

On parton number fluctuations

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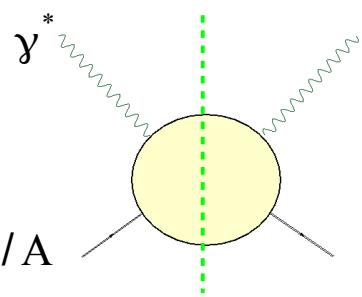
Diffraction 2014



QCD at high density

- ◆ Theoretically rich: *nonlinear physics, nontrivial fluctuations*
- ◆ Phenomenologically: *a lot of data to interpret*

Deep-inelastic scattering



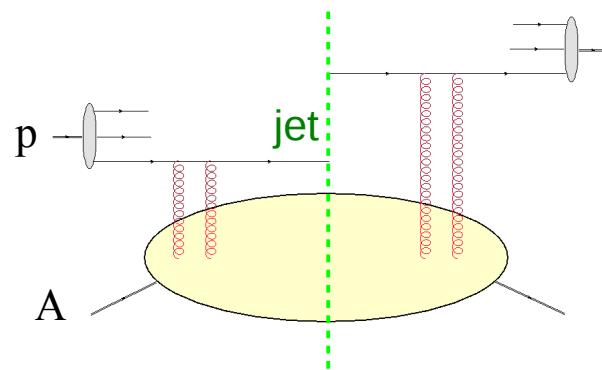
$\gamma^* p$

inclusive cross section

$\gamma^* A$

diffractive cross section

Hadron/nucleus scattering



$p A$

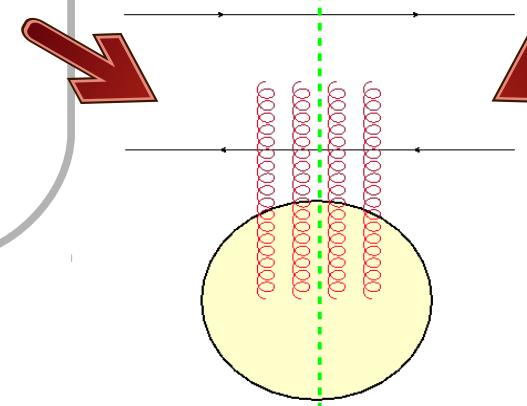
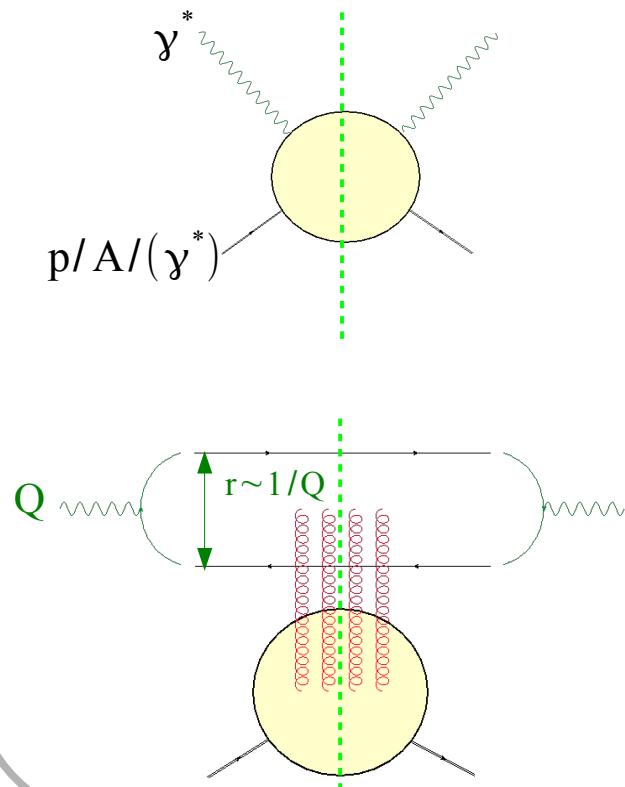
p_T broadening
dijet correlations

...

Universal object for this physics: dipole forward elastic amplitude

Dipole amplitude

Deep-inelastic scattering
HERA+future ep,eA,(ee?)

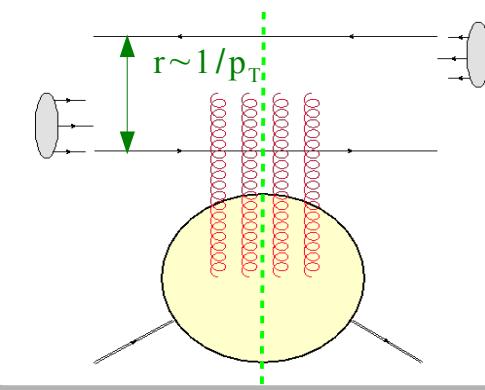
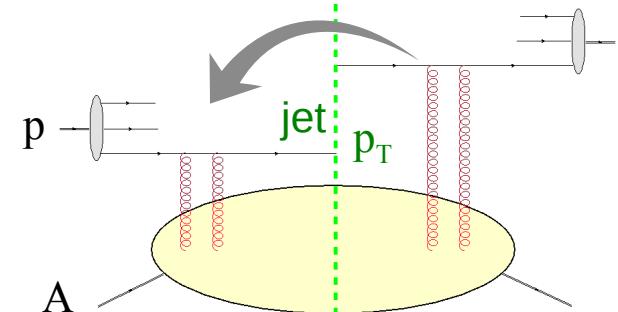


Target: *dipole, or nucleus*

(=large set of independent dipoles)

Broadening

LHC



Outline

** Dipole-nucleus scattering*

The Balitsky-Kovchegov equation

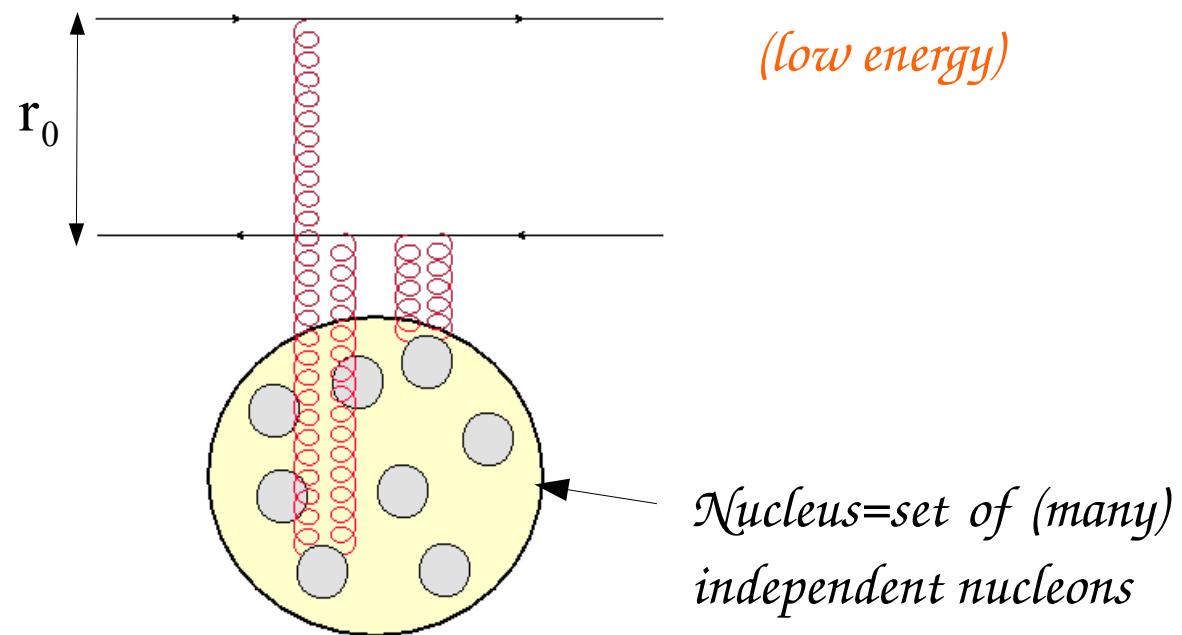
“How does it work?” Interpretation in terms of gluon number fluctuations

QCD dipole model as a branching random walk

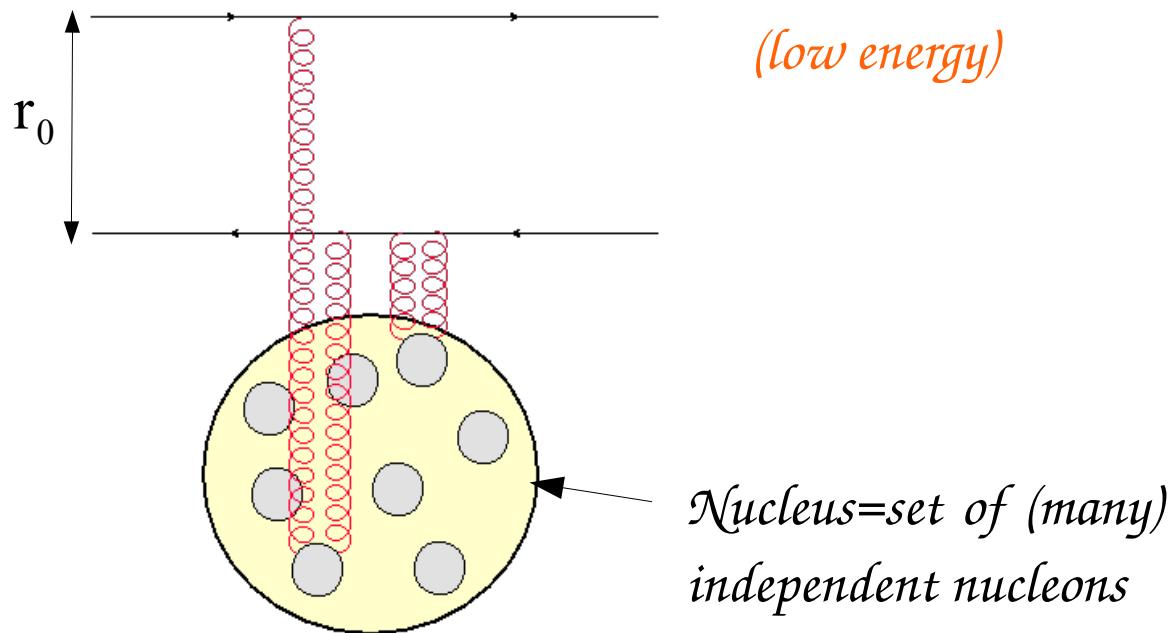
** Dipole-dipole scattering*

New results on the shape of the amplitude

Dipole-nucleus scattering

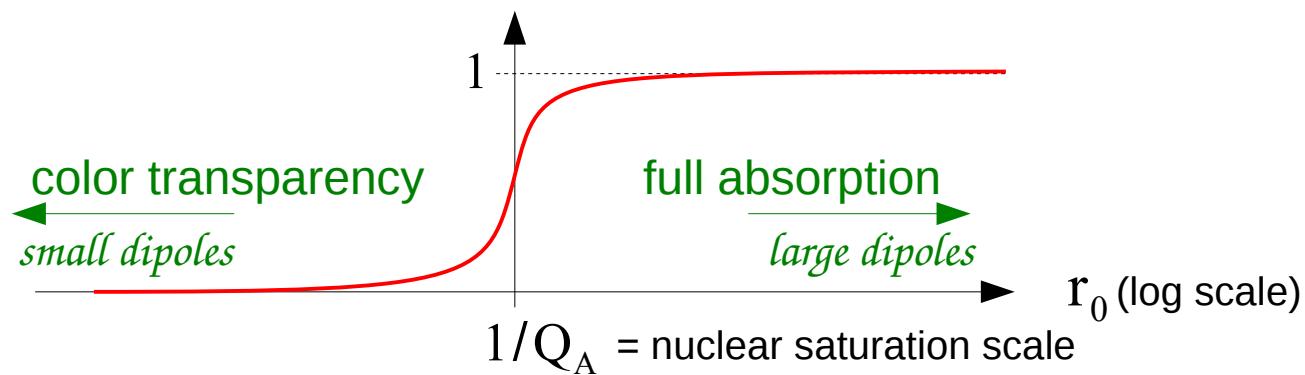


Dipole-nucleus scattering

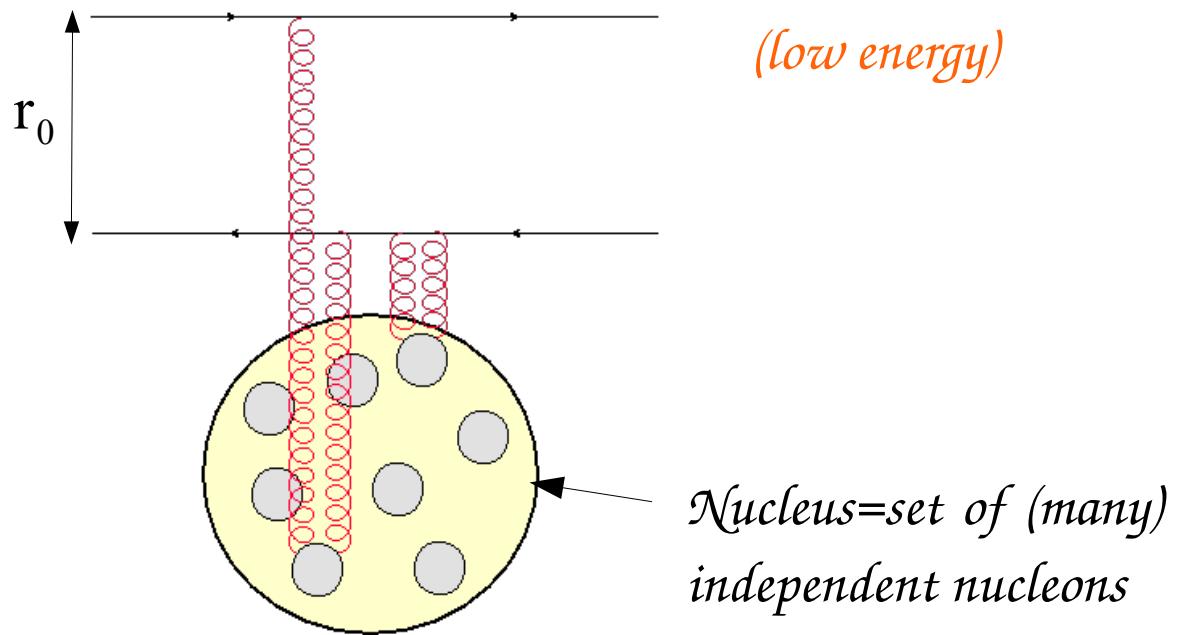


Dipole amplitude:

$$T(r_0) = 1 - e^{-\frac{r_0^2 Q_A^2}{4}} \quad (\text{McLerran-Venugopalan model})$$

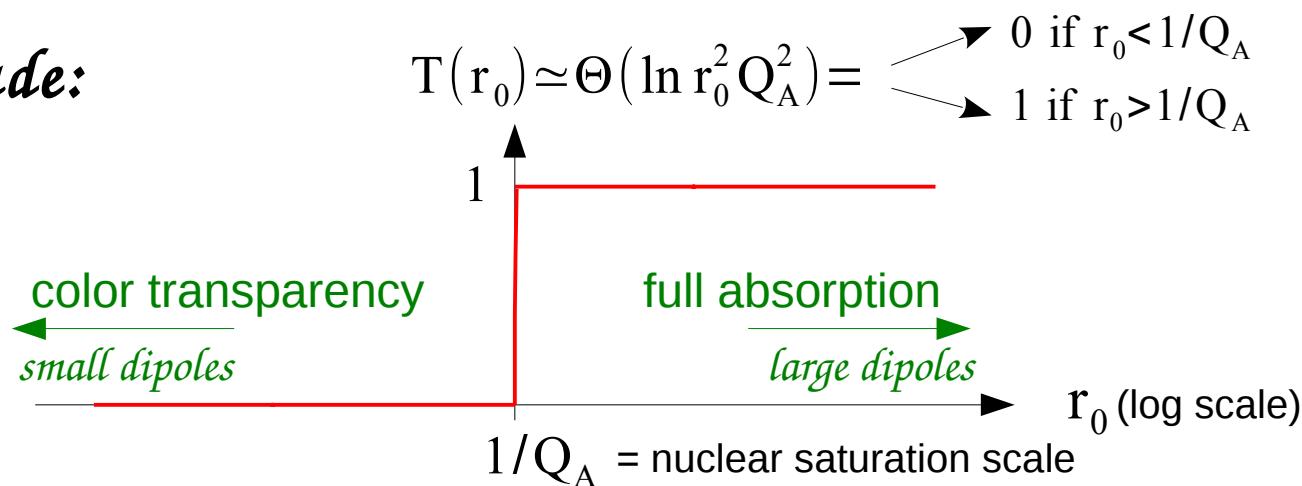


Dipole-nucleus scattering

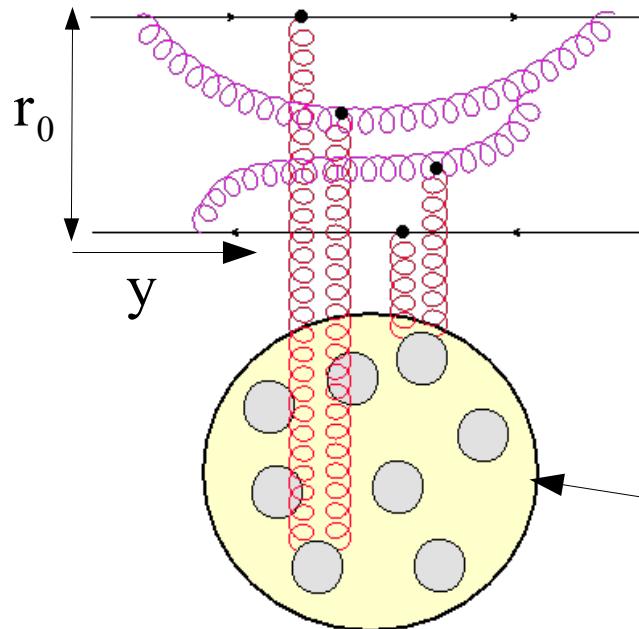


Dipole amplitude:

$$T(r_0) \simeq \Theta(\ln r_0^2 Q_A^2) = \begin{cases} 0 & \text{if } r_0 < 1/Q_A \\ 1 & \text{if } r_0 > 1/Q_A \end{cases}$$



Dipole-nucleus scattering

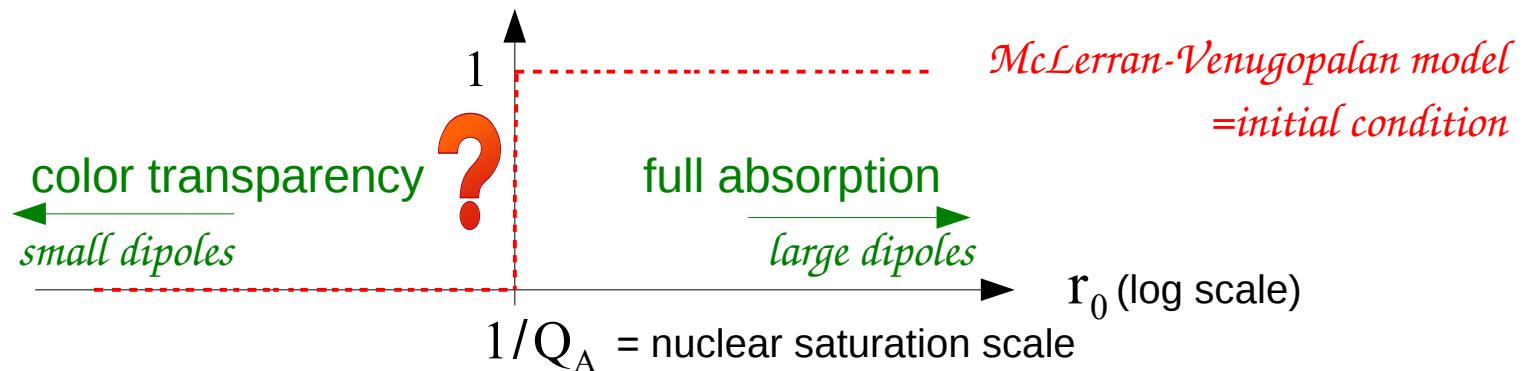


(higher energy:
the dipole has **rapidity** y)

Nucleus=set of (many)
independent nucleons

Dipole amplitude:

$T(r_0, y)$ solves the Balitsky-Kovchegov equation



Reminder: the BK equation and its solution

$$\partial_y T(r_0, y) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} [T(r_1, y) + T(r_0 - r_1, y) - T(r_0, y) - T(r_1, y)T(r_0 - r_1, y)]$$

Reminder: the BK equation and its solution

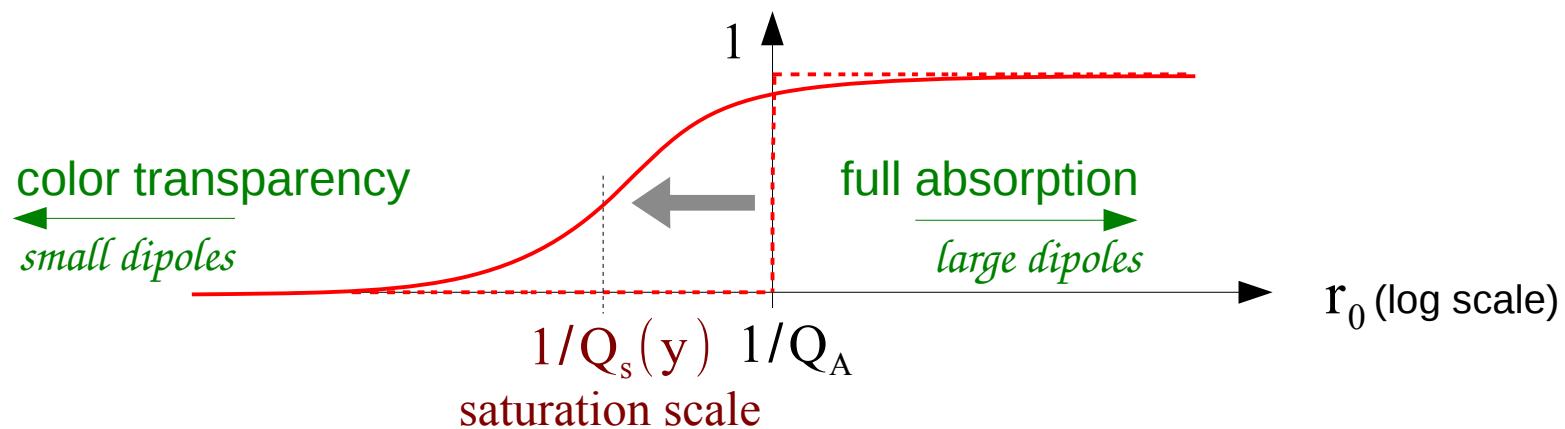
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Linear part: BFKL equation

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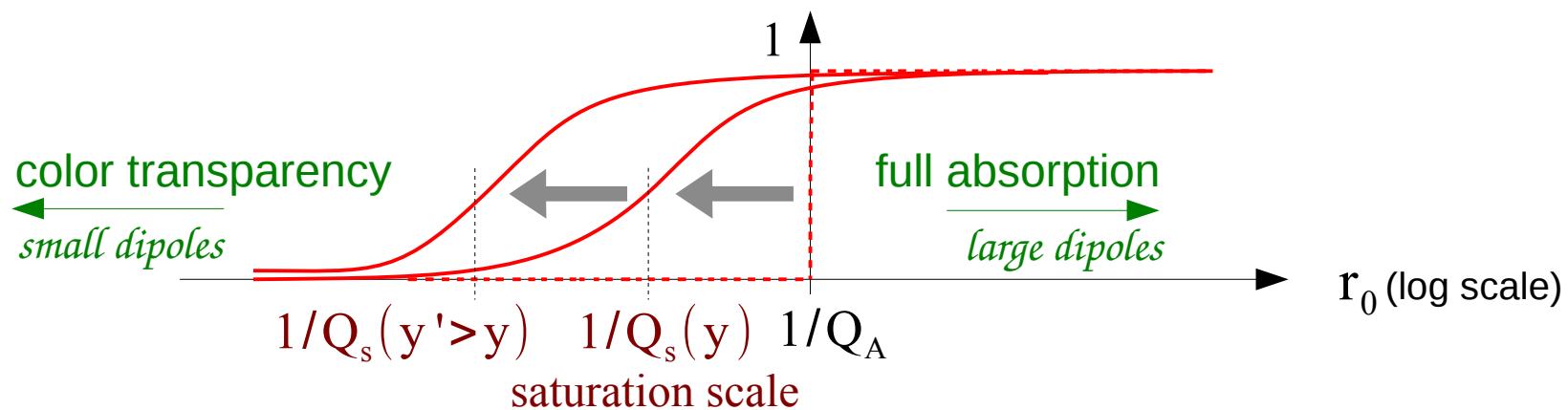
Linear part: BFKL equation



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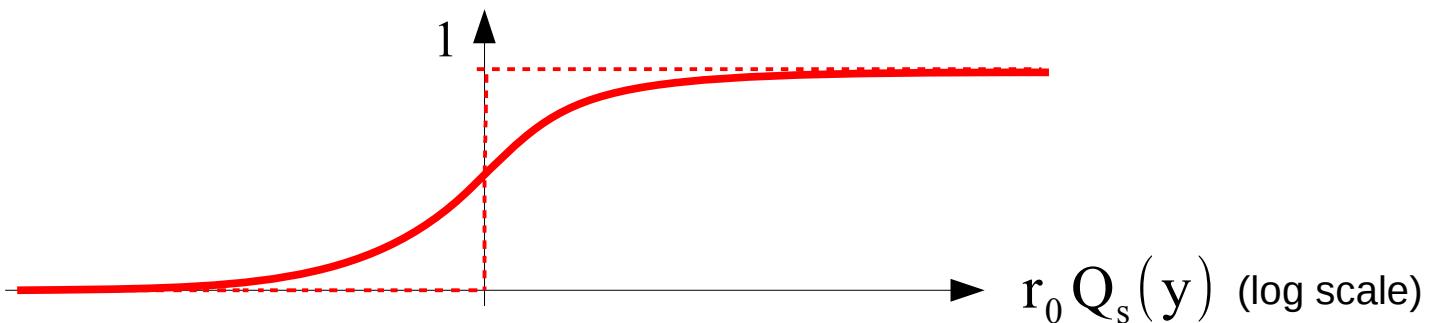
Linear part: BFKL equation



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Linear part: BFKL equation



$T(r_0, y)$ $\underset{y \gg 1}{\approx}$ function of $(r_0 Q_s(y))$

Traveling wave property = geometric scaling

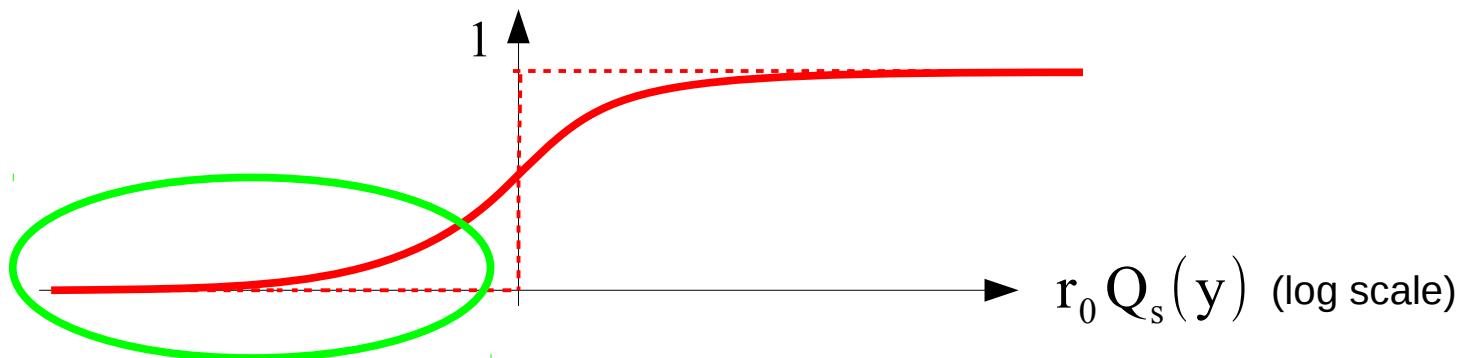
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$$\partial_y T(r_0, y) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} [T(r_1, y) + T(r_0 - r_1, y) - T(r_0, y) - T(r_1, y) T(r_0 - r_1, y)]$$

Linear part: BFKL equation

$$\text{eigenfunctions } T(r_0) = e^{y \ln r_0^2}$$

$$\text{eigenvalues } \frac{\alpha_s N_c}{\pi} \chi(y), \quad \chi(y) \equiv 2\psi(1) - \psi(y) - \psi(1-y)$$



$T(r_0, y) \underset{y \gg 1}{\approx}$ function of $(r_0 Q_s(y))$

Traveling wave property = geometric scaling

$$T(r_0, y) \underset{r_0 Q_s(y) \ll 1}{\sim} \ln \frac{1}{r_0^2 Q_s^2(y)} e^{y_0 \ln [r_0^2 Q_s^2(y)]}$$

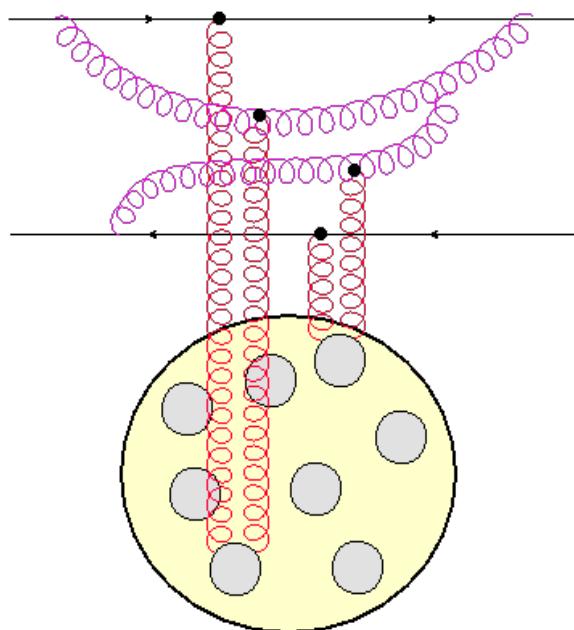
$$Q_s^2(y) \simeq Q_A^2 e^{\frac{\alpha_s N_c}{\pi} \chi'(y_0) y}$$

y_0 solves $\chi'(y_0) = \chi(y_0)$

The BK equation: how does it work?

$$\partial_y T(r_0, y) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} [T(r_1, y) + T(r_0 - r_1, y) - T(r_0, y) - T(r_1, y)T(r_0 - r_1, y)]$$

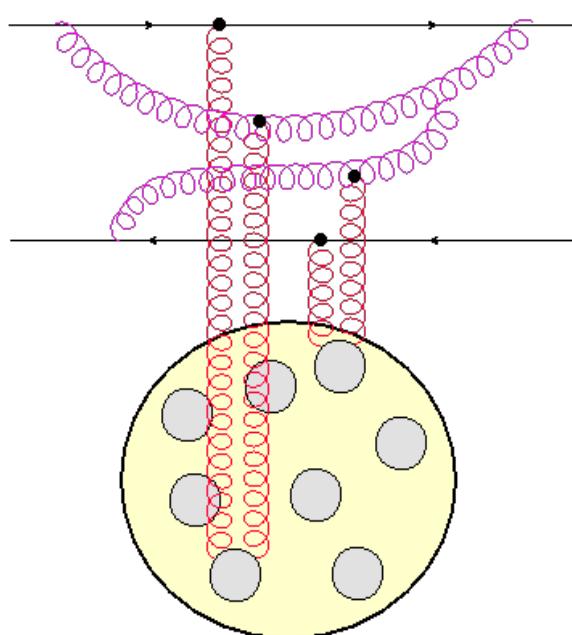
*Quantum evolution of the dipole by
gluon radiation*



The BK equation: how does it work?

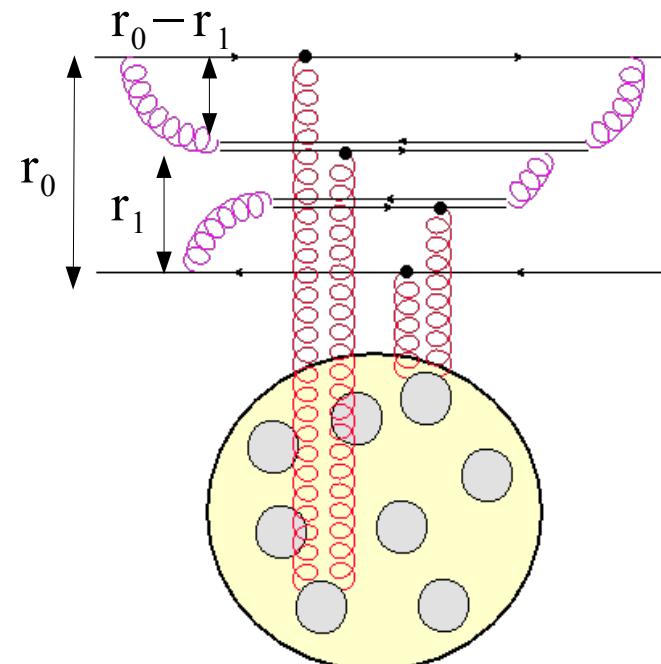
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Quantum evolution of the dipole by gluon radiation



large number-of-color limit

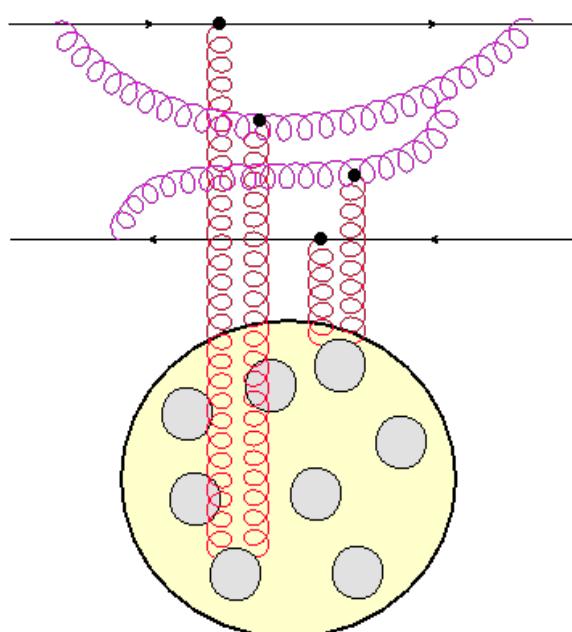
Successive splittings of dipoles



The BK equation: how does it work?

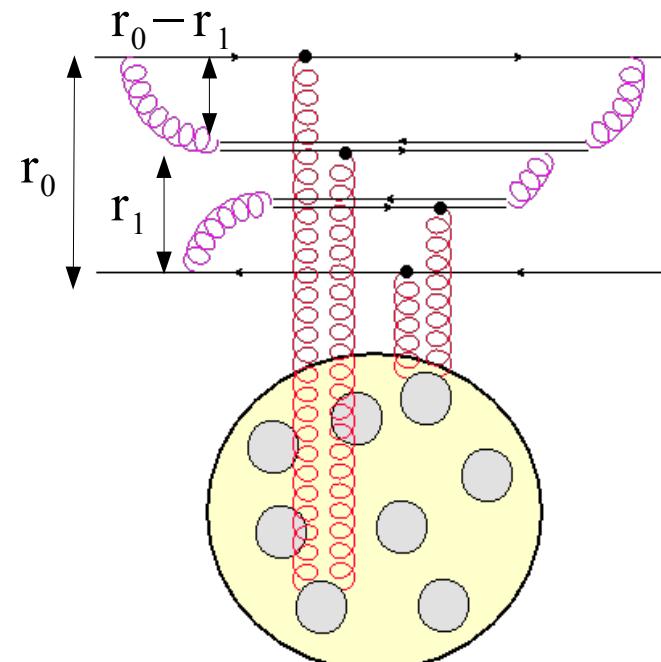
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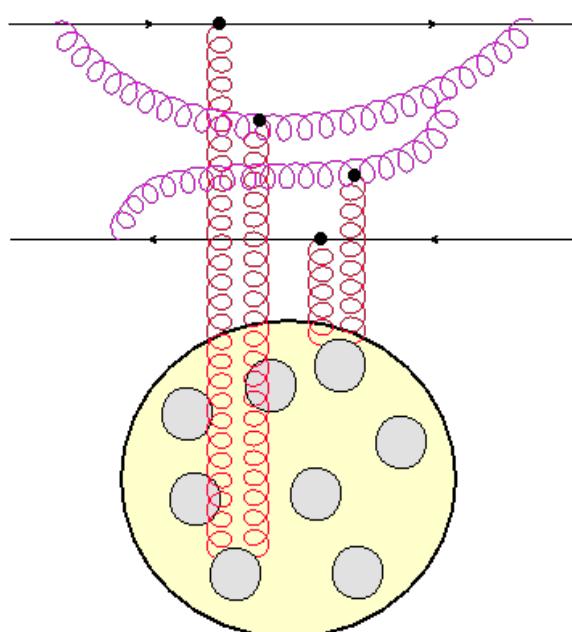
QCD dipole model: *dipole splitting rate =*

$$\frac{dp}{dy} = \frac{\alpha_s N_c}{\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \frac{d^2 r_1}{2\pi}$$

The BK equation: how does it work?

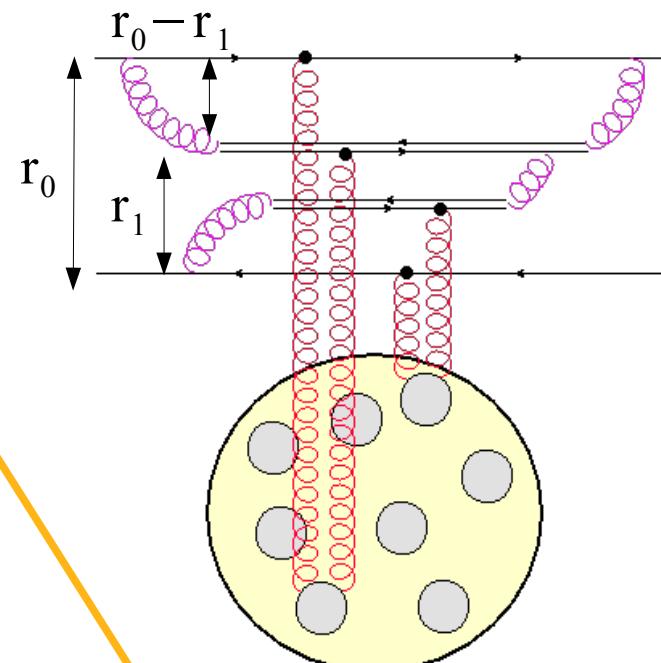
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Quantum evolution of the dipole by gluon radiation



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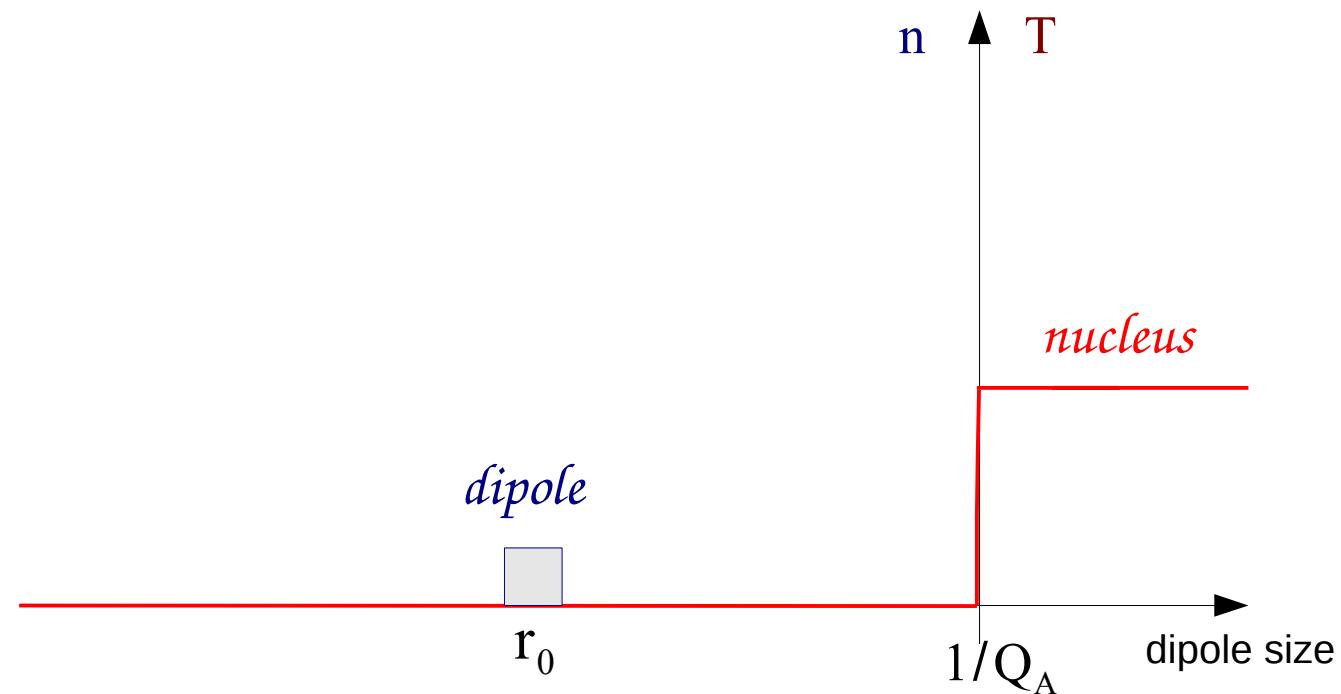
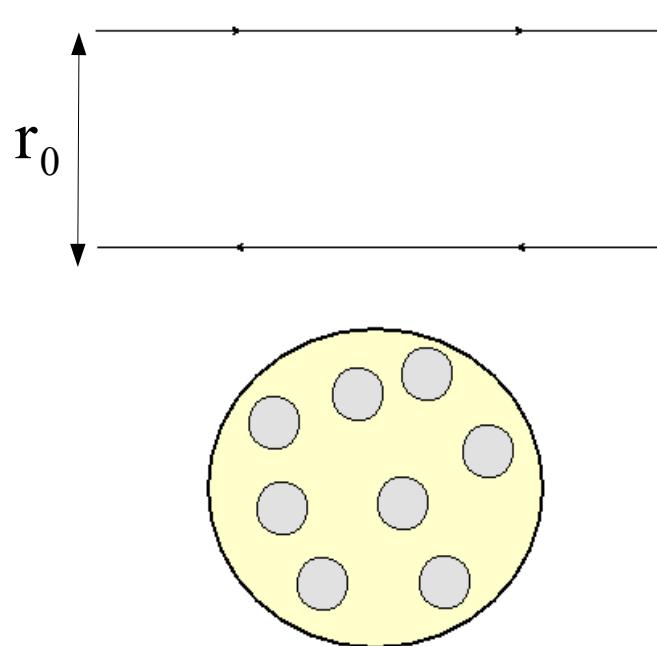
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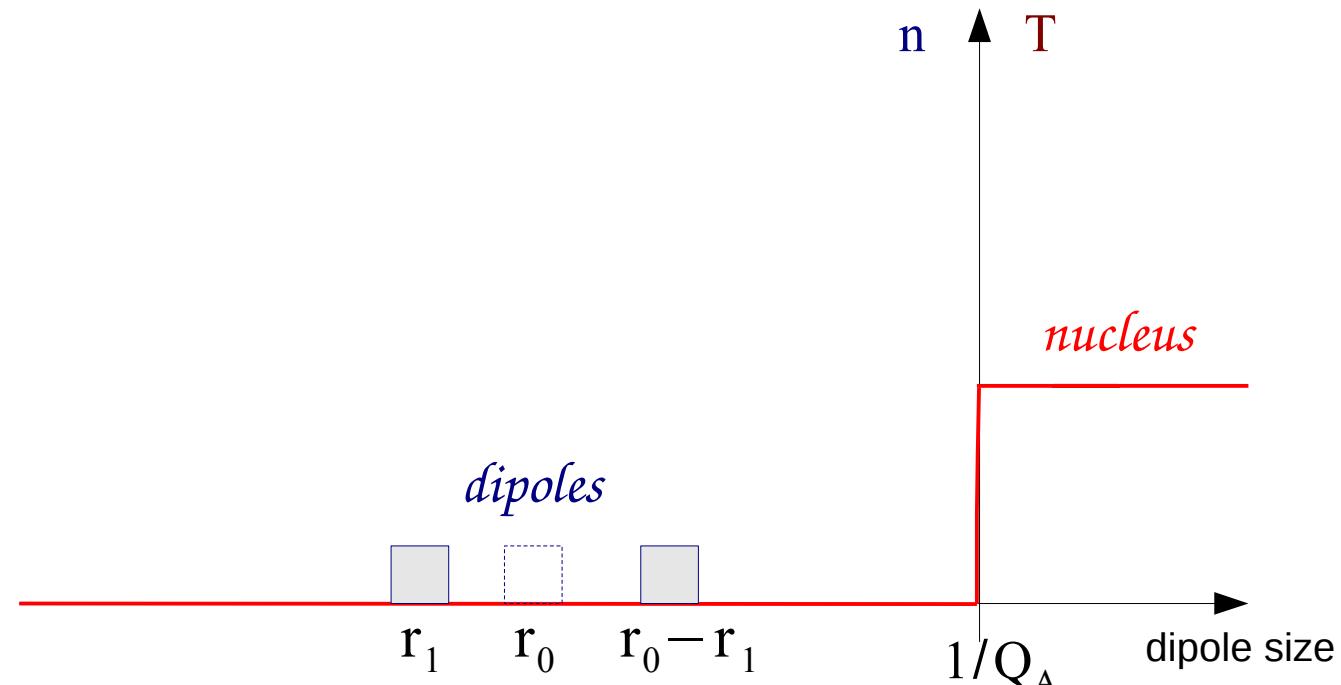
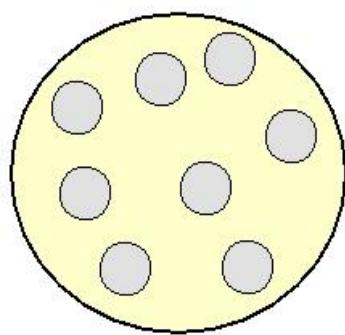
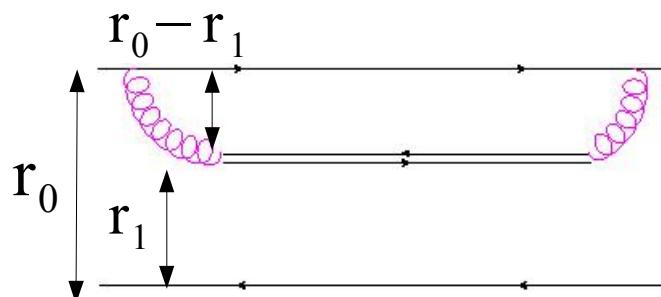
The BK equation: how does it work?

QCD dipole model:



The BK equation: how does it work?

QCD dipole model:

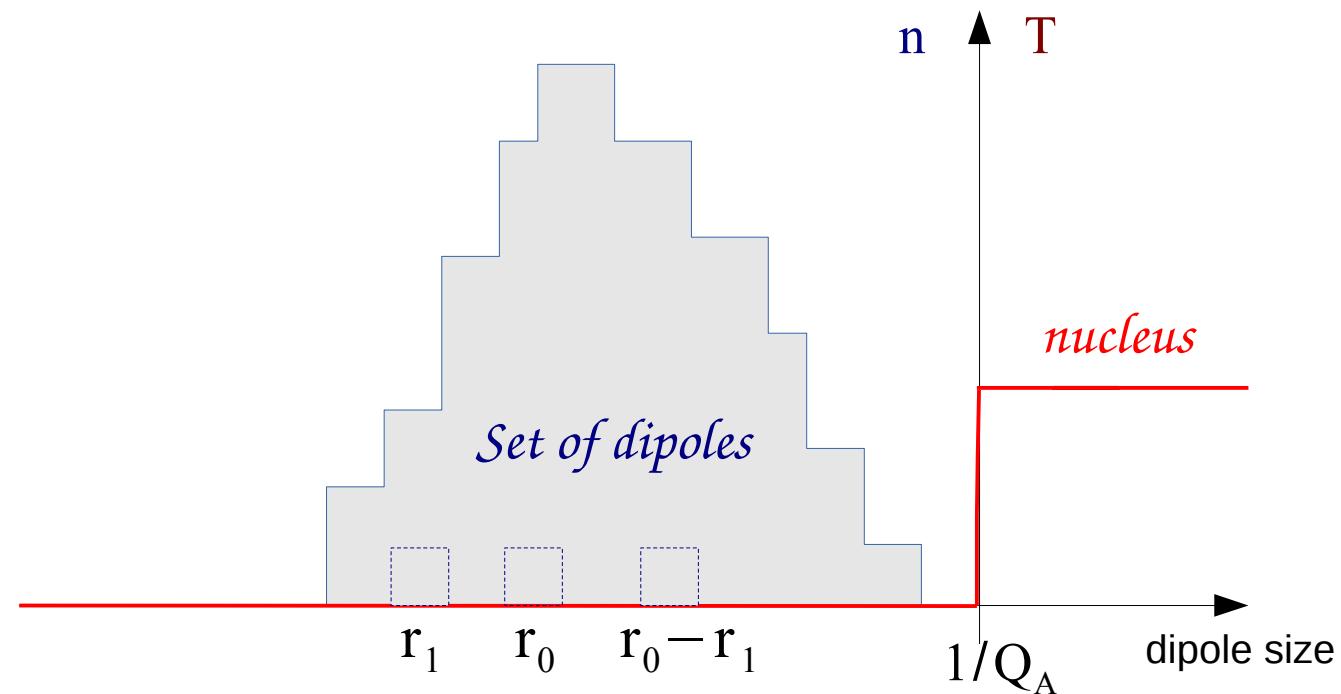
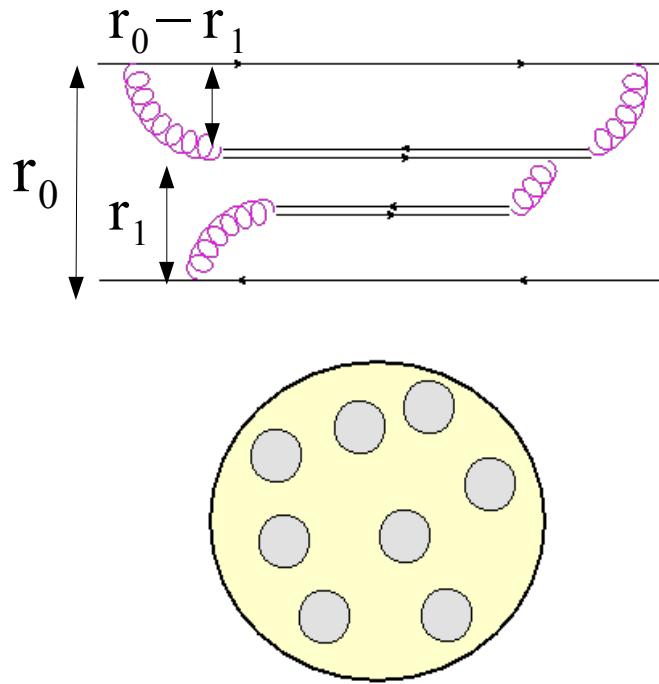


dipole splitting rate =

$$\frac{dp}{dy} = \frac{\alpha_s N_c}{\pi} \frac{r_0^2}{r_1(r_0 - r_1)^2} \frac{d^2 r_1}{2\pi}$$

The BK equation: how does it work?

QCD dipole model:

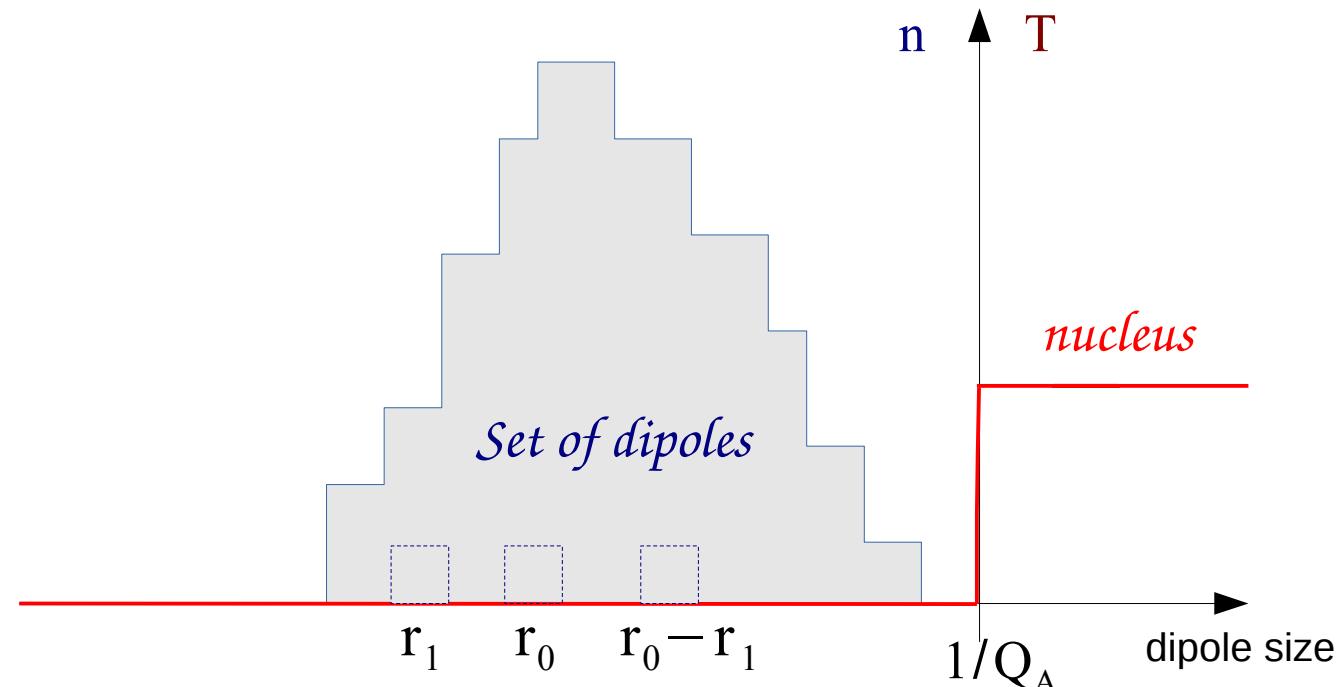
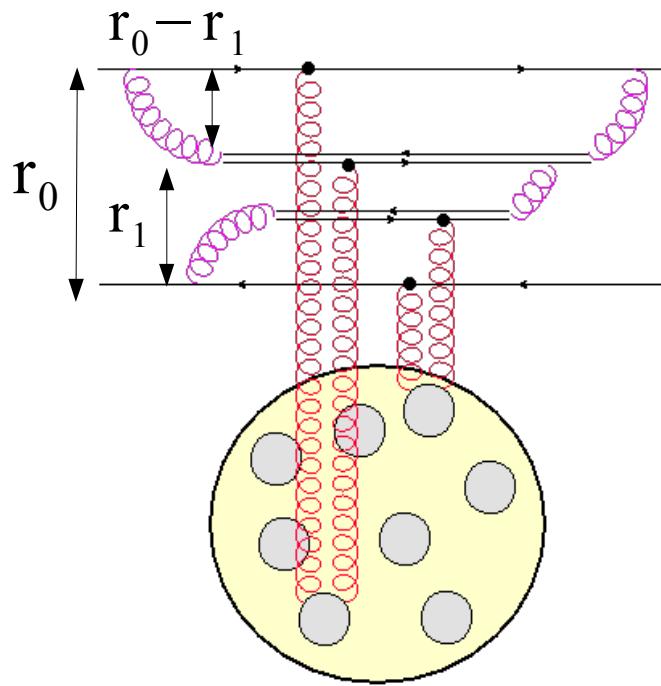


dipole splitting rate =

$$\frac{dp}{dy} = \frac{\alpha_s N_c}{\pi} \frac{r_0^2}{r_1(r_0 - r_1)^2} \frac{d^2 r_1}{2\pi}$$

The BK equation: how does it work?

QCD dipole model:



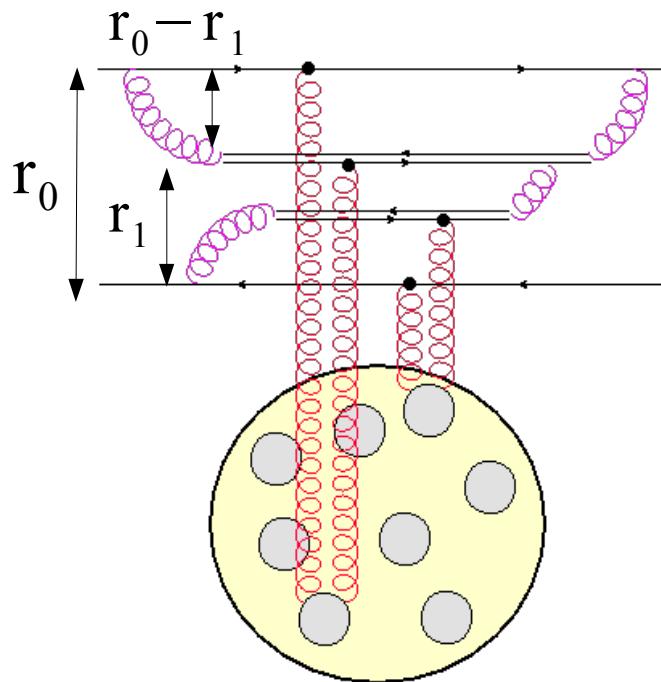
$$T_{\text{1-event}}(r_0, y) = \begin{cases} 0 & \text{if all dipoles have size } r < 1/Q_A \\ 1 & \text{if at least one dipole has size } r > 1/Q_A \end{cases}$$

dipole splitting rate =

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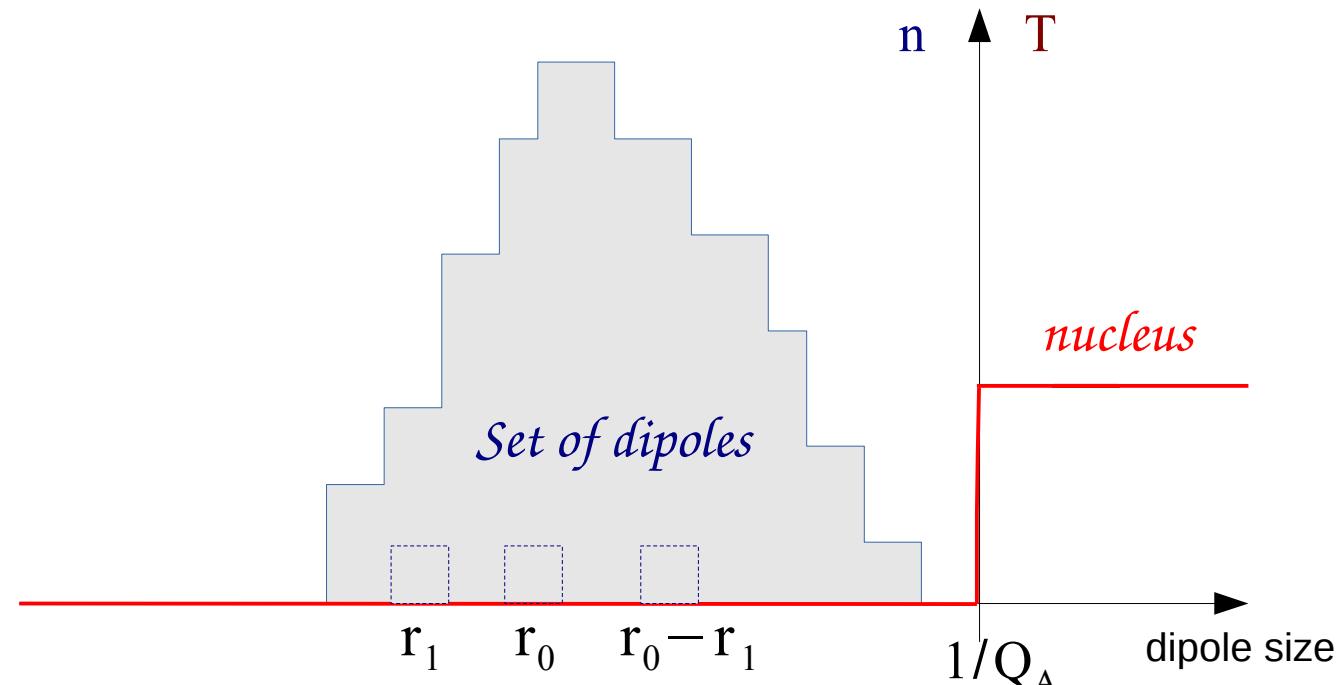
The BK equation: how does it work?

QCD dipole model:



dipole splitting rate =

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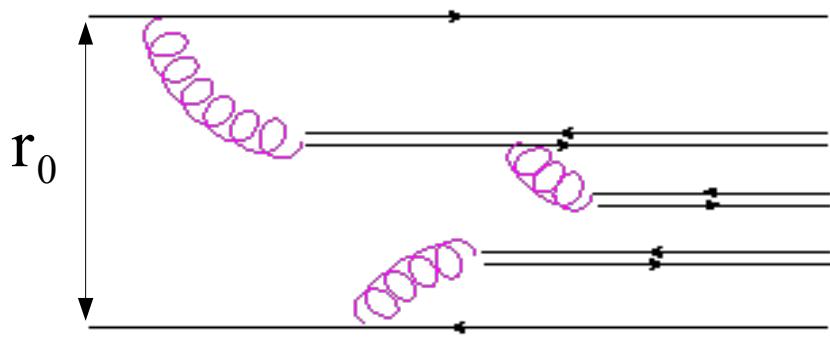


$$T_{1\text{-event}}(r_0, y) = \begin{cases} 0 & \text{if all dipoles have size } r < 1/Q_A \\ 1 & \text{if at least one dipole has size } r > 1/Q_A \end{cases}$$

$$T(r_0, y) = \langle T_{1\text{-event}}(r_0, y) \rangle \text{ solves the BK equation}$$

Interpretation: probability that the largest dipole has a size larger than the inverse nuclear saturation momentum

More on dipole evolution

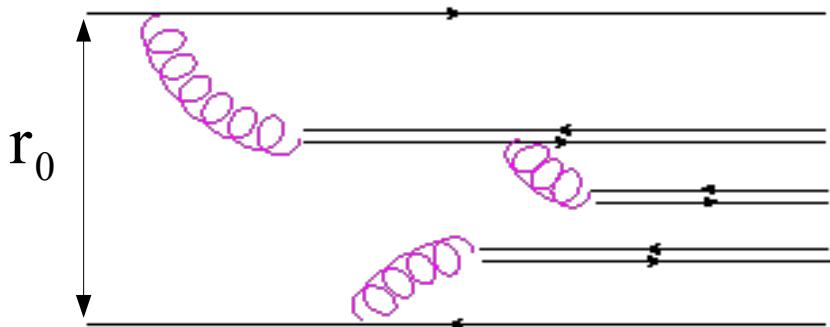


Dipole evolution

*Scattering amplitude with a nucleus: solves
the **BK** equation*

$$\partial_y T(r_0, y) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \times [T(r_1, y) + T(r_0 - r_1, y) - T(r_0, y) - T(r_1, y) T(r_0 - r_1, y)]$$

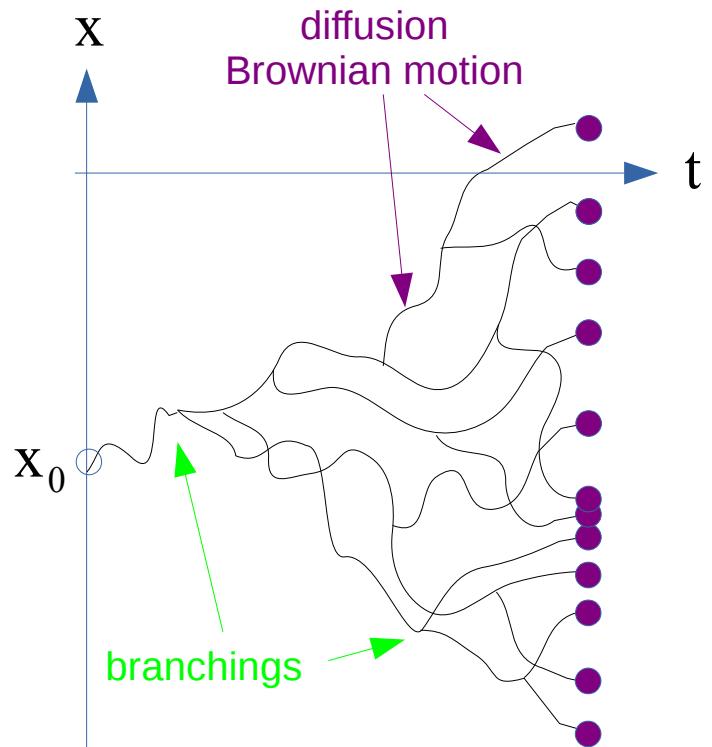
More on dipole evolution: a useful analogy



Dipole evolution

Scattering amplitude with a nucleus: solves the **BK equation**

$$\partial_y T(r_0, y) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 r_1}{2\pi} \frac{r_0^2}{r_1^2 (r_0 - r_1)^2} \times [T(r_1, y) + T(r_0 - r_1, y) - T(r_0, y) - T(r_1, y) T(r_0 - r_1, y)]$$



Branching random walk

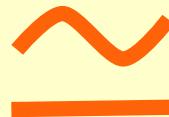
Probability P that at least one particle has a position > 0 : solves the **FKPP equation**

(Fisher-Kolmogorov-Petrovsky-Piscounov)

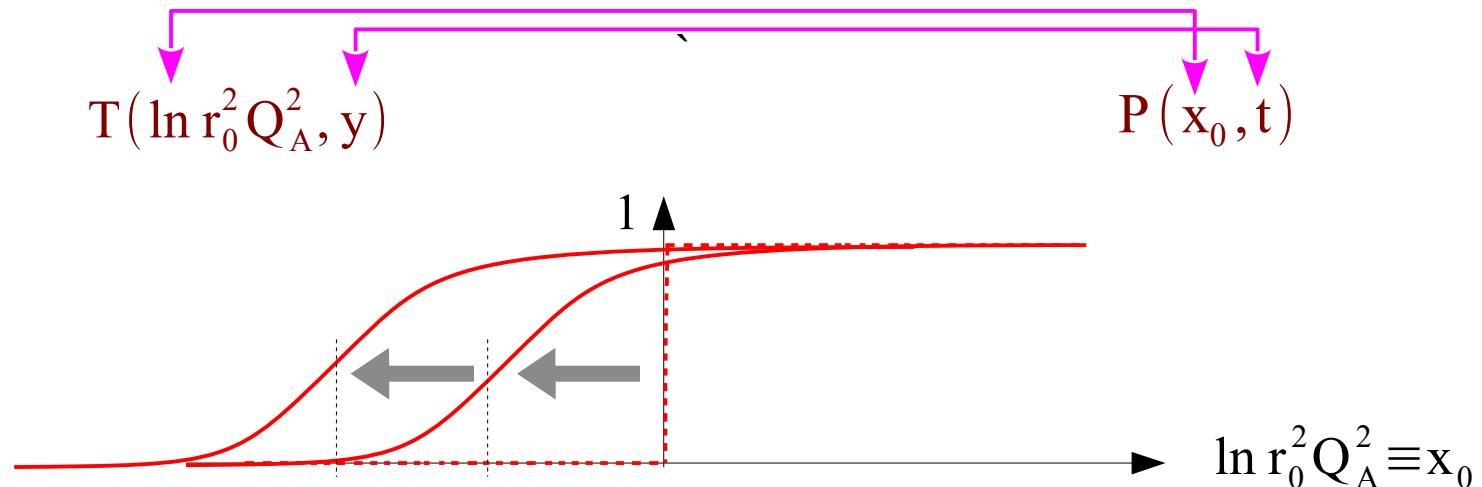
$$\partial_t P = \partial_{x_0}^2 P + P - P^2$$

(diffusion constant=1, splitting rate=1)

Rapidity evolution of dipole-nucleus amplitude



Time evolution of the boundary of a branching random walk



$$T(\ln r_0^2 Q_A^2, y) = \text{function of } (\ln r_0^2 Q_A^2 - \ln Q_A^2 / Q_s^2(y))$$

$$P(x_0, t) = \text{function of } (x_0 - X(t))$$

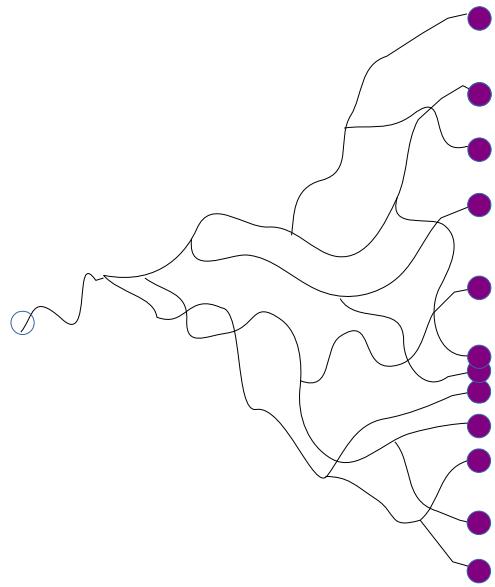
$$T(r_0, y) \underset{r_0 Q_s(y) \ll 1}{\sim} \ln \frac{1}{r_0^2 Q_s^2(y)} e^{y_0 \ln [r_0^2 Q_s^2(y)]}$$

$$\ln \frac{Q_A^2}{Q_s^2(y)} \sim -\frac{\alpha_s N_c}{\pi} \chi'(y_0) y$$

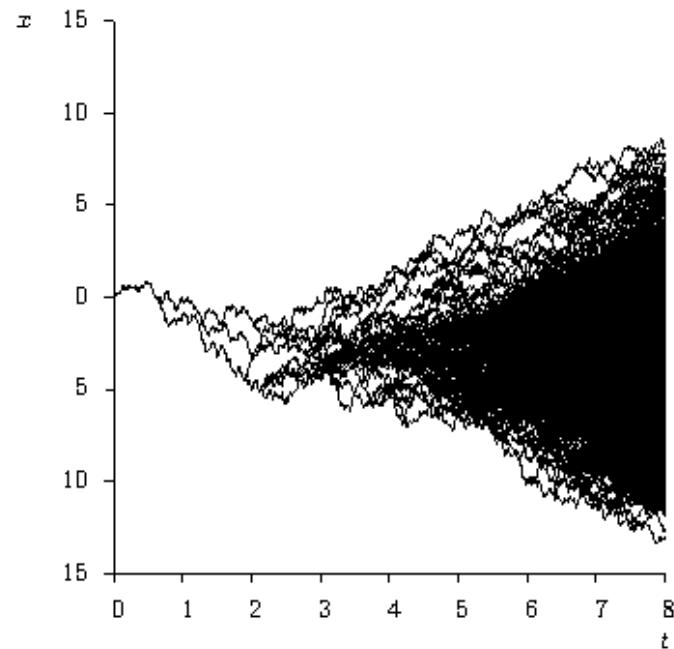
$$P(x_0, y) \sim (X(t) - x_0) e^{x_0 - X(t)}$$

$$X(t) \sim -2t$$

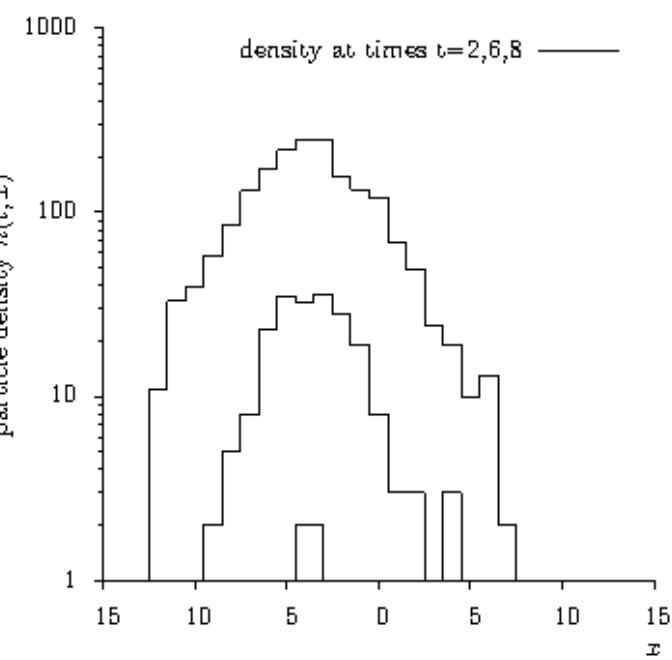
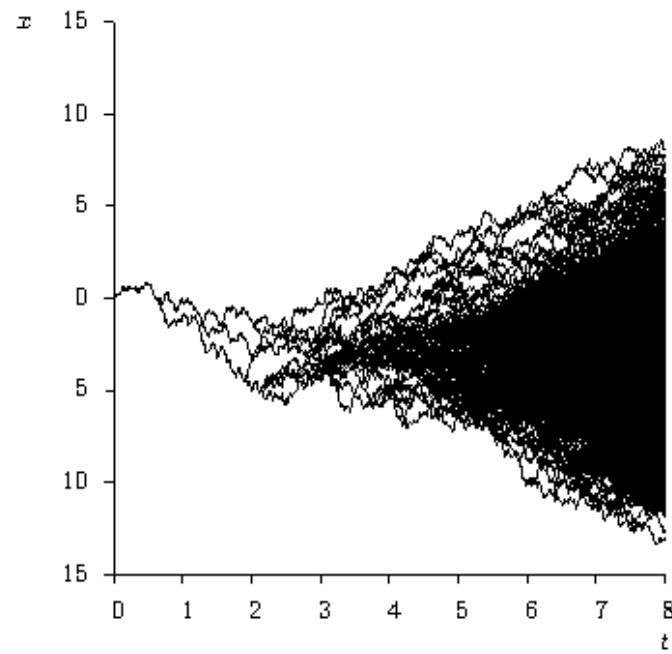
Branching random walk



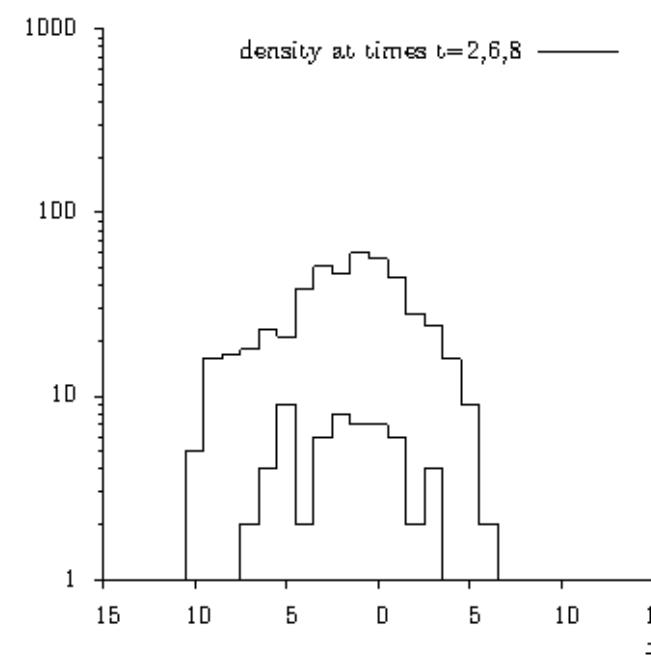
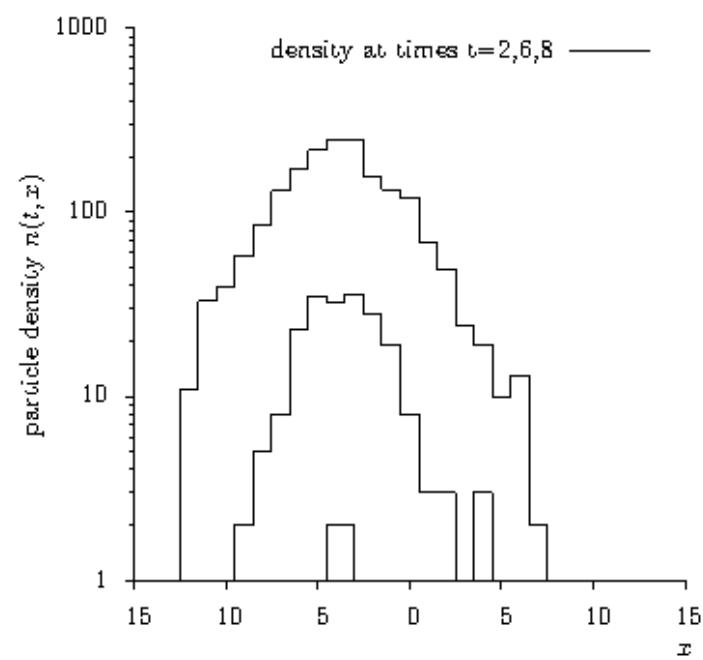
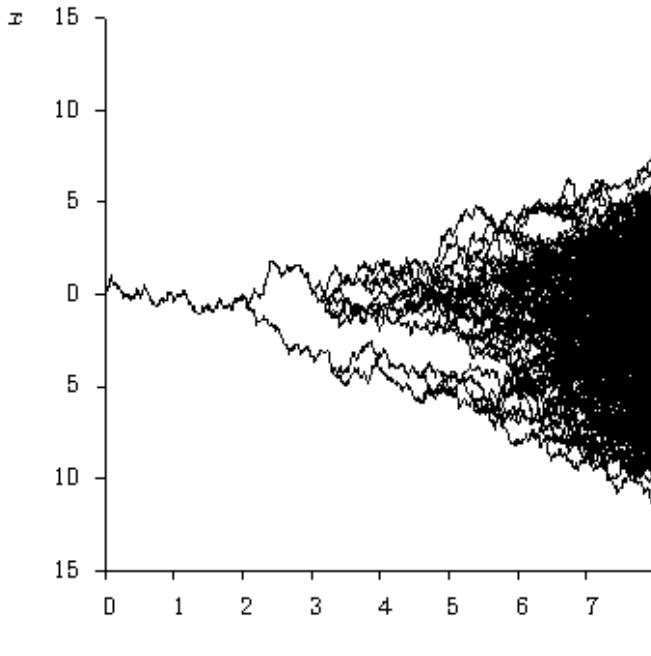
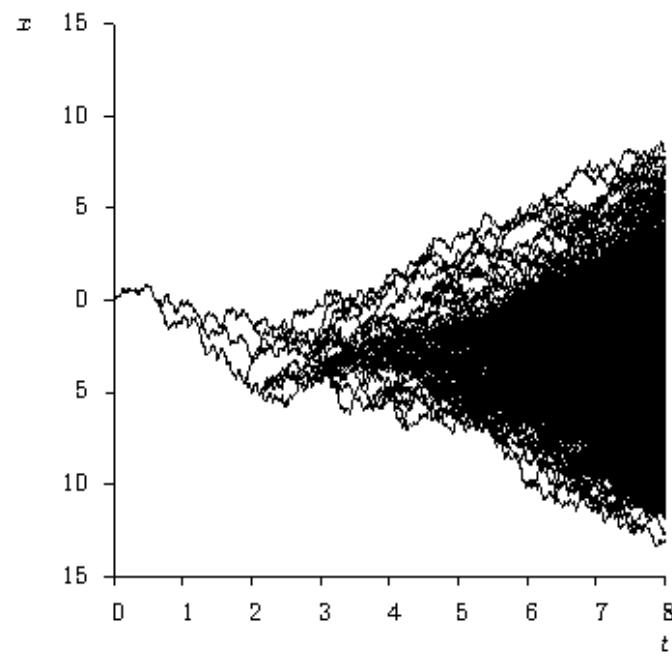
Branching random walk



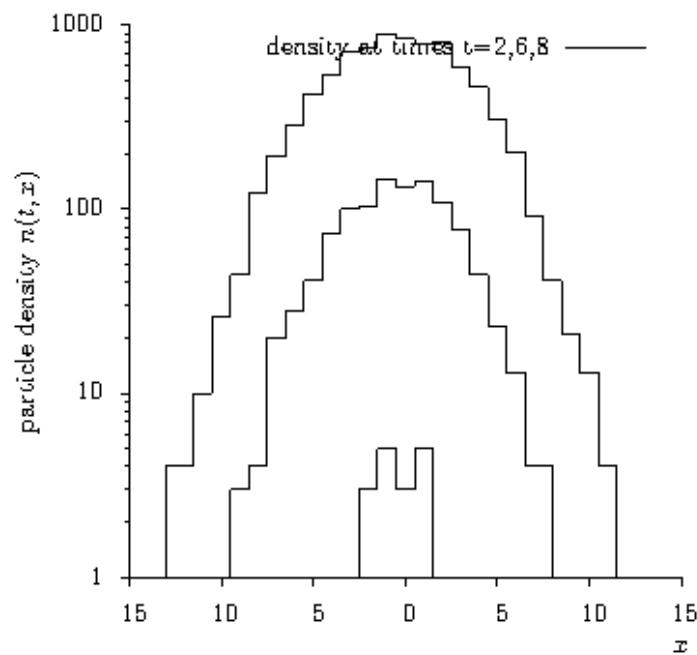
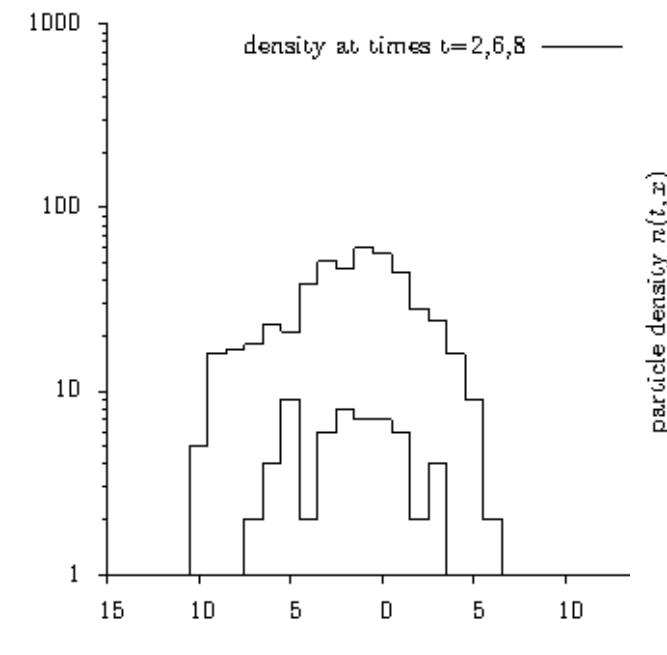
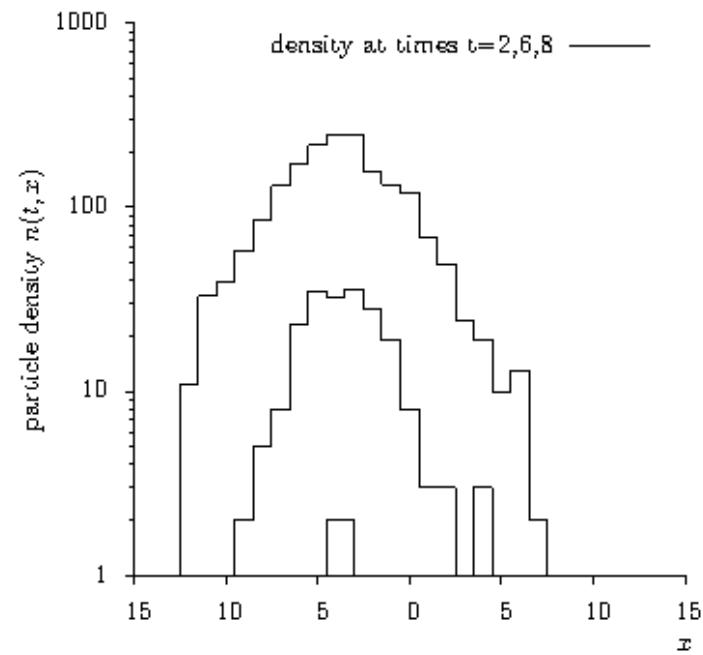
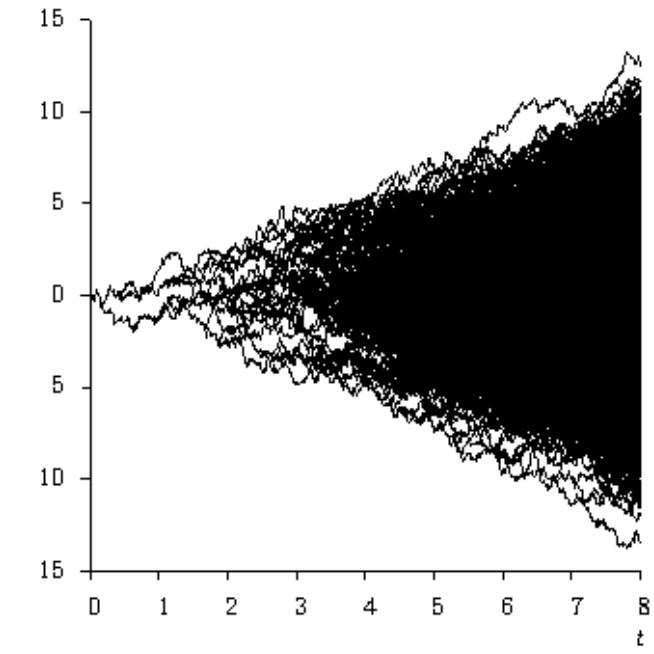
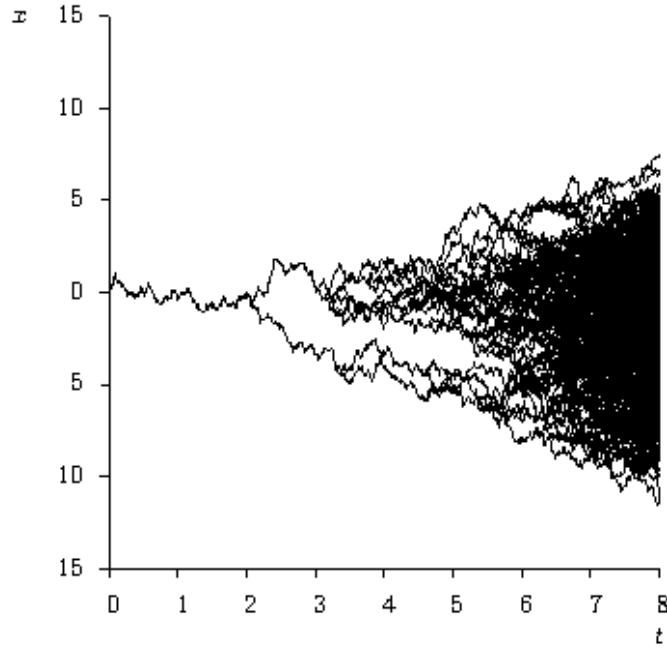
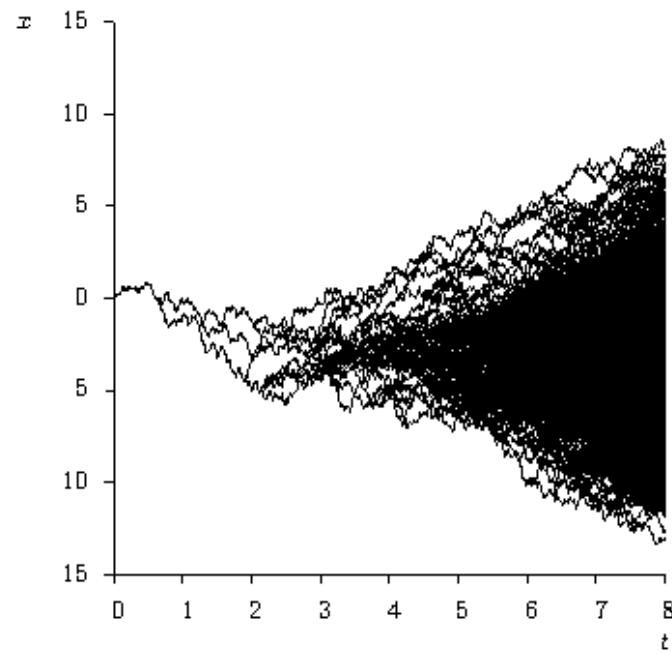
Branching random walk



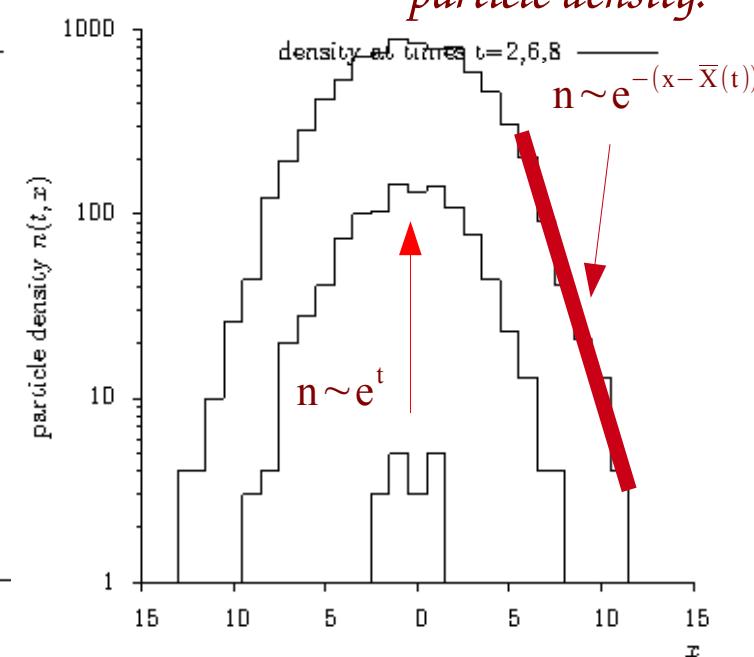
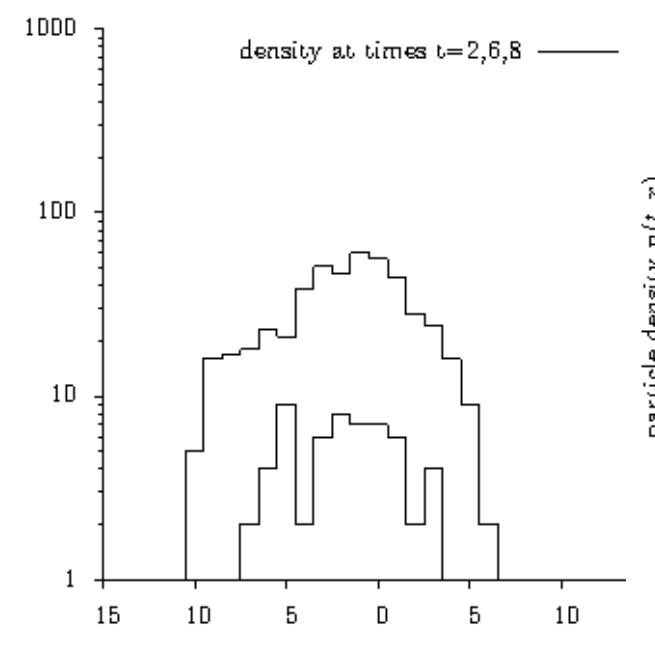
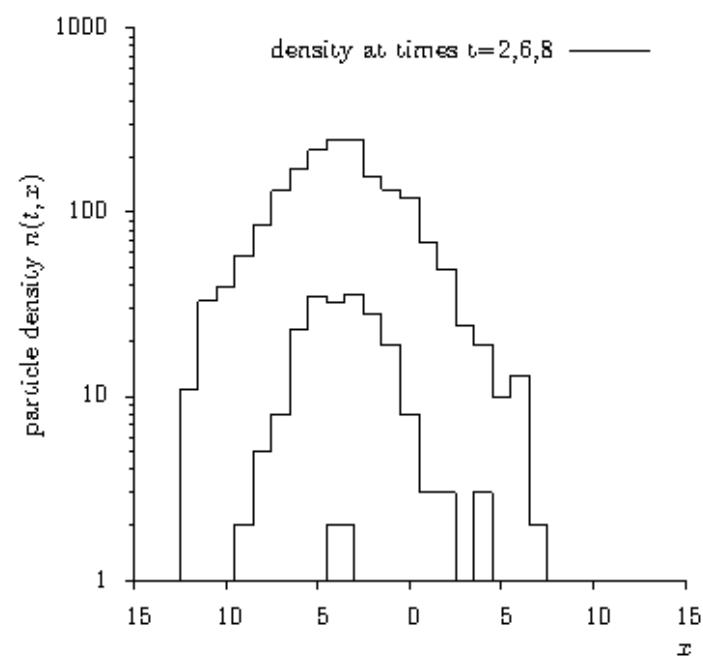
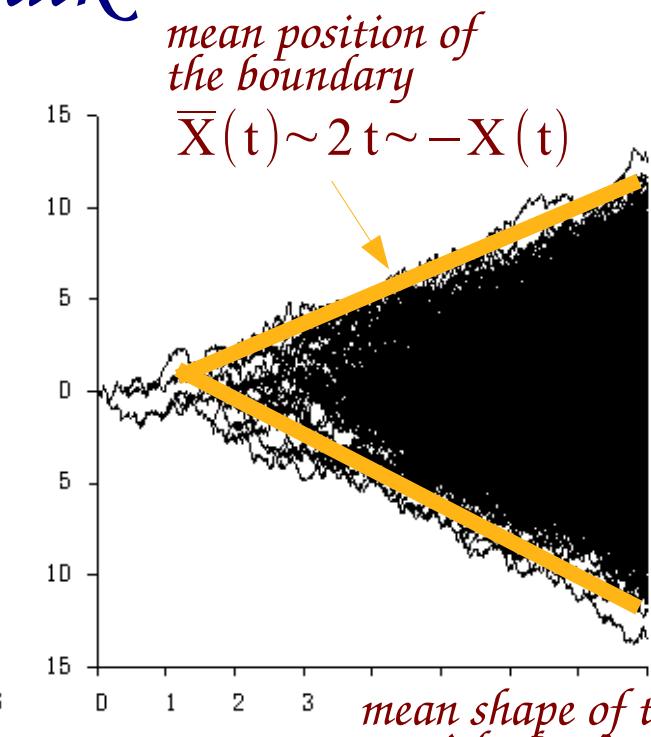
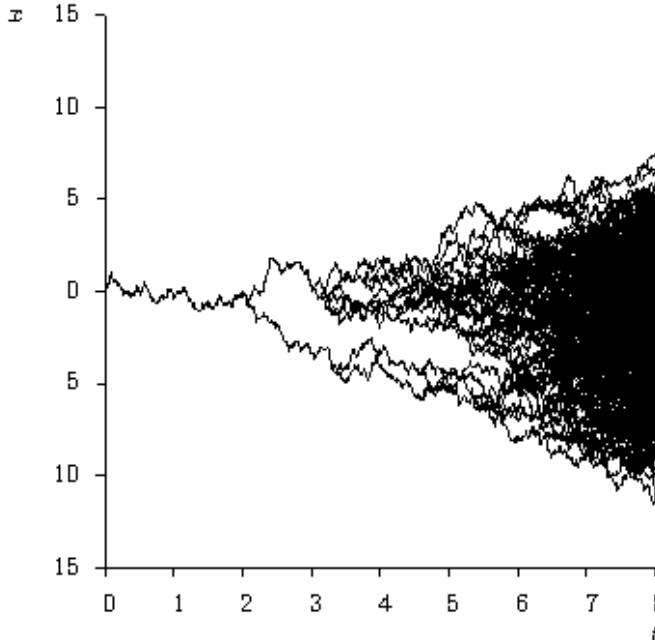
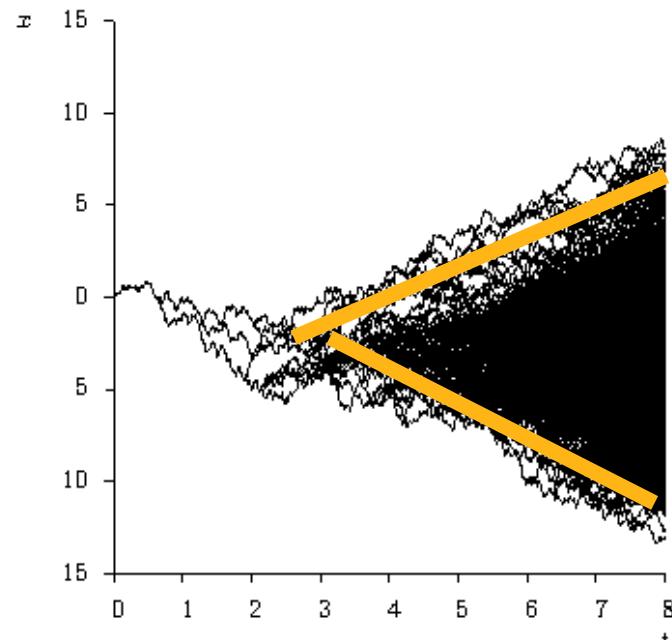
Branching random walk



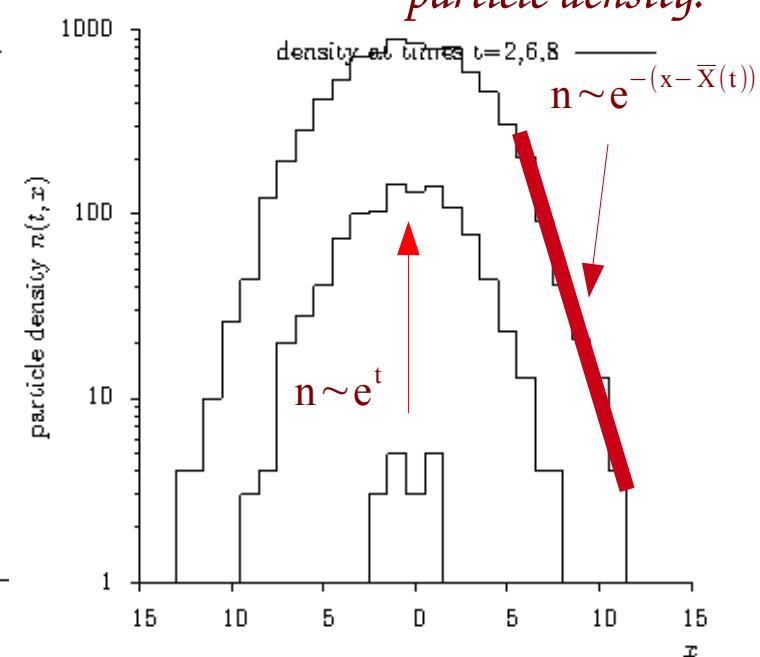
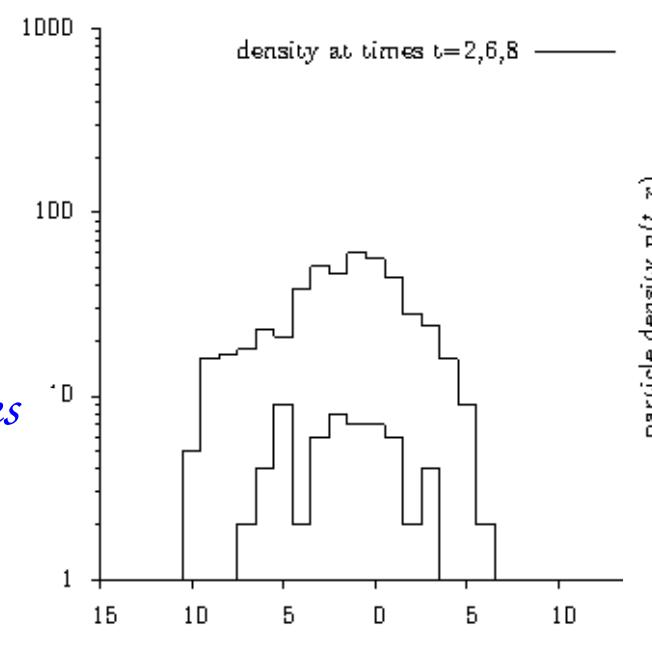
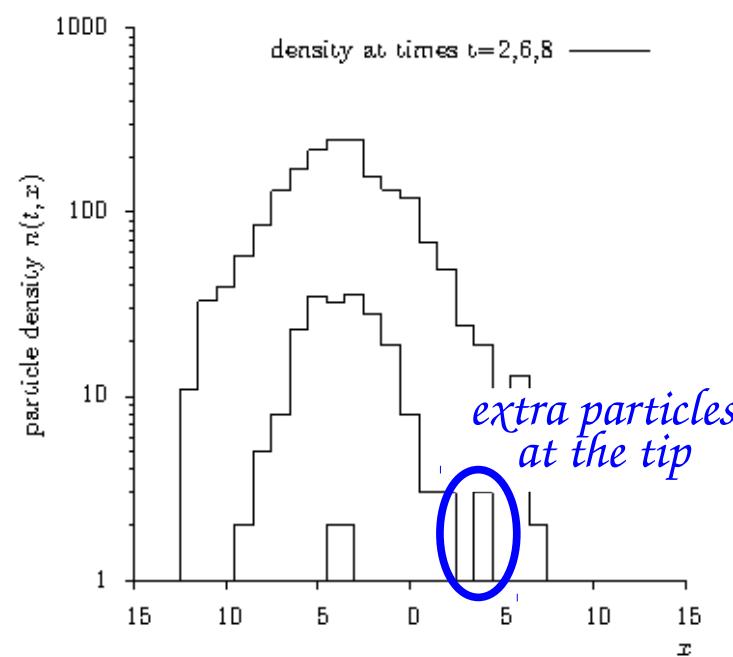
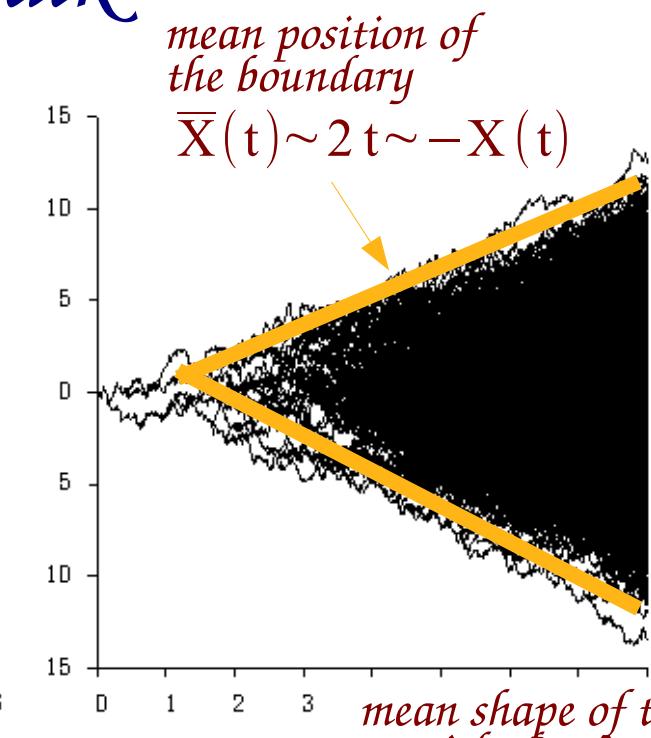
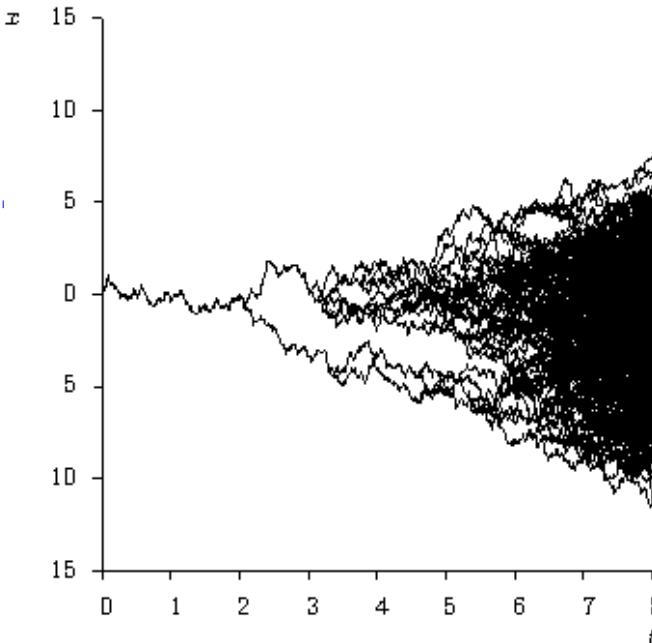
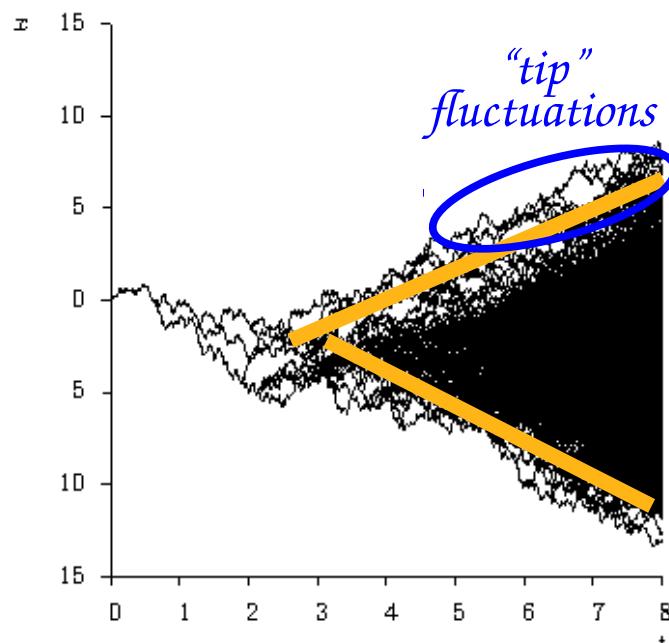
Branching random walk



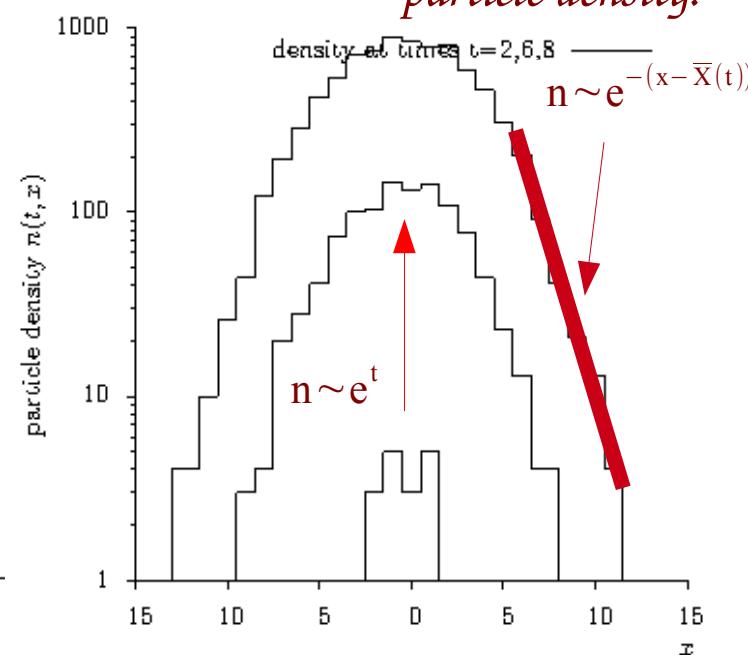
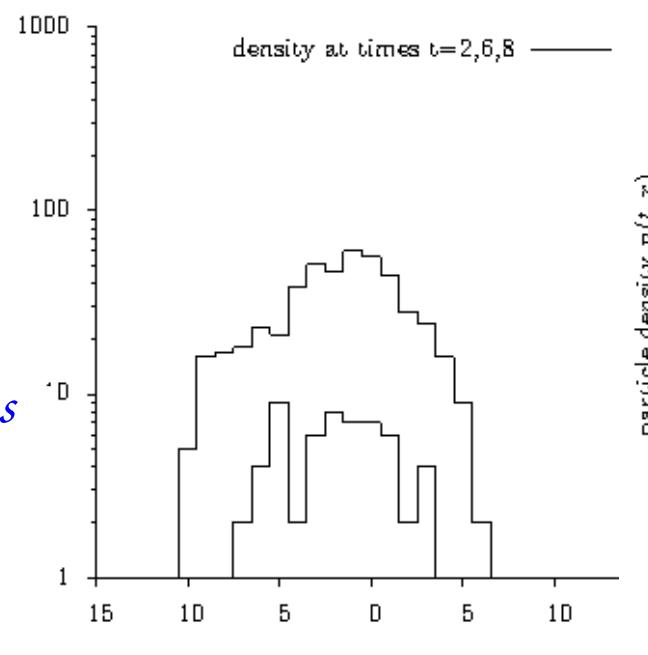
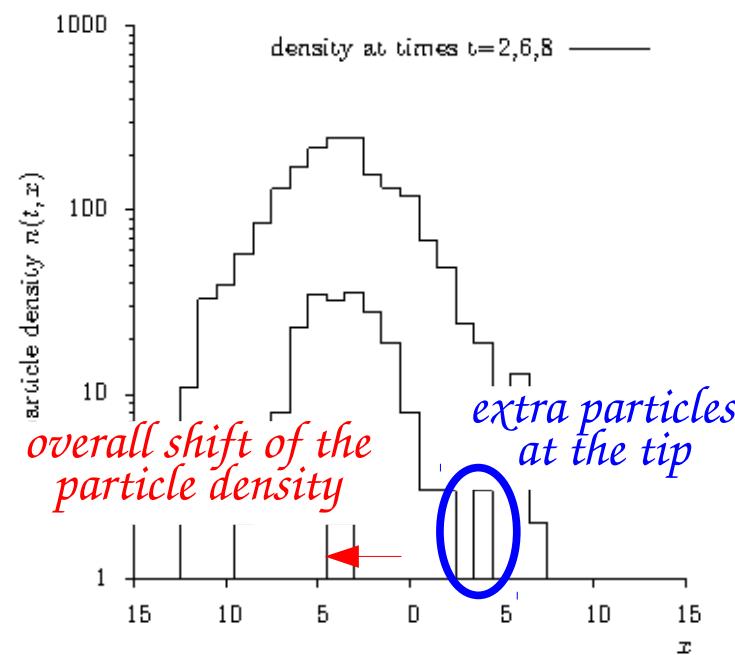
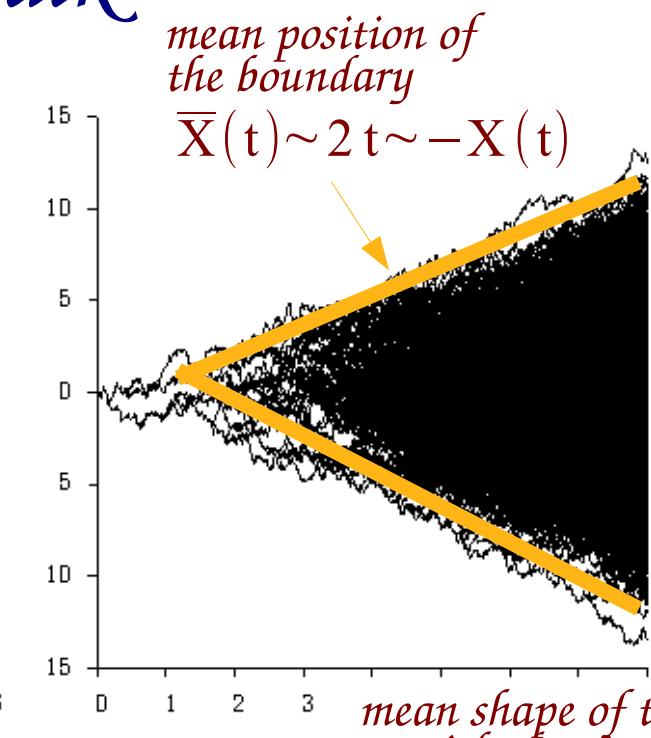
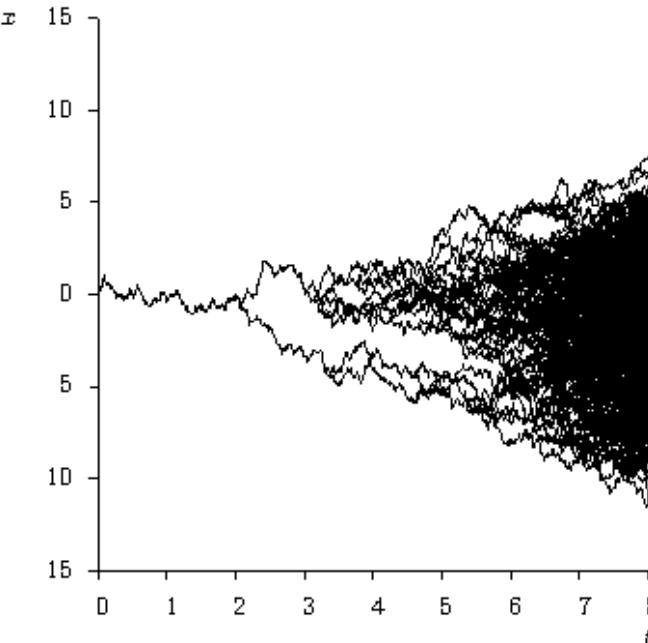
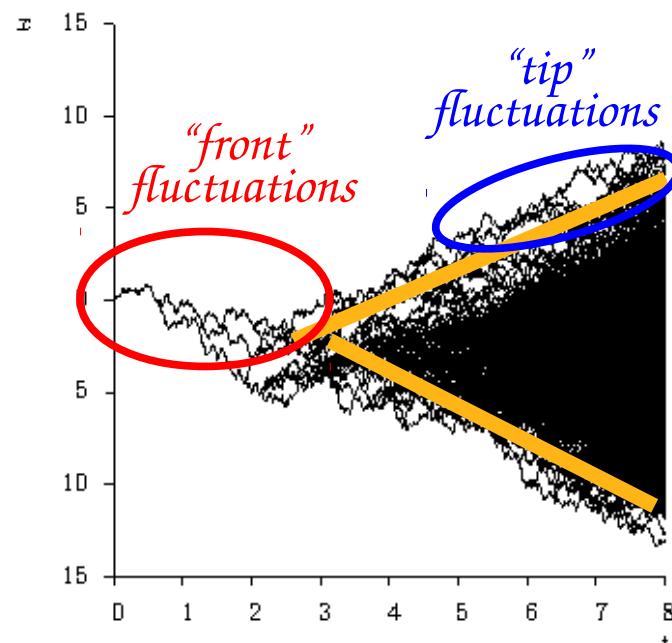
Branching random walk



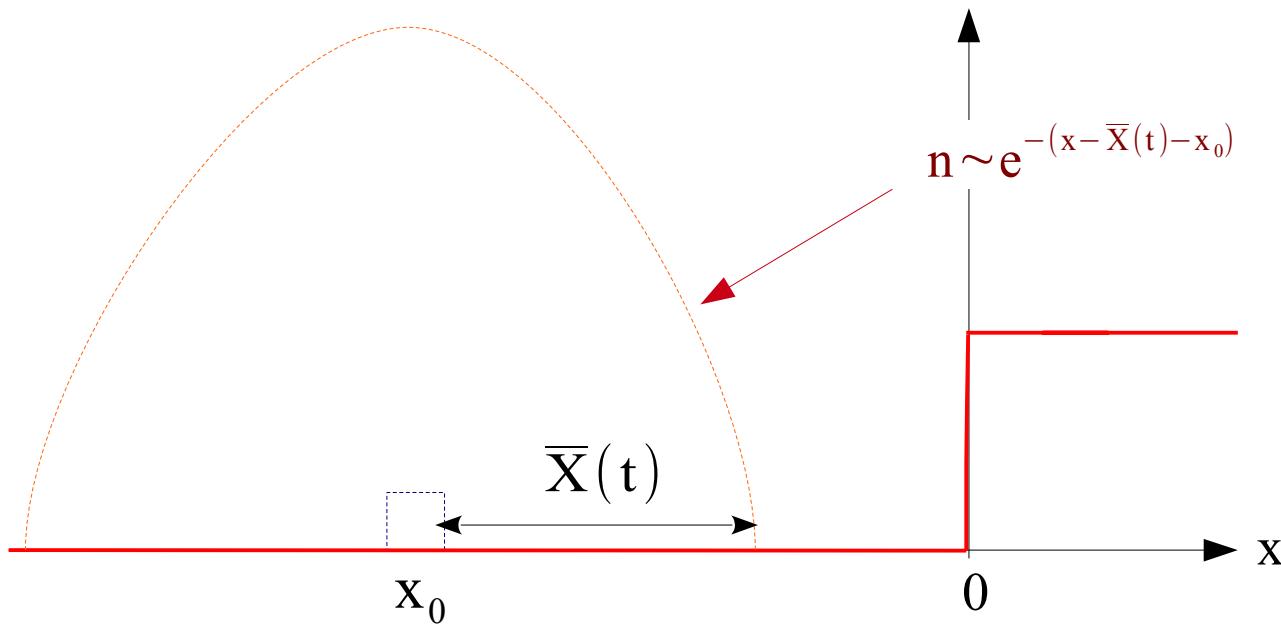
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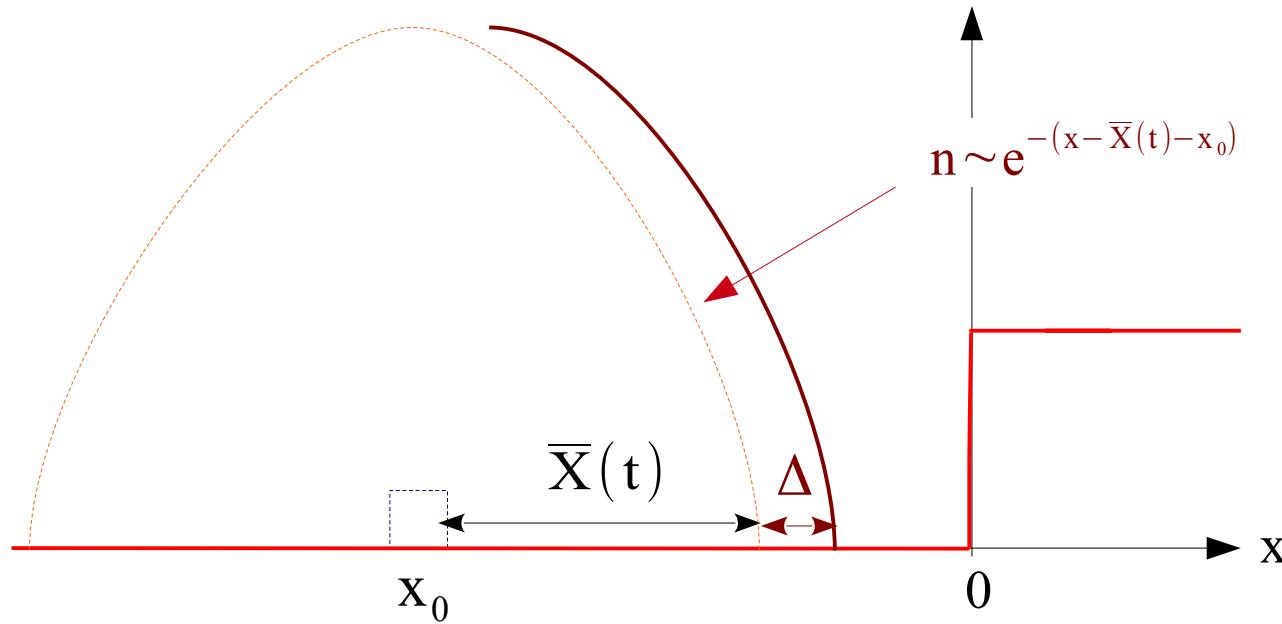
Branching random walk



Branching random walk

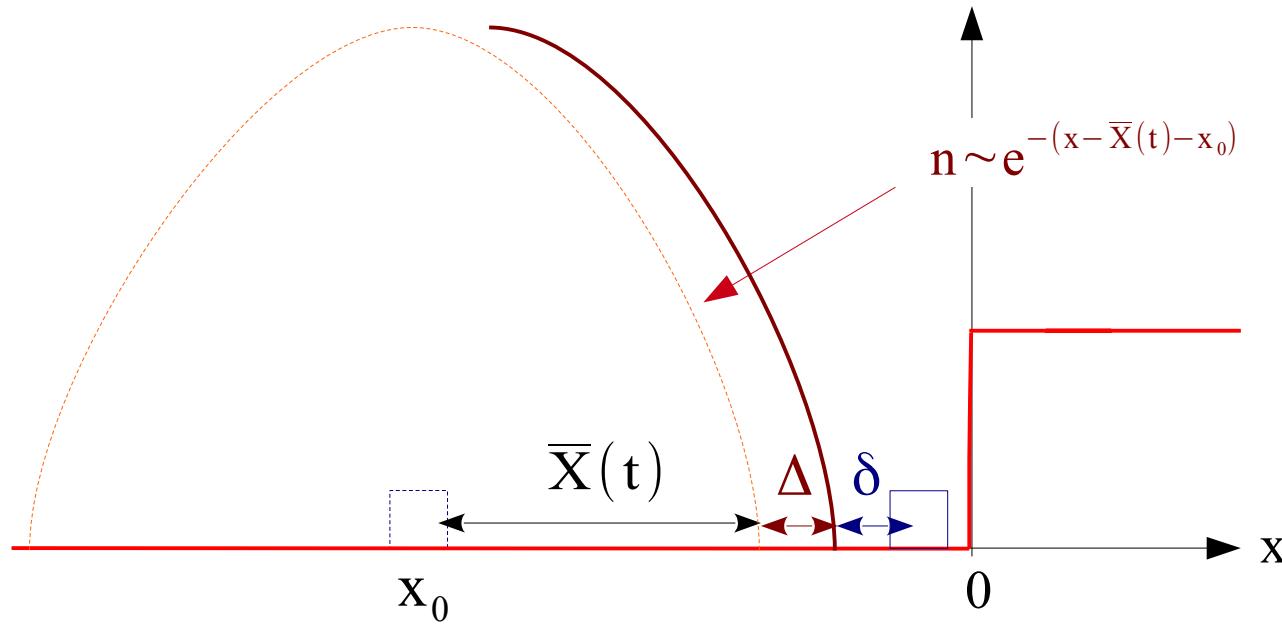


Branching random walk



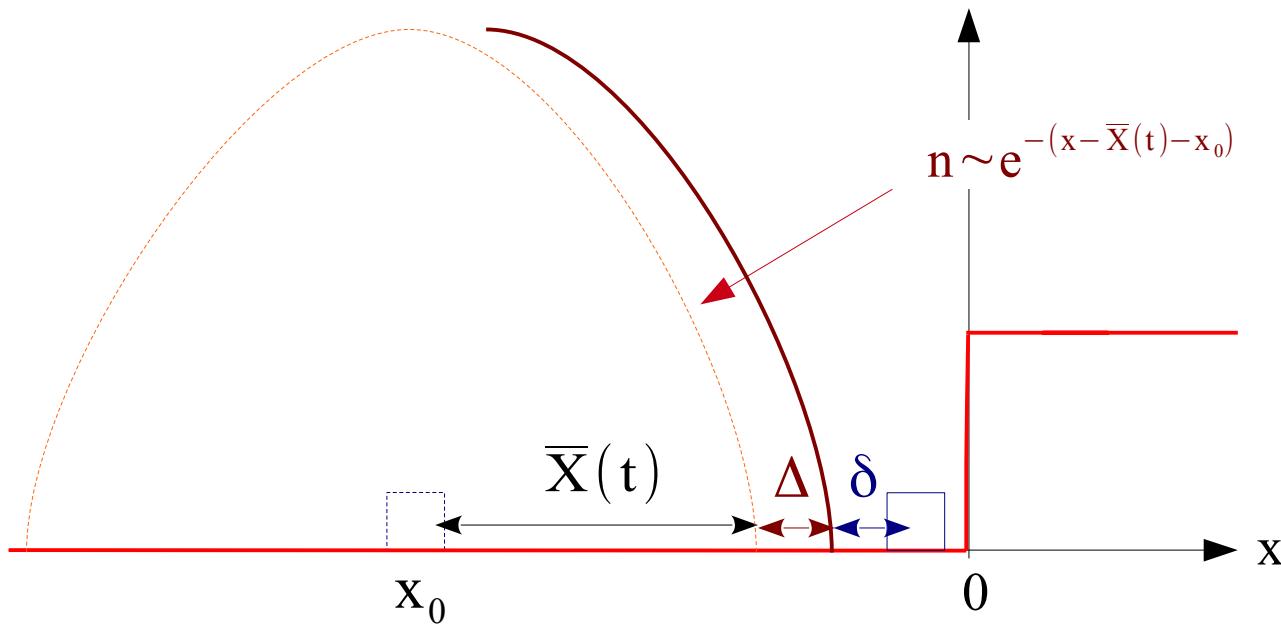
Conjecture: $p(\Delta) \sim e^{-\Delta}$

Branching random walk



Conjecture: $p(\Delta) \sim e^{-\Delta}$, $p(\delta) \sim e^{-\delta}$

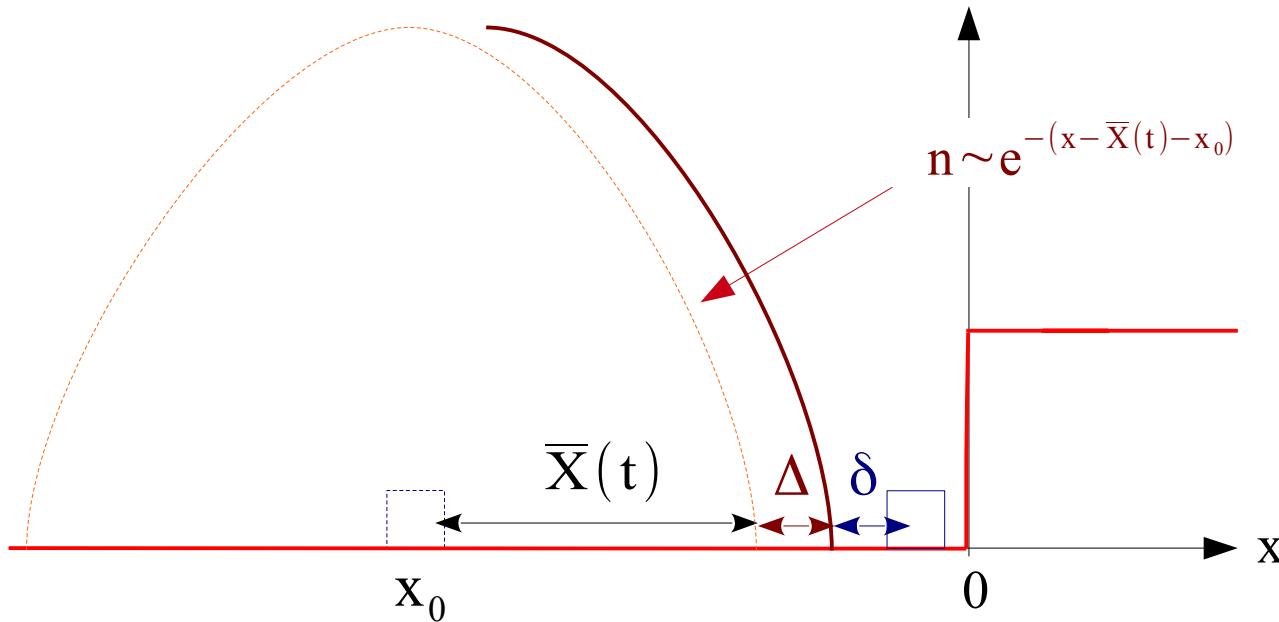
Branching random walk



$P =$ fraction of realizations in which $x_0 + \bar{X}(t) + \Delta + \delta > 0 = \langle \Theta(x_0 + \bar{X}(t) + \Delta + \delta) \rangle$

Conjecture: $p(\Delta) \sim e^{-\Delta}$, $p(\delta) \sim e^{-\delta}$

Branching random walk



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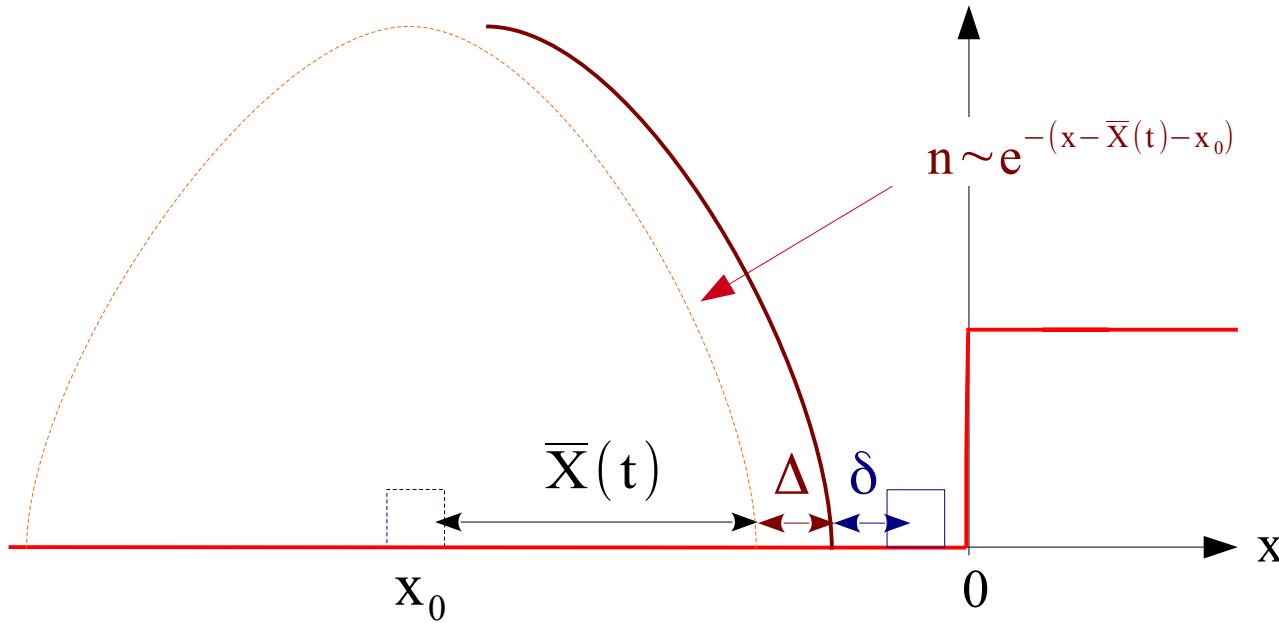
Conjecture: $p(\Delta) \sim e^{-\Delta}$, $p(\delta) \sim e^{-\delta}$

$$P(x_0, t) = \int d\Delta p(\Delta) \int d\delta p(\delta) \Theta(x_0 + \bar{X}(t) + \Delta + \delta)$$

$$\simeq (-\bar{X}(t) - x_0) e^{x_0 + \bar{X}(t)}$$

$$P(x_0, t) \simeq (X(t) - x_0) e^{x_0 - X(t)}$$

Branching random walk



$P =$ fraction of realizations in which $x_0 + \bar{X}(t) + \Delta + \delta > 0 = \langle \Theta(x_0 + \bar{X}(t) + \Delta + \delta) \rangle$

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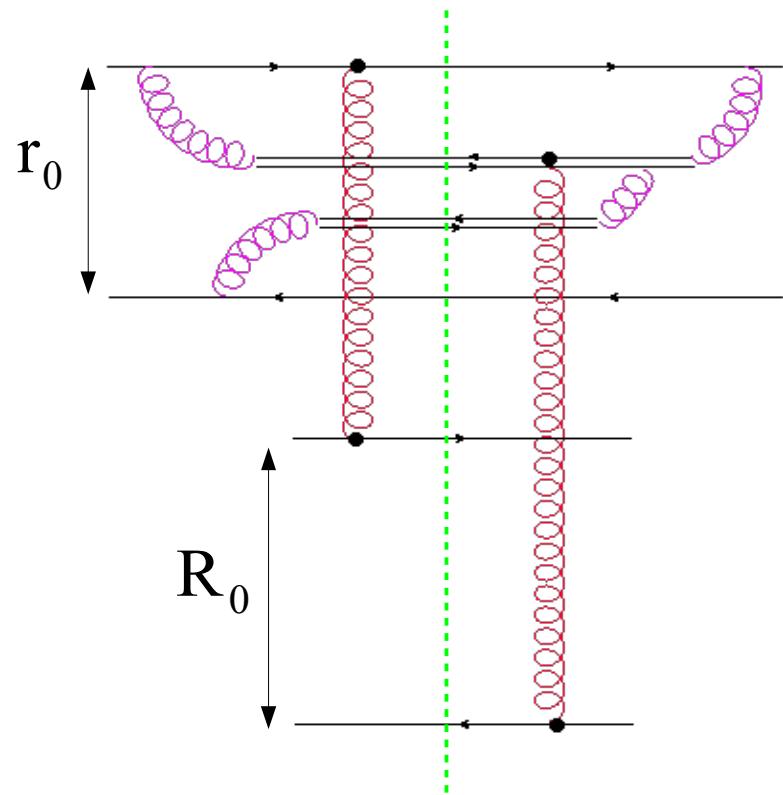
Consistent with solution to BK!

$$T(r_0, y) \simeq \ln \frac{1}{r_0^2 Q_s^2(y)} e^{\gamma_0 \ln[r_0^2 Q_s^2(y)]}$$

The shape of the dipole-nucleus scattering amplitude as a function of the dipole size is related to the fluctuations of the number of gluons in the QCD evolution

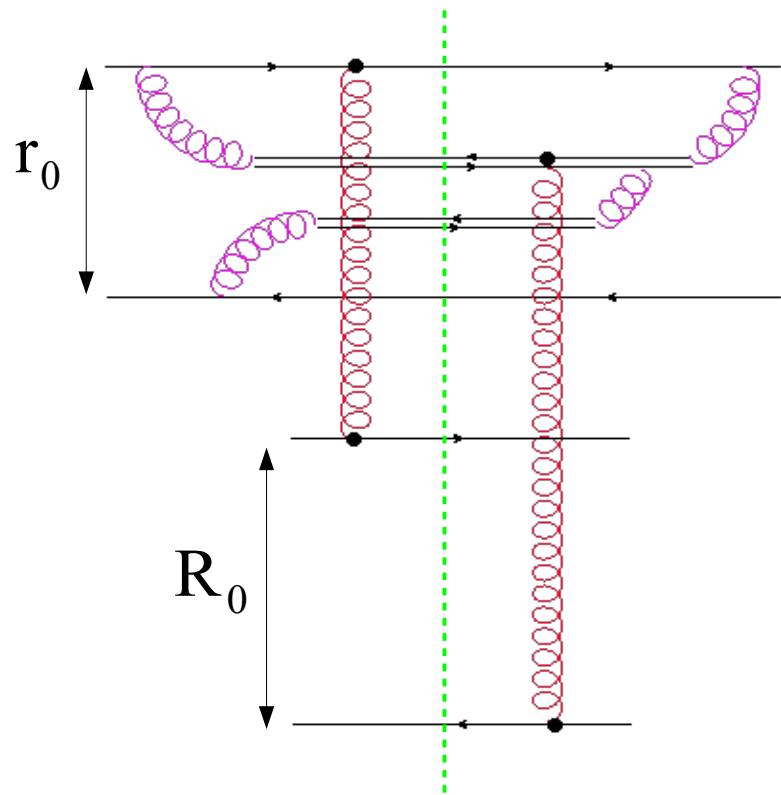
$$T(r_0, y) \underset{r_0 Q_s(y) \ll 1}{\sim} \ln \frac{1}{r_0^2 Q_s^2(y)} e^{y_0 \ln [r_0^2 Q_s^2(y)]}$$

Dipole-dipole scattering



$$T_{\text{1-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution}$$

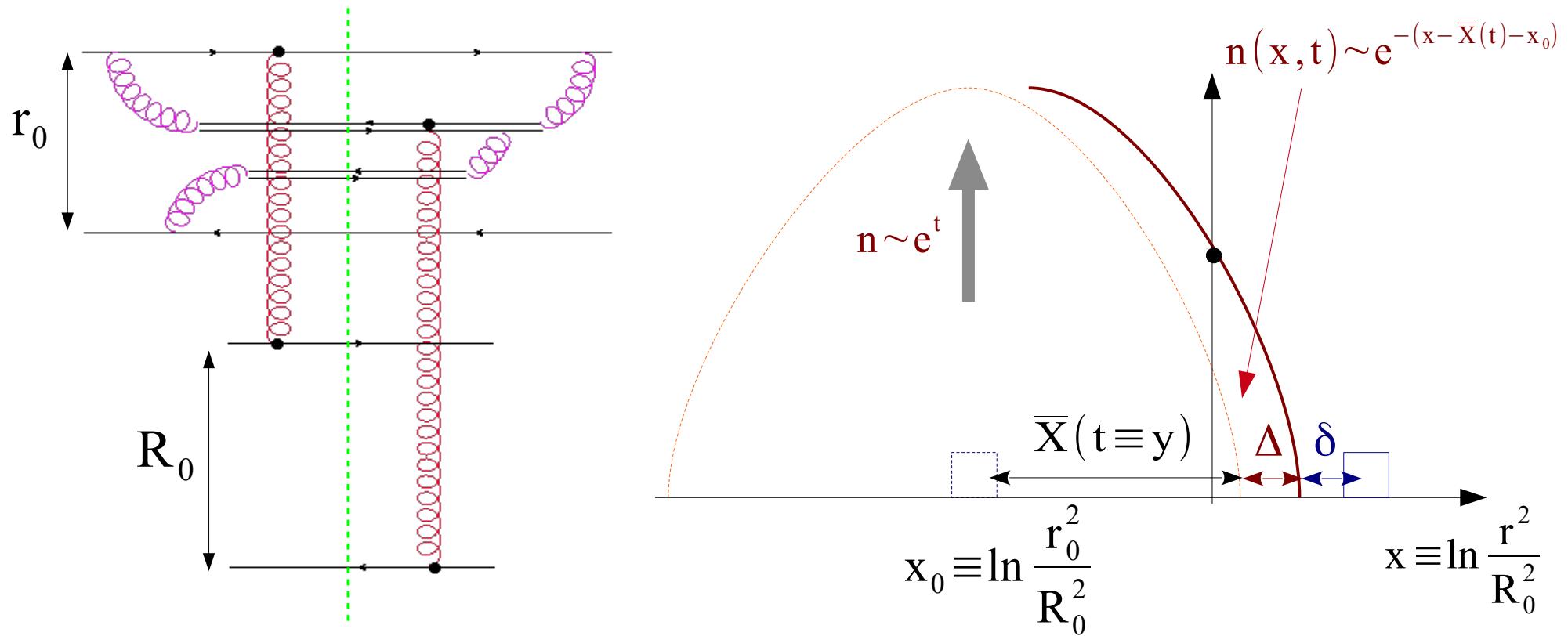
Dipole-dipole scattering



$$T_{\text{1-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution} = \alpha_s^2 \times n(x=0, y)$$

Probes the shape of the density of dipoles!

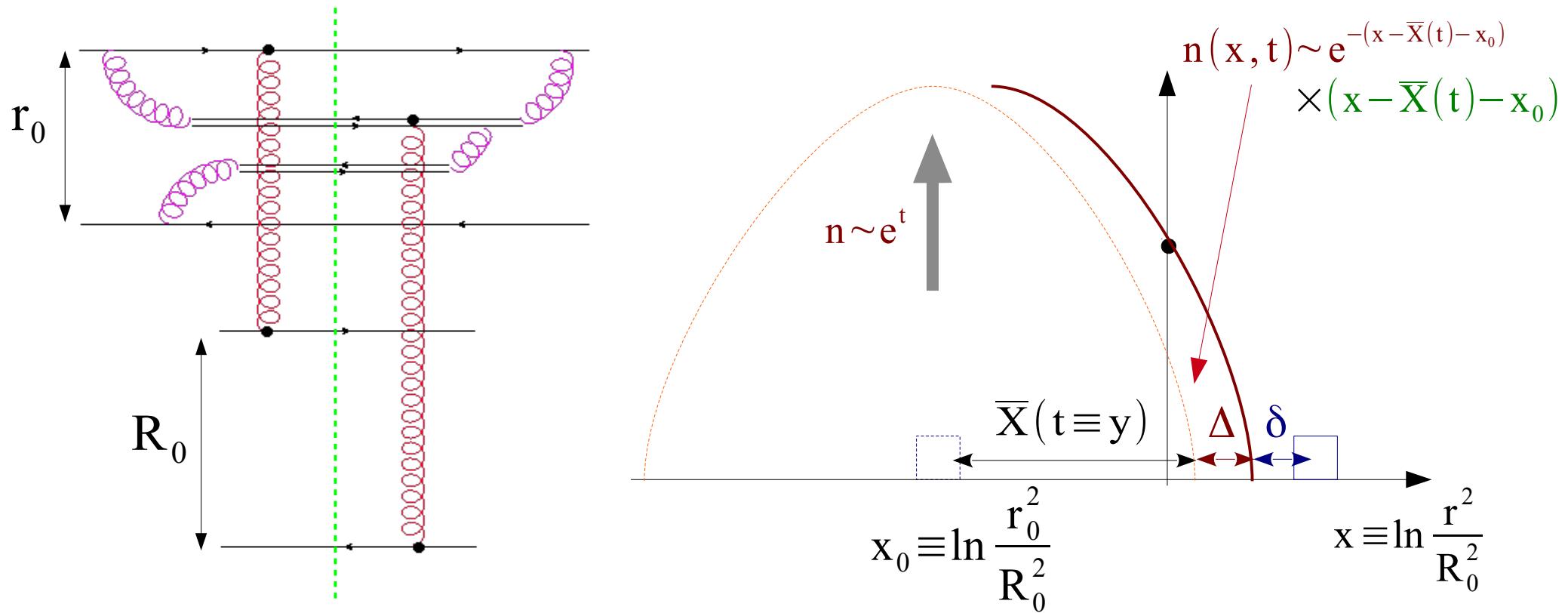
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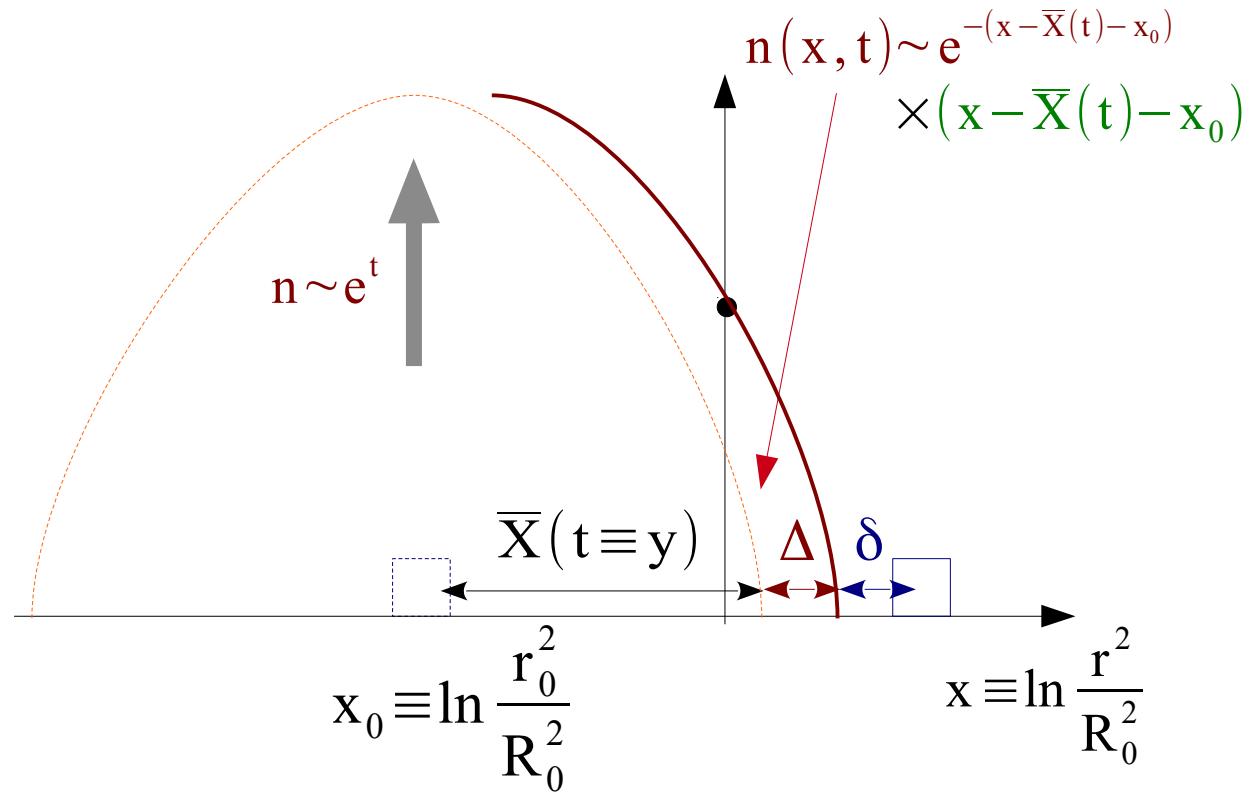
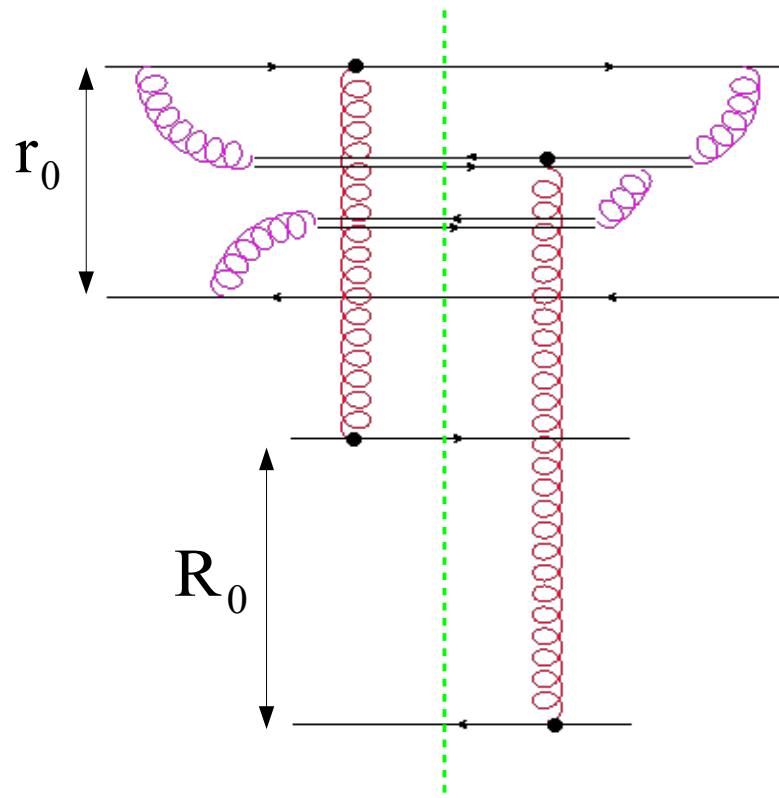
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Dipole-dipole scattering



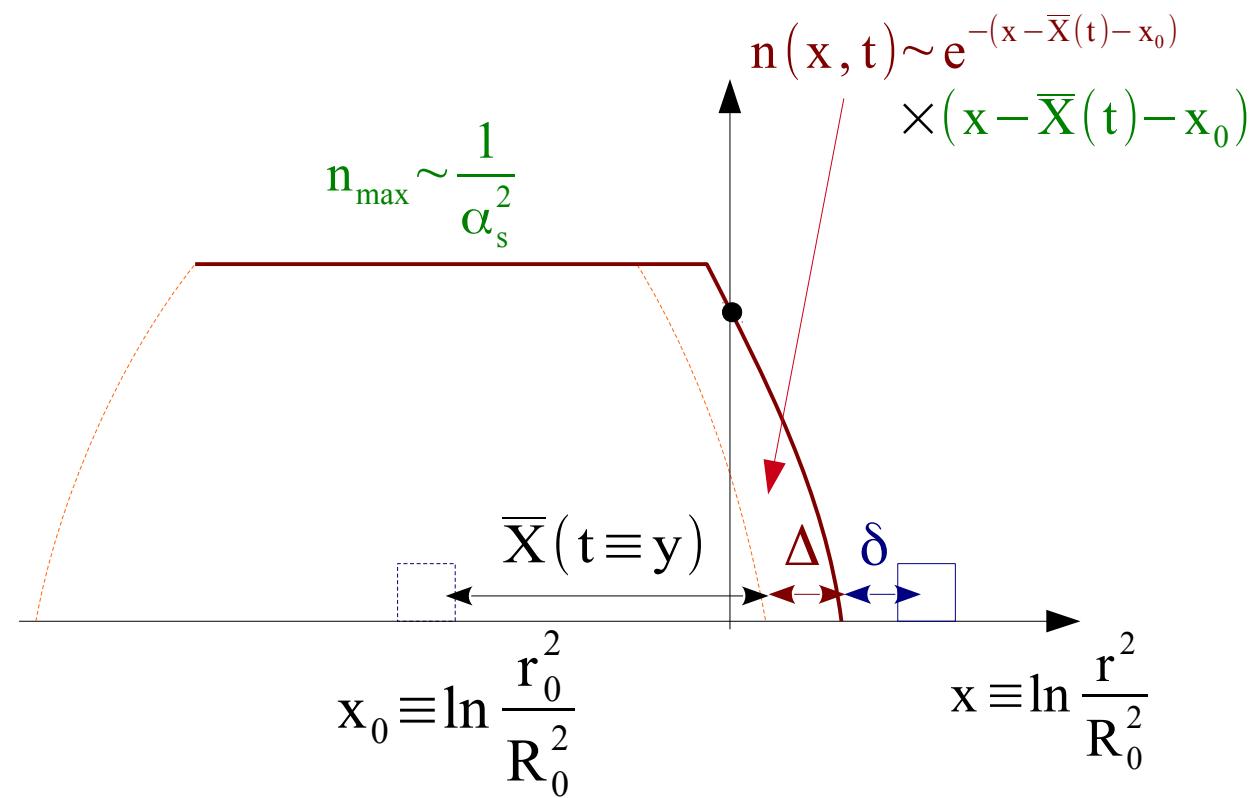
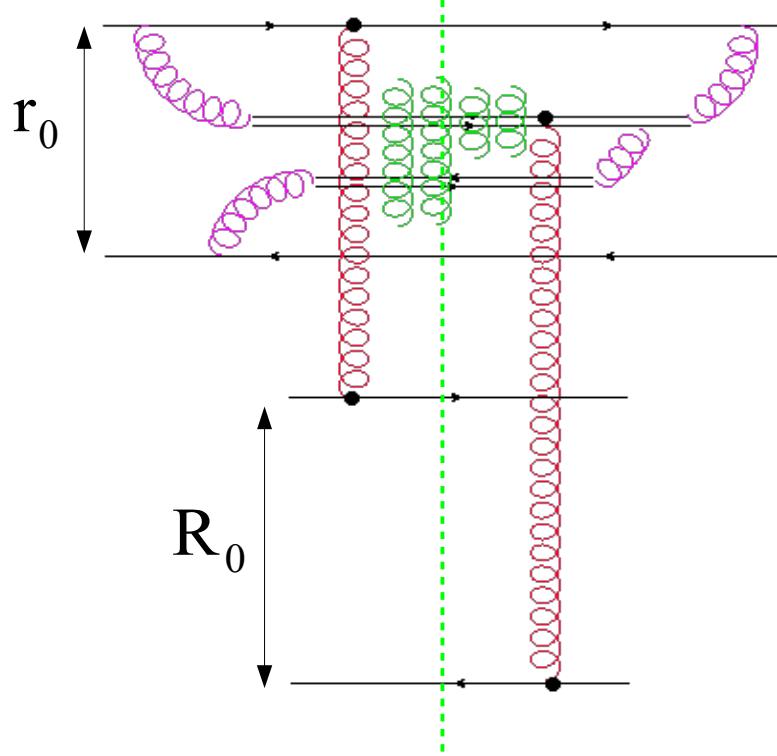
$$T_{\text{1-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution} = \alpha_s^2 \times n(x=0, y)$$

Probes the shape of the density of dipoles!

Since n grows exponentially, at some rapidity, unitarity would be violated!

Dipole-dipole scattering

Need nonlinear effects which are not described by the BK equation



$$T_{\text{1-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution} = \alpha_s^2 \times n(x=0, y)$$

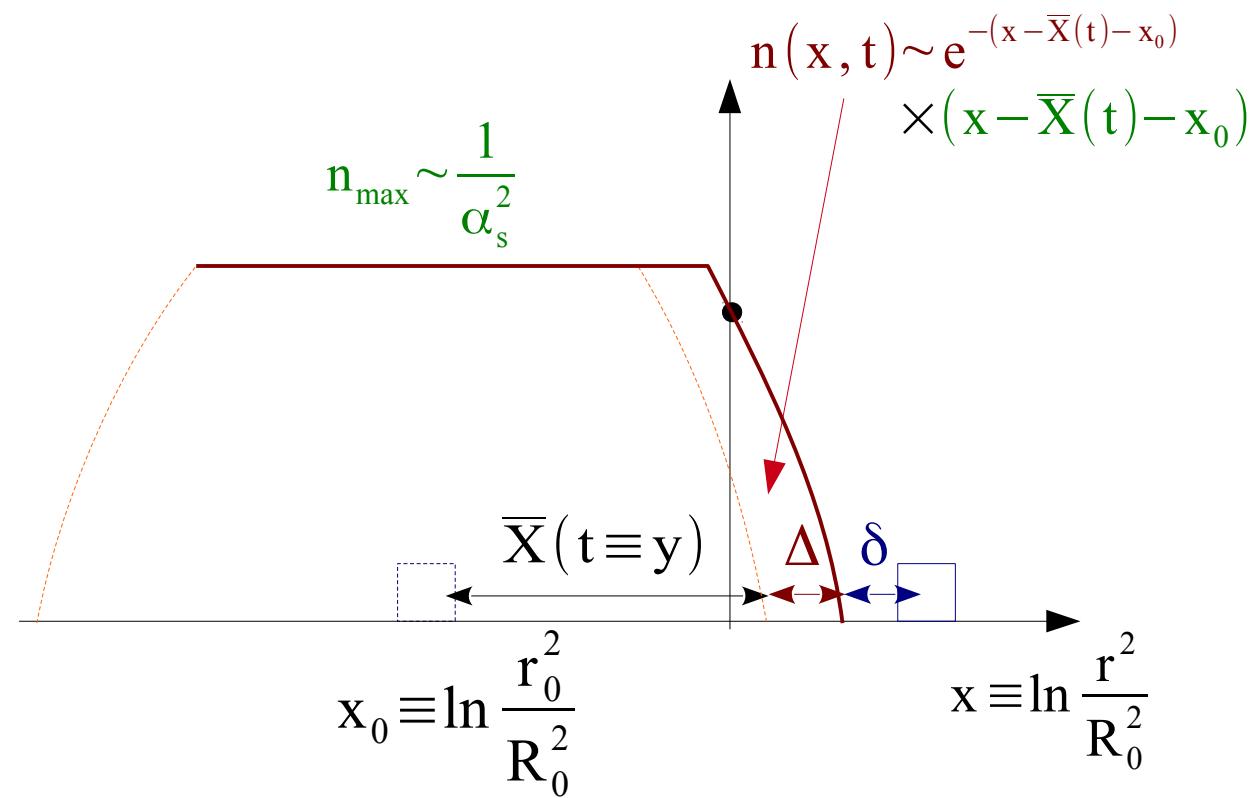
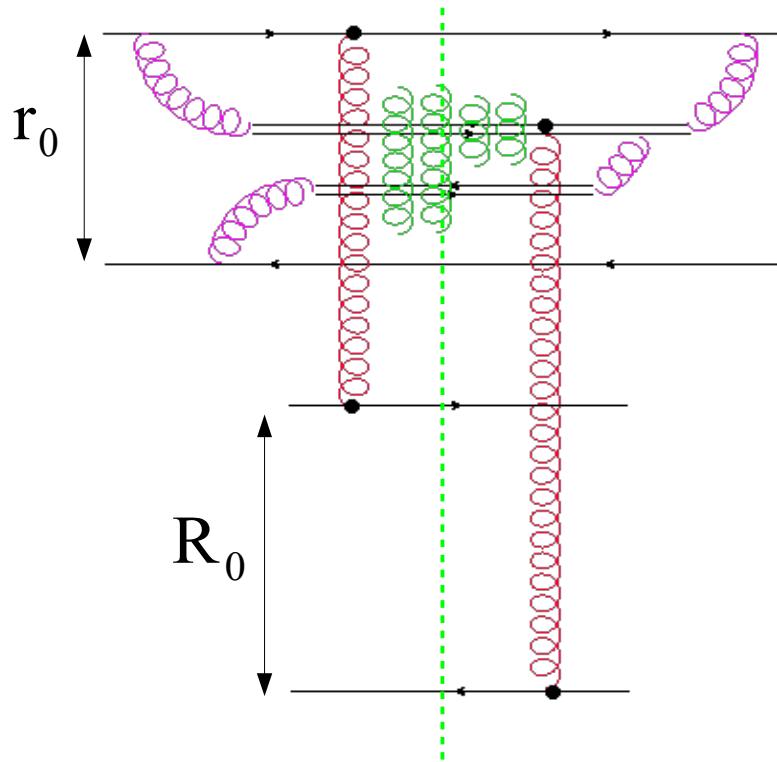
Probes the shape of the density of dipoles!

Since n grows exponentially, at some rapidity, unitarity would be violated!

One needs saturation of the gluons

Dipole-dipole scattering

Need nonlinear effects which are not described by the BK equation



$$T_{\text{1-event}}(r_0, y) = \alpha_s^2 \times \text{number of dipoles of size } R_0 \text{ after evolution} = \alpha_s^2 \times n(x=0, y)$$

$$T(r_0, y) = \langle T_{\text{1-event}}(r_0, y) \rangle \sim \left(\ln \frac{1}{r_0^2 Q_s^2(y)} \right)^2 e^{\gamma_0 \ln [r_0^2 Q_s^2(y)]}$$

The shape of the dipole-dipole scattering amplitude as a function of the dipole size is related to the shape of the gluon number density and to its fluctuations in the QCD evolution

$$T(r_0, y) \underset{r_0 Q_s(y) \ll 1}{\sim} \ln^2 \frac{1}{r_0^2 Q_s^2(y)} e^{y_0 \ln [r_0^2 Q_s^2(y)]}$$

Summary

The shape of dipole amplitudes is intimately related to the probability distribution of the fluctuations of the gluon number density, whose rapidity evolution is a branching random walk.

There is a qualitative difference between the dipole-nucleus and the dipole-dipole cases:

$$\text{Dipole-nucleus: } T(r_0, y) \underset{r_0 Q_s(y) \sim 1}{\sim} \ln \frac{1}{r_0^2 Q_s^2(y)} e^{y_0 \ln[r_0^2 Q_s^2(y)]}$$

$$\text{Dipole-dipole: } T(r_0, y) \underset{r_0 Q_s(y) \sim 1}{\sim} \ln^2 \frac{1}{r_0^2 Q_s^2(y)} e^{y_0 \ln[r_0^2 Q_s^2(y)]}$$

*valid in an intermediate rapidity regime: $y \ll \frac{1}{\alpha_s N_c} \ln^3 \frac{1}{\alpha_s^2}$. Beyond, everything becomes universal!

*Much more on fluctuations in parton evolution/branching random walks in
A.H. Mueller, S. Munier, arXiv:1404.5500, arXiv:1405.3131*

(In particular: parametric expressions for the saturation scale, picture of the fluctuations in different frames etc...)