Twist expansion of differential cross-sections of forward Drell-Yan process

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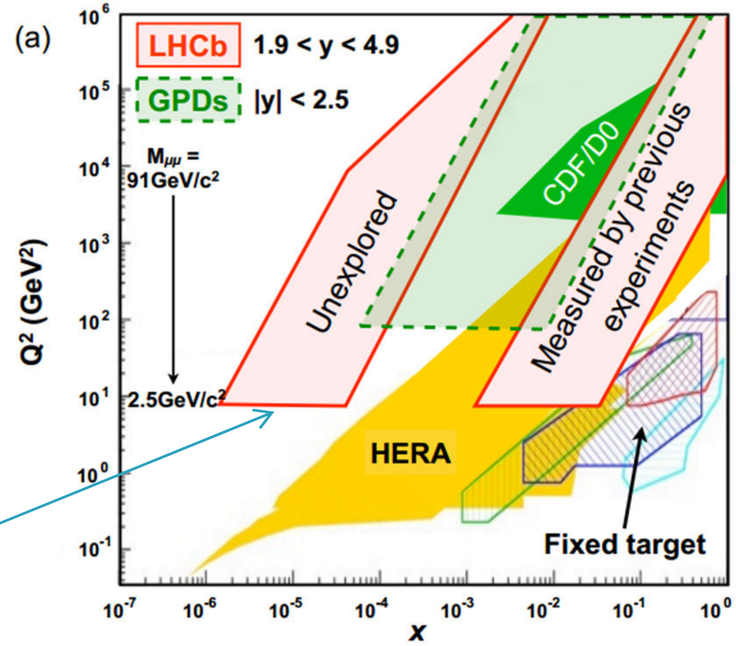
Introduction

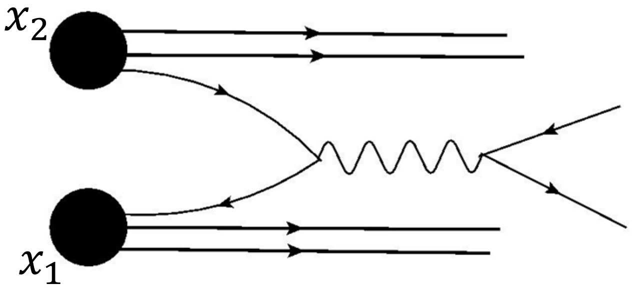
- The forward DY at the LHC provide the sensitive measurements of parton densities down to very small x.
- Kinematic region not explored before LHC.



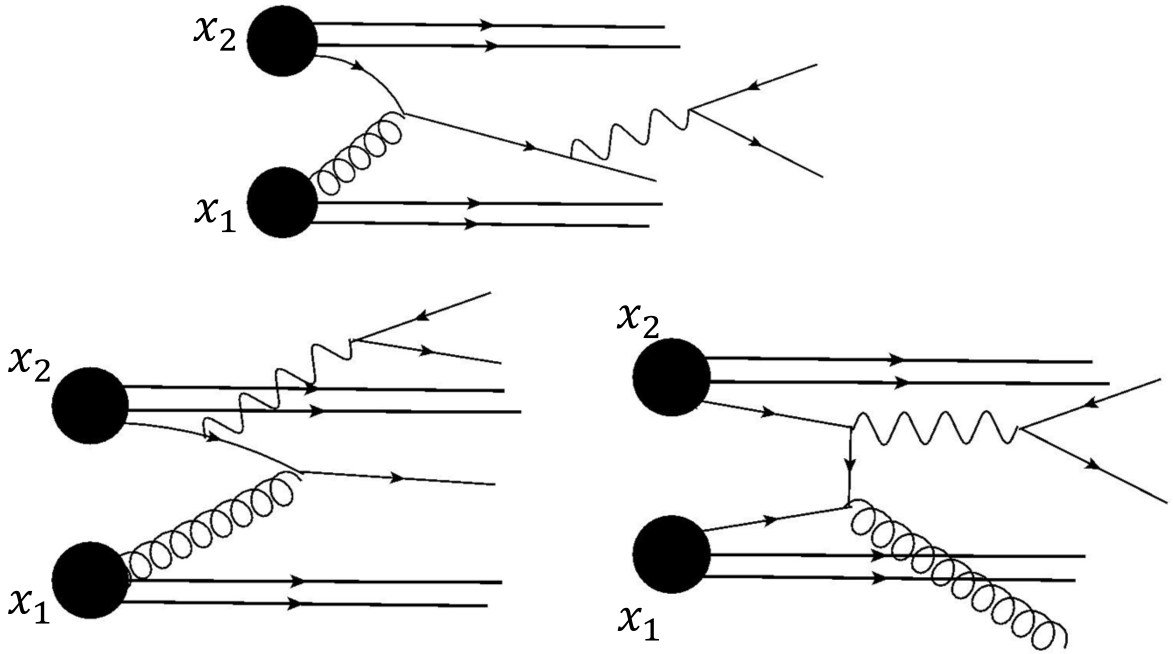
J. Anderson LHCb-CONF-2009-014 03/07/2009

- Multiple scattering, higher twist effects. Important to extract the standard twist-2 parton densities with higher precision.
- Four independent structure functions. Investigating them we are more sensitive to higher twists than in DIS (Lam-Tung relation).
- k_T-factorization framework and color dipole model.
- Extension of Golec-Biernat et al results to angular and photon transverse momentum dependent cross section.

Drell -Yan process

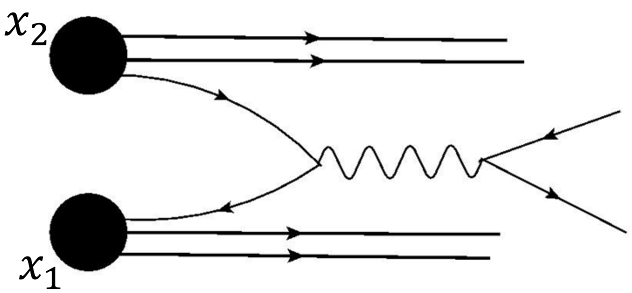


LO



NLO

Drell -Yan process



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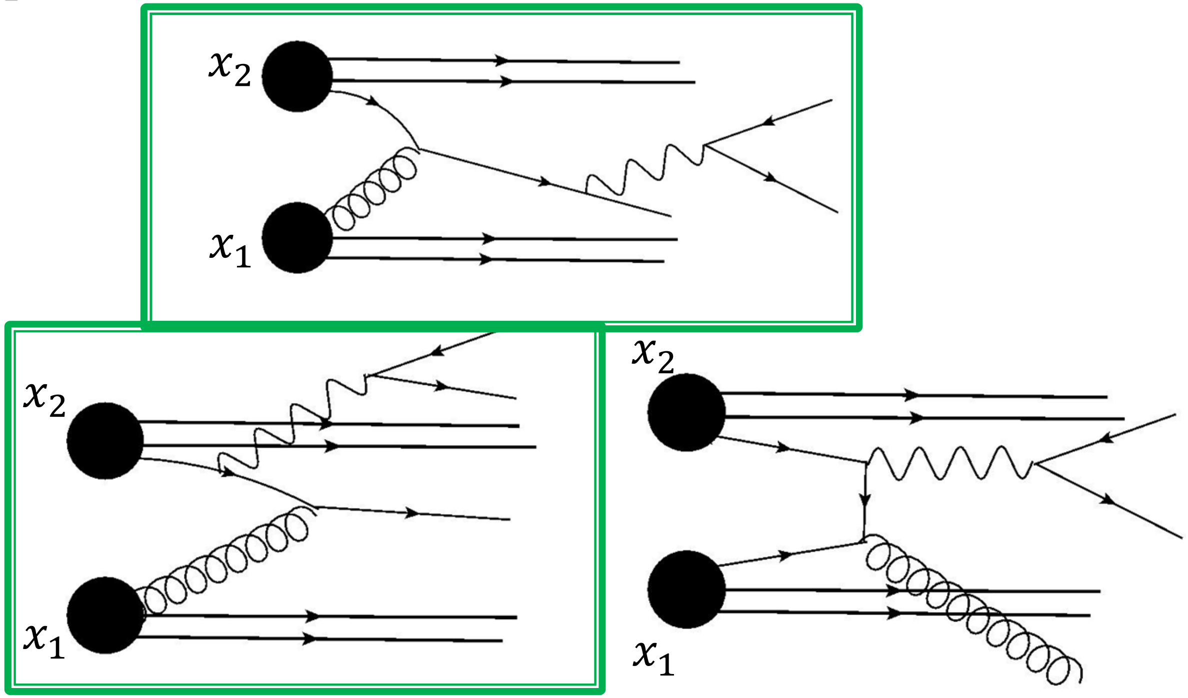


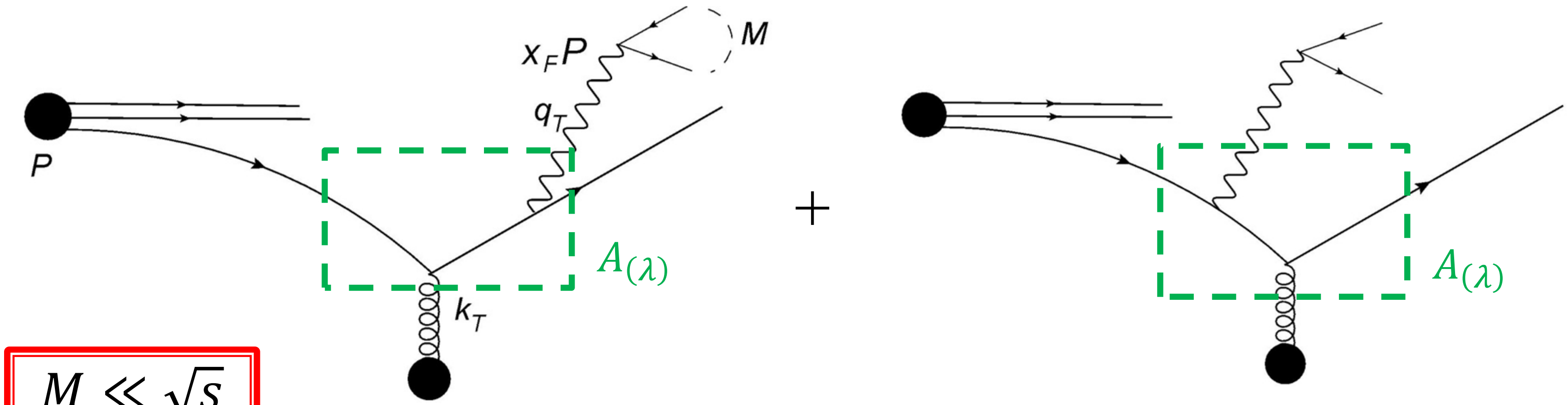
NLO

Forward Drell-Yan: $x_1 \ll x_2 \approx 0.1$

Large gluon density

NLO diagrams are dominant

Drell –Yan in target rest frame



$$\boxed{M \ll \sqrt{s}}$$

- Cross section (k_T factorization):

lepton angles $d\Omega$

$$\frac{d\sigma}{dx_F\, dM^2 d\Omega\, d^2q_T} \sim \int_{x_F}^{1} dz \frac{\wp(x_F/z)}{1-z} \int d^2k_T \frac{f(x_g, k_T)}{{k_T}^4} \overline{A_{(\sigma)}} A_{(\lambda)}\, L^{(\sigma\lambda)}$$

p.d.f.

$$\frac{f(x_g, k_T)}{{k_T}^4} = -\frac{1}{2}\int d^2r\, e^{i\overrightarrow{k_T}\vec{r}}\, \hat{\sigma}(r)$$

Kopeliovich et al. , Brodsky et al.

Invariant and helicity structure functions

Lam, Tung, Phys.Rev. D 17, 2447

Two ways of hadron tensor decomposition:

- Invariant structure functions:

$$W^{\mu\nu} = -T_1\ \tilde{g}^{\mu\nu} + T_2\ \tilde{P}^\mu \tilde{P}^\nu - T_3\ \frac{1}{2}\left(\tilde{P}^\mu \tilde{p}^\nu + \tilde{p}^\mu \tilde{P}^\nu\right) + T_4\ \tilde{p}^\mu \tilde{p}^\nu$$

$\tilde{g}^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2$, $P = P_1 + P_2$, $p = P_1 - P_2$ and $P^\mu = \tilde{g}^{\mu\nu} P_\nu / \sqrt{s}$, $\tilde{p}^\mu = \tilde{g}^{\mu\nu} p_\nu / \sqrt{s}$.

- Helicity structure functions (depend on choice of foton polarization frame):

$$W^{\mu\nu} = -\tilde{g}^{\mu\nu}(W_T + W_{TT}) - X^\mu X^\nu W_{TT} + Z^\mu Z^\nu (W_L - W_T - W_{TT}) - \left(\tilde{X}^\mu Z^\nu + Z^\mu X\nu\right) W_{LT}$$

$$\boxed{\frac{d\sigma}{dx_F dM^2 d\Omega d^2 q_\perp} = \frac{\alpha_{em}^2 \sigma_0}{2(2\pi)^4 M^4}\left[W_L(1-\cos^2\theta) + W_T(1+\cos^2\theta) + W_{TT}(\sin^2\theta\cos 2\phi) + W_{LT}(\sin 2\theta \cos\phi)\right]}$$

- One can find (linear) relation between T_i and W_j.

Mellin tansform

Bartels et al. , Golec -Biernat et al.

$$\frac{d\sigma}{dx_F\, dM^2 d\Omega\, d^2 q_T} \sim L^{(\sigma\lambda)} \int_{x_F}^{1} dz\, \frac{\wp(x_F/z)}{1-z} \int d^2 r\, \hat{\sigma}(r) \underbrace{\int d^2 k_T\, e^{i\vec{k_T}\vec{r}}\, \overline{A_{(\sigma)}} A_{(\lambda)}}_{\equiv\, \phi_{\sigma\lambda}(r)}$$

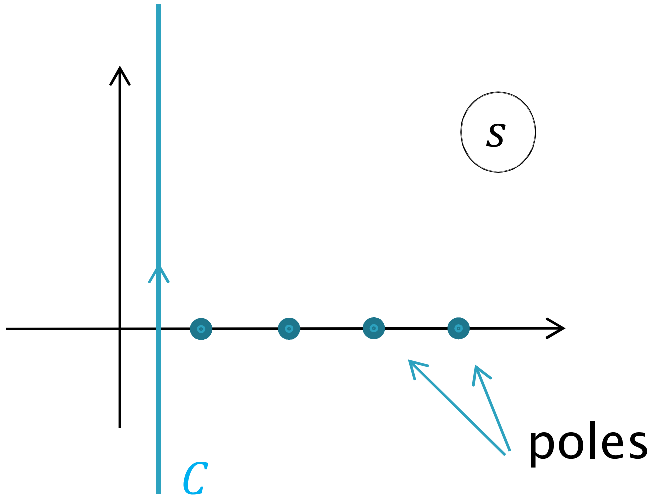
impact factor

$$\hat{\sigma}(z\vec{r}) = \int_C \frac{ds}{2\pi i} \left(\frac{z^2 Q_0^2}{4} r\right)^s \tilde{\sigma}(-s)$$

$$\int d^2 r\, \hat{\sigma}(r)\, \phi_{\sigma\lambda}(r) \Longrightarrow \int_C \frac{ds}{2\pi i} \left(\frac{{Q_o}^2}{M^2}\right)^s \tilde{\sigma}(-s)\, \tilde{\phi}_{\sigma\lambda}(s)$$

inv. Mellin transform

$$\tilde{\phi}_{\sigma\lambda}(s) = \int_0^\infty \frac{dr^2}{r^2} r^{2s}\, \phi_{\sigma\lambda}(r)$$



Twist expansion of q_T-dependent cross section

$$\frac{d\sigma_T}{dx_F dM^2 d\Omega d^2q_\perp} = \frac{\alpha_{em}^2}{2(2\pi)^4 M^4}\,(1+\cos^2\theta)\int_{x_F}^1 dz\;\wp(x_F/z)\frac{1+(1-z)^2}{z^2(1-z)}\int_C \frac{ds}{2\pi i}\left(\frac{z^2Q_0^2}{\eta_z^2}\right)^s \tilde{\sigma}(-s)\,\times$$

$$\boxed{\begin{aligned}\times\frac{1}{2}\Bigg\{&\frac{2q_\perp^2/\eta_z^2}{1+q_\perp^2/\eta_z^2}\Gamma(s+1)\Gamma(s+2)\;{}_2F_1\left(s+1,s+2,2,-\frac{q_\perp^2}{\eta_z^2}\right)-\\ &\Gamma(s+1)^2\left[{}_2F_1\left(s+1,s+1,1,-\frac{q_\perp^2}{\eta_z^2}\right)-(s+1)\;{}_2F_1\left(s+1,s+2,1,-\frac{q_\perp^2}{\eta_z^2}\right)\right]\Bigg\}\end{aligned}}$$

$$\eta_z^2 = M^2(1-z)$$

$$= \tilde{\phi}_{++}(s) + \tilde{\phi}_{--}(s)$$

We have:

- two hard scales: M, q_T
- one soft scale: Q_o

twist expansion ≡ expansion in powers of Q_o

$$\boxed{Q_o \ll M, q_T}$$

Twist expansion of q_T-dependent cross section

$$\frac{d\sigma_T}{dx_F dM^2 d\Omega d^2q_\perp} = \frac{\alpha_{em}^2}{2(2\pi)^4 M^4}\,(1+\cos^2\theta)\int_{x_F}^1 dz\ \wp(x_F/z)\frac{1+(1-z)^2}{z^2(1-z)}\int_C \frac{ds}{2\pi i}\left(\frac{z^2Q_0^2}{\eta_z^2}\right)^s \boxed{\tilde\sigma(-s)}\times$$

$$\times\frac{1}{2}\left\{\frac{2q_\perp^2/\eta_z^2}{1+q_\perp^2/\eta_z^2}\Gamma(s+1)\Gamma(s+2)\ {}_2F_1\left(s+1,s+2,2,-\frac{q_\perp^2}{\eta_z^2}\right)-\right.$$

$$\left.\Gamma(s+1)^2\left[{}_2F_1\left(s+1,s+1,1,-\frac{q_\perp^2}{\eta_z^2}\right)-(s+1)\ {}_2F_1\left(s+1,s+2,1,-\frac{q_\perp^2}{\eta_z^2}\right)\right]\right\}$$

$$\eta_z^2 = M^2(1-z)$$

Golec-Biernat and Wüsthof model:

$$\hat\sigma(\vec\rho) = \sigma_0(1-e^{-\rho^2}) \Longrightarrow \tilde\sigma(-s) = -\sigma_0\Gamma(-s)$$

Twists come from poles of Γ:

$s = 1$ – tw. 2

$s = 2$ – tw. 4

...

Twist expansion of q_T-dependent cross section

Twist 2

$$W_L^{(2)} = \frac{Q_0^2}{M^2}\int_{x_F}^1 dz\ \wp(x_F/z)\frac{4M^6\ q_\perp^2(1-z)^2}{\left[q_\perp^2+M^2(1-z)\right]^4}$$

$$W_T^{(2)} = \frac{Q_0^2}{M^2}\int_{x_F}^1 dz\ \wp(x_F/z)\left[1+(1-z)^2\right]\frac{M^4\left[q_\perp^4+M^4(1-z)^2\right]}{2\left[q_\perp^2+M^2(1-z)\right]^4}$$

$$W_{TT}^{(2)} = \frac{Q_0^2}{M^2}\int_{x_F}^1 dz\ \wp(x_F/z)\frac{2M^6\ q_\perp^2(1-z)^2}{\left[q_\perp^2+M^2(1-z)\right]^4}$$

$$W_{LT}^{(2)} = \frac{Q_0^2}{M^2}\int_{x_F}^1 dz\ \wp(x_F/z)(2-z)\ \frac{M^5\ q_\perp\left[-q_\perp^2+M^2(1-z)\right](1-z)}{\left[q_\perp^2+M^2(1-z)\right]^4}$$

Twist 4

$$W_L^{(4)} = \frac{Q_0^4}{M^4}\int_{x_F}^1 dz\ \wp(x_F/z)z^2\times$$
$$\times\frac{4M^8\left[7q_\perp^2-10M^2q_\perp^2(1-z)+M^4(1-z)^2\right](1-z)^2}{\left[q_\perp^2+M^2(1-z)\right]^6}$$

$$W_T^{(4)} = \frac{Q_0^4}{M^4}\int_{x_F}^1 dz\ \wp(x_F/z)\left[1+(1-z)^2\right]z^2\times$$
$$\times\frac{M^6\left[q_\perp^2-2M^2(1-z)\right]\left[q_\perp^4-4M^2q_\perp^2(1-z)+M^4(1-z)^2\right]}{\left[q_\perp^2+M^2(1-z)\right]^6}$$

$$W_{TT}^{(4)} = \frac{Q_0^4}{M^4}\int_{x_F}^1 dz\ \wp(x_F/z)z^2\frac{12M^8q_\perp^2\left[q_\perp^2-2M^2(1-z)\right](1-z)^2}{\left[q_\perp^2+M^2(1-z)\right]^6}$$

$$W_{LT}^{(4)} = \frac{Q_0^4}{M^4}\int_{x_F}^1 dz\ \wp(x_F/z)(2-z)\sqrt{1-z}\ z^2\times$$
$$\times\frac{2M^7\ q_\perp\left[-2q_\perp^2+M^2(1-z)\right]\left[q_\perp^2-5M^2(1-z)\right](1-z)^2}{\left[q_\perp^2+M^2(1-z)\right]^6}$$

Twist expansion of q_T–dependent cross section

Use relations between T_i and W_j to get twist expansion of T_i:

$$T_1^{(2)} = \frac{Q_0^2}{M^2}\int_{x_F}^1 dz\ \wp(x_F/z)\frac{M^4\left[4M^2q_\perp^2(1-z)^2+q_\perp^4(2-z(2-z))+M^4(1-z)^2(2-(2-z)z)\right]}{2\left[q_\perp^2+M^2(1-z)\right]^4}$$

$$T_2^{(2)} = \frac{Q_0^2}{M^2}\int_{x_F}^1 dz\ \wp(x_F/z)\frac{-M^6\left[2s^2x_F^4(1-z)^2+2sx_F^2(q_\perp^2+M^2(1-z))z(1-z)+(q_\perp^2+M^2(1-z))^2z^2\right]}{2sx_F^2\left[q_\perp^2+M^2(1-z)\right]^4}$$

$$T_3^{(2)} = \frac{Q_0^2}{M^2}\int_{x_F}^1 dz\ \wp(x_F/z)\frac{M^6\left[-2s^2x_F^4(1-z)^2+(q_\perp^2+M^2(1-z))^2z^2\right]}{sx_F^2\left[q_\perp^2+M^2(1-z)\right]^4}$$

$$T_4^{(2)} = \frac{Q_0^2}{M^2}\int_{x_F}^1 dz\ \wp(x_F/z)\frac{-M^6\left[2s^2x_F^4(1-z)^2-2sx_F^2(q_\perp^2+M^2(1-z))z(1-z)+(q_\perp^2+M^2(1-z))^2z^2\right]}{2sx_F^2\left[q_\perp^2+M^2(1-z)\right]^4}$$

Lam-Tung relation

Lam, Tung Phys.Lett. B 80, 228 :

$$T_1 + \left(\frac{q_P^2}{M^2} - 1\right) T_2 - \frac{q_P q_p}{M^2} T_3 + \left(\frac{q_p^2}{M^2} + 1\right) T_4 = 0$$

One can rewrite it using relations between T_i and W_j:

$$W_L - 2W_{TT} = 0$$

$$W_{TT}^{(2)} = \frac{Q_0^2}{M^2} \int_{x_F}^1 dz\ \wp(x_F/z) \frac{2M^6\ q_\perp^2 (1-z)^2}{\left[q_\perp^2 + M^2(1-z)\right]^4}$$

$$W_L^{(2)} = \frac{Q_0^2}{M^2} \int_{x_F}^1 dz\ \wp(x_F/z) \frac{4M^6\ q_\perp^2 (1-z)^2}{\left[q_\perp^2 + M^2(1-z)\right]^4}$$

Gelis, Jalilian-Marian

<u>Lam-Tung relation satisfied at the leading twist</u>

Next to leading twist:

$$W_L^{(4)} - 2W_{TT}^{(4)} = \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z) z^2 \frac{4M^8 (1-z)^2}{\left[q_\perp^2 + M^2(1-z)\right]^4}$$

Twist expansion of cross section integrated over q_T

$$\frac{d\sigma_T}{dx_F dM^2 d\Omega d^2 q_\perp} = \frac{\alpha_{em}^2}{2(2\pi)^4 M^4}\,(1+\cos^2\theta)\int_{x_F}^1 dz\ \wp(x_F/z)\frac{1+(1-z)^2}{z^2(1-z)}\int_C \frac{ds}{2\pi i}\left(\frac{z^2 Q_0^2}{\eta_z^2}\right)^s \tilde{\sigma}(-s)\times$$
$$\times\frac{1}{2}\left\{\frac{2q_\perp^2/\eta_z^2}{1+q_\perp^2/\eta_z^2}\Gamma(s+1)\Gamma(s+2)\ {}_2F_1\left(s+1,s+2,2,-\frac{q_\perp^2}{\eta_z^2}\right)-\right.$$
$$\left.\Gamma(s+1)^2\left[{}_2F_1\left(s+1,s+1,1,-\frac{q_\perp^2}{\eta_z^2}\right)-(s+1)\ {}_2F_1\left(s+1,s+2,1,-\frac{q_\perp^2}{\eta_z^2}\right)\right]\right\}$$

Integrate over d^2q_T:

$$\frac{d\sigma_T}{dx_F dM^2 d\Omega} = \frac{\alpha_{em}^2}{2(2\pi)^3 M^2}\,(1+\cos^2\theta)\int_{x_F}^1 dz\ \wp(x_F/z)\frac{1+(1-z)^2}{z^2}\times$$
$$\times\int_C \frac{ds}{2\pi i}\left(\frac{z^2 Q_0^2}{4\eta_z^2}\right)^s \tilde{\sigma}(-s)\left\{\frac{\sqrt{\pi}\ \Gamma(s)\Gamma(s+1)\Gamma(s+2)}{4\Gamma\left(s+\frac{3}{2}\right)}\right\}$$

Similar expresions for $\sigma_L, \sigma_{TT}, \sigma_{LT}$.

Twist expansion of cross section integrated over q_T

$$\frac{d\sigma_T}{dx_F dM^2 d\Omega} = \frac{\alpha_{em}^2}{2(2\pi)^3 M^2} \left(1+\cos^2\theta\right) \int_{x_F}^1 dz\ \wp(x_F/z) \frac{1+(1-z)^2}{z^2} \times$$
$$\times \int_C \frac{ds}{2\pi i} \left(\frac{z^2 Q_0^2}{4\eta_z^2}\right)^s \tilde{\sigma}(-s) \left\{ \frac{\sqrt{\pi}\ \Gamma(s)\Gamma(s+1)\Gamma(s+2)}{4\Gamma\left(s+\frac{3}{2}\right)} \right\}$$

Taking $s = 1$ for tw. 2:

$$\int W_T^{(2)}\ d^2 q_\perp = \frac{Q_0^2}{M^2} \int_{x_F}^1 dz\ \wp(x_F/z) \frac{1+(1-z)^2}{1-z} \frac{\pi M^2}{3}$$

Divergent!

Integrand have double pole in $s = 1$:

see Golec-Biernat, Lewandowska, Staśto; arXiv:1008.2652v1

$$\tilde{W}_T^{(2)} = \frac{Q_0^2}{4M^2} \left\{ \wp(x_F) \left[-1 + \frac{4}{3}\gamma_E - \frac{2}{3} \ln\left(\frac{Q_0^2}{4M^2(1-x_F)}\right) + \frac{2}{3}\psi(5/2) \right] + \right.$$
$$\left. + \frac{2}{3} \int_{x_F}^1 dz\ \frac{\wp(x_F/z)[1+(1-z)^2] - \wp(x_F)}{1-z} \right\}$$

$$\tilde{W}_i = \frac{1}{2\pi M^2} \int W_i\ d^2 q_\perp$$

Twist expansion of cross section integrated over q_T

Twist 2

$$\tilde{W}_L^{(2)} = \frac{Q_0^2}{3M^2}\int_{x_F}^1 dz\ \wp(x_F/z)$$

$$\tilde{W}_T^{(2)} = \frac{Q_0^2}{4M^2}\left\{\wp(x_F)\left[-1+\frac{4}{3}\gamma_E-\frac{2}{3}\ln\left(\frac{Q_0^2}{4M^2(1-x_F)}\right)+\frac{2}{3}\psi(5/2)\right]+ + \frac{2}{3}\int_{x_F}^1 dz\ \frac{\wp(x_F/z)[1+(1-z)^2]-\wp(x_F)}{1-z}\right\}$$

$$\tilde{W}_{TT}^{(2)} = \frac{Q_0^2}{6M^2}\int_{x_F}^1 dz\ \wp(x_F/z)$$

$$\tilde{W}_{LT}^{(2)} = 0$$

Twist 3

$$\tilde{W}_{LT}^{(3)} = \underbrace{const}_{\approx 0.593}\cdot\frac{Q_0^3}{M^3}\wp(x_F)$$

Twist 4

$$\tilde{W}_L^{(4)} = \frac{2}{15}\frac{Q_0^4}{M^4}\left\{\wp(x_F)\left[3-2\gamma_E+\ln\left(\frac{Q_0^2}{4M^2(1-x_F)}\right)-\psi(7/2)\right] - \int_{x_F}^1 dz\ \frac{\wp(x_F/z)z^2-\wp(x_F)}{1-z}\right\}$$

$$\tilde{W}_T^{(4)} = \ldots$$

$$\vdots$$

Summary

- Mellin representation of forward DY impact factors.
- Twist expansion of differential cross-section in the lepton angles and the DY pair transverse momentum q_T. Both for invariant and helicity structure functions.
- Twist expansion for cross-section integrated over q_T.
- Lam-Tung relation as a way of measuring higher twist effects.

... and Outlook

- Phenomenological calculation of twists and comparison with data.
- Comparison of different forms of dipole cross section.

Thank you

Relation beetween invariant and helicity structure functions

- Invariant structure functions:

$$W^{\mu\nu} = -T_1\ \tilde{g}^{\mu\nu} + T_2\ \tilde{P}^\mu \tilde{P}^\nu - T_3\ \frac{1}{2}\left(\tilde{P}^\mu \tilde{p}^\nu + \tilde{p}^\mu \tilde{P}^\nu\right) + T_4\ \tilde{p}^\mu \tilde{p}^\nu$$

$\tilde{g}^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2$, $P = P_1 + P_2$, $p = P_1 - P_2$ and $P^\mu = \tilde{g}^{\mu\nu} P_\nu / \sqrt{s}$, $\tilde{p}^\mu = \tilde{g}^{\mu\nu} p_\nu / \sqrt{s}$.

- Helicity structure functions:

$$W^{\mu\nu} = -\tilde{g}^{\mu\nu}(W_T + W_{TT}) - X^\mu X^\nu W_{TT} + Z^\mu Z^\nu (W_L - W_T - W_{TT}) - \left(\tilde{X}^\mu Z^\nu + Z^\mu X\nu\right) W_{LT}$$

$$\begin{aligned} Z^\mu &= \alpha \tilde{P}^\mu + \beta \tilde{p}^\mu \\ X^\mu &= \alpha' \tilde{P}^\mu + \beta' \tilde{p}^\mu \end{aligned}$$

choice of coefficients = choice of frame,
in our case: target rest frame

$$\frac{d\sigma}{dx_F dM^2 d\Omega d^2 q_\perp} = \frac{\alpha_{em}^2 \sigma_0}{2(2\pi)^4 M^4}\left[W_L(1-\cos^2\theta) + W_T(1+\cos^2\theta) + W_{TT}(\sin^2\theta\cos 2\phi) + W_{LT}(\sin 2\theta \cos\phi)\right]$$

Invariant and helicity structure functions

One can easily obtain relations between these two sets of structure functions:

$$
\begin{aligned}
T_1 &= W_T + W_{TT} \\
T_2 &= \frac{M^2}{x_F^2 s} W_L - \frac{M^2}{x_F^2 s} W_T - \frac{(M^2 + s x_F^2)^2 - 2 s x_F^2 q_\perp^2 + q_\perp^4}{2 x_F^2 s q_\perp^2} W_{TT} + \frac{M(M^2 + s x_F^2 - q_\perp^2)}{x_F^2 s q_\perp} W_{LT} \\
T_3 &= -\frac{2M^2}{x_F^2 s} W_L + \frac{2M^2}{x_F^2 s} W_T - \frac{M^4 - s^2 x_F^4 + q_\perp^4}{x_F^2 s q_\perp^2} W_{TT} + \frac{2M(-M^2 + q_\perp^2)}{x_F^2 s q_\perp} W_{LT} \\
T_4 &= \frac{M^2}{x_F^2 s} W_L - \frac{M^2}{x_F^2 s} W_T - \frac{(M^2 - s x_F^2)^2 + 2 s x_F^2 q_\perp^2 + q_\perp^4}{2 x_F^2 s q_\perp^2} W_{TT} + \frac{M(M^2 - s x_F^2 - q_\perp^2)}{x_F^2 s q_\perp} W_{LT}
\end{aligned}
$$

Relations in target rest frame