

Renormalization group analysis of reggeon field theory: flow equations

(Project and first results)

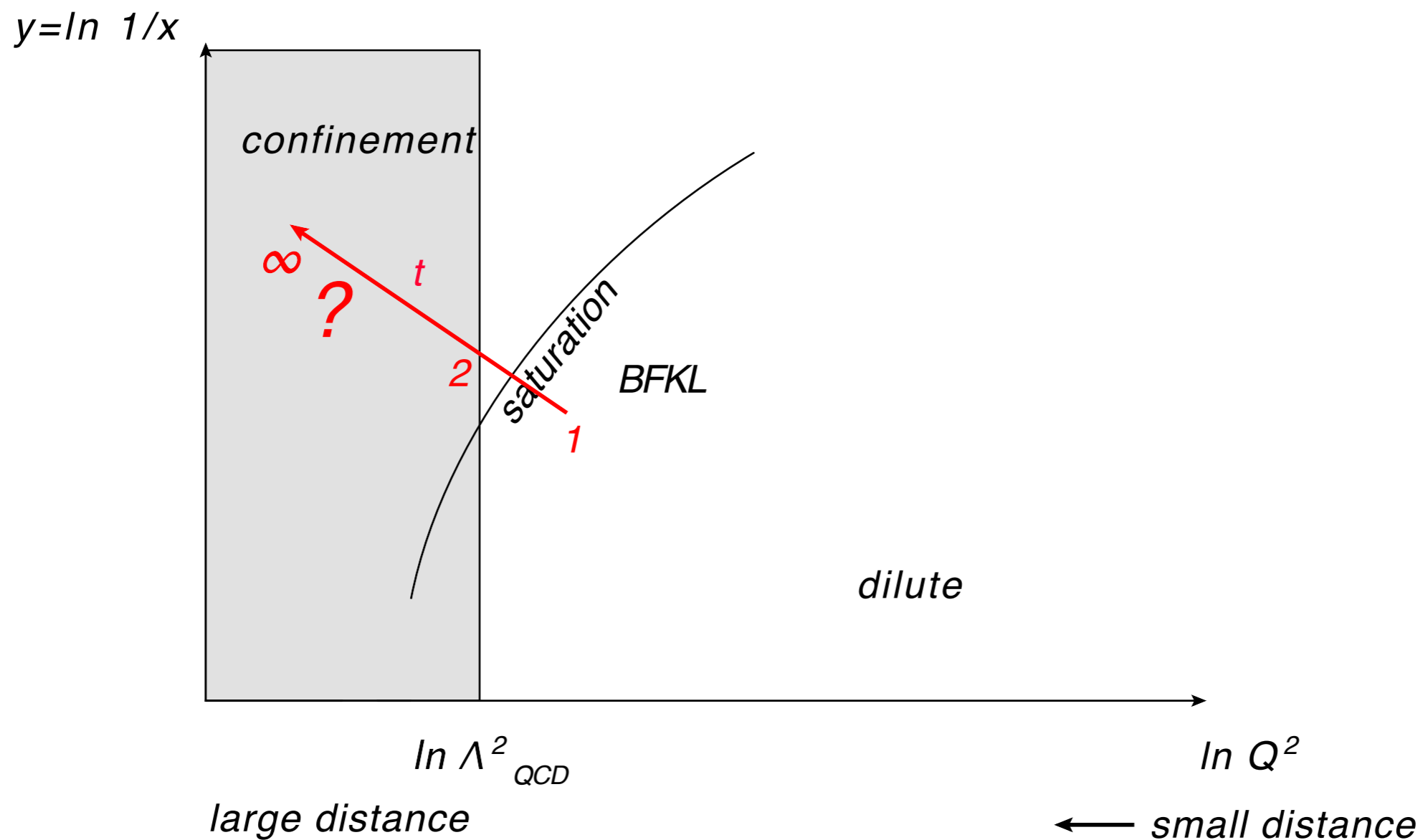
Diffraction 2014, 10. - 15. September, Primosten

Collaboration with
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- Motivation and project
- Flow equations
- First results: fixed points

I. Motivation and project

Question: how to continue small-x physics into the nonperturbative region?



Along the red line: evolution time $t = \ln k$

rapidity, transverse distance: from low to large scale

reggeon energy (angular momentum), momentum: from large to low scale
(IR problem).

1) BFKL with IR boundary conditions (no fixed cut)

Lipatov, Ross, Kowalski

2) 'Pomeron loops':

corrections, Pomeron = superposition of 2 and 4 reggeized gluons

more corrections, Pomeron = superposition of 2, 4, 6, reggeized gluons

Toy model: BK equations

...

∞) Some sort of RFT: (Nonperturbative) Pomeron with self-interactions

(Donnachie-Landshoff, GLM, KMR,...)

Important:

- Pomeron field has internal degrees of freedom (BFKL), nonlocal RFT
- Pomeron field changes as function of scale (rapidity and distance)

→ Wilson RG equation, flow equations

The formalism: functional renormalization group

Reminder: **Wilson approach**

The standard Wilsonian action is defined by an iterative change in the **UV-cutoff** induced by a partial integration of quantum fluctuations:

$$\Lambda \rightarrow \Lambda' < \Lambda$$
$$\int [d\varphi]^\Lambda e^{-S^\Lambda[\varphi]} = \int [d\varphi]^{\Lambda'} e^{-S^{\Lambda'}[\varphi]} \quad k < \Lambda$$

Alternatively: **FRG-approach (Wetterich) IR-cutoff**

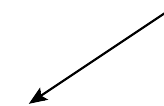
(successful use in statistical mechanics and in gravity)

define a bare theory at scale Λ .

The integration of the modes in the interval $[k, \Lambda]$ defines a k -dependent average functional.

Letting k flowing down to 0 defines a flow for the functional which leads to full theory. k -dependent effective action:

$$e^{-\Gamma_k[\phi]} = \int [d\varphi] \mu_k e^{-S[\varphi] + \int_x (\varphi - \phi)_x \frac{\delta \Gamma_k[\phi]}{\delta \phi_x} - \Delta S_k[\varphi - \phi]}$$

regulator 

Taking a derivative with respect the RG time $t = \ln k/k_0$ one obtains

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right] - \frac{\dot{\mu}_k}{\mu_k}$$

\mathcal{R} = regulator operator

which is UV and IR finite.

Quantum fluctuations \rightarrow coupled differential equations

Steps:

I) **Fixed points**: beta-functions for parameters of the potential:
zeroes determine fixed points, existence of possible theories.

Expand in powers of fields:

$$\partial_t \Gamma_k[\phi] = \mathcal{F}[\Gamma_k[\phi], \Gamma'_k[\phi], \Gamma''_k[\phi]]$$

Equate powers of field on both sides \rightarrow sets of beta functions

2) Vertex functions (physical observables: total cross section)

Taking functional derivatives in the fields:

$$\partial_t \Gamma_k = \frac{1}{2} G_{k;AB} \partial_t \mathcal{R}_{k;BA}$$

$$\partial_t \Gamma_{k;A_1}^{(1)} = -\frac{1}{2} G_{k;AB} \Gamma_{k;A_1 BC}^{(3)} G_{k;CD} \partial_t \mathcal{R}_{k;DA}$$

$$\begin{aligned} \partial_t \Gamma_{k;A_1 A_2}^{(2)} &= \frac{1}{2} G_{k;AB} \Gamma_{k;A_1 BC}^{(3)} G_{k;CD} \Gamma_{k;A_2 DE}^{(3)} G_{k;EF} \partial_t \mathcal{R}_{k;FA} \\ &\quad + \frac{1}{2} G_{k;AB} \Gamma_{k;A_2 BC}^{(3)} G_{k;AB} \Gamma_{k;A_1 BC}^{(3)} G_{k;CD} \partial_t \mathcal{R}_{k;DA} \\ &\quad - \frac{1}{2} G_{k;AB} \Gamma_{k;A_1 A_2 BC}^{(4)} G_{k;CD} \partial_t \mathcal{R}_{k;DA} \end{aligned}$$

First results

Local reggeon field theory:

Abarbanel, Bronzan;
Migdal, Polyakov, Ter-Martyrosyan
 ϵ -expansion around D=4: IR fixpoint

$$\mathcal{L} = (\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial}_y \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V(\psi, \psi^\dagger)$$

$$V(\psi, \psi^\dagger) = -\mu \psi^\dagger \psi + i\lambda \psi^\dagger (\psi^\dagger + \psi) \psi \\ + g(\psi^\dagger \psi)^2 + g' \psi^\dagger (\psi^{\dagger 2} + \psi^2) \psi + \dots \quad \mu = \alpha(0) - 1$$

J. Cardy and R. Sugar noticed in 1980 that the RFT is in the same universality class as Markov process known as Directed Percolation (DP).

Critical exponents can then be accessed also with numerical MonteCarlo computations.

Effective action with local potential:

$$\Gamma_k = \int dy d^D x \left[Z_k \left(\frac{1}{2} \psi^{dagger} \overleftrightarrow{\partial}_y \psi - \alpha'_k \psi^{dagger} \nabla^2 \psi \right) + V_k(\psi, \psi^\dagger) \right]$$

Propagator of flow equations:

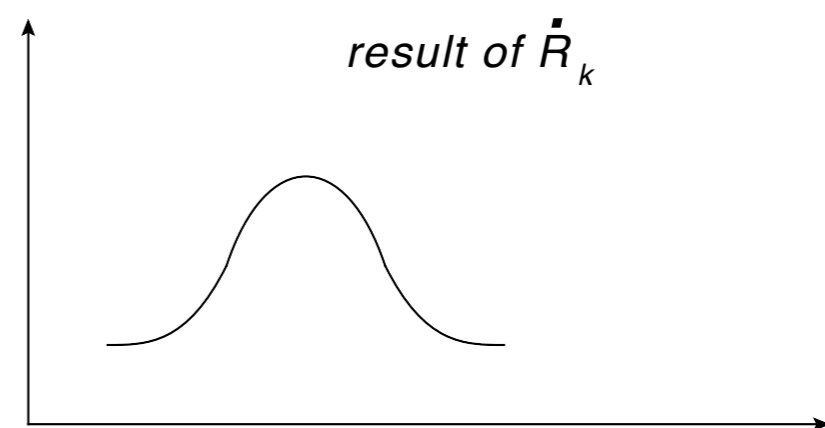
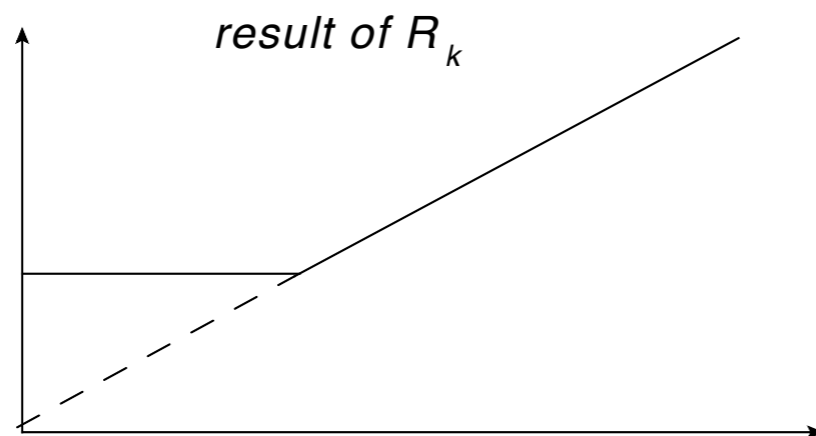
$$\Gamma_k^{(2)} + \mathbb{R} = \begin{pmatrix} V_{k\psi\psi} & -iZ_k\omega + Z_k\alpha'_k q^2 + R_k + V_{k\psi\psi^{dagger}} \\ iZ_k\omega + Z_k\alpha'_k q^2 + R_k + V_{k\psi^{dagger}\psi} & V_{k\psi^{dagger}\psi^{dagger}} \end{pmatrix}$$

Flow equation for potential:

$$\dot{V}_k(\psi, \psi^\dagger) = \frac{1}{2} \text{tr} \left\{ \int \frac{d\omega d^D q}{(2\pi)^{D+1}} \left[\left(\Gamma_k^{(2)} + \mathbb{R} \right)^{-1} \dot{\mathcal{R}}_k \right] \right\}$$

Coarse graining in momentum space:

$$R_k(q) = Z_k \alpha'_k (k^2 - q^2) \Theta(k^2 - q^2)$$



Flow equation for the potential:
 expand in powers of the fields on both sides,
 truncate after some maximal power.

Obtain coupled set of ‘beta functions’, e.g.

$$\begin{aligned}\dot{\tilde{\mu}} &= \tilde{\mu}(-2 + \zeta + \eta) + 2N_D A_D(\eta_k, \zeta_k) \frac{\tilde{\lambda}^2}{(1 - \tilde{\mu})^2} \\ \dot{\tilde{\lambda}} &= \tilde{\lambda} \left((-2 + \zeta + \frac{D}{2} + \frac{3\eta}{2}) + 2N_D A_D(\eta_k, \zeta_k) \left(\frac{4\tilde{\lambda}^2}{(1 - \tilde{\mu})^3} + \frac{(\tilde{g} + 3\tilde{g}')}{(1 - \tilde{\mu})^2} \right) \right) \\ &\quad \dots\end{aligned}$$

Fixed points: search for zeroes.

First numerical results on the existence of fixed points: preliminary (3 different truncations)

In all truncations there is always the trivial (zero coupling solution) FP.

The eigenvalues of the stability matrix in this point are:

cubic(-2,-1), quartic (add (0,0)), quintic (add (1,1)), etc

- cubic: $\text{FP}_3 = (\tilde{\mu}^*, \tilde{\lambda}^*) = (0.111, \pm 1.05)$
eigenvalues: (2.39, -1.89)
- quartic: $\text{FP}_4 = (\tilde{\mu}^*, \tilde{\lambda}^*, \tilde{g}^*, \tilde{g}'^*) = (0.27, \pm 1.35, -2.89, -1.27)$
eigenvalues: (19.99, 6.08, 2.51, -1.69)
- quintic: $\text{FP}_5 = (\tilde{\mu}^*, \tilde{\lambda}^*, \tilde{g}^*, \tilde{g}'^*, \tilde{\lambda}_5^*, \tilde{\lambda}_5'^*) = (0.39, \pm 1.35, -4.10, -1.82, -4.83, -1.34)$
eigenvalues: (59.11, 33.12, 16.26, 3.99, 2.13, -1.45)

Test: compare with Monte Carlo result.

The leading critical exponent ν (the most negative eigenvalue) (with increasing truncation) is:

$$\nu_3 = 0.52, \quad \nu_4 = 0.59, \quad \nu_5 = 0.69$$

Compare with the Monte Carlo result for Directed Percolation (same universality class): $\nu = 0.73$.

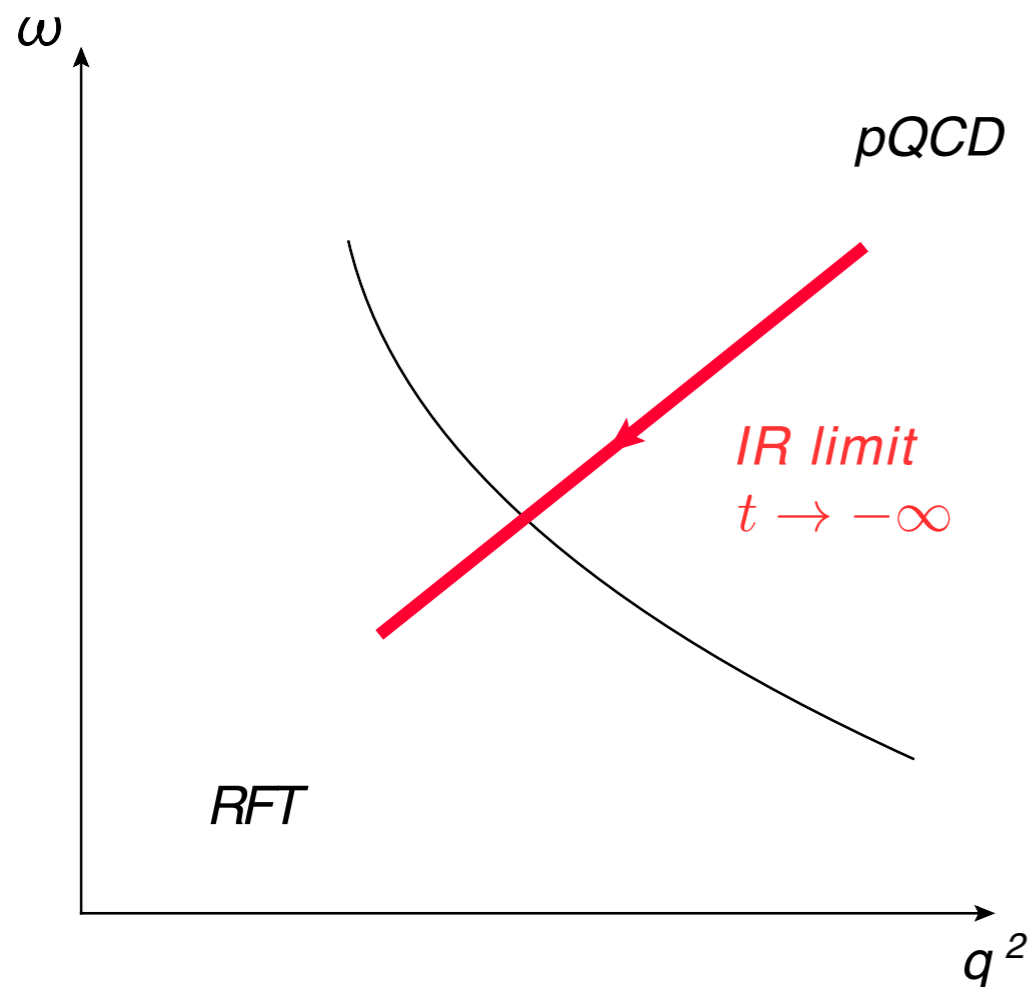
Our simple estimate in the quintic coupling truncation) is within 5%.

Interpretation:

The family of non trivial fixed points is characterized by just **one relevant direction**.

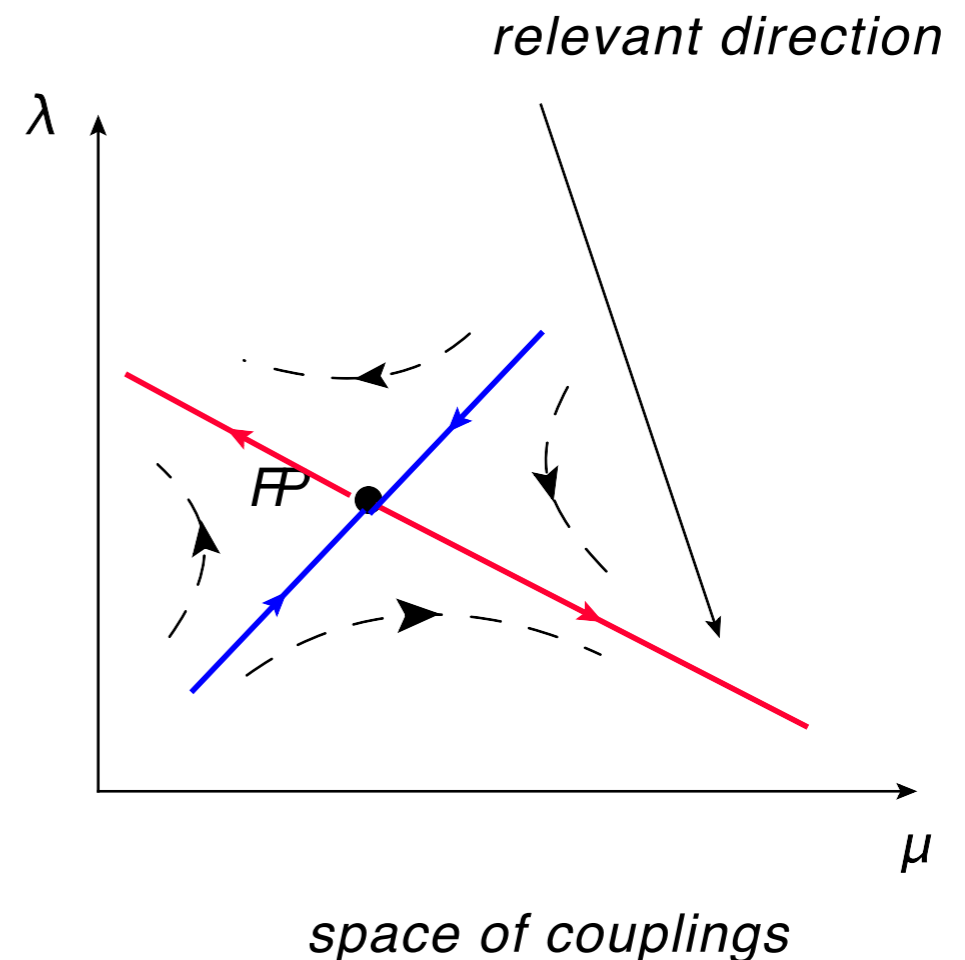
We feel tempted to interpret this as follows:

possible transition from QCD in the Regge limit to a RFT effective description.



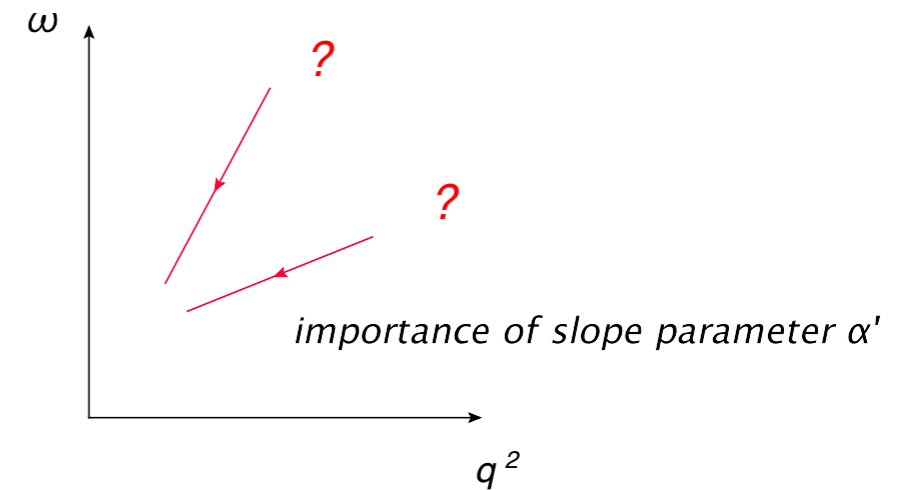
interest in the IR region:

$\left\{ \begin{array}{l} \text{small } \omega, \text{ small } q^2 \\ \text{large rapidity, large distances} \end{array} \right\}$



Next steps:

- Other course graining (cutoff) schemes:
controlling integration in energy
controlling the integration in energy and momentum
(first indications that results are consistent with first cutoff-scheme)
- expansion around nonzero fields (nonzero vacuum configuration)
encouraging new results (stability)
- Compute Green's functions: physical observables



D.Amati, L.Caneschi, R.Jengo;
A.White

Conclusions

- We have started to study features of RFT, as a candidate for a possible effective theory of the Regge limit of QCD at large rapidities/distances.
- Have set up functional renormalization group scheme (flow equations), suitable also for nonperturbative aspects.
- From preliminary results we find a nontrivial fixed point with one relevant direction: this may lead to a possible physical scenario.
- Steps under way: further course graining schemes.
- How important are possible nonzero vacuum configurations? Phase transitions?
- Physical observables: energy dependence of total cross section
- Most difficult: study the transition, from pQCD to RFT?