

# PARTICLE PRODUCTION IN DILUTE-DENSE COLLISIONS AND RAPIDITY CORRELATIONS

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<sup>†</sup> Based on work in collaboration with Edmond Iancu and Dionysis Triantafyllopoulos [work in progress]

# Eikonal Scattering

$$\hat{\mathcal{O}}_{Y+\Delta Y} \propto \int_{\Delta Y} [\mathcal{D}a^\mu] \delta(a^+) e^{iS_0[a^\mu; A^-]} \hat{\mathcal{O}}_Y [A^- + a^-]$$

RAPIDITY EVOLUTION: Integrate quantum fluctuations  $a^\mu$  with effective action obtained by expansion around classical eikonal solution (in  $A^+ = 0$  gauge)

$$\begin{aligned} S_0[a^\mu; A^-] &= S_{\text{YM}}[\delta^{\mu-} A^-] + \frac{\delta^2 S_{\text{YM}}[A^\rho + a^\rho]}{\delta a^\mu \delta a^\nu} \Big|_{a^\mu=0} a^\mu a^\nu \\ &= \frac{1}{2} \int d^4x [a^i (-D^2) a^i + (\partial^+ a^- + \partial_i a^i)^2] \\ &= -\frac{1}{2} \int_{\Lambda_0 < |p^+| < \Lambda_0} e^{\Delta Y} \frac{dp^+}{2\pi} \int_{x^+ y^+} \mathbf{x} \mathbf{y} a_b^\mu(x^+, \mathbf{x}, p^+) \\ &\quad \times G_{\mu\nu}^{-1, bc}(x^+, \mathbf{x}; y^+, \mathbf{y}; p^+) a_c^\nu(y^+, \mathbf{y}, p^+) \end{aligned}$$

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The target field is strong ( $gA^- \sim \mathcal{O}(1)$ ), but one can expand in  $ga^\mu \ll 1$  to get

$$\hat{\mathcal{O}}_{Y+\Delta Y} - \hat{\mathcal{O}}_Y \equiv \Delta \mathcal{H} \hat{\mathcal{O}}_Y$$

$$\Delta \mathcal{H} = \frac{1}{2} \int_{\Lambda_0 < |p^+| < \Lambda_0} e^{\Delta Y} \frac{dp^+}{2\pi} \int_{x_1^+ x_2^+} \mathbf{x}_1 \mathbf{x}_2 J^a(x_2^+, \mathbf{x}_2) G_{ab}^{--}(x_2^+, \mathbf{x}_2; x_1^+, \mathbf{x}_1; p^+) J^b(x_1^+, \mathbf{x}_1)$$

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Lorentz Contraction in High-Energy Scattering



Target = Shock-wave at  $x^+ = 0$

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# JIMWLK Hamiltonian

For a gluon crossing a shockwave target, the background field propagator is essentially a Wilson line

$$U_{\mathbf{x}}^\dagger = \mathcal{P} \exp \left[ ig \int dx^+ A_a^-(x^+, \mathbf{x}) T^a \right]$$

and then ( $\int dp^+/p^+ \rightarrow \ln(1/x)$ )

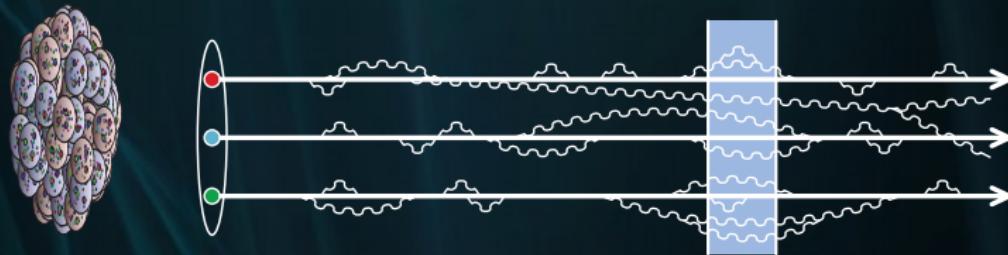
$$\Delta H = \ln \frac{1}{x} H_{\text{JIMWLK}}$$

$$H_{\text{JIMWLK}} = \frac{1}{(2\pi)^3} \int \mathcal{K}_{xyz} (U_{\mathbf{x}}^\dagger - U_{\mathbf{z}}^\dagger)^{ab} (U_{\mathbf{y}}^\dagger - U_{\mathbf{z}}^\dagger)^{ac} R_{\mathbf{x}}^b R_{\mathbf{y}}^c$$

$$R_u^a U_x^{R\dagger} = ig \delta_{ux} U_x^{R\dagger} T_R^a$$

$$\mathcal{K}_{xyz} = \mathcal{K}_{xz}^i \mathcal{K}_{yz}^i \sum \frac{d^2 k}{(2\pi)^2} e^{ik \cdot (x-z)} \underbrace{\text{Diagram with gluon-gluon vertex}}_{= 2gt^a \frac{\varepsilon_\lambda \cdot k}{k^2}} = \frac{ig t^a \varepsilon_\lambda^i}{\pi} \underbrace{\frac{(x-z)^i}{(x-z)^2}}_{\equiv \mathcal{K}_{xz}^i}$$

# Color Glass Condensate: Target Average



Lorentz contraction allows to consider the scattering with the target as an average over different field (Wilson line) configurations in the target [COLOR GLASS CONDENSATE FACTORIZATION]

$$\langle T | \hat{S} | T \rangle = \int \mathcal{D}U \mathcal{D}\bar{U} \langle T | \bar{U} \rangle \langle \bar{U} | \hat{S} | U \rangle \langle U | T \rangle$$

For *elastic or inclusive observables in the target*,  $\hat{S}$  is diagonal in Wilson line space

$$\langle T | \hat{S} | T \rangle = \int \mathcal{D}U \hat{S}_U W_Y[U]$$

$W_Y[U]$  is the target 'wavefunction squared' (classical probability amplitude)

# Multiparticle Production & Rapidity Correlations: Phenomenology

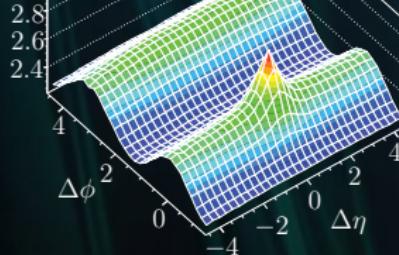
RIDGE IN  $AA$ ,  $pA$  AND  $pp$  COLLISIONS Observed dihadron correlations long-ranged in  $\Delta\eta$  and collimated in  $\Delta\phi$

CMS PbPb  $\sqrt{s_{NN}} = 2.76$  TeV,  $220 \leq N_{\text{trk}}^{\text{offline}} \leq 260$

$1 < p_T^{\text{trig}} < 3$  GeV/c

$1 < p_T^{\text{assoc}} < 3$  GeV/c

$$\frac{1}{N_{\text{trig}}} \frac{d^2 N_{\text{pair}}}{d\Delta\eta d\Delta\phi}$$

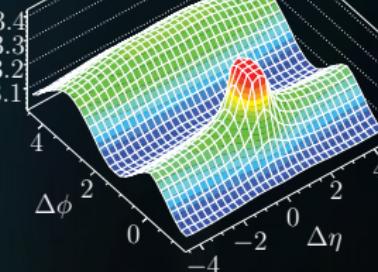


CMS pPb  $\sqrt{s_{NN}} = 5.02$  TeV,  $220 \leq N_{\text{trk}}^{\text{offline}} \leq 260$

$1 < p_T^{\text{trig}} < 3$  GeV/c

$1 < p_T^{\text{assoc}} < 3$  GeV/c

$$\frac{1}{N_{\text{trig}}} \frac{d^2 N_{\text{pair}}}{d\Delta\eta d\Delta\phi}$$

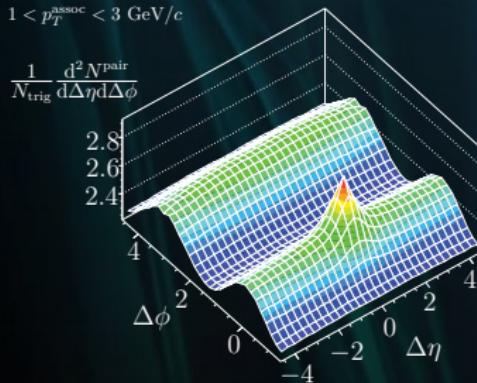


While these correlations can be attributed to final-state effects in  $AA$  that is most surely not the driving mechanism in high-multiplicity  $pA$  and  $pp$  collisions

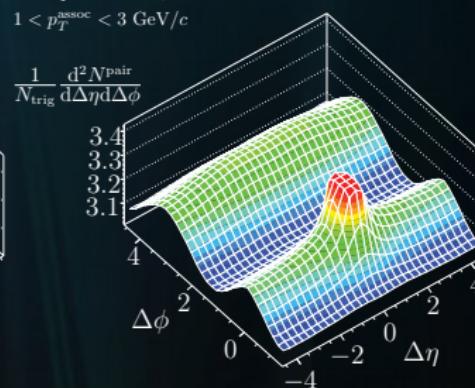
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Need to understand initial correlations

Multiparticle production at well-separated rapidities

# A Foray into the Literature

- In diffractive and semi-inclusive production processes, one needs to track independently direct and complex conjugate amplitudes
- A number of production processes computed in the literature for DIS and  $pA$  collisions, *but vast majority for forward rapidities*
  - Inclusive gluon production in quark (proton)-nucleon collisions [Kovchegov & Mueller '98; Kopeliovich, Tarasov & Schäfer '99; Dumitru & McLerran '02; Kovchegov & Tuchin '02]
  - Single inclusive quark valence production [Kovchegov & Mueller '98; Kopeliovich, Tarasov & Schäfer '99; Dumitru & McLerran '02; Kovchegov & Tuchin '02]
  - Prompt photon and Drell-Yan dilepton production [Gelis & Jalilian-Marian '02; Kopeliovich *et al.* '03; Baier *et al.* '04]
  - Inclusive gluon-gluon and gluon-valence quark production [Jalilian-Marian & Kovchegov '04; Beier *et al.* '06; Kovner & Lublinsky '06]
  - Inclusive  $q\bar{q}$  production [Blaizot, Gelis & Venugopalan '04; Kovchegov & Tuchin '06]

# Doubling the Degrees of Freedom

- JIMWLK gives rapidity evolution of amplitude  $\implies$  not directly applicable at cross-section level
- (!) However, notable relation between  $p_T$  broadening cross-section

$$\frac{dN}{d^2\mathbf{p}} = \int \frac{d^2\mathbf{x}}{(2\pi)^2} e^{-i\mathbf{p}\cdot\mathbf{x}} \tilde{S}(\mathbf{x});$$

$$\tilde{S}(\mathbf{x}) = \frac{1}{N_c} \text{Tr} \langle A | \underbrace{\bar{\mathcal{T}}(U_0^\dagger e^{-i \int d^4y \mathcal{L}_I(y)}) \mathcal{T} U_{\mathbf{x}} e^{i \int d^4y \mathcal{L}_I(y)} }_{\text{Replaceable for only one } \mathcal{T} \text{ in Schwinger-Keldysh contour}} | A \rangle$$

and amplitude for dipole-nucleus scattering

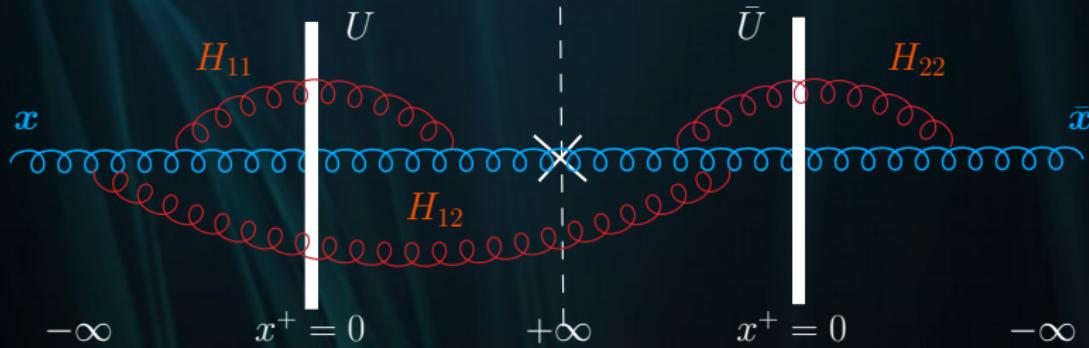
$$S(\mathbf{x}) = \frac{1}{N_c} \text{Tr} \langle A | \mathcal{T}(U_0^\dagger U_{\mathbf{x}} e^{i \int d^4y \mathcal{L}_I(y)}) | A \rangle$$

When rapidity evolution is allowed between emitted particles, these quantities are not obviously related, however they coincide at LO [Kovchegov & Tuchin '02] and NLO [Mueller & Munier '12]

# Doubling the Degrees of Freedom

The doubling of the contour in production processes suggests using independent Wilson lines for direct and conjugate amplitudes [Hentschinski,

Weigert & Schäfer '05; Kovner, Lublinsky & Weigert '06]



implementing quantum interference between direct and conjugate amplitude

$$H_{\text{evol}} = H_{\text{JIMWLK}}[U, R; U, R] + H_{\text{JIMWLK}}[U, R; \bar{U}, \bar{R}] + 2H_{\text{JIMWLK}}[\bar{U}, \bar{R}; U, R]$$

$$H_{\text{JIMWLK}}[\textcolor{blue}{U}, R; \bar{\textcolor{blue}{U}}, \bar{R}] = \frac{1}{(2\pi)^3} \int_{\mathbf{u}\mathbf{v}\mathbf{z}} \mathcal{K}_{\mathbf{u}\mathbf{v}\mathbf{z}} (\textcolor{blue}{U}_u^\dagger - \textcolor{blue}{U}_z^\dagger)^{ab} (\bar{\textcolor{blue}{U}}_v^\dagger - \bar{\textcolor{blue}{U}}_z^\dagger)^{ac} R_u^b \bar{R}_v^c.$$

# Semi-Inclusive Multiparticle Production with Rapidity Evolution

- One can take advantage of the  $U - \bar{U}$  formalism to generate on-shell gluons with a production Hamiltonian [Kovner, Lublinsky & Weigert '06; Iancu & Triantafyllopoulos '12]

$$H_{\text{prod}}(\mathbf{k}) = \frac{1}{4\pi^3} \int_{\mathbf{y}\bar{\mathbf{y}}} e^{-i\mathbf{k}\cdot(\mathbf{y}-\bar{\mathbf{y}})} \int_{\mathbf{uv}} \mathcal{K}_{\mathbf{yu}}^i \mathcal{K}_{\bar{\mathbf{y}}\mathbf{v}}^i (U_{\mathbf{u}}^\dagger - U_{\mathbf{y}}^\dagger)^{ca} (\bar{U}_{\mathbf{v}}^\dagger - \bar{U}_{\bar{\mathbf{y}}}^\dagger)^{cb} R_{\mathbf{u}}^a \bar{R}_{\mathbf{v}}^b$$

- Different coordinates in direct and conjugate amplitude needed for measuring the particle momentum
- Both  $H_{\text{evol}}$  and  $H_{\text{prod}}$  act sequentially upon a generating functional describing the projectile, e.g.

$$\hat{S}_{\mathbf{x}\bar{\mathbf{x}}}^A = \frac{1}{N_c^2 - 1} \text{Tr}[U_{\mathbf{x}}^\dagger \bar{U}_{\bar{\mathbf{x}}}] \quad (\text{gluon}); \quad \hat{Q}_{\mathbf{x}\mathbf{y}\bar{\mathbf{y}}\bar{\mathbf{x}}}^F = \frac{1}{N_c} \text{Tr}(V_{\mathbf{x}}^\dagger V_{\mathbf{y}} \bar{V}_{\bar{\mathbf{y}}}^\dagger \bar{V}_{\bar{\mathbf{x}}}) \quad (\text{quark dipole})$$

# Semi-Inclusive Multiparticle Production with Rapidity Evolution

Then, a general semi-inclusive multiproduction cross-section in the eikonal approximation can be written in the form [Kovner, Lublinsky & Weigert '06; Iancu & Triantafyllopoulos '12; Iancu, JDM & Triantafyllopoulos '14]

$$\frac{d\sigma^{\mathcal{P}(p_1, \dots, p_n) A \rightarrow \mathcal{P}(q_1, \dots, q_n) g(\mathbf{k}_1, \Delta Y_1) \cdots g(\mathbf{k}_m, \Delta Y_m) X}}{d^2 \mathbf{q}_1 dy_{q_1} \cdots d^2 \mathbf{q}_n dy_{q_n} d^2 \mathbf{k}_1 dy_{k_1} \cdots d^2 \mathbf{k}_m dy_{k_m}} = \prod_{i=1}^n p_i^+ \delta(q_i^+ - p_i^+) \\ \times \frac{1}{(2\pi)^{4+2n}} \int_{\mathbf{x}_1 \bar{\mathbf{x}}_1 \cdots \mathbf{x}_n \bar{\mathbf{x}}_n} e^{-i[\mathbf{q}_1 \cdot (\mathbf{x}_1 - \bar{\mathbf{x}}_1) + \cdots + \mathbf{q}_n \cdot (\mathbf{x}_n - \bar{\mathbf{x}}_n)]} \left\langle H_{\text{prod}}(\mathbf{k}_m) e^{H_{\text{evol}} \Delta Y_m} \right. \\ \left. \times H_{\text{prod}}(\mathbf{k}_{m-1}) e^{H_{\text{evol}} \Delta Y_{m-1}} \cdots \times H_{\text{prod}}(\mathbf{k}_1) e^{H_{\text{evol}} \Delta Y_1} \mathcal{Z}_{\mathbf{x}_i, \bar{\mathbf{x}}_i} |_{U=\bar{U}} \right\rangle_Y,$$

- Here, we are measuring the momenta of all components of the projectile, associated to the generating functional  $\mathcal{Z}_{\mathbf{x}_i, \bar{\mathbf{x}}_i}$
- After generating all emissions, we set  $U = \bar{U}$  for target average

# An Illustrative Example

Cross-Section for Gluon Production at a Rapidity Separation  $\delta Y \gtrsim \alpha_s^{-1}$  in Dipole-Nucleus Scattering (at large- $N_c$ )  
 [Iancu, JDM & Triantafyllopoulos '14]

$$\frac{d\sigma^{q(p_1)\bar{q}(p_2)A \rightarrow q(q_1)\bar{q}(q_2)g(k,\delta Y)X}}{d^2\mathbf{q}_1 dy_{q_1} d^2\mathbf{q}_2 dy_{q_2} d^2\mathbf{k} dy_k} = \frac{\prod_{i=1}^2 p_i^+ \delta(p_i^+ - q_i^+)}{(2\pi)^8} \int_{\mathbf{x}\bar{\mathbf{x}}\mathbf{y}\bar{\mathbf{y}}} e^{-i[\mathbf{q}_1 \cdot (\mathbf{x} - \bar{\mathbf{x}}) + \mathbf{q}_2 \cdot (\mathbf{y} - \bar{\mathbf{y}})]} \\ \times \langle H_{\text{prod}}(\mathbf{k}) \hat{Q}_{\mathbf{x}\mathbf{y}\bar{\mathbf{y}}\bar{\mathbf{x}}}^{F(1)} |_{V=\bar{V}} \rangle_Y,$$

$$\hat{Q}_{\mathbf{x}\mathbf{y}\bar{\mathbf{y}}\bar{\mathbf{x}}}^{F(1)} = \hat{Q}_{\mathbf{x}\mathbf{y}\bar{\mathbf{y}}\bar{\mathbf{x}}}^F + \frac{\bar{\alpha} \delta Y}{4\pi} \int_{\mathbf{z}} (\mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{z}} + \mathcal{M}_{\mathbf{x}\bar{\mathbf{x}}\mathbf{z}} - \mathcal{M}_{\bar{\mathbf{x}}\mathbf{y}\mathbf{z}}) \hat{S}_{\mathbf{x}\mathbf{z}}^F \hat{Q}_{\mathbf{z}\mathbf{y}\bar{\mathbf{y}}\bar{\mathbf{x}}}^F \\ + (\mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{z}} + \mathcal{M}_{\mathbf{y}\bar{\mathbf{y}}\mathbf{z}} - \mathcal{M}_{\mathbf{x}\bar{\mathbf{y}}\mathbf{z}}) \hat{S}_{\mathbf{z}\mathbf{y}}^F \hat{Q}_{\mathbf{x}\mathbf{z}\bar{\mathbf{y}}\bar{\mathbf{x}}}^F \\ + (\mathcal{M}_{\bar{\mathbf{x}}\bar{\mathbf{y}}\mathbf{z}} + \mathcal{M}_{\mathbf{x}\bar{\mathbf{x}}\mathbf{z}} - \mathcal{M}_{\mathbf{x}\bar{\mathbf{y}}\mathbf{z}}) \hat{S}_{\mathbf{z}\bar{\mathbf{x}}}^F \hat{Q}_{\mathbf{x}\mathbf{y}\bar{\mathbf{y}}\mathbf{z}}^F \\ + (\mathcal{M}_{\bar{\mathbf{x}}\bar{\mathbf{y}}\mathbf{z}} + \mathcal{M}_{\mathbf{y}\bar{\mathbf{y}}\mathbf{z}} - \mathcal{M}_{\bar{\mathbf{x}}\mathbf{y}\mathbf{z}}) \hat{S}_{\bar{\mathbf{y}}\mathbf{z}}^F \hat{Q}_{\mathbf{x}\mathbf{y}\mathbf{z}\bar{\mathbf{x}}}^F \\ - (\mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{z}} + \mathcal{M}_{\bar{\mathbf{x}}\bar{\mathbf{y}}\mathbf{z}} + \mathcal{M}_{\mathbf{x}\bar{\mathbf{x}}\mathbf{z}} + \mathcal{M}_{\mathbf{y}\bar{\mathbf{y}}\mathbf{z}}) \hat{Q}_{\mathbf{x}\mathbf{y}\bar{\mathbf{y}}\bar{\mathbf{x}}}^F \\ - (\mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{z}} + \mathcal{M}_{\bar{\mathbf{x}}\bar{\mathbf{y}}\mathbf{z}} - \mathcal{M}_{\bar{\mathbf{x}}\mathbf{y}\mathbf{z}} - \mathcal{M}_{\mathbf{x}\bar{\mathbf{y}}\mathbf{z}}) \hat{S}_{\mathbf{x}\mathbf{y}}^F \hat{S}_{\bar{\mathbf{y}}\bar{\mathbf{x}}}^F \\ - (\mathcal{M}_{\mathbf{x}\bar{\mathbf{x}}\mathbf{z}} + \mathcal{M}_{\mathbf{y}\bar{\mathbf{y}}\mathbf{z}} - \mathcal{M}_{\bar{\mathbf{x}}\mathbf{y}\mathbf{z}} - \mathcal{M}_{\mathbf{x}\bar{\mathbf{y}}\mathbf{z}}) \hat{Q}_{\mathbf{x}\mathbf{z}\mathbf{z}\bar{\mathbf{x}}}^F \hat{Q}_{\mathbf{z}\mathbf{y}\bar{\mathbf{y}}\mathbf{z}}^F,$$

$$\mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{z}} = \mathcal{K}_{\mathbf{x}\mathbf{x}\mathbf{z}} + \mathcal{K}_{\mathbf{y}\mathbf{y}\mathbf{z}} - 2\mathcal{K}_{\mathbf{x}\mathbf{y}\mathbf{z}}$$

# An Illustrative Example

$$\begin{aligned} \left\langle H_{\text{prod}}(\mathbf{k}) \hat{Q}_{\mathbf{x}\mathbf{y}\bar{\mathbf{y}}\bar{\mathbf{x}}}^{F(1)} \Big| V = \bar{V} \right\rangle_Y &= \frac{\bar{\alpha}}{2\pi} \int_{\eta\bar{\eta}} e^{-i\mathbf{k}\cdot(\eta-\bar{\eta})} \\ &\times \left\{ \mathcal{K}_{\eta\mathbf{x}}^i \mathcal{K}_{\bar{\eta}\bar{\mathbf{x}}}^i [S_{\bar{\mathbf{x}}\mathbf{x}\mathbf{y}\bar{\mathbf{y}}} - S_{\eta\mathbf{x}} S_{\bar{\mathbf{x}}\eta\mathbf{y}\bar{\mathbf{y}}} - S_{\bar{\mathbf{x}}\bar{\eta}} S_{\bar{\eta}\bar{\mathbf{x}}\mathbf{y}\bar{\mathbf{y}}} + S_{\bar{\mathbf{x}}\mathbf{x}\eta\bar{\eta}} S_{\mathbf{y}\bar{\mathbf{y}}\bar{\eta}\eta}] \right. \\ &+ \mathcal{K}_{\eta\mathbf{y}}^i \mathcal{K}_{\bar{\eta}\bar{\mathbf{y}}}^i [S_{\bar{\mathbf{x}}\mathbf{x}\mathbf{y}\bar{\mathbf{y}}} - S_{\mathbf{y}\eta} S_{\eta\bar{\mathbf{y}}\bar{\mathbf{x}}\mathbf{x}} - S_{\bar{\eta}\bar{\mathbf{y}}} S_{\mathbf{y}\bar{\eta}\bar{\mathbf{x}}\mathbf{x}} + S_{\bar{\mathbf{x}}\mathbf{x}\eta\bar{\eta}} S_{\mathbf{y}\bar{\mathbf{y}}\bar{\eta}\eta}] \\ &- \mathcal{K}_{\eta\mathbf{y}}^i \mathcal{K}_{\bar{\eta}\bar{\mathbf{x}}}^i [S_{\mathbf{y}\mathbf{x}} S_{\bar{\mathbf{x}}\bar{\mathbf{y}}} - S_{\eta\mathbf{x}} S_{\bar{\mathbf{x}}\eta\mathbf{y}\bar{\mathbf{y}}} - S_{\bar{\eta}\bar{\mathbf{y}}} S_{\mathbf{y}\bar{\eta}\bar{\mathbf{x}}\mathbf{x}} + S_{\bar{\mathbf{x}}\mathbf{x}\eta\bar{\eta}} S_{\mathbf{y}\bar{\mathbf{y}}\bar{\eta}\eta}] \\ &- \mathcal{K}_{\eta\mathbf{x}}^i \mathcal{K}_{\bar{\eta}\bar{\mathbf{y}}}^i [S_{\bar{\mathbf{x}}\bar{\mathbf{y}}} S_{\mathbf{y}\mathbf{x}} - S_{\mathbf{y}\eta} S_{\eta\bar{\mathbf{y}}\bar{\mathbf{x}}\mathbf{x}} - S_{\bar{\mathbf{x}}\bar{\eta}} S_{\bar{\eta}\bar{\mathbf{x}}\mathbf{y}\bar{\mathbf{y}}} + S_{\bar{\mathbf{x}}\mathbf{x}\eta\bar{\eta}} S_{\mathbf{y}\bar{\mathbf{y}}\bar{\eta}\eta}] \Big\} \\ &- \frac{\delta Y \bar{\alpha}^2}{8\pi^2} \int_{\eta\bar{\eta}\mathbf{z}} e^{-i\mathbf{k}\cdot(\eta-\bar{\eta})} \mathcal{S}; \end{aligned}$$

$$S_{i_1 i_2 i_3 i_4 \cdots i_{n-1} i_n} = \frac{1}{N_c} \left\langle \text{Tr}(V_{i_1} V_{i_2}^\dagger V_{i_3} V_{i_4}^\dagger \cdots V_{i_{n-1}} V_{i_n}^\dagger) \right\rangle_Y; \quad \bar{\alpha} = \frac{g^2 N_c}{(2\pi)^2}.$$

# An Illustrative Example

$$\begin{aligned}
 S \equiv & \mathcal{K}_{\eta x}^i \mathcal{K}_{\bar{\eta} \bar{x}}^i \left[ (\mathcal{M}_{xyz} + \mathcal{M}_{\bar{x}\bar{y}z} + \mathcal{M}_{x\bar{x}z} + \mathcal{M}_{y\bar{y}z}) \{S_{\bar{x}xy\bar{y}} - S_{\eta x} S_{\bar{x}\eta y\bar{y}} - S_{\bar{x}\bar{\eta}} S_{\bar{\eta}xy\bar{y}} + S_{\bar{x}x\eta\bar{\eta}} S_{y\bar{y}\eta\eta}\} \right. \\
 & + (\mathcal{M}_{x\bar{x}z} + \mathcal{M}_{y\bar{y}z} - \mathcal{M}_{x\bar{y}z} - \mathcal{M}_{\bar{x}yz}) S_{y\bar{y}} \{S_{\bar{x}x} - S_{\bar{x}\eta} S_{\eta x} - S_{\bar{\eta}x} S_{\bar{x}\bar{\eta}} + S_{\bar{\eta}\eta} S_{\bar{x}x\eta\bar{\eta}}\} \\
 & + (\mathcal{M}_{\bar{x}yz} - \mathcal{M}_{\bar{x}\bar{y}z} - \mathcal{M}_{y\bar{y}z}) S_{z\bar{y}} \{S_{y\bar{z}\bar{x}x} - S_{\eta x} S_{\bar{x}\eta yz} - S_{\bar{x}\bar{\eta}} S_{\bar{\eta}xyz} + S_{\bar{x}x\eta\bar{\eta}} S_{yz\bar{\eta}\eta}\} \\
 & \left. + (\mathcal{M}_{x\bar{y}z} - \mathcal{M}_{xyz} - \mathcal{M}_{y\bar{y}z}) S_{yz} \{S_{z\bar{y}\bar{x}x} - S_{\eta x} S_{\bar{x}\eta z\bar{y}} - S_{\bar{x}\bar{\eta}} S_{\bar{\eta}xz\bar{y}} + S_{\bar{x}x\eta\bar{\eta}} S_{z\bar{y}\eta\eta}\} \right] \\
 - & \mathcal{K}_{\eta x}^i \mathcal{K}_{\bar{\eta} \bar{y}}^i \left[ (\mathcal{M}_{xyz} + \mathcal{M}_{\bar{x}\bar{y}z} + \mathcal{M}_{x\bar{x}z} + \mathcal{M}_{y\bar{y}z}) \{S_{\bar{x}\bar{y}} S_{yx} - S_{y\eta} S_{\eta\bar{y}\bar{x}x} - S_{\bar{x}\bar{\eta}} S_{\bar{\eta}xy\bar{y}} + S_{\bar{x}x\eta\bar{\eta}} S_{y\bar{y}\eta\eta}\} \right. \\
 & + (\mathcal{M}_{x\bar{y}z} - \mathcal{M}_{\bar{x}\bar{y}z} - \mathcal{M}_{x\bar{x}z}) S_{\bar{x}x} \{S_{z\bar{y}} S_{yx} - S_{y\eta} S_{\eta\bar{y}z} - S_{z\bar{\eta}} S_{\bar{\eta}xy\bar{y}} + S_{z\bar{x}\eta\bar{\eta}} S_{y\bar{y}\eta\eta}\} \\
 & \left. + (\mathcal{M}_{x\bar{y}z} - \mathcal{M}_{y\bar{y}z} - \mathcal{M}_{xyz}) S_{yz} \{S_{\bar{x}\bar{y}} S_{zx} - S_{z\eta} S_{\eta\bar{y}\bar{x}x} - S_{\bar{x}\bar{\eta}} S_{\bar{\eta}xz\bar{y}} + S_{\bar{x}x\eta\bar{\eta}} S_{z\bar{y}\eta\eta}\} \right] \\
 + & \mathcal{K}_{\eta x}^i \mathcal{K}_{\bar{\eta} z}^i \left[ (\mathcal{M}_{x\bar{y}z} + \mathcal{M}_{\bar{x}yz} - \mathcal{M}_{x\bar{x}z} - \mathcal{M}_{y\bar{y}z}) S_{y\bar{y}} \{S_{z\bar{x}x} - S_{z\eta} S_{\eta z\bar{x}x} - S_{\bar{\eta}x} S_{\bar{x}\bar{\eta}} + S_{\bar{\eta}\eta} S_{\bar{x}x\eta\bar{\eta}}\} \right. \\
 & + (\mathcal{M}_{x\bar{y}z} - \mathcal{M}_{x\bar{x}z} - \mathcal{M}_{\bar{x}yz}) S_{\bar{x}x} \{S_{y\bar{y}\bar{x}x} - S_{\eta x} S_{\bar{x}\eta y\bar{y}} - S_{z\bar{\eta}} S_{\bar{\eta}xy\bar{y}} + S_{z\bar{x}\eta\bar{\eta}} S_{y\bar{y}\eta\eta}\} \\
 & \left. + (\mathcal{M}_{x\bar{y}z} + \mathcal{M}_{y\bar{y}z} - \mathcal{M}_{\bar{x}yz}) S_{z\bar{y}} \{S_{\bar{x}x} S_{yx} - S_{y\eta} S_{\eta z\bar{x}x} - S_{\bar{x}\bar{\eta}} S_{\bar{\eta}xyz} + S_{\bar{x}x\eta\bar{\eta}} S_{yz\bar{\eta}\eta}\} \right] \\
 - & \mathcal{K}_{\eta y}^i \mathcal{K}_{\bar{\eta} \bar{x}}^i \left[ (\mathcal{M}_{xyz} + \mathcal{M}_{\bar{x}\bar{y}z} + \mathcal{M}_{x\bar{x}z} + \mathcal{M}_{y\bar{y}z}) \{S_{\bar{x}\bar{y}} S_{yx} - S_{\eta x} S_{\bar{x}\eta y\bar{y}} - S_{\bar{\eta}\bar{y}} S_{y\eta\bar{x}x} + S_{\bar{x}x\eta\bar{\eta}} S_{y\bar{y}\eta\eta}\} \right. \\
 & + (\mathcal{M}_{x\bar{y}z} - \mathcal{M}_{xyz} - \mathcal{M}_{\bar{x}\bar{y}z}) S_{\bar{x}x} \{S_{y\bar{y}} S_{\bar{x}y} - S_{\eta z} S_{\bar{x}\eta y\bar{y}} - S_{\bar{\eta}y} S_{y\eta\bar{x}z} + S_{\bar{x}z\eta\bar{\eta}} S_{y\bar{y}\eta\eta}\} \\
 & \left. + (\mathcal{M}_{x\bar{y}z} - \mathcal{M}_{\bar{x}\bar{y}z} - \mathcal{M}_{y\bar{y}z}) S_{z\bar{y}} \{S_{y\bar{y}} S_{\bar{x}x} - S_{\eta x} S_{\bar{x}\eta yz} - S_{\bar{\eta}z} S_{y\eta\bar{x}x} + S_{\bar{x}x\eta\bar{\eta}} S_{yz\bar{\eta}\eta}\} \right] \\
 + & \mathcal{K}_{\eta y}^i \mathcal{K}_{\bar{\eta} \bar{y}}^i \left[ (\mathcal{M}_{xyz} + \mathcal{M}_{\bar{x}\bar{y}z} + \mathcal{M}_{x\bar{x}z} + \mathcal{M}_{y\bar{y}z}) \{S_{\bar{x}xy\bar{y}} - S_{y\eta} S_{\eta\bar{y}\bar{x}x} - S_{\bar{\eta}\bar{y}} S_{y\eta\bar{x}x} + S_{\bar{x}x\eta\bar{\eta}} S_{y\bar{y}\eta\eta}\} \right. \\
 & + (\mathcal{M}_{x\bar{x}z} + \mathcal{M}_{y\bar{y}z} - \mathcal{M}_{x\bar{y}z} - \mathcal{M}_{\bar{x}yz}) S_{\bar{x}x} \{S_{y\bar{y}} - S_{\eta y} S_{y\eta} - S_{y\bar{\eta}} S_{\bar{\eta}y} + S_{\bar{\eta}\bar{\eta}} S_{y\bar{y}\eta\eta}\} \\
 & + (\mathcal{M}_{\bar{x}yz} - \mathcal{M}_{xyz} - \mathcal{M}_{\bar{x}\bar{x}z}) S_{\bar{x}x} \{S_{\bar{x}zy\bar{y}} - S_{y\eta} S_{\eta\bar{y}\bar{x}z} - S_{\bar{\eta}y} S_{y\eta\bar{x}z} + S_{\bar{x}z\eta\bar{\eta}} S_{y\bar{y}\eta\eta}\} \\
 & \left. + (\mathcal{M}_{x\bar{y}z} - \mathcal{M}_{\bar{x}\bar{y}z} - \mathcal{M}_{x\bar{x}z}) S_{\bar{x}x} \{S_{z\bar{x}y\bar{y}} - S_{y\eta} S_{\eta\bar{y}z} - S_{\bar{\eta}y} S_{y\eta\bar{y}z} + S_{\bar{x}x\eta\bar{\eta}} S_{yz\bar{\eta}\eta}\} \right]
 \end{aligned}$$

# An Illustrative Example

$$\begin{aligned}
& + \mathcal{K}_{\eta y}^i \mathcal{K}_{\eta z}^i \left[ (\mathcal{M}_{x\bar{y}z} + \mathcal{M}_{\bar{x}yz} - \mathcal{M}_{x\bar{z}z} - \mathcal{M}_{y\bar{y}z}) S_{\bar{x}\bar{x}} \{ S_{yz} S_{z\bar{y}} - S_{\eta z} S_{z\eta y\bar{y}} - S_{y\bar{\eta}} S_{\bar{\eta}y} + S_{\eta\bar{\eta}} S_{y\bar{y}\bar{\eta}\eta} \} \right. \\
& \quad + (\mathcal{M}_{\bar{x}\bar{y}z} + \mathcal{M}_{x\bar{z}z} - \mathcal{M}_{x\bar{y}z}) S_{\bar{x}z} \{ S_{yx} S_{z\bar{y}} - S_{\eta x} S_{z\eta y\bar{y}} - S_{\bar{\eta}y} S_{y\bar{x}z} + S_{zx\eta\bar{\eta}} S_{y\bar{y}\bar{\eta}\eta} \} \\
& \quad \left. + (\mathcal{M}_{\bar{x}\bar{y}z} - \mathcal{M}_{x\bar{y}z} - \mathcal{M}_{y\bar{y}z}) S_{z\bar{y}} \{ S_{\bar{x}xy} - S_{y\eta} S_{\eta z\bar{x}x} - S_{\eta z} S_{y\bar{\eta}x} + S_{\bar{x}x\eta\bar{\eta}} S_{y\bar{y}\bar{\eta}\eta} \} \right] \\
& + \mathcal{K}_{\eta z}^i \mathcal{K}_{\bar{\eta}x}^i \left[ (\mathcal{M}_{x\bar{y}z} + \mathcal{M}_{\bar{x}yz} - \mathcal{M}_{x\bar{z}z} - \mathcal{M}_{y\bar{y}z}) S_{y\bar{y}} \{ S_{\bar{x}z} S_{z\bar{x}} - S_{\bar{x}\eta} S_{\eta x} - S_{\eta z} S_{z\bar{\eta}x} + S_{\eta\bar{\eta}} S_{\bar{x}x\eta\bar{\eta}} \} \right. \\
& \quad + (\mathcal{M}_{\bar{x}yz} - \mathcal{M}_{xyz} - \mathcal{M}_{x\bar{x}z}) S_{z\bar{x}} \{ S_{y\bar{y}x} - S_{\eta z} S_{\bar{x}\eta y\bar{y}} - S_{\bar{x}\eta} S_{\eta z y\bar{y}} + S_{\bar{x}z\eta\bar{\eta}} S_{y\bar{y}\bar{\eta}\eta} \} \\
& \quad \left. + (\mathcal{M}_{xyz} + \mathcal{M}_{y\bar{y}z} - \mathcal{M}_{x\bar{y}z}) S_{yz} \{ S_{zx} S_{\bar{x}\bar{y}} - S_{\eta x} S_{\bar{x}\eta z\bar{y}} - S_{\bar{\eta}y} S_{z\bar{\eta}x} + S_{\bar{x}x\eta\bar{\eta}} S_{z\bar{y}\bar{\eta}\eta} \} \right] \\
& + \mathcal{K}_{\eta z}^i \mathcal{K}_{\bar{\eta}y}^i \left[ (\mathcal{M}_{x\bar{y}z} + \mathcal{M}_{\bar{x}yz} - \mathcal{M}_{x\bar{z}z} - \mathcal{M}_{y\bar{y}z}) S_{\bar{x}\bar{x}} \{ S_{z\bar{y}} S_{y\bar{z}} - S_{\eta\bar{y}} S_{y\eta} - S_{z\bar{\eta}} S_{\bar{\eta}z\bar{y}} + S_{\eta\bar{\eta}} S_{y\bar{y}\bar{\eta}\eta} \} \right. \\
& \quad + (\mathcal{M}_{xyz} + \mathcal{M}_{x\bar{z}z} - \mathcal{M}_{\bar{x}yz}) S_{z\bar{x}} \{ S_{\bar{x}\bar{y}} S_{yz} - S_{y\eta} S_{\eta\bar{y}x} - S_{\bar{x}\eta} S_{\eta z y\bar{y}} + S_{\bar{x}z\eta\bar{\eta}} S_{y\bar{y}\bar{\eta}\eta} \} \\
& \quad \left. + (\mathcal{M}_{x\bar{y}z} - \mathcal{M}_{xyz} - \mathcal{M}_{y\bar{y}z}) S_{yz} \{ S_{\bar{x}xz} - S_{z\eta} S_{\eta\bar{y}x} - S_{\bar{\eta}y} S_{z\bar{\eta}x} + S_{\bar{x}x\eta\bar{\eta}} S_{z\bar{y}\bar{\eta}\eta} \} \right] \\
& - \mathcal{K}_{\eta z}^i \mathcal{K}_{\bar{\eta}z}^i \left[ (\mathcal{M}_{x\bar{y}z} + \mathcal{M}_{\bar{x}yz} - \mathcal{M}_{x\bar{z}z} - \mathcal{M}_{y\bar{y}z}) [S_{\bar{x}\bar{x}} \{ S_{y\bar{y}} - S_{\eta z} S_{z\eta y\bar{y}} - S_{z\bar{\eta}} S_{\bar{\eta}z\bar{y}} + S_{\eta\bar{\eta}} S_{y\bar{y}\bar{\eta}\eta} \} \right. \\
& \quad \left. + S_{y\bar{y}} \{ S_{\bar{x}\bar{x}} - S_{z\eta} S_{\eta z\bar{x}x} - S_{\bar{\eta}z} S_{z\bar{\eta}x} + S_{\bar{\eta}\eta} S_{\bar{x}x\eta\bar{\eta}} \}] \right],
\end{aligned}$$

Although expressed only in terms of dipoles and quadrupoles [Dominguez *et al.* '12], it seems pretty untractable... can we tame the beast?

# Langevin Reformulation: Real Computations Feasible

- From the point of view of target evolution, JIMWLK can be rephrased as a Langevin stochastic process [Blaizot, Iancu & Weigert '02]
- This is well-suited for numerics [Blaizot, Iancu & Weigert '04]: expectation value of dipole [Rummukainen & Weigert '03] and quadrupole [Dumitru *et al.* '11] obtained from solution to JIMWLK equation

## THE RECIPE

- Discretize rapidity interval,  $\Delta Y = N\epsilon$

$$\langle S_{\mathbf{x}\mathbf{y}} \rangle_Y = \frac{1}{N_c} \langle \text{Tr}[U_{N,\mathbf{x}}^\dagger U_{N,\mathbf{y}}] \rangle_\nu \quad \langle \nu_{m,\mathbf{x}}^{ia} \nu_{n,\mathbf{y}}^{jb} \rangle = \frac{1}{\epsilon} \delta_{mn} \delta^{ij} \delta^{ab} \delta_{\mathbf{x}\mathbf{y}}$$

*Random walk in space of Wilson lines with white noise*

$$U_{n,\mathbf{x}}^\dagger = e^{i\epsilon g \alpha_{L,\mathbf{x}}^n} U_{n-1,\mathbf{x}}^\dagger e^{-i\epsilon g \alpha_{R,\mathbf{x}}^n} \quad \alpha_{L,\mathbf{x}}^n = \int_{\mathbf{z}} \mathcal{K}_{\mathbf{x}\mathbf{z}}^i \nu_{n,\mathbf{z}}^{ia} T^a$$

- Initial condition  $U_{0,\mathbf{x}}^\dagger$  given by  $\alpha_{R,\mathbf{x}}^n = \int_{\mathbf{z}} \mathcal{K}_{\mathbf{x}\mathbf{z}}^i U_{n-1,\mathbf{z}}^{ab} \nu_{n,\mathbf{z}}^{ib} T^a$   
McLerran-Venugopalan model

## Langevin Reformulation: Real Computations Feasible

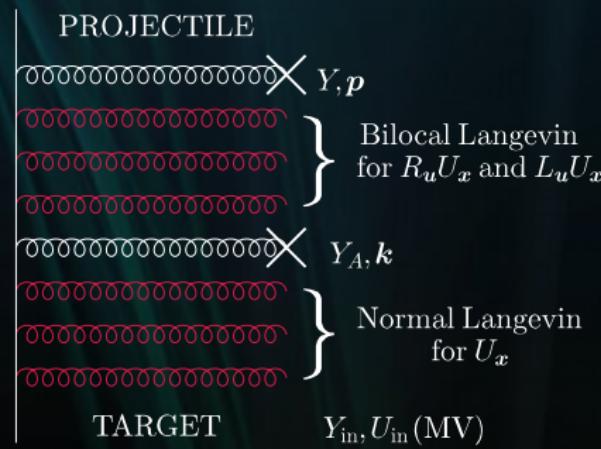
- Being able of using  $H_{\text{evol}}$  and/or  $H_{\text{prod}}$ , means keeping the functional dependence of the Wilson lines at each step of the evolution...

...but functional initial conditions *are not good for numerics*

**ALTERNATIVE:** Keep a second Langevin process in the bi-local quantity

$\mathcal{R}_{n,\mathbf{u}\mathbf{x}}^a \equiv U_{n,\mathbf{x}} R_{\mathbf{u}}^a U_{n,\mathbf{x}}^\dagger$ , for which initial conditions are no longer functional:

$$\mathcal{R}_{0,\mathbf{u}x}^a = ig\delta_{\mathbf{u}x}T^a \quad [\text{Iancu \& Triantafyllopoulos '13}]$$



# $k_T$ Factorization Is the Exception, Not the Rule

- As a general rule, evolution couples nontrivially projectile and target, braking  $k_T$  factorization
- However, for single-inclusive gluon production, one recovers factorization when the projectile is not measured, due to cancellation of final-state interactions [Chen & Mueller '95] (only  $R$ -terms remain in Hamiltonian)

$$\frac{d\sigma^{\mathcal{P}A \rightarrow g(k)X}}{d^2\mathbf{k} dy_k} = \frac{1}{(2\pi)^4} \frac{1}{4\pi^3} \int_{y\bar{y}} e^{-i\mathbf{k}\cdot(y-\bar{y})} \int_{uv} \mathcal{K}_{yu}^i \mathcal{K}_{\bar{y}v}^i \langle (U_u^\dagger - U_{\bar{y}}^\dagger)^{ca} (U_v^\dagger - U_{\bar{y}}^\dagger)^{cb} \rangle_Y \\ \times [R_u^a \bar{R}_v^b \mathcal{Z}_{\Delta Y}]_{U=\bar{U}}$$

with generic projectile generating functional  $\mathcal{Z}_{\Delta Y} = e^{H_{\text{evol}} \Delta Y} \mathcal{Z}_{Y=0}$

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