THE GREEN FUNCTION FOR THE DISCRETE BFKL POMERON

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Green function for BFKL equation with running coupling (L.O.)

$$\int dt'' \left[\omega\delta(t-t'') - \alpha_s(t)\,\mathcal{K}(t,t'')\right] \mathcal{G}_\omega(t'',t') \;=\; \delta(t-t')$$

$$t \;\equiv \ln\left(\frac{k_T^2}{\Lambda^2}\right).$$

$$\int dt'\,\mathcal{K}(t,t')\phi_\nu(t') \;=\; \chi(\nu)\phi_\nu(t)$$

$$\chi(\nu) \;=\; 2\Re e\left\{\Psi(1) - \Psi\left(\frac{1}{2}+i\nu\right)\right\}$$

Simplified Model - (diffusion approximation)

$$\chi(\nu) \;=\; a - b\nu^2, \quad a = 4\ln 2,\ b = 14\zeta(3)$$

$$\alpha_s(t) \;=\; \frac{1}{\beta t}, \quad \text{(no threshold in } \beta\text{)}$$

Equation for Green function

$$\left[(\beta\omega t - a) - b\frac{\partial^2}{\partial t^2}\right]\mathcal{G}_\omega(t,t') \;=\; \delta(t-t'), \quad \text{(Airy equation)}$$

Solutions $Ai(z),\ Bi(z)$ where

$$z \;=\; \left(\frac{\beta\omega}{b}\right)^{1/3}\left(t - \frac{a}{\beta\omega}\right)$$

$Ai(z) \stackrel{t\to\infty}{\to} 0$ (good UV solution) , $\quad Bi(z) \stackrel{t\to\infty}{\to} \infty$ (bad UV solution)

Green function which obeys the UV boundary condition

$$\mathcal{G}_\omega(t,t') = Bi(z)Ai(z')\theta(t'-t) + Ai(z)Bi(z')\theta(t-t')$$

But this is not unique
Solution may be generalized by replacing Bi by

$$\overline{Bi}(z) = Bi(z) + \frac{\cos(\phi(\omega))}{\sin(\phi(\omega))}Ai(z)$$

As $t \to -\infty$ (IR limit)

$$\overline{Bi}(z) \to \sim \frac{1}{\sin(\phi(\omega))} \sin\left(\phi(\omega) - \frac{\pi}{4} - \frac{2}{3}\left(t - a/(\beta\omega)\right)^{3/2}\right)$$

Poles wherever $\phi(\omega) = n\pi$, determined by (non-perturbative) IR phase of oscillation as $t \to -\infty$

Determination of IR Phase

Either:
Fit to Data
or
Use a model. e.g.

1. Effective gluon mass in IR limit (E.Levin, L.N. Lipatov, M. Siddikov - 2014)
2. BFKL equation at non-zero temperature, T, which serves as an IR boundary. (H. de Vega, L.N.Lipatov - 2014)

Case of real L.O. BFKL equation

$$\chi(\nu) \; = \; 2\Re e \left\{ \Psi(1) - \Psi\left(\frac{1}{2} + i\nu\right) \right\}$$

Green function may also be expressed (in semi-classical approximation) by

$$\mathcal{G}_\omega(t,t') \; = \; \overline{Bi}(z)Ai(z')\theta(t'-t) + \overline{Bi}(z')Ai(z)\theta(t-t')$$

$$z \; = \; \left[\frac{3}{2}\int_{t_c}^{t} \nu_\omega(t')dt'\right]^{2/3}$$

$$\chi(\nu_\omega(t)) \; = \; \frac{\omega}{\alpha_s(t)}, \;\; \text{and} \;\; \nu_\omega(t_c) \; = \; 0$$

Application in DIS

Unintegrated gluon density $\dot{g}(x,t) \equiv \frac{\partial}{\partial t} g(x,t)$ is given by

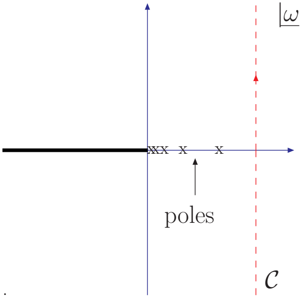
$$\dot{g}(x,t) = \int_{\mathcal{C}} d\omega dt' x^{-\omega} \mathcal{G}_\omega(t,t') \Phi_P(t')$$

where $\Phi_P(t)$ is the proton impact factor, which encodes the coupling of the BFKL pomeron to the proton (this is the only process-dependent factor)

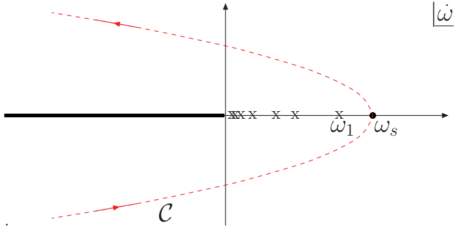
The integral is over a contour to the right of all the poles of $\mathcal{G}_\omega$.

The integrand has a saddle-point, ω_s located at

$$\ln x = \frac{\partial}{\partial\omega} \left\{ \ln\left(A_i(z_\omega(t))\right) \right\}_{|\omega=\omega_s}$$

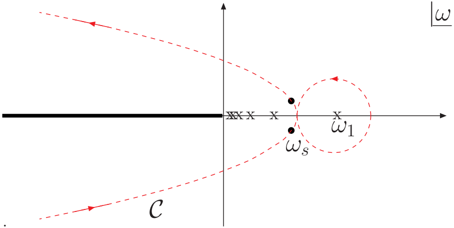


For sufficiently large t, the saddle-point is to the right of all the poles of $\mathcal{G}_\omega$



For $\omega \gg 4C_A \ln 2\alpha_s(t)/\pi$, the saddle-point approximation gives a gluon density whose t-dependence matches that of DGLAP in the DLL limit. The discrete poles affect the overall normalization of the gluon density but NOT its t-dependence

On the other hand, if t is not sufficiently large, the saddle-point(s) lies to the left of one or more of the discrete poles.



In this case, a contour which passes through the saddle-point(s) must also surround one or more poles and the expression for the unintegrated gluon density becomes

$$\dot{g}(x,t) \approx C_{\omega_s} x^{-\omega_s} \int dt' \, \mathcal{G}_{\omega_s}(t,t') \Phi_P(t') + \sum_{\omega_n > \omega_s} A_n x^{-\omega_n} A_i\left(z_{\omega_n}(t)\right)$$

The contribution from the poles is important - they dominate at sufficiently low x.

The DLL limit of DGLAP is not a good approximation in this region of t.

Numerical Example

Non-perturbative phase fixed at

$$\eta = \frac{\pi}{4}, \quad \text{for } k_0 \equiv k_T = 1\,\text{Gev}$$

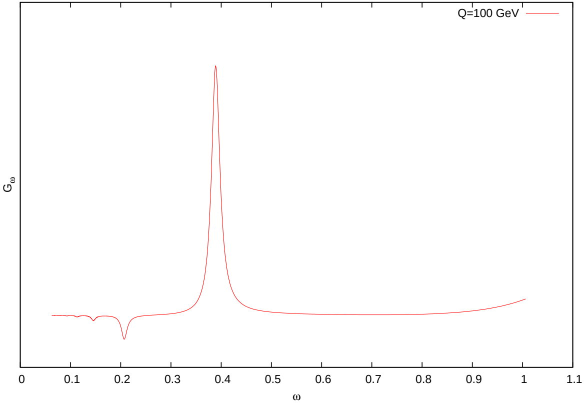
i.e. poles for values of ω satisfying

$$\int_{t_c}^{t_0} \nu_\omega(t')dt' \; = \; \left(n - \frac{1}{2}\right)$$

$$t_0 \; \equiv \ln\left(\frac{k_O^2}{\Lambda^2}\right), \quad \alpha_s(t_c) \; = \; \frac{\omega}{4\ln 2}$$

Green function is convoluted with proton impact factor which is strongly peaked at 1 GeV

Imaginary part of $x^{-\omega}\mathcal{G}_\omega$ along a contour close to real axis

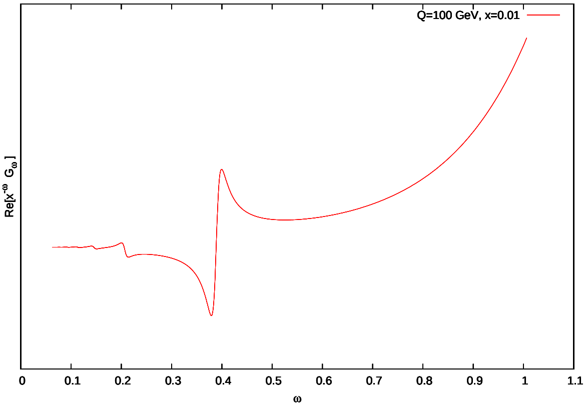


Position of poles fixed, but their residue are t-dependent and oscillate if $t < t_c$.

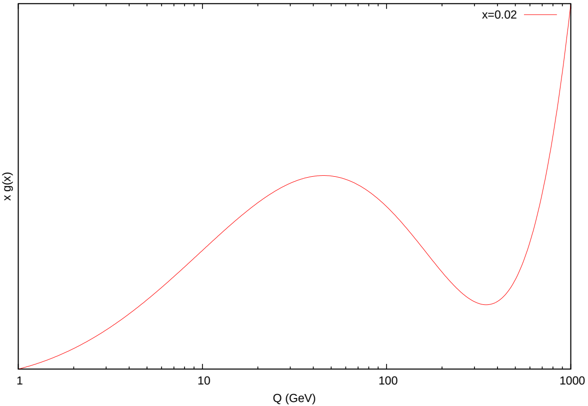
i.e. residue of pole at $\omega = \omega_n$ oscillates if

$$\alpha_s(t) \; < \; \frac{\omega}{4\ln 2}$$

Real part of $x^{-\omega}\mathcal{G}_{\omega}$ along a contour close to real axis



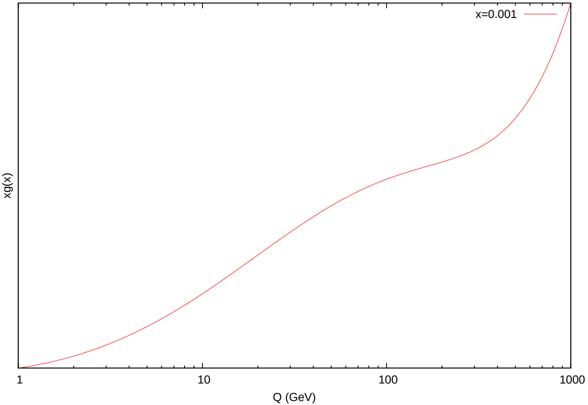
Saddle-point visible above the leading pole, but still too close to the leading pole for the saddle-point contribution to dominate.



Oscillatory behaviour for $Q < 200$ GeV arises because of influence of subsidiary poles with oscillating residues.

Asymptotic region only begins for $Q > \sim 500$ GeV.

Note threshold at $Q = 2m_t$ (350 GeV) so we cannot be "asymptotic" below that scale.



For $x = 10^{-3}$ the influence of subsidiary poles is much smaller because residue at $\omega = \omega_n$ is proportional to

$$x^{-\omega_n}$$

- falls off fast as $\omega \to 0$ for smaller x.

DGLAP in the DLL approximation

Range of validity

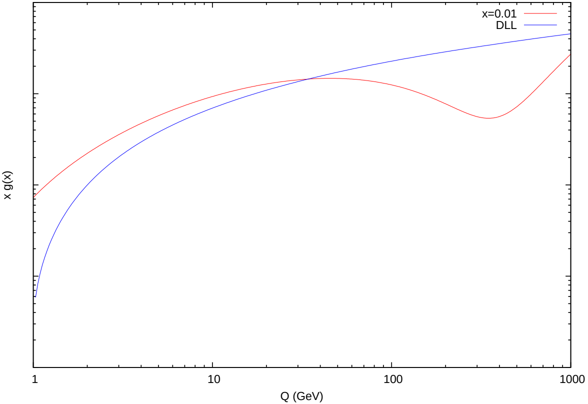
$$1 \gg \omega \gg \alpha_s(t)$$

(not a lot of room for $\alpha_s(t) \sim 0.1$)

$$\ln\left(F_2(x,Q^2)\right) = \ln\left(F_2(x,Q_0^2)\right) + \sqrt{\frac{4C_A}{\pi\beta_0}\ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right)\ln\left(\frac{1}{x}\right)}$$

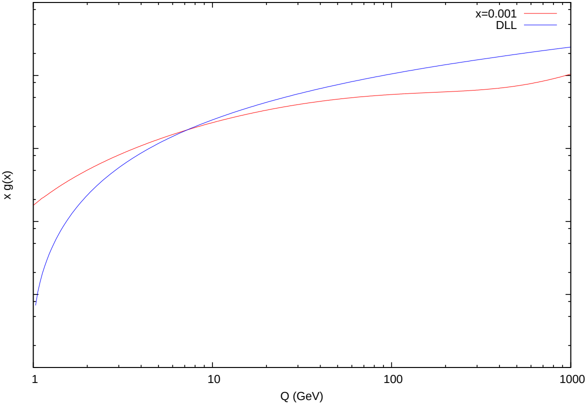
Depends on initial fit value Q_0.
β_0 has flavour thresholds up to $Q = 2m_t$.

Comparison with DLL for $x = 10^{-2}$



For $x = 10^{-2}$ the agreement with th DLL is very poor even at $Q = 1TeV$.

Comparison with DLL for $x = 10^{-3}$



For $x = 10^{-3}$ there is reasonable agreement with DLL limit for $Q > 500$ GeV.

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- We have a formalism for determining the (universal) Green function for the BFKL equation with running coupling and IR boundary condition given in terms of the phase of the oscillation phase.
- A numerical example confirms an (unintegrated) gluon density that rises with Q^2, consistent with a DGLAP approach.
- The match with DGLAP in the DLL limit is probably only numerically accurate for very large rapidities (low-x) and high Q^2. (possible even outside the reach of LHC.) - but a more careful fit using the non-perturbative phase and photon impact factor as free parameters still has to be investigated.