Discontinuities of multi-Regge amplitudes in the next-to-leading order

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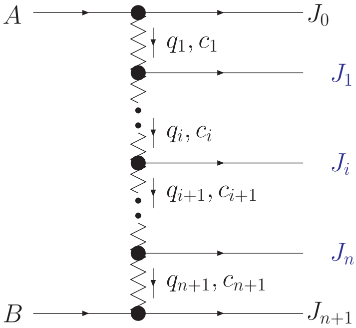
Contents

The BFKL approach is based on the multi-Regge form of MRK amplitudes.
This form can be represented by the picture



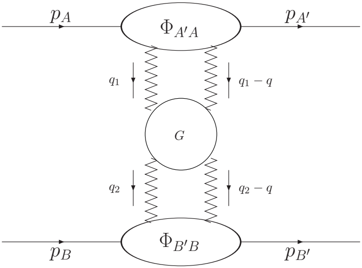
and written as

$$\Re \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1} \left(\prod_{i=1}^{n} \gamma_{c_i c_{i+1}}^{J_i}(q_i, q_{i+1}) \left(\frac{s_i}{s_0} \right)^{\omega(t_i)} \frac{1}{t_i} \right)$$

$$\frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{s_0} \right)^{\omega(t_{n+1})} \Gamma_{\tilde{B}B}^{c_{n+1}}$$

In the LLA only a gluon can be produced. In the NLA one has to account production of $Q\bar{Q}$ and GG jets.
The assumption of this form is extremely powerful since an infinite number of amplitudes is expressed in terms of the gluon Regge trajectory and several Reggeon vertices.
This simple form is correct only for real parts of the MRK amplitudes. Fortunately, only these parts are necessary for derivation of the BFKL eqution in the NLA.

The real parts of amplitudes are expressed through their imaginary parts, or discontinuities of amplitudes in s_i channels, so, it is possible to say that the discontinuities are are more important than the real parts. Remind that the BFKL Pomeron is determined by the discontinuity of elastic amplitude.

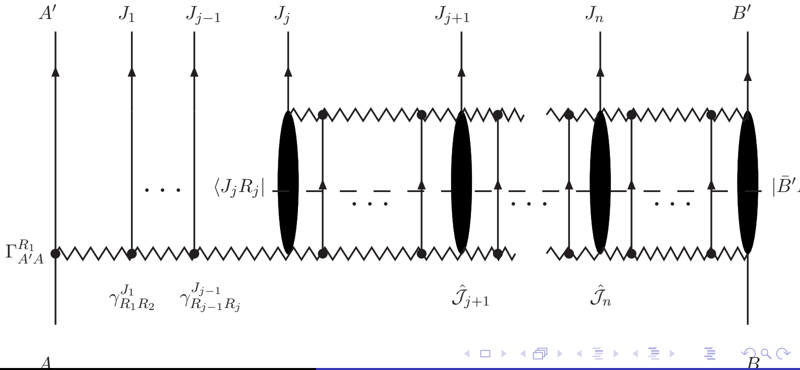


$$Im\,\mathcal{A}_{AB}^{A'B'} = \langle AA'|e^{Y\hat{\mathcal{K}}}|BB'\rangle,$$
$$\hat{\mathcal{G}} = e^{Y\hat{\mathcal{K}}},$$

$\hat{\mathcal{K}}$ is the BFKL kernel, Y is the energy evolution parameter, ($Y = \ln(s/s_0)$, s_0 is an energy scale), $\Phi_{A'A}$ and $\Phi_{B'B}$ are impact factors for particle-particle transitions.

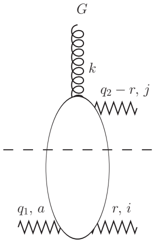
Definition of the discontinuities

The s-channel discontinuities of elastic amplitudes are the simplest ones. They are expressed in terms of particle-particle impact factors and the BFKL kernel.
Evidently, discontinuities of multi-particle amplitudes are more involved.
Schematically they are presented by the picture:



Definition of the discontinuities

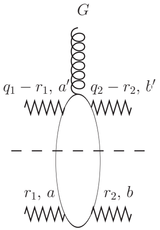
They contain two additional ingredients: impact factors for reggeon-gluon transition and gluon production contribution to the discontinuity.
The impact factors for reggeon-gluon transition describe transition of reggeized gluon into ordinary ones due to interaction with reggeized gluons:



(t-channel from left to right, s-channel from down to up)
dashed line denotes discontinuity of the reggeon $\rightarrow$ gluon scattering amplitude. The impact factor is given by the integral over squared invariant mass of particles on this discontinuity.

Definition of the discontinuities

Te gluon production contribution to the discontinuity describe *t*-channel propagation of two reggeized gluon with production of ordinary one:



(*t*-channel from left to right, *s*-channel from down to up) dashed line denotes discontinuity of the reggeon $\rightarrow$ gluon scattering amplitude.
These ingredients are expressed through effective vertices describing interaction of reggeized gluons with gluons and quarks. Now all of them are known with the NLO accuracy.

Bootstrap of the gluon Reggeization

Consideration of the s_i-channel discontinuities was performed initially for proof of the multi-Regge form of QCD amplitudes. Compatibility of the unitarity with the multi-Regge form leads to bootstrap relations connecting discontinuities of the amplitudes with products of their real parts and gluon trajectories:

$$\sum_{l=j+1}^{n+1} \Delta_{jl} - \sum_{l=0}^{j-1} \Delta_{lj} = \frac{1}{2}\left(\omega(t_{j+1}) - \omega(t_j)\right) \Re\, \mathcal{A}_{AB}^{A'B'+n} .$$

Here the s-channel discontinuities must be calculated by inserting the Reggeized form of amplitudes into the unitarity conditions.

Evidently, there is an infinite number of the bootstrap relations, because there is an infinite number of the amplitudes $\mathcal{A}_{AB}^{A'B'+n}$. It turns out that fulfillment of an infinite set of these relations guarantees the multiRegge form of scattering amplitudes.

Bootstrap of the gluon Reggeization

It occurs that an infinite number of the bootstrap relations is fulfilled if:
1. The impact factors for scattering particles satisfy the conditions

$$|A'A\rangle = \frac{g}{2}\Gamma_{A'A}|R_\omega(q)\rangle$$

where $q = p_A - p_{A'}$ and
2. $|R_\omega(q)\rangle$ is the universal (process independent) eigenstate of the kernel $\hat{\mathcal{K}}$ with the eigenvalue $\omega(t)$

$$(\hat{\mathcal{K}} - \omega(t))|R_\omega(q)\rangle$$

and the normalization

$$\frac{g^2 t N_c}{2(2\pi)^{D-1}}\langle R_\omega(q)|R_\omega(q)\rangle = \omega(t)\ , \quad t = q^2;$$

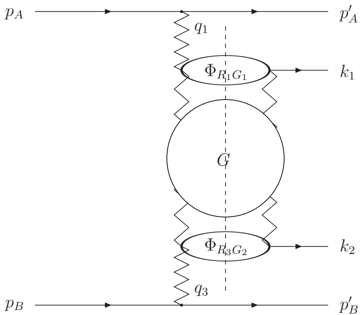
3. The Reggeon-gluon impact factors and the gluon production vertices satisfy the condition

$$\frac{gt_1}{2}\langle R_\omega(q_1)|\hat{\mathcal{G}} + \langle GR_1| = \frac{g}{2}\gamma^G(q_1, q_2)\langle R_\omega(q_2)|.$$

The conditions for the impact factors of scattering particles and the kernel were checked
M. Braun, G.P. Vacca,1999,
V.S.F., R. Fiore, M.I. Kotsky, A. Papa, 2000,
V.S.F, A. Papa, 2002
The last condition was checked recently, both in QCD
V.S.F., M.G. Kozlov, A. V. Reznichenko, 2011
V.S.F., M.G. Kozlov, A. V. Reznichenko, 2012
and in SYM
V.S.F., M.G. Kozlov, A. V. Reznichenko, 2013.

Contradiction with the BDS anzatz

There is almost evident contradiction of the expressions for s_i channel discontinuities with the BDS ansatz
Z. Bern, L.J. Dixon and V.A. Smirnov, 2005
for n gluon amplitudes with maximal helicity violation (MHV) in $N = 4$ supersymmetric Yang-Mills theory at $n \geq 6$. Indeed, let us consider the discontinuity of $A_{2\to 4}$ in the s_2- channel (s_2 is the invariant mass square of produced gluons)



Contradiction with the BDS anzatz

Let us denote

$$M_{2\to 2+n} = \frac{A_{2\to 2+n}}{A^{(B)}_{2\to 2+n}}.$$

Then

$$4(2\pi)^4\delta(\vec{q}_1 - \vec{k}_1 - \vec{k}_2 - \vec{q}_3)\Im\Delta_{s_2}\frac{M_{2\to 4}}{\Gamma_{BFKL}(t_1)\Gamma_{BFKL}(t_3)} =$$

$$= \left|\frac{s_1}{|\vec{q}_1||\vec{k}_1|}\right|^{\omega(t_1)}\left|\frac{s_3}{|\vec{k}_2||\vec{q}_3|}\right|^{\omega(t_3)}\frac{\langle G_1 R_1|e^{\hat{K}\ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)}|G_2 R_3\rangle}{\gamma^{G_1(B)}_{R_1 R_2}(\vec{q}_2^{\,2})^{-1}\gamma^{G_2(B)}_{R_2 R_3}},$$

where $\Gamma_{BFKL}(t)$- gluon-gluon-reggeon vertex at NLO, $\gamma^{G_1(B)}_{R_1 R_2}$ and $\gamma^{G_2(B)}_{R_2 R_3}$ – Born vertices for gluon production in reggeon transitions, $\langle G_1 R_1|$ and $|G_2 R_3\rangle$ – impact factors for reggeon-gluon transition.

Contradiction with the BDS anzatz

In the MRK, energy dependence of the BDS ansatz for the $M_{2\to 2+n}$ is given by the Regge factors,
J. Bartels, L. N. Lipatov and A. Sabio Vera, 2009

$$M_{2\to 2+n} \sim |s_1|^{\omega(t_1)}\, |s_2|^{\omega(t_2)}\, |s_3|^{\omega(t_3)}\,.$$

Therefore, for agreement with the BDS one needs

$$\langle G_1 R_1 | e^{\hat{K}\ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3\rangle \sim |s_2|^{\omega(t_2)}\,,$$

i.e. impact factor for reggeon-gluon transition must be the eigenfunction of the BFKL kernel with the eigenvalue equal to the gluon trajectory. But it follows from the bootstrap conditions

$$\hat{K}|A'A\rangle = \omega(t)|A'A\rangle$$

where $|A'A\rangle$ is the particle-particle impact factor, which evidently differs from the reggeon-particle impact factor.
Evidently, this evident contradiction exists only for $A_{2\to 2+n}$ at $n \geq 2$.

Explicit expression for $A_{2\to 4}$ discontinuity

For calculation of $\langle G_1 R_1 | e^{\hat{K} \ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3 \rangle$ it is convenient to transfer from the BFKL kernel $\hat{K}$ to the modified BFKL kernel $\hat{K}_m$ introduced in
J. Bartels, L. N. Lipatov and A. Sabio Vera, 2009

,

$$\hat{K}_m = \hat{K} - \omega(t).$$

An evident advantage of this kernel is non-singular infrared behavior.
Not so evident, but even more important is conformal invariance in the momentum space. In the LO this invariance is almost obvious J. Bartels, L. N. Lipatov and A. Sabio Vera, 2009
. Existence of the conformal invariant representation of the NLO kernel was proved recently
V. S. F. , R. Fiore, L. N. Lipatov and A. Papa, 2013.

Explicit expression for $A_{2\to4}$ discontinuity

The impact factors for reggeon-gluon transitions in this scheme with conformal invariant energy evolution parameter $\frac{s_2\vec{q}_2^{\,2}}{|\vec{q}_1||\vec{q}_3||\vec{k}_1||\vec{k}_2|}$ was also recently found
V. S. F. , R. Fiore, 2014.
They can be written with the NLO accuracy as

$$\langle G_1 R_1|\mathbf{G}_1\mathbf{G}_2\rangle \;=\; -\sqrt{2}g^2\delta(\vec{q}_1-\vec{k}_1-\vec{r}_1-\vec{r}_2)\frac{q_1^- r_1^+}{(q_1-r_1)^+}[1+\bar{g}^2 I(z_1)]$$
$$\times\left[1+\bar{g}^2\left(-\frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-\frac{(\vec{k}^{\,2})^\epsilon}{\epsilon^2}+2\zeta(2)\right)\right],$$

$$\langle \mathbf{G}_1'\mathbf{G}_2'|G_2 R_3\rangle \;=\; \sqrt{2}g^2\delta(\vec{r}_1{}'+\vec{r}_2{}'-\vec{q}_3-\vec{k}_2)\frac{q_3^+ r_1'^-}{(r_1'-q_3)^-}[1+\bar{g}^2 I^*(z_2)]$$
$$\times\left[1+\bar{g}^2\left(-\frac{1}{2}\ln^2\left(\frac{\vec{q}_3^{\,2}}{\vec{q}_2^{\,2}}\right)-\frac{(\vec{k}^{\,2})_2^\epsilon}{\epsilon^2}+2\zeta(2)\right)\right],$$

where $z_1 = -q_1^+ r_2^+/(k_1^+ r_1^+)$, $z_2 = q_3^+ r_2'^+/(k_2^+ r_1'^+)$

Explicit expression for $A_{2\to 4}$ discontinuity

$$I(z) = (1-z)\left(\ln\left(\frac{|1-z|^2}{|z|^2}\right)\ln\left(\frac{|1-z|^4}{|z|^6}\right) - 6Li_2(z) + 6Li_2(z^*)\right.$$

$$\left. -3\ln|z|^2\ln\frac{1-z}{1-z^*}\right) - 4\ln|1-z|^2\ln\frac{|1-z|^2}{|z|^2} - 3\ln^2|z|^2 \; .$$

In terms of eigenstates $|\nu, n\rangle$ and eigenvalues $\omega(\nu, n)$ of $\hat{K}_m$

$$\langle G_1 R_1 | e^{\hat{K}\ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3\rangle = \left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)^{\omega(t_2)}$$

$$\times \sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty} d\nu \, \langle G_1 R_1|\nu, n\rangle e^{\omega(\nu,n)\ln\left(\frac{s_2\vec{q}_2^{\,2}}{|\vec{q}_1||\vec{q}_3||\vec{k}_1||\vec{k}_2|}\right)} \langle \nu, n| G_2 R_3\rangle \; .$$

The eigenfunctions are well known. The eigenvalues $\omega(\nu, n)$ with the NLO accuracy were found two years ago V. S. F. , L. N. Lipatov, 2012.

Explicit expression for $A_{2\to 4}$ discontinuity

In an explicit form

$$\langle G_1 R_1 | e^{\hat{K}\ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3 \rangle = \delta(\vec{q}_1 - \vec{k}_1 - \vec{k}_2 - \vec{q}_3) g^2 \gamma_{R_1 R_2}^{G_1(B)} \gamma_{R_2 R_3}^{G_2(B)}$$

$$\times \left[1 + \bar{g}^2 \left(-\frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) - \frac{(\vec{k}_1^{\,2})^\epsilon}{\epsilon^2} - \frac{1}{2}\ln^2\left(\frac{\vec{q}_3^{\,2}}{\vec{q}_2^{\,2}}\right) - \frac{(\vec{k}_2^{\,2})^\epsilon}{\epsilon^2} + 4\zeta(2) \right) \right]$$

$$\times \frac{1}{2} \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} d\nu \, e^{\omega(\nu,n)\ln\left(\frac{s_2 \vec{q}_2^{\,2}}{|\vec{q}_1||\vec{q}_3||\vec{k}_1||\vec{k}_2|}\right)} w^{\frac{n}{2}+i\nu} (w^*)^{-\frac{n}{2}+i\nu}$$

$$\int \frac{dz_1}{\pi |z_1|^2} \frac{1}{1 - z_1} \left(1 + \bar{g}^2 I(z_1)\right) z_1^{\frac{n}{2}+i\nu} (z_1^*)^{-\frac{n}{2}+i\nu}$$

$$\int \frac{dz_2}{\pi |z_2|^2} \frac{1}{1 - z_2^*} \left(1 + \bar{g}^2 I^*(z_2)\right) (z_2^*)^{\frac{n}{2}-i\nu} z_2^{-\frac{n}{2}-i\nu}$$

where $w = k_2^+ q_1^+ / (k_1^+ q_3^+)$.

Summary

- Formal expressions for s_i-channel discontinuities of MRK amplitudes in the NLA are known since 2006.
- The discontinuities are in an evident contradiction with the BDS anzatz for $2 \to 2 + n$ amplitudes at $n \geq 2$.
- Now all ingredients entering in these expressions are known.
- Fulfilment of all bootstrap conditions is proved.
- Knowledge of all ingredients entering in the s_i-channel discontinuities permits to obtain the discontinuities explicitly.
- The discontinuity in invariant mass of two produced gluons is calculated in the NLA in planar $N = 4$ SYM.
- It's compatibility with the BDS ansatz corrected by the remainder factor is under consideration.