Testing General Relativity

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Subjects

- Instant General Relativity
- Historical tests
- The Equivalence principle
- Gravitomagnetic effects
  - Gyroscopes
    - Experiments in space
  - Ringlasers
    - Terrestrial experiments: GINGER
  - Atomic interferometry

General Relativity is geometry

- The universe is made out of two basic ingredients with different properties: space-time and matter/energy
- Space-time is a four-dimensional Riemannian manifold with Lorentzian signature
- The presence of matter/energy induces curvature in the manifold
- The curvature of space-time is what is commonly called the gravitational field
- The effects of gravity are described by the geometric properties of the curved manifold

The basic tools of GR

The relevant physical quantities in GR are expressed by tensors

↔ General covariance of the equations of physics

A basic ingredient is the (squared) line element (a scalar) of the world-line of a point classical particle:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

The metric tensor and its descendants

The metric tensor $g_{\mu\nu}$ incorporates the geometric properties of the manifold

$g_{\mu\nu}$ ⟶ $\Gamma_{\mu\nu}{}^{\lambda}$: the connection (not a tensor)

$R^{\alpha}{}_{\beta\mu\nu}$: the Riemann (curvature) tensor

$R_{\mu\nu}$: the Ricci tensor ⟶ R: the scalar curvature

$G_{\mu\nu}$: the Einstein tensor

Einstein’s equations

The gravitational interaction (the configuration of space-time) is governed by a tensor equation

$$G_{\mu\nu} = \frac{\kappa}{2} T_{\mu\nu}$$

Einstein tensor (geometry)

Energy momentum tensor (matter/energy)

Coupling parameter $\kappa = 16\,\pi \frac{G}{c^4}$

What can we measure?

The exterior Schwarzschild solution

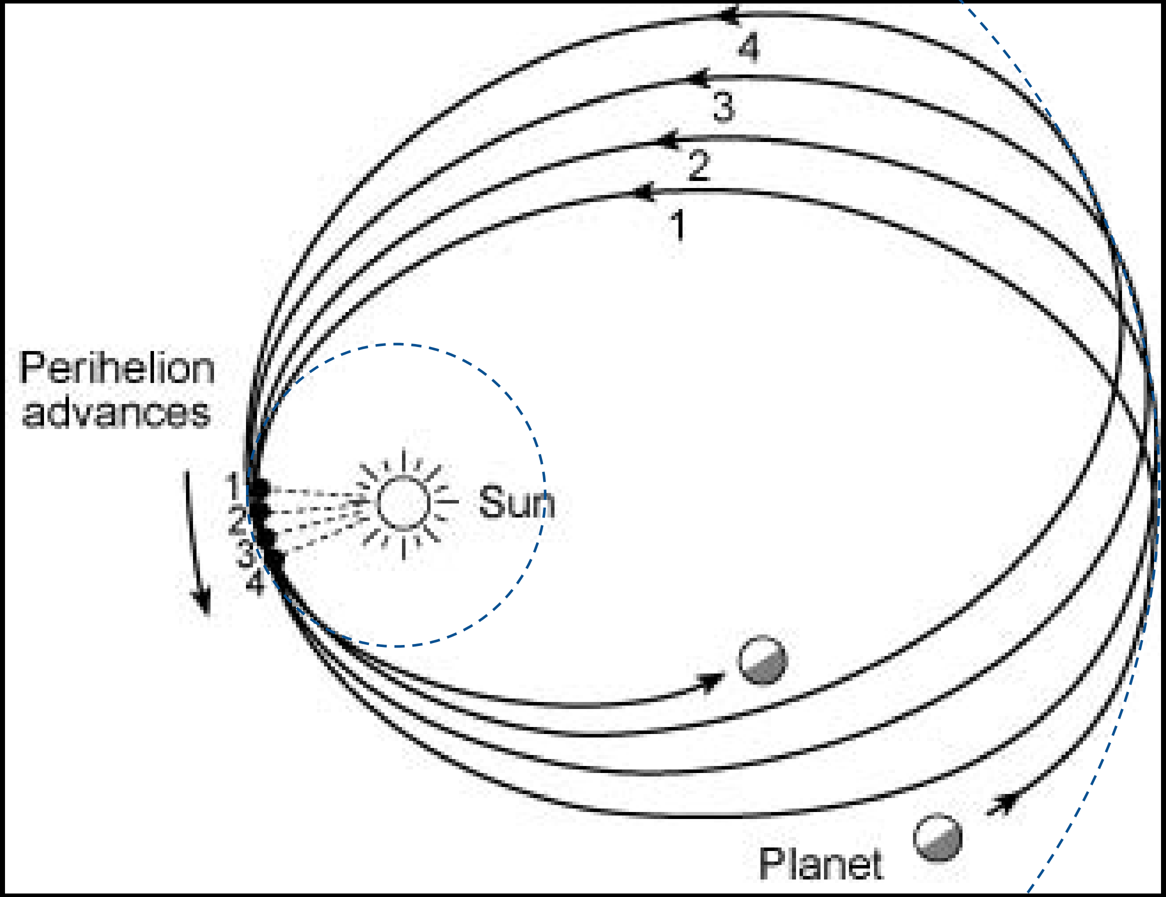
From the Einstein equations + spherical symmetry in space and independence from time

$$ds^2 = \left(1 - 2\frac{\mu}{r}\right)c^2 dt^2 - \frac{1}{\left(1 - 2\frac{\mu}{r}\right)} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2 \vartheta \, d\phi^2$$

Schwarzschild ‘polar’ coordinates

$$\mu = \frac{G}{c^2} M$$

A bounded orbit around a central mass

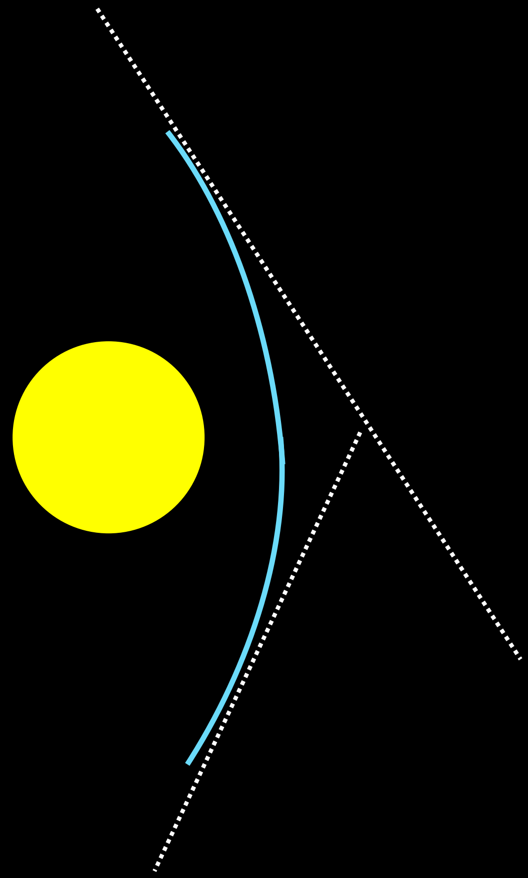


Perihelion advance per revolution

$$\Delta\phi \cong 24\pi^3 \frac{a^2}{T^2 c^2 \left(1-e^2\right)}$$

$$\cong 6\pi \frac{GM}{c^2 L}$$

Gravitational lensing



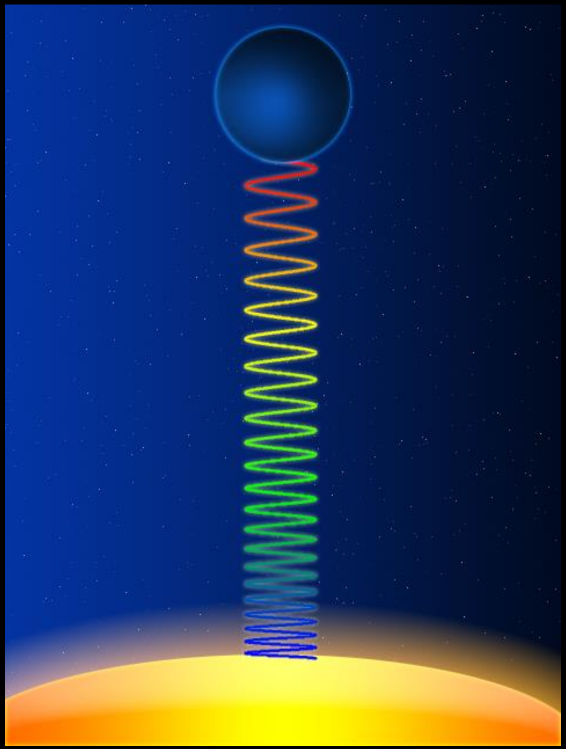
$$\delta\vartheta \simeq 4\frac{GM}{c^2 R_S}$$

Gravitational redshift

$$\mathbf{k}\cdot\mathbf{u} = g_{\mu\nu}k^{\mu}u^{\nu} = \text{constant (along a geodesic)}$$

(Four)-wavevector

Observer's four-velocity



$$\delta\nu = \left(\sqrt{\frac{g_{00}}{g_{00}{}'}} - 1\right)\nu$$

Gravitational redshift from stars and on earth

$$\frac{\delta\nu}{\nu} \cong \frac{G}{c^2}\left(\frac{M_\otimes}{R_\otimes} - \frac{M_s}{R_s}\right)$$

'Surface to surface'

$$\frac{\delta\nu}{\nu} \cong -\frac{G}{c^2}\frac{M_\otimes}{R_\otimes^{\,2}}h = -\frac{gh}{c^2}$$

Close to the ground

$$\frac{\delta\nu}{\nu} \cong -1.09\times 10^{-16}\,h$$

Historical tests

- Precession of the perihelion of Mercury
  - 1915
    - Known value: 43"/Julian century (residual after subtracting various effects from the observed value)
    - Value computed by Einstein: 43"/Julian century
- Lensing by the Sun
  - Theoretical value according to GR (1915) ~ 1.75 "
  - Observed by Eddington (1919)

$$\delta\phi = 4\frac{GM}{c^2 b}$$

- Gravitational redshift
  - Recognized in the spectral lines of Sirius B (W. S. Adams, 1925)
  - Measured on Earth: Pound and Rebka experiment (1959)

Present challenges

- (Cosmology)
- Celestial mechanics
- Lensing
- Gravitational waves
- Equivalence principle
- Rotation effects (gravitomagnetism)

Extrasolar celestial mechanics

- Compact binary systems
  - Pulsar plus compact star (White Dwarf, Neutron Star, Black Hole)
  - The Double Pulsar (PSR J0737-3029)
  - A few AU interstellar distance
  - Strong gravitational effects
- Pulsars in globular clusters
- Sagittarius A* and the surrounding stars

- Periapsis precession
  - PSR B1913+16 (Hulse and Taylor pulsar): 4.2°/year (compatible with GR)
  - PSR J0737-3029 (double pulsar): 16.9°/year
- Geodetic (de Sitter) precession
  - PSR J0737-3029B (double pulsar):
    - GR (5.0734±0.0007)°/year
    - measured $(4.77^{+0.66}_{-0.65})$°/year
  - PSR J1141–6545 (relativistic precession of the spin of the pulsar)
    - Detected (poor accuracy)

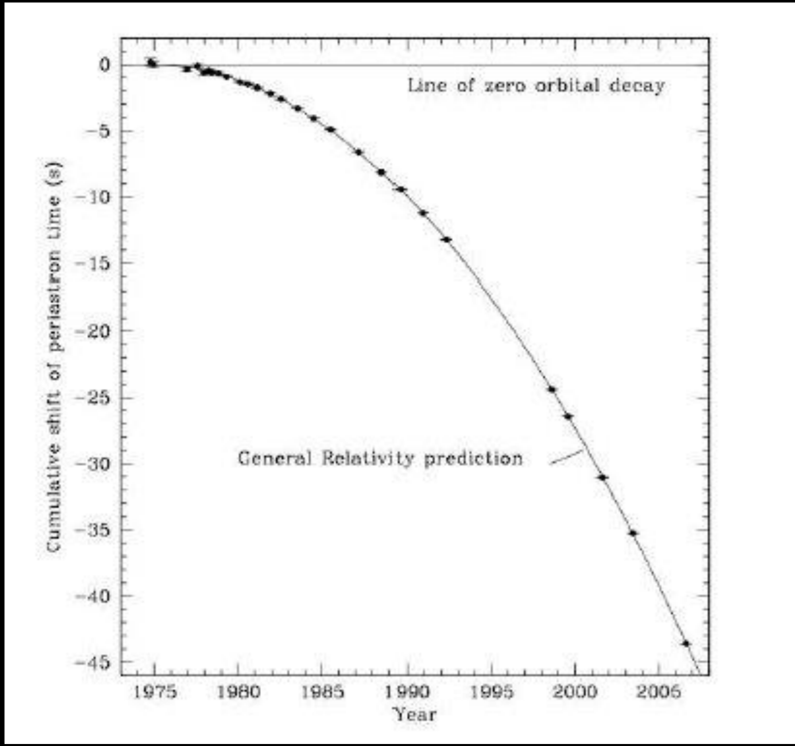
A. Possenti and M. Burgay, private communication (2011)

Period decay and GW emission

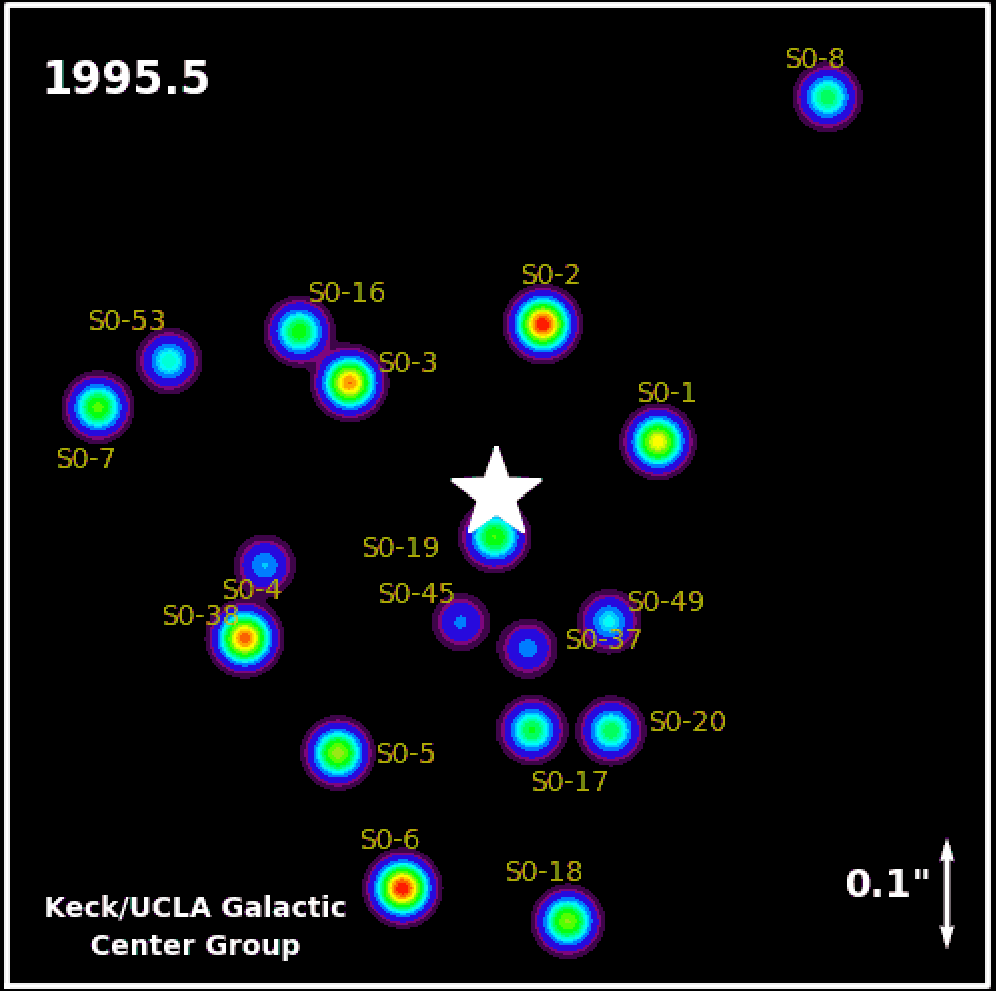
- 0.2% agreement with the quadrupole GW emission rate. PSR B1913+16 (Hulse and Taylor pulsar), after 33 years of data taking.

$$\dot{P} \cong -\frac{192\pi}{5}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)\left(1-e^2\right)^{-7/2}\left(\frac{2\pi\mathcal{M}}{P}\right)^{5/3}$$

$$\mathcal{M}=\frac{\left(m_1 m_2\right)^{3/5}}{\left(m_1+m_2\right)^{1/5}}$$



An interesting “laboratory”: Sagittarius A*



Gravitational field of a moving mass

Global line element in general coordinates (remote inertial observer)

$$ds^2 = g_{00}c^2dt^2 + g_{ij}dx^i dx^j + 2g_{0i}c\,dt\,dx^i$$

Sign depends on the sense of motion

«Acceleration» of a freely falling object (geodetic motion)

$$\frac{d^2x^i}{ds^2} = -\Gamma^i{}_{00}\left(u^0\right)^2 - \Gamma^i{}_{jk}u^j u^k - 2\Gamma^i{}_{0j}u^0 u^j$$

Christoffel symbols

$$u^i = \frac{dx^i}{ds}$$

Gravito-electric (→ Newtonian) acceleration

'Viscous' term

Gravito-magnetism

$$-2\Gamma^{1}{}_{0j}u^{0}u^{j} = -g^{1\varepsilon}\left(\frac{\partial g_{0\varepsilon}}{\partial x^{j}} + \frac{\partial g_{\varepsilon j}}{\partial x^{0}} - \frac{\partial g_{0j}}{\partial x^{\varepsilon}}\right)u^{0}u^{j}$$

Stationarity condition: $g_{\mu\nu}$'s do not depend on time

$$-2\Gamma^{i}{}_{0j}u^{0}u^{j} = -g^{ii}\left(\frac{\partial g_{0i}}{\partial x^{j}} - \frac{\partial g_{0j}}{\partial x^{i}}\right)u^{0}u^{j}$$

$$g_{0i} \rightarrow h_{i} : gravitomagnetic\ (three)vector\ \ potential$$

The ‘Lorentz force’

$$\vec{\nabla} \wedge \vec{h} = \frac{2}{c}\vec{B}_g$$

Gravito-magnetic field

$$v_i << c \rightarrow u_0 \approx 1; \quad u_i \approx \frac{v_i}{c}$$

$$F_i \cong 2m\left(v_j B_k - v_k B_j\right) \longrightarrow \vec{F} \cong 2m\vec{v} \wedge \vec{B}_g$$

Steadily rotating central mass

$$ds^2 = g_{00}c^2dt^2 + g_{rr}dr^2 + g_{\vartheta\vartheta}d\vartheta^2 + g_{\phi\phi}d\phi^2 + 2g_{0\phi}cdtd\phi$$

$g_{\mu\nu}$'s independent from both *t* and *φ*

Weak field

$$ds^2 \cong \left(1 - 2\frac{\mu}{r}\right)c^2dt^2 - \left(1 + 2\frac{\mu}{r} + \ldots\right)dr^2 - r^2d\vartheta^2 - r^2\sin^2\vartheta\, d\phi^2$$

$$+4\frac{j}{r^2}r\sin\vartheta\, cdtd\phi$$

$$j = \frac{G}{c^3}J$$

Explicit gravitomagnetic field

$$h_\phi \cong 2\frac{j}{r^2}\sin\vartheta = 2\frac{G}{c^3}\frac{J}{r^2}\sin\vartheta$$

$$\vec{h} \cong -2c\frac{\vec{j}\wedge\vec{r}}{r^3}$$

Dipole

$$\vec{B}_g \cong -2\frac{G}{c^2 r^3}\left(\vec{J} - 3\left(\vec{J}\cdot\hat{r}\right)\hat{r}\right)$$

Expected testable effects

- Torque on massive gyroscopes
- Asymmetric propagation along closed paths (in space)

Relativistic precession of a gyroscope

Angular momentum of the gyro

$$\frac{d\vec{S}}{d\tau} = \vec{\Omega} \wedge \vec{S}$$

$$\vec{\Omega} \equiv -\frac{1}{2c^2}\vec{v} \wedge \vec{a} + \frac{3}{2}\vec{v} \wedge \vec{\nabla}\Phi - \frac{c}{2}\vec{\nabla} \wedge \vec{h}; \qquad h_i = g_{0i}$$

Thomas precession

de Sitter precession

Drag of the inertial frames
(Lense-Thirring effect)

Observations and experiments

- Massive spinning stars in binaries (especially the double pulsar) → results compatible with GR
- Laser ranging of the orbit of the moon → results compatible with GR
- Precession of the orbits of terrestrial satellites
- Precession of a freely falling gyroscope in the gravitational field of the earth

A freely falling gyroscope around the Earth

$$\frac{G}{c^2}\frac{M_\oplus}{r}$$

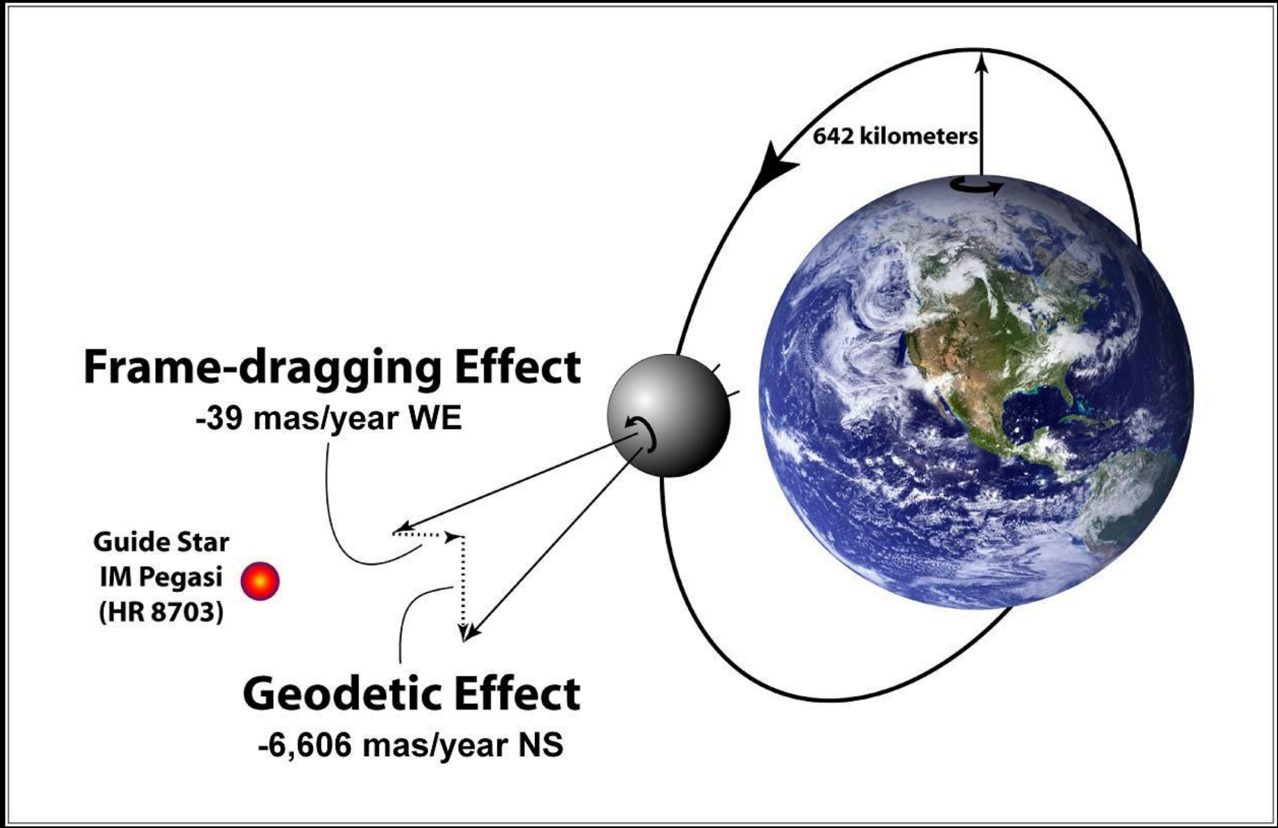
0

$$\vec{\Omega} \equiv -\frac{1}{2c^2}\vec{v}\wedge\vec{a} + \frac{3}{2}\vec{v}\wedge\vec{\nabla}\Phi - \frac{c}{2}\vec{\nabla}\wedge\vec{h}$$

$$\vec{h} \cong -2\frac{G}{c^3}\frac{\vec{J}_\oplus\wedge\vec{r}}{r^3}$$

Angular momentum of the Earth

Experiments: Gravity Probe-B

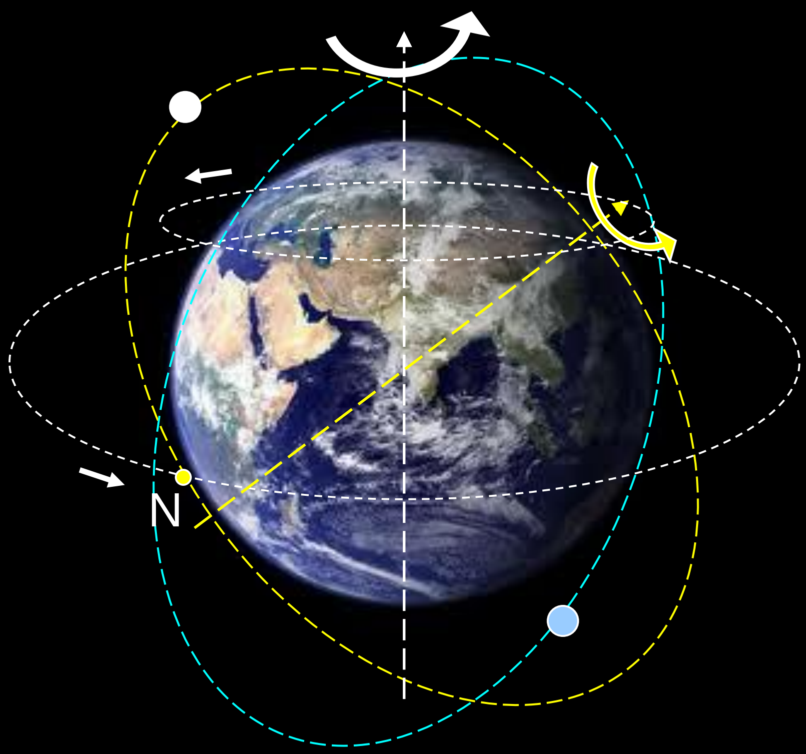


C.F.W. Everitt et al., PRL 106, 221101 (2011)

Results

- GR geodetic (de Sitter) precession confirmed within ±0.28% (previous best result ±0.7% from lunar laser ranging)
- GR frame dragging confirmed within ±19% (previous best result ±10% from the laser ranging of the LAGEOS satellites)
- Final accuracy limited by an unexpected patch effect

The LAGEOS orbit precession



The experiment

- Systematic analysis of the reconstructed orbits
- Good model of the gravitoelectric field of the Earth needed
- First results (1998): Lense-Thirring verified within 30%
- 2004-2010 results, using a model of the gravity of the Earth based on the results of the GRACE experiment: LT verified within 10%

I. Ciufolini et al., *Testing Gravitational Physics with Satellite Laser Ranging,* to appear on *Eur. Phys. J. Plus* (2011)

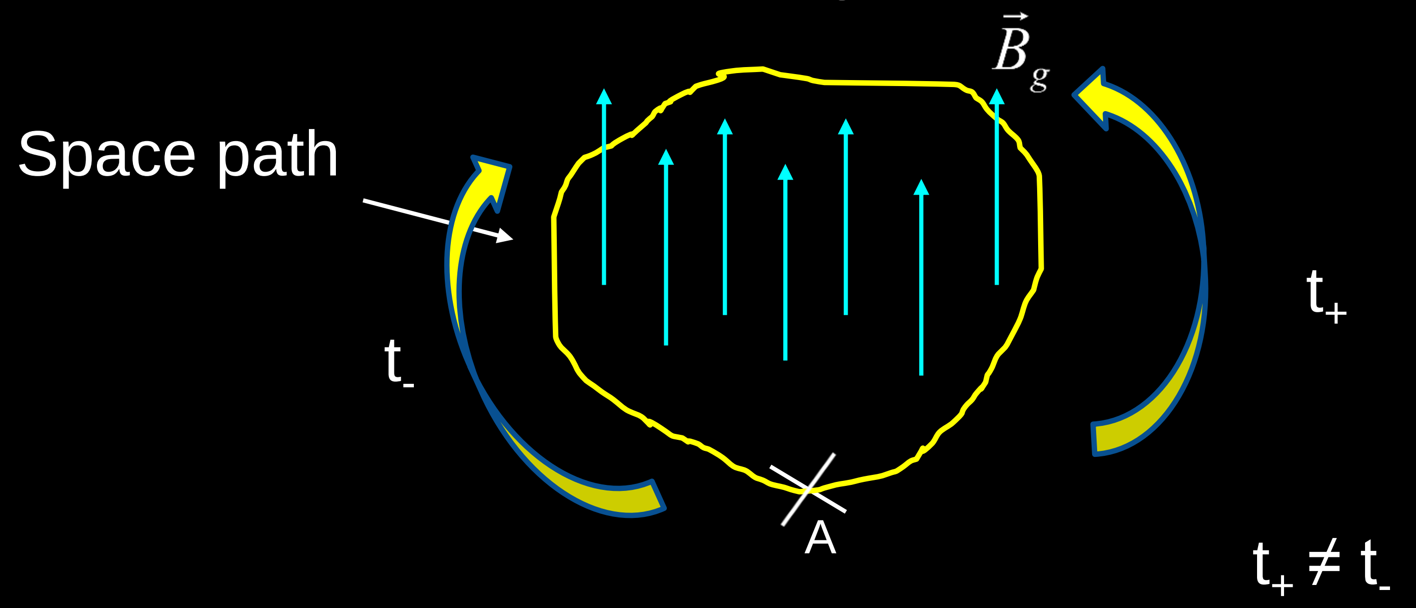
The LARES mission

- A compact sphere (36.4 cm diameter) made of a tungsten alloy; 96 retroflectors.
- Almost spherical orbit at a hight of 1450 km.
- Purpose: to allow for an LT effect measurement within a few %.
- Flying since February 2012

Courtesy of Ignazio Ciufolini

Asymmetric propagation

$$ds^2 = g_{00}c^2dt^2 + g_{ij}dx^i dx^j + 2g_{0i}cdtdx^i$$



Times of flight

$$g_{00}\left(u^0\right)^2 + g_{ij}u^i u^j + 2g_{0i}u^0u^i =$$

→ 1

→ 0 light

$$t_+ = -\frac{1}{c}\oint_+ \frac{g_{0i}}{g_{00}}dx^i + \frac{1}{c}\oint_+ \frac{\sqrt{g_{0i}{}^2 - g_{00}g_{ii}}}{g_{00}}dx^i \qquad dx^i > 0$$

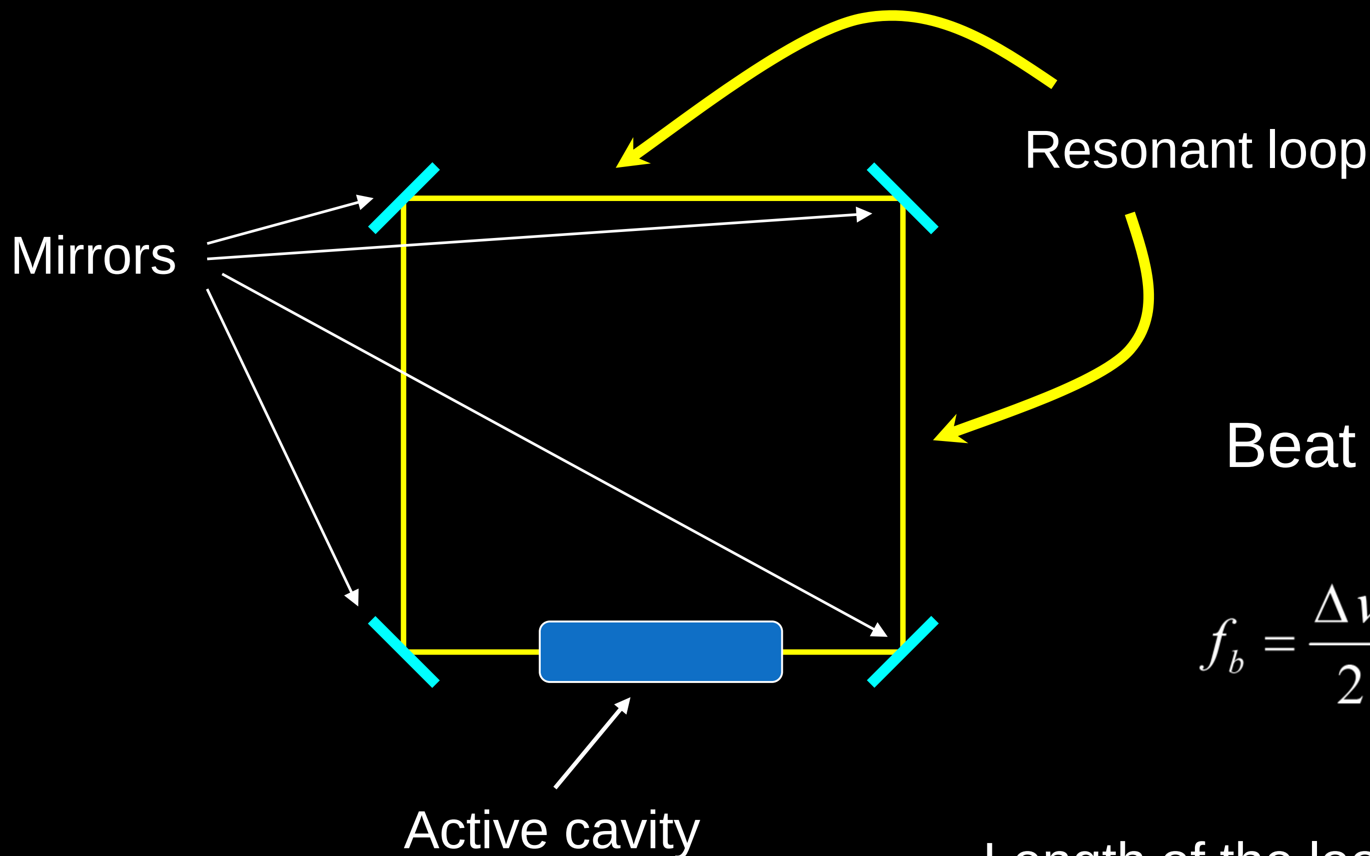
$$t_- = -\frac{1}{c}\oint_- \frac{g_{0i}}{g_{00}}dx^i - \frac{1}{c}\oint_- \frac{\sqrt{g_{0i}{}^2 - g_{00}g_{ii}}}{g_{00}}dx^i \qquad dx^i < 0$$

Time of flight difference

$$\Delta t = t_+ - t_- = -\frac{2}{c}\oint_+ \frac{g_{0i}}{g_{00}} dx^i$$ Global coordinated time

$$\Delta \tau = \tau_+ - \tau_- = -\frac{2}{c}\sqrt{g_{00}}\oint_+ \frac{g_{0i}}{g_{00}} dx^i$$ Proper laboratory time

A ringlaser



Beat frequency

$$f_b = \frac{\Delta\nu}{2} = \frac{c^2}{2\lambda P}\Delta t$$

Length of the loop

Where does the beat frequency come from?

integer

Steady state $t_+ = NT_+$ $t_- = NT_-$

periods of the waves

$$\Delta t = N\left(T_+ - T_-\right) = N\left(\frac{1}{\nu_+} - \frac{1}{\nu_-}\right) = N\frac{\Delta \nu}{\nu_+ \nu_-}$$

$$\cong N\frac{\Delta \nu}{\nu^2} = N\lambda\frac{\Delta \nu}{c^2}\lambda = P\frac{\Delta \nu}{c^2}\lambda$$

Earth-bound laboratory (lowest approximation order)

$$g_{0\phi} \cong \left(2\frac{j}{r} - r^2 \frac{\omega}{c} - 2\mu r \frac{\Omega}{c} \right) \sin^2 \vartheta$$

$$g_{00} \cong 1 - 2\frac{\mu}{r} - \frac{\omega^2 r^2}{c^2} \sin^2 \vartheta$$

$$\mu = G\frac{M_\oplus}{c^2} \approx 4.4 \times 10^{-3}\,\text{m}$$

$$j = G\frac{J_\oplus}{c^3} \approx 1.75 \times 10^{-2}\,\text{m}^2$$

Ω = angular velocity of the Earth
ω = angular velocity of the instrument
θ = colatitude of the laboratory

Expected signal

$\omega = \Omega$

$$\Delta \nu = 4\frac{A}{\lambda P}\Omega\left[\cos(\theta+\alpha) - 2\frac{\mu}{R}\sin\theta\sin\alpha + \frac{GI_{\oplus}}{c^2R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

Scale factor

$$\delta \nu = 4\frac{A}{\lambda P}\left[\vec{\Omega} - 2\frac{\mu}{R}\Omega\sin\theta\hat{u}_{\theta} + \frac{GJ_{\oplus}}{c^2R^3}\left(2\cos\theta\hat{u}_r + \sin\theta\hat{u}_{\theta}\right)\right]\cdot\hat{u}_n$$

Area of the loop

Sagnac

$\vec{\Omega}_G$

$\vec{\Omega}_B$

Orders of magnitude

$$\Omega = 7.2\times10^{-5}\ s^{-1}$$

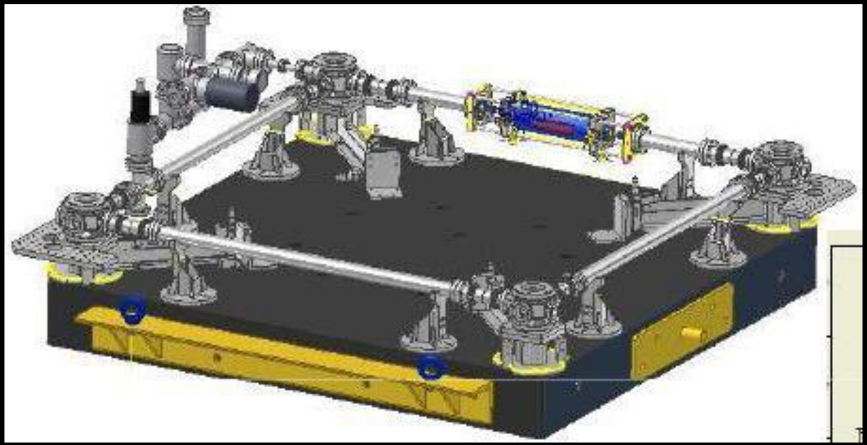
$$\Omega_G \approx \Omega_B \approx 10^{-9}\ \Omega$$

Actual gyrolasers

Commercial fiber optics gyroscope

Sensitivity: ~ 10^{-7} rad/s/$\sqrt{}$Herz

Research gyrolasers: G-Pisa



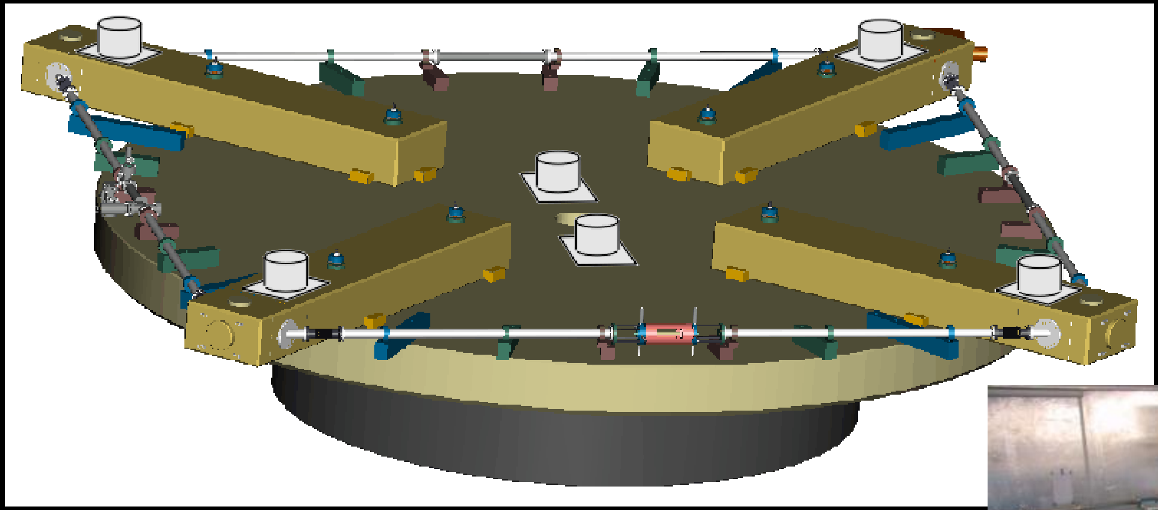
Side: 1.35 m
He-Ne laser
Granite support
Sensitivity: $10^{-10} \sim 10^{-9}$ rad/s/√Herz

The Cashmere Cavern near Christchurch (NZ)

Various configurations, side up to 20 m

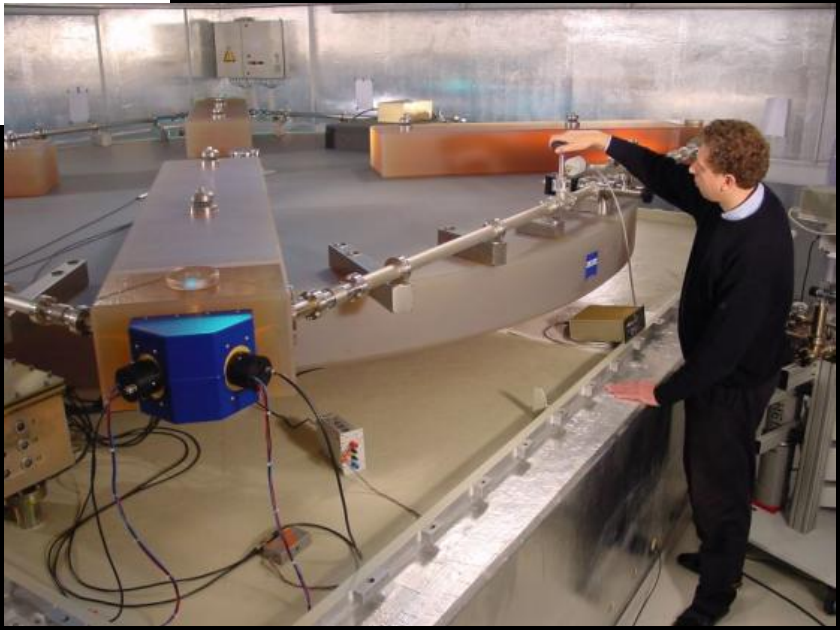
A triangular loop, 5 m side will be built

G instrument at the Geodätisches Observatorium Wettzell



Absolute sensitivity:

~ 10^{-12} rad/s/√Hz



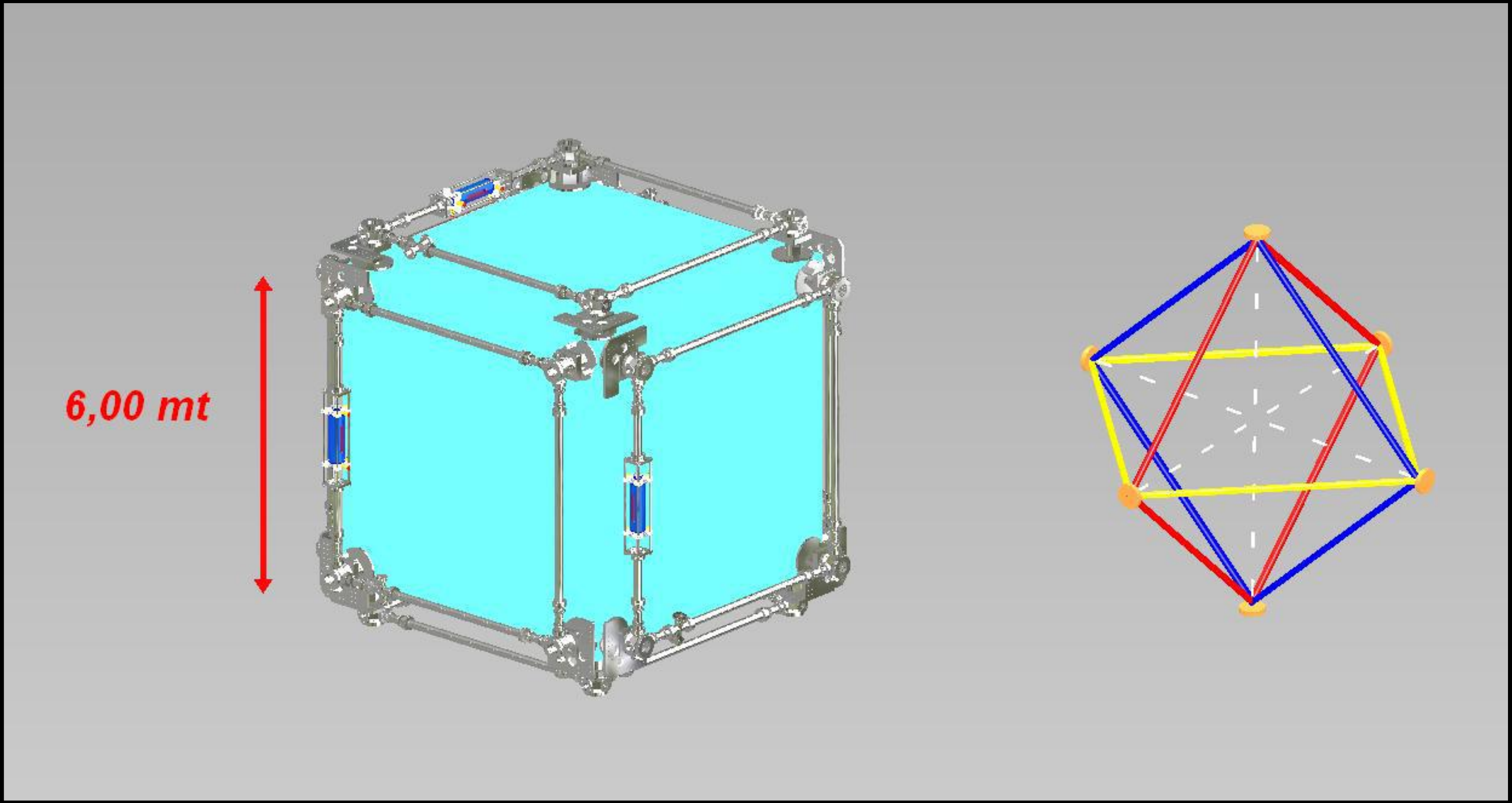
Side: 4 m
Power: 20nW
Zerodur support

GINGER: proposed and under development

- Tridimensional ring lasers array (three to six or more)
- Laser power: ~ 200 nW
- Quality factor of the cavity: $Q \geq 3\times10^{12}$
- Square loop, ≥6 m in side
- Underground location (LNGS)
- Purpose: to measure the LT effect with a 1% accuracy (one year integration time)

Bosi F. et al., Phys. Rev. D, vol. 84, p. 122002-1-122002-23 (2011)

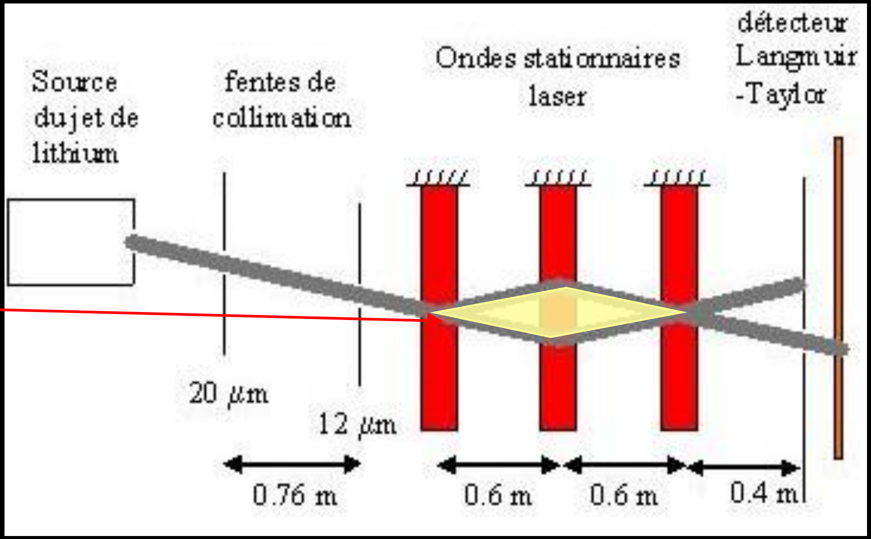
Configurations



GR and matter waves interferometry

Works like light as rotation sensor

Scale factor: $S = 4\frac{A}{\lambda P}$



Much smaller than with light

Cannot be too big

Rotations measured by means of phase shifts

Phase difference in a Sagnac-type interference experiment

$$\delta\phi = \frac{4\pi}{h} m A \hat{u}_n \cdot \vec{\Omega}$$

$$\frac{\text{Particles}}{\text{Light}} \rightarrow 10^{10}$$

$$\frac{\text{Matter interf}}{\text{Ring laser}} \rightarrow 10^{-4}$$

Problems: beam stability; duration of the experiment

Summing up

- Very good laboratories for GR:
  - Compact binaries
  - Central galactic area
  - Accuracy increasing with time
- Precision astrometry:
  - Telescopes in space; interferometric techniques
- Weak rotation effects:
  - Ring lasers; coming accuracy for LT effect: 1%