Polarization Test of Higgs' Spin and Parity

Firooz Arash
Physics Department, Tafresh University
Tafresh, Iran

Diffraction 2014, Primosten, Croatia, Sept. 15

- Introduction

I will use two basic and general concepts to distinguish spin and parity states of $J^p = 0^+ , 0^- , 2^+ , 2^-$ in $gg \rightarrow \gamma\gamma$.

A set of observables with one or both photon polarization specified will also be identified.

Usual Approach for the determination of the Spin and the parity :
In the decay of a resonance X

$$X \rightarrow ZZ, W^+W^-, \gamma\gamma \quad and \quad X \rightarrow t\bar{t}$$

Look for the dependence of cross section on the angle θ^*

Angle between momentum of one of the initial partons and the decaying particle.

Expectation is that cross-section behaves differently with θ^*
For different value of the spin of X

Vector bosons decay into 4 leptons. Their angular distribution give information on the spin and the parity of the resonance

- Investigate the invariant mass distribution of Higgs in a final state HV

These involve certain dynamical assumptions, e.g on the coupling of Higgs to other particles, and thus, making them somewhat Model dependent

- A model independent approach: Spin Observables

We restrict ourselves to $\gamma\gamma$ final state

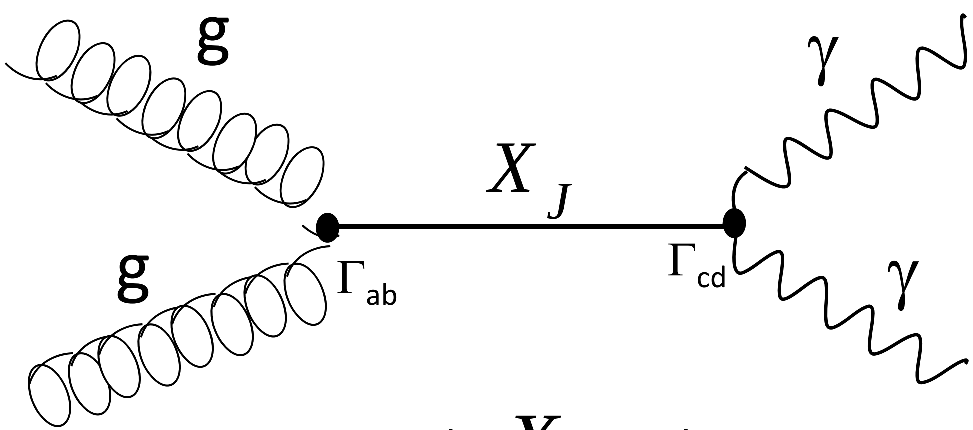
At High energies only the amplitude description of a reaction is practically feasible. Thus, one needs to choose a particular amplitude formalism. For a given reaction, and for a given set of symmetries holding for that reaction, the number of independent amplitudes describing that reaction is the same in any formalism.

Number of Amplitudes depend on the spin of the participating particles and the symmetries applied.

In the case of $gg \to \gamma\gamma$

All particles are massless spin 1 $\longrightarrow$ each particle has only 2 spin states

Polarization Test of Higgs Resonance and Its Spin and Parity In $gg \to \gamma\gamma$



$$gg \to X_J \to \gamma\gamma$$

a, b, c, d are spin components along some direction

J- constraint:

All amplitudes with $|a-b| > J$ or $|c-d| > J$ must vanish

IF two Amplitudes $D(c_1, d_1; a_1, b_1)$ *and* $D(c_2, d_2; a_2, b_2)$ are *J*-forbidden, then all eight observables formed from bilinear products of them and permuted amplitudes will vanish. The same is true if the two amplitudes are *J*-allowed but if $a_1 + b_1 + a_2 + b_2 = 2J + \beta$

Factorization constraint: *A second constraint arises from the J-constraint on the two vertices.The interaction depicted in the Figure can be visualized as the product of two non-overlapping independent parts; each containing a set of physical particles plus the resonance. Each part is a three particle vertex and at each vertex the number of vertex amplitudes is restricted by the J-constraint. A simple three- particle vertex can at most have*

$$N_{J\,s_1 s_2} = (2s_1+1)(2s_2+1)(2J+1) \quad \text{Amplitudes}$$

Then, the number of three-point amplitudes in the overall factorizable four-particle process with an arbitrary *J*- resonance is at most

$$N^f = \left[(2s_1+1)(2s_2+1)+(2s_3+1)(2s_4+1)\right](2J+1)$$

But The OVERALL reaction has only

$$N = (2s_1+1)(2s_2+1)(2s_3+1)(2s_4+1) \quad \text{Amplitudes}$$

Since one must have $N^f < N$, *Depending on the value of J, this inequality may or may not reduce the number of amplitudes.*

In fact, for J > 0 the inequality does not hold and no reduction occus.

The number of three-particle vertex amplitudes enumerated above does not take into account the *J* -constraint. In a real process in which

$$J \rightarrow s_1 + s_2 \ \textit{then}\ S_{1z} + S_{2z} \ \textit{must not exceed}\ J$$

Imposition of these considerations reduces the number of vertex amplitudes to

$$N_{J s_1 s_2} = (2s_1+1)(2s_2+1) \qquad J \geq s_1 + s_2$$

$$N_{J s_1 s_2} = (2J+1)(s_1+s_2-J)+(J+s_2-s_1+1)(J-s_2+s_1+1) \qquad s_1+s_2 \geq J \geq s_2 - s_1$$

$$N_{J s_1 s_2} = (2s_1+1)(2J+1) \qquad s_2 - s_1 \geq J$$

So, the number of independent amplitudes N_J^f from the *J*-constraint and the Factorization constraint combined reduces to

$$N_J^f = N_{J\,s_1 s_2} + N_{J\,s_3 s_4}$$

For the s-channel resonance, these constraints are most simply expressed for the center of mass helicity amplitudes

$$D(c,d\,;a,b) = \sum_J D_J\,(c,d\,;a,b)\, d^{\,J}_{c-d\,,a-b}\,(\theta)$$

For a resonance of spin *J* the spin dependence factorizes as follows

$$D(c,d\,;a,d) \propto \Gamma'^{J}_{c\,,d}\;\Gamma^{J}_{ab}$$

Γ*'s are proportional to vertex functions having the requisite number of* N_{Js1s2} *independent component of Eq.*

$$D\left(c,d\,;a,d\right) \propto \Gamma'^{J}_{c,d}\,\Gamma^{J}_{ab}$$

Gives non-linear relations:

$$D_J\left(c,d;a,b\right)D_J\left(c',d';a',b'\right)=D_J\left(c',d;a',b\right)D_J\left(c,d';a,b'\right)$$

among the helicity amplitudes that lead to complicated relations among the observables

However, when parity is conserved, the above equation simplifies

Then for s_a, s_b and for the intrinsic parities η_a, η_b the vertex satisfies

$$\Gamma^{J}_{-a,-b}=\eta_J\eta_a\eta_b\left(-1\right)^{s_a+s_b}\Gamma^{J}_{a,b}$$

So, we get

$$D_J\left(c,d\,;a,b\right)=\pm D_J\left(-c,-d\,;a,b\right)$$

For the reaction $gg \to \gamma\gamma$ With the pertinent symmetries

Only 5 helecity amplitudes

$$A_1 \equiv D(+,+;+,+) \quad , \quad A_2 \equiv D(+,+;+,-) \quad , \quad A_3 \equiv D(+,-;+,-)$$
$$A_4 \equiv D(+,+;-,-) \quad , \quad A_5 \equiv D(+,-;-,+)$$

Applying the constraints to amplitudes, leads to the following relations between the pairwise amplitudes for a resonance with $\eta_J = \pm 1$

$$\left.\begin{array}{l} D(c,d;a,b) = \sum_J D_J(c,d;a,b)\, d^J_{c-d,a-b}(\theta) \\ \Gamma^J_{-a,-b} = \eta_J \eta_a \eta_b (-1)^{s_a+s_b}\, \Gamma^J_{a,b} \end{array}\right\} \Rightarrow \quad A_1^J = \pm A_4^J \quad , \quad A_3^J = \pm A_5^J$$

Resonance signature

$$A_1 \ and \ A_4 \ both \ go \ with \ d^J_{00}(\theta)$$

$$A_1 \text{ and } A_4 \text{ both go with } d^J_{00}(\theta) \text{ so we get:}$$

$$\boxed{A_1 = \pm A_4}$$

$$\text{With no restriction on } A_2, A_3, \text{and } A_5$$

Due to *J*- constraint, for a spin zero resonance, X_0 we also have

$$A_2, = A_3 = A_5 = 0$$

$\Rightarrow$ *For* X_0 *resonance,*

there is only one independent helicity amplitude and as such, all obervables with one or both particle's polarization specified vanish uniquely. Thus, we are left with only unpolarized cross-section propotional to $|A_1|^2 = |A_4|^2$

Our amplitude test in general does not prohibit the formation of a resonance state with spin 1. However, the decay of such state into two photons are forbidden by Landau-Yang theorem
L. D. Landau, Dokl. Akad. Nawk. **USSR 60** (1948) 207.
C. N. Yang, Phys. Rev. **77** (1950) 242.
It is also true that a spin 1 color singlet state cannot be produced in gluon fusion. Therefore, we will not consider this case.

Going to *J=2* resonance, All 5 amplitudes contribute to $gg \to \gamma\gamma$ in this case, however, there are a number of observables with one or both photons polarizations are specified, which need not be zero.
Applying the condition $A_1 = \pm A_4$ to those observables, gives the relationships between observables and Amplitudes as in the Table for even and odd parity X_2 resonance

Observable	$J = 0^{\pm}$	$J = 2^{+}$	$J = 2^{-}$
σ	$4\lvert A_1\rvert^2$	$4\lvert A_1\rvert^2 + 8\lvert A_2\rvert^2 + 4\lvert A_3\rvert^2$	$4\lvert A_1\rvert^2 + 8\lvert A_2\rvert^2 + 4\lvert A_3\rvert^2$
$(A, A; A, -I)$	0	$4\,Im\left(A_1 A_2^*\right)$	$-4\,Im\left(A_2 A_3^*\right)$
$(A, A; \mathcal{R}, R)$	0	$-2\,Re\left(A_1 A_2^*\right)$	$2\,Re\left(A_2 A_3^*\right)$
$(A, A; R, R)$	0	$4\left\{\lvert A_1\rvert^2 + \lvert A_3\rvert^2\right\}$	$-4\left\{\lvert A_1\rvert^2 + \lvert A_3\rvert^2\right\}$
$(A, A; I, -I)$	0	$4\left\{\lvert A_1\rvert^2 + 2\lvert A_2\rvert^2 - \lvert A_3\rvert^2\right\}$	$-4\left\{\lvert A_1\rvert^2 - 2\lvert A_2\rvert^2 - \lvert A_3\rvert^2\right\}$
$(A, A; \Delta, R)$	0	$-4\,Re\left(A_1 A_2^*\right)$	$4\,Re\left(A_2 A_3^*\right)$

$\mathcal{R}$: *Right circular Polarization*

$\mathcal{L}$: *Left circular Polarization*

R: *Linearly polarized state* $\begin{pmatrix} \phi = 0^\circ, 180^\circ \sim -R \\ \phi = 90^\circ, 270^\circ \sim R \end{pmatrix}$

I: *Linearly polarized state* $\begin{pmatrix} \phi = 45^\circ, 225^\circ, etc. \sim I \\ \phi = 135^\circ, 315^\circ, etc. \sim -I \end{pmatrix}$

Unpolarized state: $A \equiv (++) + (--)$

Circular Asymmetry of Photon: $\Delta \equiv (++) - (--)$

Evidently, deviation of any of these observables, excluding σ ,from a null value is an indication that the spin of the observed resonance in $gg \rightarrow \gamma\gamma$ is different from zero and their sign (for most of them) determines the parity of the resonance state.

John Ellis and Hwang have used a massive Kaluza-Klein graviton type coupling J. Ellis and D. S. Hwang, JHEP **1209** (2012) 071 [arXiv:1202.6660] for $X_2 gg$ *and* $X_2\gamma\gamma$ *vertices and the relevant amplitudes are calculated. They find*

$$A_1 \propto (1+\cos\theta)^2 \,, \quad A_4 \propto (1-\cos\theta)^2$$

except at θ=90 degree, this is incompatible with our general test, namely, for a $J=2^+$ resonance one expects $A_1 = A_4$!

Conclusions:

- I presented a spin amplitude test for distinguishing the spin and the parity of the resonance state observed at LHC.
- A set of observables with only final state photons polarization specified, are identified that completely described the reaction in question.
- since the conclusions dynamics independent, any dynamical theory ought to observe those general results. Specially one needs to be careful in constructing a model in which the observed resonance is not considered to be the standard model Higgs.

EXTRA SLIDES ON THE OPTIMAL FORMALISM

The Usual Approach: Decay of resonances like

$X \rightarrow ZZ, W^+W^-$ and $X \rightarrow t\bar{t}$

- Look for the dependence of cross section on the angle between momentum of one of the initial partons and the momentum of one of the decaying particles.

Depending on the spin of X we expect the cross section to behave differently.

Given that vector bosons decay into four leptons, the angular distribution of the final state leptons provide information on the spin and the parity of the resonance state, X

- Look into invariant mass distribution of a Higgs with an associated vector boson in the final state.

- *If only the Lorentz invariance is assumed, there will be 16 independent amplitudes.*
- *Both Lorentz invariance and the parity conservation are imposed, the number reduces to 8.*
- *Imposition of time reversal and the identical particle constraint on the initial and final states brings down the number of independent amplitudes to 5.*

The spin observables depend linearly on bilinear products of the complex amplitudes, the relationship is given by a large matrix. To achieve economy and simplicity, this matrix should be as close to a diagonal one as possible.

Hermicity requirement prohibits complete diagonalization of this matrix. The class of formalisms in which the matrix is as close to diagonal as possible is called "optimal" [Moravcsik and Goldstein, Ann. Phys. N. Y. **98**(1976)128].

The observable-amplitude structure for Lorenz invariance only

Denote an Amplitude by D(c,d; a,b)
C and d spin projection of final state particles along the quantization axes. a and b are that of the initial state particles

In a general reaction A+B → C+D, the observables are denoted by ,

$$L\left(uvH_p, UVH_P; \xi\omega H_q, \Xi\Omega H_Q\right)$$

where u and v are the spin indices for particle A, the indices U and V refer to the spin of particle B, the indices ξ ,ω to particle C and finally, the indices Ξ and Ω refer to particle D and where *H* can be either real (R) or imaginary (I), provided that the subscripts of H is +1 or -1, respectively. For the process under consideration, in the argument of L we can have only four possibilities of ++, --, Re(+-) and Im(+-).

$$\begin{pmatrix} L_1 \\ L_2 \\ \\ \\ \\ \\ \\ L_n \end{pmatrix} = \begin{pmatrix} \\ \\ \\ A\ Large Matrix \\ \\ \\ \end{pmatrix}$$

Hermicity requirement prohibits complete diagonalization of this matrix
The class of formalisms in which the matrix is as close to diagonal as possible is called "*optimal* “.

In it, this matrix consists of a string of small matrices along the main diagonal and zeroes everywhere else. The size of the small matrices, for any four particle reaction, are 1-by-1, 2-by-2, 4-by-4 or 8-by-8.

- **Observables-Amplitude structure under Lorentz Invariance and Discrete symmetries**

Under the parity conservation 16 independent amplitudes reduce to 8. Since we are considering helicity, we know that the reduction from 16 to 8 will not occur by the vanishing of the 8 amplitudes, but will occur by pairwise equalities given by

$$D(c,d\;;a\;,b)=(-1)^{a+b+c+d}\,(-1)^{S_A-S_B+S_C-S_D}\,D(-c,-d\;;-a\;,-b)$$

Imposition of time reversal invariance requires that the helicity amplitudes to satisfy

$$D(c\;,d\;;a\;,b)=(-1)^{a+b+c+d}\,(-1)^{S_A+S_B+S_C+S_D}\,D(a,b;c\;,d)$$

Finally, when particles A=B and C=D, identical particle restriction also applies and provide additional relation among the helicity amplitudes according to

$$D(c\;,d\;;a,b)=(-1)^{a+b+c+d}\,(-1)^{2S_A+2S_C}\,D(d\;,c\;;b\;,a)$$

These restrictions on the amplitudes impose certain restrictions on the observables

Identical particle restriction translates into

$$\mathcal{L}\left(uvH_p, UVH_P; \xi\omega H_q, \Xi\Omega H_Q\right) = (-1)^{\xi+u+\Xi+U+\omega+v+\Omega+V} \mathcal{L}\left(UVH_P, uvVH_p; \Xi\Omega H_Q, \xi\omega H_q\right)$$

Identical Particle+Time reversal

$$\mathcal{L}\left(uvH_p, UVH_P; \xi\omega H_q, \Xi\Omega H_Q\right)$$
$$= (-1)^{\xi+u+\Xi+U+\omega+v+\Omega+V+(1/2)(p-q+P-Q)} \mathcal{L}\left(\xi\omega H_q, \Xi\Omega H_Q; uvH_p; UVH_P\right)$$

The polarization states of gluon or photon are simply denoted by

$\mathcal{R}: \quad ++ \;\; state \quad Right\ circular\ Polarization$

$\mathcal{L}: \quad -- \;\; state \quad Right\ circular\ Polarization$

$R: \quad \mathcal{R}e(+-) \;\; Linearly\ polarized\ state$

$I: \quad \mathcal{I}m(+-) \;\; Linearly\ polarized\ state$

For a photon or gluon polarized in the direction of

ϕ=90, 270, etc. degrees, we obtain $+\mathcal{R}e(+-)$

ϕ=0, 180, etc. Degrees, we get $-\mathcal{R}e(+-)$

ϕ=45, 225, etc. degrees, we have $+\mathcal{I}m(+-)$

ϕ=135, 315, etc. degrees, we obtain $-\mathcal{I}m(+-)$

Unpolarized state: $A \equiv (++) + (--)$

Circular Asymmetry of Photon: $\Delta \equiv (++) - (--)$