

# Asymptotic Scenarios for the Proton's Central Opacity: An Empirical Study

D.A. Fagundes, M.J. Menon, Paulo V. R. G. Silva  
(precchia@ifi.unicamp.br)

Hadronic Physics Group

Instituto de Física *Gleb Wataghin*  
UNICAMP - Brazil

DIFFRACTION 2014

Primošten - Croatia — 10-16 September, 2014

# Outline

- Introduction and Motivation
- Asymptotic Scenarios
- Parametrization and Fit Procedures
- Fit Results
- Summary and Conclusions

# Motivation

Why study the ratio  $X \equiv \frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} ?$

- **Central opacity** in Hadron-Hadron Collisions

*Elastic scattering amplitude (Impact Parameter representation)*

$$F(s, q) = i \int_0^\infty b db J_0(qb) \Gamma(s, b) \quad (\text{azimuthal symmetry})$$

- $\Gamma(s, b)$ : Profile function
- $\Gamma(s, b = 0) \equiv \Gamma_0(s) \longrightarrow$  Central opacity
- Naive geometrical models

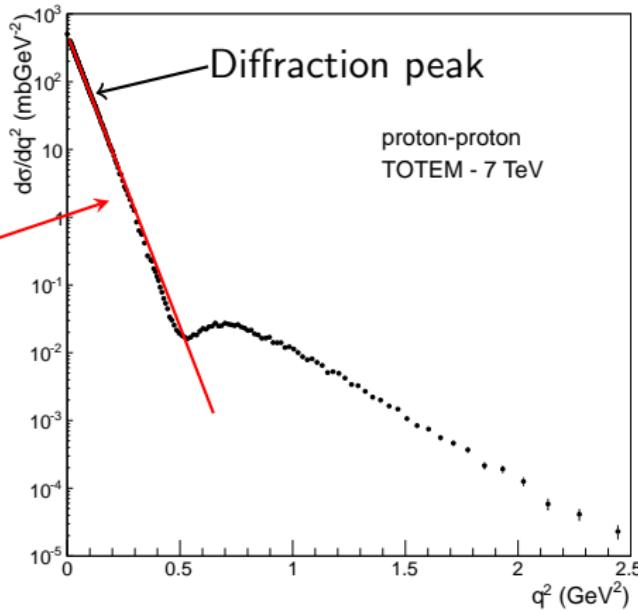
$$X(s) = \begin{cases} \Gamma_0(s)/2, & \text{Gray disk} \\ \Gamma_0(s)/4, & \text{Gaussian} \end{cases}$$

- Black disk:  $\Gamma_0 \rightarrow 1 \Rightarrow X \rightarrow 1/2$

# Motivation

- **Unitarity**
- $\sigma_{\text{tot}}$  and  $\sigma_{\text{el}}$ : from Elastic Scattering

$$\frac{d\sigma}{dq^2} = \left[ \frac{(1 + \rho^2)}{16\pi} \sigma_{\text{tot}}^2 \right] e^{-Bq^2}$$
$$\sigma_{\text{el}} = \int_0^{q_0^2} \frac{d\sigma}{dq^2} dq^2$$



- Unitarity:  $\sigma_{\text{in}} = \sigma_{\text{tot}} - \sigma_{\text{el}}$   
(Free from models assumptions)

- It gives the fraction of inelastic scattering:

$$\frac{\sigma_{\text{in}}}{\sigma_{\text{tot}}}(s) = 1 - X(s)$$

# Motivation

- Connection with ratio  $\frac{\sigma_{\text{tot}}}{B}$ 
  - Important for cosmic rays experiments (EAS studies)
  - Needs extrapolation from accelerator experiments (but it has large uncertainties).

$$X \simeq \frac{1}{16\pi} \frac{\sigma_{\text{tot}}}{B}$$

However...

# Motivation

## Elastic Scattering

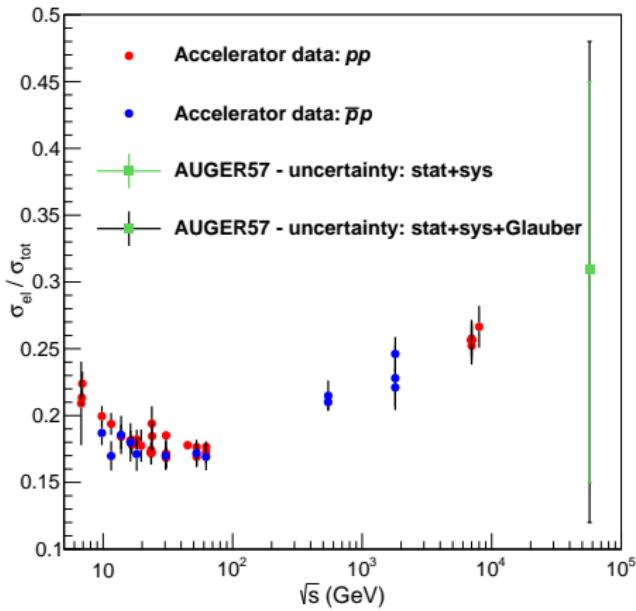
- Soft strong interaction → non-perturbative QCD
- Presently → lack theoretical description from first principles
- One way to look for phenomenological insights/inputs:

## Empirical Approach

- Empirical parametrization for  $X(s)$ ?
- Let us look to data...

# Motivation

Experimental data ( $pp$ ,  $\bar{p}p$ )  $\rightarrow X$  data **rise** at  $\sqrt{s} \gtrsim 100$  GeV



# Motivation

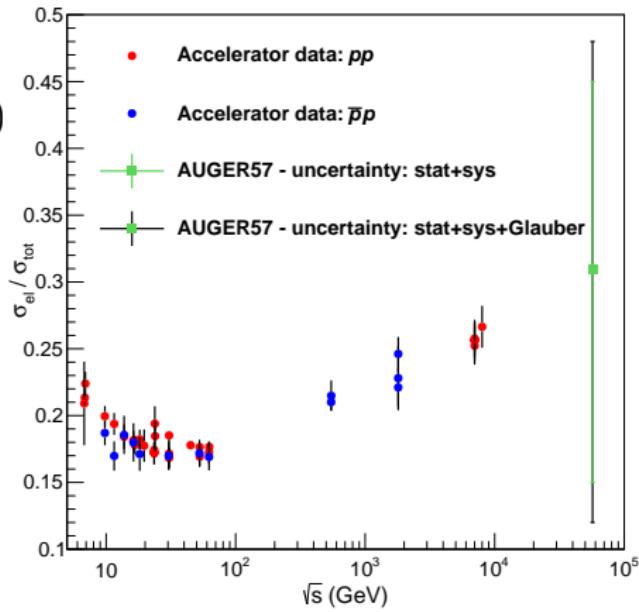
Experimental data ( $pp$ ,  $\bar{p}p$ )  $\rightarrow X$  data **rise** at  $\sqrt{s} \gtrsim 100$  GeV

- Positive curvature
- Expected Asymptotic limit (all contexts)

↓  
**a constant**

$$\boxed{\lim_{s \rightarrow \infty} X(s) = A}$$

$$A = \begin{cases} 1 & \text{(maximum unitarity)} \\ 1/2 & \text{(black disk)} \end{cases}$$



# Motivation

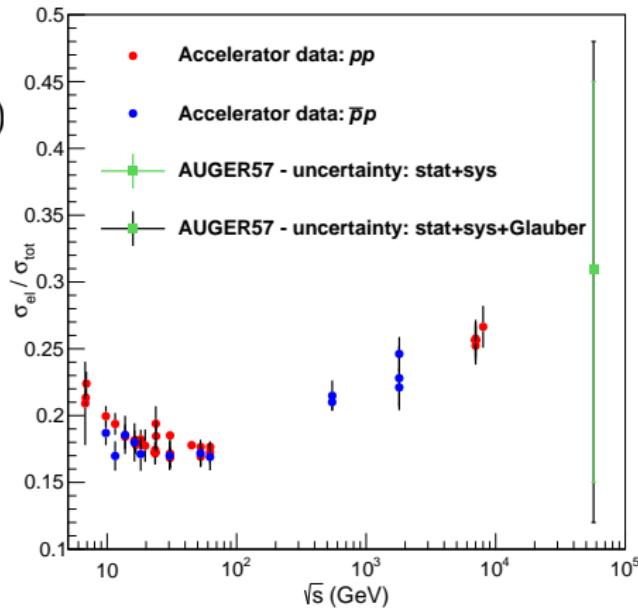
Experimental data ( $pp$ ,  $\bar{p}p$ )  $\rightarrow X$  data **rise** at  $\sqrt{s} \gtrsim 100$  GeV

- Positive curvature
- Expected Asymptotic limit (all contexts)

↓  
**a constant**

$$\boxed{\lim_{s \rightarrow \infty} X(s) = A}$$

$$A = \begin{cases} 1 & \text{(maximum unitarity)} \\ 1/2 & \text{(black disk)} \end{cases}$$



Rise and saturation (constant value)  $\xrightarrow{\text{demand}}$  **change of curvature**

# Motivation

Basic suitable assumption

$$X(s) = A f(s) \quad \text{with} \quad \lim_{s \rightarrow \infty} f(s) = 1$$

# Motivation

Basic suitable assumption

$$X(s) = A f(s) \quad \text{with} \quad \lim_{s \rightarrow \infty} f(s) = 1$$

↓      ↗ change of curvature

**A**symptotic ratio

# Motivation

Basic suitable assumption

$$X(s) = A f(s) \quad \text{with} \quad \lim_{s \rightarrow \infty} f(s) = 1$$

↓      ↗ change of curvature

**A**symptotic ratio

- Previous Empirical Ansatz: Fagundes and Menon [NPA (2012)]

$f(s)$  in terms of  $\ln s$  (standard soft variable)

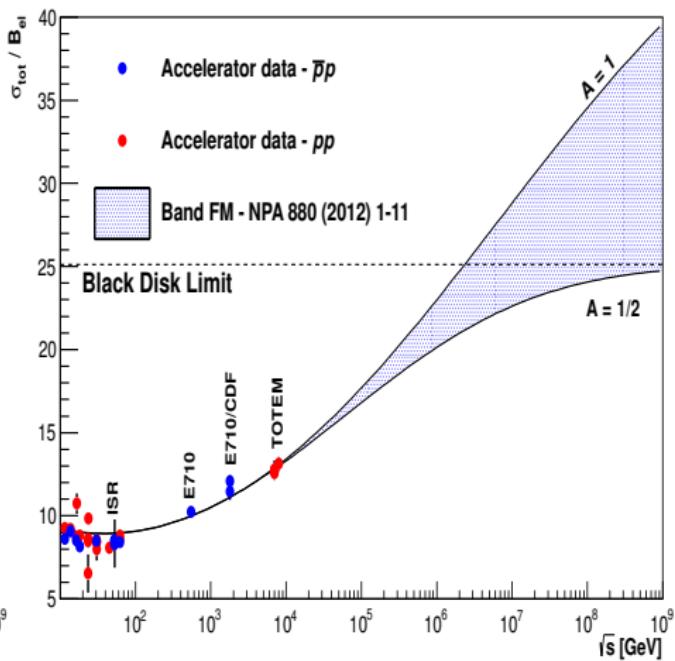
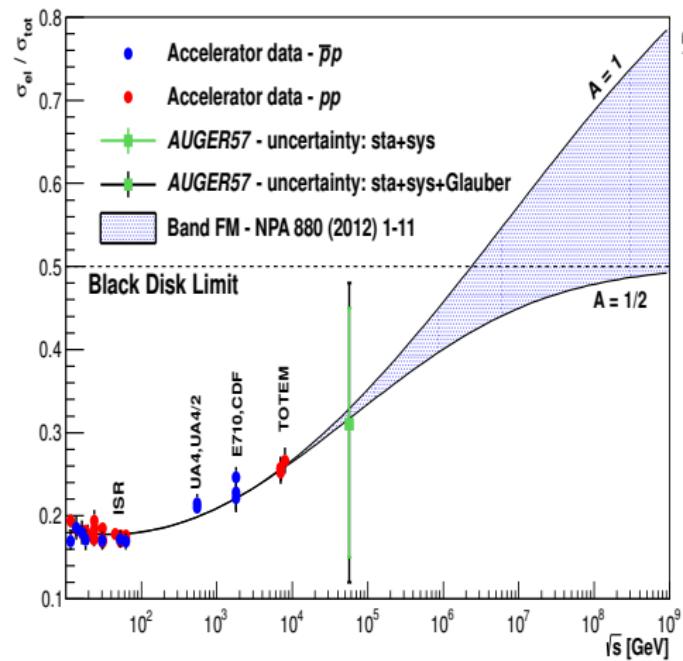
↳ Trial and error: 
$$f_{\text{FM}}(s) = \tanh[a + b \ln(s/s_0) + c \ln^2(s/s_0)]$$

- Fits  $X(s)$  data only 3 free parameters:  $a, b, c$  ( $s_0 = 1 \text{ GeV}^2$  fixed)
  - $A = 1/2$  and  $A = 1$  (fixed parameters)
  - only  $pp$  data:  $\sqrt{s}_{\min} = 10 \text{ GeV}$ ,  $\sqrt{s}_{\max} = 7 \text{ TeV}$  (1st TOTEM datum)
  - Prediction to  $\sigma_{\text{tot}}/B$  (uncertainties for EAS studies)

# Motivation

$$\sigma_{\text{el}} / \sigma_{\text{tot}}$$

$$\sigma_{\text{tot}} / B$$



# This communication

- Inclusion of all TOTEM data ( $\sigma_{\text{el}}$ ,  $\sigma_{\text{tot}}$ ) at 7 and 8 TeV
- Inclusion  $\bar{p}p$  data
- Cutoff down to  $\sqrt{s}_{\text{min}} = 5$  GeV
- **Improved** empirical parametrization for  $f(s)$
- Study on 3 scenarios: **black-disk**, **below** and **above**

# This communication

- Inclusion of all TOTEM data ( $\sigma_{\text{el}}$ ,  $\sigma_{\text{tot}}$ ) at 7 and 8 TeV
- Inclusion  $\bar{p}p$  data
- Cutoff down to  $\sqrt{s}_{\text{min}} = 5$  GeV
- **Improved** empirical parametrization for  $f(s)$
- Study on 3 scenarios: **black-disk**, **below** and **above**

## Main empirical results:

- Black disk (BD) does not represent an unique solution
- Fits favour  $\lim_{s \rightarrow \infty} X(s) = 0.36 \pm 0.08$  (below BD)

# Asymptotic Scenarios I: The Black-Disk Limit

Why all **3** scenarios?

Theoretical/phenomenological and empirical arguments:

# Asymptotic Scenarios I: The Black-Disk Limit

Why all **3** scenarios?

Theoretical/phenomenological and empirical arguments:

- **Black-Disk**

$$\lim_{s \rightarrow \infty} X(s) = \frac{1}{2}$$

- Standard phenomenological expectation
- Typical of eikonal models (unitarized by construction): Chou-Yang, Bourrely-Soffer-Wu, Block-Halzen, etc

## Asymptotic Scenarios II: Below the Black-Disk

(1) Amplitude Analyses: Fagundes, Menon, Silva [JPG, IJMPA, JPG (2013)]

$$\sigma_{\text{tot}}(s) = \text{Regge terms} + \alpha + \beta \ln^{\gamma}(s/s_h)$$

- Fits  $\sigma_{\text{tot}}$  and  $\rho$  data ( $pp, \bar{p}p$ ) using DDR,  $\sqrt{s} \geq 5$  GeV  
 $\gamma = 2$  fixed and  $\gamma$  as a free fit parameter ( $\gamma > 2$ )
- Extension to  $\sigma_{\text{el}}$  data ( $\gamma = 2$  and  $\gamma > 2$ )
- Several distinct fit variants

## Asymptotic Scenarios II: Below the Black-Disk

(1) Amplitude Analyses: Fagundes, Menon, Silva [JPG, IJMPA, JPG (2013)]

$$\sigma_{\text{tot}}(s) = \text{Regge terms} + \alpha + \beta \ln^{\gamma}(s/s_h)$$

- Fits  $\sigma_{\text{tot}}$  and  $\rho$  data ( $pp$ ,  $\bar{p}p$ ) using DDR,  $\sqrt{s} \geq 5$  GeV  
 $\gamma = 2$  fixed and  $\gamma$  as a free fit parameter ( $\gamma > 2$ )
- Extension to  $\sigma_{\text{el}}$  data ( $\gamma = 2$  and  $\gamma > 2$ )
- Several distinct fit variants
- All cases:  $\lim_{s \rightarrow \infty} X(s) < 1/2$  (within uncertainties)

Lowest central value:  $X \rightarrow 0.3$

# Asymptotic Scenarios II: Below the Black-Disk

## (2) COMPETE and TOTEM parametrizations

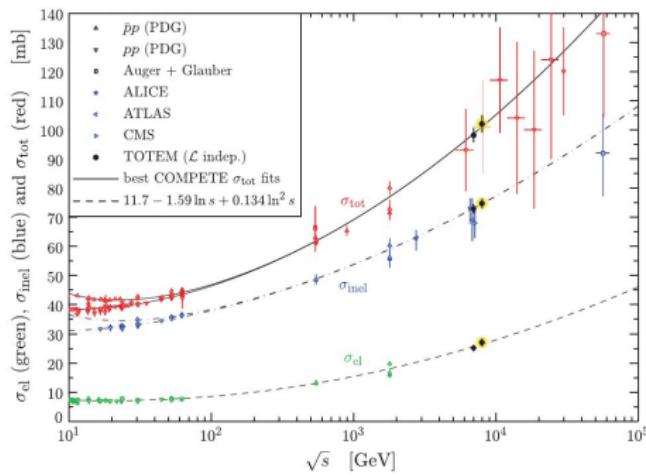
COMPETE highest-rank result for  $\sigma_{\text{tot}}(s)$  [PRL (2002)]:

$$\sigma_{\text{tot}}(s) = \text{Regge} + 35.5 + 0.307 \ln^2(s/29.1 \text{ GeV}^2)$$

TOTEM empirical fit to  $\sigma_{\text{el}}$  data:  
[PRL (2013)]

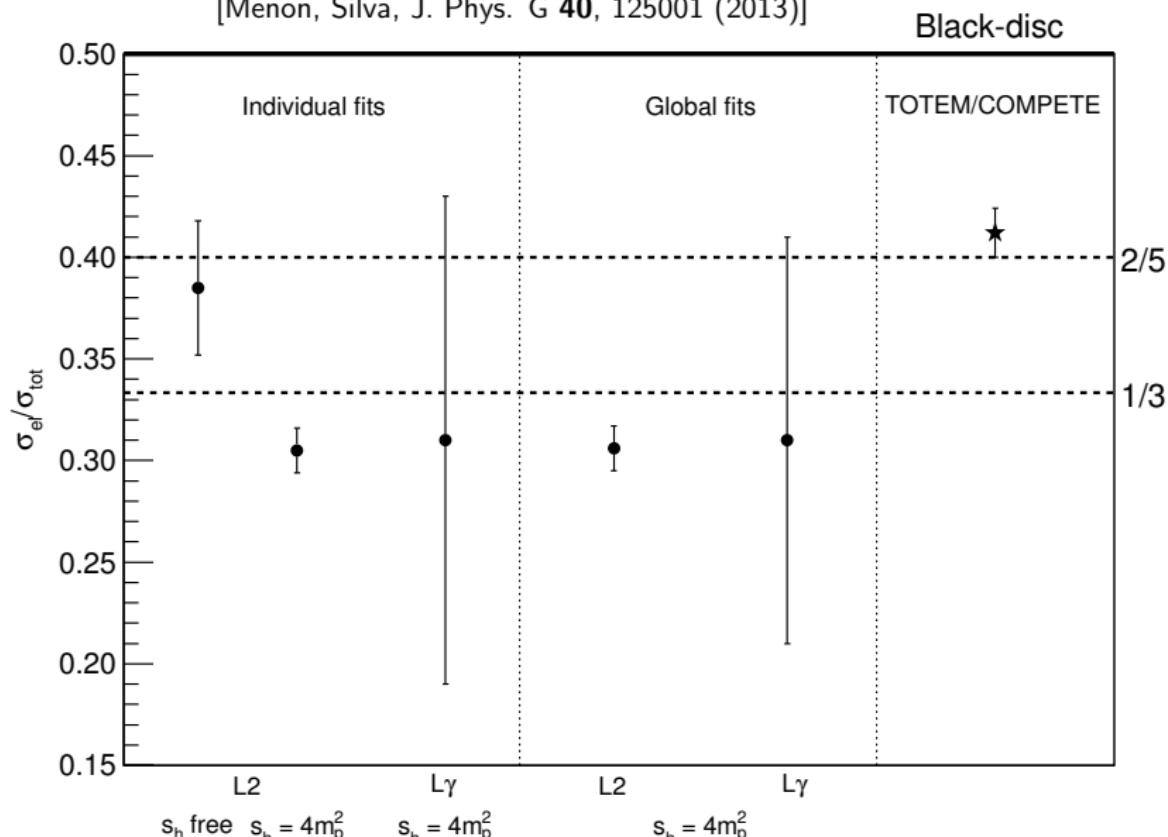
$$\sigma_{\text{el}}(s) = 11.7 - 1.59 \ln s + 0.134 \ln^2 s$$

$$\lim_{s \rightarrow \infty} X(s) = 0.436$$



# Asymptotic Scenarios II: Below the Black-Disk

[Menon, Silva, J. Phys. G **40**, 125001 (2013)]



# Asymptotic Scenarios III: Above the Black-Disk

## (1) Maximum Unitarity bound:

$$\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} + \frac{\sigma_{\text{in}}}{\sigma_{\text{tot}}} = 1 \quad \Rightarrow \quad \boxed{X \rightarrow 1} \quad (s \rightarrow \infty)$$

# Asymptotic Scenarios III: Above the Black-Disk

## (1) Maximum Unitarity bound:

$$\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} + \frac{\sigma_{\text{in}}}{\sigma_{\text{tot}}} = 1 \quad \Rightarrow \quad \boxed{X \rightarrow 1} \quad (s \rightarrow \infty)$$

## (2) Phenomenological approach

Troshin, Tyurin [PLB (1993), IJMPA (2007)]

$U$ -matrix unitarization

$$\boxed{X > \frac{1}{2}}$$

# Asymptotic Scenarios III: Above the Black-Disk

## (3) Possible Formal Limit

Formal results ( $s \rightarrow \infty$ )

[Froissart, PR (1961); Martin, Lukaszuk, NC (1966); Martin, PRD (2009)]

$$\sigma_{\text{tot}}(s) \leq \frac{\pi}{m_\pi^2} \ln^2 s$$

and

$$\sigma_{\text{in}}(s) \leq \frac{\pi}{4m_\pi^2} \ln^2 s$$

If both limits saturate as  $s \rightarrow \infty$ :

$$\frac{\sigma_{\text{in}}}{\sigma_{\text{tot}}} \rightarrow \frac{1}{4} \xrightarrow{\text{Unitarity}} X \rightarrow \frac{3}{4} = 0.75$$

# Parametrization and Fit Procedures

- Data set

- Only accelerator data on  $pp$  and  $\bar{p}p$ ,  $\sqrt{s} \geq 5$  GeV (PDG)
- TOTEM data: 4 points at 7 TeV, 1 point at 8 TeV
- Total number points: 41 ( $pp$ : 28,  $\bar{p}p$ : 13)

- Preliminary tests:

- To check consistency  $f_{\text{FM}}$  generalized:

$$f(s) = \tanh[a + b \ln(s/\textcolor{blue}{s}_0) + c \ln^{\textcolor{red}{d}}(s/\textcolor{blue}{s}_0)]$$

- Fit Procedures

- Data reduction: TMinuit of ROOT Framework ( $CL \approx 68\%$ )
- Goodness of fit:  $\chi^2/\nu$  and  $P(\chi^2, \nu)$
- Nonlinearity demands initial values for free parameters

# Parametrization and Fit Procedures

- Data set

- Only accelerator data on  $pp$  and  $\bar{p}p$ ,  $\sqrt{s} \geq 5$  GeV (PDG)
- TOTEM data: 4 points at 7 TeV, 1 point at 8 TeV
- Total number points: 41 ( $pp$ : 28,  $\bar{p}p$ : 13)

- Preliminary tests:

- To check consistency  $f_{\text{FM}}$  generalized:

New free parameter  
Search for a smooth behavior

$$f(s) = \tanh[a + b \ln(s/s_0) + c \ln^d(s/s_0)]$$

- Fit Procedures

- Data reduction: TMinuit of ROOT Framework ( $CL \approx 68\%$ )
- Goodness of fit:  $\chi^2/\nu$  and  $P(\chi^2, \nu)$
- Nonlinearity demands initial values for free parameters

# Parametrization and Fit Procedures

- Data set

- Only accelerator data on  $pp$  and  $\bar{p}p$ ,  $\sqrt{s} \geq 5$  GeV (PDG)
- TOTEM data: 4 points at 7 TeV, 1 point at 8 TeV
- Total number points: 41 ( $pp$ : 28,  $\bar{p}p$ : 13)

- Preliminary tests:

- To check consistency  $f_{\text{FM}}$  generalized:

New free parameter  
Search for a smooth behavior

$$f(s) = \tanh[a + b \ln(s/s_0) + c \ln^2(s/s_0)]$$

- Fit Procedures

Tests:  $s_0 = 1 \text{ GeV}^2$ ,  $4m_p^2$ , 25 GeV<sup>2</sup>

- Data reduction: TMinuit of ROOT Framework ( $CL \approx 68\%$ )
- Goodness of fit:  $\chi^2/\nu$  and  $P(\chi^2, \nu)$
- Nonlinearity demands initial values for free parameters

# Parametrization and Fit Procedures

$$f(s) = \tanh[a + b \ln(s/s_0) + c \ln^d(s/s_0)]$$

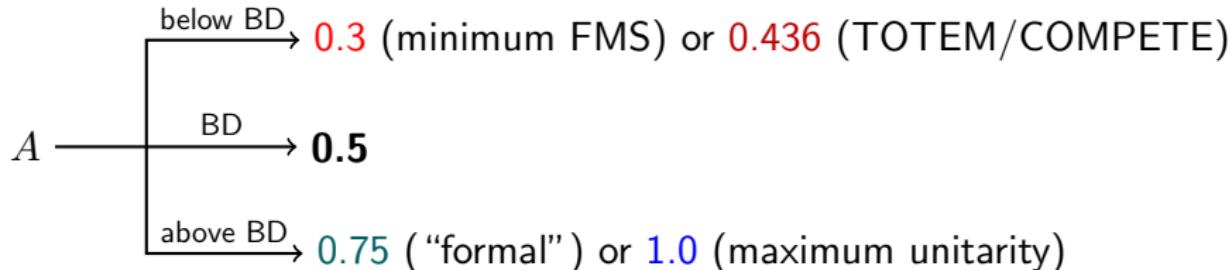
- **Best results:**  $s_0 = 25 \text{ GeV}^2$  (cutoff) and  $d = 0.5$  within uncertainties
- **Selected Parametrization:** fix  $d = 1/2$  and  $s_0 = 25 \text{ GeV}^2$

$$X(s) = A f_{\text{FMS}}(s), \quad \boxed{f_{\text{FMS}}(s) = \tanh[\alpha + \beta \ln^{1/2}(s/s_0) + \gamma \ln(s/s_0)]}$$

Fixed  $A \rightarrow$  only 3 dimensionless fit parameters:  $\alpha, \beta, \gamma$

## Two variants

- **(V1)**  $A$  **fixed** (defines asymptotic limit)  $\longrightarrow$  5 tests



- **(V2)**  $A$  **as a free parameter** (to select asymptotic limit)

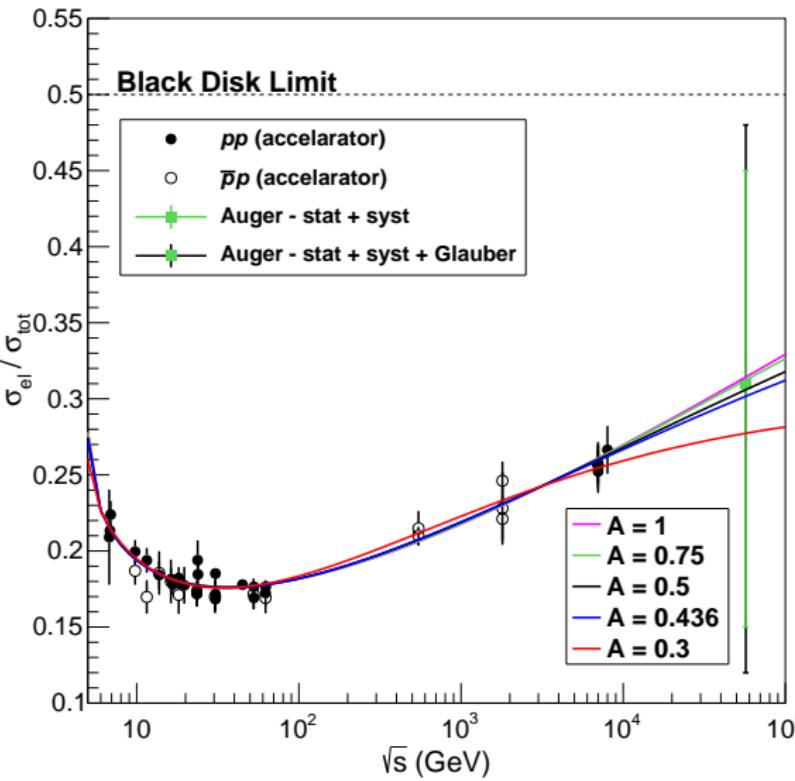
Results **(V1)** as initial values  $\xrightarrow[\text{reduction}]{\text{data}} A, \alpha, \beta, \gamma$

# Fit Results

**Variant 1:**  $A$  fixed

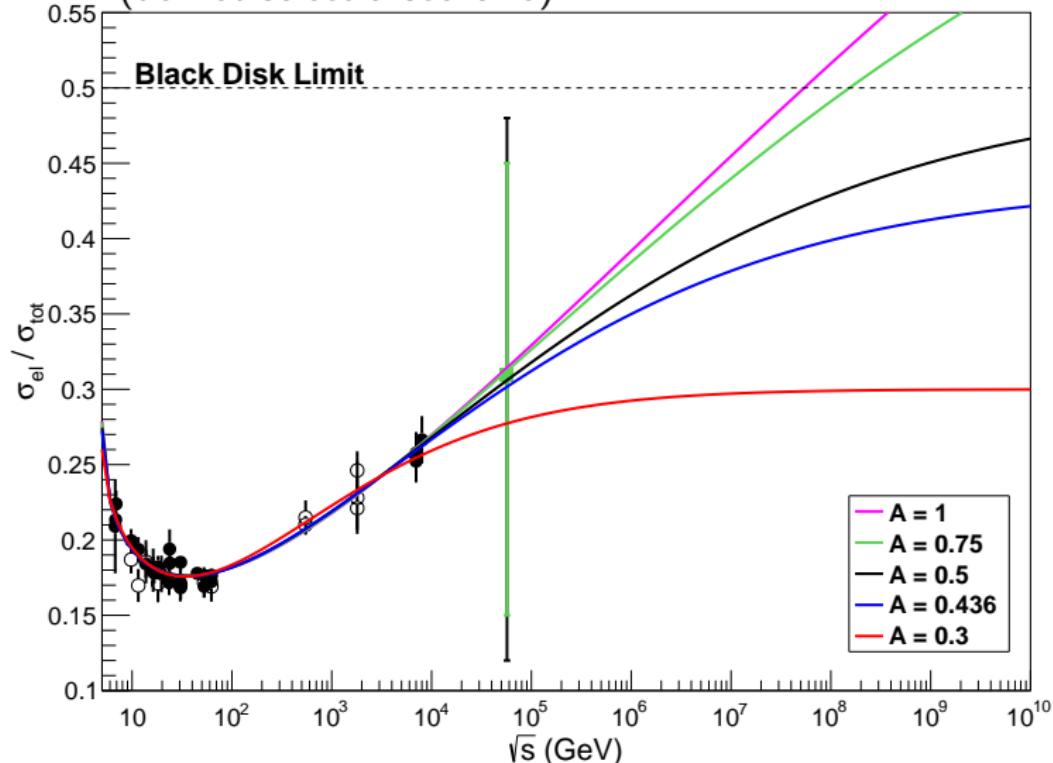
$A$ (fixed)	$\chi^2/\nu$	$P(\chi^2, \nu)$
0.3	0.789	0.812
0.436	0.774	0.840
0.5	0.778	0.834
0.75	0.787	0.823
1.0	0.790	0.818

$(\nu = 38)$



# Results with $A$ fixed

All cases  $\rightarrow$  consistent description of experimental data (analyzed)  
(do not select a scenario)



# $A$ as a free parameter

**Variant 2:**  $A$  free

$A$ (initial values)	$A$ (fit result)
0.3	
0.436	
0.5	
0.75	
1	

# $A$ as a free parameter

**Variant 2:**  $A$  free

$A$   
(initial values)

$A$   
(fit result)

$$0.3 \xrightarrow{\hspace{1cm}} 0.360 \pm 0.078$$

0.436

0.5

0.75

1

# $A$ as a free parameter

**Variant 2:**  $A$  free

$A$ (initial values)	$A$ (fit result)
0.3	$0.360 \pm 0.078$
0.436	$0.361 \pm 0.078$
0.5	
0.75	
1	

# $A$ as a free parameter

**Variant 2:**  $A$  free

$A$   
(initial values)

$A$   
(fit result)

$$0.3 \xrightarrow{\quad} 0.360 \pm 0.078$$

$$0.436 \xrightarrow{\quad} 0.361 \pm 0.078$$

$$0.5$$

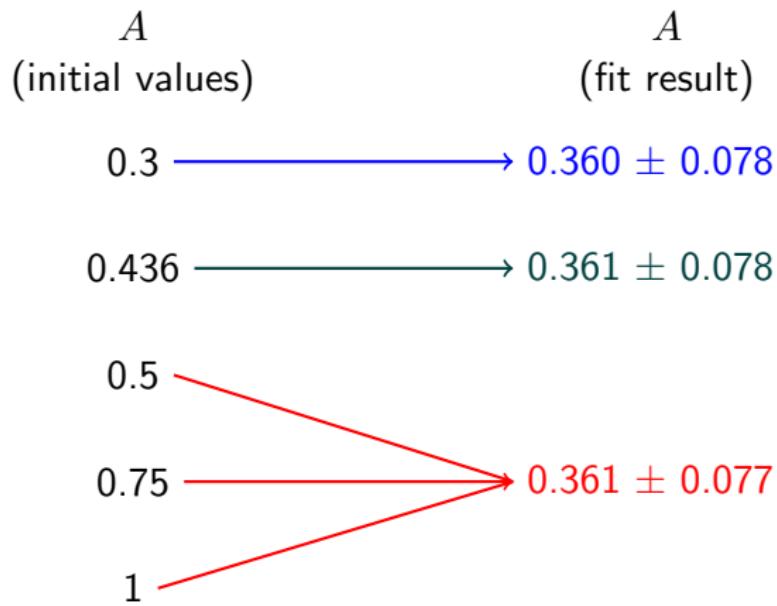
$$0.75$$

$$1$$

$$0.361 \pm 0.077$$

# $A$ as a free parameter

**Variant 2:**  $A$  free



**All cases:**

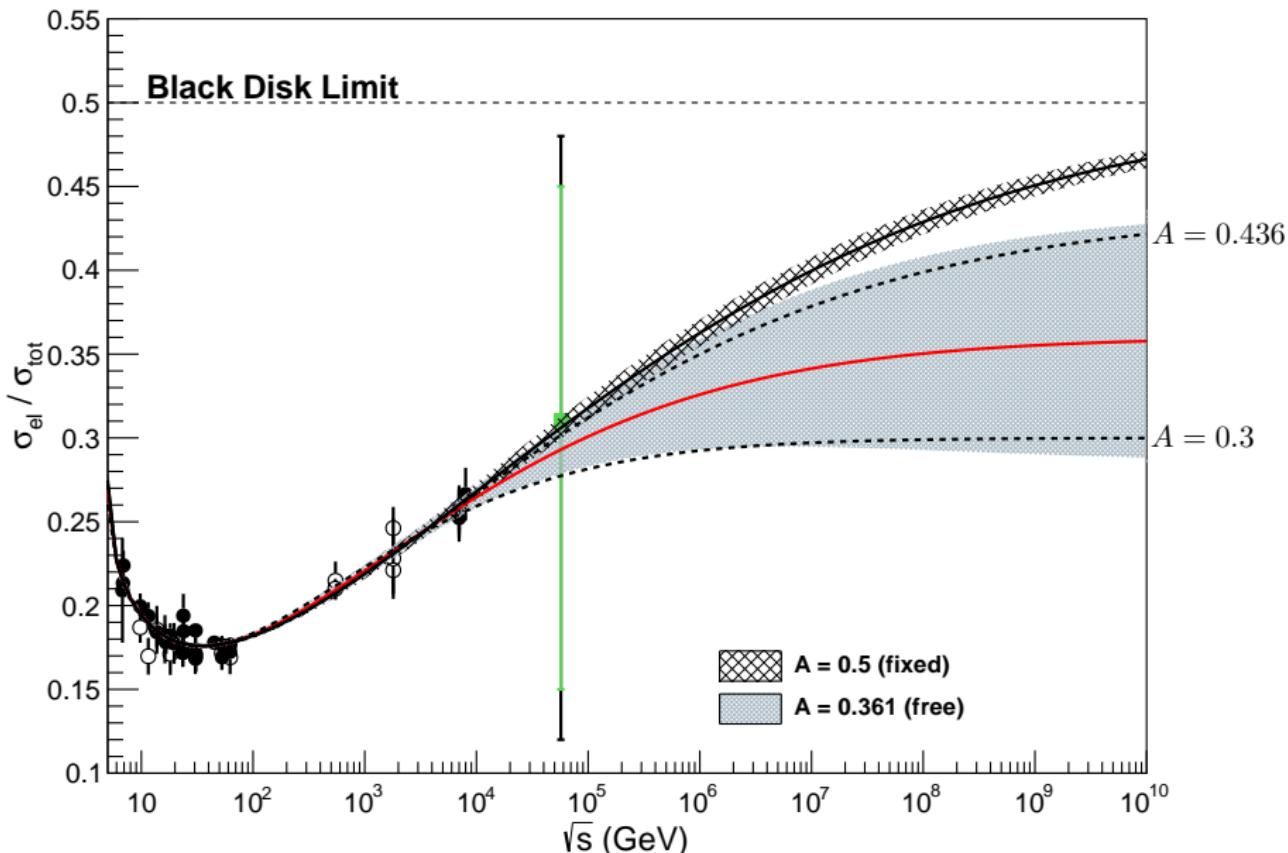
$$\begin{aligned}\chi^2/\nu &= 0.791 \\ P(\chi^2, \nu) &= 0.814\end{aligned}$$

$$(\nu = 37)$$

**Same central values:**

$$\begin{aligned}\alpha &= 0.96 \pm 0.32 \\ \beta &= -0.43 \pm 0.19 \\ \gamma &= 0.109 \pm 0.048 \\ A &= 0.361 \pm 0.078\end{aligned}$$

# $A$ as a free parameter



# Predictions and Possible Implications

- If **Pumplin bound** [PRD (1973)]:  $\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} + \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \leq \frac{1}{2}$   
 $(\sigma_{\text{diff}} = \sigma_{\text{SD}} + \sigma_{\text{DD}})$

is **saturated**:

$$\frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \rightarrow 0.14 \pm 0.08$$

- **Predictions** ( $1\sigma$ )

$\sqrt{s}$ (TeV)	$\sigma_{\text{el}}/\sigma_{\text{tot}}$	$\sigma_{\text{diff}}/\sigma_{\text{tot}}$	$\sigma_{\text{in}}/\sigma_{\text{tot}}$
2.76	$0.2406 \pm 0.0041$	$0.2594 \pm 0.0041$	$0.7594 \pm 0.0041$
13	$0.2693 \pm 0.0072$	$0.2307 \pm 0.0072$	$0.7307 \pm 0.0072$
14	$0.2706 \pm 0.0075$	$0.2294 \pm 0.0075$	$0.7294 \pm 0.0075$
57	$0.293 \pm 0.014$	$0.207 \pm 0.014$	$0.707 \pm 0.014$

# Summary and Conclusions

- **Empirical Parametrization**

$$X(s) = A f_{\text{FMS}}(s) \text{ and } f_{\text{FMS}}(s) = \tanh[\alpha + \beta \ln^{1/2}(s/s_0) + \gamma \ln(s/s_0)]$$

with  $s_0 = 25 \text{ GeV}^2$  (energy cutoff)

- Only **4** free parameters

# Summary and Conclusions

- **Empirical Parametrization**

$$X(s) = A f_{\text{FMS}}(s) \text{ and } f_{\text{FMS}}(s) = \tanh[\alpha + \beta \ln^{1/2}(s/s_0) + \gamma \ln(s/s_0)]$$

with  $s_0 = 25 \text{ GeV}^2$  (energy cutoff)

- Only **4** free parameters
- **Fit results to  $pp$  and  $\bar{p}p$  data,  $\sqrt{s} : 5 \text{ GeV} - 8 \text{ TeV}$**

- $A$  fixed

0.3, 0.436, 0.5, 0.75, 1.0 → all results consistent with exp. data

Cannot discriminate asymptotic scenarios → still open problem

Black-disk limit → does not represent an unique solution

# Summary and Conclusions

- Fit results to  $pp$  and  $\bar{p}p$  data,  $\sqrt{s} : 5 \text{ GeV} - 8 \text{ TeV}$

- $A$  free

Unique solution  $\longrightarrow$  below the black-disk

$$X(s) \rightarrow 0.36 \pm 0.08 \quad (s \rightarrow \infty)$$

- Within uncertainties  $\longrightarrow$  agreement with:
  - TOTEM/COMPETE parametrization ( $\sigma_{\text{tot}}$ ,  $\rho$ ,  $\sigma_{\text{el}}$ )
  - FMS amplitude analyses ( $\sigma_{\text{tot}}$ ,  $\rho$ ,  $\sigma_{\text{el}}$ )
  - Recent phenomenological analysis by Kohara-Ferreira-Kodama:  
[arXiv (2014); this conference]

$$X \rightarrow \frac{1}{3} \quad (s \rightarrow \infty)$$

# Summary and Conclusions

- Analyses in progress:
  - Inclusion of new ATLAS datum at 7 TeV [arXiv (2014)]
  - Discussion on the change of curvature (position and change in dynamics)
  - Extensions to  $\sigma_{\text{tot}}/B$  (extrapolation/uncertainties)

## Quoted References (order presentation)

- D.A. Fagundes, M.J. Menon, Nucl. Phys. A **880**, 1 (2012)
- D.A. Fagundes, M.J. Menon, P.V.R.G. Silva, J. Phys. G **40**, 065005 (2013)
- M.J. Menon, P.V.R.G. Silva, Int. J. Mod. Phys A **28**, 1350099 (2013)
- M.J. Menon, P.V.R.G. Silva, J. Phys. G **40**, 125001 (2013)
- J.R. Cudell *et al* (COMPETE Collab.), Phys. Rev. Lett. **89**, 201801 (2002)
- G. Antchev *et al* (TOTEM Collab.), Phys. Rev. Lett. **111**, 012001 (2013)
- M. Froissart, Phys. Rev. **123**, 1053 (1961)
- A. Martin, Il Nuovo Cimento **42**, 930 (1966)
- L. Lukaszuk, A. Martin, Il Nuovo Cimento **52**, 122 (1967)
- A. Martin, Phys. Rev. D **80**, 065013 (2009)
- A. K. Kohara, E. Ferreira, T. Kodama, arXiv:1408.1599, 1406.5773 [hep-ph]
- The ATLAS Collaboration, arXiv:1408.5778 [hep-ex]

## Sponsors



*Conselho Nacional de Desenvolvimento  
Científico e Tecnológico*



THANK YOU!!

# Backup Slides

# Central Opacity

- Profile function:  $\Gamma(s, b) = 1 - e^{i\chi(s, b)}$
- Eikonal:  $\chi(s, b)$
- Opacity:  $\Omega(s, b) = \text{Im } \chi(s, b)$
- Considering only the imaginary part of the eikonal:

$$e^{-\Omega(s, b)} = 1 - \Gamma(s, b)$$

- Central collisions:  $e^{-\Omega(s, b=0)} = 1 - \Gamma(s, b=0)$

## Relation between $\sigma_{\text{tot}}/B$ and $\sigma_{\text{el}}/\sigma_{\text{tot}}$

- Differential cross section (forward peak):  $\frac{d\sigma}{dq^2} = \left. \frac{d\sigma}{dq^2} \right|_{q^2=0} e^{-Bq^2}$
- Optical point:  $\left. \frac{d\sigma}{dq^2} \right|_{q^2=0} = \frac{(1 + \rho^2)}{16\pi} \sigma_{\text{tot}}^2$
- Integrated elastic cross section:  $\sigma_{\text{el}} = \int_0^{q_0^2} \frac{d\sigma}{dq^2} dq^2$
- With assumption  $1 + \rho^2 \approx 1$ , taking limit  $q_0^2 \rightarrow \infty$  and using the optical point:

$$\sigma_{\text{el}}(s) = \frac{1}{B(s)} \frac{\sigma_{\text{tot}}^2(s)}{16\pi} \Rightarrow \boxed{\frac{\sigma_{\text{tot}}(s)}{B(s)} = 16\pi \frac{\sigma_{\text{el}}(s)}{\sigma_{\text{tot}}(s)}}$$

# Black-disk model

- Impact parameter formalism (azimuthal symmetry)

$$F(s, q) = i \int_0^\infty b db J_0(qb) \Gamma(s, b)$$

Gray-disk (Profile function):

$$\Gamma(s, b) = \begin{cases} \Gamma_0(s), & b \leq R(s) \\ 0, & b > R(s) \end{cases}$$

$$\sigma_{\text{tot}} = 2\pi R^2 |\Gamma_0|^2$$

$$\sigma_{\text{el}} = \pi R^2 \operatorname{Re} \Gamma_0$$

$$\frac{d\sigma}{dq^2} = \frac{|\Gamma_0|^2 R^4}{16\pi s^2} \left| \frac{J_1(qR)}{qR} \right|^2$$

- Black-disk:  $\Gamma_0 \rightarrow 1$

$$\boxed{\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} = \frac{1}{2}}$$

(black-disk limit)

# Estimation<sup>1</sup> of $\sigma_{\text{diff}}/\sigma_{\text{tot}}$ at 7 TeV

- TOTEM (indep. lum.):

$$\sigma_{\text{tot}} = 98.0 \pm 2.5 \text{ mb}, \sigma_{\text{el}} = 25.1 \pm 1.1 \text{ mb}, \sigma_{\text{in}} = 72.9 \pm 1.5 \text{ mb},$$
$$\sigma_{\text{el}}/\sigma_{\text{tot}} = 0.256 \pm 0.013$$

- ALICE: Fraction of single (SD) and double (DD) diffraction in inelastic collisions:

$$\frac{\sigma_{\text{SD}}}{\sigma_{\text{in}}} = 0.20^{+0.04}_{-0.07} \quad \text{and} \quad \frac{\sigma_{\text{DD}}}{\sigma_{\text{in}}} = 0.12^{+0.05}_{-0.04}$$

With  $\sigma_{\text{diff}} = \sigma_{\text{SD}} + \sigma_{\text{DD}}$ :

$$\frac{\sigma_{\text{diff}}}{\sigma_{\text{in}}} = 0.32^{+0.06}_{-0.08}$$

---

<sup>1</sup>P. Lipari, M. Lusignoli, Eur. Phys. J. C **73**, 2630 (2013)

# Estimation<sup>2</sup> of $\sigma_{\text{diff}}/\sigma_{\text{tot}}$ at 7 TeV

- Combining TOTEM and ALICE results:

$$\sigma_{\text{diff}} = 23.3^{+4.4}_{-5.9} \text{ mb} \quad \text{and} \quad \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \simeq 0.24^{+0.05}_{-0.06}$$

$$\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} + \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} = 0.496^{+0.05}_{-0.06}$$

- Indicates saturation of Pumplin bound at LHC energy

---

<sup>2</sup>P. Lipari, M. Lusignoli, Eur. Phys. J. C **73**, 2630 (2013)

# Parameters

$A$	$\alpha$	$\beta$	$\gamma$	$P(\chi^2, \nu)$
0.3 (fixed)	1.36	-0.66	0.17	0.82
0.361	0.96	-0.43	0.11	0.81
0.436 (fixed)	0.73	-0.31	0.078	0.84
0.361	0.96	-0.43	0.11	0.81
0.5 (fixed)	0.62	-0.25	0.063	0.83
0.361	0.96	-0.43	0.11	0.81
0.75 (fixed)	0.39	-0.15	0.038	0.82
0.361	0.96	-0.43	0.11	0.81

Ref.: A. Martin, Phys. Rev. D **80**, 065013 (2009)

After obtaining the limit  $\sigma_{\text{in}} < \frac{\pi}{4m_\pi^2} \ln^2 s$  (pag. 3):

**"This ends the rigorous part of this paper. Now comes the fact that most theoreticians believe that the worse that can happen at high energies is that the elastic cross section reaches half of the total cross section, which corresponds to an expanding black disk."**

End of the paper (pag. 3), concerning the limit  $\sigma_{\text{el}}/\sigma_{\text{tot}} > 1/2$ ,  $s \rightarrow \infty$ :

*"To say the least, this seems to me extremely unlikely and, therefore, I tend to believe that we have"*

$$\sigma_{\text{tot}} < \frac{\pi}{2m_\pi^2} \ln^2 s.$$

Some authors assume this result:

- M.M. Block, F. Halzen, Phys. Rev. Lett. **107**, 212002 (2011)
- N. Cartiglia, *Measurement of the proton-proton total, elastic, inelastic and diffractive cross sections at 2, 7, 8 and 57 TeV*, arXiv:1305.6131v3 [hep-ex]
- I.M. Dremin, *Hadron structure and elastic scattering*, arXiv:1311.4159 [hep-ph]

# Predictions and Possible Implications

