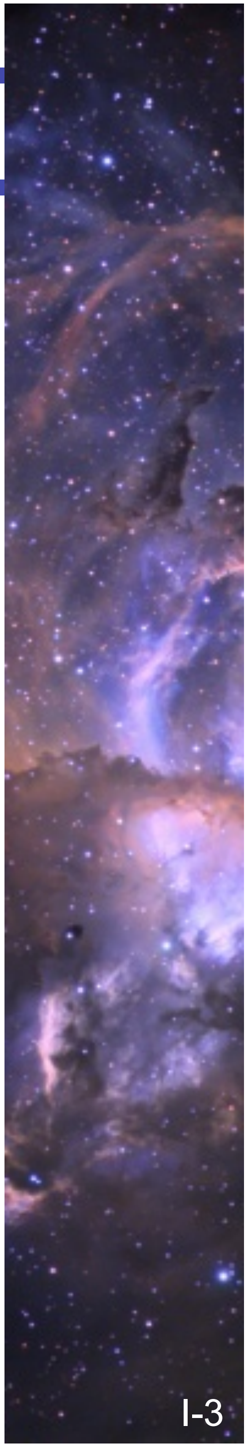
Unit: 1 barn=10^{-28} m^2

Example: $^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$ (first step of pp chain)

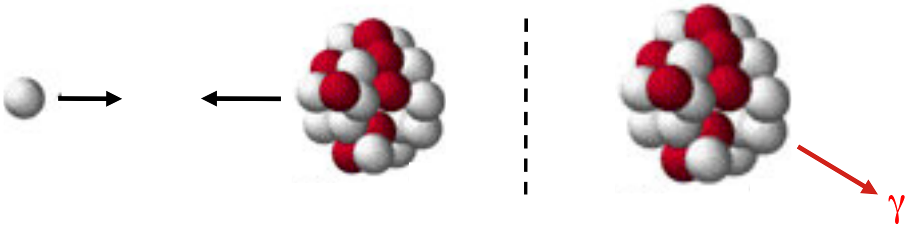
$\sigma_{theo}=8\times10^{-48}$ cm^2 at E_{lab}=1 MeV [E_{cm}=0.5 MeV]

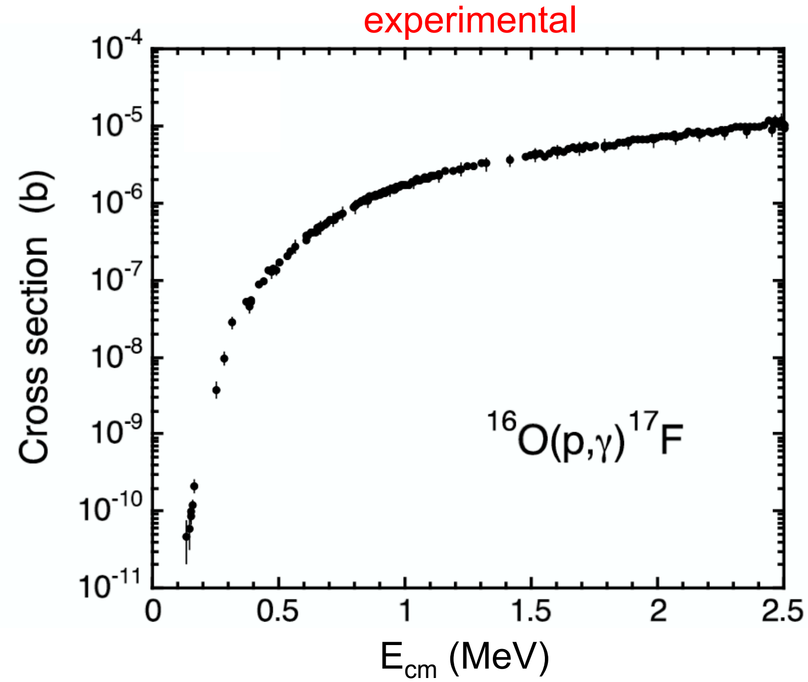
1 ampere (A) proton beam (6×10^{18} p/s) on dense proton target (10^{20} p/cm^2)

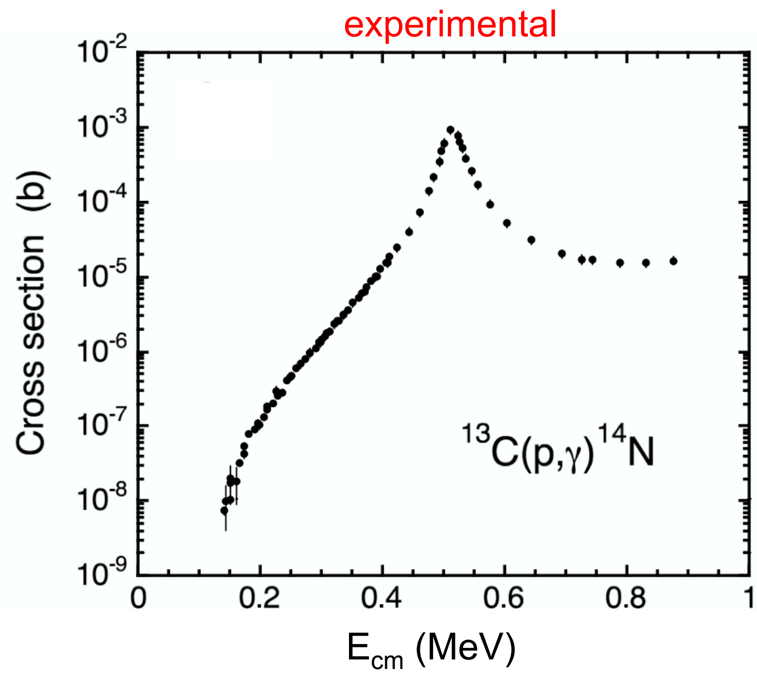
gives only 1 reaction in 6 years of measurement!



Cross Sections

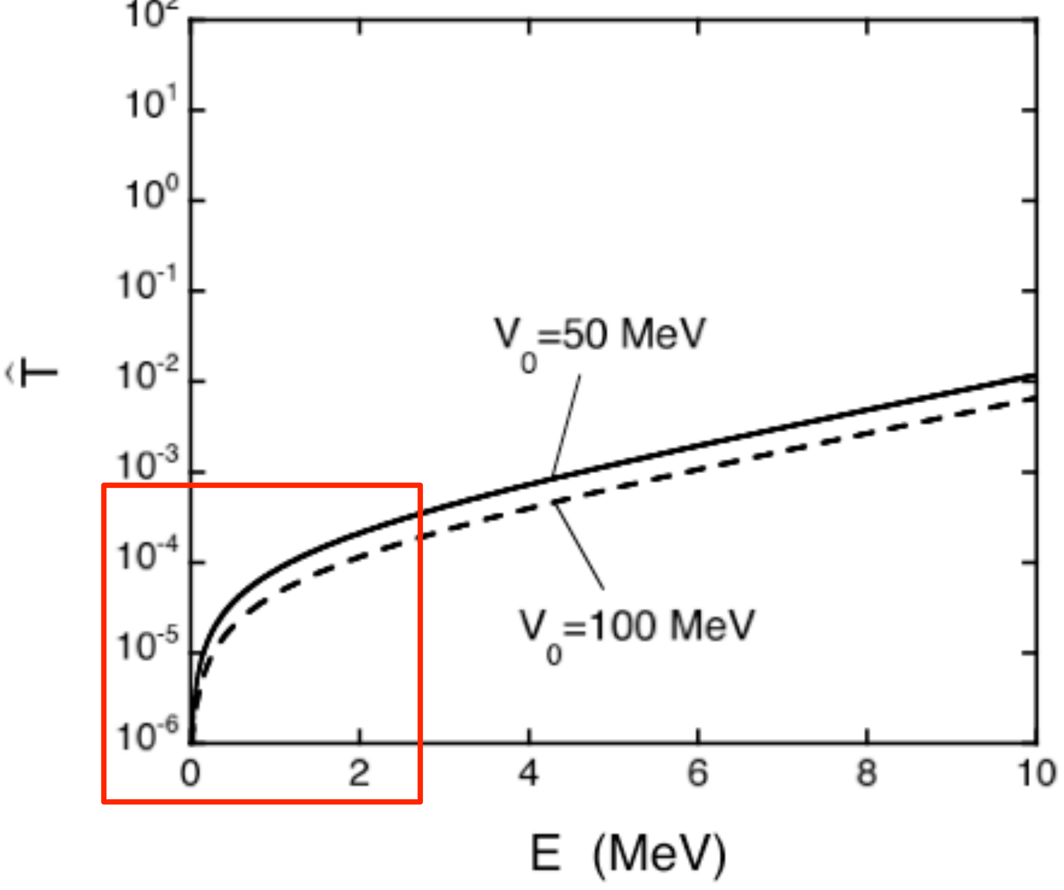




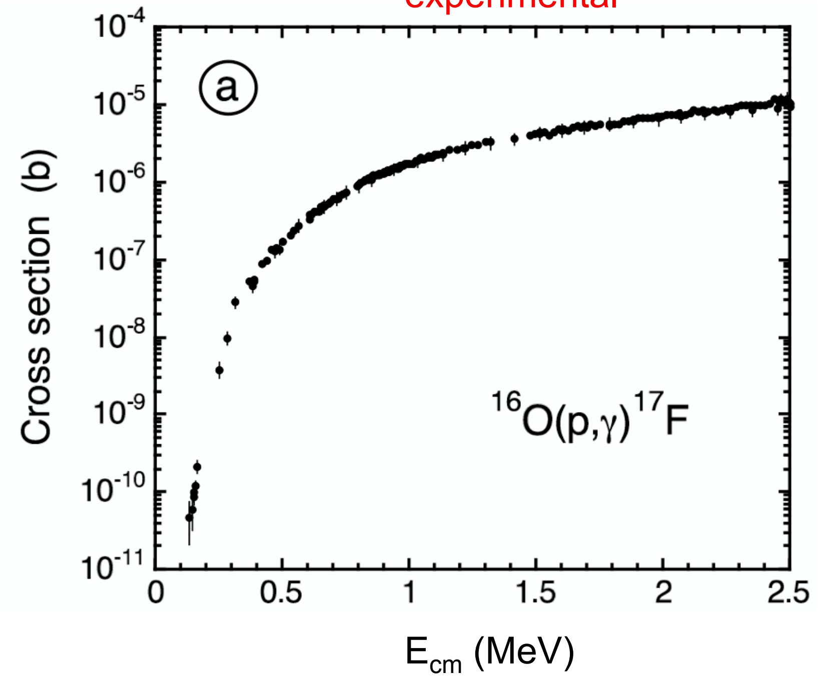


(i) why does the cross section fall drastically at low energies?
(ii) where is the peak in the cross section coming from?

calculated



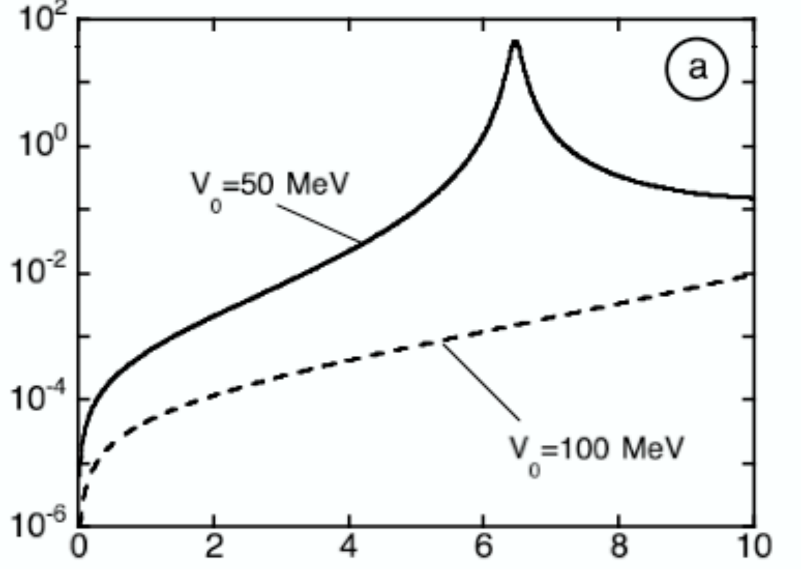
experimental



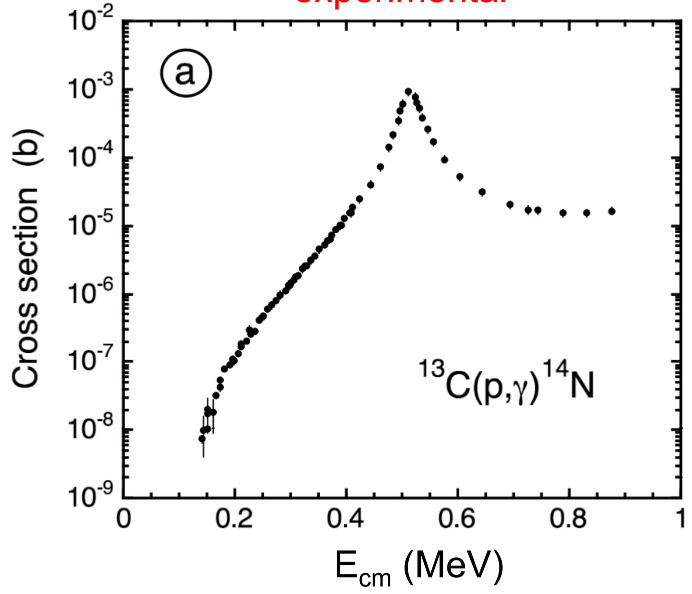
Tunnel effect is the reason for the strong drop in cross section at low energies!

$$\frac{|A'|^2}{|F'|^2}$$

calculated



experimental



E (MeV)

Transmission Through the Coulomb Barrier

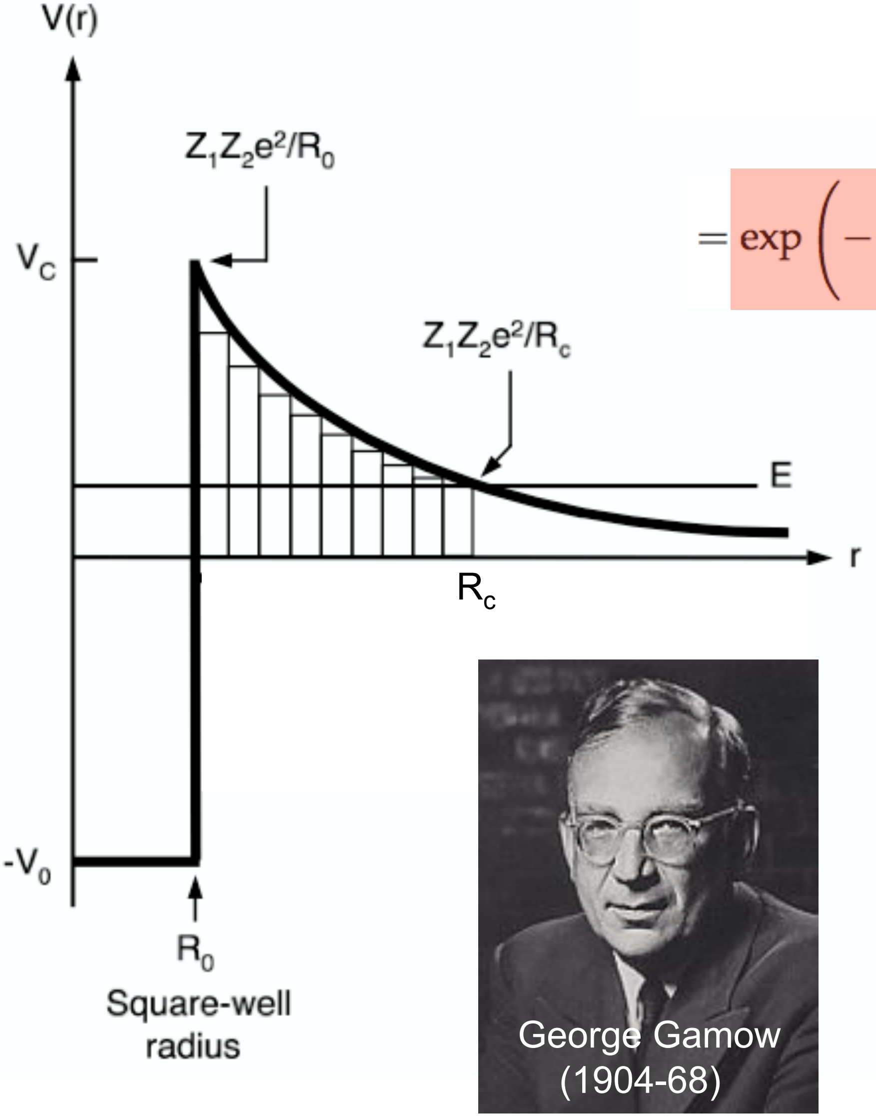
$$\hat{T} = \hat{T}_1 \cdot \hat{T}_2 \cdot \ldots \cdot \hat{T}_n \approx \exp\left[-\frac{2}{\hbar}\sum_i \sqrt{2m(V_i - E)}(R_{i+1} - R_i)\right]$$

$$\xrightarrow[n\ \text{large}]{} \exp\left[-\frac{2}{\hbar}\int_{R_0}^{R_c} \sqrt{2m[V(r) - E]}\,dr\right]$$

$$= \exp\left(-\frac{2\pi}{\hbar}\sqrt{\frac{m}{2E}}Z_0 Z_1 e^2 \left[1 + \frac{2}{3\pi}\left(\frac{E}{V_C}\right)^{3/2}\right] + \frac{4}{\hbar}\sqrt{2mZ_0 Z_1 e^2 R_0}\right)$$

[for low energies and zero angular momentum]

"Gamow factor" $e^{-2\pi\eta}$

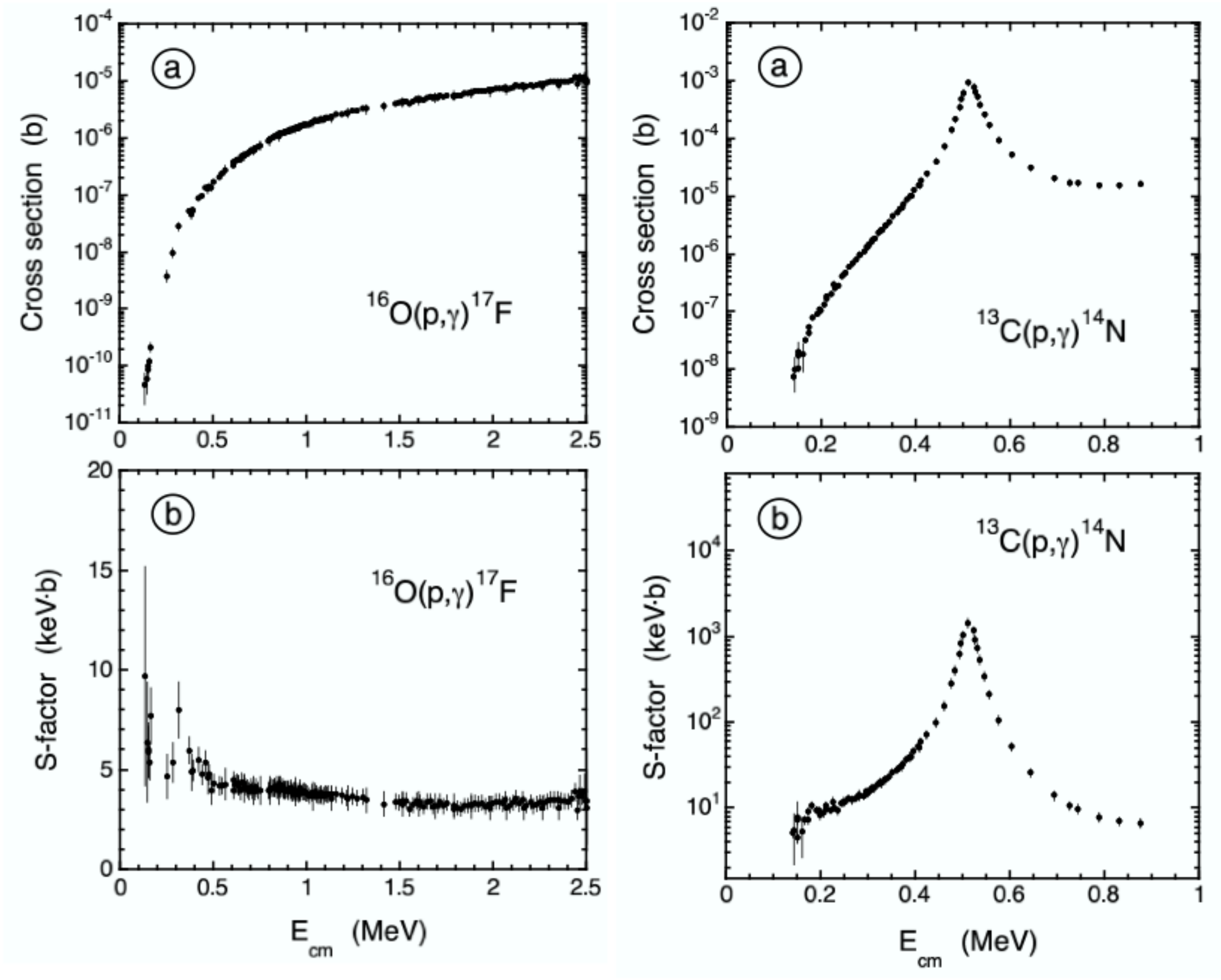


George Gamow (1904-68)

Comparison: S-factors and Cross Sections

cross sections →

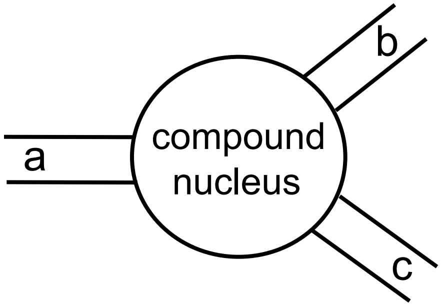
S-factors →



Formal Reaction Theory: Breit-Wigner Formula

Eugene Wigner
(1902-95)
Nobel Prize 1963

Example: $^{22}\mathrm{Ne}(\alpha,\gamma)^{26}\mathrm{Mg}$
$^{22}\mathrm{Ne}(\alpha,n)^{25}\mathrm{Mg}$



de Broglie wavelength

partial widths for incoming and outgoing channel

$$\sigma_{\mathrm{BW}}(E) = \frac{\lambda^2}{4\pi}\underbrace{\frac{(2J+1)}{(2j_0+1)(2j_1+1)}}\frac{\Gamma_a\Gamma_b}{(E_r-E)^2+\Gamma^2/4}$$

spin factor

resonance energy

total width

Thermonuclear Reactions

For a reaction 0 + 1 → 2 + 3 we find from the definition of σ (see earlier) a “reaction rate”:

$$r_{01} = N_0 N_1 \int_0^{\infty} v P(v) \sigma(v)\, dv \equiv N_0 N_1 \langle \sigma v \rangle_{01}$$

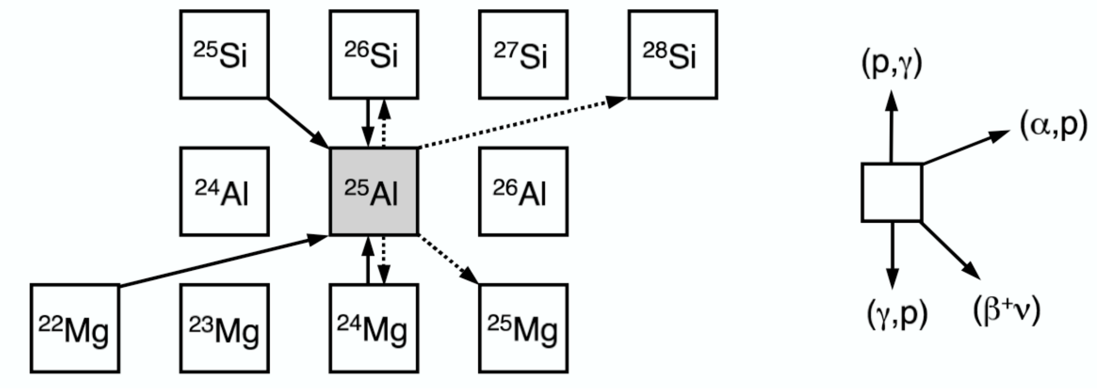
For a stellar plasma: kinetic energy for reaction derives from thermal motion:

“Thermonuclear reaction”

For a Maxwell-Boltzmann distribution:

$$\boxed{\langle \sigma v \rangle_{01} = \left(\frac{8}{\pi m_{01}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^{\infty} E\, \sigma(E)\, e^{-E/kT}\, dE}$$

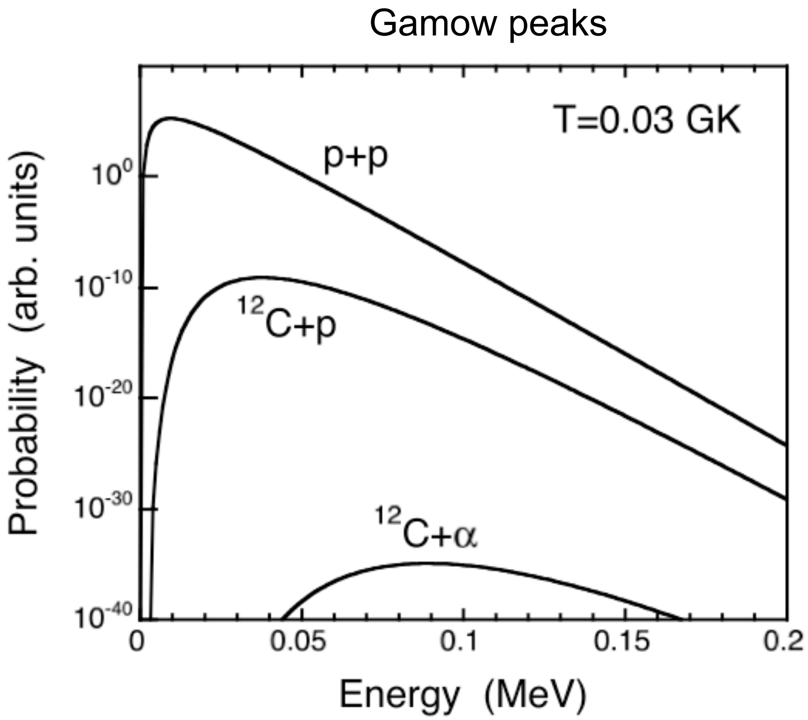
Interplay of Many Different Nuclear Reactions in Stellar Plasma



$$
\begin{aligned}
\frac{d(N_{^{25}\mathrm{Al}})}{dt} = {} & \left.\begin{aligned} & N_\mathrm{H} N_{^{24}\mathrm{Mg}} \langle\sigma v\rangle_{^{24}\mathrm{Mg}(\mathrm{p},\gamma)} + N_{^{4}\mathrm{He}} N_{^{22}\mathrm{Mg}} \langle\sigma v\rangle_{^{22}\mathrm{Mg}(\alpha,\mathrm{p})} \\ & + N_{^{25}\mathrm{Si}} \lambda_{^{25}\mathrm{Si}(\beta^+\nu)} + N_{^{26}\mathrm{Si}} \lambda_{^{26}\mathrm{Si}(\gamma,\mathrm{p})} + \ldots \end{aligned}\right\} \text{production} \\
& \left.\begin{aligned} & - N_\mathrm{H} N_{^{25}\mathrm{Al}} \langle\sigma v\rangle_{^{25}\mathrm{Al}(\mathrm{p},\gamma)} - N_{^{4}\mathrm{He}} N_{^{25}\mathrm{Al}} \langle\sigma v\rangle_{^{25}\mathrm{Al}(\alpha,\mathrm{p})} \\ & - N_{^{25}\mathrm{Al}} \lambda_{^{25}\mathrm{Al}(\beta^+\nu)} - N_{^{25}\mathrm{Al}} \lambda_{^{25}\mathrm{Al}(\gamma,\mathrm{p})} - \ldots \end{aligned}\right\} \text{destruction}
\end{aligned}
$$

System of coupled differential equations: "nuclear reaction network"

Solved numerically
[Arnett, "Supernovae and Nucleosynthesis", Princeton University Press, 1996]

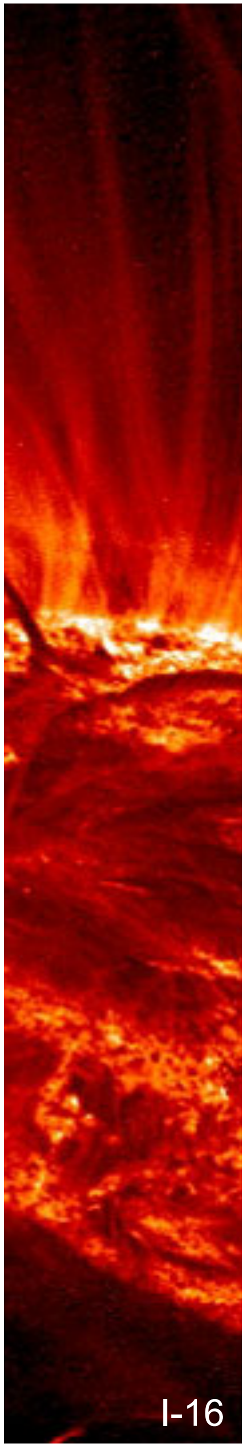

Gamow peaks

Important aspects:

(i) Gamow peak shifts to higher energy for increasing charges Z_p and Z_t

(ii) at same time, area under Gamow peak decreases drastically

Conclusion: for a mixture of different nuclei in a plasma, those reactions with the smallest Coulomb barrier produce most of the energy and are consumed most rapidly



Special case #2: Reaction Rates for “Narrow” Resonances (Γ_i const over total Γ)

Breit-Wigner formula (energy-**independent** partial widths)

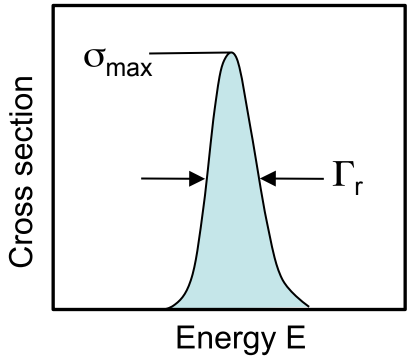
$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty E\,\sigma(E)\,e^{-E/kT}\,dE$$

$$= N_A \frac{\sqrt{2\pi}\hbar^2}{(m_{01}kT)^{3/2}} e^{-E_r/kT}\,\omega\frac{\Gamma_a\Gamma_b}{\Gamma}\,2\pi$$

- resonance energy needs to be known rather precisely
- takes into account only rate contribution at E_r

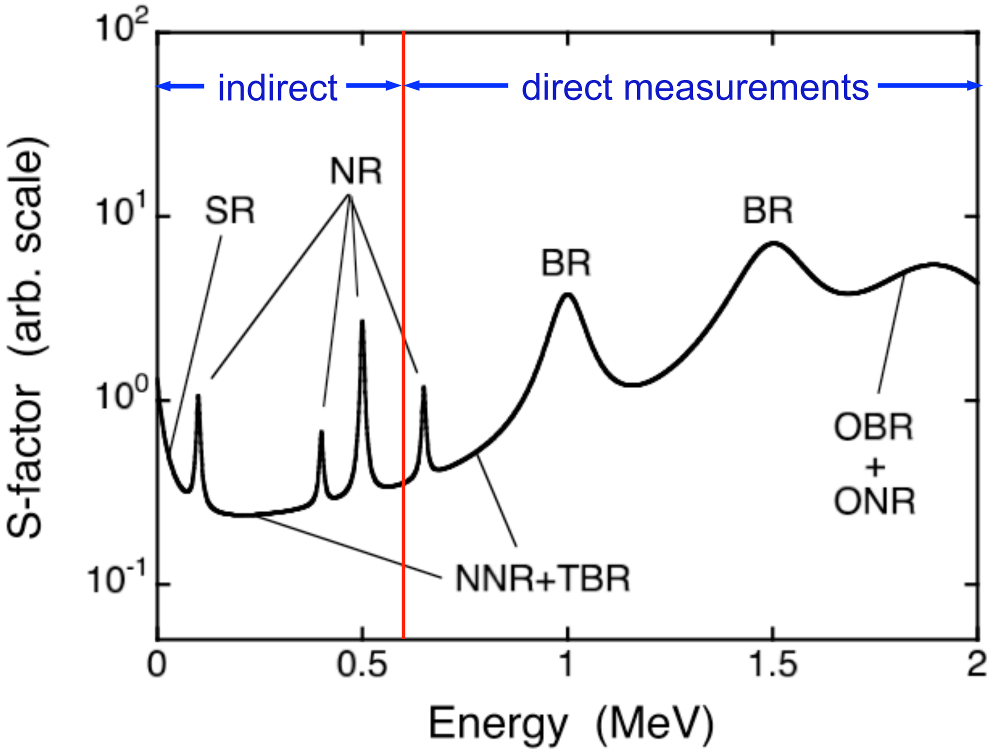
“resonance strength” $\omega\gamma$:

- proportional to area under narrow resonance curve
- energy-dependence of σ not important



$$\omega\gamma \propto \sigma_{\max}\cdot\Gamma_r$$

Total Thermonuclear Reaction Rate



Need to consider:

- non-resonant processes
- narrow resonances
- broad resonances
- subthreshold resonances
- interferences
- continuum

every nuclear reaction represents a special case !

Stellar Burning Stages

A first impression:

Globular Cluster
M10

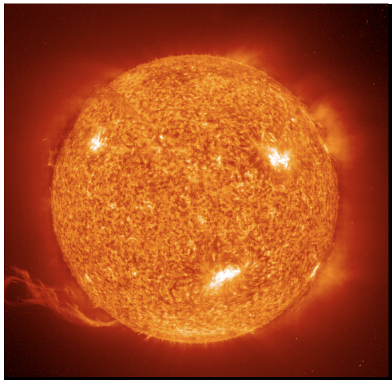
Red giant stars:
H→He via CNO cycles in
H shell surrounding He core

Horizontal branch stars:
He→C, O in core
H→He in shell

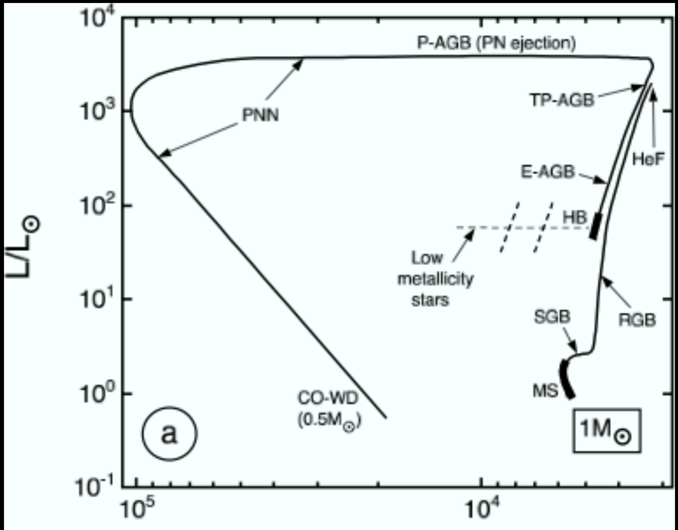
Main sequence stars:
H→He via pp chains in core

Credner & Kohle, Sternwarte Bonn

Hertzsprung-Russell Diagram: “Tracks”



Cat’s Eye nebula
(NGC 6543)



Effective surface temperature T_{eff} (K)

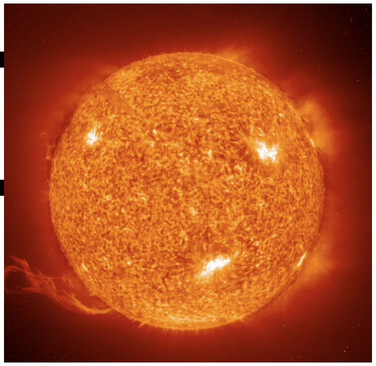
- heavy lines: major nuclear burning stages in stellar core

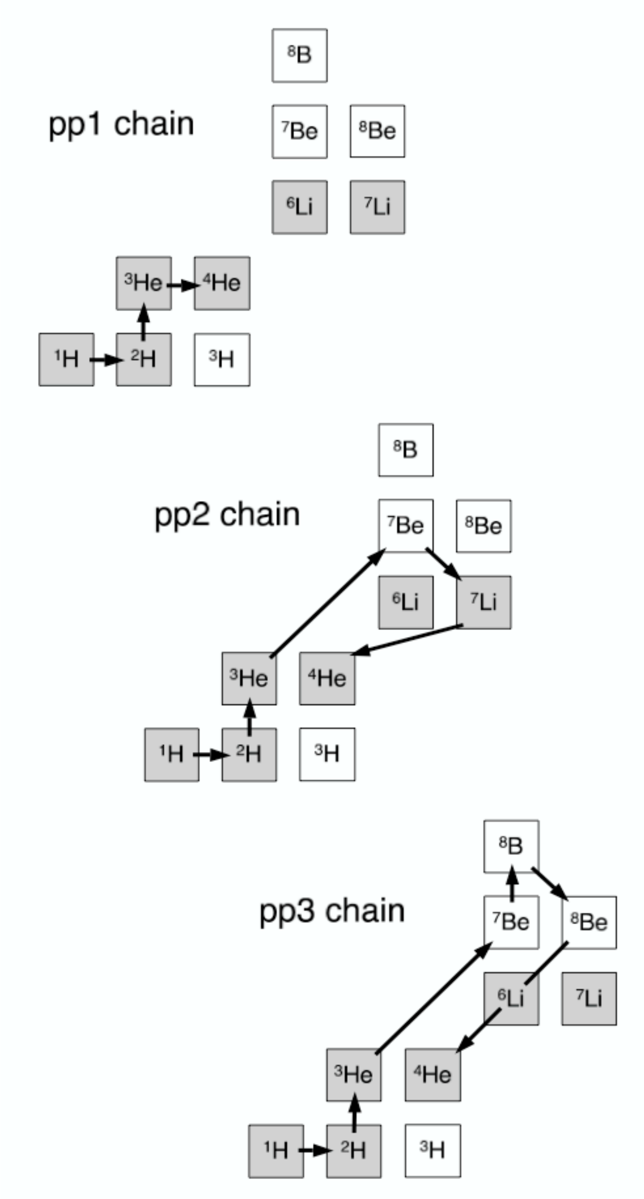
- evolution of star mainly determined by its mass:

  larger mass:
  → larger T and P in core
  → faster nuclear energy generation
  → larger luminosity
  → faster fuel consumption
  → shorter lifetime

Betelgeuse (α Orionis)

Hydrostatic Hydrogen Burning: Sun (T=15.6 MK), stellar core (T=8-55 MK), shell of AGB stars (T=45-100 MK)

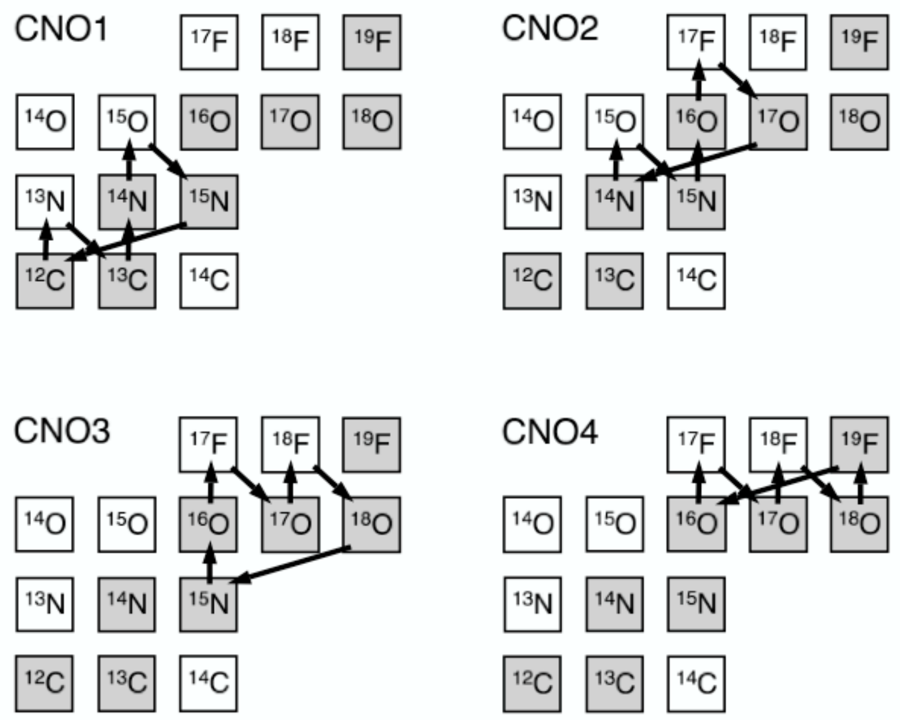




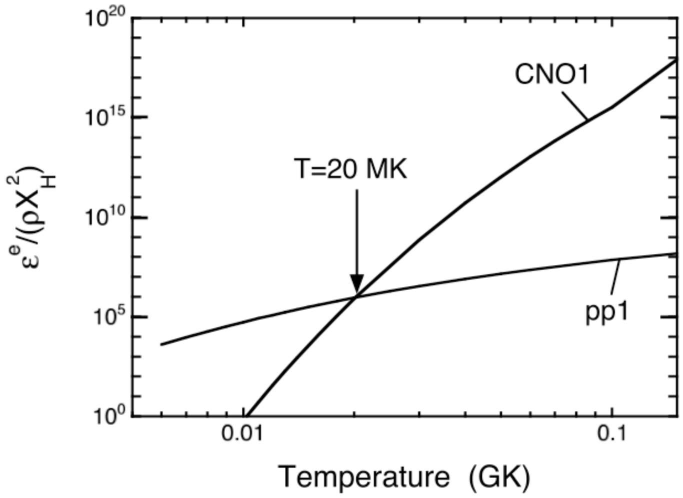
pp1 chain	pp2 chain	pp3 chain
$p(p,e^+\nu)d$	$p(p,e^+\nu)d$	$p(p,e^+\nu)d$
$d(p,\gamma)^3$He	$d(p,\gamma)^3$He	$d(p,\gamma)^3$He
^{3}He(^{3}He,2p)α	^{3}He$(\alpha,\gamma)^7$Be	^{3}He$(\alpha,\gamma)^7$Be
	^{7}Be$(e^-,\nu)^7$Li	^{7}Be$(p,\gamma)^8$B
	^{7}Li$(p,\alpha)\alpha$	^{8}B$(\beta^+\nu)^8$Be
		^{8}Be$(\alpha)\alpha$

- 4H→^{4}He releases 26.7 MeV
- reactions are non-resonant at low energies
- p+p [slowest reaction] has not been measured
- d+p, ^{3}He+^{3}He, ^{3}He+α have been measured recently by LUNA collaboration
- 90% of Sun's energy produced by pp1 chain

Hydrostatic Hydrogen Burning: Sun (T=15.6 MK), stellar core (T=8-55 MK), shell of AGB stars (T=45-100 MK)

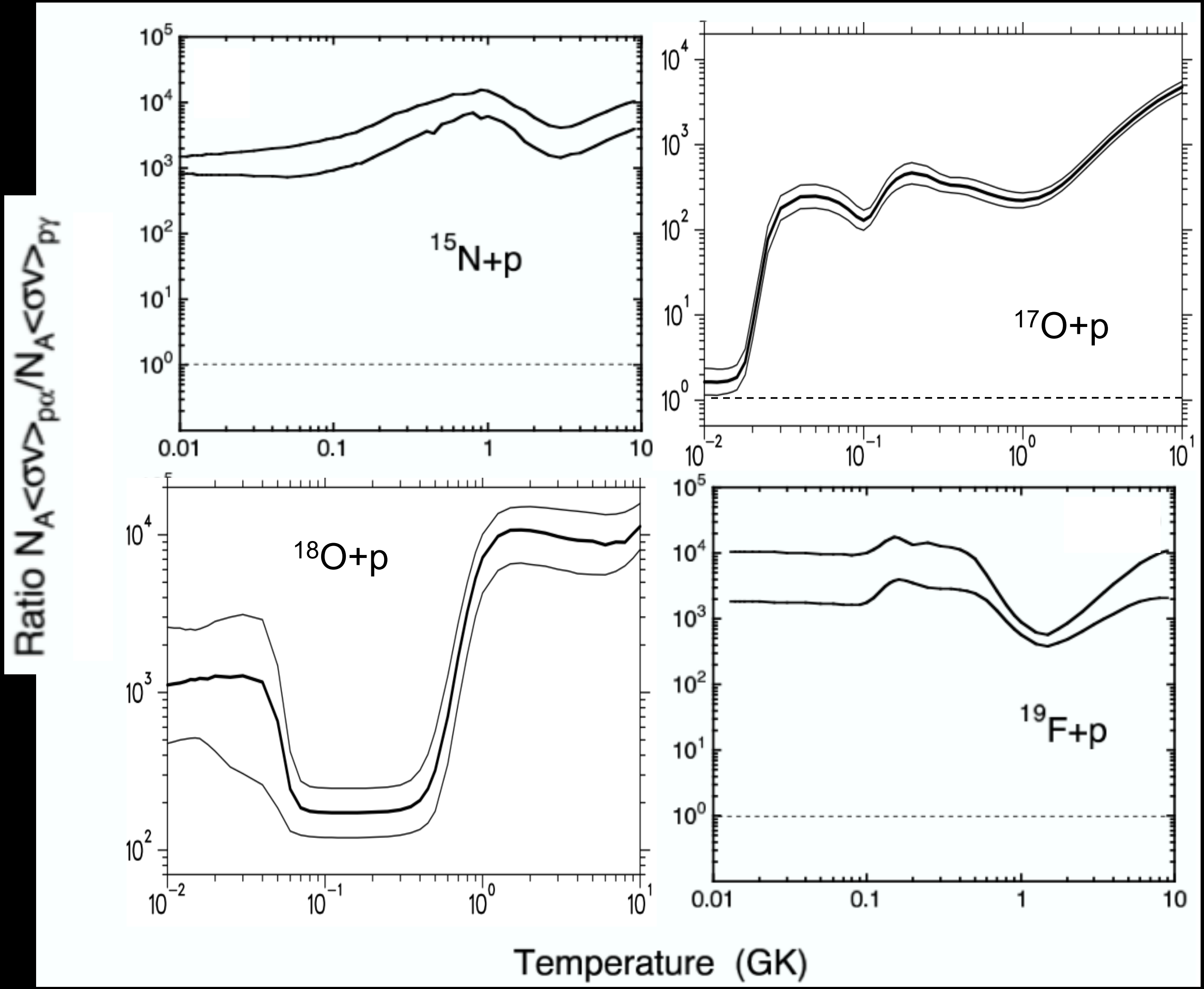


- ^{12}C and ^{16}O nuclei act as catalysts
- branchings: (p,α) stronger than (p,γ)
- ^{14}N(p,γ)^{15}O slowest reaction in CNO1 has been measured by LUNA/LENA
- solar: ^{13}C/^{12}C=0.01; CNO1: ^{13}C/^{12}C=0.25 (“steady state”)
- T>20 MK: CNO1 faster than pp1



CNO1	CNO2	CNO3	CNO4
^{12}C(p,γ)^{13}N	^{14}N(p,γ)^{15}O	^{15}N(p,γ)^{16}O	^{16}O(p,γ)^{17}F
^{13}N($\beta^+\nu$)^{13}C	^{15}O($\beta^+\nu$)^{15}N	^{16}O(p,γ)^{17}F	^{17}F($\beta^+\nu$)^{17}O
^{13}C(p,γ)^{14}N	^{15}N(p,γ)^{16}O	^{17}F($\beta^+\nu$)^{17}O	^{17}O(p,γ)^{18}F
^{14}N(p,γ)^{15}O	^{16}O(p,γ)^{17}F	^{17}O(p,γ)^{18}F	^{18}F($\beta^+\nu$)^{18}O
^{15}O($\beta^+\nu$)^{15}N	^{17}F($\beta^+\nu$)^{17}O	^{18}F($\beta^+\nu$)^{18}O	^{18}O(p,γ)^{19}F
^{15}N(p,α)^{12}C	^{17}O(p,α)^{14}N	^{18}O(p,α)^{15}N	^{19}F(p,α)^{16}O

The Four Reaction Branchings in the CNO Cycles

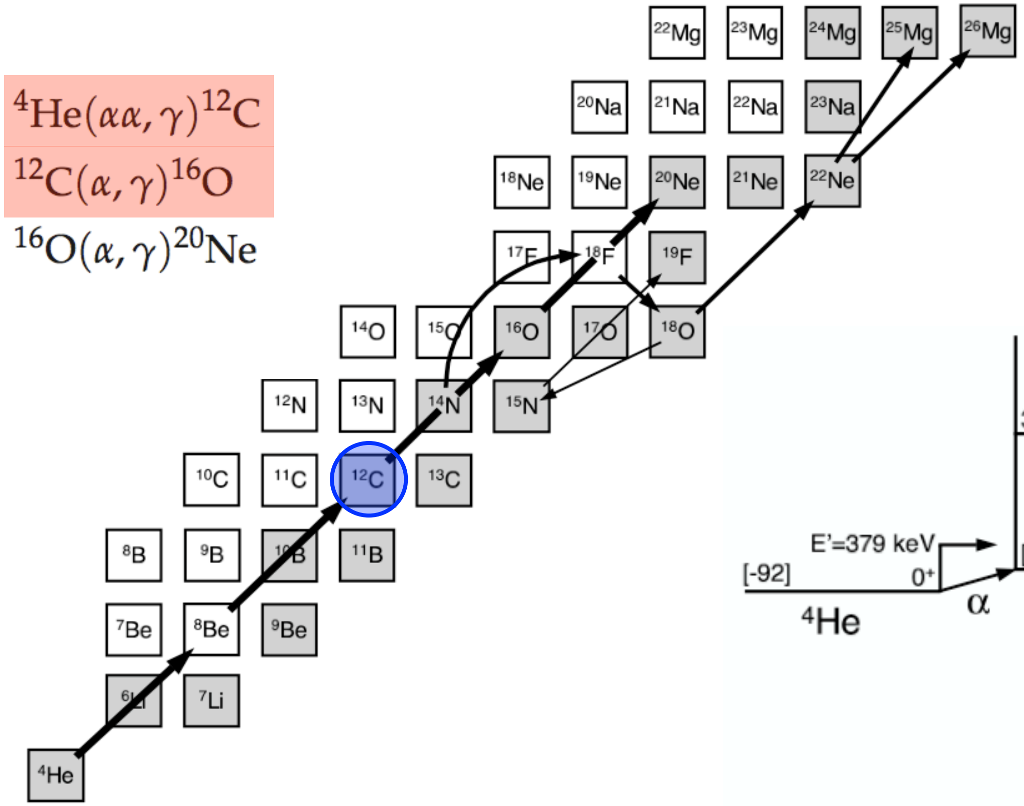


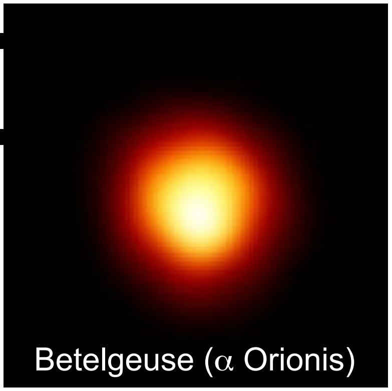
Helium Burning: Massive stars (T=100-400 MK)

$^{4}\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$

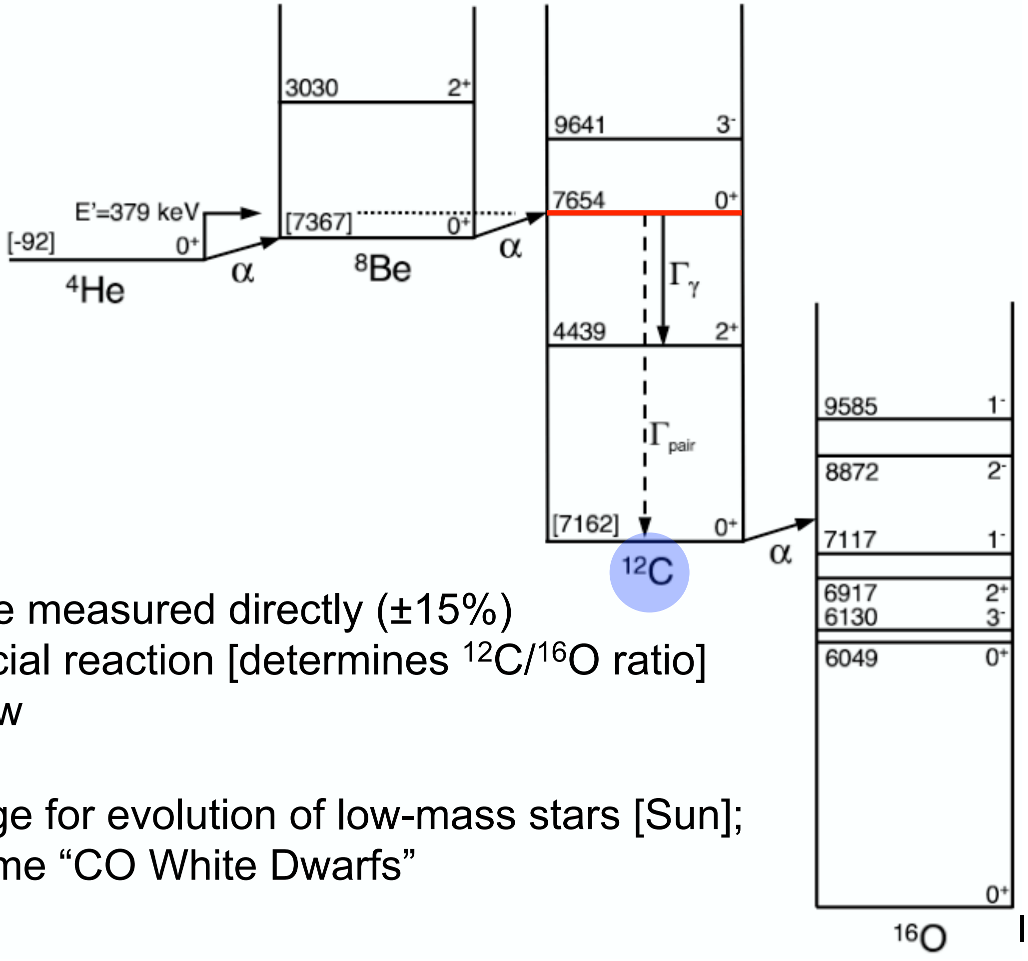
$^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$

$^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$

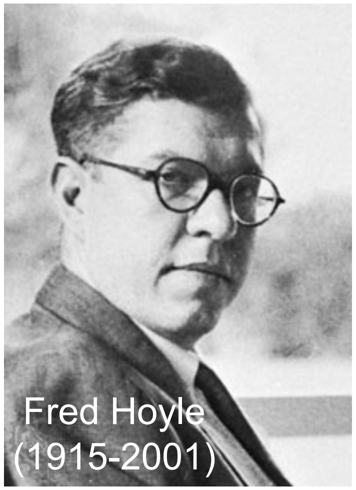




Betelgeuse (α Orionis)



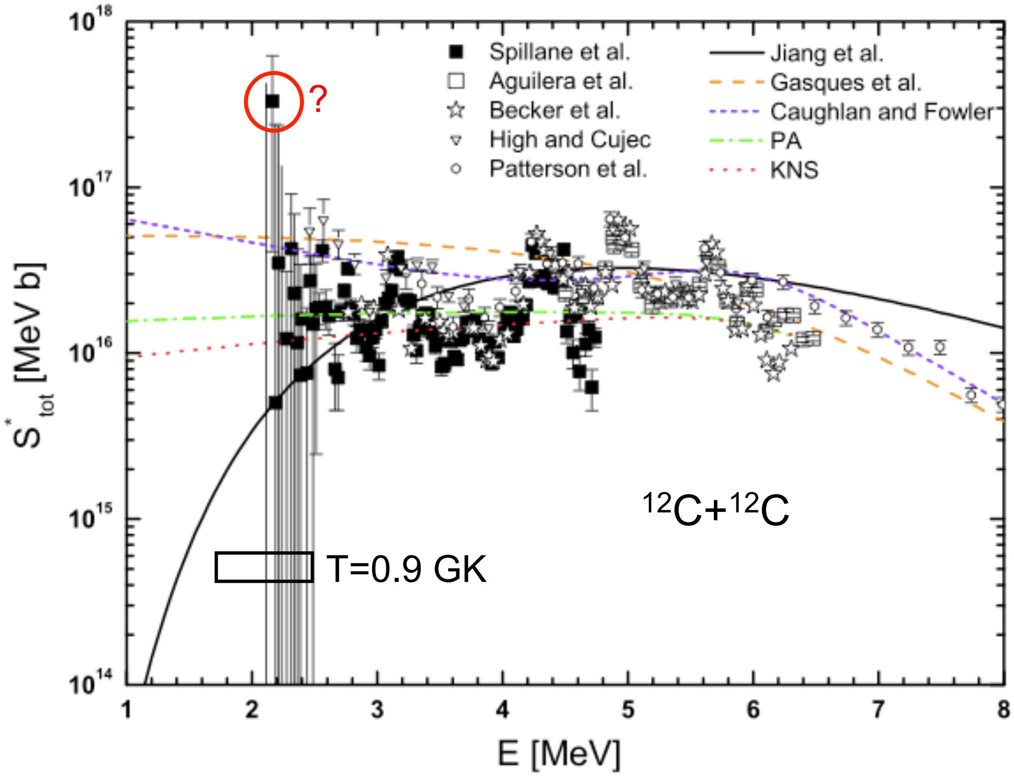
- 3α reaction cannot be measured directly (±15%)
- $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ slow, crucial reaction [determines $^{12}\mathrm{C}/^{16}\mathrm{O}$ ratio]
- $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ very slow
- ashes: $^{12}\mathrm{C}$, $^{16}\mathrm{O}$
- last core burning stage for evolution of low-mass stars [Sun]; they eventually become "CO White Dwarfs"



Fred Hoyle (1915-2001)

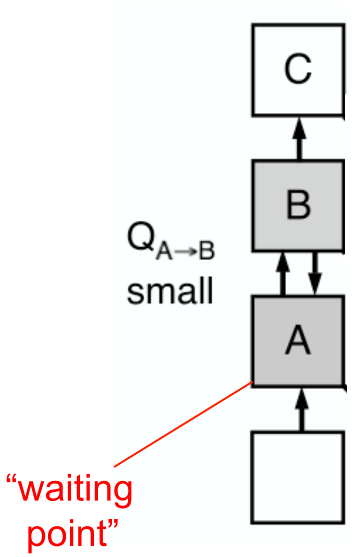
A Closer Look at $^{12}C+^{12}C$

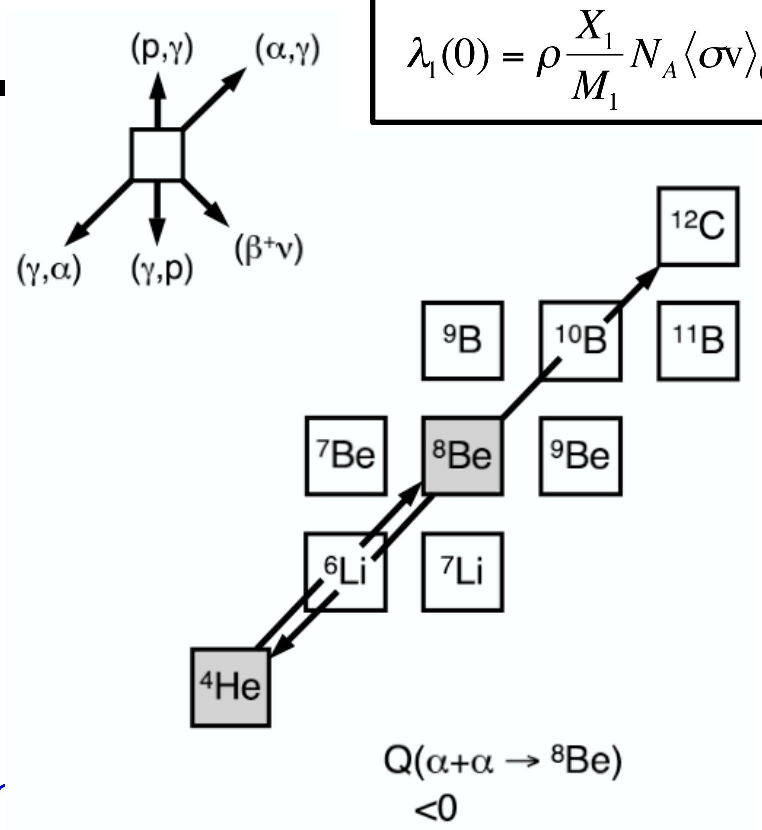
Costantini et al., Rep. Prog. Phys. 72, 086301 (2009)



Reaction Rate Equilibria: $r = r_{A\to B} - r_{B\to A} =$

$$\lambda_1(0) = \rho \frac{X_1}{M_1} N_A \langle \sigma v \rangle_{01}$$





From Saha statistical equation and reciprocity theorem

$$\lambda_{A\to B\to C} = \frac{\lambda_{A\to B}}{\lambda_{B\to A}} \lambda_{B\to C}$$

$$= N_a \left(\frac{h^2}{2\pi}\right)^{3/2} \frac{1}{(m_{Aa}kT)^{3/2}} \frac{(2j_B+1)}{(2j_A+1)(2j_a+1)}$$

$$\times \frac{G_B^{\mathrm{norm}}}{G_A^{\mathrm{norm}} G_a^{\mathrm{norm}}} e^{Q_{A\to B}/kT} \lambda_{B\to C}$$

independent of reaction rate for A→B!

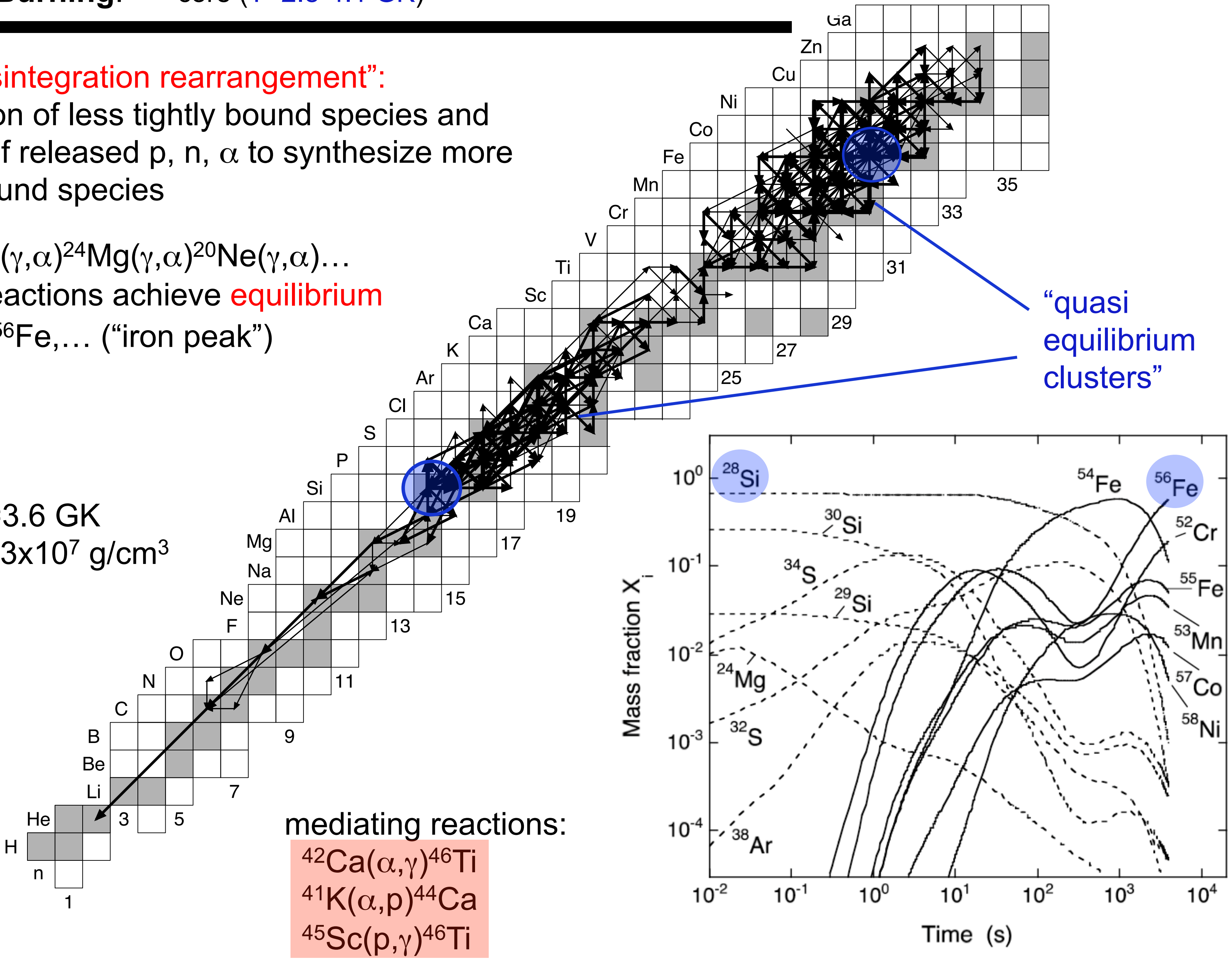
Meghnad Saha
1893-1956

$$\lambda_{3\alpha} = 0.23673 \frac{(\rho X_\alpha)^2}{T_9^3} e^{-11.6048E'/T_9} \omega\gamma_{^8\mathrm{Be}(\alpha,\gamma)} \qquad (\mathrm{s}^{-1})$$

Silicon Burning: core (T=2.8-4.1 GK)

"Photodisintegration rearrangement":
destruction of less tightly bound species and capture of released p, n, α to synthesize more tightly bound species

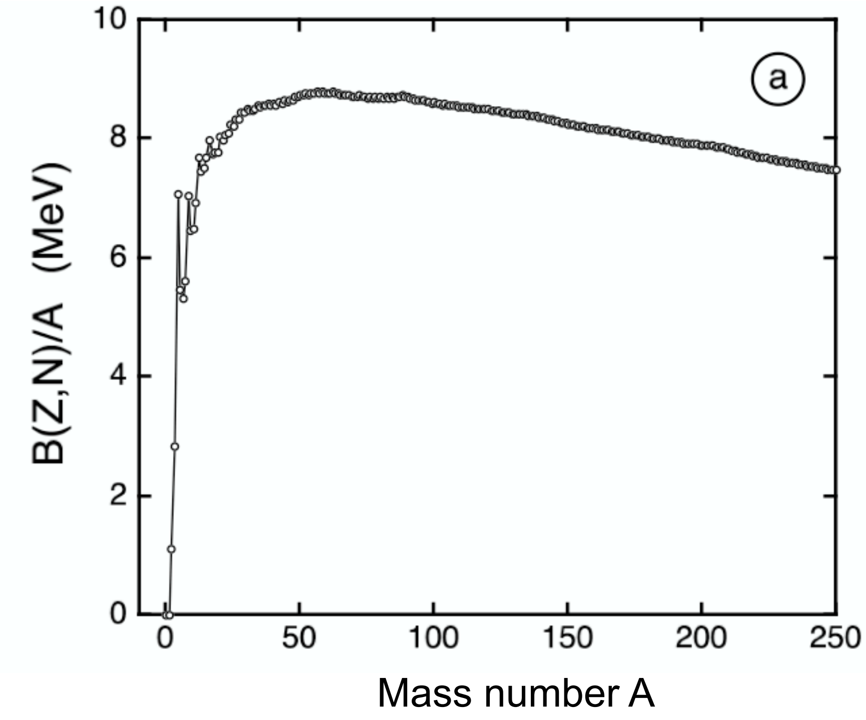
start: $^{28}Si(\gamma,\alpha)^{24}Mg(\gamma,\alpha)^{20}Ne(\gamma,\alpha)$…
- many reactions achieve equilibrium
- ashes: ^{56}Fe,… ("iron peak")

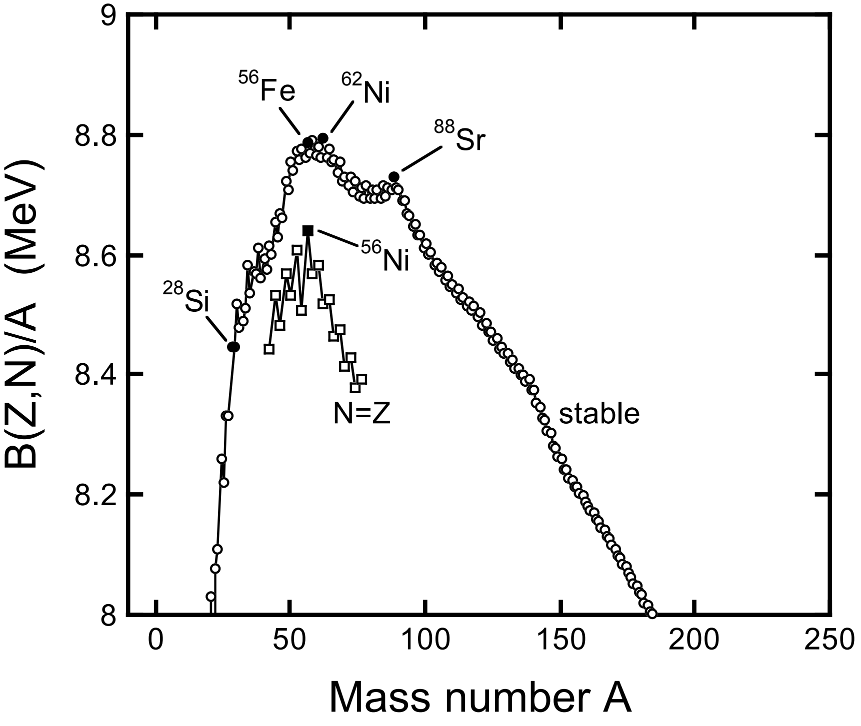


T=3.6 GK
$\rho=3\times10^{7}$ g/cm^{3}

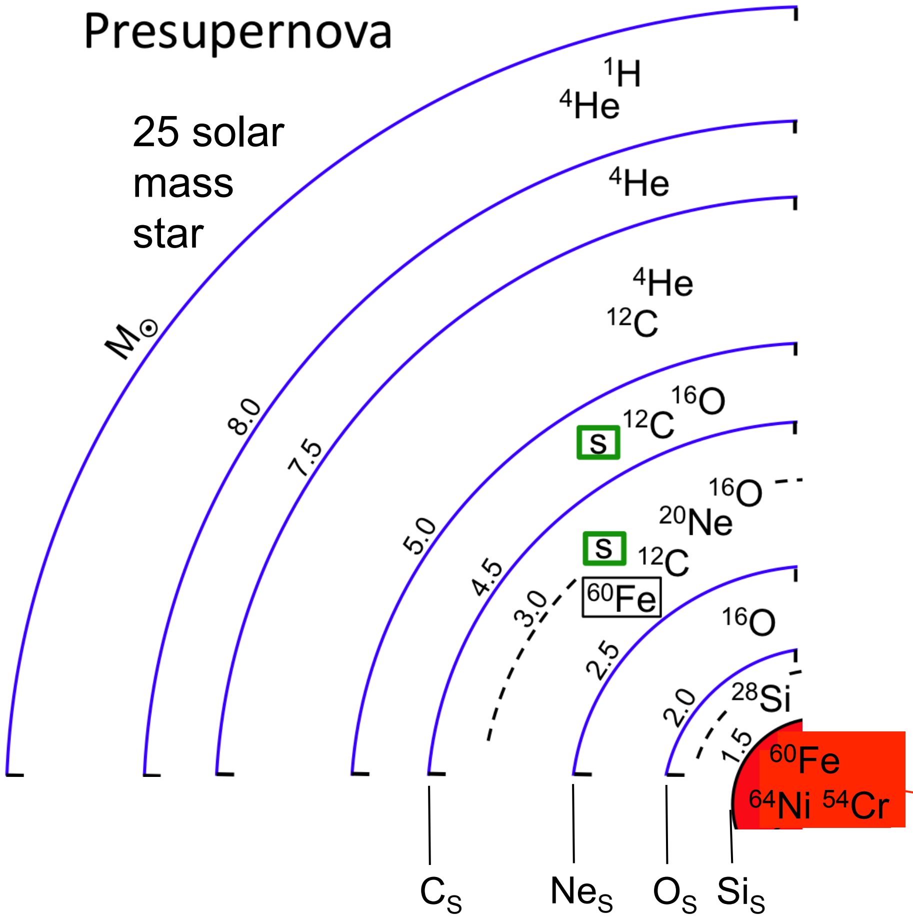
mediating reactions:
$^{42}Ca(\alpha,\gamma)^{46}Ti$
$^{41}K(\alpha,p)^{44}Ca$
$^{45}Sc(p,\gamma)^{46}Ti$

Experimental binding energies per nucleon





Onion Shell Structure of Massive Star at Instant Before Core Collapse

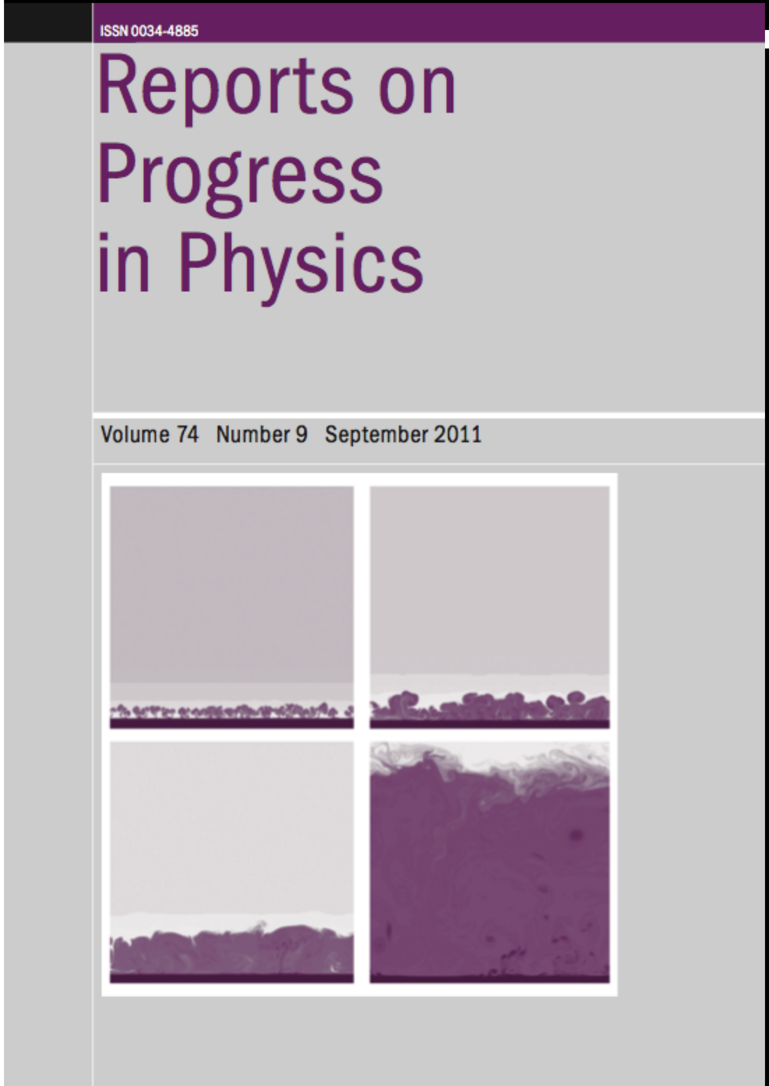


- we discussed burning in the core
- burning also takes place in thin regions (burning shells) at the interface of different compositional layers
- each burning shell migrated outward to the position indicated by blue lines

25M$_{sol}$, solar metallicity star:
η=0.13
T$_c$=5.5e^9 K
ρ_c=1.6e^9 g/cm^3

J. Jose & C. Iliadis, “The Unfinished Quest for the Origin of the Elements”, Rep. Prog. Phys. 74, 096901 (2011)

Recent review article

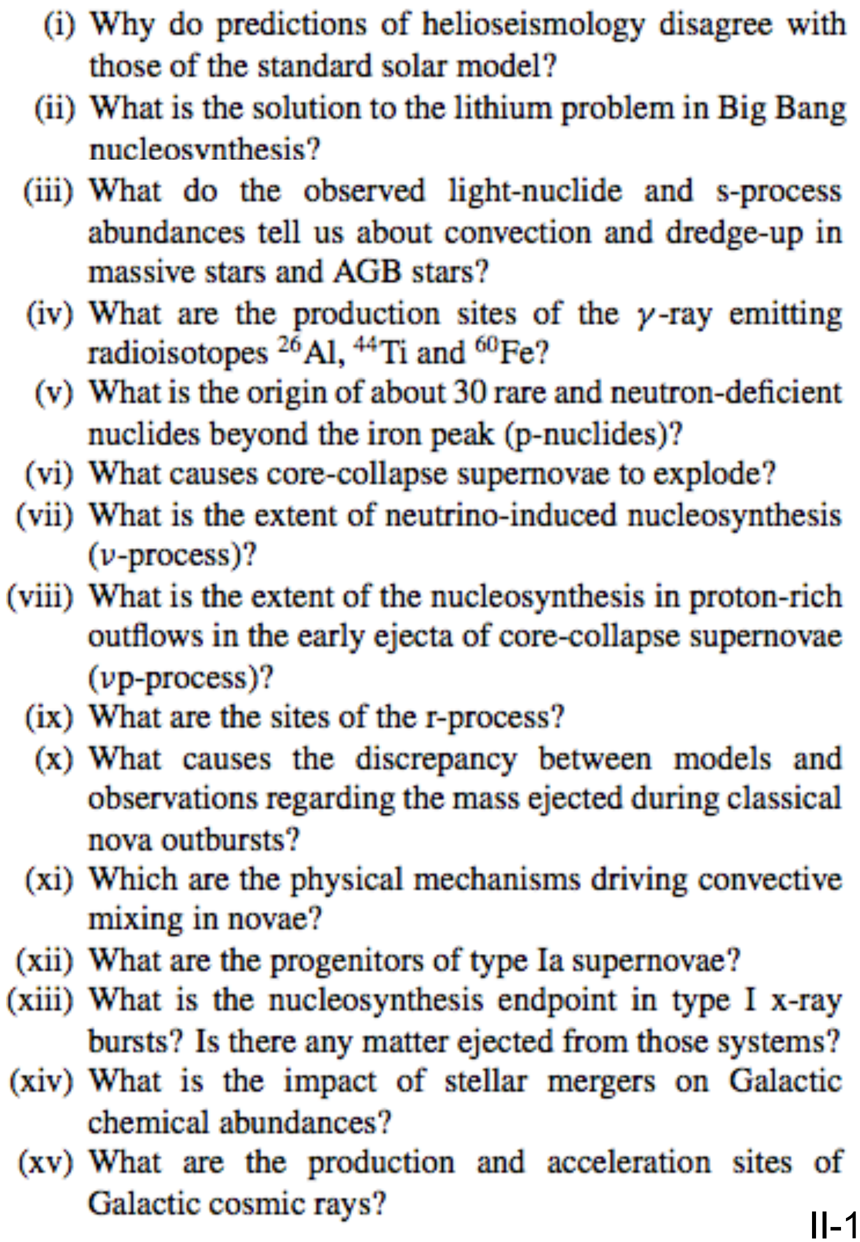


ISSN 0034-4885

Reports on
Progress
in Physics

Volume 74 Number 9 September 2011

Jose & Iliadis, “Nuclear Astrophysics: the Unfinished Quest for the Origin of the Elements”, Reports on Progress in Physics 74, 096901 (2011)



(i) Why do predictions of helioseismology disagree with those of the standard solar model?
(ii) What is the solution to the lithium problem in Big Bang nucleosynthesis?
(iii) What do the observed light-nuclide and s-process abundances tell us about convection and dredge-up in massive stars and AGB stars?
(iv) What are the production sites of the γ-ray emitting radioisotopes ^{26}Al, ^{44}Ti and ^{60}Fe?
(v) What is the origin of about 30 rare and neutron-deficient nuclides beyond the iron peak (p-nuclides)?
(vi) What causes core-collapse supernovae to explode?
(vii) What is the extent of neutrino-induced nucleosynthesis (ν-process)?
(viii) What is the extent of the nucleosynthesis in proton-rich outflows in the early ejecta of core-collapse supernovae (νp-process)?
(ix) What are the sites of the r-process?
(x) What causes the discrepancy between models and observations regarding the mass ejected during classical nova outbursts?
(xi) Which are the physical mechanisms driving convective mixing in novae?
(xii) What are the progenitors of type Ia supernovae?
(xiii) What is the nucleosynthesis endpoint in type I x-ray bursts? Is there any matter ejected from those systems?
(xiv) What is the impact of stellar mergers on Galactic chemical abundances?
(xv) What are the production and acceleration sites of Galactic cosmic rays?